

Strong CP problem and spontaneous generation of the CP violating phase in the CKM matrix

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(Received 22 January 2017; published 22 May 2017)

We show that in a complementary two-Higgs doublet model (C2HDM) the CP violating phase in the Cabibbo-Kobayashi-Maskawa matrix can be generated spontaneously, dangerous flavor changing neutral currents (FCNC) can be naturally suppressed and the strong CP problem can also be avoided. The two Higgs doublets in the model are complementary in the sense that none of them is enough to describe masses of a given type of quarks. We find that the strength of FCNC is suppressed by the strength of Yukawa couplings of the first generation quark and the tree-level FCNC is sufficiently small. Using an explicit example, we show that radiative correction to the assumed Yukawa couplings can modify the discussion about the strong θ . The correction to the strong θ is estimated to be less than around $10^{-12} \sim 10^{-10}$ which can be tested in future experiments.

DOI: [10.1103/PhysRevD.95.095020](https://doi.org/10.1103/PhysRevD.95.095020)

I. INTRODUCTION

One of the deep mysteries of particle physics is the origin of CP violation. On one hand, CP symmetry is found to be broken in flavor changing processes of K and B mesons. CP violating phenomena so far measured are successfully explained by the CP violating phase in the complex Cabibbo-Kobayashi-Maskawa (CKM) matrix in the standard model (SM). On the other hand, the strong CP phase θ in

$$\Delta\mathcal{L} = \frac{\alpha_s}{8\pi} \theta G_{\mu\nu} \tilde{G}^{\mu\nu}, \quad (1)$$

another possible source of CP violation in SM, has not been observed in experiment. On the contrary, this strong θ is found to be $\theta \lesssim 10^{-10} \sim 10^{-9}$, in measurements of electric dipole moment (EDM) of neutron, mercury etc. [1].

The problem is quite challenging in view of the fact that the CP violating phase in the CKM matrix arises from complex Yukawa couplings of quarks. These complex Yukawa couplings are natural to have nonzero flavor diagonal phases which can contribute to the strong θ . More specifically, after spontaneous breaking of the $SU(2)_L \times U(1)_Y$ gauge symmetry, the mass terms of quarks in the SM are generated as

$$M^{u,d} = Y^{u,d} v, \quad (2)$$

where $Y^{u,d}$ is the Yukawa coupling of up(down)-type quarks, $v = 246/\sqrt{2}$ GeV the vacuum expectation value of the Higgs doublet in the SM. Performing redefinitions of left-handed and right-handed fields separately and diagonalizing the mass terms, one can get the CKM matrix in charged current interaction of left-handed quarks. Meanwhile, the chiral $U(1)$ part of the field-redefinition would transform the θ term so that the presence of these

complex mass terms or Yukawa terms would give a contribution to the physical strong θ

$$\theta = \theta_0 + \arg(\det(M^u M^d)), \quad (3)$$

where θ_0 is the θ term before receiving correction. One would naturally expect the second term in (3) is not zero if $Y^{u,d}$ (or $M^{u,d}$) are complex matrices. So a very large fine-tuning between the two terms in (3) is required to achieve a value of θ as small as $\lesssim 10^{-9}$. In particular, a large fine-tuning seems unavoidable if CP symmetry is broken explicitly as in the SM.¹ This is the so-called strong CP problem [3].

One approach to understand the origin of CP violation is spontaneous breaking of CP symmetry [4]. In this approach, CP symmetry is exact and the θ term is zero before the symmetry is broken spontaneously. So spontaneous breaking of CP symmetry is a possible solution to the strong CP problem [5–7], and the strong θ can be calculable in some new physics models. Moreover, it was shown by some authors that the CP violating phase in the CKM matrix can be generated spontaneously [8,9]. Although this is a very interesting approach to understand the origin of CP violation, it is not straightforward to see whether the complex quark mass matrices generated in this kind of model can still give a zero contribution to the θ term

¹If making an extra assumption that there is no flavor-diagonal phase in complex Yukawa couplings, one can ignore this fine-tuning problem. In this case, flavor nondiagonal phase in CKM matrix can still contribute to strong θ through radiative correction. But the first nonzero correction appears in 4th order in loop and is of order 10^{-16} [2]. We do not study this case concerning the CP violation in the SM.

after the CP violating phase in the CKM matrix is generated spontaneously.

In this paper, we are going to pursue the idea of spontaneous generation of flavor nondiagonal CP violating phase in the CKM matrix and study the strong θ term in this approach. We will show that the CP violating phase in the CKM matrix and a zero or very small strong θ can be obtained simultaneously in a model of spontaneous generation of CP violation. First, we will show, using an explicit toy model, that the flavor nondiagonal CP violating phase in the CKM matrix can be generated spontaneously. Since more than one Higgs doublets are needed in order to implement spontaneous CP violation, Yukawa couplings can be very complicated in general and dangerous flavor changing neutral current (FCNC) processes could be generated. We show that there are some cases for which FCNC processes can be naturally suppressed. Since CP symmetry is exact before spontaneous breaking, the initial strong θ_0 is zero. We show that the strong θ can still be zero in such kind of model even when the CP violating phase in the CKM matrix is generated spontaneously. We also study radiative corrections to Yukawa couplings and check the robustness of the above statement against possible radiative corrections. We find that θ is smaller than around $10^{-12} \sim 10^{-10}$ in a particular model.

II. SPONTANEOUS GENERATION OF FLAVOR CHANGING CP VIOLATING PHASE

In this section we show that the CP violating phase in the CKM matrix can be generated spontaneously. We assume that CP symmetry is exact before spontaneous symmetry breaking. So Yukawa couplings are real and initial strong θ_0 is zero.

Spontaneous CP violation in general involves more than one Higgs doublets, e.g. the two Higgs doublets ϕ_1 and ϕ_2 . A general Lagrangian with two Higgs doublets which can give rise to a spontaneous generation of CP violating phase has been discussed in literature, e.g. in a recent paper [10]. In the present article we are not going to elaborate on this Lagrangian. Instead, we assume that a spontaneous breaking of CP symmetry and a suitable CP violating phase can be achieved with a suitable Lagrangian. We assume that ϕ_1 and ϕ_2 , both having hypercharge $-\frac{1}{2}$, develop vacuum expectation values after spontaneous symmetry breaking

$$\langle \phi_1 \rangle = (v_1, 0)^T, \quad \langle \phi_2 \rangle = (v_2, 0)^T, \quad (4)$$

where v_1 and v_2 are complex in general and they satisfy $|v_1|^2 + |v_2|^2 = v^2$. ϕ_1 and ϕ_2 can both couple to quarks:

$$-\Delta\mathcal{L} = \bar{Q}Y_1^u\phi_1u_R + \bar{Q}Y_2^u\phi_2u_R + \bar{Q}Y_1^d\tilde{\phi}_1d_R + \bar{Q}Y_2^d\tilde{\phi}_2d_R, \quad (5)$$

where Q is the field of the left-handed quark doublet, u_R and d_R the fields of right-handed up-type and down-type quarks. $\tilde{\phi}_{1,2} = i\sigma^2\phi_{1,2}^*$. Flavor indices have been

suppressed in (5). $Y^{u,d}$ in (5) are all real matrices so that CP symmetry is not broken explicitly. After spontaneous symmetry breaking the mass matrices of quarks are obtained as

$$M^u = Y_1^u v_1 + Y_2^u v_2, \quad (6)$$

and

$$M^d = Y_1^d v_1^* + Y_2^d v_2^*. \quad (7)$$

Complex values of v_1 and v_2 in general make $M^{u,d}$ complex. So there is a possibility to get the flavor nondiagonal phase in the CKM matrix from this setup [8,9]. However, it is complicated to show that this can be achieved in general cases. In the following, we will use an explicit example to show that this can be achieved.

An explicit example of spontaneous generation of the flavor nondiagonal CP violating phase in the CKM matrix can be given by couplings as follows

$$Y_1^u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} K_{23}^\dagger g^u, \\ Y_2^u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} K_{23}^\dagger h^u, \quad (8)$$

and

$$Y_1^d = K_{13} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} K_{12} g^d, \\ Y_2^d = K_{13} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} K_{12} h^d, \quad (9)$$

where $g^{u,d}$ and $h^{u,d}$ are diagonal matrices with real eigenvalues and they are taken as

$$g^{u,d} = y^{u,d} v / |v_1|, \quad h^{u,d} = y^{u,d} v / |v_2| \quad (10)$$

where y^u and y^d are diagonalized real Yukawa couplings appearing in the SM. $K_{12,13,23}$ are standard rotation matrices in the standard parametrization of the CKM matrix with rotation angles appearing in 1–2, 1–3 or 2–3 entries. In the following, we will take s_{ij} and c_{ij} as the sine and cosine of the rotation angle θ_{ij} in the matrix K_{ij} . We note that Yukawa couplings in (8) and (9) can be redefined subject to rotations as $Y_i^u \rightarrow O_L Y_i^u O_u$ and $Y_i^d \rightarrow O_L Y_i^d O_d$ where $O_{L,u,d}$ are real rotation matrices, and results presented in this article are kept intact under this redefinition.

We can see in (8), (9), and (10) that the strength of the Yukawa couplings can be much larger than that in the SM. In particular, if $|v_1| \ll v$, e.g. $|v_1| \sim 0.1v$, strength of Y_1 can be ten times larger than in the SM, while the strength of Y_2 , the couplings with the second and third generation of quarks would remain almost the same as in the SM. Measurements of the Yukawa couplings of the third generation quark, which are so far consistent with the SM prediction [1], would put a constraint on this model. This constraint says that $|v_2|$ should be much larger than $|v_1|$. So we can conclude in this model that the strength of the Yukawa couplings of the first generation quark should be much larger than that in the SM.

The CP phases in v_1 and v_2 are subject to rephasing of scalar fields and can be taken in v_1 :

$$v_1 = |v_1|e^{-i\delta}, \quad v_2 = |v_2|. \quad (11)$$

So we can find that

$$M^u = V_L^u m^u, \quad m^u = y^u v, \quad (12)$$

and

$$V_L^u = \begin{pmatrix} e^{-i\delta} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} K_{23}^\dagger \quad (13)$$

for up-type quarks, and

$$M^d = V_L^d m^d, \quad m^d = y^d v, \quad (14)$$

and

$$V_L^d = K_{13} \begin{pmatrix} e^{i\delta} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} K_{12}, \quad (15)$$

for down-type quarks. In writing out (12) and (14), (10) has been used.

So we can get the CKM matrix

$$K = V_L^{u\dagger} V_L^d = K_{23} \begin{pmatrix} e^{i\delta} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} K_{13} \begin{pmatrix} e^{i\delta} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} K_{12}. \quad (16)$$

It is equivalent to the standard parametrization of the CKM matrix [1,11] via a vectorlike transformation of up quark which does not change θ . That is, with

$$u_L \rightarrow e^{2i\delta} u_L, \quad u_R \rightarrow e^{2i\delta} u_R \quad (17)$$

we can get the standard parametrization of CKM matrix

$$K \rightarrow K = K_{23} \begin{pmatrix} e^{-i\delta} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} K_{13} \begin{pmatrix} e^{i\delta} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} K_{12}. \quad (18)$$

We conclude that spontaneous generation of the CP violating phase in the CKM matrix can be achieved in model described in this article. We use an explicit example to show this possibility. In particular, we show that spontaneous generation of the CP violating phase in the CKM matrix can be achieved using matrices of Yukawa couplings with rank less than three.

III. FCNC AND STRONG θ TERM WITH SPONTANEOUS GENERATION OF CP VIOLATION

In this section we show how FCNC can be suppressed in the model presented in the present article. We then show how the strong CP problem is avoided.

One of the main problems associated with (6) is the that unitary transformations that diagonalize the mass matrix (6) do not necessarily diagonalize the Higgs couplings to quarks in (5). So tree level FCNC could be present and this type of theory of spontaneous generation of CP violation may encounter difficulty in this aspect, as pointed out in [12] for model presented in [8].

We show how dangerous FCNC can be avoided. Taking

$$M^{u,d} = M_1^{u,d} + M_2^{u,d} \quad (19)$$

where $M_{1,2}^u = Y_{1,2}^u v_{1,2}$ and $M_{1,2}^d = Y_{1,2}^d v_{1,2}^*$, we assume $M_2^{u,d}$ is the dominant contribution to $M^{u,d}$, i.e.

$$\|M_1^{u,d}\| \ll \|M_2^{u,d}\|. \quad (20)$$

Note that M_1 or M_2 is proportional to real matrix Y_1 or Y_2 and diagonalizing M_1 or M_2 does not need complex matrices. So diagonalizing M_1 or M_2 does not change strong θ and we can work in a base that one of M_1 and M_2 is diagonalized. In this base nonzero elements of $M_1^{u,d}$ can be taken at most at order of $m_{u,d}$ with $m_{u,d}$ being the masses of up and down quarks.

Taking $M_2^{u,d}$ as rank two and working in the base that M_2^u is diagonalized

$$M_2^u = \text{diag}\{0, x_2, x_3\} \quad (21)$$

we can write

$$M^u = \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} + x_2 & x_{23} \\ x_{31} & x_{32} & x_{33} + x_3 \end{pmatrix}, \quad (22)$$

where x_{ij} comes from M_1^u . In this form of mass matrix, (20) means $|x_{ij}| \ll x_{2,3}$ for M_1^u and M_2^u . In particular, nonzero x_{ij} would be at most of order m_u . $x_2 \approx m_c$ and $x_3 \approx m_t$ with $m_{c,t}$ being the masses of charm and top quarks.

If $x_{ij} = 0$ except x_{11} , (22) is already diagonalized and no extra flavor mixing is needed. In this case, M_1^u and M_2^u can be diagonalized simultaneously. In general, off-diagonal matrix element x_{ij} may not be zero, and M_1^u and M_2^u cannot be diagonalized simultaneously. So tree level FCNC can be present. However, for $|x_{ij}| \ll x_{2,3}$ the matrices V_L and V_R that further diagonalize M^u in (22) are all close to unit matrix. More specifically, we can write $M^u = V_L m^u V_R^\dagger$ where m^u is the diagonalized mass matrix of up-type quarks with real eigenvalues $m_{1,2,3}$ and possible $U(1)$ factors, which do not change the conclusion about FCNC, have been suppressed. To first order we find

$$V_L \approx \begin{pmatrix} 1 & a_{12} & a_{13} \\ -a_{12}^* & 1 & a_{23} \\ -a_{13}^* & -a_{23}^* & 1 \end{pmatrix},$$

$$V_R \approx \begin{pmatrix} 1 & b_{12} & b_{13} \\ -b_{12}^* & 1 & b_{23} \\ -b_{13}^* & -b_{23}^* & 1 \end{pmatrix}, \quad (23)$$

where a_{ij} and b_{ij} satisfy $|a_{ij}| \ll 1$ and $|b_{ij}| \ll 1$. a_{ij} and b_{ij} are found to be

$$a_{ij} = \frac{x_{ij}m_j + x_{ji}^*m_i}{m_j^2 - m_i^2}, \quad b_{ij} = \frac{x_{ij}m_i + x_{ji}^*m_j}{m_j^2 - m_i^2}, \quad \text{for } i < j. \quad (24)$$

For eigenvalues in m^u , we have $m_{2,3} \approx x_{2,3}$ and $m_1 \approx x_{11}$. Since $m_j \gg m_i$ for $j > i$, we can find $a_{ij} \approx x_{ij}/m_j$ and $b_{ij} \approx x_{ji}^*/m_j$. We can see that nonzero off-diagonal elements in $V_{L,R}$ are either of order m_u/m_c or of order m_u/m_t .

After diagonalizing the mass matrix M^u , the coupling of ϕ_1 with up-type quarks, which originally mixes flavors, still mixes flavors. The strength of this FCNC coupling is $x_{ij}/|v_1| \sim m_u/|v_1|$. After diagonalizing the mass matrix M^u , the coupling of ϕ_2 with up-type quarks, which is originally flavor diagonal, gives rise to new FCNC couplings. For example, a flavor diagonal coupling $(x_i/v_2)\bar{u}_L^i u_R^i \phi_2^0$, with ϕ_2^0 being the neutral component of ϕ_2 , becomes $(x_i/v_2)(V_L)_{ij}^*(V_R)_{ik}\bar{u}_L^j u_R^k \phi_2^0$ after field redefinition using $V_{L,R}$. At first order, it gives rise to FCNC couplings $(x_i/v_2)(V_L)_{ij}^*\bar{u}_L^j u_R^i \phi_2^0$ and $(x_i/v_2)(V_R)_{ij}\bar{u}_L^i u_R^j \phi_2^0$ for $j \neq i$. As shown above, the off-diagonal matrix elements $(V_L)_{ij}$ and $(V_R)_{ij}$ with $i \neq j$ all have a strength $\sim |x_{ij}|/\max(m_i, m_j)$ or $\sim |x_{ji}|/\max(m_i, m_j)$. So we can find that these FCNC couplings induced in couplings with ϕ_2 have strength $\sim x_i/v_2 \times (|x_{ij}| \text{ or } |x_{ji}|)/\max(m_i, m_j) \lesssim m_u/v_2$. Summarizing these two cases, the strengths of the

FCNC couplings with up-type quarks are suppressed to be $\lesssim m_u/|v_1|$.

Similarly, one can show that possible FCNC interactions of Higgs with down-type quarks are also suppressed to be less than order $m_d/|v_1|$ if taking M_2^d as the dominant contribution to M^d . If magnitude of v_1 is not extremely small, FCNC couplings given by Yukawa couplings in (5) can be safely neglected. For example, if $|v_1| \sim 0.1 \times v$, FCNC Higgs coupling with up-type quarks would be at most at order 10^{-4} . Any possible FCNC processes induced by these couplings would be suppressed by the square of this FCNC amplitude, i.e. suppressed by a factor of order 10^{-8} . However, if the magnitude of v_1 is extremely small, e.g. $|v_1| \sim 10^{-3}v$, the magnitude of Y_1 would be large and there could be dangerous FCNC couplings arising from it. To avoid possibly large FCNC couplings a hierarchy between Y_1 and Y_2 is preferred. This implies that $|v_1|$ should not be very small. We assume $|v_1| > 0.01v$.

We note that our arguments for suppressing FCNC are based on the assumption that a hierarchy can exist in Y_1 and Y_2 . For this assumption to hold, radiative corrections should not change the hierarchy. In fact, the radiative correction to Y_1 from couplings of ϕ_2 will be proportional to elements of Y_1 , so that the hierarchy between Y_1 and Y_2 is not affected by the radiative corrections. This means that the suppression of FCNC is robust against quantum correction. This is because we have taken $Y_2^{u,d}$ as rank two and if setting $Y_1^{u,d} = 0$ there is a chiral symmetry appearing in the Lagrangian which protects the hierarchy.

Now we come to explain that the strong θ is naturally zero in this model. Using (6) one can easily show that the determinant $\det(M^u M^d)$ is real and zero correction to θ can be achieved if taking the rank of Y_1 and Y_2 both less than 3. For example, if taking Y_1 or M_1 as rank one and Y_2 or M_2 as rank two we can write them as

$$M_1 = \begin{pmatrix} x_{11}^u v_1 & x_{12}^u v_1 & x_{13}^u v_1 \\ a_u x_{11}^u v_1 & a_u x_{12}^u v_1 & a_u x_{13}^u v_1 \\ b_u x_{11}^u v_1 & b_u x_{12}^u v_1 & b_u x_{13}^u v_1 \end{pmatrix}, \quad (25)$$

$$M_2 = \text{diag}\{0, y_2^u v_2, y_3^u v_2\}, \quad (26)$$

where x_{ij}^u, y_i^u, a_u and b_u are all real numbers and $y_{2,3}^u$ are eigenvalues of Y_2^u . We can find

$$\det(M^u) = \det(M_1^u + M_2^u) = x_{11}^u y_2^u y_3^u v_1 v_2 v_2. \quad (27)$$

A similar expression holds for M^d

$$M^d = Y_1^d v_1^* + Y_2^d v_2^*. \quad (28)$$

and we get

$$\det(M^d) = x_{11}^d y_2^d y_3^d v_1^* v_2^*. \quad (29)$$

So we get

$$\det(M^u M^d) = x_{11}^u y_2^u y_3^u x_{11}^d y_2^d y_3^d |v_1|^2 |v_2|^4. \quad (30)$$

(30) is real and $\arg(\det(M^u M^d)) = 0$. So the correction to θ is zero as can be seen in (3). This statement relies on the fact that Y_1 and Y_2 all have ranks less than three, and in particular the sum of the ranks of Y_1 and Y_2 equals to three. Radiative corrections can change this feature and give rise to nonzero correction to θ .

We conclude that in model presented in this article the QCD θ is zero at tree level after spontaneous generation of the CP violating phase in the CKM matrix and FCNC can be naturally suppressed. In the model presented here we have assumed that the two matrices of Yukawa couplings both have rank less than three, and the sum of the ranks of two matrices of Yukawa couplings equals to three. So the two Higgs doublets complement to each other in the sense that they together give rise to the complete quark mass matrices and none of them is enough without the help of other Higgs doublet. In this sense, we can call this model of two Higgs doublets as the complementary two Higgs doublet model (C2HDM).

As discussed in the previous section, spontaneous generation of the CP violating phase in the CKM matrix can be achieved using matrices of Yukawa couplings with rank less than three and in particular in C2HDM. Moreover, the Yukawa couplings used in the last section, (8) and (9), satisfy the assumption in this section and indeed lead to zero contribution to θ at tree level.

Since the discussion on strong θ in this section depends on the assumption of Yukawa couplings, it is natural to ask what is the effect of radiative correction on the assumed Yukawa couplings and what is the effect on the size of the induced strong θ . In the next section we will study this question.

IV. RADIATIVE CORRECTION TO YUKAWA COUPLING AND STRONG θ

In this section we study radiative correction to Yukawa couplings and the correction to θ term arising from it. A general discussion on the correction to θ term seems very complicated. We are not going to do a general discussion, but rather to show that the radiative correction to (8) and (9) would lead to a correction to strong θ at order $10^{-12} \sim 10^{-10}$ at one-loop level. With this example, we illustrate that a small enough strong θ , being consistent with experimental bound, can be achieved in model of spontaneous generation of the CP violating phase in the CKM matrix.

One-loop radiative correction to Yukawa couplings can be read out in their renormalization group equation (RGE) as shown in (A1) and (A2). Yukawa couplings in (8) and (9) have a nice feature

$$\text{Tr}[Y_i^u Y_j^{u\dagger}] = \text{Tr}[Y_i^d Y_j^{d\dagger}] = 0, \quad \text{for } i \neq j. \quad (31)$$

If we further assume Y_i^l , the Yukawa coupling of charged leptons, also has a similar feature

$$\text{Tr}[Y_i^{l\dagger} Y_j^l] = 0, \quad \text{for } i \neq j, \quad (32)$$

RGEs in (A1) and (A2) can be simplified. In particular, the second term in (A1) or (A2) can be combined with the first term and part of the last term can be combined with the third term. Moreover, in this case Y_i^l would appear in factor $A_i^{u,d}$ and can be omitted in future discussion. So we arrive at (A4) and (A5).

Using (A4) and (A5) we can see that the one loop corrected Yukawa couplings are

$$Y_1^{uu} = Y_1^u + \epsilon[-A_1^u Y_1^u + B_1^u Y_1^u + Y_1^u C^u - 2Y_2^d Y_1^{d\dagger} Y_2^u], \quad (33)$$

$$Y_2^{uu} = Y_2^u + \epsilon[-A_2^u Y_2^u + B_2^u Y_2^u + Y_2^u C^u], \quad (34)$$

$$Y_1^{dd} = Y_1^d + \epsilon[-A_1^d Y_1^d + B_1^d Y_1^d + Y_1^d C^d], \quad (35)$$

$$Y_2^{dd} = Y_2^d + \epsilon[-A_2^d Y_2^d + B_2^d Y_2^d + Y_2^d C^d], \quad (36)$$

where we have used (B2) and $\epsilon = \log(\mu/\Lambda)/(16\pi^2)$. ϵ is a small number and ϵ^2 would be smaller than around 10^{-3} for Λ lower than around 10^5 GeV which means the new physics scale is no more than three orders of magnitude higher than the electroweak scale. $Y_2^d Y_1^{d\dagger} Y_2^u$ in (33) is found to be rank one and is given in (B1). Other $Y_i^d Y_j^{d\dagger} Y_i^u$ and $Y_i^u Y_j^{u\dagger} Y_i^d$ terms in (A4) and (A5) are found to be zero. The mass matrices of up-type and down-type quarks are obtained as

$$\begin{aligned} M^{uu} &= Y_1^{uu} v_1 + Y_2^{uu} v_2 \\ &= X_1^u Y_1^u v_1 + \epsilon Y_1^u C^u v_1 + X_2^u Y_2^u v_2 + \epsilon Y_2^u C^u v_2 \\ &\quad - 2\epsilon Y_2^d Y_1^{d\dagger} Y_2^u v_1, \end{aligned} \quad (37)$$

$$\begin{aligned} M^{dd} &= Y_1^{dd} v_1^* + Y_2^{dd} v_2^* \\ &= X_1^d Y_1^d v_1^* + \epsilon Y_1^d C^d v_1^* + X_2^d Y_2^d v_2^* + \epsilon Y_2^d C^d v_2^*, \end{aligned} \quad (38)$$

where $X_i^{u,d} = 1 - \epsilon A_i^{u,d} + \epsilon B_i^{u,d}$ ($i = 1, 2$) are real matrices with rank three.

Now we can compute the determinant of M^{uu} and M^{dd} . The leading term of $\det(M^{uu})$ is proportional to $v_1 v_2^2$ and is $v_1 v_2^2 g_1^u h_2^u h_3^u$. Possible corrections to the $v_1 v_2^2$ term do not change the conclusion of the discussion below and will be omitted. Subleading terms in $\det(M^{uu})$ can be proportional to v_1^3 , $v_1^2 v_2$ or v_2^3 . The term proportional to v_2^3 comes from $\det(v_2 X_2^u Y_2^u + \epsilon v_2 Y_2^u C^u)$ and as shown in Appendix B it is zero. Similarly, one can show that the term proportional to v_1^3 vanishes. The leading nonzero correction is the term proportional to $v_1^2 v_2$ as given in (B7) and (B8). Thus, we obtain $\det(M^{uu})$ as

$$\begin{aligned} \det(M'^u) &= v_1 v_2^2 g_1^u h_2^u h_3^u \\ &\quad - 3\epsilon^2 v_1^2 v_2 (g_2^d h_2^d - g_1^d h_1^d) s_{13} c_{12} s_{12} c_{23} s_{23} \\ &\quad \times [(h_3^u)^2 - (h_2^u)^2] g_1^u h_2^u h_3^u, \end{aligned} \quad (39)$$

where the smaller correction in (B8) has been neglected.

The leading term of $\det(M'^d)$ is proportional to $v_1^*(v_2^*)^2$ and is $v_1^*(v_2^*)^2 g_1^d h_2^d h_3^d$. Possible corrections to the $v_1^*(v_2^*)^2$ term do not change the conclusion of the discussion below and will be omitted. Subleading terms in $\det(M'^d)$ can be proportional to $(v_1^*)^3$, $(v_1^*)^2 v_2^*$, or $(v_2^*)^3$. The term proportional to $(v_1^*)^3$ comes from $\det(v_1^* X_1^d Y_1^d + \epsilon v_1^* Y_1^d C^d)$ and as shown in Appendix B it is zero. Terms proportional to $(v_2^*)^3$ and $(v_1^*)^2 v_2^*$ are calculated in (B10) and (B11). Combining these results we can get

$$\begin{aligned} \det(M'^d) &= v_1^*(v_2^*)^2 g_1^d h_2^d h_3^d \\ &\quad + \frac{1}{2} \epsilon^2 [(v_2^*)^3 + (v_1^*)^2 v_2^*] s_{13} c_{12} s_{12} c_{23} s_{23} g_1^d h_2^d h_3^d \\ &\quad \times [(h_3^d)^2 - (h_2^d)^2] (g_2^d h_2^d - g_1^d h_1^d). \end{aligned} \quad (40)$$

A common factor in (39) and (40) is $s_{13} c_{12} s_{12} c_{23} s_{23} (g_2^d h_2^d - g_1^d h_1^d) \sim 10^{-5} (m_s/v)^2 (v^2/|v_1 v_2|) \sim 10^{-11} \times (v^2/|v_1 v_2|)$. One can see that for $|v_1| \ll |v_2|$ correction in (39) is of order $10^{-11} \epsilon^2$ and correction in (40) is of order $10^{-11} (v^2/|v_1|^2) \epsilon^2$. So we have

$$\begin{aligned} \det(M'^u M'^d) &= |v_1|^2 |v_2|^4 g_1^u h_2^u h_3^u g_1^d h_2^d h_3^d \\ &\quad \times \left[1 + \mathcal{O}(10^{-11}) \times \frac{v^2}{|v_1|^2} \times \epsilon^2 \right]. \end{aligned} \quad (41)$$

We can see that the radiative correction to the determinant of $M^u M^d$ is of order 10^{-12} for $|v_1| \sim 0.1 v$ and for $\epsilon^2 \approx 10^{-3}$ which corresponds to the new physics scale being three orders of magnitude higher than the electroweak scale. The radiative correction would be smaller if new physics scale is closer to the electroweak scale. One can also see that if $|v_1|$ is too small, e.g. $|v_1| \lesssim 10^{-2} v$, the radiative correction would be too large and it could give rise to a strong θ reaching the experimental bound. Since the Yukawa couplings of the first generation fermions are proportional to $1/v_1$, $|v_1|$ should not be too small as argued for naturally suppressing possible FCNC couplings. In particular we have assumed that $|v_1| > 0.01 v$. We can conclude that for reasonable values of parameters, the radiative correction to the determinant of $M^u M^d$ is smaller than order of $10^{-12} \sim 10^{-10}$. So its correction to strong θ is also smaller than order $10^{-12} \sim 10^{-10}$.

Combining the conclusions in the last section and this section, we can see that in the scenario discussed in this article, i.e. with (8) and (9), strong θ is zero at leading order and can be generated at one-loop level, but is smaller than around $10^{-12} \sim 10^{-10}$. This prediction can be tested in future EDM experiment [13].

V. CONCLUSION

In summary, we have presented a model of spontaneous CP violation. CP symmetry is exact before the spontaneous symmetry breaking. The CP violating phase in the CKM matrix is generated spontaneously in this model. We show that it is possible to achieve a zero strong θ even after the spontaneous breaking of CP symmetry in such kind of model.

We show that zero strong θ term can be achieved if the two Higgs doublets involved in the setup are complementary in the sense that they are both needed to describe the quark masses and none of them is enough. To be specific, the ranks of the two Yukawa couplings with a specific type of quark, say up-type quark or down-type quark, can be rank two and rank one and their sum is three. We have called this kind of model of two Higgs doublets as C2HMD, the complementary 2HDM. It is straightforward to show that similar conclusion can be achieved if there are three Higgs doublets and each them couple to the quark fields with a rank one Yukawa coupling, similar to the case that each Higgs doublet coupled with one generation of quarks.

In a specific model with specific Yukawa couplings, we have studied the radiative correction to the assumed Yukawa couplings and have discussed the robustness of the above statement on strong θ . We find in this example that correction to strong θ can vary from 10^{-12} , much smaller than the experimental bound, to 10^{-10} reaching the experimental bound, depending on the Yukawa couplings of the first generation quarks. Using this example, we demonstrate that it is possible to have a very small strong θ term in model of spontaneous generation of the CP violating phase in the CKM matrix even if radiative correction to the assumed scenario is considered into account. A general discussion on this part seems complicated and we have left it to future study.

We have shown that in the setup discussed in the present article, say C2HDM, not only the CP violating phase in CKM matrix can be generated spontaneously and strong θ is naturally zero or very small, but also the dangerous FCNC can be naturally suppressed. The point is that one of the Higgs doublets in the complementary pair of the two Higgs doublets can be the dominant one and FCNC would naturally vanish without the other complementary Higgs doublet. So the appearance of FCNC coupling would be proportional to the strength of the other Yukawa coupling and is suppressed by quantities $\sim m_u/|v_1|$ or $m_d/|v_1|$.

We note that one interesting consequence of the model is that the Yukawa coupling of the first generation is around $\sim m_q/|v_1|$ which can be much larger than the corresponding Yukawa couplings in the SM. For example, the Yukawa couplings of the first generation quarks can be ten times larger than that in the SM if $|v_1| \sim 0.1 v$, or even larger if $|v_1|$ is even smaller. The strength of the coupling of the light neutral Higgs with first generation quarks would be proportional to $\sim \sin \alpha m_q/|v_1|$ with α being the mixing

angle of neutral Higgs field. The strength of the coupling of the heavy neutral Higgs would be proportional to $\sim \cos am_q/|v_1|$. They both have a possibility to be much larger than what usually expected in 2HDMs. This may give rise to interesting implications for Higgs phenomenology. Since the radiative correction to strong θ also depends on the Yukawa couplings of the first generation quarks, measuring and testing the Yukawa couplings seem to be a very interesting subject to study.

We note that the SM cannot give a successful explanation of the baryon-number generation in the universe. Physics beyond the SM and a new source of CP violation are needed to implement a baryon-number generation in the early universe. Since CP symmetry is broken spontaneously in our model, sufficient baryon-number generation should also be implemented by the model. Some other interesting topics include the CP violating phase in the leptonic sector and the impact to neutrino mixings [14]. In the present paper we do not discuss all these related issues and leave them to future research.

ACKNOWLEDGMENTS

This work is supported by National Science Foundation of China (NSFC), Grant No. 11375065, and Shanghai Key Laboratory of Particle Physics and Cosmology, Grant No. 15DZ2272100.

APPENDIX A: RGE OF YUKAWA COUPLINGS FOR 2HDM

RGEs for Yukawa coupling Y_i^u and Y_i^d for a general two-Higgs doublet model are [15]:

$$16\pi^2 \frac{d}{d\ln\mu} Y_i^u = -A^u Y_i^u + \sum_j \text{Tr}[N_c(Y_i^u Y_j^{u\dagger} + Y_j^d Y_i^{d\dagger}) + Y_i^l Y_j^l] Y_j^u + \frac{1}{2} \sum_j (Y_j^u Y_j^{u\dagger} + Y_j^d Y_j^{d\dagger}) Y_i^u + Y_i^u \sum_j Y_j^{u\dagger} Y_j^u - 2 \sum_j Y_j^d Y_i^{d\dagger} Y_j^u, \quad (\text{A1})$$

$$16\pi^2 \frac{d}{d\ln\mu} Y_i^d = -A^d Y_i^d + \sum_j \text{Tr}[N_c(Y_i^d Y_j^{d\dagger} + Y_j^u Y_i^{u\dagger}) + Y_i^l Y_j^l] Y_j^d + \frac{1}{2} \sum_j (Y_j^u Y_j^{u\dagger} + Y_j^d Y_j^{d\dagger}) Y_i^d + Y_i^d \sum_j Y_j^{d\dagger} Y_j^d - 2 \sum_j Y_j^u Y_i^{u\dagger} Y_j^d, \quad (\text{A2})$$

where Y_i^l is the Yukawa coupling of charged lepton, $N_c = 3$, and

$$A^u = 8g_3^2 + \frac{9}{4}g_2^2 + \frac{17}{12}g_1^2, \quad A^d = 8g_3^2 + \frac{9}{4}g_2^2 + \frac{5}{12}g_1^2 \quad (\text{A3})$$

with $g_{1,2,3}$ the gauge couplings of $U(1)_Y$, $SU(2)_L$, and $SU(3)_C$ groups respectively.

With assumption of (31) and (32), (A1) and (A2) can be written as

$$16\pi^2 \frac{d}{d\ln\mu} Y_i^u = -A_i^u Y_i^u + B_i^u Y_i^u + Y_i^u C^u - 2 \sum_{j \neq i} Y_j^d Y_i^{d\dagger} Y_j^u, \quad (\text{A4})$$

$$16\pi^2 \frac{d}{d\ln\mu} Y_i^d = -A_i^d Y_i^d + B_i^d Y_i^d + Y_i^d C^d - 2 \sum_{j \neq i} Y_j^u Y_i^{u\dagger} Y_j^d, \quad (\text{A5})$$

where using (8) and (9) $A_i^{u,d}$, $B_i^{u,d}$, and $C_i^{u,d}$ are given as

$$A_i^{u,d} = A^{u,d} - \text{Tr}[N_c(Y_i^u Y_i^{u\dagger} + Y_i^d Y_i^{d\dagger}) + Y_i^l Y_i^{l\dagger}], \quad (\text{A6})$$

$$B_1^u = B - 2K_{13} \text{diag}\{(g_1^d)^2 c_{12}^2 + (g_2^d)^2 s_{12}^2, 0, 0\} K_{13}^\dagger, \quad (\text{A7})$$

$$B_2^u = B - 2K_{13} \text{diag}\{0, (h_1^d)^2 s_{12}^2 + (h_2^d)^2 c_{12}^2, (h_3^d)^2\} K_{13}^\dagger, \quad (\text{A8})$$

$$B_1^d = B - 2K_{23}^\dagger \text{diag}\{(g_1^u)^2, 0, 0\} K_{23}, \quad (\text{A9})$$

$$B_2^d = B - 2K_{23}^\dagger \text{diag}\{0, (h_2^u)^2, (h_3^u)^2\} K_{23}, \quad (\text{A10})$$

where

$$B = \frac{1}{2} K_{23}^\dagger \text{diag}\{(g_1^u)^2, (h_2^u)^2, (h_3^u)^2\} K_{23} + \frac{1}{2} K_{13} \text{diag}\{(g_1^d)^2 c_{12}^2 + (g_2^d)^2 s_{12}^2, (h_1^d)^2 s_{12}^2 + (h_2^d)^2 c_{12}^2, (h_3^d)^2\} K_{13}^\dagger, \quad (\text{A11})$$

and

$$C^u = \text{diag}\{(g_1^u)^2, (h_2^u)^2, (h_3^u)^2\}, \quad (\text{A12})$$

$$C^d = \begin{pmatrix} (g_1^d)^2 c_{12}^2 + (h_1^d)^2 s_{12}^2 & (g_1^d g_2^d - h_1^d h_2^d) s_{12} c_{12} & 0 \\ (g_1^d g_2^d - h_1^d h_2^d) s_{12} c_{12} & (g_2^d)^2 s_{12}^2 + (h_2^d)^2 c_{12}^2 & 0 \\ 0 & 0 & (h_3^d)^2 \end{pmatrix}. \quad (\text{A13})$$

APPENDIX B: CORRECTION TO DETERMINANT OF QUARK MASS MATRIX

Using (8) and (9) one can find that the last terms in (A4) and (A5) are

$$Y_2^d Y_1^{d\dagger} Y_2^u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -s_d \\ 0 & 0 & 0 \end{pmatrix} K_{23}^\dagger h^u, \quad (\text{B1})$$

where $s_d = s_{13}c_{12}s_{12}(h_2^d g_2^d - h_1^d g_1^d)$, and

$$Y_1^d Y_2^{d\dagger} Y_1^u = Y_1^u Y_2^{u\dagger} Y_1^d = Y_2^u Y_1^{u\dagger} Y_2^d = 0. \quad (\text{B2})$$

One can also find from $(Y_2^u)_{1i} = (Y_2^u)_{i1} = 0$ that

$$(Y_2^u C^u)_{1i} = (Y_2^u C^u)_{i1} = (X_2^u Y_2^u)_{i1} = 0, \quad i = 1, 2, 3. \quad (\text{B3})$$

Determinant of M^u in (37) is calculated as follows.

(1) v_2^3 term in $\det(M^u)$ comes from $\det(X_2^u Y_2^u v_2 + \epsilon Y_2^u C^u v_2)$. Using the fact that Y_2^u is rank two and $(Y_2^u C^u)_{1i} = 0$ we can get

$$\begin{aligned} & \det(X_2^u Y_2^u v_2 + \epsilon Y_2^u C^u v_2) \\ &= v_2^3 \epsilon^{ijk} [\epsilon (X_2^u Y_2^u)_{1i} (X_2^u Y_2^u)_{2j} (Y^u C^u)_{3k} \\ & \quad + \epsilon (X_2^u Y_2^u)_{1i} (Y^u C^u)_{2j} (X_2^u Y_2^u)_{3k} \\ & \quad + \epsilon^2 (X_2^u Y_2^u)_{1i} (Y^u C^u)_{2j} (Y^u C^u)_{3k}]. \end{aligned} \quad (\text{B4})$$

Since one of the i, j, k has to be 1, according to (B3), none of the three terms in (B4) is nonzero. So the term proportional to v_2^3 is zero.

(2) v_1^3 terms in $\det(M^u)$ comes from $\det(X^u Y_1^u v_1 + \epsilon Y_1^u C^u v_1 - 2\epsilon Y_2^d Y_1^{d\dagger} Y_2^u v_1)$. Since $(Y_2^d Y_1^{d\dagger} Y_2^u)_{1i} = (Y_2^d Y_1^{d\dagger} Y_2^u)_{3i} = 0$ as shown in (B1), we can get

$$\begin{aligned} & \det[X_1^u Y_1^u v_1 + \epsilon Y_1^u C^u v_1 - 2\epsilon Y_2^d Y_1^{d\dagger} Y_2^u v_1] \\ &= -2\epsilon^2 v_1^3 \epsilon^{ijk} [(X_1^u Y_1^u)_{1i} (Y_1^u C^u)_{3k} \\ & \quad + (Y_1^u C^u)_{1i} (X_1^u Y_1^u)_{3k}] (Y_2^d Y_1^{d\dagger} Y_2^u)_{2j}. \end{aligned} \quad (\text{B5})$$

Since $(Y_1^u C^u)_{3k} = 0$, $(X_1^u Y_1^u)_{3k} = 0$ for $k = 2$ or 3 and $(Y_1^u C^u)_{1i} = 0$ for $i = 2$ or 3 , we can see that (B5) is zero.

(3) $v_1^2 v_2$ term $\det(M^u)$ have two factors of v_1 . Similar to discussion above for (B5), we can find that $\epsilon^{ijk} (X_1^u Y_1^u)_{ai} (Y_1^u C^u)_{bj} = 0$ and these two factors of v_1 cannot both come from $X_1^u Y_1^u + \epsilon Y_1^u C^u$. One can find that the $v_1^2 v_2$ term is

$$\begin{aligned} & -2\epsilon v_1^2 v_2 \epsilon^{ijk} [(X_1^u Y_1^u + \epsilon Y_1^u C^u)_{1i} (Y_2^d Y_1^{d\dagger} Y_2^u)_{2j} (X_2^u Y_2^u + \epsilon Y_2^u C^u)_{3k} \\ & \quad + (X_2^u Y_2^u + \epsilon Y_2^u C^u)_{1i} (Y_2^d Y_1^{d\dagger} Y_2^u)_{2j} (X_1^u Y_1^u + \epsilon Y_1^u C^u)_{3k}]. \end{aligned} \quad (\text{B6})$$

Using $(Y_2^d Y_1^{d\dagger} Y_2^u)_{2j} = s_d (Y_2^u)_{3j}$ as shown in (B1), the first term in (B6) can be found to be

$$\begin{aligned} & -2\epsilon v_1^2 v_2 \epsilon^{ijk} (X_1^u Y_1^u + \epsilon Y_1^u C^u)_{1i} s_d (Y_2^u)_{3j} [(1 - \epsilon A_2^u) Y_{3k}^u + (\epsilon B_2^u Y_2^u + \epsilon Y_2^u C^u)_{3k}] \\ &= -2\epsilon^2 v_1^2 v_2 \epsilon^{ijk} (X_1^u Y_1^u + \epsilon Y_1^u C^u)_{1i} s_d (Y_2^u)_{3j} (B_2^u Y_2^u + \epsilon Y_2^u C^u)_{3k} \\ &\approx -2\epsilon^2 v_1^2 v_2 s_d \epsilon^{ijk} (Y_1^u)_{11} \delta_{i1} (Y_2^u)_{3j} [(B_2^u)_{32} (Y_2^u)_{2k} + (Y_2^u)_{3k} (C^u)_{kk}] \\ &= -2\epsilon^2 v_1^2 v_2 s_d (B_2^u)_{32} (Y_1^u)_{11} [\epsilon^{1jk} (Y_2^u)_{3j} (Y_2^u)_{2k} + (Y_2^u)_{32} (Y_2^u)_{33} ((C^u)_{33} - (C^u)_{22})] \\ &= -3\epsilon^2 s_d v_1^2 v_2 [(h_3^u)^2 - (h_2^u)^2] g_1^u h_2^u h_3^u c_{23} s_{23}, \end{aligned} \quad (\text{B7})$$

where s_d is given after (B1). The second term in (B6) can be found to be

$$\begin{aligned} & -2\epsilon^3 v_1^2 v_2 \epsilon^{ijk} s_d (B_2^u Y_2^u + Y_2^u C^u)_{1i} (B_1^u Y_1^u + Y_1^u C^u)_{3k} \\ &= -2\epsilon^3 v_1^2 v_2 \epsilon^{ijk} s_d (B_2^u)_{12} (B_1^u)_{31} \\ & \quad \times (Y_2^u)_{2i} (Y_2^u)_{3j} (Y_1^u)_{11} \delta_{k1} \\ &= -2\epsilon^3 v_1^2 v_2 s_d (B_2^u)_{12} (B_1^u)_{31} g_1^u h_2^u h_3^u, \end{aligned} \quad (\text{B8})$$

where we have used $(Y_1^u)_{3k} = (Y_2^u)_{1i} = 0$. Comparing with (B7), (B8) is at higher order and can be neglected.

Determinant of M^d in (38) is calculated as follows.

$\det(M^d)$ can be computed using $\hat{M}^d = K_{13}^\dagger M^d$

$$\hat{M}^d = \hat{X}_1^d \hat{Y}_1^d + \epsilon \hat{Y}_1^d C^d v_1^* + \hat{X}_2^d \hat{Y}_2^d v_2^* + \epsilon \hat{Y}_2^d C^d v_2^*, \quad (\text{B9})$$

where $\hat{X}_{1,2}^d = K_{13}^\dagger \hat{X}_{1,2}^d K_{13} = 1 - \epsilon A_{1,2}^d + \epsilon \hat{B}_{1,2}^d$ with $\hat{B}_{1,2}^d = K_{13}^\dagger B_{1,2}^d K_{13}$, $\hat{Y}_{1,2}^d = K_{13}^\dagger Y_{1,2}^d$. As can be seen in (9), we would have $(\hat{Y}_1^d)_{2i} = (\hat{Y}_1^d)_{3i} = (\hat{Y}_1^d)_{13} = 0$ and $(\hat{Y}_2^d)_{1i} = (\hat{Y}_2^d)_{23} = (\hat{Y}_2^d)_{31} = (\hat{Y}_2^d)_{32} = 0$.

(4) $(v_1^*)^3$ term in $\det(\hat{M}^d)$ comes from $\det(v_1^* \hat{X}_1^d \hat{Y}_1^d + \epsilon v_1^* \hat{Y}_1^d C^d)$. Two matrices appearing in it are both rank one. So the determinant of this 3×3 matrix must be zero.

- (5) $(v_1^*)^2 v_2^*$ term in $\det(\hat{M}^d)$ should have contributions from both $v_1^* \hat{X}_1^d \hat{Y}_1^d$ and $\epsilon v_1^* \hat{Y}_1^d C^d$ since they are both rank one. Since $(\hat{Y}_1^d C^d)_{2i} = (\hat{Y}_1^d C^d)_{3i} = 0$ and $(\hat{Y}_1^d)_{13} = (\hat{Y}_2^d)_{23} = (\hat{Y}_1^d C^d)_{13} = 0$, this part of the determinant is

$$\begin{aligned}
& \epsilon (v_1^*)^2 v_2^* \epsilon^{ijk} (\hat{Y}_1^d C^d)_{1i} [(\hat{X}_1^d \hat{Y}_1^d)_{2j} (\hat{X}_2^d \hat{Y}_2^d + \epsilon \hat{Y}_2^d C^d)_{3k} + (\hat{X}_2^d \hat{Y}_2^d + \epsilon \hat{Y}_2^d C^d)_{2j} (\hat{X}_1^d \hat{Y}_1^d)_{3k}] \\
& = \epsilon^2 (v_1^*)^2 v_2^* \epsilon^{ijk} (\hat{Y}_1^d C^d)_{1i} [(\hat{B}_1^d)_{21} (\hat{Y}_1^d)_{1j} (\hat{X}_2^d \hat{Y}_2^d + \epsilon \hat{Y}_2^d C^d)_{3k} + (\hat{X}_2^d \hat{Y}_2^d + \epsilon \hat{Y}_2^d C^d)_{2j} (\hat{B}_1^d)_{31} (\hat{Y}_1^d)_{1k}] \\
& \approx \epsilon^2 (v_1^*)^2 v_2^* \epsilon^{ijk} (\hat{Y}_1^d C^d)_{1i} [(\hat{B}_1^d)_{21} (\hat{Y}_1^d)_{1j} (\hat{Y}_2^d)_{3k} + (\hat{Y}_2^d)_{2j} (\hat{B}_1^d)_{31} (\hat{Y}_1^d)_{1k}] \\
& = \epsilon^2 (v_1^*)^2 v_2^* \epsilon^{ijk} (\hat{B}_1^d)_{21} [(\hat{Y}_1^d C^d)_{11} (\hat{Y}_1^d)_{12} - (\hat{Y}_1^d C^d)_{12} (\hat{Y}_1^d)_{11}] (\hat{Y}_2^d)_{33} \\
& = \frac{1}{2} \epsilon^2 (v_1^*)^2 v_2^* s_{23} c_{23} s_{13} s_{12} c_{12} h_3^d h_2^d g_1^d (g_2^d h_2^d - g_1^d h_1^d) [(h_3^u)^2 - (h_2^u)^2].
\end{aligned} \tag{B10}$$

In the fourth line in (B10) we have used the fact that the factors times $(\hat{B}_1^d)_{31}$ is zero since if any one of the i, j, k indices equals to 3, an associated factor would be zero.

- (6) $(v_2^*)^3$ term comes from $\det(v_2^* \hat{X}_2^d \hat{Y}_2^d + v_2^* \epsilon \hat{Y}_2^d C^d)$. The leading term in it is $\det[(1 - \epsilon A_2^d) \hat{Y}_2^d + \epsilon \hat{Y}_2^d C^d] = \det[\hat{Y}_2^d (1 - \epsilon A_2^d \epsilon C^d)] = \det(\hat{Y}_2^d) \det(1 - \epsilon A_2^d + \epsilon C^d) = 0$. Since $(\hat{Y}_2^d)_{1i} = 0$, $(\hat{Y}_2^d)_{31} = (\hat{Y}_2^d)_{32} = 0$ and $(\hat{Y}_2^d C^d)_{31} = (\hat{Y}_2^d C^d)_{32} = 0$, the leading nonzero term is

$$\begin{aligned}
& \epsilon^2 (v_2^*)^3 \epsilon^{ijk} (\hat{B}_2^d \hat{Y}_2^d)_{1i} [(\hat{Y}_2^d)_{2j} (\hat{Y}_2^d C^d)_{3k} + (\hat{Y}_2^d C^d)_{2j} (\hat{Y}_2^d)_{3k}] \\
& = \epsilon^2 (v_2^*)^3 \epsilon^{ijk} [(\hat{B}_2^d)_{13} (\hat{Y}_2^d)_{3i} (\hat{Y}_2^d)_{2j} (\hat{Y}_2^d C^d)_{3k} + (\hat{B}_2^d)_{12} (\hat{Y}_2^d)_{2i} (\hat{Y}_2^d C^d)_{2j} (\hat{Y}_2^d)_{3k}] \\
& = \epsilon^2 (v_2^*)^3 \epsilon^{ijk} [(\hat{B}_2^d)_{13} (\hat{Y}_2^d C^d)_{3k} + (\hat{B}_2^d)_{12} (\hat{Y}_2^d C^d)_{2k}] (\hat{Y}_2^d)_{3i} (\hat{Y}_2^d)_{2j} \\
& = \epsilon^2 (v_2^*)^3 \epsilon^{3jk} (\hat{B}_2^d)_{12} (\hat{Y}_2^d)_{2k'} (C^d)_{k'k} (\hat{Y}_2^d)_{33} (\hat{Y}_2^d)_{2j} \\
& = \frac{1}{2} \epsilon^2 (v_2^*)^3 s_{13} c_{12} s_{12} c_{23} s_{23} g_1^d h_2^d h_3^d [(h_3^u)^2 - (h_2^u)^2] (g_2^d h_2^d - g_1^d h_1^d),
\end{aligned} \tag{B11}$$

where in the fourth line we have used $\epsilon^{ijk} (\hat{Y}_2^d C^d)_{3k} (\hat{Y}_2^d)_{3i} = 0$.

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- [1] C. Patrignani *et al.* (Particle Data Group), *Chin. Phys. C* **40**, 100001 (2016).
[2] J. Ellis and M. K. Gaillard, *Nucl. Phys.* **B150**, 141 (1979).
[3] H. Y. Cheng, *Phys. Rep.* **158**, 1 (1988).
[4] T. D. Lee, *Phys. Rev. D* **8**, 1226 (1973); *Phys. Rep.* **9**, 143 (1974).
[5] M. A. B. Beg and H. S. Tsao, *Phys. Rev. Lett.* **41**, 278 (1978); R. N. Mohapatra and G. Senjanovic, *Phys. Lett.* **79B**, 283 (1978); H. Georgi, *Hadronic J.* **1**, 155 (1978).
[6] A. Nelson, *Phys. Lett.* **136B**, 387 (1984); S. M. Barr, *Phys. Rev. Lett.* **53**, 329 (1984).
[7] S. M. Barr, D. Chang, and G. Senjanovic, *Phys. Rev. Lett.* **67**, 2765 (1991).
[8] R. Friedberg and T. D. Lee, *Chin. Phys. C* **32**, 939 (2008); *Ann. Phys. (N.Y.)* **324**, 2196 (2009).
[9] S.-L. Chen, N. G. Deshpande, X.-G. He, J. Jiang, and L.-H. Tsai, *Eur. Phys. J. C* **53**, 607 (2008).
[10] Y.-N. Mao and S.-H. Zhu, *Phys. Rev. D* **90**, 115024 (2014); **94**, 055008 (2016); **94**, 059904(E) (2016).
[11] L. L. Chau and W. Y. Keung, *Phys. Rev. Lett.* **53**, 1802 (1984).
[12] T. Araki and C. Q. Geng, *Phys. Rev. D* **79**, 077301 (2009).
[13] J. L. Hewett *et al.*, arXiv:1205.2671; W. Dekens, J. de Vries, J. Bsaisou, W. Bernreuther, C. Hanhart, U.-G. Meißner, A. Nogga, and A. Wirzba, *J. High Energy Phys.* **07** (2014) 069.
[14] X.-G. He and L.-H. Tsai, *Eur. Phys. J. C* **71**, 1598 (2011).
[15] P. M. Ferreira, L. Lavoura, and J. P. Silva, *Phys. Lett. B* **688**, 341 (2010); W. Grimus and L. Lavoura, *Eur. Phys. J. C* **39**, 219 (2005).