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# Lorentz invariance violation as an explanation of the muon excess in Auger data

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The Auger Collaboration has observed the number of muons, which is higher than its prediction by existing hadronic interaction models. We explain this excess of muons by using Lorentz invariance violation (LIV) in the photon sector. As an outcome of Lorentz invariance violation, the dispersion relation of the photon gets modified, which we use for the calculation of $\pi^0$ decay width. In the Auger data of primary energy $10^{9.8} < E(\mathrm{GeV}) < 10^{10.2}$, we find that the neutral pion decay width is suppressed in comparison to its standard model (SM) counterpart. As a result, we get a large number of muons explaining the observed muon excess. We consider Planck-suppressed LIV at order $\mathcal{O}(p^2/M_{\mathrm{Pl}}^2)$ for studying the photon sector, which is in agreement with the current bounds and not as tightly constrained as LIV at order $\mathcal{O}(p/M_{\mathrm{Pl}})$.

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## I. INTRODUCTION

Ultrahigh-energy (UHE) cosmic rays with energy of $\sim\mathcal{O}(10^{11})$ GeV are the most energetic particles observed on the Earth. After their collisions with the Earth's atmosphere, a huge cascade of secondary particles with low energy is created. As the collision energy is roughly ten times higher than the one at LHC, it can be a suitable window for new physics. These cascades of particles or showers are explored by large arrays like Yakutsk Extensive Air Shower Array and Pierre Auger Observatory (PAO). A new study from the Auger Collaboration [1,2] suggests that the number of muons produced in UHE showers is higher in comparison to the one predicted by existing models [3–5]. Basically, the hadronic component of showers with primary energy $10^{9.8} < E(\mathrm{GeV}) < 10^{10.2}$ has 30–60% more muons than expected [1,2,6]. The explanation of the muon excess in PAO data is challenged by the distribution of the depth of shower maximum, $X_{\mathrm{max}}$, which should be independently fitted. To fit the data, the density of the muon at 1 km from the shower core, which is denoted as $N_\mu$, should increase. The properties of hadronic interactions which affect $N_\mu$ and $X_{\mathrm{max}}$ are cross section, elasticity, multiplicity, primary mass, and $\pi^0$ energy fraction. The variation of $N_\mu$ and $X_{\mathrm{max}}$ with them is shown in Ref. [7], where it is noted that changing the $\pi^0$ energy fraction is the only viable option for increasing $N_\mu$. Any other change in $N_\mu$ without affecting the longitudinal profile $X_{\mathrm{max}}$, which is tightly constrained, is not possible (see Fig. 1 of Ref. [7]). If a hadronic shower carries $f_{\mathrm{had}}$ energy fraction of the total primary cosmic-ray energy $E$, then it scales as

$$f_{\mathrm{had}} \sim (1 - f_{\mathrm{EM}})^{n_{\mathrm{gen}}}, \tag{1.1}$$

where $f_{\mathrm{EM}}$ is the fraction of energy transferred into electromagnetic particles per generation and $n_{\mathrm{gen}}$ is the number of generations required for most pions to have energy below ∼100 GeV. Below 100 GeV energy, most of the charged pions decay rather than interact, terminating the energy transfer to the electromagnetic component of the shower, while the charged pions interact instead of decay above 100 GeV energy, persisting the hadronic shower. The best way to increase $f_{\mathrm{had}}$ is to reduce either $n_{\mathrm{gen}}$ or $f_{\mathrm{EM}}$. The estimated values of $n_{\mathrm{gen}}$ needed to reach pion energy below ∼100 GeV are $n_{\mathrm{gen}} = 3, 4, 5, 6$ for primary energies $E = 10^5, 10^6, 10^7, 10^8$ GeV, respectively [8]. But the required $n_{\mathrm{gen}}$ for getting the desired result also reduces $X_{\mathrm{max}}$, which is tightly constrained. So, the best option for increasing the muon density is to reduce $f_{\mathrm{EM}}$ ($\pi^0$ energy fraction).[1] There are many proposals for reducing the $\pi^0$ energy fraction such as chiral symmetry restoration, pion decay suppression, and pion production suppression [7,10]. The string percolation models [11] and strange fireball mechanism [12] are other approaches used for the explanation of the observed muon excess. In this work, we focus on the decay suppression of $\pi^0$, which can occur from Lorentz invariance violation in the photon sector. As the lifetime of $\pi^0$ is very small $\sim\mathcal{O}(10^{-17}$ sec), it decays immediately into two photons after its production. We modify the photon dispersion relation in the spirit of Refs. [13–15] and calculate the neutral pion decay width. At high energy, as a result of the modified dispersion relation, the photon becomes massive enough to suppress the $\pi^0$ decay into two photons. The possible Lorentz invariance violation is motivated from quantum gravity

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[1] In Ref. [9], the variation of $X_{\mathrm{max}}$ as a function of photon energy is discussed in the LIV framework. We will examine this point in Sec. IV.

 

[16–18], and in many studies [19–25], it has been shown that LIV becomes important at a very-high-energy scale. There are stringent constraints on LIV in the photon [13–15,26] and fermion [27–30]. Specifically, LIV in the photon sector is tightly constrained for Planck-mass-suppressed dimension-5 operators, and even dimension-6 operators are constrained to an unprecedented level [13,14,26]. For dimension-6 operators, the bound on the photon LIV parameter $\eta$ is $\eta \gtrsim -10^{-7}$ [13], which comes from the stringent upper bound on photon flux above $10^{11}$ GeV [31], and if the photon is observed at $10^{10}$ GeV, then $\eta \lesssim 10^{-8}$ [14]. We consider a dimension-6 scenario [LIV at order $\mathcal{O}(p^2/M_{\rm Pl}^2)$] in this work and find that for getting the desired muon excess the LIV parameter is $\eta \sim 10^{-2}$, which seems to be in tension with the upper bound mentioned in Refs. [14,15]. But we want to emphasize that the upper limit quoted in Refs. [14,15] is based on the assumption of the observation of the photon with $10^{10}$ GeV energy. In cosmic rays, a photon with this much energy is a question of discussion [32,33], and the upper bound can be avoided at present. The rest of the paper is as follows. In Sec. II, we discuss the modified dispersion relation. We give the neutral pion decay calculation in the LIV framework in Sec. III and our discussion and conclusion in Secs. IV and V, respectively.

## II. DISPERSION RELATION

The Lorentz invariance violation modifies the dispersion relation of the photon. As we mentioned before, the LIV at order $\mathcal{O}(p/M_{\rm Pl})$ corresponds to a cubic dispersion relation which arises from dimension-5 operator. The LIV at order $\mathcal{O}(p/M_{\rm Pl})$ is tightly constrained with the required suppression scale well above the Planck mass. In the following, we consider the underlying theory to be $CPT$ invariant by taking LIV at order $\mathcal{O}(p^2/M_{\rm Pl}^2)$. We denote the 4-momentum of the photon $\gamma(p_1)$ by $(E_1, \vec{p_1})$ and consider the following dispersion relation for photon

$$E_1^2 = p_1^2 + \eta p_1^2 \left(\frac{p_1}{M_{\rm Pl}}\right)^n, \tag{2.1}$$

where $\eta$ is a LIV parameter and Planck mass $M_{\rm Pl} = 1.2 \times 10^{19}$ GeV. This dispersion relation can be obtained from the Lagrangian given in Ref. [34]. As the $n = 1$ scenario of Eq. (2.1) arises from $CPT$-odd contributions [35–37], it is tightly constrained [13,26,38]. In the following, we assume that theory is $CPT$ even by taking $n = 2$.

## III. NEUTRAL PION DECAY

We calculate the neutral pion decay width using modified dispersion relation of Eq. (2.1) considering $n = 2$. We compute the amplitude for the neutral pion decay process $\pi^0(q) \to \gamma(p_1)\gamma(p_2)$, which is dominated by chiral anomaly and reads [39]

$$\mathcal{M} = \frac{e^2}{4\pi^2 f_\pi} \epsilon_{\mu\nu\alpha\beta} \epsilon_1^\mu \epsilon_2^\nu p_1^\alpha p_2^\beta, \tag{3.1}$$

where $f_\pi$ is the pion decay constant. We calculate the average amplitude square, which is

$$|\mathcal{M}|^2 = \frac{e^4}{64\pi^4 f_\pi^2} (m_\pi^2 - \eta' p_1^4 - \eta' p_2^4)^2, \tag{3.2}$$

where $\eta' \equiv \eta/M_{\rm Pl}^2$. The decay width of $\pi^0$ is then given as

$$\Gamma = \frac{\alpha^2}{64\pi^3 f_\pi^2 E_\pi} \int \frac{p_1 dp_1 d\cos\theta}{\sqrt{|\vec{p} - \vec{p_1}|^2 + \eta' |\vec{p} - \vec{p_1}|^4}} \times \delta\left(E_\pi - E_1 - \sqrt{|\vec{p} - \vec{p_1}|^2 + \eta' |\vec{p} - \vec{p_1}|^4}\right) \times (m_\pi^2 - \eta' p_1^4 - \eta' (p - p_1)^4)^2, \tag{3.3}$$

where $\alpha = e^2/4\pi$ is the fine structure constant and $E_1$ is the photon energy which is defined as $E_1 = \sqrt{p_1^2 + \eta' p_1^4}$. The momentum of the photon is defined as $|\vec{p} - \vec{p_1}|^2 = p^2 + p_1^2 - 2pp_1 \cos\theta$. From the argument of the delta function in Eq. (3.3), one reads

$$\sqrt{|\vec{p} - \vec{p_1}|^2 + \eta' |\vec{p} - \vec{p_1}|^4} = E_\pi - E_1, \tag{3.4}$$

which after solving gives

$$\cos\theta = \frac{2p_1 E_\pi - m_\pi^2 + \eta'(E_\pi^4 - 4E_\pi^3 p_1 + 6E_\pi^2 p_1^2 - 3E_\pi p_1^3)}{2pp_1}. \tag{3.5}$$

We reduce the argument of the $\delta$ function in terms of $\cos\theta$ by taking

$$\left|\frac{d}{d\cos\theta}\left(E_\pi - E_1 - \sqrt{|\vec{p} - \vec{p_1}|^2 + \eta' |\vec{p} - \vec{p_1}|^4}\right)\right| = \frac{pp_1}{\sqrt{p^2 + p_1^2 - 2pp_1\cos\theta + \eta' |\vec{p} - \vec{p_1}|^4}}. \tag{3.6}$$

After these simplifications, we get the decay width of the neutral pion,

$$\Gamma = \frac{\alpha^2}{64\pi^3 f_\pi^2 E_\pi} \int \frac{dp_1}{p} (m_\pi^2 - \eta' p_1^4 - \eta' \tilde{p}_2^4)^2, \tag{3.7}$$

where $\tilde{p}_2^4 = m_\pi^4 + 2m_\pi^2 p^2 + p^4 - 4E_\pi m_\pi^2 p_1 - 4E_\pi p^2 p_1 + 4E_\pi^2 p_1^2 + 2m_\pi^2 p_1^2 + 2p^2 p_1^2 - 4E_\pi p_1^3 + p_1^4$. We perform the integration of Eq. (3.7) in the allowed limits of $p_1$, which are fixed by taking $\cos\theta = \pm 1$ in Eq. (3.5), and it gives

$$p_{1_{\max}} = \frac{m_\pi^2 - \eta'(E_\pi^4 - 4E_\pi^3 p_{1_{\max}} + 6E_\pi^2 p_{1_{\max}}^2 - 3E_\pi p_{1_{\max}}^3)}{2(E_\pi - p)}, \tag{3.8}$$

$$p_{1_{\min}} = \frac{m_\pi^2 - \eta'(E_\pi^4 - 4E_\pi^3 p_{1_{\min}} + 6E_\pi^2 p_{1_{\min}}^2 - 3E_\pi p_{1_{\min}}^3)}{2(E_\pi + p)}. \tag{3.9}$$

By solving these equations numerically, we get the allowed limits on the photon momentum. Using these limits, we solve Eq. (3.7) to get the decay width of $\pi^0$ and then compare it with the standard model result of pion decay in a moving frame, which is given as

$$\Gamma_{\rm SM}(\pi^0 \to \gamma\gamma) = \frac{\alpha^2 m_\pi^4}{64\pi^3 f_\pi^2 E_\pi}. \tag{3.10}$$

In Fig. 1, we have shown the deviation of the $\pi^0$ decay width from its SM prediction [see Eq. (3.10)]. We find that, as a result of phase space and $|\mathcal{M}|^2$ suppression, the decay width of $\pi^0$ (electromagnetic energy transferred per generation, $f_{\rm EM}$) decreases with large pion momentum. As a result, $f_{\rm had}$ increases [see Eq. (1.1)], which can enhance the number of muons by 30–60% in the desired energy range. We have shown the Auger muon excess region for the primary cosmic-ray energy $E$ $(10^{3.8} < E({\rm PeV}) < 10^{4.2})$, which translates into neutral pion energy with $\sim 25\% E$ [40]. Here, it is important to mention that we also checked our calculation for the $n = 1$ scenario and found that $\eta \sim 10^{-12}$ is required to explain the Auger muon excess, which is 3 orders of magnitude higher than the current bounds [26].

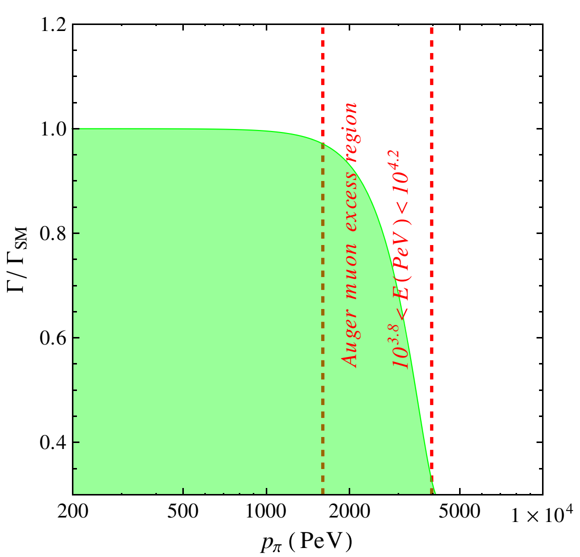


FIG. 1. The ratio $\Gamma/\Gamma_{\rm SM}$ for the $\pi^0 \to \gamma\gamma$ process in the Lorentz invariance violating framework to its SM counterpart as a function of pion momentum $p_\pi$ by considering $\eta = 10^{-2}$. The Auger region where the muon excess is observed also shown.

So, it is not possible to address the observed muon excess in Auger data for the $n = 1$ scenario.

## IV. DISCUSSION

In the previous sections, we discussed how the modified dispersion relation gives rise to a massive photon, which stops $\pi^0$ decay at energy denoted as $E_\pi^{\rm cutoff}$. We mentioned that $\pi^0$ decay does not contribute to the shower maximum depth $X_{\rm max}$, but as a result of LIV, it is possible that the photon becomes massive enough to decay into $e^+e^-$ pairs, which can modify $X_{\rm max}$. We contemplate this idea in the spirit of Ref. [9] and check our LIV scenario against that. The threshold energy for photon decay into $e^+e^-$ pairs is

$$E_{1_{\rm th}} \simeq \sqrt{\frac{2m_e M_{\rm Pl}}{\eta^{1/2}}}, \tag{4.1}$$

where $m_e$ is the mass of the electron. We get $E_{1_{\rm th}}$ after considering the condition $m_\gamma \simeq 2m_e$. If the initial photon energy $E_1 > E_{1_{\rm th}}$, then the photon starts decaying into the pair $e^+e^-$. As a result, the shower maximum gets modified and can be written as [9]

$$\tilde{X}_{\rm max} = \lambda_r \beta \ln\left(\frac{E_1/2}{E_{1_{\rm th}}}\right) + \lambda_r \ln\left(\frac{E_{1_{\rm th}}}{E_c}\right), \tag{4.2}$$

where $E_c$ is the critical energy at which ionization starts dominating over radiative processes, $\lambda_r$ is the radiation length in the medium, and $\beta \equiv \ln 2/\ln 3$. In the standard scenario $(\eta = 0)$, $X_{\rm max} = \lambda_r \ln(E_1/E_c)$ [41]. In Fig. 2, we have shown the behavior of modified $\tilde{X}_{\rm max}$ as a function of initial photon energy $E_1$ by using $E_c \approx 80$ MeV, $\lambda_r \approx 37\ {\rm g/cm^3}$ [41], and $\eta = 10^{-2}$. The standard Lorentz invariant $(\eta = 0)$ scenario is also shown for comparison. By taking $\eta = 10^{-2}$, Eq. (4.1) gives $E_{1_{\rm th}} \approx 315$ PeV. It is

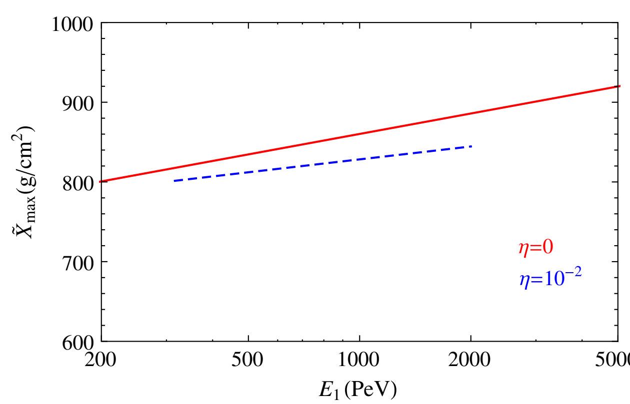


FIG. 2. The electromagnetic shower depth $\tilde{X}_{\rm max}$ as a function of initial photon energy $E_1$ for LIV parameter $\eta = 10^{-2}$. The Lorentz invariant standard result $(\eta = 0)$ is also shown for comparison.

clear from Fig. 1 that neutral pion decay for $\eta = 10^{-2}$ stops at $E_\pi^{\rm cutoff} \sim 4000$ PeV, so there should not be any photon with energy $E_1 > E_\pi^{\rm cutoff}/2$. Analyzing Fig. 2, we find that in the allowed region $E_{1_{\rm th}} < E_1 < E_\pi^{\rm cutoff}/2$ the modified shower maximum depth $\tilde{X}_{\rm max}$ varies between 10–40 g/cm$^3$. The precise measurement of $X_{\rm max}$ in the future can be used to probe this model.

## V. CONCLUSION

In this paper, we discuss the Lorentz invariance violation explaining the muon excess observed by the Auger Collaboration. The relative number of muons can be increased either by reducing the energy fraction in electromagnetic decay, i.e., suppressing the neutral pion decay or reducing the $n_{\rm gen}$. As the variation of $n_{\rm gen}$ is tightly constrained from the independent observation of $X_{\rm max}$, a change in $f_{\rm EM}$ is the best option for increasing the number of muons. We reduce the energy fraction $f_{\rm EM}$ by suppressing the $\pi^0$ decay width. We consider the modified dispersion relation for the photon by taking LIV at order $\mathcal{O}(p^2/M_{\rm Pl}^2)$ from a $CPT$-even dimension-6 operator and calculate the neutral pion decay width in this scenario. We find that, at high energies, Lorentz invariance violation starts playing an important role and suppresses the decay width of $\pi^0$, which depends on the value of LIV parameter $\eta$. We find that by taking the Plank-mass-square-suppressed $\eta \sim 10^{-2}$ it is possible to suppress the decay width of $\pi^0$ in the desired energy range. As a result of neutral pion decay width suppression, the energy fraction in the electromagnetic shower reduces, and it gives rise to the relative number of muons observed by the Auger Collaboration.

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