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Analysis of $P_c^+(4380)$ and $P_c^+(4450)$ as pentaquark states in the molecular picture with QCD sum rules

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To better understand the nature and internal structure of the exotic states discovered by many collaborations, more information on their electromagnetic properties and their strong and weak interactions with other hadrons is needed. The residue or current coupling constant of these states together with their mass are the main inputs in determinations of such properties. We perform QCD sum rules analyses on the hidden-charm pentaquark states with spin parities $J^P = \frac{3}{2}^{\pm}$ and $J^P = \frac{5}{2}^{\pm}$ to calculate their residue and mass. In the calculations, we adopt a molecular picture for $J^P = \frac{3}{2}^{\pm}$ states and a mixed current in a molecular form for $J^P = \frac{5}{2}^{\pm}$. Our analyses show that the $P_c^+(4380)$ and $P_c^+(4450)$, observed by the LHCb Collaboration, can be considered as hidden-charm pentaquark states with $J^P = \frac{3}{2}^{-}$ and $J^P = \frac{5}{2}^{+}$, respectively.

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I. INTRODUCTION

The recent experimental progress resulting in the observation of exotic hadrons has made this subject a focus of interest. These hadrons have an internal structure that is more complex than those containing usual $q\bar{q}$ or qqq quark contents. The existence of these types of hadrons is not forbidden in either the naive quark model, which provides a good description of the observed conventional hadrons, or in quantum chromodynamics (QCD), which describes the interactions among quarks and gluons. Starting from the observation of $X(3872)$ in 2003 by the Belle Collaboration [1], many experiments have been designed to identify and measure the parameters of the nonconventional particles, especially the XYZ states. These experimental attempts have been accompanied by many theoretical works on tetraquarks, pentaquarks, hybrids, glueballs, etc.

The first detailed theoretical analysis on the exotic states provided by Jaffe [2] was followed by a vast amount of theoretical studies that investigated the properties of these particles. Among these states are the pentaquarks, for which the first claim of observation was in 2003 through the interaction $\gamma n \to n K^+ K^-$ [3], suggesting a possible quark content $uudd\bar{s}$ (Θ^+) with strangeness $S = +1$. Even before this claim, there were several works on the properties of pentaquarks (see, for instance, Refs. [4–14]). Later, two other experiments also found some positive signatures [15,16]. With the motivation provided by those results, there came another estimation on the anticharmed analogue of Θ^+ with quark content $uudd\bar{c}$, denoted as Θ_c. Its mass together with the mass of its b partner Θ_b were predicted as 2985 ± 50 and 6398 ± 50 MeV, respectively [17]. The masses of the Θ^+, Θ_c, and Θ_b states were also predicted in Ref. [18]. The masses and other properties of Θ^+, Θ_c, and Θ_b were then extensively examined via various methods (see, for instance, Refs. [19–50] and references therein). In the mean time, the observation of Θ_c was announced later by the H1 Collaboration at HERA [51]. However, despite all of these positive experimental results and related theoretical studies, some experiments announced negative results regarding the existence of these particles [52–62]. All of these controversial results have made the subject more intriguing from a theoretical point of view, since theoretical works might provide valuable insights into the experimental searches.

All of effort in searching for exotic states finally resulted in success on the experimental side. With the report of the observation of Z_c [63] in 2013, which might be an indication of the existence of pentaquark states, the pentaquark once again became a focus of interest. However, some experimental searches for pentaquarks still gave null results, such as the result of the ALICE Collaboration investigating the $\phi(1869)$ pentaquark [64] and the J-PARC E19 Collaboration searching for the Θ^+ state [65]. On the other hand, the theoretical studies indicated that the search for the pentaquark containing heavy quark constituents is still necessary [66] due to the effect of such a structure on the stability of the hadronic structures beyond the traditional hadrons [67]. In 2015, the observation of two pentaquark states—$P_c^+(4380)$ and $P_c^+(4450)$—was finally reported by the LHCb Collaboration in the $\Lambda_b^0 \to J/\psi K^- p$ decays. The reported masses were $4380 \pm 8 \pm 29$ and $4449.8 \pm 1.7 \pm 2.5$ MeV, with corresponding spins $3/2$ and $5/2$ and decay widths $205 \pm 18 \pm 86$ and $39 \pm 5 \pm 19$ MeV, respectively [68].

The observation by the LHCb Collaboration put these particles at the focus of intense theoretical works which aimed to explain the properties of these states. To explain their substructure, different models were proposed. Their nature was examined using the meson baryon molecular model [69–78], diquark-triquark model [78–80], diquark-diquark-antiquark model [78,81–86], and topological soliton model [87]. They were also investigated by taking into account the possibility of their being a kinematical effect or a real resonance state considering the triangle singularity mechanism [88–90]. In Ref. [91], however, it was concluded that with the presently claimed experimental quantum numbers, the triangle singularity cannot be the explanation for the peaks. One can find a review on the multiquark states including pentaquarks in Ref. [92].

All of these developments make it necessary to study pentaquarks more deeply to gain information on their nature and substructure. Theoretical investigations on their spectroscopic and electromagnetic properties together with their strong and weak decays may provide valuable insights for future experimental searches. Moreover, a comparison between new theoretical findings and existing experimental and theoretical results may lead to a better understanding of the nature of these particles as well as the dynamics of the strong interaction. With this motivation, in this paper we investigate the residue and mass of the hidden-charm pentaquark states with the spin parities $J^P = \frac{3}{2}^{\pm}$ and $J^P = \frac{5}{2}^{\pm}$. To fulfill this aim, we apply the QCD sum rule method [93,94] via a choice of interpolating current in the molecular form. Here we shall remark that the QCD sum rule approach in its standard form was formulated to reproduce the mass of the lowest hadronic state in a given channel, assuming that there are no other resonances close to the lowest one. We apply this method to reproduce the experimental data in the channels under consideration with the assumption that there are no other prominent resonances close to the lowest states with $J = \frac{3}{2}$ and $J = \frac{5}{2}$. In principle, there can be many interpolating currents with the same quantum numbers and flavor contents to investigate the states under consideration, and there are no preferable interpolating currents. We choose a molecular picture and investigate these states by considering their interpolating currents in the anticharmed meson-charmed baryon form. For the states with $J = \frac{5}{2}$, we consider an admixture of $[\bar{D}\Sigma_c^*]$ and $[\bar{D}^*\Lambda_c]$ and use a mixed anticharmed meson-charmed baryon molecular current. In choosing this current we consider the discussion given in Ref. [95], which stated that a choice of a mixed molecular current provides a mass result consistent with the experimental data. For $J = \frac{3}{2}$ states we also use an anticharmed meson-charmed baryon molecular current, namely, $\bar{D}^*\Sigma_c$. As the residue is the main input in the analysis of the width, electromagnetic properties, and strong and weak decays of these particles, the main goal in this work is to calculate the residue of these pentaquarks with both parities considering the molecular and mixed molecular currents for $J = \frac{3}{2}$ and $J = \frac{5}{2}$ states, respectively. We also calculate the masses of these states in the same pictures. Here we shall remark that in Refs. [75,85] the authors used the QCD sum rule method to investigate these pentaquark states as well. In Ref. [75] the authors calculated only the masses of the $J^P = \frac{3}{2}^-$ and $J^P = \frac{5}{2}^+$ pentaquark states with the same currents and internal quark organizations as in the present work. In Ref. [85], however, the diquark-diquark-antiquark type interpolating currents were used to calculate the mass and residue of the $J^P = \frac{3}{2}^-$ and $J^P = \frac{5}{2}^+$ pentaquark states.

The paper is organized as follows. In Secs. II and III we present the details of the mass and residue calculations for the hidden-charm pentaquark states with $J = \frac{3}{2}$ and $J = \frac{5}{2}$, respectively. Section IV is devoted to the numerical analysis and discussion of the results. The last section is devoted to the summary and outlook.

II. THE HIDDEN-CHARM PENTAQUARK STATES WITH $J = \frac{3}{2}$

This section is devoted to presenting the details of the calculations of the mass and residue of the pentaquark states with spin $3/2$ and both the positive and negative parities. The starting point is to consider the following two-point correlation function:

$$\Pi_{1\mu\nu}(p) = i\int d^4x e^{ip\cdot x}\langle 0|\mathcal{T}\{J_\mu^{\bar{D}^*\Sigma_c}(x)\bar{J}_\nu^{\bar{D}^*\Sigma_c}(0)\}|0\rangle, \qquad (1)$$

where $J_\mu^{\bar{D}^*\Sigma_c}(x)$ is the interpolating current with $J^P = \frac{3}{2}^-$ that couples to both the negative- and positive-parity particles [75],

$$J_\mu^{\bar{D}^*\Sigma_c} = [\bar{c}_d\gamma_\mu d_d][\epsilon_{abc}(u_a^T C\gamma_\theta u_b)\gamma^\theta\gamma_5 c_c]. \qquad (2)$$

The first step is to calculate the correlation function in terms of hadronic degrees of freedom containing the physical parameters of the states under consideration. This requires the insertion of a complete set of the hadronic states into Eq. (1), which is followed by an integration over x. This leads to

$$\Pi_{1\mu\nu}^{\rm Phys}(p) = \frac{\langle 0|J_\mu|\frac{3}{2}^+(p)\rangle\langle\frac{3}{2}^+(p)|\bar{J}_\nu|0\rangle}{m_+^2 - p^2} + \frac{\langle 0|J_\mu|\frac{3}{2}^-(p)\rangle\langle\frac{3}{2}^-(p)|\bar{J}_\nu|0\rangle}{m_-^2 - p^2} + \cdots, \qquad (3)$$

where $m_\pm$ are the masses of the positive- and negative-parity particles. The dots appearing in the last equation represent the contributions coming from the higher states and continuum resonances. The matrix elements in Eq. (3) are parametrized in terms of the residues λ_+ and λ_- as well as the corresponding spinors as

$$\left\langle 0|J_\mu|\frac{3}{2}^+(p)\right\rangle = \lambda_+\gamma_5 u_\mu(p),$$
$$\left\langle 0|J_\mu|\frac{3}{2}^-(p)\right\rangle = \lambda_- u_\mu(p), \tag{4}$$

where the negative-parity nature of the current under consideration has been imposed. Here we should remark that the J_μ current couples not only to the spin-3/2 states, but also to the spin-1/2 states with both parities. We will choose appropriate structures to take into account only the particles with spin 3/2. The summation over the Rarita-Schwinger spinor is applied in the form

$$\sum_s u_\mu(p,s)\bar{u}_\nu(p,s) = -(\not{p}+m)\left[g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{2p_\mu p_\nu}{3m^2} + \frac{p_\mu\gamma_\nu - p_\nu\gamma_\mu}{3m}\right]. \tag{5}$$

After applying of the Borel transformation, the hadronic side gets its final form in terms of different structures,

$$\begin{aligned}\mathcal{B}_{p^2}\Pi^{\rm Phys}_{1\mu\nu}(p) = &-\lambda_+^2 e^{-\frac{m_+^2}{M^2}}(-\gamma_5)(\not{p}+m_+)\\ &\times\left[g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{2p_\mu p_\nu}{3m_+^2} + \frac{p_\mu\gamma_\nu - p_\nu\gamma_\mu}{3m_+}\right]\gamma_5\\ &-\lambda_-^2 e^{-\frac{m_-^2}{M^2}}(\not{p}+m_-)\\ &\times\left[g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{2p_\mu p_\nu}{3m_-^2} + \frac{p_\mu\gamma_\nu - p_\nu\gamma_\mu}{3m_-}\right] + \cdots,\end{aligned} \tag{6}$$

where M^2 is the Borel parameter that should be fixed later. To avoid the unwanted contributions coming from the spin-1/2 states, we select the $g_{\mu\nu}$ and $\not{p}g_{\mu\nu}$ structures after ordering of the Dirac matrices.

To get the QCD sum rules one also needs to calculate the same correlation function on the QCD side in terms of quark-gluon degrees of freedom in the deep Euclidean region using the operator product expansion (OPE). This requires the contraction of the heavy and light quark fields, which leads to the result

$$\begin{aligned}\Pi^{\rm QCD}_{1\mu\nu}(p) = &-i\int d^4x e^{ip\cdot x}\epsilon^{abc}\epsilon^{a'b'c'}\\ &\times {\rm Tr}[\gamma_\mu S_d^{dd'}(x)\gamma_\nu S_c^{d'd}(-x)](\gamma\theta\gamma_5 S_c^{cc'}(x)\gamma_5\gamma\beta)\\ &\times\{{\rm Tr}[\gamma_\beta \tilde{S}_u^{aa'}(x)\gamma_\theta S_u^{bb'}(x)]\\ &-{\rm Tr}[\gamma_\beta\tilde{S}_u^{ba'}(x)\gamma_\theta S_u^{ab'}(x)]\},\end{aligned} \tag{7}$$

where $\tilde{S}_{u(d)}(x) = CS^T_{u(d)}(x)C$, and $S^{ab}_{u(d)}(x)$ and $S_c^{ab}(x)$ appearing in Eq. (7) are the propagators of the light $u(d)$ and heavy c quarks, respectively. The explicit expression for light quark propagator has the form

$$\begin{aligned}S_q^{ab}(x) = &i\delta_{ab}\frac{\not{x}}{2\pi^2x^4} - \delta_{ab}\frac{m_q}{4\pi^2x^2} - \delta_{ab}\frac{\langle\bar{q}q\rangle}{12}\\ &+ i\delta_{ab}\frac{\not{x}m_q\langle\bar{q}q\rangle}{48} - \delta_{ab}\frac{x^2}{192}\langle\bar{q}g_s\sigma Gq\rangle + i\delta_{ab}\frac{x^2\not{x}m_q}{1152}\\ &\times\langle\bar{q}g_s\sigma Gq\rangle - i\frac{g_sG^{\alpha\beta}_{ab}}{32\pi^2x^2}[\not{x}\sigma_{\alpha\beta}+\sigma_{\alpha\beta}\not{x}]\\ &- i\delta_{ab}\frac{x^2\not{x}g_s^2\langle\bar{q}q\rangle^2}{7776} + \cdots,\end{aligned} \tag{8}$$

and the heavy quark propagator is given as [96]

$$\begin{aligned}S_c^{ab}(x) = i\int\frac{d^4k}{(2\pi)^4}e^{-ikx}\Bigg\{&\frac{\delta_{ab}(\not{k}+m_c)}{k^2-m_c^2}\\ &- \frac{g_sG^{\alpha\beta}_{ab}}{4}\frac{\sigma_{\alpha\beta}(\not{k}+m_c)+(\not{k}+m_c)\sigma_{\alpha\beta}}{(k^2-m_c^2)^2}\\ &+ \frac{g_s^2G^2}{12}\delta_{ab}m_c\frac{k^2+m_c\not{k}}{(k^2-m_c^2)^4} + \cdots\Bigg\},\end{aligned} \tag{9}$$

where we used the short-hand notation

$$G^{\alpha\beta}_{ab} = G^{\alpha\beta}_A t^A_{ab}, \qquad G^2 = G^A_{\alpha\beta}G^A_{\alpha\beta}, \tag{10}$$

in which $A = 1, 2, \ldots, 8$ and $a, b = 1, 2, 3$ are color indices and $t^A = \lambda^A/2$, with λ^A being the Gell-Mann matrices.

The calculations on the OPE side proceed by writing the correlation function in a dispersion integral form,

$$\Pi^{\rm QCD}_{1\mu\nu}(p^2) = \int_{(2m_c)^2}^{s_0}\frac{\rho^{\rm QCD}_{\frac{3}{2}}(s)}{s-p^2}ds + \cdots, \tag{11}$$

where $\rho^{\rm QCD}_{\frac{3}{2}}(s)$ is the two-point spectral density, which is found via the imaginary part of the correlation function following the standard procedures. Here, s_0 is the continuum threshold. The calculations are very lengthy. For details, we refer the interested reader to, e.g., Refs. [97,98]. The explicit expression of the spectral density $\rho^{\rm QCD}_{\frac{3}{2}}(s)$ (for instance, for the $g_{\mu\nu}$ structure) is given in the Appendix. With the aim of suppressing the contributions of the higher states and continuum, we also apply the Borel transformation to this side to find the correlation function in its final form in the Borel scheme.

Now, we match the coefficients of the structures $g_{\mu\nu}$ and $\not{p}g_{\mu\nu}$ from both the hadronic and OPE sides and apply a continuum subtraction supported by the quark-hadron duality assumption. This leads to the sum rules

$$\begin{aligned}m_+\lambda_+^2e^{-m_+^2/M^2} - m_-\lambda_-^2e^{-m_-^2/M^2} &= \Pi_1^1,\\ -\lambda_+^2e^{-m_+^2/M^2} - \lambda_-^2e^{-m_-^2/M^2} &= \Pi_1^2,\end{aligned} \tag{12}$$

including the masses and residues of the $\frac{3}{2}^+$ and $\frac{3}{2}^-$ states. In the last equation, Π_1^1 and Π_1^2 are the invariant functions obtained from the OPE side and correspond to the coefficients of the structures $g_{\mu\nu}$ and $\not{p} g_{\mu\nu}$, respectively.

Note that Eq. (12) contains two sum rules with four unknowns: two masses m_+ and m_-, as well as two residues λ_+ and λ_-. Hence, to find these four unknowns, we need two more equations, which are found by applying the derivatives with respect to $\frac{1}{M^2}$ to both sides of the above sum rules. By simultaneously solving the four resulting equations, one can find the four unknowns in terms of QCD degrees of freedom as well as the continuum threshold and Borel mass parameter.

III. THE HIDDEN-CHARM PENTAQUARK STATES WITH $J = \frac{5}{2}$

In this section we follow similar steps as in the previous section. In this case, the following two-point correlation function is used:

$$\Pi_{2\mu\nu\rho\sigma}(p) = i \int d^4 x e^{ip\cdot x} \langle 0|\mathcal{T}\{J_{\mu\nu}(x)\bar{J}_{\rho\sigma}(0)\}|0\rangle, \quad (13)$$

where $J_{\mu\nu}(x)$ is the interpolating current with quantum numbers $J^P = \frac{5}{2}^+$. This current is defined in terms of the mixed currents of $J_{\mu\nu}^{\bar{D}\Sigma_c^*}$ and $J_{\mu\nu}^{\bar{D}^*\Lambda_c}$ via the expression [75]

$$J_{\mu\nu}(x) = \sin\theta \times J_{\mu\nu}^{\bar{D}\Sigma_c^*} + \cos\theta \times J_{\mu\nu}^{\bar{D}^*\Lambda_c}, \quad (14)$$

where θ is a mixing angle and

$$J_{\mu\nu}^{\bar{D}\Sigma_c^*} = [\bar{c}_d \gamma_\mu \gamma_5 d_d][\epsilon_{abc}(u_a^T C\gamma_\nu u_b)c_c] + \{\mu \leftrightarrow \nu\},$$
$$J_{\mu\nu}^{\bar{D}^*\Lambda_c} = [\bar{c}_d \gamma_\mu u_d][\epsilon_{abc}(u_a^T C\gamma_\nu \gamma_5 d_b)c_c] + \{\mu \leftrightarrow \nu\}. \quad (15)$$

In Ref. [75] it was found that the above current with the mixing angle $\theta = (-51 \pm 5)^\circ$ gives a result consistent with the experimental mass of the $P_c(4450)$ state.[1]

The hadronic side, after integration over x, is obtained as

$$\Pi_{2\mu\nu\rho\sigma}^{\mathrm{Phys}}(p) = \frac{\langle 0|J_{\mu\nu}|\frac{5}{2}^+(p)\rangle\langle\frac{5}{2}^+(p)|\bar{J}_{\rho\sigma}|0\rangle}{m_+^2 - p^2} + \frac{\langle 0|J_{\mu\nu}|\frac{5}{2}^-(p)\rangle\langle\frac{5}{2}^-(p)|\bar{J}_{\rho\sigma}|0\rangle}{m_-^2 - p^2} + \cdots, \quad (16)$$

with $m_\pm$ being the masses of the $\frac{5}{2}$ states with positive and negative parities. The contributions of the higher and continuum state resonances to the correlation function are represented via the dots appearing in the last equation. For the matrix elements presented in Eq. (16), the following parameterizations in terms of the residues and spinors are used:

$$\left\langle 0|J_{\mu\nu}|\frac{5^+}{2}(p)\right\rangle = \lambda_+ u_{\mu\nu}(p),$$
$$\left\langle 0|J_{\mu\nu}|\frac{5^-}{2}(p)\right\rangle = \lambda_- \gamma_5 u_{\mu\nu}(p). \quad (17)$$

The current $J_{\mu\nu}(x)$ also couples to the states with spin $3/2$ and $1/2$ with both parities. Again, we will choose the structures that only give contributions to the spin-$5/2$ particles. By using the summation [85]

$$\sum_s u_{\mu\nu}\bar{u}_{\rho\sigma} = (\not{p} + m)\Big\{\frac{\tilde{g}_{\mu\rho}\tilde{g}_{\nu\sigma} + \tilde{g}_{\mu\sigma}\tilde{g}_{\nu\rho}}{2} - \frac{\tilde{g}_{\mu\nu}\tilde{g}_{\rho\sigma}}{5}$$
$$- \frac{1}{10}\left(\gamma_\mu\gamma_\rho + \frac{\gamma_\mu p_\rho - \gamma_\rho p_\mu}{\sqrt{p^2}} - \frac{p_\mu p_\rho}{p^2}\right)\tilde{g}_{\nu\sigma}$$
$$- \frac{1}{10}\left(\gamma_\nu\gamma_\rho + \frac{\gamma_\nu p_\rho - \gamma_\rho p_\nu}{\sqrt{p^2}} - \frac{p_\nu p_\rho}{p^2}\right)\tilde{g}_{\mu\sigma}$$
$$- \frac{1}{10}\left(\gamma_\mu\gamma_\sigma + \frac{\gamma_\mu p_\sigma - \gamma_\sigma p_\mu}{\sqrt{p^2}} - \frac{p_\mu p_\sigma}{p^2}\right)\tilde{g}_{\nu\rho}$$
$$- \frac{1}{10}\left(\gamma_\nu\gamma_\sigma + \frac{\gamma_\nu p_\sigma - \gamma_\sigma p_\nu}{\sqrt{p^2}} - \frac{p_\nu p_\sigma}{p^2}\right)\tilde{g}_{\mu\rho}\Big\}, \quad (18)$$

where $\tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}$, the correlation function takes the form

$$\Pi_{2\mu\nu\rho\sigma}^{\mathrm{Phys}}(p) = \frac{\lambda_+^2}{(m_+^2 - p^2)}(\not{p} + m_+)\frac{g_{\mu\rho}g_{\nu\sigma} + g_{\mu\sigma}g_{\nu\rho}}{2} + \frac{\lambda_-^2}{(m_-^2 - p^2)}(\not{p} - m_-)\frac{g_{\mu\rho}g_{\nu\sigma} + g_{\mu\sigma}g_{\nu\rho}}{2} + \cdots, \quad (19)$$

in terms of m_+, m_-, λ_+, and λ_-. In the last result there are other Lorentz structures giving contributions to the correlation function; however, those structures mainly include contributions that also come from other pentaquark states with spin-$1/2$ and spin-$3/2$. To exclude this type of contributions, in the remaining part of the calculations we use the presented structures to extract the mass and residue of the states under consideration. Therefore, the dots in Eq. (19) represent both the contributions coming from other Lorentz structures that are not written explicitly here, as well as the contributions of higher states and continuum. Applying the Borel transformation to Eq. (19) results in

[1]Our analyses show that the results do not considerably depend on θ. Hence, an optimization such as that advised in Ref. [99] does not work in this case.

$$\mathcal{B}_{p^2}\Pi^{\mathrm{Phys}}_{2\mu\nu\rho\sigma}(p)=\lambda_+^2 e^{-\frac{m_+^2}{M^2}}(\not{p}+m_+)\left(\frac{g_{\mu\rho}g_{\nu\sigma}+g_{\mu\sigma}g_{\nu\rho}}{2}\right)+\lambda_-^2 e^{-\frac{m_-^2}{M^2}}(\not{p}-m_-)\left(\frac{g_{\mu\rho}g_{\nu\sigma}+g_{\mu\sigma}g_{\nu\rho}}{2}\right)+\cdots. \tag{20}$$

In order to obtain the QCD side of the correlation function, we contract the heavy and light quark fields using Wick's theorem, which leads to $\sin^2$

$$\begin{aligned}\Pi^{\mathrm{QCD}}_{2\mu\nu\rho\sigma}(p)=i\int d^4x e^{ipx}\epsilon^{abc}\epsilon^{a'b'c'}\{&\sin^2\theta S_c^{cc'}(x)\{\mathrm{Tr}[\gamma_\mu\gamma_5 S_d^{dd'}(x)\gamma_5\gamma_\rho S_c^{d'd}(-x)](\mathrm{Tr}[\gamma_\nu S_u^{ba'}(x)\gamma_\sigma\\&\times\tilde{S}_u^{ab'}(x)]-\mathrm{Tr}[\gamma_\nu S_u^{bb'}(x)\gamma_\sigma\tilde{S}_u^{aa'}(x)])\}+\cos^2\theta S_c^{cc'}(x)\{\mathrm{Tr}[\gamma_\mu S_u^{da'}(x)\gamma_\sigma\gamma_5\tilde{S}_d^{bb'}(x)\gamma_5\gamma_\nu S_u^{ad'}(x)\gamma_\rho\\&\times S_c^{d'd}(-x)]-\mathrm{Tr}[\gamma_\mu S_u^{dd'}(x)\gamma_\rho S_c^{d'd}(-x)]\mathrm{Tr}[\gamma_\nu\gamma_5 S_d^{bb'}(x)\gamma_5\gamma_\sigma\tilde{S}_u^{aa'}(x)]\}+\sin\theta\cos\theta S_c^{cc'}(x)\\&\times\{\mathrm{Tr}[\gamma_\mu\gamma_5 S_d^{db'}(x)\gamma_5\gamma_\sigma\tilde{S}_u^{aa'}(x)\gamma_\nu S_u^{bd'}(x)\gamma_\rho S_c^{d'd}(-x)]-\mathrm{Tr}[\gamma_\mu\gamma_5 S_d^{db'}(x)\gamma_5\gamma_\sigma\tilde{S}_u^{ba'}(x)\gamma_\nu\\&\times S_u^{ad'}(x)\gamma_\rho S_c^{d'd}(-x)]+\mathrm{Tr}[\gamma_\mu S_u^{db'}(x)\gamma_\sigma\tilde{S}_u^{aa'}(x)\gamma_\nu\gamma_5 S_d^{bd'}(x)\gamma_5\gamma_\rho S_c^{d'd}(-x)]-\mathrm{Tr}[\gamma_\mu S_u^{da'}(x)\gamma_\sigma\\&\times\tilde{S}_u^{ab'}(x)\gamma_\nu\gamma_5 S_d^{bd'}(x)\gamma_5\gamma_\rho S_c^{d'd}(-x)]\}+(\mu\leftrightarrow\nu)+(\rho\leftrightarrow\sigma)+(\mu\leftrightarrow\nu,\rho\leftrightarrow\sigma)\}.\end{aligned} \tag{21}$$

In this step, we have used the expressions for the heavy and light propagators and transformed the calculations to momentum space. By using the dispersion relation, we find the imaginary part of the correlation function to extract the corresponding spectral density of the $\frac{5}{2}$ state. Omitting the details of very lengthy calculations, we show the spectral density $\rho^{\mathrm{QCD}}_{\frac{5}{2}}(s)$ defining the state under consideration (for instance, for the $\frac{g_{\mu\rho}g_{\nu\sigma}+g_{\mu\sigma}g_{\nu\rho}}{2}$ structure) in the Appendix.

By matching the coefficients of the selected structures from both sides, we find the sum rules

$$\begin{aligned}m_+\lambda_+^2 e^{-m_+^2/M^2}-m_-\lambda_-^2 e^{-m_-^2/M^2}&=\Pi_2^1,\\ \lambda_+^2 e^{-m_+^2/M^2}+\lambda_-^2 e^{-m_-^2/M^2}&=\Pi_2^2,\end{aligned} \tag{22}$$

where Π_2^1 and Π_2^2 correspond to the coefficients of the structures $\frac{g_{\mu\rho}g_{\nu\sigma}+g_{\mu\sigma}g_{\nu\rho}}{2}$ and $\not{p}\frac{g_{\mu\rho}g_{\nu\sigma}+g_{\mu\sigma}g_{\nu\rho}}{2}$ on the OPE side, respectively. The four unknowns m_+ and m_-, λ_+, and λ_- can be obtained using the above two sum rules and two extra sum rules obtained via applying the derivatives with respect to $\frac{1}{M^2}$ to both sides.

IV. NUMERICAL RESULTS

The QCD sum rules for the physical quantities under consideration contain certain parameters, such as quark, gluon, and mixed condensates, and the mass of the c quark. We collect their values in Table I. We set the light quark masses m_u and m_d to zero. In addition to the above parameters, there are two auxiliary parameters that should be fixed before going further, namely, the continuum threshold s_0 and Borel parameter M^2. We find their working windows such that the physical quantities under consideration are roughly independent of these parameters. To determine the working interval of the Borel parameter one needs to consider two criteria: the convergence of the series of the OPE, and an adequate suppression of the higher states and continuum. Considering these criteria in the analysis leads to the intervals

$$4\ \mathrm{GeV}^2\le M^2\le 7\ \mathrm{GeV}^2. \tag{23}$$

To determine the working regions of the continuum threshold, we impose the conditions of pole dominance and OPE convergence. This leads to the interval

$$22\ \mathrm{GeV}^2\le s_0\le 24\ \mathrm{GeV}^2 \tag{24}$$

for $\frac{3}{2}$ states with both parities, and

$$22.5\ \mathrm{GeV}^2\le s_0\le 24.5\ \mathrm{GeV}^2 \tag{25}$$

for $\frac{5}{2}$ states with negative and positive parities.

As examples, the variations of the mass and residue of the hidden-charm pentaquark with $J=\frac{5}{2}$ and positive parity with respect to the Borel parameter (continuum threshold) at different fixed values of the continuum threshold (Borel parameter) are depicted in Figs. 1 and 2. From these figures, we see that the corresponding mass and residue demonstrate an overall weak dependence on the variations of the Borel mass parameter and continuum threshold in their working intervals.

Having determined the suitable intervals for the parameters s_0 and M^2, the next stage is to use them in the determination of the mass and residue of the considered pentaquarks. The average values obtained from our calculations are presented in Table II. The errors in the given results arise from the input parameters and the uncertainties

TABLE I. Some input parameters used in the calculations.

Parameters	Values
m_c	(1.27 ± 0.03) GeV
$\langle\bar{q}q\rangle$	$(-0.24\pm0.01)^3$ GeV^3
m_0^2	(0.8 ± 0.1) GeV^2
$\langle\bar{q}g_s\sigma Gq\rangle$	$m_0^2\langle\bar{q}q\rangle$
$\langle\frac{\alpha_s G^2}{\pi}\rangle$	(0.012 ± 0.004) GeV^4

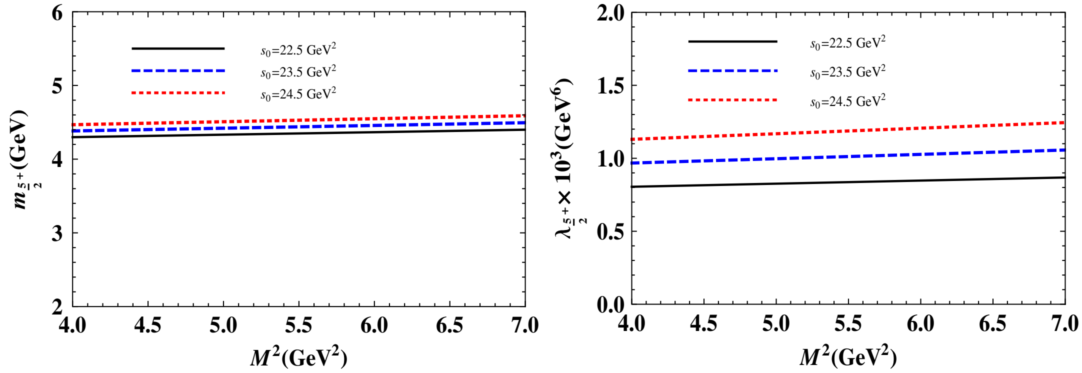

FIG. 1. Left: The mass of the pentaquark with $J^P = \frac{5}{2}^+$ as a function of the Borel parameter M^2 at different fixed values of the continuum threshold. Right: The residue of the pentaquark with $J^P = \frac{5}{2}^+$ as a function of the Borel parameter M^2 at different fixed values of the continuum threshold.

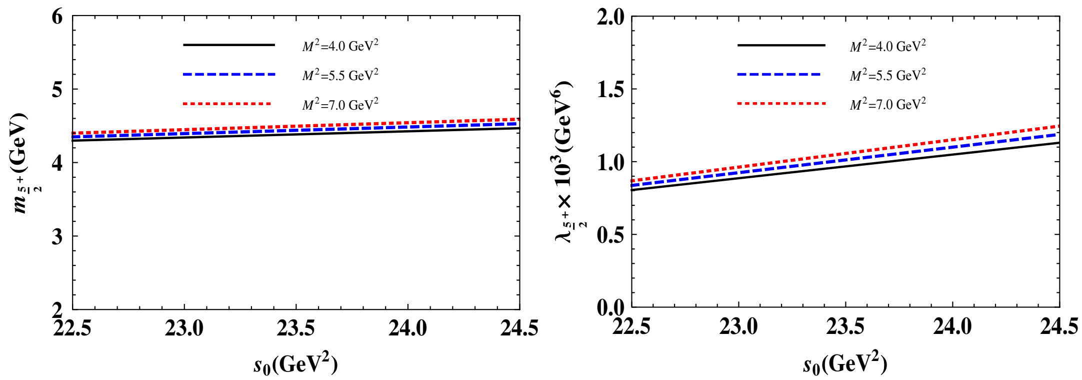

FIG. 2. Left: The mass of the pentaquark with $J^P = \frac{5}{2}^+$ as a function of s_0 at different fixed values of the Borel parameter. Right: The residue of the pentaquark with $J^P = \frac{5}{2}^+$ as a function of s_0 at different fixed values of the Borel parameter.

coming from the determination of the working windows of the auxiliary parameters s_0 and M^2. Comparison of the results on the masses with the experimental data of the LHCb Collaboration, i.e., $m_{P_c^+(4380)} = 4380 \pm 8 \pm 29$ MeV and $m_{P_c^+(4450)} = 4449.8 \pm 1.7 \pm 2.5$ MeV [68], reveals that the $\frac{3}{2}^-$ state can be assigned to the $P_c^+(4380)$ observed at LHCb. Our prediction for the mass of the $\frac{5}{2}^+$ state is also consistent with the experimental data on the mass of $P_c^+(4450)$. Our results on the masses of the $\frac{3}{2}^-$ and $\frac{5}{2}^+$ states are also in a good agreement with the results of the theoretical works [75,85]. Our predictions for the residues of the $\frac{3}{2}^-$ and $\frac{5}{2}^+$ states, within the errors, are also comparable with the predictions of Ref. [85], where diquark-diquark-antiquark type interpolating currents were used to calculate the mass and residue of the pentaquark states with $J^P = \frac{3}{2}^-$ and $J^P = \frac{5}{2}^+$. Here we note that by using the experimental data for the mass of $\frac{3}{2}^-$ and $\frac{5}{2}^+$ states in our sum rules, we find the residues $\lambda_{\frac{3}{2}^-} = (0.98 \pm 0.05) \times 10^{-3}$ GeV^6 and $\lambda_{\frac{5}{2}^+} = (1.02 \pm 0.23) \times 10^{-3}$ GeV^6, which are very close to the related values in Table II, and we do not see considerable differences. Our results for the masses of the opposite-parity states, i.e., $\frac{3}{2}^+$ and $\frac{5}{2}^-$, as well as our predictions for the residues may be verified via different approaches.

TABLE II. The results of QCD sum rules calculations for the mass and residue of the pentaquark states.

J^P	m (GeV)	λ (GeV^6)
$\frac{3}{2}^+$	4.24 ± 0.16	$(0.59 \pm 0.07) \times 10^{-3}$
$\frac{3}{2}^-$	4.30 ± 0.10	$(0.94 \pm 0.05) \times 10^{-3}$
$\frac{5}{2}^+$	4.44 ± 0.15	$(1.01 \pm 0.23) \times 10^{-3}$
$\frac{5}{2}^-$	4.20 ± 0.15	$(0.51 \pm 0.09) \times 10^{-3}$

V. SUMMARY AND OUTLOOK

We performed QCD sum rules analyses to compute the mass and residue of the hidden-charm pentaquark states with $J = \frac{3}{2}$ and $J = \frac{5}{2}$ and both positive and negative parities. We adopted interpolating currents in an anti-charmed meson-charmed baryon molecular form of $\bar{D}^*\Sigma_c$ for states with $J = \frac{3}{2}$ and a mixed anticharmed meson-charmed baryon molecular current of $[\bar{D}\Sigma_c^*]$ and $[\bar{D}^*\Lambda_c]$ for the states with $J = \frac{5}{2}$. By fixing the auxiliary parameters entering into the calculations, we obtained the values of the masses and residues for all of the considered states. Our predictions for the masses of the $J^P = \frac{3}{2}^-$ and $J^P = \frac{5}{2}^+$ states are consistent with the experimental data of the LHCb Collaboration for the masses of the $P_c^+(4380)$ and $P_c^+(4450)$ states, respectively. Our results are also consistent with the predictions of the theoretical works [75,85] on the masses. As we previously said, the authors of Ref. [75] used the same picture and method as in the present work, but they only predicted the masses of the $J^P = \frac{3}{2}^-$ and $J^P = \frac{5}{2}^+$ states. However, in Ref. [85] a different quark organization was used to also predict the masses of the $J^P = \frac{3}{2}^-$ and $J^P = \frac{5}{2}^+$ states.

Using the currents adopted in the present study, we also derived the values of the residues for the considered states with both parities. Our results for the residues of $J^P = \frac{3}{2}^-$ and $J^P = \frac{5}{2}^+$ states are comparable to those of Ref. [85] within the errors. The residues can be used as the main inputs in the analyses of the electromagnetic properties and strong decays of the pentaquark states $P_c^+(4380)$ and $P_c^+(4450)$. Such analyses are needed and would be very important in the determination of the internal structures, geometric shapes, charge distribution, and multipole moments of these states and the strong interactions inside them. In our future works, we aim to analyze the strong, electromagnetic, and weak decay channels of the pentaquark states considered in the present study to calculate the corresponding strong coupling constants as well as the widths of these states. A comparison of the theoretical results on the many parameters of the pentaquarks with present and future experimental data would help us better understand their quark organizations, and will provide us with useful knowledge on the quantum chromodynamics of the exotic baryons.

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APPENDIX: THE TWO-POINT SPECTRAL DENSITIES

In this appendix we present the results for the two-point spectral densities obtained from QCD sum rules calculations. As examples, we only present those spectral densities corresponding to the structures $g_{\mu\nu}$ and $\frac{g_{\mu\rho}g_{\nu\sigma}+g_{\mu\sigma}g_{\nu\rho}}{2}$ for the states with $J=\frac{3}{2}$ and $J=\frac{5}{2}$, respectively. They are obtained as

$$\rho_i^{\mathrm{QCD}}(s) = \rho_i^{\mathrm{pert}}(s) + \sum_{k=3}^{6} \rho_{i,k}(s), \tag{A1}$$

with i being $\frac{3}{2}$ or $\frac{5}{2}$. In Eq. (A1), $\rho_{i,k}(s)$ denotes the nonperturbative contributions to spectral densities $\rho_i^{\mathrm{QCD}}(s)$. The explicit expressions for $\rho_i^{\mathrm{pert}}(s)$ and $\rho_{i,k}(s)$ are obtained in terms of the integrals of the Feynman parameters x and y as

$$\begin{aligned}
\rho_{\frac{3}{2}}^{\mathrm{pert}}(s) &= \frac{m_c}{5\times 2^{15}\pi^8}\int_0^1 dx \int_0^{1-x} dy \frac{(6hsxy - m_c^2 r)(hsxy - m_c^2 r)^4}{h^3 t^8}\Theta[L],\\
\rho_{\frac{3}{2},3}(s) &= \frac{m_c^2}{2^9\pi^6}\langle\bar{d}d\rangle \int_0^1 dx \int_0^{1-x} dy \frac{(hsxy - m_c^2 t(x+y))^3}{h^2 t^5}\Theta[L],\\
\rho_{\frac{3}{2},4}(s) &= \langle\frac{\alpha_s}{\pi}G^2\rangle \frac{1}{3^2\times 2^{15}\pi^6}\int_0^1 dx \int_0^{1-x} dy \frac{[hsxy - m_c^2 t(x+y)]}{h^3 t^7}\{12hm_c sxy^3(h^2sx^3 + m_c^2t^2y)\\
&\quad - 6m_c y(m_c^2 t(x+y) - hsxy)[2h^2sx^3y + m_c^2t^2y^2 + hsx(34x^4 + 2y(y-1)^2(16y-9)\\
&\quad + x^3(105y-88) + x^2(72-209y+137y^2) + 2x(50y^3-102y^2+61y-9))]\\
&\quad + m_c(hsxy - m_c^2t(x+y))^2[6h^2y^2 + (68x^4 + 3y(y-1)^2(17y-12)\\
&\quad + x^3(197y-176) + 8x^2(18-49y+31y^2) + 3x(58y^3-123y^2+77y-12))]\}\Theta[L],\\
\rho_{\frac{3}{2},5}(s) &= \frac{3m_c^2}{2^{10}\pi^6}m_0^2\langle\bar{d}d\rangle \int_0^1 dx \int_0^{1-x} dy \frac{(hsxy - m_c^2 t(x+y))^2}{ht^4}\Theta[L],\\
\rho_{\frac{3}{2},6}(s) &= \frac{m_c}{3^3\times 2^8\pi^6}(2g_s^2\langle\bar{u}u\rangle^2 + g_s^2\langle\bar{d}d\rangle^2)\int_0^1 dx \int_0^{1-x} dy \frac{x(m_c^2r - 3hsxy)(m_c^2r - hsxy)}{t^5}\Theta[L]\\
&\quad + \frac{m_c}{2^4\pi^4}\langle\bar{u}u\rangle^2 \int_0^1 dx \int_0^{1-x} dy \frac{x(m_c^2r - 3hsxy)(m_c^2r - hsxy)}{t^5}\Theta[L],\\
\rho_{\frac{5}{2}}^{\mathrm{pert}}(s) &= \frac{m_c(5\cos^2\theta - 4\cos\theta\sin\theta + 12\sin^2\theta)}{2^{17}\times 3\times 5^2\pi^8}\int_0^1 dx \int_0^{1-x} dy \frac{x(5x^2 + x(y+5z) + 5zy)}{h^3 t^9}\\
&\quad \times (sxyh - m_c^2 r)^4 (m_c^2 r - 6sxyh)\Theta[L],
\end{aligned}$$

$$\rho_{\frac{5}{2},3}(s) = -\frac{m_c^2(\cos^2\theta(\langle\bar{d}d\rangle + 4\langle\bar{u}u\rangle) + 4\cos\theta\sin\theta(\langle\bar{d}d\rangle - 2\langle\bar{u}u\rangle))}{2^{11}\times 3^2\times\pi^6}\int_0^1 dx\int_0^{1-x}dy\frac{(3x^2+x(y+3z)+3yz)}{h^2t^6}$$
$$\times(m_c^2 r - sxyh)^3\Theta[L],$$

$$\rho_{\frac{5}{2},4}(s) = -\langle\frac{\alpha_s}{\pi}G^2\rangle\frac{m_c}{2^{17}\times 3^3\times 5\pi^6}\int_0^1 dx\int_0^{1-x}dy\frac{x(m_c^2 r - sxyh)}{h^3t^8}\{4\cos\theta\sin\theta(4s^2x^2y^2h^2(20x^6+100z^3y^3+4x^5(31y+10z)$$
$$+5xz^2y^2(56z+27y)+40x^3y(13-33y+20y^2)+x^4(20-504y+505y^2)+5x^2y\times(219y-337y^2+154y^3-36))$$
$$+m_c^4t^2(20x^8+10z^2y^5(22z+3y)+x^7(40z+314y)+x^6(20-1004y)+1639y^2+2x^5y(475-2192y+1956y^2)$$
$$+3xy^4(1095y-1232y^2+457y^3-320)+x^2y^3(6525y-8537y^2+3572y^3-1560)$$
$$+x^3y^2(-1120+6865y-11362y^2+5623y^3)+x^4y(-300+3865y-9221y^2+5779y^3))$$
$$-m_c^2sxy(100x^{10}+10z^4y^5(62z+9y)+10x^9(40y+93y)+xz^3y^4(2880-7505y+4733y^2)$$
$$+x^8(600-6220y+6633y^2)+x^2z^2y^3(23645y-4920-34196y^2+15489y^3)$$
$$+x^7(11700y-400-29884y^2+18857y^3)+x^3z^2y^2(-3680+29025y-56766y^2+31998y^3)$$
$$+2x^6(50-5540y+26777y^2-39055y^3+17777y^4)+2x^5y(2645-23834y+63097y^2-65643y^3+23735y^4)$$
$$+x^4y(-1020+21045y-98406y^2+183623y^3-151144y^4+45902y^5)))$$
$$+24\sin^2\theta(m_c^4t^2(20x^8+x^7(83z-17)-15z^2y^4(3y^2-2)+x^3y(120-750y+895y^2+256y^3-524y^4)$$
$$+x^6(170-398y+113y^2)-x^5(120-665y+623y^2+36y^3)+x^2y^2(180-630y+345y^2+541y^3-436y^4)$$
$$+x^4(30-470y+1080y^2-347y^3-332y^4)+xy^3(120-290y+20y^2+353y^3-203y^4))$$
$$+4s^2x^2y^2h^2(20x^6-30z^3y^2+4x^5(22z-3)+10xz^2y(6-11y+y^2)+x^4(170-318y+145y^2)$$
$$+5x^3(-24+86y-89y^2+27y^3)+10x^2(3-26y+50y^2-33y^3+6y^4))$$
$$-m_c^2sxy(100x^{10}+35x^9(19z-1)-15z^4y^4(8y+5y^2-10)+6x^8(325-690y+336y^2)$$
$$-2xz^3y^3(300-740y+300y^2+167y^3)+2x^7(-1400+5225y-5704y^2+1837y^3)$$
$$-x^3z^2y(5580y-11835y^2+6772y^3+319y^4-600)-x^2z^2y^2(4620y-6505y^2 2152y^3+642y^4$$
$$-900)+x^6(2200-13760y+26198y^2-19000y^3+4353y^4)+x^5(10005y-31216y^2+39533y^3-900$$
$$-20672y^4+3250y^5)+2x^4(75-1910y+10145y^2-20991y^3+19413y^4-7324y^5+592y^6)))$$
$$+5\cos^2\theta(4s^2x^2y^2h^2(52x^6+4z^3y^2(5z-13)+4x^5(61z-1)+xz^2y(144-320y+107y^2)+x^4$$
$$\times(412-864y+449y^2)-4x^3(72-284y+333y^2-121y^3)+x^2(72-660y+1419y^2-1129y^3+298y^4))$$
$$+m_c^4t^2(52x^8+x^7(270z+22)-2z^2y^4(22y+29y^2)+x^6(412-1156y+599y^2-36)$$
$$+2x^5(893y-1186y^2+348y^3-144)+x^2y^2(432-1824y+2133y^2-409y^3-332y^4)$$
$$+x^3y(288-2024y+3521y^2-1658y^3-133y^4)-3xy^3(296y-235y^2-36y^3+71y^4-96)$$
$$+x^4(72-1188y+3365y^2-2677y^3+359y^4))-m_c^2sxy(260x^{10}+2x^9(-880+931y)$$
$$-2z^4y^4(206y+19y^2-180)+5x^8(960-2236y+1233y^2)+xz^3y^3(4128y-2941y^2-1440$$
$$+145y^3)+x^7(27420y-33356y^2+12589y^3-6800)+x^2z^2y^2(2160-12072y+20341y^2-12004y^3$$
$$+1557y^4)+x^3z^2y(1440-14128y+34209y^2-27606y^3+5634y^4)+2x^6(2650-17620y$$
$$+36793y^2-30611y^3+8779y^4)+2x^5(12535y-42226y^2+60059y^3-37935y^4+8647y^5-1080)$$
$$+x^4(360-9372y+52905y^2-120438y^3+129907y^4-65384y^5+12022y^6)))\}\Theta[L],$$

$$
\begin{aligned}
\rho_{\frac{5}{2},5}(s) &= \frac{(\cos\theta - 2\sin\theta)m_c^2 m_0^2(6\sin\theta\langle\bar{d}d\rangle + \cos\theta(\langle\bar{d}d\rangle + 4\langle\bar{u}u\rangle))}{2^{13}\pi^6}\int_0^1 dx\int_0^{1-x} dy\frac{(sxyh - m_c^2 t(x+y))^2}{ht^5}\\
&\quad\times(2x^2 + x(3z+1) + 2yz)\Theta[L],\\
\rho_{\frac{5}{2},6}(s) &= \int_0^1 dx\int_0^{1-x} dy\bigg\{(2g_s^2\langle\bar{u}u\rangle^2 + g_s^2\langle\bar{d}d\rangle^2)\frac{m_c(5\cos^2\theta - 4\cos\theta\sin\theta + 12\sin^2\theta)}{2^{11}\times 3^4\pi^6}(m_c^2 t(x+y) - 3shxy)\\
&\quad\times(m_c^2 t(x+y) - shxy)(2xyz + x^2(2x+3y-2)) - \frac{m_c(\cos\theta - 2\sin\theta)}{3\times 2^8\pi^4 t^5}[\langle\bar{u}u\rangle^2(\cos\theta + 6\sin\theta) + 4\langle\bar{u}u\rangle\langle\bar{d}d\rangle\cos\theta]\\
&\quad\times(m_c^2 tx(x+y) - 3shx^2 y)(m_c^2 t(x+y) - shxy)\bigg\}\Theta[L],
\end{aligned}
\tag{A2}
$$

where $\Theta[L]$ is the usual unit-step function and we have used the shorthand notations

$$
\begin{aligned}
z &= y - 1,\\
h &= x + y - 1,\\
t &= x^2 + (x+y)(y-1),\\
r &= x^3 + x^2(2y-1) + y(y-1)(2x+y),\\
L &= \frac{z}{t^2}[sxyh - m_c^2(x+y)t].
\end{aligned}
\tag{A3}
$$

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