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Decay properties of *P*-wave charmed baryons from light-cone QCD sum rules

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We study decay properties of the *P*-wave charmed baryons using the method of light-cone QCD sum rules, including the *S*-wave decays of the flavor $\bar{\mathbf{3}}_F$ *P*-wave charmed baryons into ground-state charmed baryons accompanied by a pseudoscalar meson (π or K) or a vector meson (ρ or K^*), and the *S*-wave decays of the flavor $\mathbf{6}_F$ *P*-wave charmed baryons into ground-state charmed baryons accompanied by a pseudoscalar meson (π or K). We study both two-body and three-body decays which are kinematically allowed. We find two mixing solutions from internal ρ- and λ-mode excitations, which can well describe both masses and decay properties of the $\Lambda_c(2595)$, $\Lambda_c(2625)$, $\Xi_c(2790)$ and $\Xi_c(2815)$. We also discuss the possible interpretations of *P*-wave charmed baryons for the $\Sigma_c(2800)$, $\Xi_c(2930)$, $\Xi_c(2980)$, and the recently observed $\Omega_c(3000)$, $\Omega_c(3050)$, $\Omega_c(3066)$, $\Omega_c(3090)$, and $\Omega_c(3119)$.

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I. INTRODUCTION

Recently, the LHCb Collaboration observed five excited Ω_c states in the $\Xi_c^+ K^-$ mass spectrum [1], i.e., the $\Omega_c(3000)^0$, $\Omega_c(3050)^0$, $\Omega_c(3066)^0$, $\Omega_c(3090)^0$, and $\Omega_c(3119)^0$. Their masses and widths were measured to be

$$\Omega_c(3000)^0\colon\ M = 3000.4 \pm 0.2 \pm 0.1^{+0.3}_{-0.5}\ \text{MeV},\quad \Gamma = 4.5 \pm 0.6 \pm 0.3\ \text{MeV},$$

$$\Omega_c(3050)^0\colon\ M = 3050.2 \pm 0.1 \pm 0.1^{+0.3}_{-0.5}\ \text{MeV},\quad \Gamma = 0.8 \pm 0.2 \pm 0.1\ \text{MeV},$$

$$\Omega_c(3066)^0\colon\ M = 3065.6 \pm 0.1 \pm 0.3^{+0.3}_{-0.5}\ \text{MeV},\quad \Gamma = 3.5 \pm 0.4 \pm 0.2\ \text{MeV},$$

$$\Omega_c(3090)^0\colon\ M = 3090.2 \pm 0.3 \pm 0.5^{+0.3}_{-0.5}\ \text{MeV},\quad \Gamma = 8.7 \pm 1.0 \pm 0.8\ \text{MeV},$$

$$\Omega_c(3119)^0\colon\ M = 3119.1 \pm 0.3 \pm 0.9^{+0.3}_{-0.5}\ \text{MeV},\quad \Gamma = 1.1 \pm 0.8 \pm 0.4\ \text{MeV}.$$

These excited Ω_c states are good *P*-wave charmed baryon candidates. Besides them, the $\Lambda_c(2595)$ ($J^P = 1/2^-$), $\Lambda_c(2625)$ ($J^P = 3/2^-$), $\Xi_c(2790)$ ($J^P = 1/2^-$) and $\Xi_c(2815)$ ($J^P = 3/2^-$) can be well interpreted as the *P*-wave charmed baryons completing two flavor $\bar{\mathbf{3}}_F$ multiplets of $J^P = 1/2^-$ and $3/2^-$ [2–6]; the $\Sigma_c(2800)$ ($J^P = ?^?$), $\Xi_c(2930)$ ($J^P = ?^?$) and $\Xi_c(2980)$ ($J^P = ?^?$) are also *P*-wave charmed baryon candidates of the flavor $\mathbf{6}_F$ [7–13].

These charmed baryons are interesting in a theoretical point of view, and many phenomenological methods/models were proposed to study them [14], such as various quark models [15–20], various dynamical models [21–26], the hyperfine interaction [27,28], and the Lattice QCD [29–31], etc. [32–36]. Their productions and decay properties were studied in Refs. [37–43]. See reviews in Refs. [14,44–49] for their recent progress.

We have also systematically studied the mass spectra of these excited heavy baryons [50–53], using the method of QCD sum rules [54,55] in the framework of heavy-quark effective theory (HQET) [56–58]. The HQET works well in the bottom sector but not so good in the charm sector. Hence, in Refs. [50–53] we have taken into account the $\mathcal{O}(1/m_Q)$ corrections (m_Q is the heavy-quark mass), and did find them to be non-negligible. In Ref. [59] it was also found that the finite quark mass corrections to the form

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factors and the rates of semileptonic transitions are important for heavy-to-light (charm-to-strange) transitions and not negligible for heavy-to-heavy (bottom-to-charm) transitions. In a different framework based on the Dyson-Schwinger equation [60] it was similarly concluded that the heavy-quark expansion is accurate for the bottom quark while it provides a poor approximation for the charm quark. More studies on heavy mesons and baryons using this scheme can be found in Refs. [61–80], and others using the method of QCD sum rules but not in HQET can be found in Refs. [81–85].

Based on the heavy-quark effective theory, we can classify the P-wave charmed baryons into eight charmed baryon multiplets, including four of the flavor $\bar{\mathbf{3}}_F$ ($[\bar{\mathbf{3}}_F, 0, 1, \rho]$, $[\bar{\mathbf{3}}_F, 1, 1, \rho]$, $[\bar{\mathbf{3}}_F, 2, 1, \rho]$ and $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$) and four of the flavor $\mathbf{6}_F$ ($[\mathbf{6}_F, 1, 0, \rho]$, $[\mathbf{6}_F, 0, 1, \lambda]$, $[\mathbf{6}_F, 1, 1, \lambda]$ and $[\mathbf{6}_F, 2, 1, \lambda]$). See Sec. II for the explanations of these symbols. For each set of multiples, the first one of $j_l = 0$ forms a heavy-quark singlet, while the other three of $j_l = 1$ form heavy-quark doublets. These multiplets provide lots of P-wave charmed baryons. For example, there can be as many as seven P-wave Ω_c states theoretically, including three $J^P = 1/2^-$, three $J^P = 3/2^-$, and one $J^P = 5/2^-$ states. Previously, it seems impossible to observe all these P-wave Ω_c states experimentally. However, the recent LHCb experiment observed as many as five excited Ω_c states at the same time [1] [actually this number is six if the $\Omega_c(3188)^0$ is included], suggesting that "an ideal platform to study these structures (the gross, fine and hyperfine structures of the strong interaction) is the heavy hadrons containing one charm or bottom quark [14]".

In this paper we shall further use the method of light-cone QCD sum rules [86–105] to study the decay properties of these P-wave charmed baryons, the method of which is also based on the HQET. We shall only study their S-wave decay properties, and note that their D-wave decays can also happen but these contributions may not be significant. Our sum rule calculations will be done separately for the above eight charmed baryon multiplets. However, because the heavy-quark symmetry is not perfect, the physical states are probably mixed states containing various components with different inner quantum numbers. Hence, we shall also discuss the mixing of these charmed baryon multiplets in this paper, where we shall find that the decay properties of the P-wave charmed baryons are quite sensitive to this. We refer to Refs. [106–113] for earlier studies using the method of light-cone QCD sum rules in the framework of HQET. The decay properties of heavy baryons have also been studied using many other methods, such as in Ref. [114] where the one-pion transitions between heavy baryons were investigated in the constituent quark model based on the heavy-quark symmetry.

This paper is organized as follows. First in Sec. II we reevaluated and listed the input parameters for the present study, including the masses, decay constants, and interpolating fields of the ground-state and P-wave charmed baryons. Then in Sec. III we investigate the decay properties of the flavor $\bar{\mathbf{3}}_F$ P-wave charmed baryons within the method of light-cone QCD sum rules, and we shall study their S-wave decays into ground-state charmed baryons accompanied by a pseudoscalar meson (π or K) or a vector meson (ρ or K^*). Using the same procedures, in Sec. IV we investigate the decay properties of the flavor $\mathbf{6}_F$ P-wave charmed baryons, and we shall study their S-wave decays into ground-state charmed baryons accompanied by a pseudoscalar meson (π or K). The results are summarized and discussed in Sec. V.

II. GROUND-STATE AND P-WAVE CHARMED BARYONS

To study the decay properties of charmed baryons in the method of light-cone QCD sum rules, we need some parameters of these states, such as their masses ($m_{\mathcal{B}}$), interpolating fields ($J^{\alpha_1\cdots\alpha_{j-1/2}}$), decay constants ($f_{\mathcal{B}}$), and threshold values (ω_c), etc. These parameters are defined below, while their values are listed separately in the following three subsections for both ground-state charmed baryons, P-wave charmed baryons of flavor $\bar{\mathbf{3}}_F$ and P-wave charmed baryons of flavor $\mathbf{6}_F$.

The coupling of the interpolating field $J^{\alpha_1\cdots\alpha_{j-1/2}}(x)$ to the charmed baryon $\mathcal{B}$ of spin j is defined to be

$$\langle 0|J^{\alpha_1\cdots\alpha_{j-1/2}}(x)|\mathcal{B}\rangle = f_{\mathcal{B}} u^{\alpha_1\cdots\alpha_{j-1/2}}(x), \tag{1}$$

where $f_{\mathcal{B}}$ is the decay constant, and $u^{\alpha_1\cdots\alpha_j}$ is the relevant spinor.

Then the two-point correlation function at the hadron level can be written as

$$\begin{aligned}\Pi^{\alpha_1\cdots\alpha_{j-1/2},\beta_1\cdots\beta_{j-1/2}}(\omega) &= i\int d^4x e^{ikx}\langle 0|T[J^{\alpha_1\cdots\alpha_{j-1/2}}(x)\bar{J}^{\beta_1\cdots\beta_{j-1/2}}(0)]|0\rangle \\ &= \mathbb{S}[g_t^{\alpha_1\beta_1}\dots g_t^{\alpha_{j-1/2}\beta_{j-1/2}}]\times\frac{1+\slashed{v}}{2}\times\Pi_{F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}(\omega), \\ &= \mathbb{S}[g_t^{\alpha_1\beta_1}\dots g_t^{\alpha_{j-1/2}\beta_{j-1/2}}]\times\frac{1+\slashed{v}}{2}\times\left(\frac{f_{\mathcal{B}}^2}{\bar{\Lambda}_{\mathcal{B}}-\omega}+\text{higher states}\right).\end{aligned} \tag{2}$$

Here $\mathbb{S}[\cdots]$ denotes symmetrization and subtracting the trace terms in the sets $(\alpha_1 \ldots \alpha_{j-1/2})$ and $(\beta_1 \ldots \beta_{j-1/2})$; ω is the external off-shell energy $\omega = v \cdot k$; $\bar{\Lambda}_\mathcal{B}$ is defined to be

$$\bar{\Lambda}_\mathcal{B} \equiv \lim_{m_Q \to \infty} (m_\mathcal{B} - m_Q), \quad (3)$$

where m_Q is the heavy-quark mass, and $m_\mathcal{B}$ is the mass of the lowest-lying charmed baryon state coupling with $J^{\alpha_1 \ldots \alpha_{j-1/2}}(x)$.

At the quark-gluon level, one can calculate the two-point correlation function, Eq. (2), using the method of QCD operator product expansion (OPE). By assuming the contribution from the continuum states (higher states) can be approximated well by the OPE spectral density above a threshold value ω_c, one can arrive at the mass sum rule relation which can be used to calculate masses and decay constants of charmed baryons. See Refs. [50–52] for detailed discussions, and their results are reevaluated and listed below.

A. Ground-state charmed baryons

The masses and decay constants of the *S*-wave bottom baryons have been systematically investigated in Ref. [50] using the method of QCD sum rules in HQET. We replace the bottom quark by the charm quark, reevaluate their parameters, and shortly summarize the results here. For completeness, we first list masses of the ground-state charmed baryons from PDG [2]:

$$\begin{aligned}
\Lambda_c(1/2^+)&: m = 2286.46 \text{ MeV},\\
\Xi_c(1/2^+)&: m = 2469.34 \text{ MeV},\\
\Sigma_c(1/2^+)&: m = 2453.54 \text{ MeV},\\
&\quad \Gamma = 1.86 \text{ MeV},\\
&\quad g_{\Sigma_c\Lambda_c\pi} = 3.94 \text{ GeV}^{-1},\\
\Xi'_c(1/2^+)&: m = 2576.8 \text{ MeV},\\
\Omega_c(1/2^+)&: m = 2695.2 \text{ MeV},\\
\Sigma^*_c(3/2^+)&: m = 2518.1 \text{ MeV},\\
&\quad \Gamma = 14.7 \text{ MeV},\\
&\quad g_{\Sigma^*_c\Lambda_c\pi} = 7.39 \text{ GeV}^{-1},\\
\Xi^*_c(3/2^+)&: m = 2645.9 \text{ MeV},\\
&\quad \Gamma \le 4.3 \text{ MeV},\\
&\quad g_{\Xi^*_c\Xi_c\pi} < 4.90 \text{ GeV}^{-1},\\
\Omega^*_c(3/2^+)&: m = 2765.9 \text{ MeV},
\end{aligned} \quad (4)$$

whose values have been averaged over isospin. We also list masses of the ground-state pseudoscalar and vector mesons [2]:

$$\begin{aligned}
\pi(0^-)&: m = 138.04 \text{ MeV},\\
K(0^-)&: m = 495.65 \text{ MeV},\\
\rho(1^-)&: m = 775.21 \text{ MeV},\\
&\quad \Gamma = 148.2 \text{ MeV},\\
&\quad g_{\rho\pi\pi} = 5.94,\\
K^*(1^-)&: m = 893.57 \text{ MeV},\\
&\quad \Gamma = 49.1 \text{ MeV},\\
&\quad g_{K^*K\pi} = 6.40.
\end{aligned} \quad (5)$$

In these equations there are five coupling constants, $g_{\Sigma_c\Lambda_c\pi}$, $g_{\Sigma^*_c\Lambda_c\pi}$, $g_{\Xi^*_c\Xi_c\pi}$, $g_{\rho\pi\pi}$, and $g_{K^*K\pi}$, which are evaluated using the experimental decay widths of the $\Sigma_c(1/2^+)$, $\Sigma^*_c(3/2^+)$, $\Xi^*_c(3/2^+)$, $\rho(1^-)$ and $K^*(1^-)$ [2] through the following Lagrangians:

$$\begin{aligned}
\mathcal{L}_{\Sigma_c\Lambda_c\pi} &= g_{\Sigma_c\Lambda_c\pi}\bar{\Sigma}_c^+\gamma_\mu\gamma_5\Lambda_c^+\partial^\mu\pi^0 + \cdots,\\
\mathcal{L}_{\Sigma^*_c\Lambda_c\pi} &= g_{\Sigma^*_c\Lambda_c\pi}\bar{\Sigma}_{c\mu}^{*+}\Lambda_c^+\partial^\mu\pi^0 + \cdots,\\
\mathcal{L}_{\Xi^*_c\Xi_c\pi} &= g_{\Xi^*_c\Xi_c\pi}\bar{\Xi}_{c\mu}^{*+}\Xi_c^+\partial^\mu\pi^0 + \cdots,\\
\mathcal{L}_{\rho\pi\pi} &= g_{\rho\pi\pi} \times (\rho^0_\mu\pi^+\partial^\mu\pi^- + \rho^0_\mu\pi^-\partial^\mu\pi^+) + \cdots,\\
\mathcal{L}_{K^*K\pi} &= g_{K^*K\pi}K_\mu^{*+}K^-\partial^\mu\pi^0 + \cdots,
\end{aligned} \quad (6)$$

where $\cdots$ contain their isospin partners as well as their hermitian conjugate.

The ground-state charmed baryons have been systematically investigated in Ref. [50], which compose one flavor $\bar{\mathbf{3}}_F$ multiplet of $J^P = 1/2^+$, one flavor $\mathbf{6}_F$ multiplet of $J^P = 1/2^+$, and one flavor $\mathbf{6}_F$ multiplet of $J^P = 3/2^+$. The two flavor $\mathbf{6}_F$ multiplets of $J^P = 1/2^+$ and $3/2^+$ compose one charmed baryon multiplet where the spin of the two light quarks is $s_l = 1$, and the flavor $\bar{\mathbf{3}}_F$ multiplet of $J^P = 1/2^+$ composes another charmed baryon multiplet where the spin of the two light quarks is $s_l = 0$. The results of their mass sum rules are [50]

(1) The flavor $\bar{\mathbf{3}}_F$ multiplet of $J^P = 1/2^+$ contains $\Lambda_c^+(1/2^+)$, $\Xi_c^+(1/2^+)$ and $\Xi_c^0(1/2^+)$, which can be well coupled by the following interpolating fields:

$$J_{\Lambda_c^+}(x) = \epsilon_{abc}[u^{aT}(x)C\gamma_5 d^b(x)]h_v^c(x), \quad (7)$$

$$J_{\Xi_c^+}(x) = \epsilon_{abc}[u^{aT}(x)C\gamma_5 s^b(x)]h_v^c(x), \quad (8)$$

$$J_{\Xi_c^0}(x) = \epsilon_{abc}[d^{aT}(x)C\gamma_5 s^b(x)]h_v^c(x), \quad (9)$$

where a, b and c are color indices; ϵ_{abc} is the totally antisymmetric tensor; C is the charge-conjugation operator; the superscript T represents the transpose of the Dirac indices only; $u(x)$, $d(x)$ and $s(x)$ are the light quark fields at location x; $h_v(x)$ is the heavy-quark field at location x. Based on the results of

Ref. [50], we evaluate their parameters to be $\bar{\Lambda}_{\Lambda_c^+} = 0.773$ GeV, $\bar{\Lambda}_{\Xi_c^+} = \bar{\Lambda}_{\Xi_c^0} = 0.908$ GeV, $f_{\Lambda_c^+} = 0.0255$ GeV^3 and $f_{\Xi_c^+} = f_{\Xi_c^0} = 0.0258$ GeV^3, with the threshold values $\omega_{\Lambda_c^+} = 1.1$ GeV and $\omega_{\Xi_c^+} = \omega_{\Xi_c^0} = 1.25$ GeV.

(2) The flavor $\mathbf{6}_F$ multiplet of $J^P = 1/2^+$ contains $\Sigma_c^{++}(1/2^+)$, $\Sigma_c^+(1/2^+)$, $\Sigma_c^0(1/2^+)$, $\Xi_c^{\prime +}(1/2^+)$, $\Xi_c^{\prime 0}(1/2^+)$ and $\Omega_c^0(1/2^+)$, which can be well coupled by

$$J_{\Sigma_c^{++}}(x) = \epsilon_{abc}[u^{aT}(x)C\gamma_\mu u^b(x)]\gamma_t^\mu\gamma_5 h_v^c(x), \quad (10)$$

$$J_{\Sigma_c^{+}}(x) = \epsilon_{abc}[u^{aT}(x)C\gamma_\mu d^b(x)]\gamma_t^\mu\gamma_5 h_v^c(x), \quad (11)$$

$$J_{\Sigma_c^{0}}(x) = \epsilon_{abc}[d^{aT}(x)C\gamma_\mu d^b(x)]\gamma_t^\mu\gamma_5 h_v^c(x), \quad (12)$$

$$J_{\Xi_c^{\prime +}}(x) = \epsilon_{abc}[u^{aT}(x)C\gamma_\mu s^b(x)]\gamma_t^\mu\gamma_5 h_v^c(x), \quad (13)$$

$$J_{\Xi_c^{\prime 0}}(x) = \epsilon_{abc}[d^{aT}(x)C\gamma_\mu s^b(x)]\gamma_t^\mu\gamma_5 h_v^c(x), \quad (14)$$

$$J_{\Omega_c^{0}}(x) = \epsilon_{abc}[s^{aT}(x)C\gamma_\mu s^b(x)]\gamma_t^\mu\gamma_5 h_v^c(x). \quad (15)$$

Based on the results of Ref. [50], we evaluate their parameters to be $\bar{\Lambda}_{\Sigma_c^{++}} = \bar{\Lambda}_{\Sigma_c^{+}} = \bar{\Lambda}_{\Sigma_c^{0}} = 0.950$ GeV, $\bar{\Lambda}_{\Xi_c^{\prime +}} = \bar{\Lambda}_{\Xi_c^{\prime 0}} = 1.042$ GeV, $\bar{\Lambda}_{\Omega_c^{0}} = 1.169$ GeV, $\frac{1}{\sqrt{2}} f_{\Sigma_c^{++}} = f_{\Sigma_c^{+}} = \frac{1}{\sqrt{2}} f_{\Sigma_c^{0}} = 0.0432$ GeV^3, $f_{\Xi_c^{\prime +}} = f_{\Xi_c^{\prime 0}} = 0.0435$ GeV^3 and $\frac{1}{\sqrt{2}} f_{\Omega_c^{0}} = 0.0438$ GeV^3, with the threshold values $\omega_{\Sigma_c^{++}} = \omega_{\Sigma_c^{+}} = \omega_{\Sigma_c^{0}} = 1.3$ GeV, $\omega_{\Xi_c^{\prime +}} = \omega_{\Xi_c^{\prime 0}} = 1.4$ GeV and $\omega_{\Omega_c^{0}} = 1.55$ GeV.

(3) The flavor $\mathbf{6}_F$ multiplet of $J^P = 3/2^+$ contains $\Sigma_c^{*++}(3/2^+)$, $\Sigma_c^{*+}(3/2^+)$, $\Sigma_c^{*0}(3/2^+)$, $\Xi_c^{*+}(3/2^+)$, $\Xi_c^{*0}(3/2^+)$ and $\Omega_c^{*0}(3/2^+)$, which can be well coupled by

$$J^\mu_{\Sigma_c^{*++}}(x) = \epsilon_{abc}[u^{aT}(x)C\gamma_\nu u^b(x)] \times \left(-g_t^{\mu\nu} + \frac{1}{3}\gamma_t^\mu\gamma_t^\nu\right) h_v^c(x), \quad (16)$$

$$J^\mu_{\Sigma_c^{*+}}(x) = \epsilon_{abc}[u^{aT}(x)C\gamma_\nu d^b(x)] \times \left(-g_t^{\mu\nu} + \frac{1}{3}\gamma_t^\mu\gamma_t^\nu\right) h_v^c(x), \quad (17)$$

$$J^\mu_{\Sigma_c^{*0}}(x) = \epsilon_{abc}[d^{aT}(x)C\gamma_\nu d^b(x)] \times \left(-g_t^{\mu\nu} + \frac{1}{3}\gamma_t^\mu\gamma_t^\nu\right) h_v^c(x), \quad (18)$$

$$J^\mu_{\Xi_c^{*+}}(x) = \epsilon_{abc}[u^{aT}(x)C\gamma_\nu s^b(x)] \left(-g_t^{\mu\nu} + \frac{1}{3}\gamma_t^\mu\gamma_t^\nu\right) h_v^c(x), \quad (19)$$

$$J^\mu_{\Xi_c^{*0}}(x) = \epsilon_{abc}[d^{aT}(x)C\gamma_\nu s^b(x)] \times \left(-g_t^{\mu\nu} + \frac{1}{3}\gamma_t^\mu\gamma_t^\nu\right) h_v^c(x), \quad (20)$$

$$J^\mu_{\Omega_c^{*0}}(x) = \epsilon_{abc}[s^{aT}(x)C\gamma_\nu s^b(x)] \times \left(-g_t^{\mu\nu} + \frac{1}{3}\gamma_t^\mu\gamma_t^\nu\right) h_v^c(x). \quad (21)$$

Based on the results of Ref. [50], we evaluate their parameters to be $\bar{\Lambda}_{\Sigma_c^{*++}} = \bar{\Lambda}_{\Sigma_c^{*+}} = \bar{\Lambda}_{\Sigma_c^{*0}} = 0.950$ GeV, $\bar{\Lambda}_{\Xi_c^{*+}} = \bar{\Lambda}_{\Xi_c^{*0}} = 1.042$ GeV, $\bar{\Lambda}_{\Omega_c^{*0}} = 1.169$ GeV, $\frac{1}{\sqrt{2}} f_{\Sigma_c^{*++}} = f_{\Sigma_c^{*+}} = \frac{1}{\sqrt{2}} f_{\Sigma_c^{*0}} = \frac{1}{\sqrt{3}} 0.0432$ GeV^3, $f_{\Xi_c^{*+}} = f_{\Xi_c^{*0}} = \frac{1}{\sqrt{3}} 0.0435$ GeV^3 and $\frac{1}{\sqrt{2}} f_{\Omega_c^{*0}} = \frac{1}{\sqrt{3}} 0.0438$ GeV^3, with the threshold values $\omega_{\Sigma_c^{*++}} = \omega_{\Sigma_c^{*+}} = \omega_{\Sigma_c^{*0}} = 1.3$ GeV, $\omega_{\Xi_c^{*+}} = \omega_{\Xi_c^{*0}} = 1.4$ GeV and $\omega_{\Omega_c^{*0}} = 1.55$ GeV.

We list all these values in Table I.

B. P-wave charmed baryons of flavor $\bar{\mathbf{3}}_F$

The four observed states $\Lambda_c(2595)$ ($J^P = 1/2^-$), $\Lambda_c(2625)$ ($J^P = 3/2^-$), $\Xi_c(2790)$ ($J^P = 1/2^-$) and $\Xi_c(2815)$ ($J^P = 3/2^-$) probably complete two flavor $\bar{\mathbf{3}}_F$ multiplets of $J^P = 1/2^-$ and $3/2^-$ [2]. Accordingly, in the present study we assume masses of the P-wave charmed baryon states of flavor $\bar{\mathbf{3}}_F$ to be

$$\begin{aligned} &\Lambda_c(1/2^-):\ m = 2592.25\ \text{MeV}, \\ &\Lambda_c(3/2^-):\ m = 2628.11\ \text{MeV}, \\ &\Xi_c(1/2^-):\ m = 2790.5\ \text{MeV}, \\ &\Lambda_c(3/2^-):\ m = 2818.1\ \text{MeV}, \end{aligned} \quad (22)$$

which values will be used in Sec. III to evaluate their decay widths.

The masses and decay constants of the flavor $\bar{\mathbf{3}}_F$ P-wave charmed baryons have been systematically investigated in Refs. [51,52] using the method of QCD sum rules in HQET, and our results suggested that the $\Lambda_c(2595)$, $\Lambda_c(2625)$, $\Xi_c(2790)$ and $\Xi_c(2815)$ can be well described by the baryon doublet $[\bar{\mathbf{3}}_F, 1, 1, \rho]$ and they complete two $\bar{\mathbf{3}}_F$ multiplets of $J^P = 1/2^-$ and $3/2^-$; while the results obtained using the baryon doublet $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$ seem also consistent with the data. We note that the definition of the external off-shell energy ω in the present study is different from that used in Refs. [51,52]. Hence, in the present study we also reevaluate their parameters, and shortly summarize their results in the following.

Based on HQET, we use $J^{\alpha_1\cdots\alpha_{j-1/2}}_{j,P,F,j_l,s_l,\rho/\lambda}$ to denote the P-wave charmed baryon field coupling to $|j, P, F, j_l, s_l, \rho/\lambda\rangle$, where j, P, and F are the total angular momentum, parity and flavor representation (either $\bar{\mathbf{3}}_F$ or $\mathbf{6}_F$) of the charmed baryons, and

TABLE I. The parameters of the ground-state charmed baryons. The two flavor $\mathbf{6}_F$ multiplets of $J^P = 1/2^+$ and $3/2^+$ compose one charmed baryon multiplet where the spin of the two light quarks is $s_l = 1$, while the flavor $\bar{\mathbf{3}}_F$ multiplet of $J^P = 1/2^+$ composes another charmed baryon multiplet where the spin of the two light quarks is $s_l = 0$.

Multiplets	Baryon	Mass (MeV)	ω_c (GeV)	$\bar{\Lambda}$ (GeV)	f (GeV^3)
$[\bar{\mathbf{3}}_F, \frac{1}{2}^+]$	$\Lambda_c^+(1/2^+)$	2286.46	1.10	0.773	0.0255
	$\Xi_c^+(1/2^+)$	2467.8	1.25	0.908	0.0258
	$\Xi_c^0(1/2^+)$	2470.88	1.25	0.908	0.0258
$[\mathbf{6}_F, \frac{1}{2}^+]$	$\Sigma_c^{++}(1/2^+)$	2453.98	1.30	0.950	$\sqrt{2}\times 0.0432$
	$\Sigma_c^+(1/2^+)$	2452.9	1.30	0.950	0.0432
	$\Sigma_c^0(1/2^+)$	2453.74	1.30	0.950	$\sqrt{2}\times 0.0432$
	$\Xi_c^{\prime +}(1/2^+)$	2575.6	1.40	1.042	0.0435
	$\Xi_c^{\prime 0}(1/2^+)$	2577.9	1.40	1.042	0.0435
	$\Omega_c^+(1/2^+)$	2695.2	1.55	1.169	$\sqrt{2}\times 0.0438$
$[\mathbf{6}_F, \frac{3}{2}^+]$	$\Sigma_c^{*++}(1/2^+)$	2517.9	1.30	0.950	$\sqrt{\frac{2}{3}}\times 0.0432$
	$\Sigma_c^{*+}(1/2^+)$	2517.5	1.30	0.950	$\sqrt{\frac{1}{3}}\times 0.0432$
	$\Sigma_c^{*0}(1/2^+)$	2518.8	1.30	0.950	$\sqrt{\frac{2}{3}}\times 0.0432$
	$\Xi_c^{*+}(1/2^+)$	2645.9	1.40	1.042	$\sqrt{\frac{1}{3}}\times 0.0435$
	$\Xi_c^{*0}(1/2^+)$	2645.9	1.40	1.042	$\sqrt{\frac{1}{3}}\times 0.0435$
	$\Omega_c^{*+}(1/2^+)$	2765.9	1.55	1.169	$\sqrt{\frac{2}{3}}\times 0.0438$

j_l and s_l are the total angular momentum and spin angular momentum of the light components. We use l_ρ to denote the orbital angular momentum between the two light quarks, l_λ to denote the orbital angular momentum between the charm quark and the two-light-quark system, and then ρ to denote $l_\rho = 1$ and $l_\lambda = 0$, while λ to denote $l_\rho = 0$ and $l_\lambda = 1$. We have the relations $L = l_\lambda \otimes l_\rho$, $j_l = L \otimes s_l$ and $j = j_l \otimes s_Q$, where $s_Q = 1/2$ is the spin of the heavy quark. This field $J^{\alpha_1\ldots\alpha_{j-1/2}}_{j,P,F,j_l,s_l,\rho/\lambda}$ belongs to the baryon multiplet $[F, j_l, s_l, \rho/\lambda]$.

There are altogether four *P*-wave charmed baryon multiplets of the flavor $\bar{\mathbf{3}}_F$, $[\bar{\mathbf{3}}_F, 0, 1, \rho]$, $[\bar{\mathbf{3}}_F, 1, 1, \rho]$, $[\bar{\mathbf{3}}_F, 2, 1, \rho]$, and $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$. The results of their mass sum rules are [51,52]

(1) The $[\bar{\mathbf{3}}_F, 0, 1, \rho]$ multiplet contains $\Lambda_c^+(\frac{1}{2}^-)$ and $\Xi_c^{+,0}(\frac{1}{2}^-)$, which are coupled by

$$J_{1/2,-,\bar{\mathbf{3}}_F,0,1,\rho} = i\epsilon_{abc}([\mathcal{D}_t^\mu q^{aT}]C\gamma_t^\mu q^b - q^{aT}C\gamma_t^\mu[\mathcal{D}_t^\mu q^b])h_v^c, \qquad (23)$$

where $D^\mu = \partial^\mu - igA^\mu$ is the gauge-covariant derivative. We can further explicitly denote the quark contents by simply replacing $\bar{\mathbf{3}}_F$ by Λ_c and Ξ_c. For example, we use $J_{1/2,-,\Xi_c^+,0,1,\rho}$ to denote $J_{1/2,-,\bar{\mathbf{3}}_F,0,1,\rho}$ with the quark contents *usc*:

$$J_{1/2,-,\Xi_c^+,0,1,\rho} = i\epsilon_{abc}([\mathcal{D}_t^\mu u^{aT}]C\gamma_t^\mu s^b - u^{aT}C\gamma_t^\mu[\mathcal{D}_t^\mu s^b])h_v^c. \qquad (24)$$

Based on the results of Refs. [51,52], we evaluate their parameters to be $\bar{\Lambda}_{\Lambda_c^+(\frac{1}{2}^-)} = 0.987$ GeV, $\bar{\Lambda}_{\Xi_c^{+,0}(\frac{1}{2}^-)} = 1.181$ GeV, $f_{\Lambda_c^+(\frac{1}{2}^-)} = 0.0198\,\mathrm{GeV}^4$ and $f_{\Xi_c^{+,0}(\frac{1}{2}^-)} = 0.0296\,\mathrm{GeV}^4$, with the threshold values $\omega_{\Lambda_c^+(\frac{1}{2}^-)} = 1.75$ GeV and $\omega_{\Xi_c^{+,0}(\frac{1}{2}^-)} = 1.55$ GeV.

(2) The $[\bar{\mathbf{3}}_F, 1, 1, \rho]$ multiplet contains $\Lambda_c^+(\frac{1}{2}^-/\frac{3}{2}^-)$ and $\Xi_c^{+,0}(\frac{1}{2}^-/\frac{3}{2}^-)$, which are coupled by

$$J_{1/2,-,\bar{\mathbf{3}}_F,1,1,\rho} = i\epsilon_{abc}([\mathcal{D}_t^\mu q^{aT}]C\gamma_t^\nu q^b - q^{aT}C\gamma_t^\nu[\mathcal{D}_t^\mu q^b])\sigma_t^{\mu\nu}h_v^c, \qquad (25)$$

$$J^\alpha_{3/2,-,\bar{\mathbf{3}}_F,1,1,\rho} = i\epsilon_{abc}([\mathcal{D}_t^\mu q^{aT}]C\gamma_t^\nu q^b - q^{aT}C\gamma_t^\nu[\mathcal{D}_t^\mu q^b]) \times \left(g_t^{\alpha\mu}\gamma_t^\nu\gamma_5 - g_t^{\alpha\nu}\gamma_t^\mu\gamma_5 - \frac{1}{3}\gamma_t^\alpha\gamma_t^\mu\gamma_t^\nu\gamma_5 + \frac{1}{3}\gamma_t^\alpha\gamma_t^\nu\gamma_t^\mu\gamma_5\right)h_v^c. \qquad (26)$$

Based on the results of Refs. [51,52], we evaluate their parameters to be $\bar{\Lambda}_{\Lambda_c^+(\frac{1}{2}^-/\frac{3}{2}^-)} = 1.164\,\mathrm{GeV}$, $\bar{\Lambda}_{\Xi_c^{+,0}(\frac{1}{2}^-/\frac{3}{2}^-)} = 1.349\,\mathrm{GeV}$, $f_{\Lambda_c^+(\frac{1}{2}^-)} = 0.0523$ GeV^4, $f_{\Xi_c^{+,0}(\frac{1}{2}^-)} = 0.0788\,\mathrm{GeV}^4$, $f_{\Lambda_c^+(\frac{3}{2}^-)} = 0.0523\,\mathrm{GeV}^4$ and $f_{\Xi_c^{+,0}(\frac{3}{2}^-)} = 0.0788$ GeV^4, with the threshold values $\omega_{\Lambda_c^+(\frac{1}{2}^-/\frac{3}{2}^-)} = 1.55\,\mathrm{GeV}$ and $\omega_{\Xi_c^{+,0}(\frac{1}{2}^-/\frac{3}{2}^-)} = 1.8$ GeV.

(3) The $[\bar{\mathbf{3}}_F, 2, 1, \rho]$ multiplet contains $\Lambda_c^+(\frac{3}{2}^-/\frac{5}{2}^-)$ and $\Xi_c^{+,0}(\frac{3}{2}^-/\frac{5}{2}^-)$, which are coupled by

$$J^{\alpha}_{3/2,-,\bar{\mathbf{3}}_F,2,1,\rho} = i\epsilon_{abc}([\mathcal{D}^{\mu}_t q^{aT}]C\gamma^{\nu}_t q^b - q^{aT}C\gamma^{\nu}_t[\mathcal{D}^{\mu}_t q^b]) \times \left(g^{\alpha\mu}_t\gamma^{\nu}_t\gamma_5 + g^{\alpha\nu}_t\gamma^{\mu}_t\gamma_5 - \frac{2}{3}g^{\mu\nu}_t\gamma^{\alpha}_t\gamma_5\right)h^c_v, \tag{27}$$

$$J^{\alpha_1\alpha_2}_{5/2,-,\bar{\mathbf{3}}_F,2,1,\rho} = i\epsilon_{abc}([\mathcal{D}^{\mu}_t q^{aT}]C\gamma^{\nu}_t q^b - q^{aT}C\gamma^{\nu}_t[\mathcal{D}^{\mu}_t q^b]) \times \Gamma^{\alpha_1\alpha_2,\mu\nu}h^c_v, \tag{28}$$

where $\Gamma^{\alpha_1\alpha_2,\mu\nu}$ is the projection operator:

$$\Gamma^{\alpha\beta,\mu\nu} = g^{\alpha\mu}_t g^{\beta\nu}_t + g^{\alpha\nu}_t g^{\beta\mu}_t - \frac{2}{15}g^{\alpha\beta}_t g^{\mu\nu}_t - \frac{1}{3}g^{\alpha\mu}_t\gamma^{\beta}_t\gamma^{\nu}_t - \frac{1}{3}g^{\alpha\nu}_t\gamma^{\beta}_t\gamma^{\mu}_t - \frac{1}{3}g^{\beta\mu}_t\gamma^{\alpha}_t\gamma^{\nu}_t - \frac{1}{3}g^{\beta\nu}_t\gamma^{\alpha}_t\gamma^{\mu}_t + \frac{1}{15}\gamma^{\alpha}_t\gamma^{\mu}_t\gamma^{\beta}_t\gamma^{\nu}_t + \frac{1}{15}\gamma^{\alpha}_t\gamma^{\nu}_t\gamma^{\beta}_t\gamma^{\mu}_t + \frac{1}{15}\gamma^{\beta}_t\gamma^{\mu}_t\gamma^{\alpha}_t\gamma^{\nu}_t + \frac{1}{15}\gamma^{\beta}_t\gamma^{\nu}_t\gamma^{\alpha}_t\gamma^{\mu}_t. \tag{29}$$

Based on the results of Refs. [51,52], we evaluate their parameters to be $\bar{\Lambda}_{\Lambda_c^+(\frac{3}{2}^-/\frac{5}{2}^-)} = 1.339$ GeV, $\bar{\Lambda}_{\Xi_c^{+,0}(\frac{3}{2}^-/\frac{5}{2}^-)} = 1.510$ GeV, $f_{\Lambda_c^+(\frac{3}{2}^-)} = 0.0578\,\mathrm{GeV}^4$, $f_{\Xi_c^{+,0}(\frac{3}{2}^-)} = 0.0901\,\mathrm{GeV}^4$, $f_{\Lambda_c^+(\frac{5}{2}^-)} = \frac{1}{\sqrt{5}}0.0578\,\mathrm{GeV}^4$ and $f_{\Xi_c^{+,0}(\frac{5}{2}^-)} = \frac{1}{\sqrt{5}}0.0901\,\mathrm{GeV}^4$, with the threshold values $\omega_{\Lambda_c^+(\frac{3}{2}^-/\frac{5}{2}^-)} = 1.8$GeV and $\omega_{\Xi_c^{+,0}(\frac{3}{2}^-/\frac{5}{2}^-)} = 2.0$GeV.

(4) The $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$ multiplet contains $\Lambda_c^+(\frac{1}{2}^-/\frac{3}{2}^-)$ and $\Xi_c^{+,0}(\frac{1}{2}^-/\frac{3}{2}^-)$, which are coupled by

$$J_{1/2,-,\bar{\mathbf{3}}_F,1,0,\lambda} = i\epsilon_{abc}([\mathcal{D}^{\mu}_t q^{aT}]C\gamma_5 q^b + q^{aT}C\gamma_5[\mathcal{D}^{\mu}_t q^b]) \times \gamma^{\mu}_t\gamma_5 h^c_v, \tag{30}$$

$$J^{\alpha}_{3/2,-,\bar{\mathbf{3}}_F,1,0,\lambda} = i\epsilon_{abc}([\mathcal{D}^{\mu}_t q^{aT}]C\gamma_5 q^b + q^{aT}C\gamma_5[\mathcal{D}^{\mu}_t q^b]) \times \left(g^{\alpha\mu}_t - \frac{1}{3}\gamma^{\alpha}_t\gamma^{\mu}_t\right)h^c_v. \tag{31}$$

Based on the results of Refs. [51,52], we evaluate their parameters to be $\bar{\Lambda}_{\Lambda_c^+(\frac{1}{2}^-/\frac{3}{2}^-)} = 0.961$ GeV, $\bar{\Lambda}_{\Xi_c^{+,0}(\frac{1}{2}^-/\frac{3}{2}^-)} = 1.057$ GeV, $f_{\Lambda_c^+(\frac{1}{2}^-)} = 0.0201$ GeV^4, $f_{\Xi_c^{+,0}(\frac{1}{2}^-)} = 0.0255$ GeV^4, $f_{\Lambda_c^+(\frac{3}{2}^-)} = \frac{1}{\sqrt{3}}0.0201$ GeV^4 and $f_{\Xi_c^{+,0}(\frac{3}{2}^-)} = \frac{1}{\sqrt{3}}0.0255$ GeV^4, with the threshold values $\omega_{\Lambda_c^+(\frac{1}{2}^-/\frac{3}{2}^-)} = 1.45$ GeV and $\omega_{\Xi_c^{+,0}(\frac{1}{2}^-/\frac{3}{2}^-)} = 1.55$ GeV.

We also list these values in Table II.

C. *P*-wave charmed baryons of flavor $\mathbf{6}_F$

The three observed states $\Sigma_c(2800)$ ($J^P = ?^?$), $\Xi_c(2930)$ ($J^P = ?^?$) and $\Xi_c(2980)$ ($J^P = ?^?$) may be *P*-wave charmed baryon states of the flavor $\mathbf{6}_F$. Besides them, there are five excited Ω_c states recently observed by LHCb [1]: the $\Omega_c(3000)$, $\Omega_c(3050)$, $\Omega_c(3066)$, $\Omega_c(3090)$, and $\Omega_c(3119)$, which may also be interpreted as *P*-wave charmed baryon states of the flavor $\mathbf{6}_F$. Accordingly, in the present study we assume the masses of the *P*-wave charmed baryon fields to be

$$\Sigma_c\left(\frac{1^-}{2}\Big/\frac{3^-}{2}\right):\ m \approx 2800\ \mathrm{MeV},$$

$$\Xi'_c\left(\frac{1^-}{2}\Big/\frac{3^-}{2}\right):\ m \approx 2950\ \mathrm{MeV},$$

$$\Omega_c\left(\frac{1^-}{2}\right):\ m \approx 3100\ \mathrm{MeV},$$

$$\Omega_c\left(\frac{3^-}{2}\right):\ m \approx 3120\ \mathrm{MeV},$$

which values will be used in Sec. IV to evaluate their decay widths.

The masses and decay constants of the flavor $\mathbf{6}_F$ *P*-wave charmed baryons have been systematically investigated in Refs. [51,52] using the method of QCD sum rules in HQET, and our results suggested that the baryon doublet $[\mathbf{6}_F, 1, 0, \rho]$ contains $\Sigma_c(1/2^-, 3/2^-)$, $\Xi'_c(1/2^-, 3/2^-)$, and $\Omega_c(1/2^-, 3/2^-)$, and its obtained results are consistent with the observed states $\Sigma_c(2800)$ ($J^P = ?^?$) and $\Xi_c(2980)$ ($J^P = ?^?$), while the results obtained by using the baryon doublet $[\mathbf{6}_F, 2, 1, \lambda]$ are also consistent with them. We shortly summarize their results in the following.

There are altogether four *P*-wave charmed baryon multiplets of the flavor $\mathbf{6}_F$, $[\mathbf{6}_F, 1, 0, \rho]$, $[\mathbf{6}_F, 0, 1, \lambda]$, $[\mathbf{6}_F, 1, 1, \lambda]$ and $[\mathbf{6}_F, 2, 1, \lambda]$, and the results of their mass sum rules are [51,52]

(1) The $[\mathbf{6}_F, 1, 0, \rho]$ multiplet contains $\Sigma_c^{++,+,0}(\frac{1}{2}^-/\frac{3}{2}^-)$, $\Xi_c^{\prime+,0}(\frac{1}{2}^-/\frac{3}{2}^-)$ and $\Omega_c^0(\frac{1}{2}^-/\frac{3}{2}^-)$, which are coupled by

$$J_{1/2,-,\mathbf{6}_F,1,0,\rho} = i\epsilon_{abc}([\mathcal{D}^{\mu}_t q^{aT}]C\gamma_5 q^b - q^{aT}C\gamma_5[\mathcal{D}^{\mu}_t q^b]) \times \gamma^{\mu}_t\gamma_5 h^c_v, \tag{32}$$

$$J^{\alpha}_{3/2,-,\mathbf{6}_F,1,0,\rho} = i\epsilon_{abc}([\mathcal{D}^{\mu}_t q^{aT}]C\gamma_5 q^b - q^{aT}C\gamma_5[\mathcal{D}^{\mu}_t q^b]) \times \left(g^{\alpha\mu}_t - \frac{1}{3}\gamma^{\alpha}_t\gamma^{\mu}_t\right)h^c_v. \tag{33}$$

Based on the results of Refs. [51,52], we evaluate their parameters to be $\bar{\Lambda}_{\Sigma_c^{++,+,0}(\frac{1}{2}^-/\frac{3}{2}^-)} = 1.224$ GeV, $\bar{\Lambda}_{\Xi_c^{\prime+,0}(\frac{1}{2}^-/\frac{3}{2}^-)} = 1.422$ GeV, $\bar{\Lambda}_{\Omega_c^0(\frac{1}{2}^-/\frac{3}{2}^-)} = 1.641$ GeV,

TABLE II. The parameters of the P-wave charmed baryons of flavor $\bar{\mathbf{3}}_F$. In Ref. [51] we have systematically evaluated masses of the P-wave charmed baryons, and our results suggested the four observed states $\Lambda_c(2595)$ ($J^P = 1/2^-$), $\Lambda_c(2625)$ ($J^P = 3/2^-$), $\Xi_c(2790)$ ($J^P = 1/2^-$) and $\Xi_c(2815)$ ($J^P = 3/2^-$) can be well described by the baryon doublet $[\bar{\mathbf{3}}_F, 1, 1, \rho]$ and they complete two $\bar{\mathbf{3}}_F$ multiplets of $J^P = 1/2^-$ and $3/2^-$, while the currents belonging to the baryon doublet $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$ seem also consistent with the data.

Multiplets	Baryon	ω_c (GeV)	T (GeV)	$\bar{\Lambda}$ (GeV)	f (GeV^4)
$[\bar{\mathbf{3}}_F, 0, 1, \rho]$	$\Lambda_c^+(\frac{1}{2}^-)$	1.75	~0.37	0.987	0.0198
	$\Xi_c^+(\frac{1}{2}^-)$	1.55	~0.38	1.181	0.0296
	$\Xi_c^0(\frac{1}{2}^-)$	1.55	~0.38	1.181	0.0296
$[\bar{\mathbf{3}}_F, 1, 1, \rho]$	$\Lambda_c^+(\frac{1}{2}^-/\frac{3}{2}^-)$	1.55	$0.27 < T < 0.30$	1.164	0.0523/0.0523
	$\Xi_c^+(\frac{1}{2}^-/\frac{3}{2}^-)$	1.80	$0.27 < T < 0.32$	1.349	0.0788/0.0788
	$\Xi_c^0(\frac{1}{2}^-/\frac{3}{2}^-)$	1.80	$0.27 < T < 0.32$	1.349	0.0788/0.0788
$[\bar{\mathbf{3}}_F, 2, 1, \rho]$	$\Lambda_c^+(\frac{3}{2}^-/\frac{5}{2}^-)$	1.80	~0.30	1.339	$0.0578/\sqrt{\frac{1}{5}}\times 0.0578$
	$\Xi_c^+(\frac{3}{2}^-/\frac{5}{2}^-)$	2.00	~0.33	1.510	$0.0901/\sqrt{\frac{1}{5}}\times 0.0901$
	$\Xi_c^0(\frac{3}{2}^-/\frac{5}{2}^-)$	2.00	~0.33	1.510	$0.0901/\sqrt{\frac{1}{5}}\times 0.0901$
$[\bar{\mathbf{3}}_F, 1, 0, \lambda]$	$\Lambda_c^+(\frac{1}{2}^-/\frac{3}{2}^-)$	1.45	~0.30	0.961	$0.0201/\sqrt{\frac{1}{3}}\times 0.0201$
	$\Xi_c^+(\frac{1}{2}^-/\frac{3}{2}^-)$	1.55	~0.32	1.057	$0.0255/\sqrt{\frac{1}{3}}\times 0.0255$
	$\Xi_c^0(\frac{1}{2}^-/\frac{3}{2}^-)$	1.55	~0.32	1.057	$0.0255/\sqrt{\frac{1}{3}}\times 0.0255$

$\frac{1}{\sqrt{2}} f_{\Sigma_c^{++,0}(\frac{1}{2}^-)} = f_{\Sigma_c^+(\frac{1}{2}^-)} = 0.0437$ GeV^4, $f_{\Xi_c^{\prime +,0}(\frac{1}{2}^-)} = 0.0680$ GeV^4, $\frac{1}{\sqrt{2}} f_{\Omega_c^0(\frac{1}{2}^-)} = 0.108$ GeV^4, $\frac{1}{\sqrt{2}} f_{\Sigma_c^{++,0}(\frac{3}{2}^-)} = f_{\Sigma_c^+(\frac{3}{2}^-)} = \frac{1}{\sqrt{3}} 0.0437$ GeV^4, $f_{\Xi_c^{\prime +,0}(\frac{3}{2}^-)} = \frac{1}{\sqrt{3}} 0.0680$ GeV^4 and $\frac{1}{\sqrt{2}} f_{\Omega_c^0(\frac{3}{2}^-)} = \frac{1}{\sqrt{3}} 0.108$ GeV^4, with the threshold values $\omega_{\Sigma_c^{++,+,0}(\frac{1}{2}^-/\frac{3}{2}^-)} = 1.7$ GeV, $\omega_{\Xi_c^{\prime +,0}(\frac{1}{2}^-/\frac{3}{2}^-)} = 1.95$ GeV and $\omega_{\Omega_c^0(\frac{1}{2}^-/\frac{3}{2}^-)} = 2.2$ GeV.

(2) The $[\mathbf{6}_F, 0, 1, \lambda]$ multiplet contains $\Sigma_c^{++,+,0}(\frac{1}{2}^-)$, $\Xi_c^{\prime +,0}(\frac{1}{2}^-)$, and $\Omega_c^0(\frac{1}{2}^-)$, which are coupled by

$$J_{1/2,-,\mathbf{6}_F,0,1,\lambda} = i\epsilon_{abc}([\mathcal{D}_t^\mu q^{aT}]C\gamma_t^\mu q^b + q^{aT}C\gamma_t^\mu[\mathcal{D}_t^\mu q^b]) \times h_v^c. \tag{34}$$

Based on the results of Refs. [51,52], we evaluate their parameters to be $\bar{\Lambda}_{\Sigma_c^{++,+,0}(\frac{1}{2}^-)} = 1.100$ GeV, $\bar{\Lambda}_{\Xi_c^{\prime +,0}(\frac{1}{2}^-)} = 1.295$ GeV, $\bar{\Lambda}_{\Omega_c^0(\frac{1}{2}^-)} = 1.504$ GeV, $\frac{1}{\sqrt{2}} f_{\Sigma_c^{++,0}(\frac{1}{2}^-)} = f_{\Sigma_c^+(\frac{1}{2}^-)} = 0.0344$ GeV^4, $f_{\Xi_c^{\prime +,0}(\frac{1}{2}^-)} = 0.0512$ GeV^4 and $\frac{1}{\sqrt{2}} f_{\Omega_c^0(\frac{1}{2}^-)} = 0.0804$ GeV^4, with the threshold values $\omega_{\Sigma_c^{++,+,0}(\frac{1}{2}^-)} = 1.45$ GeV, $\omega_{\Xi_c^{\prime +,0}(\frac{1}{2}^-)} = 1.7$ GeV and $\omega_{\Omega_c^0(\frac{1}{2}^-)} = 1.95$ GeV.

(3) The $[\mathbf{6}_F, 1, 1, \lambda]$ multiplet $\Sigma_c^{++,+,0}(\frac{1}{2}^-/\frac{3}{2}^-)$, $\Xi_c^{\prime +,0}(\frac{1}{2}^-/\frac{3}{2}^-)$, and $\Omega_c^0(\frac{1}{2}^-/\frac{3}{2}^-)$, which are coupled by

$$J_{1/2,-,\mathbf{6}_F,1,1,\lambda} = i\epsilon_{abc}([\mathcal{D}_t^\mu q^{aT}]C\gamma_t^\nu q^b + q^{aT}C\gamma_t^\nu[\mathcal{D}_t^\mu q^b]) \times \sigma_t^{\mu\nu} h_v^c, \tag{35}$$

$$J^\alpha_{3/2,-,\mathbf{6}_F,1,1,\lambda} = i\epsilon_{abc}([\mathcal{D}_t^\mu q^{aT}]C\gamma_t^\nu q^b + q^{aT}C\gamma_t^\nu[\mathcal{D}_t^\mu q^b]) \times \left(g_t^{\alpha\mu}\gamma_t^\nu\gamma_5 - g_t^{\alpha\nu}\gamma_t^\mu\gamma_5 - \frac{1}{3}\gamma_t^\alpha\gamma_t^\mu\gamma_t^\nu\gamma_5 + \frac{1}{3}\gamma_t^\alpha\gamma_t^\nu\gamma_t^\mu\gamma_5\right) h_v^c. \tag{36}$$

Based on the results of Refs. [51,52], we evaluate their parameters to be $\bar{\Lambda}_{\Sigma_c^{++,+,0}(\frac{1}{2}^-/\frac{3}{2}^-)} = 1.066$ GeV, $\bar{\Lambda}_{\Xi_c^{\prime +,0}(\frac{1}{2}^-/\frac{3}{2}^-)} = 1.181$ GeV, $\bar{\Lambda}_{\Omega_c^0(\frac{1}{2}^-/\frac{3}{2}^-)} = 1.270$ GeV, $\frac{1}{\sqrt{2}} f_{\Sigma_c^{++,0}(\frac{1}{2}^-)} = f_{\Sigma_c^+(\frac{1}{2}^-)} = 0.0349$ GeV^4, $f_{\Xi_c^{\prime +,0}(\frac{1}{2}^-)} = 0.0451$ GeV^4, $\frac{1}{\sqrt{2}} f_{\Omega_c^0(\frac{1}{2}^-)} = 0.0546$ GeV^4, $\frac{1}{\sqrt{2}} f_{\Sigma_c^{++,0}(\frac{3}{2}^-)} = f_{\Sigma_c^+(\frac{3}{2}^-)} = 0.0349$ GeV^4, $f_{\Xi_c^{\prime +,0}(\frac{3}{2}^-)} = 0.0451$ GeV^4 and $\frac{1}{\sqrt{2}} f_{\Omega_c^0(\frac{3}{2}^-)} = 0.0546$ GeV^4, with the threshold values $\omega_{\Sigma_c^{++,+,0}(\frac{1}{2}^-/\frac{3}{2}^-)} = 1.75$ GeV, $\omega_{\Xi_c^{\prime +,0}(\frac{1}{2}^-/\frac{3}{2}^-)} = 1.75$ GeV and $\omega_{\Omega_c^0(\frac{1}{2}^-/\frac{3}{2}^-)} = 1.75$ GeV.

(4) The $[\mathbf{6}_F, 2, 1, \lambda]$ multiplet $\Sigma_c^{++,+,0}(\frac{3}{2}^-/\frac{5}{2}^-)$, $\Xi_c^{\prime +,0}(\frac{3}{2}^-/\frac{5}{2}^-)$, and $\Omega_c^0(\frac{3}{2}^-/\frac{5}{2}^-)$, which are coupled by

$$J^\alpha_{3/2,-,\mathbf{6}_F,2,1,\lambda} = i\epsilon_{abc}([\mathcal{D}_t^\mu q^{aT}]C\gamma_t^\nu q^b + q^{aT}C\gamma_t^\nu[\mathcal{D}_t^\mu q^b]) \times \left(g_t^{\alpha\mu}\gamma_t^\nu\gamma_5 + g_t^{\alpha\nu}\gamma_t^\mu\gamma_5 - \frac{2}{3}g_t^{\mu\nu}\gamma_t^\alpha\gamma_5\right) h_v^c, \tag{37}$$

$$J^{\alpha_1\alpha_2}_{5/2,-,\mathbf{6}_F,2,1,\lambda} = i\epsilon_{abc}([\mathcal{D}_t^\mu q^{aT}]C\gamma_t^\nu q^b + q^{aT}C\gamma_t^\nu[\mathcal{D}_t^\mu q^b]) \times \Gamma^{\alpha_1\alpha_2,\mu\nu} h_v^c. \tag{38}$$

Based on the results of Refs. [51,52], we evaluate their parameters to be $\bar{\Lambda}_{\Sigma_c^{++,+,0}(\frac{3}{2}^-/\frac{5}{2}^-)} = 1.099$ GeV, $\bar{\Lambda}_{\Xi_c^{\prime +,0}(\frac{3}{2}^-/\frac{5}{2}^-)} = 1.254$ GeV, $\bar{\Lambda}_{\Omega_c^0(\frac{3}{2}^-/\frac{5}{2}^-)} = 1.461$ GeV, $\frac{1}{\sqrt{2}} f_{\Sigma_c^{++,0}(\frac{3}{2}^-)} = f_{\Sigma_c^+(\frac{3}{2}^-)} = 0.0395\ \mathrm{GeV}^4$, $f_{\Xi_c^{\prime +,0}(\frac{3}{2}^-)} = 0.0599\ \mathrm{GeV}^4$, $\frac{1}{\sqrt{2}} f_{\Omega_c^0(\frac{3}{2}^-)} = 0.0976\ \mathrm{GeV}^4$, $\frac{1}{\sqrt{2}} f_{\Sigma_c^{++,0}(\frac{5}{2}^-)} = f_{\Sigma_c^+(\frac{5}{2}^-)} = \frac{1}{\sqrt{5}} 0.0395\ \mathrm{GeV}^4$, $f_{\Xi_c^{\prime +,0}(\frac{5}{2}^-)} = \frac{1}{\sqrt{5}} 0.0599\ \mathrm{GeV}^4$ and $\frac{1}{\sqrt{2}} f_{\Omega_c^0(\frac{5}{2}^-)} = \frac{1}{\sqrt{5}} 0.0976\ \mathrm{GeV}^4$, with the threshold values $\omega_{\Sigma_c^{++,+,0}(\frac{3}{2}^-/\frac{5}{2}^-)} = 1.5$ GeV, $\omega_{\Xi_c^{\prime +,0}(\frac{3}{2}^-/\frac{5}{2}^-)} = 1.75$ GeV and $\omega_{\Omega_c^0(\frac{3}{2}^-/\frac{5}{2}^-)} = 2.0$ GeV. We also list all these values in Table III.

III. DECAY PROPERTIES OF FLAVOR $\bar{\mathbf{3}}_F$ P-WAVE CHARMED BARYONS

In this section we use the method of light-cone QCD sum rules to study decay properties of the flavor $\bar{\mathbf{3}}_F$ P-wave charmed baryons. We only study their S-wave decays into ground-state charmed baryons accompanied by a pseudoscalar meson (π or K) or a vector meson (ρ or K^*). Because the masses of the flavor $\bar{\mathbf{3}}_F$ P-wave charmed baryons are sometimes below the two-body decay thresholds (such as $\Lambda_c(3/2^-) \to \Sigma_c^*(3/2^+) + \pi$), we also study their three-body decays, which are kinematically allowed [such as $\Lambda_c(3/2^-) \to \Sigma_c^*(3/2^+) + \pi \to \Lambda_c(1/2^+) + \pi + \pi$].

TABLE III. The parameters of the P-wave charmed baryons of flavor 6. In Ref. [51] we have systematically evaluated the masses of the P-wave charmed baryons, and our results suggested that the baryon doublet $[\mathbf{6}_F, 1, 0, \rho]$ contains $\Sigma_c(1/2^-, 3/2^-)$, $\Xi_c^\prime(1/2^-, 3/2^-)$, and $\Omega_c(1/2^-, 3/2^-)$, and its obtained results are consistent with the observed states $\Sigma_c(2800)$ ($J^P = ?^?$) and $\Xi_c(2980)$ ($J^P = ?^?$), while the results obtained by using the baryon doublet $[\mathbf{6}_F, 2, 1, \lambda]$ are also consistent with them.

Multiplets	Baryon	ω_c (GeV)	T (GeV)	$\bar{\Lambda}$ (GeV)	f (GeV^4)
$[\mathbf{6}_F, 1, 0, \rho]$	$\Sigma_c^{++}(\frac{1}{2}^-/\frac{3}{2}^-)$	1.70	$0.26 < T < 0.32$	1.224	$\sqrt{2} \times 0.0437/\sqrt{\frac{2}{3}} \times 0.0437$
	$\Sigma_c^{+}(\frac{1}{2}^-/\frac{3}{2}^-)$	1.70	$0.26 < T < 0.32$	1.224	$0.0437/\sqrt{\frac{1}{3}} \times 0.0437$
	$\Sigma_c^{0}(\frac{1}{2}^-/\frac{3}{2}^-)$	1.70	$0.26 < T < 0.32$	1.224	$\sqrt{2} \times 0.0437/\sqrt{\frac{2}{3}} \times 0.0437$
	$\Xi_c^{\prime +}(\frac{1}{2}^-/\frac{3}{2}^-)$	1.95	$0.26 < T < 0.35$	1.422	$0.0680/\sqrt{\frac{1}{3}} \times 0.0680$
	$\Xi_c^{\prime 0}(\frac{1}{2}^-/\frac{3}{2}^-)$	1.95	$0.26 < T < 0.35$	1.422	$0.0680/\sqrt{\frac{1}{3}} \times 0.0680$
	$\Omega_c^{+}(\frac{1}{2}^-/\frac{3}{2}^-)$	2.20	$0.25 < T < 0.39$	1.641	$\sqrt{2} \times 0.108/\sqrt{\frac{2}{3}} \times 0.108$
$[\mathbf{6}_F, 0, 1, \lambda]$	$\Sigma_c^{++}(\frac{1}{2}^-)$	1.45	~ 0.29	1.100	$\sqrt{2} \times 0.0344$
	$\Sigma_c^{+}(\frac{1}{2}^-)$	1.45	~ 0.29	1.100	0.0344
	$\Sigma_c^{0}(\frac{1}{2}^-)$	1.45	~ 0.29	1.100	$\sqrt{2} \times 0.0344$
	$\Xi_c^{\prime +}(\frac{1}{2}^-)$	1.70	$0.27 < T < 0.32$	1.295	0.0512
	$\Xi_c^{\prime 0}(\frac{1}{2}^-)$	1.70	$0.27 < T < 0.32$	1.295	0.0512
	$\Omega_c^{+}(\frac{1}{2}^-)$	1.95	$0.27 < T < 0.33$	1.504	$\sqrt{2} \times 0.0804$
$[\mathbf{6}_F, 1, 1, \lambda]$	$\Sigma_c^{++}(\frac{1}{2}^-/\frac{3}{2}^-)$	1.75	$0.32 < T < 0.34$	1.066	$\sqrt{2} \times 0.0349/\sqrt{2} \times 0.0349$
	$\Sigma_c^{+}(\frac{1}{2}^-/\frac{3}{2}^-)$	1.75	$0.32 < T < 0.34$	1.066	0.0349/0.0349
	$\Sigma_c^{0}(\frac{1}{2}^-/\frac{3}{2}^-)$	1.75	$0.32 < T < 0.34$	1.066	$\sqrt{2} \times 0.0349/\sqrt{2} \times 0.0349$
	$\Xi_c^{\prime +}(\frac{1}{2}^-/\frac{3}{2}^-)$	1.75	~ 0.35	1.181	0.0451/0.0451
	$\Xi_c^{\prime 0}(\frac{1}{2}^-/\frac{3}{2}^-)$	1.75	~ 0.35	1.181	0.0451/0.0451
	$\Omega_c^{+}(\frac{1}{2}^-/\frac{3}{2}^-)$	1.75	~ 0.36	1.270	$\sqrt{2} \times 0.0546/\sqrt{2} \times 0.0546$
$[\mathbf{6}_F, 2, 1, \lambda]$	$\Sigma_c^{++}(\frac{3}{2}^-/\frac{5}{2}^-)$	1.50	$0.27 < T < 0.29$	1.099	$\sqrt{2} \times 0.0395/\sqrt{\frac{2}{5}} \times 0.0395$
	$\Sigma_c^{+}(\frac{3}{2}^-/\frac{5}{2}^-)$	1.50	$0.27 < T < 0.29$	1.099	$0.0395/\sqrt{\frac{1}{5}} \times 0.0395$
	$\Sigma_c^{0}(\frac{3}{2}^-/\frac{5}{2}^-)$	1.50	$0.27 < T < 0.29$	1.099	$\sqrt{2} \times 0.0395/\sqrt{\frac{2}{5}} \times 0.0395$
	$\Xi_c^{\prime +}(\frac{3}{2}^-/\frac{5}{2}^-)$	1.75	$0.26 < T < 0.32$	1.254	$0.0599/\sqrt{\frac{1}{5}} \times 0.0599$
	$\Xi_c^{\prime 0}(\frac{3}{2}^-/\frac{5}{2}^-)$	1.75	$0.26 < T < 0.32$	1.254	$0.0599/\sqrt{\frac{1}{5}} \times 0.0599$
	$\Omega_c^{+}(\frac{3}{2}^-/\frac{5}{2}^-)$	2.00	$0.26 < T < 0.36$	1.461	$\sqrt{2} \times 0.0976/\sqrt{\frac{2}{5}} \times 0.0976$

The possible decay channels are

$$(a)\ \ \mathbf{\Gamma}[\Lambda_c(1/2^-)\to\Sigma_c(1/2^+)+\pi] \\ =\mathbf{\Gamma}[\Lambda_c^+(1/2^-)\to\Sigma_c^+(1/2^+)+\pi^0]+2\times\mathbf{\Gamma}[\Lambda_c^+(1/2^-)\to\Sigma_c^{++}(1/2^+)+\pi^-\to\Lambda_c^+(1/2^+)+\pi^++\pi^-], \tag{39}$$

$$(b)\ \ \mathbf{\Gamma}[\Lambda_c(3/2^-)\to\Sigma_c^*(3/2^+)+\pi\to\Lambda_c(1/2^+)+\pi+\pi] \\ =3\times\mathbf{\Gamma}[\Lambda_c^+(3/2^-)\to\Sigma_c^{*++}(3/2^+)+\pi^-\to\Lambda_c^+(1/2^+)+\pi^++\pi^-], \tag{40}$$

$$(c)\ \ \mathbf{\Gamma}[\Xi_c(1/2^-)\to\Xi_c(1/2^+)+\pi]=\frac{3}{2}\times\mathbf{\Gamma}[\Xi_c^0(1/2^-)\to\Xi_c^+(1/2^+)+\pi^-], \tag{41}$$

$$(d)\ \ \mathbf{\Gamma}[\Xi_c(1/2^-)\to\Lambda_c(1/2^+)+K]=\mathbf{\Gamma}[\Xi_c^0(1/2^-)\to\Lambda_c^+(1/2^+)+K^-], \tag{42}$$

$$(e)\ \ \mathbf{\Gamma}[\Xi_c(1/2^-)\to\Xi_c(1/2^+)+\rho\to\Xi_c(1/2^+)+\pi+\pi] \\ =\frac{3}{2}\times\mathbf{\Gamma}[\Xi_c^0(1/2^-)\to\Xi_c^+(1/2^+)+\rho^-\to\Xi_c^+(1/2^+)+\pi^0+\pi^-], \tag{43}$$

$$(f)\ \ \mathbf{\Gamma}[\Xi_c(1/2^-)\to\Xi_c'(1/2^+)+\pi]=\frac{3}{2}\times\mathbf{\Gamma}[\Xi_c^0(1/2^-)\to\Xi_c'^+(1/2^+)+\pi^-], \tag{44}$$

$$(g)\ \ \mathbf{\Gamma}[\Xi_c(3/2^-)\to\Xi_c(1/2^+)+\rho^-\to\Xi_c(1/2^+)+\pi+\pi] \\ =\frac{3}{2}\times\mathbf{\Gamma}[\Xi_c^0(3/2^-)\to\Xi_c^+(1/2^+)+\rho^-\to\Xi_c^+(1/2^+)+\pi^0+\pi^-], \tag{45}$$

$$(h)\ \ \mathbf{\Gamma}[\Xi_c(3/2^-)\to\Xi_c^*(3/2^+)+\pi]=\frac{3}{2}\times\mathbf{\Gamma}[\Xi_c^0(3/2^-)\to\Xi_c^{*+}(3/2^+)+\pi^-], \tag{46}$$

which can be calculated through the following Lagrangians:

$$\begin{aligned}
(a)\quad & \mathcal{L}_{\Lambda_c[\frac{1}{2}^-]\to\Sigma_c\pi}=g_{\Lambda_c^+[\frac{1}{2}^-]\to\Sigma_c^{++}\pi^-}\bar{\Lambda}_c^+(1/2^-)\Sigma_c^{++}\pi^-+\cdots,\\
(b)\quad & \mathcal{L}_{\Lambda_c[\frac{3}{2}^-]\to\Sigma_c^*\pi}=g_{\Lambda_c^+[\frac{3}{2}^-]\to\Sigma_c^{*++}\pi^-}\bar{\Lambda}_{c\mu}^+(3/2^-)\Sigma_{c\mu}^{*++}\pi^-+\cdots,\\
(c)\quad & \mathcal{L}_{\Xi_c[\frac{1}{2}^-]\to\Xi_c\pi}=g_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^+\pi^-}\bar{\Xi}_c^0(1/2^-)\Xi_c^+\pi^-+\cdots,\\
(d)\quad & \mathcal{L}_{\Xi_c[\frac{1}{2}^-]\to\Lambda_cK}=g_{\Xi_c^0[\frac{1}{2}^-]\to\Lambda_c^+K^-}\bar{\Xi}_c^0(1/2^-)\Lambda_c^+K^-+\cdots,\\
(e)\quad & \mathcal{L}_{\Xi_c[\frac{1}{2}^-]\to\Xi_c\rho}=g_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^+\rho^-}\bar{\Xi}_c^0(1/2^-)\gamma_\mu\gamma_5\Xi_c^+\rho_\mu^-+\cdots,\\
(f)\quad & \mathcal{L}_{\Xi_c[\frac{1}{2}^-]\to\Xi_c'\pi}=g_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c'^+\pi^-}\bar{\Xi}_c^0(1/2^-)\Xi_c'^+\pi^-+\cdots,\\
(g)\quad & \mathcal{L}_{\Xi_c[\frac{3}{2}^-]\to\Xi_c\rho}=g_{\Xi_c^0[\frac{3}{2}^-]\to\Xi_c^+\rho^-}\bar{\Xi}_{c\mu}^0(3/2^-)\Xi_c^+\rho_\mu^-+\cdots,\\
(h)\quad & \mathcal{L}_{\Xi_c[\frac{3}{2}^-]\to\Xi_c^*\pi}=g_{\Xi_c^0[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-}\bar{\Xi}_{c\mu}^0(3/2^-)\Xi_{c\mu}^{*+}\pi^-+\cdots.
\end{aligned} \tag{47}$$

We note that the mass of the $\Lambda_c(2595)$ is above the threshold of $\Sigma_c^+\pi^0$ but below the thresholds of $\Sigma_c^{++}\pi^-$ and $\Sigma_c^0\pi^+$, so we evaluate both its two-body decay $\Lambda_c(2595)\to\Sigma_c^+\pi^0$ and three-body decays $\Lambda_c(2595)\to\Sigma_c^{++}\pi^-\to\Lambda_c^+\pi^+\pi^-$ and $\Lambda_c(2595)\to\Sigma_c^0\pi^+\to\Lambda_c^+\pi^+\pi^-$, using [2]:

$$\Sigma_c^{++,0}:\ m\approx 2453.86\ \text{MeV},\qquad \Sigma_c^+:\ m=2452.9\ \text{MeV},\\ \pi^\pm:\ m=139.57\ \text{MeV},\qquad \pi^0:\ m=134.98\ \text{MeV}.$$

Besides these channels, we also assume masses of the $\Lambda_c(5/2^-)$ and $\Xi_c(5/2^-)$ to be around

$$\Lambda_c(5/2^-):\ m\sim 2850\ \text{MeV},\\ \Xi_c(5/2^-):\ m\sim 3000\ \text{MeV}, \tag{48}$$

so that the following decay channels are kinematically allowed:

$$(i)\ \ \mathbf{\Gamma}[\Lambda_c(5/2^-)\to\Sigma_c^*(3/2^+)+\rho\to\Sigma_c^*(3/2^+)+\pi+\pi] \\ =3\times\mathbf{\Gamma}[\Lambda_c^+(5/2^-)\to\Sigma_c^{*++}(3/2^+)+\rho^- \\ \to\Sigma_c^{*++}(3/2^+)+\pi^0+\pi^-], \tag{49}$$

$$(j)\ \ \mathbf{\Gamma}[\Xi_c(5/2^-)\to\Xi_c^*(3/2^+)+\rho\to\Xi_c^*(3/2^+)+\pi+\pi] \\ =\frac{3}{2}\times\mathbf{\Gamma}[\Xi_c^0(5/2^-)\to\Xi_c^{*+}(3/2^+)+\rho^- \\ \to\Xi_c^{*+}(3/2^+)+\pi^0+\pi^-], \tag{50}$$

and can be calculated through the following Lagrangians:

$$(i)\quad \mathcal{L}_{\Lambda_c[\frac{5}{2}^-]\to\Sigma_c^*\rho} = g_{\Lambda_c^+[\frac{5}{2}^-]\to\Sigma_c^{*++}\rho^-}\bar{\Lambda}^+_{c\mu\nu}(5/2^-)\Sigma^{*++}_{c\mu}\rho^-_\nu + \cdots,$$
$$(j)\quad \mathcal{L}_{\Xi_c[\frac{5}{2}^-]\to\Xi_c^*\rho} = g_{\Xi_c^0[\frac{5}{2}^-]\to\Xi_c^{*+}\rho^-}\bar{\Xi}^0_{c\mu\nu}(5/2^-)\Xi^{*+}_{c\mu}\rho^-_\nu + \cdots. \tag{51}$$

As an example, we shall first study the S-wave decay of the $\Xi_c^0(1/2^-)$ belonging to $[\bar{\mathbf{3}}_F,1,1,\rho]$ into $\Xi_c^{\prime+}(1/2^+)$ and $\pi^-(0^-)$ in the next subsection, and then separately investigate the four charmed baryon multiplets of flavor $\bar{\mathbf{3}}_F$, $[\bar{\mathbf{3}}_F,0,1,\rho]$, $[\bar{\mathbf{3}}_F,1,1,\rho]$, $[\bar{\mathbf{3}}_F,2,1,\rho]$ and $[\bar{\mathbf{3}}_F,1,0,\lambda]$, in the following subsections.

A. $\Xi_c^0(1/2^-)$ of $[\bar{\mathbf{3}}_F,1,1,\rho]$ decaying into $\Xi_c^{\prime+}(1/2^+)$ and $\pi^-(0^-)$

As an example, we evaluate the following three-point correlation function to study the S-wave decay of the $\Xi_c^0(1/2^-)$ belonging to $[\bar{\mathbf{3}}_F,1,1,\rho]$ into $\Xi_c^{\prime+}(1/2^+)$ and $\pi^-(0^-)$:

$$\Pi(\omega,\omega') = \int d^4x e^{-ik\cdot x}\langle 0|J_{1/2,-,\Xi_c^0,1,1,\rho}(0)\bar{J}_{\Xi_c^{\prime+}}(x)|\pi^-\rangle = \frac{1+\not{v}}{2}G_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^{\prime+}\pi^-}(\omega,\omega'), \tag{52}$$

where

$$k' = k+q,\qquad \omega' = v\cdot k',\qquad \omega = v\cdot k. \tag{53}$$

The currents $J_{1/2,-,\Xi_c^0,1,1,\rho}$ and $J_{\Xi_c^{\prime+}}$ have been defined in Eqs. (25) and (10), and couple to $\Xi_c^0(1/2^-)$ belonging to $[\bar{\mathbf{3}}_F,1,1,\rho]$ and $\Xi_c^{\prime+}(1/2^+)$, respectively. The function $G_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^{\prime+}\pi^-}$ has the following pole terms at the hadronic level from double dispersion relation:

$$G_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^{\prime+}\pi^-}(\omega,\omega') = g_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^{\prime+}\pi^-}\times\frac{f_{\Xi_c^0[\frac{1}{2}^-]}f_{\Xi_c^{\prime+}}}{(\bar{\Lambda}_{\Xi_c^0[\frac{1}{2}^-]}-\omega')(\bar{\Lambda}_{\Xi_c^{\prime+}}-\omega)} + \frac{c'}{\bar{\Lambda}_{\Xi_c^0[\frac{1}{2}^-]}-\omega'} + \frac{c}{\bar{\Lambda}_{\Xi_c^{\prime+}}-\omega}, \tag{54}$$

where the S-wave coupling constants $g_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^{\prime+}\pi^-}$ is defined through the following Lagrangian:

$$\mathcal{L}_{\Xi_c[\frac{1}{2}^-]\to\Xi_c^\prime\pi} = g_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^{\prime+}\pi^-}\bar{\Xi}_c^0(1/2^-)\Xi_c^{\prime+}\pi^- + \cdots. \tag{55}$$

c and c' in Eq. (54) are free parameters which can be suppressed by the Borel transformation. The three-point correlation function $\Pi(\omega,\omega')$ can also be calculated at the quark-gluon level using the QCD operator product expansion (in our calculations we have used the software Mathematica with the FeynCalc package [115]):

$$\begin{aligned}
G_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^{\prime+}\pi^-}(\omega,\omega') &= g_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^{\prime+}\pi^-}\times\frac{f_{\Xi_c^0[\frac{1}{2}^-]}f_{\Xi_c^{\prime+}}}{(\bar{\Lambda}_{\Xi_c^0[\frac{1}{2}^-]}-\omega')(\bar{\Lambda}_{\Xi_c^{\prime+}}-\omega)}\\
&= \int_0^\infty dt\int_0^1 du e^{i(1-u)\omega' t}e^{iu\omega t}\times 4\times\bigg(\frac{3if_\pi v\cdot q}{2\pi^2t^4}\phi_{2;\pi}(u) + \frac{3if_\pi v\cdot q}{32\pi^2t^2}\phi_{4;\pi}(u) + \frac{3if_\pi}{2\pi^2t^4v\cdot q}\psi_{4;\pi}(u)\\
&\quad + \frac{if_\pi m_\pi^2 v\cdot q}{24(m_u+m_d)}\langle\bar{s}s\rangle\phi^\sigma_{3;\pi}(u) + \frac{if_\pi m_\pi^2t^2 v\cdot q}{384(m_u+m_d)}\langle g_s\bar{s}\sigma Gs\rangle\phi^\sigma_{3;\pi}(u) + \frac{if_\pi m_\pi^2 v\cdot q}{8\pi^2t^2(m_u+m_d)}m_s\phi^\sigma_{3;\pi}(u)\\
&\quad + \frac{if_\pi v\cdot q}{16}m_s\langle\bar{s}s\rangle\phi_{2;\pi}(u) + \frac{if_\pi t^2 v\cdot q}{256}m_s\langle\bar{s}s\rangle\phi_{4;\pi}(u) + \frac{if_\pi}{16v\cdot q}m_s\langle\bar{s}s\rangle\psi_{4;\pi}(u)\bigg)\\
&\quad + \int_0^\infty dt\int_0^1 du\int\mathcal{D}\underline{\alpha}e^{i\omega' t(\alpha_2+u\alpha_3)}e^{i\omega t(1-\alpha_2-u\alpha_3)}\times 4\times\bigg(\frac{3if_\pi v\cdot q}{8\pi^2t^2}\Phi_{4;\pi}(\underline{\alpha}) - \frac{if_\pi v\cdot q}{4\pi^2t^2}\Psi_{4;\pi}(\underline{\alpha})\\
&\quad + \frac{if_\pi v\cdot q}{8\pi^2t^2}\tilde{\Phi}_{4;\pi}(\underline{\alpha}) + \frac{if_\pi v\cdot q}{4\pi^2t^2}\tilde{\Psi}_{4;\pi}(\underline{\alpha}) + \frac{if_\pi u v\cdot q}{4\pi^2t^2}\Phi_{4;\pi}(\underline{\alpha}) + \frac{if_\pi u v\cdot q}{2\pi^2t^2}\Psi_{4;\pi}(\underline{\alpha})\bigg),
\end{aligned}\tag{56}$$

which contains many light-cone distribution amplitudes, whose definitions and explicit forms can be found in Refs. [89–96]. As examples, we list the light-cone distribution amplitudes of the K meson in Appendix B. Their values can also be found in these references, and in the present study we work at the renormalization scale 1 GeV. The condensates contained in this sum rules take the following values [2,116–123]:

$$\begin{aligned} \langle \bar{q}q \rangle &= -(0.24\ \text{GeV})^3, \\ \langle \bar{s}s \rangle &= (0.8 \pm 0.1) \times \langle \bar{q}q \rangle, \\ \langle g_s^2 GG \rangle &= (0.48 \pm 0.14)\ \text{GeV}^4, \\ \langle g_s \bar{q}\sigma G q \rangle &= M_0^2 \times \langle \bar{q}q \rangle, \\ \langle g_s \bar{s}\sigma G s \rangle &= M_0^2 \times \langle \bar{s}s \rangle, \\ M_0^2 &= 0.8\ \ \text{GeV}^2. \end{aligned} \tag{57}$$

After Wick rotations and making double Borel transformation with the variables ω and ω' to be T_1 and T_2, we obtain

$$\begin{aligned} & g_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c'^+\pi^-} f_{\Xi_c^0[\frac{1}{2}^-]} f_{\Xi_c'^+} e^{-\frac{\bar{\Lambda}_{\Xi_c^0[\frac{1}{2}^-]}}{T_1}} e^{-\frac{\bar{\Lambda}_{\Xi_c'^+}}{T_2}} \\ &= 4 \times \Bigg(-\frac{3 f_\pi}{2\pi^2} T^6 f_5\left(\frac{\omega_c}{T}\right) \frac{d\phi_{2;\pi}(u_0)}{du} + \frac{3 f_\pi}{32\pi^2} T^4 f_3\left(\frac{\omega_c}{T}\right) \frac{d\phi_{4;\pi}(u_0)}{du} - \frac{3 f_\pi}{2\pi^2} T^4 f_3\left(\frac{\omega_c}{T}\right) \int_0^{u_0} \psi_{4;\pi}(u) du \\ &\quad - \frac{f_\pi m_\pi^2}{24(m_u + m_d)} \langle \bar{s}s \rangle T^2 f_1\left(\frac{\omega_c}{T}\right) \frac{d\phi_{3;\pi}^\sigma(u_0)}{du} + \frac{f_\pi m_\pi^2}{384(m_u + m_d)} \langle g_s \bar{s}\sigma G s \rangle \frac{d\phi_{3;\pi}^\sigma(u_0)}{du} \\ &\quad + \frac{f_\pi m_\pi^2 m_s}{8\pi^2(m_u + m_d)} T^4 f_3\left(\frac{\omega_c}{T}\right) \frac{d\phi_{3;\pi}^\sigma(u_0)}{du} - \frac{f_\pi m_s}{16} \langle \bar{s}s \rangle T^2 f_1\left(\frac{\omega_c}{T}\right) \frac{d\phi_{2;\pi}(u_0)}{du} \\ &\quad + \frac{f_\pi m_s}{256} \langle \bar{s}s \rangle \frac{d\phi_{4;\pi}(u_0)}{du} - \frac{f_\pi m_s}{16} \langle \bar{s}s \rangle \frac{d\psi_{4;\pi}(u_0)}{du} \Bigg) \\ &\quad + \int_0^{u_0} d\alpha_2 \times \frac{f_\pi}{2\pi^2 \alpha_3} T^4 f_3\left(\frac{\omega_c}{T}\right) \times (3\Phi_{4;\pi}(\underline{\alpha}) - 2\Psi_{4;\pi}(\underline{\alpha}) + \tilde{\Phi}_{4;\pi}(\underline{\alpha}) + 2\tilde{\Psi}_{4;\pi}(\underline{\alpha}))|_{\alpha_1 = u_0, \alpha_3 = 1 - u_0 - \alpha_2} \\ &\quad - \int_0^{u_0} d\alpha_1 \times \frac{f_\pi}{2\pi^2 \alpha_3} T^4 f_3\left(\frac{\omega_c}{T}\right) \times (3\Phi_{4;\pi}(\underline{\alpha}) - 2\Psi_{4;\pi}(\underline{\alpha}) + \tilde{\Phi}_{4;\pi}(\underline{\alpha}) + 2\tilde{\Psi}_{4;\pi}(\underline{\alpha}))|_{\alpha_2 = u_0, \alpha_3 = 1 - u_0 - \alpha_1} \\ &\quad + \int_0^{u_0} d\alpha_2 \times \frac{f_\pi}{\pi^2 \alpha_3} T^4 f_3\left(\frac{\omega_c}{T}\right) \times (\Phi_{4;\pi}(\underline{\alpha}) + 2\Psi_{4;\pi}(\underline{\alpha}))|_{\alpha_1 = u_0, \alpha_3 = 1 - u_0 - \alpha_2} \\ &\quad - \int_0^{u_0} d\alpha_2 \int_{u_0 - \alpha_2}^{1 - \alpha_2} d\alpha_3 \times \frac{f_\pi}{\pi^2 \alpha_3^2} T^4 f_3\left(\frac{\omega_c}{T}\right) \times (\Phi_{4;\pi}(\underline{\alpha}) + 2\Psi_{4;\pi}(\underline{\alpha}))|_{\alpha_1 = u_0}, \end{aligned} \tag{58}$$

where $u_0 = \frac{T_1}{T_1+T_2}$, $T = \frac{T_1 T_2}{T_1+T_2}$ and $f_n(x) = 1 - e^{-x} \sum_{k=0}^n \frac{x^k}{k!}$.

We work at the symmetric point $T_1 = T_2 = 2T$, so $u_0 = 1/2$. Now the coupling constant $g_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c'^+\pi^-}$ only depends on two free parameters, the threshold value ω_c and the Borel mass T. After choosing $\omega_c = 1.60$ GeV [the average of the thresholds of the $\Xi_c(1/2^-)$ and $\Xi_c'^+$ mass sum rules], we show $g_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c'^+\pi^-}$ as a function of T in Fig. 1. The working region for T has been reevaluated and listed in Table II to be $0.27\ \text{GeV} < T < 0.32\ \text{GeV}$, where we obtain

$$(f) g_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c'^+\pi^-} = 0.21\,^{-0.07}_{+0.15} = 0.21\,^{+0.03}_{-0.01}\,^{+0.06}_{-0.04}\,^{+0.13}_{-0.06}\,^{+0.00}_{-0.00}. \tag{59}$$

Using this value and the parameters listed in Sec. II, we further obtain

$$\begin{aligned} (f)\Gamma_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c'^+\pi^-} &= 1.6\,^{+2.7}_{-0.9}\ \text{MeV} \\ &= 1.6\,^{+0.5}_{-0.1}\,^{+1.0}_{-0.5}\,^{+2.5}_{-0.8}\,^{+0.00}_{-0.00}\ \text{MeV}, \end{aligned} \tag{60}$$

where the uncertainties mainly come from the Borel mass ($0.27\ \text{GeV} < T < 0.32\ \text{GeV}$), the parameters of the $\Xi_c'^+$ ($\omega_{\Xi_c'^+} = 1.4 \pm 0.1$ GeV, $\bar{\Lambda}_{\Xi_c'^+} = 1.042 \pm 0.080$ GeV, and $f_{\Xi_c'^+} = 0.0435 \pm 0.0080\,\text{GeV}^3$), the parameters of the $\Xi_c^0[\frac{1}{2}^-]$

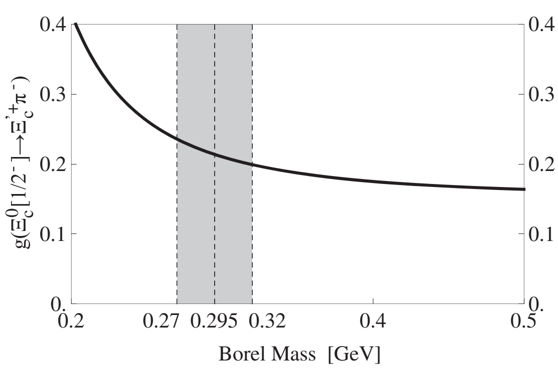

FIG. 1. The coupling constant $g_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c'^+\pi^-}$ as a function of the Borel mass T. The current $J_{1/2,-,\Xi_c^0,1,1,\rho}$ belonging to the baryon doublet $[\bar{\mathbf{3}}_F, 1, 1, \rho]$ is used here.

($\omega_{\Xi_c^0[\frac{1}{2}^-]} = 1.8 \pm 0.1$ GeV, $\bar{\Lambda}_{\Xi_c^0[\frac{1}{2}^-]} = 1.349 \pm 0.130$ GeV, and $f_{\Xi_c^0[\frac{1}{2}^-]} = 0.0788 \pm 0.0280$ GeV^4), and various quark masses and condensates listed in Eq. (57), respectively. We note that the $\mathcal{O}(1/m_Q)$ corrections (m_Q is the heavy-quark mass) have not been considered in the present study, which can cause some theoretical uncertainties [but the $\mathcal{O}(1/m_Q)$ corrections to the masses of the heavy baryons have been taken into account in Refs. [50–53]]. Totally, the results can be 3 times larger or smaller than those we have obtained, i.e., $\Gamma_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^{\prime+}\pi^-} = 1.6^{+200\%}_{-67\%}$ MeV. We shall not estimate the uncertainties of other coupling constants, but just note that their uncertainties are at the same level.

Following these procedures, we separately investigate the four multiplets of flavor $\bar{\mathbf{3}}_F$, $[\bar{\mathbf{3}}_F, 0, 1, \rho]$, $[\bar{\mathbf{3}}_F, 1, 1, \rho]$, $[\bar{\mathbf{3}}_F, 2, 1, \rho]$ and $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$, in the following subsections.

B. The baryon singlet $[\bar{\mathbf{3}}_F, \mathbf{0}, \mathbf{1}, \boldsymbol{\rho}]$

The $[\bar{\mathbf{3}}_F, 0, 1, \rho]$ multiplet contains $\Lambda_c(\frac{1}{2}^-)$ and $\Xi_c(\frac{1}{2}^-)$. Their sum rules are listed in Appendix C 1, suggesting that their possible decay channels are (c) and (d), while the other three channels (a), (e) and (f) vanish. We show the two coupling constants, $g_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^+\pi^-}$ and $g_{\Xi_c^0[\frac{1}{2}^-]\to\Lambda_c^+K^-}$, as functions of the Borel mass T in Fig. 2. Using the values of T listed in Table II, we obtain

$$(c) \quad g_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^+\pi^-} = 2.3,$$
$$(d) \quad g_{\Xi_c^0[\frac{1}{2}^-]\to\Lambda_c^+K^-} = 2.7. \tag{61}$$

Using these values and the parameters listed in Sec. II, we further obtain

$$(c) \quad \Gamma_{\Xi_c[\frac{1}{2}^-]\to\Xi_c\pi} = 300 \text{ MeV},$$
$$(d) \quad \Gamma_{\Xi_c[\frac{1}{2}^-]\to\Lambda_cK} = 82 \text{ MeV}. \tag{62}$$

C. The baryon doublet $[\bar{\mathbf{3}}_F, \mathbf{1}, \mathbf{1}, \boldsymbol{\rho}]$

The $[\bar{\mathbf{3}}_F, 1, 1, \rho]$ multiplet contains $\Lambda_c(\frac{1}{2}^-/\frac{3}{2}^-)$ and $\Xi_c(\frac{1}{2}^-/\frac{3}{2}^-)$. Their sum rules are listed in Appendix C 2, suggesting that their possible decay channels are (a), (b), (e), (f), (g) and (h), while the other two channels (c) and (d) vanish. We show the six coupling constants, $g_{\Lambda_c^+[\frac{1}{2}^-]\to\Sigma_c^{++}\pi^-}$, $g_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^{\prime+}\pi^-}$, $g_{\Lambda_c^+[\frac{3}{2}^-]\to\Sigma_c^{*++}\pi^-}$, $g_{\Xi_c^0[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-}$, $g_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^+\rho^-}$ and $g_{\Xi_c^0[\frac{3}{2}^-]\to\Xi_c^+\rho^-}$, as functions of the Borel mass T in Fig. 3. Using the values of T listed in Table II, we obtain

$$(a) \quad g_{\Lambda_c^+[\frac{1}{2}^-]\to\Sigma_c^{++}\pi^-} = 0.25,$$
$$(f) \quad g_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^{\prime+}\pi^-} = 0.21,$$
$$(b) \quad g_{\Lambda_c^+[\frac{3}{2}^-]\to\Sigma_c^{*++}\pi^-} = 0.033,$$
$$(h) \quad g_{\Xi_c^0[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-} = 0.024,$$
$$(e) \quad g_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^+\rho^-} = 0.11,$$
$$(g) \quad g_{\Xi_c^0[\frac{3}{2}^-]\to\Xi_c^+\rho^-} = 0.074. \tag{63}$$

Using these values and the parameters listed in Sec. II, we further obtain

$$(a, a') \quad \Gamma_{\Lambda_c[\frac{1}{2}^-]\to\Sigma_c\pi(\to\Lambda\pi\pi)} = 0.39 \text{ MeV},$$
$$(f) \quad \Gamma_{\Xi_c[\frac{1}{2}^-]\to\Xi_c'\pi} = 1.6 \text{ MeV},$$
$$(b) \quad \Gamma_{\Lambda_c[\frac{3}{2}^-]\to\Sigma_c^*\pi\to\Lambda_c\pi\pi} = 4\times10^{-4} \text{ MeV},$$
$$(h) \quad \Gamma_{\Xi_c[\frac{3}{2}^-]\to\Xi_c^*\pi} = 0.01 \text{ MeV},$$
$$(e) \quad \Gamma_{\Xi_c[\frac{1}{2}^-]\to\Xi_c\rho\to\Xi_c\pi\pi} = 3\times10^{-5} \text{ MeV},$$
$$(g) \quad \Gamma_{\Xi_c[\frac{3}{2}^-]\to\Xi_c\rho\to\Xi_c\pi\pi} = 5\times10^{-5} \text{ MeV}. \tag{64}$$

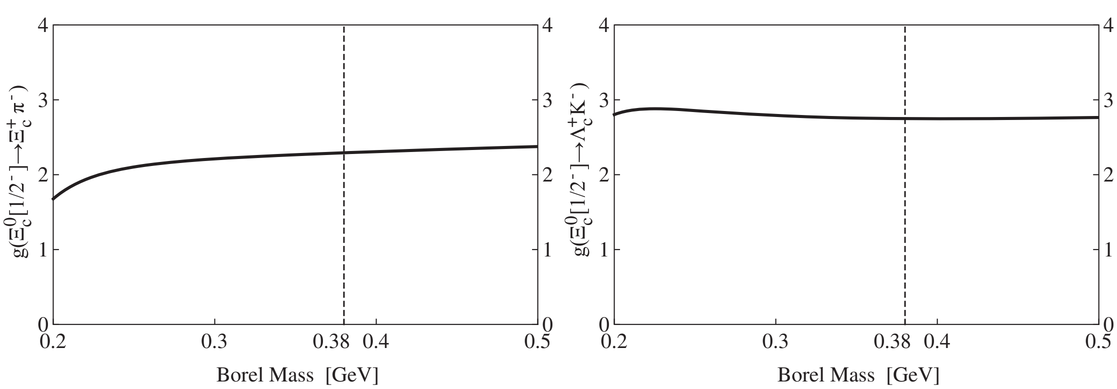

FIG. 2. The coupling constants $g_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^+\pi^-}$ and $g_{\Xi_c^0[\frac{1}{2}^-]\to\Lambda_c^+K^-}$ as functions of the Borel mass T. The currents belonging to the baryon singlet $[\bar{\mathbf{3}}_F, 0, 1, \rho]$ are used here.

D. The baryon doublet $[\bar{\mathbf{3}}_F, 2, 1, \rho]$

The $[\bar{\mathbf{3}}_F, 2, 1, \rho]$ multiplet contains $\Lambda_c(\frac{3}{2}^-/\frac{5}{2}^-)$ and $\Xi_c^+(\frac{3}{2}^-/\frac{5}{2}^-)$. Their sum rules are listed in Appendix C 3, suggesting that their possible decay channels are (b), (h), (i) and (j), while the other channel (g) vanishes. We show the four coupling constants, $g_{\Lambda_c^+[\frac{3}{2}^-]\to\Sigma_c^{*++}\pi^-}$, $g_{\Xi_c^0[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-}$, $g_{\Lambda_c^+[\frac{5}{2}^-]\to\Sigma_c^{*++}\rho^-}$ and $g_{\Xi_c^0[\frac{5}{2}^-]\to\Xi_c^{*+}\rho^-}$, as functions of the Borel mass T in Fig. 4. Using the values of T listed in Table II, we obtain

$$
\begin{aligned}
(b)\quad & g_{\Lambda_c^+[\frac{3}{2}^-]\to\Sigma_c^{*++}\pi^-} = 0.25,\\
(h)\quad & g_{\Xi_c^0[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-} = 0.17,\\
(i)\quad & g_{\Lambda_c^+[\frac{5}{2}^-]\to\Sigma_c^{*++}\rho^-} = 2.3,\\
(j)\quad & g_{\Xi_c^0[\frac{5}{2}^-]\to\Xi_c^{*+}\rho^-} = 2.0.
\end{aligned}
\tag{65}
$$

Using these values and the parameters listed in Sec. II, we further obtain

$$
\begin{aligned}
(b)\quad & \Gamma_{\Lambda_c[\frac{3}{2}^-]\to\Sigma_c^*\pi\to\Lambda_c\pi\pi} = 0.03\ \mathrm{MeV},\\
(h)\quad & \Gamma_{\Xi_c[\frac{3}{2}^-]\to\Xi_c^*\pi} = 0.69\ \mathrm{MeV},\\
(i)\quad & \Gamma_{\Lambda_c[\frac{5}{2}^-]\to\Sigma_c^*\rho\to\Sigma_c^*\pi\pi} = 11\ \mathrm{MeV},\\
(j)\quad & \Gamma_{\Xi_c[\frac{5}{2}^-]\to\Xi_c^*\rho\to\Xi_c^*\pi\pi} = 12\ \mathrm{MeV}.
\end{aligned}
\tag{66}
$$

We note that the two decay widths, $\Gamma_{\Lambda_c[\frac{5}{2}^-]\to\Sigma_c^*\rho}$ and $\Gamma_{\Xi_c[\frac{5}{2}^-]\to\Xi_c^*\rho}$, do depend significantly on the masses of the $\Lambda_c(5/2^-)$ and $\Xi_c(5/2^-)$, which we assumed to be around 2850 and 3000 MeV in Eq. (48). Because their physical masses (if they exist) are possibly smaller than these values, these two decay channels might be kinematically forbidden.

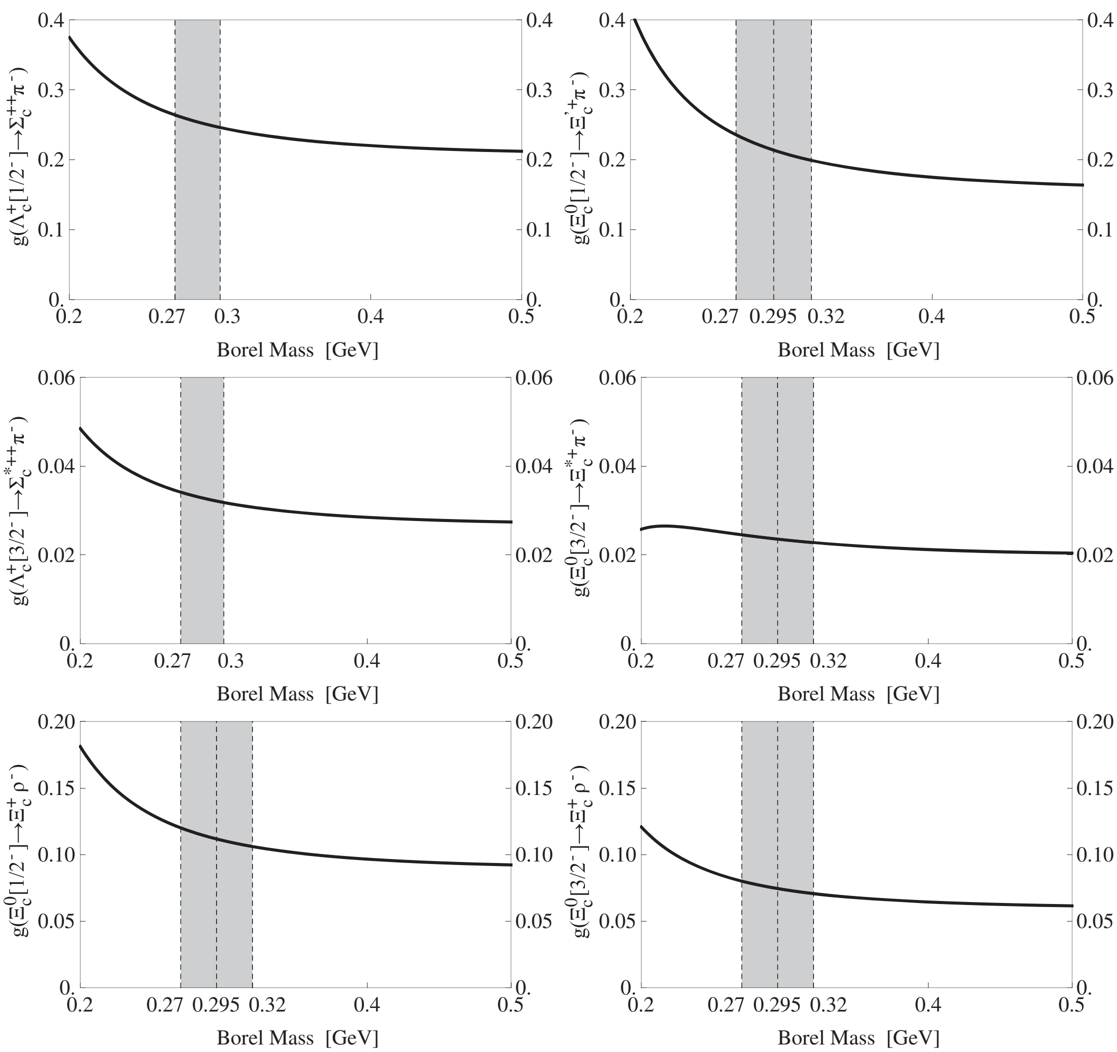

FIG. 3. The coupling constants $g_{\Lambda_c^+[\frac{1}{2}^-]\to\Sigma_c^{++}\pi^-}$ (top-left), $g_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^{\prime+}\pi^-}$ (top-right), $g_{\Lambda_c^+[\frac{3}{2}^-]\to\Sigma_c^{*++}\pi^-}$ (middle-left), $g_{\Xi_c^0[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-}$ (middle-right), $g_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^+\rho^-}$ (bottom-left) and $g_{\Xi_c^0[\frac{3}{2}^-]\to\Xi_c^+\rho^-}$ (bottom-right) as functions of the Borel mass T. The currents belonging to the baryon doublet $[\bar{\mathbf{3}}_F, 1, 1, \rho]$ are used here.

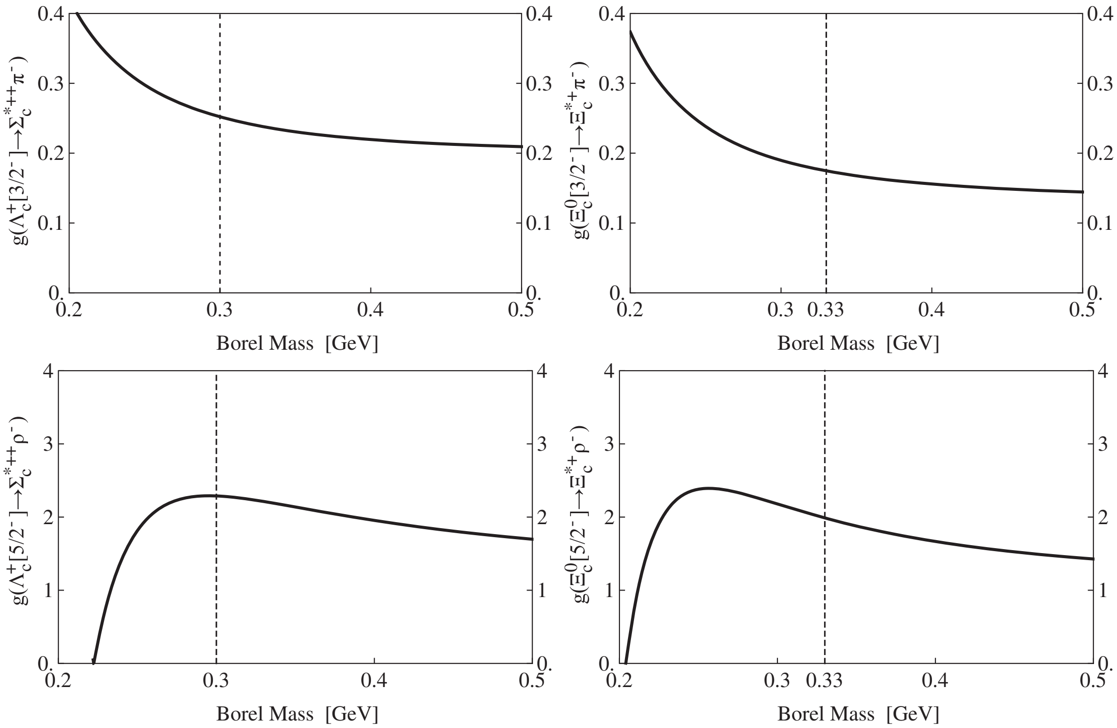

FIG. 4. The coupling constants $g_{\Lambda_c^+[\frac{3}{2}^-]\to\Sigma_c^{*++}\pi^-}$ (top-left), $g_{\Xi_c^0[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-}$ (top-right), $g_{\Lambda_c^+[\frac{5}{2}^-]\to\Sigma_c^{*++}\rho^-}$ (bottom-left) and $g_{\Xi_c^0[\frac{5}{2}^-]\to\Xi_c^{*+}\rho^-}$ (bottom-right) as functions of the Borel mass T. The currents belonging to the baryon doublet $[\bar{\mathbf{3}}_F, 2, 1, \rho]$ are used here.

E. The baryon doublet $[\bar{\mathbf{3}}_F, \mathbf{1}, \mathbf{0}, \boldsymbol{\lambda}]$

The $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$ multiplet contains $\Lambda_c(\frac{1}{2}^-/\frac{3}{2}^-)$ and $\Xi_c(\frac{1}{2}^-/\frac{3}{2}^-)$. Their sum rules are listed in Appendix C 4, suggesting that their possible decay channels are (a), (b), (e), (f), (g) and (h), while the other two channels (c) and (d) vanish. We show the six coupling constants, $g_{\Lambda_c^+[\frac{1}{2}^-]\to\Sigma_c^{++}\pi^-}$, $g_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^{\prime+}\pi^-}$, $g_{\Lambda_c^+[\frac{3}{2}^-]\to\Sigma_c^{*++}\pi^-}$, $g_{\Xi_c^0[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-}$, $g_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^+\rho^-}$ and $g_{\Xi_c^0[\frac{3}{2}^-]\to\Xi_c^+\rho^-}$, as functions of the Borel mass T in Fig. 5. Using the values of T listed in Table II, we obtain

$$\begin{aligned}
(a)&\quad g_{\Lambda_c^+[\frac{1}{2}^-]\to\Sigma_c^{++}\pi^-} = 2.3,\\
(f)&\quad g_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^{\prime+}\pi^-} = 1.7,\\
(b)&\quad g_{\Lambda_c^+[\frac{3}{2}^-]\to\Sigma_c^{*++}\pi^-} = 1.5,\\
(h)&\quad g_{\Xi_c^0[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-} = 1.2,\\
(e)&\quad g_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^+\rho^-} = 4.5,\\
(g)&\quad g_{\Xi_c^0[\frac{3}{2}^-]\to\Xi_c^+\rho^-} = 5.2.
\end{aligned}\tag{67}$$

Using these values and the parameters listed in Sec. II, we further obtain

$$\begin{aligned}
(a,a')&\quad \Gamma_{\Lambda_c[\frac{1}{2}^-]\to\Sigma_c\pi(\to\Lambda\pi\pi)} = 32\text{ MeV},\\
(f)&\quad \Gamma_{\Xi_c[\frac{1}{2}^-]\to\Xi_c'\pi} = 100\text{ MeV},\\
(b)&\quad \Gamma_{\Lambda_c[\frac{3}{2}^-]\to\Sigma_c^*\pi\to\Lambda_c\pi\pi} = 0.96\text{ MeV},\\
(h)&\quad \Gamma_{\Xi_c[\frac{3}{2}^-]\to\Xi_c^*\pi} = 30\text{ MeV},\\
(e)&\quad \Gamma_{\Xi_c[\frac{1}{2}^-]\to\Xi_c\rho\to\Xi_c\pi\pi} = 0.04\text{ MeV},\\
(g)&\quad \Gamma_{\Xi_c[\frac{3}{2}^-]\to\Xi_c\rho\to\Xi_c\pi\pi} = 0.23\text{ MeV}.
\end{aligned}\tag{68}$$

IV. DECAY PROPERTIES OF FLAVOR $\mathbf{6}_F$ P-WAVE CHARMED BARYONS

In this section we use the method of light-cone QCD sum rules to study decay properties of the flavor $\mathbf{6}_F$ P-wave charmed baryons. We only study their S-wave decays into ground-state charmed baryons accompanied by a pseudoscalar meson (π or K), including both two-body and three-body decays which are kinematically allowed. We shall study their S-wave decays into ground-state charmed baryons accompanied by a vector meson (ρ or K^*) in our future work, but note that the widths of these decays are probably quite small (see the results of the flavor $\bar{\mathbf{3}}_F$ P-wave charmed baryons).

The possible decay channels are

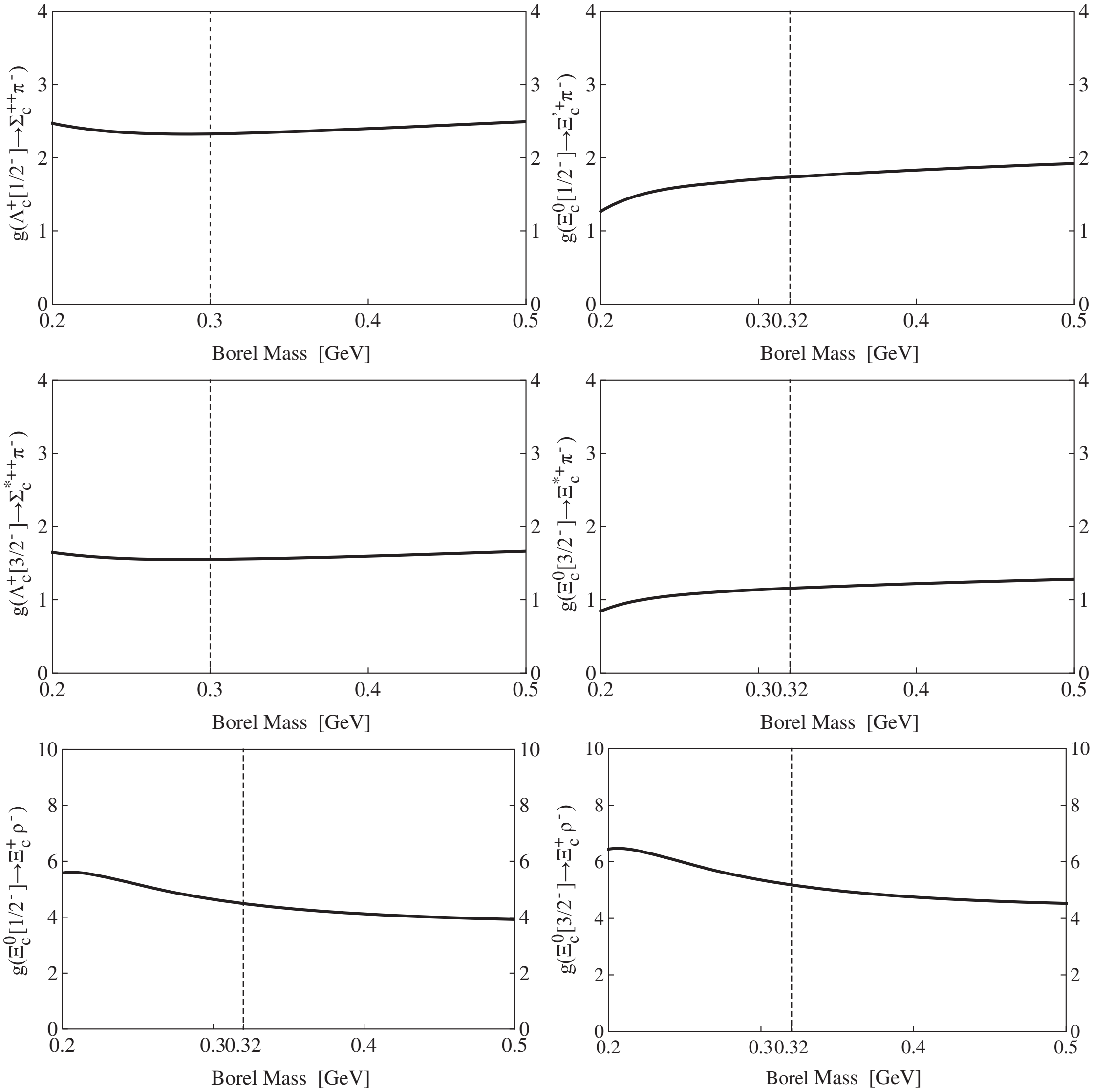

FIG. 5. The coupling constants $g_{\Lambda_c^+[\frac{1}{2}^-]\to\Sigma_c^{++}\pi^-}$ (top-left), $g_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^{\prime +}\pi^-}$ (top-right), $g_{\Lambda_c^+[\frac{3}{2}^-]\to\Sigma_c^{*++}\pi^-}$ (middle-left), $g_{\Xi_c^0[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-}$ (middle-right), $g_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^+\rho^-}$ (bottom-left) and $g_{\Xi_c^0[\frac{3}{2}^-]\to\Xi_c^+\rho^-}$ (bottom-right) as functions of the Borel mass T. The currents belonging to the baryon doublet $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$ are used here.

$$(k)\quad \mathbf{\Gamma}[\Sigma_c(1/2^-)\to\Lambda_c(1/2^+)+\pi] = \mathbf{\Gamma}[\Sigma_c^0(1/2^-)\to\Lambda_c^+(1/2^+)+\pi^-], \tag{69}$$

$$(l)\quad \mathbf{\Gamma}[\Sigma_c(1/2^-)\to\Sigma_c(1/2^+)+\pi] = 2\times\mathbf{\Gamma}[\Sigma_c^0(1/2^-)\to\Sigma_c^+(1/2^+)+\pi^-], \tag{70}$$

$$(m)\quad \mathbf{\Gamma}[\Xi_c'(1/2^-)\to\Xi_c(1/2^+)+\pi] = \frac{3}{2}\times\mathbf{\Gamma}[\Xi_c'^0(1/2^-)\to\Xi_c^+(1/2^+)+\pi^-], \tag{71}$$

$$(n)\quad \mathbf{\Gamma}[\Xi_c'(1/2^-)\to\Lambda_c(1/2^+)+K] = \mathbf{\Gamma}[\Xi_c'^0(1/2^-)\to\Lambda_c^+(1/2^+)+K^-], \tag{72}$$

$$(o)\quad \mathbf{\Gamma}[\Xi_c'(1/2^-)\to\Xi_c'(1/2^+)+\pi] = \frac{3}{2}\times\mathbf{\Gamma}[\Xi_c'^0(1/2^-)\to\Xi_c'^+(1/2^+)+\pi^-], \tag{73}$$

$$(p)\quad \mathbf{\Gamma}[\Xi_c'(1/2^-)\to\Sigma_c(1/2^+)+K] = 3\times\mathbf{\Gamma}[\Xi_c'^0(1/2^-)\to\Sigma_c^+(1/2^+)+K^-], \tag{74}$$

$$(q)\quad \mathbf{\Gamma}[\Omega_c(1/2^-)\to\Xi_c(1/2^+)+K] = 2\times\mathbf{\Gamma}[\Omega_c^0(1/2^-)\to\Xi_c^+(1/2^+)+K^-], \tag{75}$$

$$(r)\quad \mathbf{\Gamma}[\Omega_c(1/2^-)\to\Xi_c'(1/2^+)+K]=2\times\mathbf{\Gamma}[\Omega_c^0(1/2^-)\to\Xi_c'^+(1/2^+)+K^-],\tag{76}$$

$$(s)\quad \mathbf{\Gamma}[\Sigma_c(3/2^-)\to\Sigma_c^*(3/2^+)+\pi]=2\times\mathbf{\Gamma}[\Sigma_c^0(3/2^-)\to\Sigma_c^{*+}(3/2^+)+\pi^-],\tag{77}$$

$$(t)\quad \mathbf{\Gamma}[\Xi_c'(3/2^-)\to\Xi_c^*(3/2^+)+\pi]=\frac{3}{2}\times\mathbf{\Gamma}[\Xi_c'^0(3/2^-)\to\Xi_c^{*+}(3/2^+)+\pi^-].\tag{78}$$

$$(u)\quad \mathbf{\Gamma}[\Xi_c'(3/2^-)\to\Sigma_c^*(3/2^+)+K\to\Lambda_c(1/2^+)+\pi+K] \\ =3\times\mathbf{\Gamma}[\Xi_c'^0(3/2^-)\to\Sigma_c^{*+}(3/2^+)+K^-\to\Lambda_c^+(3/2^+)+\pi^0+K^-].\tag{79}$$

$$(v)\quad \mathbf{\Gamma}[\Omega_c(3/2^-)\to\Xi_c^*(3/2^+)+K\to\Xi_c(1/2^+)+\pi+K] \\ =6\times\mathbf{\Gamma}[\Omega_c^0(3/2^-)\to\Xi_c^{*+}(3/2^+)+K^-\to\Xi_c^+(1/2^+)+\pi^0+K^-].\tag{80}$$

Their widths can be simply calculated through the two Lagrangians $\mathcal{L}_{X(1/2^-)\to Y(1/2^+)P}=g\bar{X}YP$ and $\mathcal{L}_{X(3/2^-)\to Y(3/2^+)P}=g\bar{X}_\mu Y_\mu P$. Especially, we assume the mass of the $\Omega_c(3/2^-)$ state to be 3120 MeV in the case (v) in order to make this decay channel kinematically allowed, but still use 3100 MeV for other cases.

In the following subsections, we shall separately investigate the four P-wave charmed baryon multiplets of flavor $\mathbf{6}_F$, $[\mathbf{6}_F,1,0,\rho]$, $[\mathbf{6}_F,0,1,\lambda]$, $[\mathbf{6}_F,1,1,\lambda]$ and $[\mathbf{6}_F,2,1,\lambda]$.

A. The baryon doublet $[\mathbf{6}_F,1,0,\rho]$

The $[\mathbf{6}_F,1,0,\rho]$ multiplet contains $\Sigma_c(\frac{1}{2}^-/\frac{3}{2}^-)$, $\Xi_c'(\frac{1}{2}^-/\frac{3}{2}^-)$ and $\Omega_c(\frac{1}{2}^-/\frac{3}{2}^-)$. Their sum rules are listed in Appendix C 5, suggesting that their possible decay channels are (l), (o), (p), (r), (s), (t), (u) and (v), while the other four channels (k), (m), (n) and (q) vanish. We show the eight coupling constants, $g_{\Sigma_c^0[\frac{1}{2}^-]\to\Sigma_c^+\pi^-}$, $g_{\Xi_c'^0[\frac{1}{2}^-]\to\Xi_c'^+\pi^-}$, $g_{\Xi_c'^0[\frac{1}{2}^-]\to\Sigma_c^+K^-}$, $g_{\Omega_c^0[\frac{1}{2}^-]\to\Xi_c'^+K^-}$, $g_{\Sigma_c^0[\frac{3}{2}^-]\to\Sigma_c^{*+}\pi^-}$, $g_{\Xi_c'^0[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-}$, $g_{\Xi_c'^0[\frac{3}{2}^-]\to\Sigma_c^{*+}K^-}$ and $g_{\Omega_c^0[\frac{3}{2}^-]\to\Xi_c^{*+}K^-}$, as functions of the Borel mass T in Fig. 6. Using the values of T listed in Table III, we obtain

$$\begin{aligned}
(l)&\quad g_{\Sigma_c^0[\frac{1}{2}^-]\to\Sigma_c^+\pi^-}=1.9,\\
(o)&\quad g_{\Xi_c'^0[\frac{1}{2}^-]\to\Xi_c'^+\pi^-}=1.4,\\
(p)&\quad g_{\Xi_c'^0[\frac{1}{2}^-]\to\Sigma_c^+K^-}=1.7,\\
(r)&\quad g_{\Omega_c^0[\frac{1}{2}^-]\to\Xi_c'^+K^-}=2.5,\\
(s)&\quad g_{\Sigma_c^0[\frac{3}{2}^-]\to\Sigma_c^{*+}\pi^-}=1.3,\\
(t)&\quad g_{\Xi_c'^0[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-}=0.95,\\
(u)&\quad g_{\Xi_c'^0[\frac{3}{2}^-]\to\Sigma_c^{*+}K^-}=1.1,\\
(v)&\quad g_{\Omega_c^0[\frac{3}{2}^-]\to\Xi_c^{*+}K^-}=1.7.
\end{aligned}\tag{81}$$

Using these values and the parameters listed in Sec. II, we further obtain

$$\begin{aligned}
(l)&\quad \Gamma_{\Sigma_c[\frac{1}{2}^-]\to\Sigma_c\pi}=300\text{ MeV},\\
(o)&\quad \Gamma_{\Xi_c'[\frac{1}{2}^-]\to\Xi_c'\pi}=140\text{ MeV},\\
(p)&\quad \Gamma_{\Xi_c'[\frac{1}{2}^-]\to\Sigma_cK}=29\text{ MeV},\\
(r)&\quad \Gamma_{\Omega_c[\frac{1}{2}^-]\to\Xi_c'K}=250\text{ MeV},\\
(s)&\quad \Gamma_{\Sigma_c[\frac{3}{2}^-]\to\Sigma_c^*\pi}=110\text{ MeV},\\
(t)&\quad \Gamma_{\Xi_c'[\frac{3}{2}^-]\to\Xi_c^*\pi}=50\text{ MeV},\\
(u)&\quad \Gamma_{\Xi_c'[\frac{3}{2}^-]\to\Sigma_c^*K\to\Lambda_c\pi K}=0.03\text{ MeV},\\
(v)&\quad \Gamma_{\Omega_c[\frac{3}{2}^-]\to\Xi_c^*K\to\Xi_c\pi K}=0.07\text{ MeV}.
\end{aligned}\tag{82}$$

B. The baryon doublet $[\mathbf{6}_F,0,1,\lambda]$

The $[\mathbf{6}_F,0,1,\lambda]$ multiplet contains $\Sigma_c(\frac{1}{2}^-)$, $\Xi_c'(\frac{1}{2}^-)$ and $\Omega_c(\frac{1}{2}^-)$. Their sum rules are listed in Appendix C 6, suggesting that their possible decay channels are (k), (m), (n) and (q), while the other four channels (l), (o), (p) and (r) vanish. We show the four coupling constants, $g_{\Sigma_c^0[\frac{1}{2}^-]\to\Lambda_c^+\pi^-}$, $g_{\Xi_c'^0[\frac{1}{2}^-]\to\Xi_c^+\pi^-}$, $g_{\Xi_c'^0[\frac{1}{2}^-]\to\Lambda_c^+K^-}$ and $g_{\Omega_c^0[\frac{1}{2}^-]\to\Xi_c^+K^-}$, as functions of the Borel mass T in Fig. 7. Using the values of T listed in Table III, we obtain

$$\begin{aligned}
(k)&\quad g_{\Sigma_c^0[\frac{1}{2}^-]\to\Lambda_c^+\pi^-}=1.8,\\
(m)&\quad g_{\Xi_c'^0[\frac{1}{2}^-]\to\Xi_c^+\pi^-}=1.7,\\
(n)&\quad g_{\Xi_c'^0[\frac{1}{2}^-]\to\Lambda_c^+K^-}=1.8,\\
(q)&\quad g_{\Omega_c^0[\frac{1}{2}^-]\to\Xi_c^+K^-}=3.0.
\end{aligned}\tag{83}$$

Using these values and the parameters listed in Sec. II, we further obtain

$$\begin{aligned}
(k)&\quad \Gamma_{\Sigma_c[\frac{1}{2}^-]\to\Lambda_c\pi}=200\text{ MeV},\\
(m)&\quad \Gamma_{\Xi_c'[\frac{1}{2}^-]\to\Xi_c\pi}=230\text{ MeV},\\
(n)&\quad \Gamma_{\Xi_c'[\frac{1}{2}^-]\to\Lambda_cK}=160\text{ MeV},\\
(q)&\quad \Gamma_{\Omega_c[\frac{1}{2}^-]\to\Xi_cK}=820\text{ MeV}.
\end{aligned}\tag{84}$$

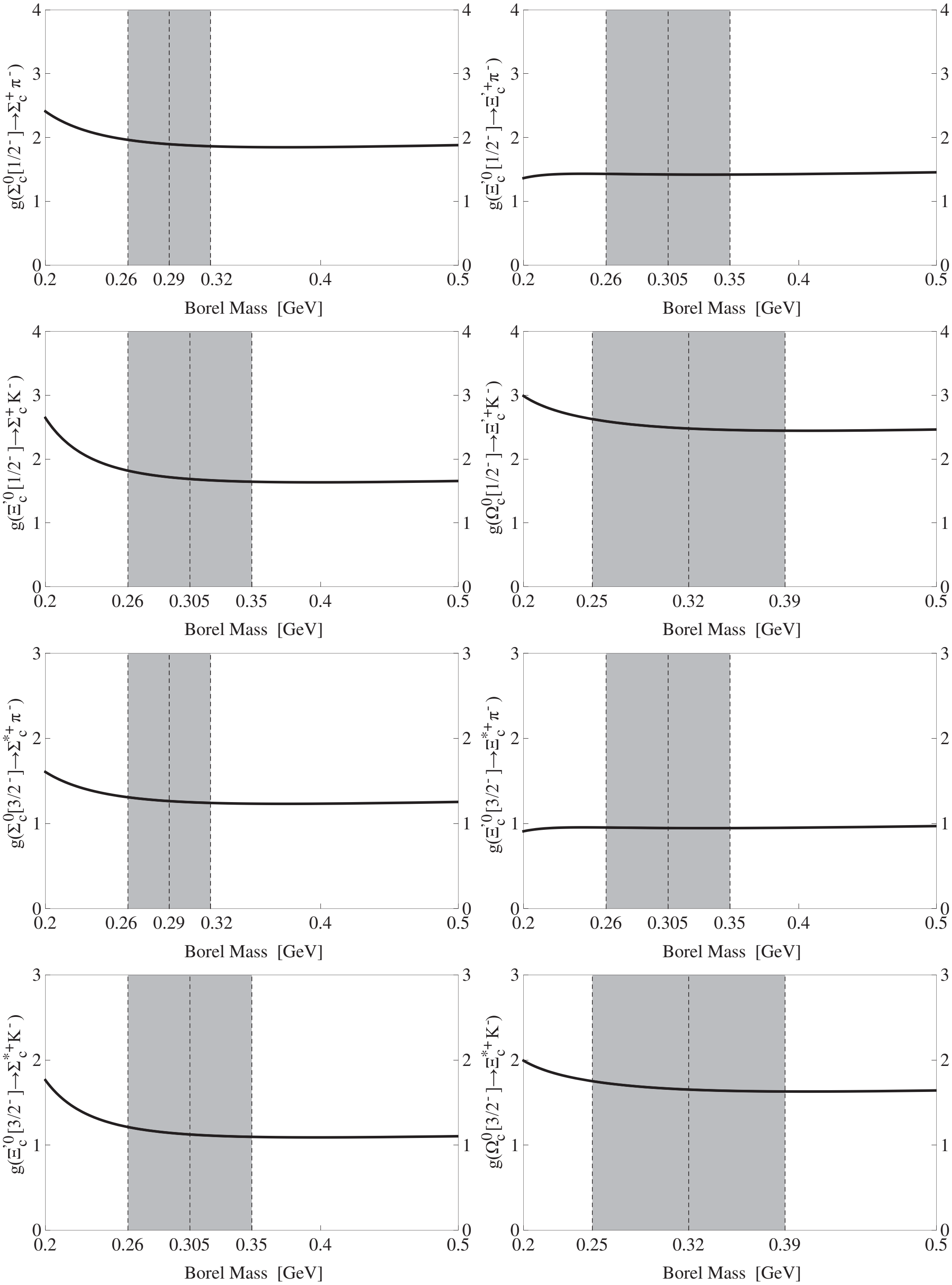

FIG. 6. The coupling constants $g_{\Sigma_c^0[\frac{1}{2}^-]\to\Sigma_c^+\pi^-}$ (top-left), $g_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Xi_c^{\prime +}\pi^-}$ (top-right), $g_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Sigma_c^+K^-}$ (middle-left), $g_{\Omega_c^0[\frac{1}{2}^-]\to\Xi_c^{\prime +}K^-}$ (middle-right), $g_{\Sigma_c^0[\frac{3}{2}^-]\to\Sigma_c^{*+}\pi^-}$ (middle-left), $g_{\Xi_c^{\prime 0}[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-}$ (middle-right), $g_{\Xi_c^{\prime 0}[\frac{3}{2}^-]\to\Sigma_c^{*+}K^-}$ (bottom-left) and $g_{\Omega_c^0[\frac{3}{2}^-]\to\Xi_c^{*+}K^-}$ (bottom-right) as functions of the Borel mass T. The currents belonging to the baryon doublet $[\mathbf{6}_F, 1, 0, \rho]$ are used here.

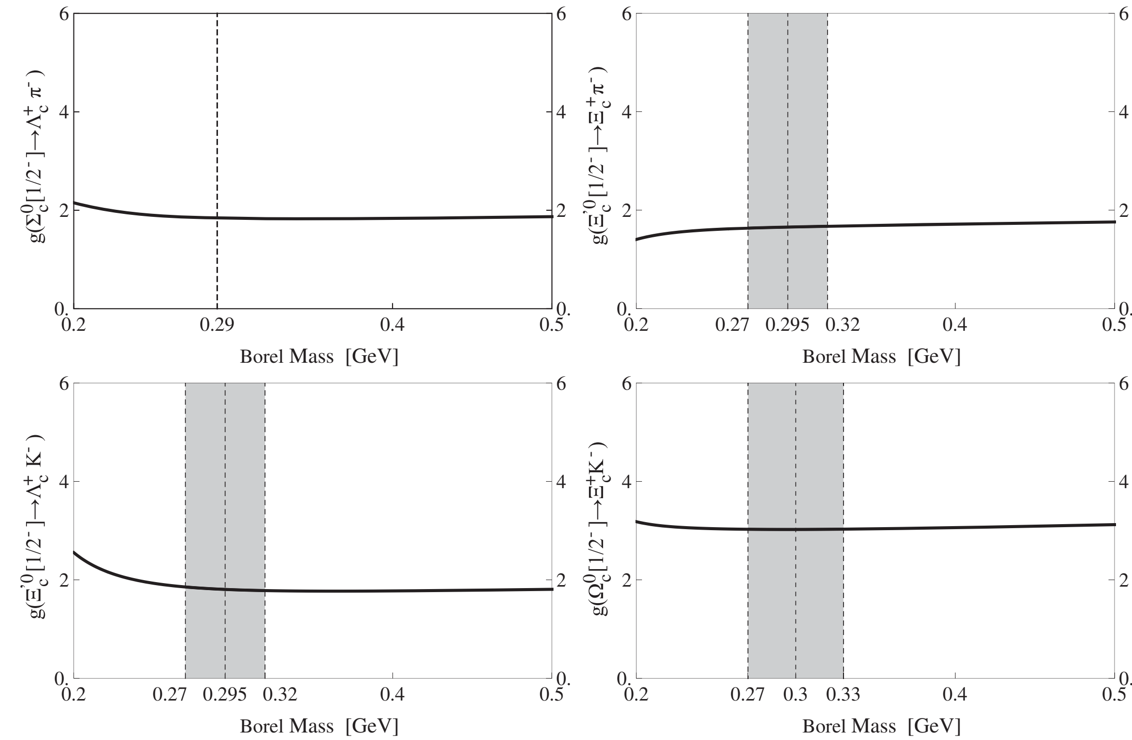

FIG. 7. The coupling constants $g_{\Sigma_c^0[\frac{1}{2}^-]\to\Lambda_c^+\pi^-}$ (top-left), $g_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Xi_c^+\pi^-}$ (top-right), $g_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Lambda_c^+K^-}$ (bottom-left) and $g_{\Omega_c^0[\frac{1}{2}^-]\to\Xi_c^+K^-}$ (bottom-right) as functions of the Borel mass T. The currents belonging to the baryon doublet $[\mathbf{6}_F, 0, 1, \lambda]$ are used here.

C. The baryon doublet $[\mathbf{6}_F,\mathbf{1},\mathbf{1},\lambda]$

The $[\mathbf{6}_F, 1, 1, \lambda]$ multiplet contains $\Sigma_c(\frac{1}{2}^-/\frac{3}{2}^-)$, $\Xi_c^\prime(\frac{1}{2}^-/\frac{3}{2}^-)$ and $\Omega_c(\frac{1}{2}^-/\frac{3}{2}^-)$. Their sum rules are listed in Appendix C 7, suggesting that their possible decay channels are (l), (o), (p), (r), (s), (t), (u) and (v), while the other four channels (k), (m), (n) and (q) vanish. We show the eight coupling constants, $g_{\Sigma_c^0[\frac{1}{2}^-]\to\Sigma_c^+\pi^-}$, $g_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Xi_c^{\prime +}\pi^-}$, $g_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Sigma_c^+K^-}$, $g_{\Omega_c^0[\frac{1}{2}^-]\to\Xi_c^{\prime +}K^-}$, $g_{\Sigma_c^0[\frac{3}{2}^-]\to\Sigma_c^{*+}\pi^-}$, $g_{\Xi_c^{\prime 0}[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-}$, $g_{\Xi_c^{\prime 0}[\frac{3}{2}^-]\to\Sigma_c^{*+}K^-}$ and $g_{\Omega_c^0[\frac{3}{2}^-]\to\Xi_c^{*+}K^-}$, as functions of the Borel mass T in Fig. 8. Using the values of T listed in Table III, we obtain

$$\begin{aligned}
&(l) \quad g_{\Sigma_c^0[\frac{1}{2}^-]\to\Sigma_c^+\pi^-} = 0.31,\\
&(o) \quad g_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Xi_c^{\prime +}\pi^-} = 0.23,\\
&(p) \quad g_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Sigma_c^+K^-} = 0.59,\\
&(r) \quad g_{\Omega_c^0[\frac{1}{2}^-]\to\Xi_c^{\prime +}K^-} = 0.85,\\
&(s) \quad g_{\Sigma_c^0[\frac{3}{2}^-]\to\Sigma_c^{*+}\pi^-} = 0.12,\\
&(t) \quad g_{\Xi_c^{\prime 0}[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-} = 0.09,\\
&(u) \quad g_{\Xi_c^{\prime 0}[\frac{3}{2}^-]\to\Sigma_c^{*+}K^-} = 0.056,\\
&(v) \quad g_{\Omega_c^0[\frac{3}{2}^-]\to\Xi_c^{*+}K^-} = 0.064. \qquad (85)
\end{aligned}$$

Using these values and the parameters listed in Sec. II, we further obtain

$$\begin{aligned}
&(l) \quad \Gamma_{\Sigma_c[\frac{1}{2}^-]\to\Sigma_c\pi} = 7.9\ \text{MeV},\\
&(o) \quad \Gamma_{\Xi_c^\prime[\frac{1}{2}^-]\to\Xi_c^\prime\pi} = 3.7\ \text{MeV},\\
&(p) \quad \Gamma_{\Xi_c^\prime[\frac{1}{2}^-]\to\Sigma_cK} = 3.6\ \text{MeV},\\
&(r) \quad \Gamma_{\Omega_c[\frac{1}{2}^-]\to\Xi_c^\prime K} = 29\ \text{MeV},\\
&(s) \quad \Gamma_{\Sigma_c[\frac{3}{2}^-]\to\Sigma_c^*\pi} = 0.95\ \text{MeV},\\
&(t) \quad \Gamma_{\Xi_c^\prime[\frac{3}{2}^-]\to\Xi_c^*\pi} = 0.45\ \text{MeV},\\
&(u) \quad \Gamma_{\Xi_c^\prime[\frac{3}{2}^-]\to\Sigma_c^*K\to\Lambda_c\pi K} = 7\times 10^{-5}\ \text{MeV},\\
&(v) \quad \Gamma_{\Omega_c[\frac{3}{2}^-]\to\Xi_c^*K\to\Xi_c\pi K} = 1\times 10^{-4}\ \text{MeV}. \qquad (86)
\end{aligned}$$

D. The baryon doublet $[\mathbf{6}_F,\mathbf{2},\mathbf{1},\lambda]$

The $[\mathbf{6}_F, 2, 1, \lambda]$ multiplet contains $\Sigma_c(\frac{3}{2}^-/\frac{5}{2}^-)$, $\Xi_c^\prime(\frac{3}{2}^-/\frac{5}{2}^-)$ and $\Omega_c(\frac{3}{2}^-/\frac{5}{2}^-)$. Their sum rules are listed in Appendix C 8, suggesting that their possible decay channels are (s), (t), (u) and (v). We show the four coupling constants, $g_{\Sigma_c^0[\frac{3}{2}^-]\to\Sigma_c^{*+}\pi^-}$, $g_{\Xi_c^{\prime 0}[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-}$, $g_{\Xi_c^{\prime 0}[\frac{3}{2}^-]\to\Sigma_c^{*+}K^-}$ and $g_{\Omega_c^0[\frac{3}{2}^-]\to\Xi_c^{*+}K^-}$, as functions of the Borel mass T in Fig. 9. Using the values of T listed in Table III, we obtain

$$\begin{aligned}
&(s) \quad g_{\Sigma_c^0[\frac{3}{2}^-]\to\Sigma_c^{*+}\pi^-} = 0.005,\\
&(t) \quad g_{\Xi_c^{\prime 0}[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-} = 0.004,\\
&(u) \quad g_{\Xi_c^{\prime 0}[\frac{3}{2}^-]\to\Sigma_c^{*+}K^-} = 0.013,\\
&(v) \quad g_{\Omega_c^0[\frac{3}{2}^-]\to\Xi_c^{*+}K^-} = 0.019. \qquad (87)
\end{aligned}$$

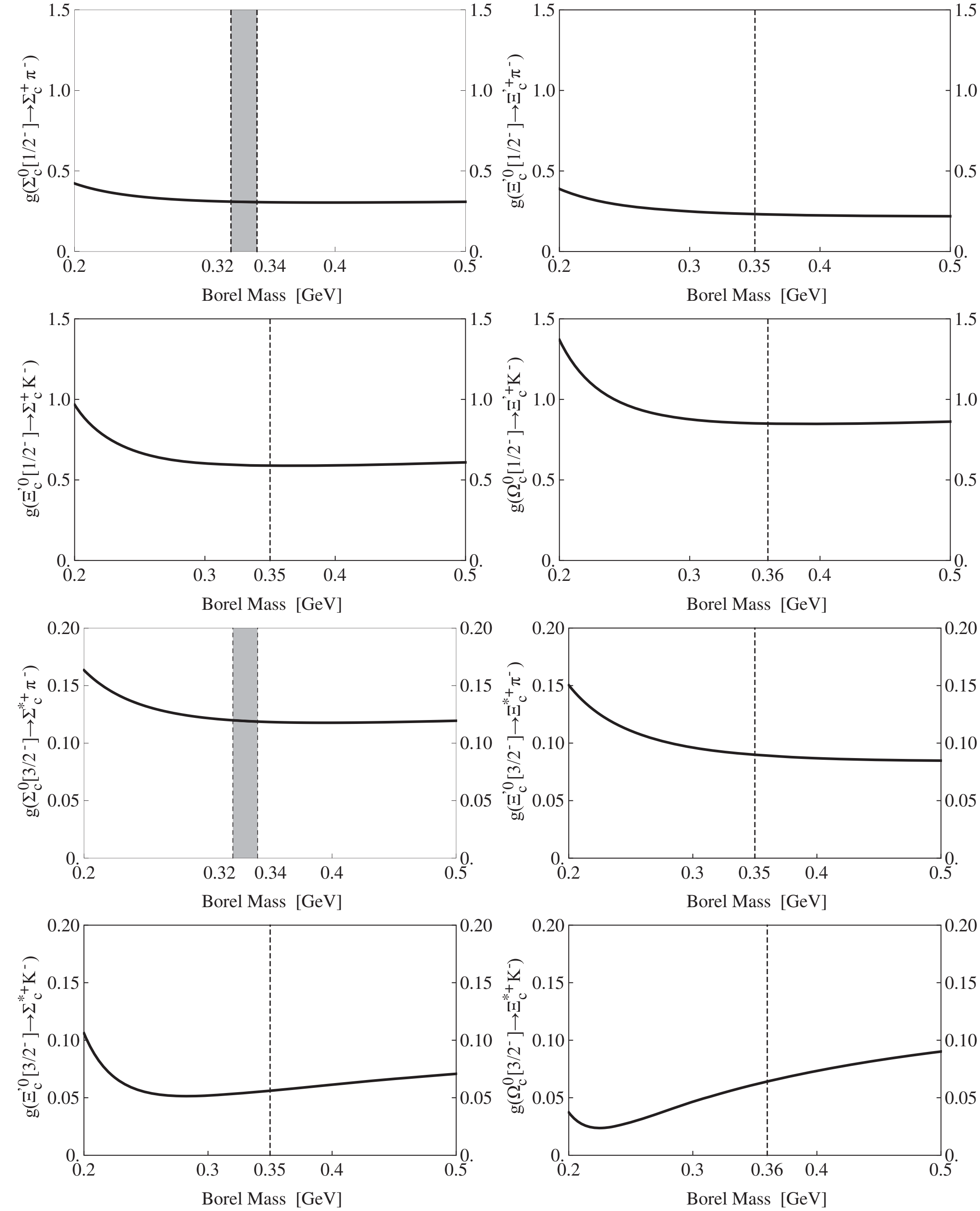

FIG. 8. The coupling constants $g_{\Sigma_c^0[\frac{1}{2}^-]\to\Sigma_c^+\pi^-}$ (top-left), $g_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Xi_c^{\prime +}\pi^-}$ (top-right), $g_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Sigma_c^+K^-}$ (middle-left), $g_{\Omega_c^0[\frac{1}{2}^-]\to\Xi_c^{\prime +}K^-}$ (middle-right), $g_{\Sigma_c^0[\frac{3}{2}^-]\to\Sigma_c^{*+}\pi^-}$ (middle-left), $g_{\Xi_c^{\prime 0}[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-}$ (middle-right), $g_{\Xi_c^{\prime 0}[\frac{3}{2}^-]\to\Sigma_c^{*+}K^-}$ (bottom-left) and $g_{\Omega_c^0[\frac{3}{2}^-]\to\Xi_c^{*+}K^-}$ (bottom-right) as functions of the Borel mass T. The currents belonging to the baryon doublet $[\mathbf{6}_F, 1, 1, \lambda]$ are used here.

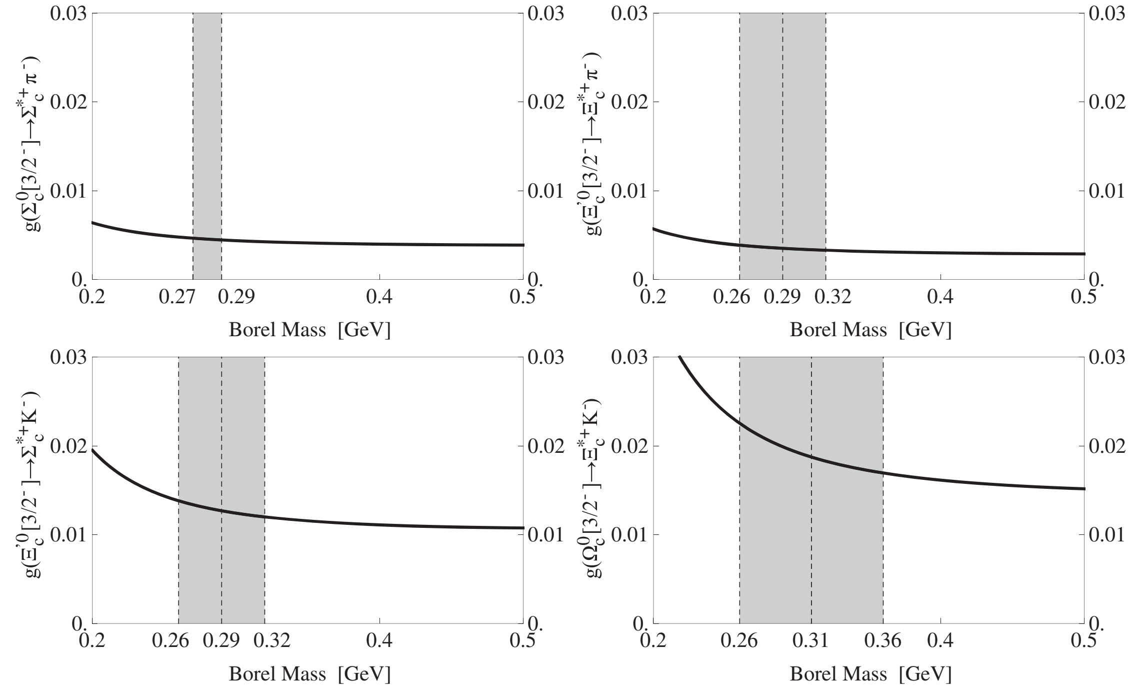

FIG. 9. The coupling constants $g_{\Sigma_c^0[\frac{3}{2}^-]\to\Sigma_c^{*+}\pi^-}$ (top-left), $g_{\Xi_c^{\prime 0}[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-}$ (top-right), $g_{\Xi_c^{\prime 0}[\frac{3}{2}^-]\to\Sigma_c^{*+}K^-}$ (bottom-left) and $g_{\Omega_c^0[\frac{3}{2}^-]\to\Xi_c^{*+}K^-}$ (bottom-right) as functions of the Borel mass T. The currents belonging to the baryon doublet $[\mathbf{6}_F, 2, 1, \lambda]$ are used here.

Using these values and the parameters listed in Sec. II, we further obtain

$$
\begin{aligned}
(s)\quad & \Gamma_{\Sigma_c[\frac{3}{2}^-]\to\Sigma_c^*\pi} = 1\times10^{-3}\ \text{MeV},\\
(t)\quad & \Gamma_{\Xi_c'[\frac{3}{2}^-]\to\Xi_c^*\pi} = 7\times10^{-4}\ \text{MeV},\\
(u)\quad & \Gamma_{\Xi_c'[\frac{3}{2}^-]\to\Sigma_c^*K\to\Lambda_c\pi K} = 3\times10^{-6}\ \text{MeV},\\
(v)\quad & \Gamma_{\Omega_c[\frac{3}{2}^-]\to\Xi_c^*K\to\Xi_c\pi K} = 9\times10^{-6}\ \text{MeV}. \qquad (88)
\end{aligned}
$$

V. SUMMARY AND DISCUSSIONS

To summarize this paper, we have used the method of light-cone QCD sum rules to study the decay properties of the P-wave charmed baryons. Firstly we summarize our results on the flavor $\bar{\mathbf{3}}_F$ P-wave charmed baryons. We have studied their S-wave decays into ground-state charmed baryons accompanied by a pseudoscalar meson (π or K) or a vector meson (ρ or K^*), including both two-body and three-body decays which are kinematically allowed. The results are listed in Table IV, where the possible decay channels are $(a)\Lambda_c[\frac{1}{2}^-]\to\Sigma_c\pi(\to\Lambda_c\pi\pi)$, $(b)\Lambda_c[\frac{3}{2}^-]\to\Sigma_c^*\pi\to\Lambda_c\pi\pi$, $(c)\Xi_c[\frac{1}{2}^-]\to\Xi_c\pi$, $(d)\Xi_c[\frac{1}{2}^-]\to\Lambda_c K$, $(e)\Xi_c[\frac{1}{2}^-]\to\Xi_c\rho\to\Xi_c\pi\pi$, $(f)\Xi_c[\frac{1}{2}^-]\to\Xi_c'\pi$, $(g)\Xi_c[\frac{3}{2}^-]\to\Xi_c\rho\to\Xi_c\pi\pi$, $(h)\Xi_c[\frac{3}{2}^-]\to\Xi_c^*\pi$, $(i)\Lambda_c[\frac{5}{2}^-]\to\Sigma_c^*\rho\to\Sigma_c^*\pi\pi$, and $(j)\Xi_c[\frac{5}{2}^-]\to\Xi_c^*\rho\to\Xi_c^*\pi\pi$. We note that the uncertainties can be as large as $\Gamma^{+200\%}_{-67\%}$.

Our calculations are performed based on the HQET and separately for the four charmed baryon multiplets of flavor $\bar{\mathbf{3}}_F$, $[\bar{\mathbf{3}}_F, 0, 1, \rho]$, $[\bar{\mathbf{3}}_F, 1, 1, \rho]$, $[\bar{\mathbf{3}}_F, 2, 1, \rho]$ and $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$. We find that none of these four multiplets can independently well describe the experimental decay data of the $\Lambda_c(2595)$. This is somehow in contrast with quark model calculations which describe some of the decay rates, but not all, in a reasonable manner [41]. It would be a future issue to see further relations of various approaches. See also Refs. [24,25] for other possible interpretations of the $\Lambda_c(2595)$. In the present sum rule study, considering the fact that the heavy-quark symmetry is not perfect, the physical states are probably mixed states containing various components with different inner quantum numbers. It is then possible that the $\Lambda_c(2595)$ is an admixture of the above four multiplets. Thus we try to use the mixture of $[\bar{\mathbf{3}}_F, 1, 1, \rho]$ and $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$ as an explanation and assume the physical state to be

$$
\begin{aligned}
|\Lambda_c(1/2^-)\rangle = {}& \cos\theta\times|1/2,-,\Lambda_c,1,1,\rho\rangle\\
& + \sin\theta\times|1/2,-,\Lambda_c,1,0,\lambda\rangle, \qquad (89)
\end{aligned}
$$

so that we have

$$
\begin{aligned}
g_{\Lambda_c[\frac{1}{2}^-]\to\Sigma_c\pi} = {}& \cos\theta\times g_{|1/2,-,\Lambda_c,1,1,\rho\rangle\to\Sigma_c\pi}\\
& + \sin\theta\times g_{|1/2,-,\Lambda_c,1,0,\lambda\rangle\to\Sigma_c\pi}, \qquad (90)
\end{aligned}
$$

and we can further obtain

TABLE IV. Nonvanishing decay widths of the flavor $\bar{\mathbf{3}}_F$ P-wave charmed baryons, in units of MeV. The two mass values with $*$ are our assumptions, so that the decay channels (i) and (j) are kinematically allowed. The possible decay channels are (a) $\Lambda_c[\frac{1}{2}^-] \to \Sigma_c\pi(\to \Lambda_c\pi\pi)$, (b) $\Lambda_c[\frac{3}{2}^-] \to \Sigma_c^*\pi \to \Lambda_c\pi\pi$, (c) $\Xi_c[\frac{1}{2}^-] \to \Xi_c\pi$, (d) $\Xi_c[\frac{1}{2}^-] \to \Lambda_c K$, (e) $\Xi_c[\frac{1}{2}^-] \to \Xi_c\rho \to \Xi_c\pi\pi$, (f) $\Xi_c[\frac{1}{2}^-] \to \Xi_c'\pi$, (g) $\Xi_c[\frac{3}{2}^-] \to \Xi_c\rho \to \Xi_c\pi\pi$, (h) $\Xi_c[\frac{3}{2}^-] \to \Xi_c^*\pi$, (i) $\Lambda_c[\frac{5}{2}^-] \to \Sigma_c^*\rho \to \Sigma_c^*\pi\pi$, and (j) $\Xi_c[\frac{5}{2}^-] \to \Xi_c^*\rho \to \Xi_c^*\pi\pi$. We use the two-body middle/final states to denote them in the table.

Baryon	Experiments [2]	$[\bar{\mathbf{3}}_F,0,1,\rho]$	$[\bar{\mathbf{3}}_F,1,1,\rho]$	$[\bar{\mathbf{3}}_F,2,1,\rho]$	$[\bar{\mathbf{3}}_F,1,0,\lambda]$	Mix-$\vert B_1\rangle$	Mix-$\vert B_2\rangle$
$\Lambda_c(2595)$ $J^P=\frac{1}{2}^-$	$\Gamma_{\Lambda_c(2595)}=2.59$ $[\Sigma_c^{++,0}\pi^\mp]:48\%$ $[\Lambda_c\pi\pi]_{3\text{body}}:18\%$	…	$[\Sigma_c\pi]=0.39$	…	$[\Sigma_c\pi]=32$	Input: $\frac{\Gamma(\Sigma_c\pi)}{\Gamma(\Lambda_c(2595))}=0.66$ $\theta_1=-20°$	$\theta_2=7°$
$\Xi_c(2790)$ $J^P=\frac{1}{2}^-$	$\Gamma_{\Xi_c^+(2790)}<15$ $\Gamma_{\Xi_c^0(2790)}<12$ $[\Xi_c'\pi]_{2\text{body}}:\text{seen}$	$[\Xi_c\pi]=300$ $[\Lambda_cK]=82$	$[\Xi_c'\pi]=1.6$ $[\Xi_c\rho]=0.00$	…	$[\Xi_c'\pi]=100$ $[\Xi_c\rho]=0.04$	$[\Xi_c'\pi]=4.7$ $[\Xi_c\rho]=0.00$	$[\Xi_c'\pi]=6.1$ $[\Xi_c\rho]=0.00$
$\Lambda_c(2625)$ $J^P=\frac{3}{2}^-$	$\Gamma_{\Lambda_c(2625)}<0.97$ $[\Sigma_c\pi]_{2\text{body}}<10\%$ $[\Lambda_c\pi\pi]_{3\text{body}}:\text{large}$	…	$[\Sigma_c^*\pi]=0.00$	$[\Sigma_c^*\pi]=0.03$	$[\Sigma_c^*\pi]=0.96$	$[\Sigma_c^*\pi]=0.11$	$[\Sigma_c^*\pi]=0.01$
$\Xi_c(2815)$ $J^P=\frac{3}{2}^-$	$\Gamma_{\Xi_c^+(2815)}<3.5$ $\Gamma_{\Xi_c^0(2815)}<6.5$ $[\Xi_c\pi\pi]_{3\text{body}}:\text{seen}$	…	$[\Xi_c^*\pi]=0.01$ $[\Xi_c\rho]=0.00$	$[\Xi_c^*\pi]=0.69$	$[\Xi_c^*\pi]=30$ $[\Xi_c\rho]=0.23$	$[\Xi_c^*\pi]=3.0$ $[\Xi_c\rho]=0.03$	$[\Xi_c^*\pi]=0.59$ $[\Xi_c\rho]=0.00$
$\Lambda_c(5/2^-)$	$M_{\Lambda_c(5/2^-)}\sim 2850^*$	…	…	$[\Sigma_c^*\rho]=11$	…	…	…
$\Xi_c(5/2^-)$	$M_{\Xi_c(5/2^-)}\sim 3000^*$	…	…	$[\Xi_c^*\rho]=12$	…	…	…

$$\sqrt{\Gamma_{\Lambda_c[\frac{1}{2}^-]\to\Sigma_c\pi(\to\Lambda_c\pi\pi)}} = \cos\theta\times\sqrt{\Gamma_{|1/2,-,\Lambda_c,1,1,\rho\rangle\to\Sigma_c\pi(\to\Lambda_c\pi\pi)}} + \sin\theta\times\sqrt{\Gamma_{|1/2,-,\Lambda_c,1,0,\lambda\rangle\to\Sigma_c\pi(\to\Lambda_c\pi\pi)}}. \tag{91}$$

Other channels can be similarly evaluated. The mixing angle θ can be estimated by assuming [2,124]

$$\frac{\Gamma(\Sigma_c\pi)}{\Gamma_{\Lambda_c(2595)}} \approx \frac{\Gamma(\Sigma_c^{++}\pi^- + \Sigma_c^0\pi^+)}{\Gamma(\Lambda_c^+\pi^+\pi^-)} = 0.66^{+0.13}_{-0.16}\pm 0.07. \tag{92}$$

There are two possible solutions: $\theta_1=-20°$ and $\theta_2=7°$, which we denote as Mix-$|B_1\rangle$ and Mix-$|B_2\rangle$, respectively. Assuming the mixing angle to be an overall parameter, we evaluate decay widths of the $\Lambda_c(2625)$, $\Xi_c(2790)$ and $\Xi_c(2815)$. The results are listed in Table IV, which are consistent with their experimental decay data, while the Mix-$|B_1\rangle$ seems a bit better. Recall that both $[\bar{\mathbf{3}}_F,1,1,\rho]$ and $[\bar{\mathbf{3}}_F,1,0,\lambda]$ can also describe the masses of these states [51], so these two mixing solutions can well describe both masses and decay properties of the $\Lambda_c(2595)$, $\Lambda_c(2625)$, $\Xi_c(2790)$ and $\Xi_c(2815)$ at the same time. We would like to suggest the Belle/KEK, LHCb, and J-PARC experiments to further examine these values.

Using the same method, we have also studied the decay properties of the flavor $\mathbf{6}_F$ P-wave charmed baryons. We have studied their S-wave decays into ground-state charmed baryons accompanied by a pseudoscalar meson (π or K), including both two-body and three-body decays which are kinematically allowed. The results are listed in Table V, where the possible decay channels are $(k)\Sigma_c[\frac{1}{2}^-]\to\Lambda_c\pi$, $(l)\Sigma_c[\frac{1}{2}^-]\to\Sigma_c\pi$, $(m)\Xi_c'[\frac{1}{2}^-]\to\Xi_c\pi$, $(n)\Xi_c'[\frac{1}{2}^-]\to\Lambda_cK$, $(o)\Xi_c'[\frac{1}{2}^-]\to\Xi_c'\pi$, $(p)\Xi_c'[\frac{1}{2}^-]\to\Sigma_cK$, $(q)\Omega_c[\frac{1}{2}^-]\to\Xi_cK$, $(r)\Omega_c[\frac{1}{2}^-]\to\Xi_c'K$, $(s)\Sigma_c[\frac{3}{2}^-]\to\Sigma_c^*\pi$, $(t)\Xi_c'[\frac{3}{2}^-]\to\Xi_c^*\pi$, $(u)\Xi_c'[\frac{3}{2}^-]\to\Sigma_c^*K\to\Lambda_c\pi K$, and $(v)\Omega_c[\frac{3}{2}^-]\to\Xi_c^*K\to\Xi_c\pi K$. We note again that the uncertainties can be as large as $\Gamma^{+200\%}_{-67\%}$.

Our calculations are done separately for the four charmed baryon multiplets of flavor $\mathbf{6}_F$, $[\mathbf{6}_F,1,0,\rho]$, $[\mathbf{6}_F,0,1,\lambda]$, $[\mathbf{6}_F,1,1,\lambda]$ and $[\mathbf{6}_F,2,1,\lambda]$. The situation in this case is more ambiguous that the previous case of the flavor $\bar{\mathbf{3}}_F$ charmed baryons:

(1) The $\Sigma_c(2800)$ is a good P-wave charmed baryon candidate of flavor $\mathbf{6}_F$. It has a large width around 70 MeV and was observed in the $\Lambda_c\pi$ decay channel. Our results suggest that it may be interpreted as a $J^P=1/2^-$ state belonging to the $[\mathbf{6}_F,0,1,\lambda]$ multiplet, and it can be better interpreted as a $J^P=1/2^-$ state containing both $[\mathbf{6}_F,0,1,\lambda]$ and $[\mathbf{6}_F,1,1,\lambda]$ components.

(2) The $\Xi_c(2930)$ has a width around 36 MeV, and it was only observed by the *BABAR* experiment in the Λ_cK decay channel [125]. Our results suggest that it may be interpreted as a $J^P=1/2^-$ state containing both $[\mathbf{6}_F,0,1,\lambda]$ and $[\mathbf{6}_F,1,1,\lambda]$ components.

(3) The $\Xi_c(2980)$ has a width around 20 MeV. It was observed in the $\Sigma_c(2455)K$ and $\Xi_c(2645)\pi$ decay

channels, but was not seen in the $\Lambda_c K$ decay channel. Our results suggest that it may be interpreted as a $J^P = 1/2^-$ state belonging to the $[\mathbf{6}_F, 1, 1, \lambda]$ multiplet but it does not contain $[\mathbf{6}_F, 0, 1, \lambda]$ component.

At present, the J^P quantum number of some states has not been measured. They could also be the candidates of the radial excitations or D-wave states. More experiments are also necessary to understand them. Especially, the five excited Ω_c states recently observed by LHCb [1], $\Omega_c(3000)$, $\Omega_c(3050)$, $\Omega_c(3066)$, $\Omega_c(3090)$, and $\Omega_c(3119)$, are very helpful to improve our understanding of the excited charmed baryons. Their widths are quite small and were all observed in the $\Xi_c K$ decay channel. We use their masses as inputs and redo the previous calculations. The results are shown in Table VI (note that the $\Xi_c' K$ threshold is 3072 MeV and the $\Xi_c \pi K$ threshold is 3103 MeV):

(1) We may use the $[\mathbf{6}_F, 1, 1, \lambda]$ multiplet together with a tiny $[\mathbf{6}_F, 0, 1, \lambda]$ component to interpret one of these Ω_c states ($\Omega_c(3000)$, $\Omega_c(3050)$ or $\Omega_c(3066)$) as a $J^P = 1/2^-$ state.
(2) The $[\mathbf{6}_F, 2, 1, \lambda]$ multiplet may be used to interpret two of these Ω_c states as one $J^P = 3/2^-$ state and one $J^P = 5/2^-$ state, but we still need to study their D-wave decays into $\Xi_c K$ to check this possibility.
(3) Two of these excited Ω_c states may be interpreted as two $2S$ states of $J^P = 1/2^+$ and $3/2^+$. See the recent reference [126] for more discussions.

To end this work, we note that we have only investigated the S-wave decay properties of these excited charmed baryons in the present study, but their D-wave decays can also happen and contribute (although these contributions may be not large). Hence, in our following study we plan to further study their D-wave decay properties. We also plan to study the S-wave decays of the flavor $\mathbf{6}_F$ P-wave charmed baryons into ground-state charmed baryons accompanied by a vector meson (ρ or K^*), which have not been done in the present work. We would like to suggest the Belle/KEK, LHCb, and J-PARC experiments to investigate the decays of these excited Ω_c states into $\Xi_c' K$ to further understand them.

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APPENDIX A: FORMULAS OF DECAY AMPLITUDES AND DECAY WIDTHS

The decay widths of P-wave charmed baryons can be evaluated based on the Lagrangians (a)–(j) listed in Eqs. (6), (51) and (47):

(1) The decay amplitude of the two-body decay $(a)\Lambda_c^+(1/2^-) \to \Sigma_c^+ \pi^0$ is

TABLE V. Nonvanishing decay widths of the flavor $\mathbf{6}_F$ P-wave charmed baryons, in units of MeV. The possible decay channels are: $(k)\Sigma_c[\frac{1}{2}^-] \to \Lambda_c\pi$, $(l)\Sigma_c[\frac{1}{2}^-] \to \Sigma_c\pi$, $(m)\Xi_c'[\frac{1}{2}^-] \to \Xi_c\pi$, $(n)\Xi_c'[\frac{1}{2}^-] \to \Lambda_c K$, $(o)\Xi_c'[\frac{1}{2}^-] \to \Xi_c'\pi$, $(p)\Xi_c'[\frac{1}{2}^-] \to \Sigma_c K$, $(q)\Omega_c[\frac{1}{2}^-] \to \Xi_c K$, $(r)\Omega_c[\frac{1}{2}^-] \to \Xi_c' K$, $(s)\Sigma_c[\frac{3}{2}^-] \to \Sigma_c^*\pi$, $(t)\Xi_c'[\frac{3}{2}^-] \to \Xi_c^*\pi$, $(u)\Xi_c'[\frac{3}{2}^-] \to \Sigma_c^* K \to \Lambda_c\pi K$, and $(v)\Omega_c[\frac{3}{2}^-] \to \Xi_c^* K \to \Xi_c\pi K$. We use the two-body middle/final states to denote them in the table.

Baryon	Mass	$[\mathbf{6}_F, 1, 0, \rho]$	$[\mathbf{6}_F, 0, 1, \lambda]$	$[\mathbf{6}_F, 1, 1, \lambda]$	$[\mathbf{6}_F, 2, 1, \lambda]$
$\Sigma_c[\frac{1}{2}^-]$	~2800	$[\Sigma_c\pi] = 300$	$[\Lambda_c\pi] = 200$	$[\Sigma_c\pi] = 7.9$	…
		$[\Xi_c'\pi] = 140$	$[\Xi_c\pi] = 230$	$[\Xi_c'\pi] = 3.7$	
$\Xi_c'[\frac{1}{2}^-]$	~2950				…
		$[\Sigma_c K] = 29$	$[\Lambda_c K] = 160$	$[\Sigma_c K] = 3.6$	
$\Omega_c[\frac{1}{2}^-]$	~3100	$[\Xi_c' K] = 250$	$[\Xi_c K] = 820$	$[\Xi_c' K] = 29$	…
$\Sigma_c[\frac{3}{2}^-]$	~2800	$[\Sigma_c^*\pi] = 110$	…	$[\Sigma_c^*\pi] = 0.95$	$[\Sigma_c^*\pi] = 0.00$
		$[\Xi_c^*\pi] = 50$		$[\Xi_c^*\pi] = 0.45$	$[\Xi_c^*\pi] = 0.00$
$\Xi_c'[\frac{3}{2}^-]$	~2950		…		
		$[\Sigma_c^* K] = 0.03$		$[\Sigma_c^* K] = 0.00$	$[\Sigma_c^* K] = 0.00$
$\Omega_c[\frac{3}{2}^-]$	~3120	$[\Xi_c^* K] = 0.07$	…	$[\Xi_c^* K] = 0.00$	$[\Xi_c^* K] = 0.00$
$\Sigma_c[\frac{5}{2}^-]$	…	…	…	…	…
$\Xi_c'[\frac{5}{2}^-]$	…	…	…	…	…
$\Omega_c[\frac{5}{2}^-]$	…	…	…	…	…

TABLE VI. Decay widths of the five excited Ω_c states recently observed by LHCb [1], assuming they are P-wave charmed baryons. The results are in units of MeV. The possible decay channels are $(q)\Omega_c[\frac{1}{2}^-] \to \Xi_c K$, $(r)\Omega_c[\frac{1}{2}^-] \to \Xi_c' K$, and $(v)\Omega_c[\frac{3}{2}^-] \to \Xi_c^* K \to \Xi_c \pi K$. We use the two-body middle/final states to denote them in the table.

	$[\mathbf{6}_F, 1, 0, \rho]$		$[\mathbf{6}_F, 0, 1, \lambda]$	$[\mathbf{6}_F, 1, 1, \lambda]$		$[\mathbf{6}_F, 2, 1, \lambda]$	
Experiments	$1/2^-$	$3/2^-$	$1/2^-$	$1/2^-$	$3/2^-$	$3/2^-$	$5/2^-$
$\Omega_c(3000)$	⋯	⋯	$[\Xi_c K] = 420$	⋯	⋯	⋯	⋯
$\Omega_c(3050)$	⋯	⋯	$[\Xi_c K] = 650$	⋯	⋯	⋯	⋯
$\Omega_c(3066)$	⋯	⋯	$[\Xi_c K] = 700$	⋯	⋯	⋯	⋯
$\Omega_c(3090)$	$[\Xi_c' K] = 200$	⋯	$[\Xi_c K] = 790$	$[\Xi_c' K] = 23$	⋯	⋯	⋯
$\Omega_c(3119)$	$[\Xi_c' K] = 320$	$[\Xi_c^* K] = 0.06$	$[\Xi_c K] = 870$	$[\Xi_c' K] = 38$	$[\Xi_c^* K] = 0.00$	$[\Xi_c^* K] = 0.00$	⋯

$$\mathcal{M}(0\to 2+1) \equiv \mathcal{M}(\Lambda_c^+(1/2^-) \to \Sigma_c^+(1/2^+) + \pi^0) = g_{0\to 2+1}\bar{u}_0 u_2, \quad \text{(A1)}$$

where 0 denotes the initial state $\Lambda_c^+(1/2^-)$; 1 and 2 denote the finial states π^0 and $\Sigma_c^+(1/2^+)$, respectively. This amplitude can be used to further evaluate its decay width $\not{p}$

$$\Gamma(0\to 2+1) \equiv \Gamma(\Lambda_c^+(1/2^-) \to \Sigma_c^+(1/2^+) + \pi^0) = \frac{|\vec{p}_1|}{8\pi m_0^2} \times g_{0\to 2+1}^2 \times \frac{1}{2}\mathrm{Tr}[(\not{p}_0 + m_0)(\not{p}_2 + m_2)], \quad \text{(A2)}$$

where we have used the following formula for the baryon field of spin 1/2:

$$\sum_{\text{spin}} u(p)\bar{u}(p) = (\not{p} + m). \quad \text{(A3)}$$

The two-body decays, (c), (d), (f), and (k)–(r), can be similarly evaluated.

(2) The decay amplitude of the two-body decay $(h)\Xi_c(3/2^-) \to \Xi_c^*\pi$ is

$$\mathcal{M}(0\to 2+1) \equiv \mathcal{M}(\Xi_c^0(3/2^-) \to \Xi_c^{*+}(3/2^+) + \pi^-) = g_{0\to 2+1}\bar{u}_{0,\mu}u_{2,\mu}, \quad \text{(A4)}$$

where 0 denotes the initial state $\Xi_c^0(3/2^-)$; 1 and 2 denote the finial states π^- and $\Xi_c^{*+}(3/2^+)$, respectively. This amplitude can be used to further evaluate its decay width

$$\Gamma(0\to 2+1) \equiv \Gamma(\Xi_c^0(3/2^-) \to \Xi_c^{*+}(3/2^+) + \pi^-) = \frac{|\vec{p}_1|}{8\pi m_0^2} \times g_{0\to 2+1}^2 \times \frac{1}{4}\mathrm{Tr}\left[\left(g_{\mu'\mu} - \frac{1}{3}\gamma_{\mu'}\gamma_\mu - \frac{p_{2,\mu'}\gamma_\mu - p_{2,\mu}\gamma_{\mu'}}{3m_2} - \frac{2p_{2,\mu'}p_{2,\mu}}{3m_2^2}\right)(\not{p}_2 + m_2) \times \left(g_{\mu\mu'} - \frac{1}{3}\gamma_\mu\gamma_{\mu'} - \frac{p_{0,\mu}\gamma_{\mu'} - p_{0,\mu'}\gamma_\mu}{3m_0} - \frac{2p_{0,\mu}p_{0,\mu'}}{3m_0^2}\right)(\not{p}_0 + m_0)\right], \quad \text{(A5)}$$

where we have used the following formula for the baryon field of spin 3/2:

$$\sum_{\text{spin}} u_\mu(p)\bar{u}_{\mu'}(p) = \left(g_{\mu\mu'} - \frac{1}{3}\gamma_\mu\gamma_{\mu'} - \frac{p_\mu\gamma_{\mu'} - p_{\mu'}\gamma_\mu}{3m} - \frac{2p_\mu p_{\mu'}}{3m^2}\right)(\not{p} + m). \quad \text{(A6)}$$

The two-body decays, (s) and (t), can be similarly evaluated.

(3) The decay amplitude of the three-body decay $(a')\Lambda_c^+(1/2^-) \to \Sigma_c^{++}\pi^- \to \Lambda_c^+\pi^+\pi^-$ is

$$\mathcal{M}(0\to 4+1\to 3+2+1) \equiv \mathcal{M}(\Lambda_c^+(1/2^-) \to \Sigma_c^{++}(1/2^+) + \pi^- \to \Lambda_c^+(1/2^+) + \pi^+ + \pi^-) = g_{0\to 4+1} \times g_{4\to 3+2} \times \bar{u}_0 \times \frac{\not{p}_4 + m_4}{p_4^2 - m_4^2 + im_4\Gamma_4} \times \gamma_\mu\gamma_5 \times u_3 \times p_{2,\mu}, \quad \text{(A7)}$$

where 0 denotes the initial state $\Lambda_c^+(1/2^-)$; 4 denotes the middle state $\Sigma_c^{++}(1/2^+)$; 1, 2 and 3 denote the finial states π^-, π^+ and $\Lambda_c^+(1/2^+)$, respectively. This amplitude can be used to further evaluate its decay width

$$\begin{aligned}\Gamma(0\to 4+1\to 3+2+1) &\equiv \Gamma(\Lambda_c^+(1/2^-)\to\Sigma_c^{++}(1/2^+)+\pi^-\to\Lambda_c^+(1/2^+)+\pi^++\pi^-)\\ &=\frac{1}{(2\pi)^3}\times\frac{1}{32m_0^3}\times g^2_{0\to4+1}\times g^2_{4\to3+2}\times\int dm_{12}dm_{23}\\ &\times\frac{1}{2}\mathrm{Tr}[(\not p_3+m_3)\gamma_{\mu'}\gamma_5(\not p_4+m_4)(\not p_0+m_0)(\not p_4+m_4)\gamma_\mu\gamma_5]\\ &\times\frac{1}{|p_4^2-m_4^2+im_4\Gamma_4|^2}\times p_{2,\mu}p_{2,\mu'},\end{aligned} \tag{A8}$$

where we have used the standard Dalitz integration [2].

(4) The decay amplitude of the three-body decay $(b)\Lambda_c(3/2^-)\to\Sigma_c^*\pi\to\Lambda_c\pi\pi$ is

$$\begin{aligned}\mathcal{M}(0\to 4+1\to 3+2+1) &\equiv \mathcal{M}(\Lambda_c^+(3/2^-)\to\Sigma_c^{*++}(3/2^+)+\pi^-\to\Lambda_c^+(1/2^+)+\pi^++\pi^-)\\ &= g_{0\to4+1}\times g_{4\to3+2}\\ &\times\bar u_{0,\mu}\times\left(g_{\mu\nu}-\frac{1}{3}\gamma_\mu\gamma_\nu-\frac{p_{4,\mu}\gamma_\nu-p_{4,\nu}\gamma_\mu}{3m_4}-\frac{2p_{4,\mu}p_{4,\nu}}{3m_4^2}\right)\\ &\times\frac{\not p_4+m_4}{p_4^2-m_4^2+im_4\Gamma_4}\times u_3\times p_{2,\nu},\end{aligned} \tag{A9}$$

where 0 denotes the initial state $\Lambda_c^+(3/2^-)$; 4 denotes the middle state $\Sigma_c^{*++}(3/2^+)$; 1, 2 and 3 denote the finial states π^-, π^+ and $\Lambda_c^+(1/2^+)$, respectively. This amplitude can be used to further evaluate its decay width

$$\begin{aligned}\Gamma(0\to 4+1\to 3+2+1) &\equiv \Gamma(\Lambda_c^+(3/2^-)\to\Sigma_c^{*++}(3/2^+)+\pi^-\to\Lambda_c^+(1/2^+)+\pi^++\pi^-)\\ &=\frac{1}{(2\pi)^3}\times\frac{1}{32m_0^3}\times g^2_{0\to4+1}\times g^2_{4\to3+2}\times\int dm_{12}dm_{23}\\ &\times\frac{1}{4}\mathrm{Tr}[(\not p_3+m_3)\times\left(g_{\nu'\mu'}-\frac{1}{3}\gamma_{\nu'}\gamma_{\mu'}-\frac{p_{4,\nu'}\gamma_{\mu'}-p_{4,\mu'}\gamma_{\nu'}}{3m_4}-\frac{2p_{4,\nu'}p_{4,\mu'}}{3m_4^2}\right)(\not p_4+m_4)\\ &\times\left(g_{\mu'\mu}-\frac{1}{3}\gamma_{\mu'}\gamma_\mu-\frac{p_{0,\mu'}\gamma_\mu-p_{0,\mu}\gamma_{\mu'}}{3m_0}-\frac{2p_{0,\mu'}p_{0,\mu}}{3m_0^2}\right)(\not p_0+m_0)\\ &\times\left(g_{\mu\nu}-\frac{1}{3}\gamma_\mu\gamma_\nu-\frac{p_{4,\mu}\gamma_\nu-p_{4,\nu}\gamma_\mu}{3m_4}-\frac{2p_{4,\mu}p_{4,\nu}}{3m_4^2}\right)(\not p_4+m_4)]\\ &\times\frac{1}{|p_4^2-m_4^2+im_4\Gamma_4|^2}\times p_{2,\nu}p_{2,\nu'}.\end{aligned} \tag{A10}$$

The three-body decays, (u) and (v), can be similarly evaluated.

(5) The decay amplitude of the three-body decay $(e)\Xi_c(1/2^-)\to\Xi_c\rho\to\Xi_c\pi\pi$ is

$$\begin{aligned}\mathcal{M}(0\to 3+4\to 3+2+1) &\equiv \mathcal{M}(\Xi_c^0(1/2^-)\to\Xi_c^+(1/2^+)+\rho^-\to\Xi_c^+(1/2^+)+\pi^0+\pi^-)\\ &= g_{0\to3+4}\times g_{4\to2+1}\times\bar u_0\gamma_\mu\gamma_5 u_3\times\left(g_{\mu\nu}-\frac{p_{4,\mu}p_{4,\nu}}{m_4^2}\right)\\ &\times\frac{1}{p_4^2-m_4^2+im_4\Gamma_4}\times(p_{1,\nu}+p_{2,\nu}),\end{aligned} \tag{A11}$$

where 0 denotes the initial state $\Xi_c^0(1/2^-)$; 4 denotes the middle state ρ^-; 1, 2 and 3 denote the finial states π^-, π^0 and $\Xi_c^+(1/2^+)$, respectively. This amplitude can be used to further evaluate its decay width

$$\Gamma(0\to 3+4\to 3+2+1)\equiv\Gamma(\Xi_c^0(1/2^-)\to\Xi_c^+(1/2^+)+\rho^-\to\Xi_c^+(1/2^+)+\pi^0+\pi^-)$$
$$=\frac{1}{(2\pi)^3}\times\frac{1}{32m_0^3}\times g^2_{0\to 3+4}\times g^2_{4\to 2+1}\times\int dm_{12}dm_{23}$$
$$\times\frac{1}{2}\mathrm{Tr}[(\not p_3+m_3)\gamma_{\mu'}\gamma_5(\not p_0+m_0)\gamma_\mu\gamma_5]$$
$$\times\left(g_{\mu\nu}-\frac{p_{4,\mu}p_{4,\nu}}{m_4^2}\right)\left(g_{\mu'\nu'}-\frac{p_{4,\mu'}p_{4,\nu'}}{m_4^2}\right)\times\frac{1}{|p_4^2-m_4^2+im_4\Gamma_4|^2}$$
$$\times(p_{1,\nu}+p_{2,\nu})(p_{1,\nu'}+p_{2,\nu'}).\tag{A12}$$

(6) The decay amplitude of the three-body decay $(g)\Xi_c(3/2^-)\to\Xi_c\rho\to\Xi_c\pi\pi$ is

$$\mathcal{M}(0\to 3+4\to 3+2+1)\equiv\mathcal{M}(\Xi_c^0(3/2^-)\to\Xi_c^+(1/2^+)+\rho^-\to\Xi_c^+(1/2^+)+\pi^0+\pi^-)$$
$$=g_{0\to 3+4}\times g_{4\to 2+1}\times\bar u_{0,\mu}u_3\times\left(g_{\mu\nu}-\frac{p_{4,\mu}p_{4,\nu}}{m_4^2}\right)\times\frac{1}{p_4^2-m_4^2+im_4\Gamma_4}$$
$$\times(p_{1,\nu}+p_{2,\nu}),\tag{A13}$$

where 0 denotes the initial state $\Xi_c^0(3/2^-)$; 4 denotes the middle state ρ^-; 1, 2 and 3 denote the finial states π^-, π^0 and $\Xi_c^+(1/2^+)$, respectively. This amplitude can be used to further evaluate its decay width

$$\Gamma(0\to 3+4\to 3+2+1)\equiv\Gamma(\Xi_c^0(3/2^-)\to\Xi_c^+(1/2^+)+\rho^-\to\Xi_c^+(1/2^+)+\pi^0+\pi^-)$$
$$=\frac{1}{(2\pi)^3}\times\frac{1}{32m_0^3}\times g^2_{0\to 3+4}\times g^2_{4\to 2+1}\times\int dm_{12}dm_{23}$$
$$\times\frac{1}{4}\mathrm{Tr}\left[(\not p_3+m_3)\left(g_{\mu'\mu}-\frac{1}{3}\gamma_{\mu'}\gamma_\mu-\frac{p_{0,\mu'}\gamma_\mu-p_{0,\mu}\gamma_{\mu'}}{3m_0}-\frac{2p_{0,\mu'}p_{0,\mu}}{3m_0^2}\right)(\not p_0+m_0)\right]$$
$$\times\left(g_{\mu\nu}-\frac{p_{4,\mu}p_{4,\nu}}{m_4^2}\right)\left(g_{\mu'\nu'}-\frac{p_{4,\mu'}p_{4,\nu'}}{m_4^2}\right)\times\frac{1}{|p_4^2-m_4^2+im_4\Gamma_4|^2}$$
$$\times(p_{1,\nu}+p_{2,\nu})(p_{1,\nu'}+p_{2,\nu'}).\tag{A14}$$

(7) The decay amplitude of the three-body decay $(i)\quad\Lambda_c(5/2^-)\to\Sigma_c^*\rho\to\Sigma_c^*\pi\pi$ is

$$\mathcal{M}(0\to 3+4\to 3+2+1)\equiv\mathcal{M}(\Lambda_c^+(5/2^-)\to\Sigma_c^{*++}(3/2^+)+\rho^-\to\Sigma_c^{*++}(3/2^+)+\pi^0+\pi^-)$$
$$=g_{0\to 3+4}\times g_{4\to 2+1}\times\bar u_{0,\mu\rho}u_{3,\rho}\times\left(g_{\mu\nu}-\frac{p_{4,\mu}p_{4,\nu}}{m_4^2}\right)$$
$$\times\frac{1}{p_4^2-m_4^2+im_4\Gamma_4}\times(p_{1,\nu}+p_{2,\nu}),\tag{A15}$$

where 0 denotes the initial state $\Lambda_c^+(5/2^-)$; 4 denotes the middle state ρ^-; 1, 2 and 3 denote the finial states π^-, π^0 and $\Sigma_c^{*++}(3/2^+)$, respectively. This amplitude can be used to further evaluate its decay width

$$\begin{aligned}\Gamma(0\to 3+4\to 3+2+1) \equiv{}& \Gamma(\Lambda_c^+(5/2^-)\to\Sigma_c^{*++}(3/2^+)+\rho^-\to\Sigma_c^{*++}(3/2^+)+\pi^0+\pi^-)\\ ={}& \frac{1}{(2\pi)^3}\times\frac{1}{32m_0^3}\times g^2_{0\to3+4}\times g^2_{4\to2+1}\times\int dm_{12}dm_{23}\\ &\times\frac{1}{6}\mathrm{Tr}\left[\left(g_{\rho'\rho}-\frac{1}{3}\gamma_{\rho'}\gamma_\rho-\frac{p_{3,\rho'}\gamma_\rho-p_{3,\rho}\gamma_{\rho'}}{3m_3}-\frac{2p_{3,\rho'}p_{3,\rho}}{3m_3^2}\right)(\not p_3+m_3)\right.\\ &\left.\times\left(\frac{1}{2}g_{\mu\mu'}g_{\rho\rho'}+\frac{1}{2}g_{\mu\rho'}g_{\rho\mu'}-\frac{1}{3}g_{\mu\rho}g_{\mu'\rho'}\right)(\not p_0+m_0)\right]\\ &\times\left(g_{\mu\nu}-\frac{p_{4,\mu}p_{4,\nu}}{m_4^2}\right)\left(g_{\mu'\nu'}-\frac{p_{4,\mu'}p_{4,\nu'}}{m_4^2}\right)\times\frac{1}{|p_4^2-m_4^2+im_4\Gamma_4|^2}\\ &\times(p_{1,\nu}+p_{2,\nu})(p_{1,\nu'}+p_{2,\nu'}),\end{aligned} \tag{A16}$$

where we have simply used the following formula for the baryon field of spin 5/2:

$$\sum_{\mathrm{spin}}u_{\mu\nu}(p)\bar{u}_{\mu'\nu'}(p)=\left(\frac{1}{2}g_{\mu\mu'}g_{\nu\nu'}+\frac{1}{2}g_{\mu\nu'}g_{\nu\mu'}-\frac{1}{3}g_{\mu\nu}g_{\mu'\nu'}\right)(\not p+m). \tag{A17}$$

The three-body decay, $(j)\Xi_c(5/2^-)\to\Xi_c^*\rho\to\Xi_c^*\pi\pi$, can be similarly evaluated.

APPENDIX B: LIGHT-CONE DISTRIBUTION AMPLITUDES OF THE K MESON

In this appendix we list the light-cone distribution amplitudes of the K meson as examples. They are taken from Ref. [90], and we refer interested readers to read Refs. [89–96] for details. The light-cone distribution amplitudes of the K meson used in the present study are

$$\langle 0|\bar{q}(z)\gamma_\mu\gamma_5 s(-z)|K(q)\rangle = if_K q_\mu\int_0^1 du e^{i(2u-1)q\cdot z}\left(\phi_{2;K}(u)+\frac{1}{4}z^2\phi_{4;K}(u)\right)+\frac{i}{2}f_K\frac{1}{q\cdot z}z_\mu\int_0^1 du e^{i(2u-1)q\cdot z}\psi_{4;K}(u), \tag{B1}$$

$$\langle 0|\bar{q}(z)i\gamma_5 s(-z)|K(q)\rangle = \frac{f_K m_K^2}{m_s+m_q}\int_0^1 du e^{i(2u-1)q\cdot z}\phi^p_{3;K}(u), \tag{B2}$$

$$\langle 0|\bar{q}(z)\sigma_{\alpha\beta}\gamma_5 s(-z)|K(q)\rangle = -\frac{i}{3}\frac{f_K m_K^2}{m_s+m_q}(q_\alpha z_\beta-q_\beta z_\alpha)\int_0^1 du e^{i(2u-1)q\cdot z}\phi^\sigma_{3;K}(u), \tag{B3}$$

$$\langle 0|\bar{q}(z)\gamma_\mu\gamma_5 gG_{\alpha\beta}(vz)s(-z)|K(q)\rangle = q_\mu(q_\alpha z_\beta-q_\beta z_\alpha)\frac{1}{q\cdot z}f_K\Phi_{4;K}(v,q\cdot z)+(q_\beta g^\perp_{\alpha\mu}-q_\alpha g^\perp_{\beta\mu})f_K\Psi_{4;K}(v,q\cdot z), \tag{B4}$$

$$\langle 0|\bar{q}(z)\gamma_\mu ig\tilde{G}_{\alpha\beta}(vz)s(-z)|K(q)\rangle = q_\mu(q_\alpha z_\beta-q_\beta z_\alpha)\frac{1}{q\cdot z}f_K\tilde{\Phi}_{4;K}(v,q\cdot z)+(q_\beta g^\perp_{\alpha\mu}-q_\alpha g^\perp_{\beta\mu})f_K\tilde{\Psi}_{4;K}(v,q\cdot z), \tag{B5}$$

$$\langle 0|\bar{q}(z)\sigma_{\mu\nu}\gamma_5 gG_{\alpha\beta}(vz)s(-z)|K(q)\rangle = if_{3K}(q_\alpha q_\mu g^\perp_{\nu\beta}-q_\alpha q_\nu g^\perp_{\mu\beta}-(\alpha\leftrightarrow\beta))\times\int\mathcal{D}\underline{\alpha}e^{-iq\cdot z(\alpha_2-\alpha_1+v\alpha_3)}\Phi_{3;K}(\alpha_1,\alpha_2,\alpha_3). \tag{B6}$$

where $\tilde{G}_{\mu\nu}=\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}G^{\rho\sigma}$.

APPENDIX C: OTHER SUM RULES

In this appendix we show the sum rules for other currents with different quark contents.

1. $[\bar{\mathbf{3}}_F, \mathbf{0}, \mathbf{1}, \boldsymbol{\rho}]$

The sum rule for $\Lambda_c^+(\frac{1}{2}^-)$ belonging to $[\bar{\mathbf{3}}_F, 0, 1, \rho]$ is

$$G_{\Lambda_c^+[\frac{1}{2}^-]\to\Sigma_c^{++}\pi^-}(\omega,\omega') = \frac{g_{\Lambda_c^+[\frac{1}{2}^-]\to\Sigma_c^{++}\pi^-} f_{\Lambda_c^+[\frac{1}{2}^-]} f_{\Sigma_c^{++}}}{(\bar{\Lambda}_{\Lambda_c^+[\frac{1}{2}^-]} - \omega')(\bar{\Lambda}_{\Sigma_c^{++}} - \omega)} = 0. \tag{C1}$$

The sum rules for $\Xi_c^0(\frac{1}{2}^-)$ belonging to $[\bar{\mathbf{3}}_F, 0, 1, \rho]$ are

$$G_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c'^+\pi^-}(\omega,\omega') = \frac{g_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c'^+\pi^-} f_{\Xi_c^0[\frac{1}{2}^-]} f_{\Xi_c'^+}}{(\bar{\Lambda}_{\Xi_c^0[\frac{1}{2}^-]} - \omega')(\bar{\Lambda}_{\Xi_c'^+} - \omega)} = 0, \tag{C2}$$

$$\begin{aligned}
G_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^+\pi^-}(\omega,\omega') &= \frac{g_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^+\pi^-} f_{\Xi_c^0[\frac{1}{2}^-]} f_{\Xi_c^+}}{(\bar{\Lambda}_{\Xi_c^0[\frac{1}{2}^-]} - \omega')(\bar{\Lambda}_{\Xi_c^+} - \omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \left(-\frac{3 f_\pi m_\pi^2}{4\pi^2 t^4 (m_u + m_d)} \phi_{3;\pi}^p(u) + \frac{i f_\pi m_\pi^2 v\cdot q}{8\pi^2 t^3 (m_u + m_d)} \phi_{3;\pi}^\sigma(u) \right. \\
&\quad - \frac{i f_\pi}{16 t v\cdot q} \langle \bar{s}s \rangle \psi_{4;\pi}(u) - \frac{i f_\pi t}{256 v\cdot q} \langle g_s \bar{s}\sigma G s \rangle \psi_{4;\pi}(u) - \frac{3 i f_\pi}{16\pi^2 t^3 v\cdot q} m_s \psi_{4;\pi}(u) \\
&\quad \left. - \frac{f_\pi m_\pi^2}{32(m_u + m_d)} m_s \langle \bar{s}s \rangle \phi_{3;\pi}^p(u) + \frac{i f_\pi m_\pi^2 t v\cdot q}{192(m_u + m_d)} m_s \langle \bar{s}s \rangle \phi_{3;\pi}^\sigma(u) \right),
\end{aligned} \tag{C3}$$

$$\begin{aligned}
G_{\Xi_c^0[\frac{1}{2}^-]\to\Lambda_c^+K^-}(\omega,\omega') &= \frac{g_{\Xi_c^0[\frac{1}{2}^-]\to\Lambda_c^+K^-} f_{\Xi_c^0[\frac{1}{2}^-]} f_{\Lambda_c^+}}{(\bar{\Lambda}_{\Xi_c^0[\frac{1}{2}^-]} - \omega')(\bar{\Lambda}_{\Lambda_c^+} - \omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \left(-\frac{3 f_K m_K^2}{4\pi^2 t^4 (m_u + m_s)} \phi_{3;K}^p(u) + \frac{i f_K m_K^2 v\cdot q}{8\pi^2 t^3 (m_u + m_s)} \phi_{3;K}^\sigma(u) \right. \\
&\quad \left. - \frac{i f_K}{16 t v\cdot q} \langle \bar{q}q \rangle \psi_{4;K}(u) - \frac{i f_K t}{256 v\cdot q} \langle g_s \bar{q}\sigma G q \rangle \psi_{4;K}(u) \right),
\end{aligned} \tag{C4}$$

$$G_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^+\rho^-}(\omega,\omega') = \frac{g_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^+\rho^-} f_{\Xi_c^0[\frac{1}{2}^-]} f_{\Xi_c^+}}{(\bar{\Lambda}_{\Xi_c^0[\frac{1}{2}^-]} - \omega')(\bar{\Lambda}_{\Xi_c^+} - \omega)} = 0. \tag{C5}$$

2. $[\bar{\mathbf{3}}_F, \mathbf{1}, \mathbf{1}, \boldsymbol{\rho}]$

The sum rule for $\Lambda_c^+(\frac{1}{2}^-)$ belonging to $[\bar{\mathbf{3}}_F, 1, 1, \rho]$ is

$$\begin{aligned}
G_{\Lambda_c^+[\frac{1}{2}^-]\to\Sigma_c^{++}\pi^-}(\omega,\omega') &= \frac{g_{\Lambda_c^+[\frac{1}{2}^-]\to\Sigma_c^{++}\pi^-} f_{\Lambda_c^+[\frac{1}{2}^-]} f_{\Sigma_c^{++}}}{(\bar{\Lambda}_{\Lambda_c^+[\frac{1}{2}^-]} - \omega')(\bar{\Lambda}_{\Sigma_c^{++}} - \omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 8 \times \left(\frac{3 i f_\pi v\cdot q}{2\pi^2 t^4} \phi_{2;\pi}(u) + \frac{3 i f_\pi v\cdot q}{32\pi^2 t^2} \phi_{4;\pi}(u) + \frac{3 i f_\pi}{2\pi^2 t^4 v\cdot q} \psi_{4;\pi}(u) \right. \\
&\quad \left. + \frac{i f_\pi m_\pi^2 v\cdot q}{24(m_u + m_d)} \langle \bar{q}q \rangle \phi_{3;\pi}^\sigma(u) + \frac{i f_\pi m_\pi^2 t^2 v\cdot q}{384(m_u + m_d)} \langle g_s \bar{q}\sigma G q \rangle \phi_{3;\pi}^\sigma(u) \right) \\
&\quad + \int_0^\infty dt \int_0^1 du \int \mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2 + u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times 8 \times \left(\frac{3 i f_\pi v\cdot q}{8\pi^2 t^2} \Phi_{4;\pi}(\underline{\alpha}) - \frac{i f_\pi v\cdot q}{4\pi^2 t^2} \Psi_{4;\pi}(\underline{\alpha}) \right. \\
&\quad \left. + \frac{i f_\pi v\cdot q}{8\pi^2 t^2} \tilde{\Phi}_{4;\pi}(\underline{\alpha}) + \frac{i f_\pi v\cdot q}{4\pi^2 t^2} \tilde{\Psi}_{4;\pi}(\underline{\alpha}) + \frac{i f_\pi u v\cdot q}{4\pi^2 t^2} \Phi_{4;\pi}(\underline{\alpha}) + \frac{i f_\pi u v\cdot q}{2\pi^2 t^2} \Psi_{4;\pi}(\underline{\alpha}) \right).
\end{aligned} \tag{C6}$$

One of the sum rules for $\Xi_c^0(\frac{1}{2}^-)$ belonging to $[\bar{\mathbf{3}}_F, 1, 1, \rho]$ has been given in Eq. (56), and the others are

$$G_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^+\pi^-}(\omega,\omega') = \frac{g_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^+\pi^-} f_{\Xi_c^0[\frac{1}{2}^-]} f_{\Xi_c^+}}{(\bar{\Lambda}_{\Xi_c^0[\frac{1}{2}^-]}-\omega')(\bar{\Lambda}_{\Xi_c^+}-\omega)} = 0, \tag{C7}$$

$$G_{\Xi_c^0[\frac{1}{2}^-]\to\Lambda_c^+K^-}(\omega,\omega') = \frac{g_{\Xi_c^0[\frac{1}{2}^-]\to\Lambda_c^+K^-} f_{\Xi_c^0[\frac{1}{2}^-]} f_{\Lambda_c^+}}{(\bar{\Lambda}_{\Xi_c^0[\frac{1}{2}^-]}-\omega')(\bar{\Lambda}_{\Lambda_c^+}-\omega)} = 0, \tag{C8}$$

$$\begin{aligned}
G_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^+\rho^-}(\omega,\omega') &= \frac{g_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^+\rho^-} f_{\Xi_c^0[\frac{1}{2}^-]} f_{\Xi_c^+}}{(\bar{\Lambda}_{\Xi_c^0[\frac{1}{2}^-]}-\omega')(\bar{\Lambda}_{\Xi_c^+}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \Big(-\frac{f_\rho^\perp v\cdot q}{2\pi^2 t^4}\phi_{2;\rho}^\perp(u) + \frac{f_\rho^\perp m_\rho^2}{2\pi^2 t^4 v\cdot q}\phi_{2;\rho}^\perp(u) - \frac{f_\rho^\perp m_\rho^2}{2\pi^2 t^4 v\cdot q}\psi_{4;\rho}^\perp(u) \\
&\quad - \frac{f_\rho^\perp m_\rho^2 v\cdot q}{32\pi^2 t^2}\phi_{4;\rho}^\perp(u) + \frac{f_\rho^\parallel m_\rho v\cdot q}{48}\langle\bar{s}s\rangle\psi_{3;\rho}^\perp(u) + \frac{f_\rho^\parallel m_\rho t^2 v\cdot q}{768}\langle g_s\bar{s}\sigma Gs\rangle\psi_{3;\rho}^\perp(u) + \frac{f_\rho^\parallel m_\rho v\cdot q}{16\pi^2 t^2} m_s\psi_{3;\rho}^\perp(u) \\
&\quad - \frac{f_\rho^\perp v\cdot q}{48} m_s\langle\bar{s}s\rangle\phi_{2;\rho}^\perp(u) + \frac{f_\rho^\perp m_\rho^2}{48 v\cdot q} m_s\langle\bar{s}s\rangle\phi_{2;\rho}^\perp(u) \\
&\quad - \frac{f_\rho^\perp m_\rho^2}{48 v\cdot q} m_s\langle\bar{s}s\rangle\psi_{4;\rho}^\perp(u) - \frac{f_\rho^\perp m_\rho^2 t^2 v\cdot q}{768} m_s\langle\bar{s}s\rangle\phi_{4;\rho}^\perp(u)\Big) \\
&\quad + \int_0^\infty dt \int_0^1 du \int \mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times 4 \times \Big(-\frac{f_\rho^\perp m_\rho^2 v\cdot q}{8\pi^2 t^2}\Psi_{4;\rho}^\perp(\underline{\alpha}) + \frac{f_\rho^\perp m_\rho^2 v\cdot q}{8\pi^2 t^2}\tilde{\Psi}_{4;\rho}^\perp(\underline{\alpha}) \\
&\quad - \frac{f_\rho^\perp m_\rho^2 u v\cdot q}{8\pi^2 t^2}\Phi_{4;\rho}^{\perp 1}(\underline{\alpha}) + \frac{f_\rho^\perp m_\rho^2 u v\cdot q}{8\pi^2 t^2}\Phi_{4;\rho}^{\perp 2}(\underline{\alpha}) - \frac{f_\rho^\perp m_\rho^2 u v\cdot q}{4\pi^2 t^2}\tilde{\Psi}_{4;\rho}^\perp(\underline{\alpha})\Big).
\end{aligned} \tag{C9}$$

The sum rule for $\Lambda_c^+[\frac{3}{2}^-]$ belonging to $[\bar{\mathbf{3}}_F, 1, 1, \rho]$ is

$$\begin{aligned}
G_{\Lambda_c^+[\frac{3}{2}^-]\to\Sigma_c^{*++}\pi^-}(\omega,\omega') &= \frac{g_{\Lambda_c^+[\frac{3}{2}^-]\to\Sigma_c^{*++}\pi^-} f_{\Lambda_c^+[\frac{3}{2}^-]} f_{\Sigma_c^{*++}}}{(\bar{\Lambda}_{\Lambda_c^+[\frac{3}{2}^-]}-\omega')(\bar{\Lambda}_{\Sigma_c^{*++}}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 8 \times \Big(-\frac{f_\pi v\cdot q}{3\pi^2 t^4}\phi_{2;\pi}(u) - \frac{f_\pi v\cdot q}{48\pi^2 t^2}\phi_{4;\pi}(u) - \frac{f_\pi}{3\pi^2 t^4 v\cdot q}\psi_{4;\pi}(u) \\
&\quad - \frac{f_\pi m_\pi^2 v\cdot q}{108(m_u+m_d)}\langle\bar{q}q\rangle\phi_{3;\pi}^\sigma(u) - \frac{f_\pi m_\pi^2 t^2 v\cdot q}{1728(m_u+m_d)}\langle g_s\bar{q}\sigma Gq\rangle\phi_{3;\pi}^\sigma(u)\Big) \\
&\quad + \int_0^\infty dt \int_0^1 du \int \mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times 8 \times \Big(-\frac{f_\pi v\cdot q}{24\pi^2 t^2}\Phi_{4;\pi}(\underline{\alpha}) + \frac{5 f_\pi v\cdot q}{72\pi^2 t^2}\Psi_{4;\pi}(\underline{\alpha}) \\
&\quad - \frac{f_\pi v\cdot q}{72\pi^2 t^2}\tilde{\Phi}_{4;\pi}(\underline{\alpha}) - \frac{5 f_\pi v\cdot q}{72\pi^2 t^2}\tilde{\Psi}_{4;\pi}(\underline{\alpha}) - \frac{f_\pi u v\cdot q}{36\pi^2 t^2}\Phi_{4;\pi}(\underline{\alpha}) - \frac{7 f_\pi u v\cdot q}{72\pi^2 t^2}\Psi_{4;\pi}(\underline{\alpha}) \\
&\quad + \frac{f_\pi u v\cdot q}{12\pi^2 t^2}\tilde{\Phi}_{4;\pi}(\underline{\alpha}) + \frac{f_\pi u v\cdot q}{24\pi^2 t^2}\tilde{\Psi}_{4;\pi}(\underline{\alpha})\Big).
\end{aligned} \tag{C10}$$

The sum rules for $\Xi_c^0(\frac{3}{2}^-)$ belonging to $[\bar{\mathbf{3}}_F, 1, 1, \rho]$ are

$$
\begin{aligned}
G_{\Xi_c^0[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-}(\omega,\omega') &= \frac{g_{\Xi_c^0[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-} f_{\Xi_c^0[\frac{3}{2}^-]} f_{\Xi_c^{*+}}}{(\bar{\Lambda}_{\Xi_c^0[\frac{3}{2}^-]}-\omega')(\bar{\Lambda}_{\Xi_c^{*+}}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \bigg(-\frac{f_\pi v\cdot q}{3\pi^2 t^4}\phi_{2;\pi}(u) - \frac{f_\pi v\cdot q}{48\pi^2 t^2}\phi_{4;\pi}(u) - \frac{f_\pi}{3\pi^2 t^4 v\cdot q}\psi_{4;\pi}(u) \\
&\quad - \frac{f_\pi m_\pi^2 v\cdot q}{108(m_u+m_d)}\langle\bar{s}s\rangle\phi_{3;\pi}^\sigma(u) - \frac{f_\pi m_\pi^2 t^2 v\cdot q}{1728(m_u+m_d)}\langle g_s\bar{s}\sigma Gs\rangle\phi_{3;\pi}^\sigma(u) - \frac{f_\pi m_\pi^2 v\cdot q}{36\pi^2 t^2(m_u+m_d)} m_s\phi_{3;\pi}^\sigma(u) \\
&\quad - \frac{f_\pi v\cdot q}{72} m_s\langle\bar{s}s\rangle\phi_{2;\pi}(u) - \frac{f_\pi t^2 v\cdot q}{1152} m_s\langle\bar{s}s\rangle\phi_{4;\pi}(u) - \frac{f_\pi}{72 v\cdot q} m_s\langle\bar{s}s\rangle\psi_{4;\pi}(u)\bigg) \\
&\quad + \int_0^\infty dt \int_0^1 du \int \mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times 4 \times \bigg(-\frac{f_\pi v\cdot q}{24\pi^2 t^2}\Phi_{4;\pi}(\underline{\alpha}) + \frac{5 f_\pi v\cdot q}{72\pi^2 t^2}\Psi_{4;\pi}(\underline{\alpha}) \\
&\quad - \frac{f_\pi v\cdot q}{72\pi^2 t^2}\tilde{\Phi}_{4;\pi}(\underline{\alpha}) - \frac{5 f_\pi v\cdot q}{72\pi^2 t^2}\tilde{\Psi}_{4;\pi}(\underline{\alpha}) - \frac{f_\pi u v\cdot q}{36\pi^2 t^2}\Phi_{4;\pi}(\underline{\alpha}) - \frac{7 f_\pi u v\cdot q}{72\pi^2 t^2}\Psi_{4;\pi}(\underline{\alpha}) + \frac{f_\pi u v\cdot q}{12\pi^2 t^2}\tilde{\Phi}_{4;\pi}(\underline{\alpha}) \\
&\quad + \frac{f_\pi u v\cdot q}{24\pi^2 t^2}\tilde{\Psi}_{4;\pi}(\underline{\alpha})\bigg), \qquad \text{(C11)}
\end{aligned}
$$

$$
\begin{aligned}
G_{\Xi_c^0[\frac{3}{2}^-]\to\Xi_c^{+}\rho^-}(\omega,\omega') &= \frac{g_{\Xi_c^0[\frac{3}{2}^-]\to\Xi_c^{+}\rho^-} f_{\Xi_c^0[\frac{3}{2}^-]} f_{\Xi_c^{+}}}{(\bar{\Lambda}_{\Xi_c^0[\frac{3}{2}^-]}-\omega')(\bar{\Lambda}_{\Xi_c^{+}}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \bigg(\frac{f_\rho^\perp v\cdot q}{3\pi^2 t^4}\phi_{2;\rho}^\perp(u) - \frac{f_\rho^\perp m_\rho^2}{3\pi^2 t^4 v\cdot q}\phi_{2;\rho}^\perp(u) + \frac{f_\rho^\perp m_\rho^2}{3\pi^2 t^4 v\cdot q}\psi_{4;\rho}^\perp(u) \\
&\quad + \frac{f_\rho^\perp m_\rho^2 v\cdot q}{48\pi^2 t^2}\phi_{4;\rho}^\perp(u) - \frac{f_\rho^\parallel m_\rho v\cdot q}{72}\langle\bar{s}s\rangle\psi_{3;\rho}^\perp(u) - \frac{f_\rho^\parallel m_\rho t^2 v\cdot q}{1152}\langle g_s\bar{s}\sigma Gs\rangle\psi_{3;\rho}^\perp(u) - \frac{f_\rho^\parallel m_\rho v\cdot q}{24\pi^2 t^2} m_s\psi_{3;\rho}^\perp(u) \\
&\quad + \frac{f_\rho^\perp v\cdot q}{72} m_s\langle\bar{s}s\rangle\phi_{2;\rho}^\perp(u) - \frac{f_\rho^\perp m_\rho^2}{72 v\cdot q} m_s\langle\bar{s}s\rangle\phi_{2;\rho}^\perp(u) \\
&\quad + \frac{f_\rho^\perp m_\rho^2}{72 v\cdot q} m_s\langle\bar{s}s\rangle\psi_{4;\rho}^\perp(u) + \frac{f_\rho^\perp m_\rho^2 t^2 v\cdot q}{1152} m_s\langle\bar{s}s\rangle\phi_{4;\rho}^\perp(u)\bigg) \\
&\quad + \int_0^\infty dt \int_0^1 du \int \mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times 4 \times \bigg(\frac{f_\rho^\perp m_\rho^2 v\cdot q}{12\pi^2 t^2}\Psi_{4;\rho}^\perp(\underline{\alpha}) - \frac{f_\rho^\perp m_\rho^2 v\cdot q}{12\pi^2 t^2}\tilde{\Psi}_{4;\rho}^\perp(\underline{\alpha}) \\
&\quad + \frac{f_\rho^\perp m_\rho^2 u v\cdot q}{12\pi^2 t^2}\Phi_{4;\rho}^{\perp(1)}(\underline{\alpha}) - \frac{f_\rho^\perp m_\rho^2 u v\cdot q}{12\pi^2 t^2}\Phi_{4;\rho}^{\perp(2)}(\underline{\alpha}) + \frac{f_\rho^\perp m_\rho^2 u v\cdot q}{6\pi^2 t^2}\tilde{\Psi}_{4;\rho}^\perp(\underline{\alpha})\bigg). \qquad \text{(C12)}
\end{aligned}
$$

3. $[\bar{\mathbf{3}}_F, 2, 1, \rho]$

The sum rule for $\Lambda_c^+[\frac{3}{2}^-]$ belonging to $[\bar{\mathbf{3}}_F, 2, 1, \rho]$ is

$$
\begin{aligned}
G_{\Lambda_c^+[\frac{3}{2}^-]\to\Sigma_c^{*++}\pi^-}(\omega,\omega') &= \frac{g_{\Lambda_c^+[\frac{3}{2}^-]\to\Sigma_c^{*++}\pi^-} f_{\Lambda_c^+[\frac{3}{2}^-]} f_{\Sigma_c^{*++}}}{(\bar{\Lambda}_{\Lambda_c^+[\frac{3}{2}^-]}-\omega')(\bar{\Lambda}_{\Sigma_c^{*++}}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du \int \mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times 8 \times \bigg(\frac{f_\pi v\cdot q}{24\pi^2 t^2}\Phi_{4;\pi}(\underline{\alpha}) + \frac{f_\pi v\cdot q}{24\pi^2 t^2}\Psi_{4;\pi}(\underline{\alpha}) \\
&\quad - \frac{f_\pi v\cdot q}{24\pi^2 t^2}\tilde{\Phi}_{4;\pi}(\underline{\alpha}) - \frac{f_\pi v\cdot q}{24\pi^2 t^2}\tilde{\Psi}_{4;\pi}(\underline{\alpha}) - \frac{f_\pi u v\cdot q}{12\pi^2 t^2}\Phi_{4;\pi}(\underline{\alpha}) - \frac{f_\pi u v\cdot q}{24\pi^2 t^2}\Psi_{4;\pi}(\underline{\alpha}) + \frac{f_\pi u v\cdot q}{12\pi^2 t^2}\tilde{\Phi}_{4;\pi}(\underline{\alpha}) \\
&\quad + \frac{f_\pi u v\cdot q}{24\pi^2 t^2}\tilde{\Psi}_{4;\pi}(\underline{\alpha})\bigg). \qquad \text{(C13)}
\end{aligned}
$$

The sum rules for $\Xi_c^0(\frac{3}{2}^-)$ belonging to $[\bar{\mathbf{3}}_F, 2, 1, \rho]$ are

$$\begin{aligned} G_{\Xi_c^0[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-}(\omega,\omega') &= \frac{g_{\Xi_c^0[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-} f_{\Xi_c^0[\frac{3}{2}^-]} f_{\Xi_c^{*+}}}{(\bar{\Lambda}_{\Xi_c^0[\frac{3}{2}^-]}-\omega')(\bar{\Lambda}_{\Xi_c^{*+}}-\omega)} \\ &= \int_0^\infty dt \int_0^1 du \int \mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times 4 \times \left(\frac{f_\pi v\cdot q}{24\pi^2 t^2} \Phi_{4;\pi}(\underline{\alpha}) + \frac{f_\pi v\cdot q}{24\pi^2 t^2} \Psi_{4;\pi}(\underline{\alpha}) \right. \\ &\quad - \frac{f_\pi v\cdot q}{24\pi^2 t^2} \tilde{\Phi}_{4;\pi}(\underline{\alpha}) - \frac{f_\pi v\cdot q}{24\pi^2 t^2} \tilde{\Psi}_{4;\pi}(\underline{\alpha}) - \frac{f_\pi u v\cdot q}{12\pi^2 t^2} \Phi_{4;\pi}(\underline{\alpha}) - \frac{f_\pi u v\cdot q}{24\pi^2 t^2} \Psi_{4;\pi}(\underline{\alpha}) + \frac{f_\pi u v\cdot q}{12\pi^2 t^2} \tilde{\Phi}_{4;\pi}(\underline{\alpha}) \\ &\quad \left. + \frac{f_\pi u v\cdot q}{24\pi^2 t^2} \tilde{\Psi}_{4;\pi}(\underline{\alpha}) \right), \end{aligned} \tag{C14}$$

$$G_{\Xi_c^0[\frac{3}{2}^-]\to\Xi_c^{+}\rho^-}(\omega,\omega') = \frac{g_{\Xi_c^0[\frac{3}{2}^-]\to\Xi_c^{+}\rho^-} f_{\Xi_c^0[\frac{3}{2}^-]} f_{\Xi_c^{+}}}{(\bar{\Lambda}_{\Xi_c^0[\frac{3}{2}^-]}-\omega')(\bar{\Lambda}_{\Xi_c^{+}}-\omega)} = 0. \tag{C15}$$

The sum rule for $\Lambda_c^+[\frac{5}{2}^-]$ belonging to $[\bar{\mathbf{3}}_F, 2, 1, \rho]$ is

$$\begin{aligned} G_{\Lambda_c^+[\frac{5}{2}^-]\to\Sigma_c^{*++}\rho^-}(\omega,\omega') &= \frac{g_{\Lambda_c^+[\frac{5}{2}^-]\to\Sigma_c^{*++}\rho^-} f_{\Lambda_c^+[\frac{5}{2}^-]} f_{\Sigma_c^{*++}}}{(\bar{\Lambda}_{\Lambda_c^+[\frac{5}{2}^-]}-\omega')(\bar{\Lambda}_{\Sigma_c^{*++}}-\omega)} \\ &= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 8 \times \left(-\frac{3if_\rho^\parallel m_\rho}{10\pi^2 t^4} \phi_{2;\rho}^\parallel(u) + \frac{3if_\rho^\parallel m_\rho^3}{20\pi^2 t^4 (v\cdot q)^2} \phi_{2;\rho}^\parallel(u) + \frac{3if_\rho^\parallel m_\rho}{10\pi^2 t^4} \phi_{3;\rho}^\perp(u) \right. \\ &\quad - \frac{3if_\rho^\parallel m_\rho^3}{10\pi^2 t^4 (v\cdot q)^2} \phi_{3;\rho}^\perp(u) - \frac{3f_\rho^\parallel m_\rho v\cdot q}{40\pi^2 t^3} \psi_{3;\rho}^\perp(u) - \frac{3if_\rho^\parallel m_\rho^3}{160\pi^2 t^2} \phi_{4;\rho}^\parallel(u) + \frac{3if_\rho^\parallel m_\rho^3}{20\pi^2 t^4 (v\cdot q)^2} \psi_{4;\rho}^\parallel(u) \\ &\quad + \frac{f_\rho^\perp m_\rho^2}{40 t v\cdot q} \langle\bar{q}q\rangle \phi_{2;\rho}^\perp(u) + \frac{if_\rho^\perp m_\rho^2}{40} \langle\bar{q}q\rangle \psi_{3;\rho}^\parallel(u) - \frac{f_\rho^\perp m_\rho^2}{40 t v\cdot q} \langle\bar{q}q\rangle \psi_{4;\rho}^\perp(u) \\ &\quad \left. + \frac{f_\rho^\perp m_\rho^2 t}{640 v\cdot q} \langle g_s\bar{q}\sigma Gq\rangle \phi_{2;\rho}^\perp(u) + \frac{if_\rho^\perp m_\rho^2 t^2}{640} \langle g_s\bar{q}\sigma Gq\rangle \psi_{3;\rho}^\parallel(u) - \frac{f_\rho^\perp m_\rho^2 t}{640 v\cdot q} \langle g_s\bar{q}\sigma Gq\rangle \psi_{4;\rho}^\perp(u) \right) \\ &\quad + \int_0^\infty dt \int_0^1 du \int \mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times 8 \\ &\quad \times \left(\frac{3if_\rho^\parallel m_\rho^3}{40\pi^2 t^2} \Psi_{4;\rho}^\parallel(\underline{\alpha}) - \frac{3if_\rho^\parallel m_\rho^3}{40\pi^2 t^2} \tilde{\Psi}_{4;\rho}^\parallel(\underline{\alpha}) + \frac{3if_\rho^\parallel m_\rho^3 u}{20\pi^2 t^2} \Psi_{4;\rho}^\parallel(\underline{\alpha}) \right). \end{aligned} \tag{C16}$$

The sum rule for $\Xi_c^0(\frac{5}{2}^-)$ belonging to $[\bar{\mathbf{3}}_F, 2, 1, \rho]$ is

$$
\begin{aligned}
G_{\Xi_c^0[\frac{5}{2}^-]\to\Xi_c^{*+}\rho^-}(\omega,\omega') &= \frac{g_{\Xi_c^0[\frac{5}{2}^-]\to\Xi_c^{*+}\rho^-} f_{\Xi_c^0[\frac{5}{2}^-]} f_{\Xi_c^{*+}}}{(\bar\Lambda_{\Xi_c^0[\frac{5}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c^{*+}}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 8 \times \bigg(-\frac{3if_\rho^\parallel m_\rho}{10\pi^2 t^4}\phi_{2;\rho}^\parallel(u) + \frac{3if_\rho^\parallel m_\rho^3}{20\pi^2 t^4 (v\cdot q)^2}\phi_{2;\rho}^\parallel(u) + \frac{3if_\rho^\parallel m_\rho}{10\pi^2 t^4}\phi_{3;\rho}^\perp(u) \\
&\quad - \frac{3if_\rho^\parallel m_\rho^3}{10\pi^2 t^4 (v\cdot q)^2}\phi_{3;\rho}^\perp(u) - \frac{3f_\rho^\parallel m_\rho v\cdot q}{40\pi^2 t^3}\psi_{3;\rho}^\perp(u) - \frac{3if_\rho^\parallel m_\rho^3}{160\pi^2 t^2}\phi_{4;\rho}^\parallel(u) + \frac{3if_\rho^\parallel m_\rho^3}{20\pi^2 t^4 (v\cdot q)^2}\psi_{4;\rho}^\parallel(u) \\
&\quad + \frac{f_\rho^\perp m_\rho^2}{40 t v\cdot q}\langle\bar s s\rangle\phi_{2;\rho}^\perp(u) + \frac{if_\rho^\perp m_\rho^2}{40}\langle\bar s s\rangle\psi_{3;\rho}^\parallel(u) - \frac{f_\rho^\perp m_\rho^2}{40 t v\cdot q}\langle\bar s s\rangle\psi_{4;\rho}^\perp(u) + \frac{f_\rho^\perp m_\rho^2 t}{640 v\cdot q}\langle g_s\bar s\sigma G s\rangle\phi_{2;\rho}^\perp(u) \\
&\quad + \frac{if_\rho^\perp m_\rho^2 t^2}{640}\langle g_s\bar s\sigma G s\rangle\psi_{3;\rho}^\parallel(u) - \frac{f_\rho^\perp m_\rho^2 t}{640 v\cdot q}\langle g_s\bar s\sigma G s\rangle\psi_{4;\rho}^\perp(u) + \frac{3f_\rho^\perp m_\rho^2}{40\pi^2 t^3 v\cdot q} m_s\phi_{2;\rho}^\perp(u) \\
&\quad + \frac{3if_\rho^\perp m_\rho^2}{40\pi^2 t^2} m_s\psi_{3;\rho}^\parallel(u) - \frac{3f_\rho^\perp m_\rho^2}{40\pi^2 t^3 v\cdot q} m_s\psi_{4;\rho}^\perp(u) - \frac{if_\rho^\parallel m_\rho}{80} m_s\langle\bar s s\rangle\phi_{2;\rho}^\parallel(u) + \frac{if_\rho^\parallel m_\rho^3}{160(v\cdot q)^2} m_s\langle\bar s s\rangle\phi_{2;\rho}^\parallel(u) \\
&\quad + \frac{if_\rho^\parallel m_\rho}{80} m_s\langle\bar s s\rangle\phi_{3;\rho}^\perp(u) - \frac{if_\rho^\parallel m_\rho^3}{80(v\cdot q)^2} m_s\langle\bar s s\rangle\phi_{3;\rho}^\perp(u) - \frac{f_\rho^\parallel m_\rho t v\cdot q}{320} m_s\langle\bar s s\rangle\psi_{3;\rho}^\perp(u) \\
&\quad - \frac{if_\rho^\parallel m_\rho^3 t^2}{1280} m_s\langle\bar s s\rangle\phi_{4;\rho}^\parallel(u) + \frac{if_\rho^\parallel m_\rho^3}{160(v\cdot q)^2} m_s\langle\bar s s\rangle\psi_{4;\rho}^\parallel(u)\bigg) \\
&\quad + \int_0^\infty dt \int_0^1 du \int \mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times 8 \\
&\quad \times \bigg(\frac{3if_\rho^\parallel m_\rho^3}{40\pi^2 t^2}\Psi_{4;\rho}^\parallel(\underline{\alpha}) - \frac{3if_\rho^\parallel m_\rho^3}{40\pi^2 t^2}\tilde\Psi_{4;\rho}^\parallel(\underline{\alpha}) + \frac{3if_\rho^\parallel m_\rho^3 u}{20\pi^2 t^2}\Psi_{4;\rho}^\parallel(\underline{\alpha})\bigg). \qquad \text{(C17)}
\end{aligned}
$$

4. $[\bar{\mathbf{3}}_F, \mathbf{1}, \mathbf{0}, \lambda]$

The sum rule for $\Lambda_c^+(\frac{1}{2}^-)$ belonging to $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$ is

$$
\begin{aligned}
G_{\Lambda_c^+[\frac{1}{2}^-]\to\Sigma_c^{++}\pi^-}(\omega,\omega') &= \frac{g_{\Lambda_c^+[\frac{1}{2}^-]\to\Sigma_c^{++}\pi^-} f_{\Lambda_c^+[\frac{1}{2}^-]} f_{\Sigma_c^{++}}}{(\bar\Lambda_{\Lambda_c^+[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Sigma_c^{++}}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 8 \times \bigg(\frac{3f_\pi m_\pi^2}{4\pi^2 t^4 (m_u+m_d)}\phi_{3;\pi}^p(u) - \frac{if_\pi m_\pi^2 v\cdot q}{8\pi^2 t^3 (m_u+m_d)}\phi_{3;\pi}^\sigma(u) \\
&\quad + \frac{if_\pi}{16 t v\cdot q}\langle\bar q q\rangle\psi_{4;\pi}(u) + \frac{if_\pi t}{256 v\cdot q}\langle g_s\bar q\sigma G q\rangle\psi_{4;\pi}(u)\bigg). \qquad \text{(C18)}
\end{aligned}
$$

The sum rules for $\Xi_c^0(\frac{1}{2}^-)$ belonging to $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$ are

$$
\begin{aligned}
G_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^{\prime +}\pi^-}(\omega,\omega') &= \frac{g_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^{\prime +}\pi^-} f_{\Xi_c^0[\frac{1}{2}^-]} f_{\Xi_c^{\prime +}}}{(\bar\Lambda_{\Xi_c^0[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c^{\prime +}}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \bigg(\frac{3f_\pi m_\pi^2}{4\pi^2 t^4 (m_u+m_d)}\phi_{3;\pi}^p(u) - \frac{if_\pi m_\pi^2 v\cdot q}{8\pi^2 t^3 (m_u+m_d)}\phi_{3;\pi}^\sigma(u) \\
&\quad + \frac{if_\pi}{16 t v\cdot q}\langle\bar s s\rangle\psi_{4;\pi}(u) + \frac{if_\pi t}{256 v\cdot q}\langle g_s\bar s\sigma G s\rangle\psi_{4;\pi}(u) \\
&\quad + \frac{3if_\pi}{16\pi^2 t^3 v\cdot q} m_s\psi_{4;\pi}(u) + \frac{f_\pi m_\pi^2}{32(m_u+m_d)} m_s\langle\bar s s\rangle\phi_{3;\pi}^p(u) - \frac{if_\pi m_\pi^2 t v\cdot q}{192(m_u+m_d)} m_s\langle\bar s s\rangle\phi_{3;\pi}^\sigma(u)\bigg), \qquad \text{(C19)}
\end{aligned}
$$

$$G_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^+\pi^-}(\omega,\omega') = \frac{g_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^+\pi^-} f_{\Xi_c^0[\frac{1}{2}^-]} f_{\Xi_c^+}}{(\bar\Lambda_{\Xi_c^0[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c^+}-\omega)} = 0, \tag{C20}$$

$$G_{\Xi_c^0[\frac{1}{2}^-]\to\Lambda_c^+K^-}(\omega,\omega') = \frac{g_{\Xi_c^0[\frac{1}{2}^-]\to\Lambda_c^+K^-} f_{\Xi_c^0[\frac{1}{2}^-]} f_{\Lambda_c^+}}{(\bar\Lambda_{\Xi_c^0[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Lambda_c^+}-\omega)} = 0, \tag{C21}$$

$$\begin{aligned}
G_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^+\rho^-}(\omega,\omega') &= \frac{g_{\Xi_c^0[\frac{1}{2}^-]\to\Xi_c^+\rho^-} f_{\Xi_c^0[\frac{1}{2}^-]} f_{\Xi_c^+}}{(\bar\Lambda_{\Xi_c^0[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c^+}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \bigg(\frac{if_\rho^\parallel m_\rho}{4\pi^2 t^4} \phi_{2;\rho}^\parallel(u) - \frac{if_\rho^\parallel m_\rho^3}{8\pi^2 t^4 (v\cdot q)^2} \phi_{2;\rho}^\parallel(u) + \frac{if_\rho^\parallel m_\rho^3}{4\pi^2 t^4 (v\cdot q)^2} \phi_{3;\rho}^\perp(u) \\
&\quad - \frac{if_\rho^\parallel m_\rho^3}{8\pi^2 t^4 (v\cdot q)^2} \psi_{4;\rho}^\parallel(u) + \frac{if_\rho^\parallel m_\rho^3}{64\pi^2 t^2} \phi_{4;\rho}^\parallel(u) - \frac{if_\rho^\perp m_\rho^2}{48} \langle \bar s s\rangle \psi_{3;\rho}^\parallel(u) - \frac{if_\rho^\perp m_\rho^2 t^2}{768} \langle g_s \bar s \sigma G s\rangle \psi_{3;\rho}^\parallel(u) \\
&\quad - \frac{if_\rho^\perp m_\rho^2}{16\pi^2 t^2} m_s \psi_{3;\rho}^\parallel(u) + \frac{if_\rho^\parallel m_\rho}{96} m_s \langle \bar s s\rangle \phi_{2;\rho}^\parallel(u) - \frac{if_\rho^\parallel m_\rho^3}{192 (v\cdot q)^2} m_s \langle \bar s s\rangle \phi_{2;\rho}^\parallel(u) + \frac{if_\rho^\parallel m_\rho^3}{96 (v\cdot q)^2} m_s \langle \bar s s\rangle \phi_{3;\rho}^\perp(u) \\
&\quad + \frac{if_\rho^\parallel m_\rho^3 t^2}{1536} m_s \langle \bar s s\rangle \phi_{4;\rho}^\parallel(u) - \frac{if_\rho^\parallel m_\rho^3}{192 (v\cdot q)^2} m_s \langle \bar s s\rangle \psi_{4;\rho}^\parallel(u) \bigg) \\
&\quad + \int_0^\infty dt \int_0^1 du \int \mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2 + u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times 4 \times \bigg(- \frac{if_\rho^\parallel m_\rho^3}{8\pi^2 t^2} \Phi_{4;\rho}^\parallel(\underline{\alpha}) - \frac{if_\rho^\parallel m_\rho^3}{16\pi^2 t^2} \Psi_{4;\rho}^\parallel(\underline{\alpha}) \\
&\quad - \frac{if_\rho^\parallel m_\rho^3}{8\pi^2 t^2} \tilde\Phi_{4;\rho}^\parallel(\underline{\alpha}) - \frac{if_\rho^\parallel m_\rho^3}{16\pi^2 t^2} \tilde\Psi_{4;\rho}^\parallel(\underline{\alpha}) - \frac{if_\rho^\parallel m_\rho^3 u}{4\pi^2 t^2} \Phi_{4;\rho}^\parallel(\underline{\alpha}) - \frac{if_\rho^\parallel m_\rho^3 u}{8\pi^2 t^2} \Psi_{4;\rho}^\parallel(\underline{\alpha}) \bigg).
\end{aligned} \tag{C22}$$

The sum rule for $\Lambda_c^+[\frac{3}{2}^-]$ belonging to $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$ is

$$\begin{aligned}
G_{\Lambda_c^+[\frac{3}{2}^-]\to\Sigma_c^{*++}\pi^-}(\omega,\omega') &= \frac{g_{\Lambda_c^+[\frac{3}{2}^-]\to\Sigma_c^{*++}\pi^-} f_{\Lambda_c^+[\frac{3}{2}^-]} f_{\Sigma_c^{*++}}}{(\bar\Lambda_{\Lambda_c^+[\frac{3}{2}^-]}-\omega')(\bar\Lambda_{\Sigma_c^{*++}}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 8 \times \bigg(- \frac{f_\pi m_\pi^2}{6\pi^2 t^4 (m_u + m_d)} \phi_{3;\pi}^p(u) + \frac{if_\pi m_\pi^2 v\cdot q}{36\pi^2 t^3 (m_u + m_d)} \phi_{3;\pi}^\sigma(u) \\
&\quad - \frac{if_\pi}{72 t v\cdot q} \langle \bar q q\rangle \psi_{4;\pi}(u) - \frac{if_\pi t}{1152 v\cdot q} \langle g_s \bar q \sigma G q\rangle \psi_{4;\pi}(u) \bigg).
\end{aligned} \tag{C23}$$

The sum rules for $\Xi_c^0(\frac{3}{2}^-)$ belonging to $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$ are

$$\begin{aligned}
G_{\Xi_c^0[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-}(\omega,\omega') &= \frac{g_{\Xi_c^0[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-} f_{\Xi_c^0[\frac{3}{2}^-]} f_{\Xi_c^{*+}}}{(\bar\Lambda_{\Xi_c^0[\frac{3}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c^{*+}}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \bigg(- \frac{f_\pi m_\pi^2}{6\pi^2 t^4 (m_u + m_d)} \phi_{3;\pi}^p(u) + \frac{if_\pi m_\pi^2 v\cdot q}{36\pi^2 t^3 (m_u + m_d)} \phi_{3;\pi}^\sigma(u) \\
&\quad - \frac{if_\pi}{72 t v\cdot q} \langle \bar s s\rangle \psi_{4;\pi}(u) - \frac{if_\pi t}{1152 v\cdot q} \langle g_s \bar s \sigma G s\rangle \psi_{4;\pi}(u) - \frac{if_\pi}{24\pi^2 t^3 v\cdot q} m_s \psi_{4;\pi}(u) \\
&\quad - \frac{f_\pi m_\pi^2}{144 (m_u + m_d)} m_s \langle \bar s s\rangle \phi_{3;\pi}^p(u) + \frac{if_\pi m_\pi^2 t v\cdot q}{864 (m_u + m_d)} m_s \langle \bar s s\rangle \phi_{3;\pi}^\sigma(u) \bigg),
\end{aligned} \tag{C24}$$

$$G_{\Xi_c^0[\frac{3}{2}^-]\to\Xi_c^+\rho^-}(\omega,\omega') = \frac{g_{\Xi_c^0[\frac{3}{2}^-]\to\Xi_c^+\rho^-} f_{\Xi_c^0[\frac{3}{2}^-]} f_{\Xi_c^+}}{(\bar\Lambda_{\Xi_c^0[\frac{3}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c^+}-\omega)}$$
$$= \int_0^\infty dt\int_0^1 du e^{i(1-u)\omega' t}e^{iu\omega t}\times 4\times\Bigg(\frac{if_\rho^\parallel m_\rho}{6\pi^2t^4}\phi_{2;\rho}^\parallel(u) - \frac{if_\rho^\parallel m_\rho^3}{12\pi^2t^4(v\cdot q)^2}\phi_{2;\rho}^\parallel(u) + \frac{if_\rho^\parallel m_\rho^3}{6\pi^2t^4(v\cdot q)^2}\phi_{3;\rho}^\perp(u)$$
$$-\frac{if_\rho^\parallel m_\rho^3}{12\pi^2t^4(v\cdot q)^2}\psi_{4;\rho}^\parallel(u) + \frac{if_\rho^\parallel m_\rho^3}{96\pi^2t^2}\phi_{4;\rho}^\parallel(u) - \frac{if_\rho^\perp m_\rho^2}{72}\langle\bar ss\rangle\psi_{3;\rho}^\parallel(u) - \frac{if_\rho^\perp m_\rho^2t^2}{1152}\langle g_s\bar s\sigma Gs\rangle\psi_{3;\rho}^\parallel(u)$$
$$-\frac{if_\rho^\perp m_\rho^2}{24\pi^2t^2}m_s\psi_{3;\rho}^\parallel(u) + \frac{if_\rho^\parallel m_\rho}{144}m_s\langle\bar ss\rangle\phi_{2;\rho}^\parallel(u) - \frac{if_\rho^\parallel m_\rho^3}{288(v\cdot q)^2}m_s\langle\bar ss\rangle\phi_{2;\rho}^\parallel(u) + \frac{if_\rho^\parallel m_\rho^3}{144(v\cdot q)^2}m_s\langle\bar ss\rangle\phi_{3;\rho}^\perp(u)$$
$$+\frac{if_\rho^\parallel m_\rho^3t^2}{2304}m_s\langle\bar ss\rangle\phi_{4;\rho}^\parallel(u) - \frac{if_\rho^\parallel m_\rho^3}{288(v\cdot q)^2}m_s\langle\bar ss\rangle\psi_{4;\rho}^\parallel(u)\Bigg)$$
$$+\int_0^\infty dt\int_0^1 du\int\mathcal{D}\underline{\alpha}e^{i\omega' t(\alpha_2+u\alpha_3)}e^{i\omega t(1-\alpha_2-u\alpha_3)}\times 4\times\Bigg(-\frac{if_\rho^\parallel m_\rho^3}{12\pi^2t^2}\Phi_{4;\rho}^\parallel(\underline{\alpha}) - \frac{if_\rho^\parallel m_\rho^3}{24\pi^2t^2}\Psi_{4;\rho}^\parallel(\underline{\alpha})$$
$$-\frac{if_\rho^\parallel m_\rho^3}{12\pi^2t^2}\tilde\Phi_{4;\rho}^\parallel(\underline{\alpha}) - \frac{if_\rho^\parallel m_\rho^3}{24\pi^2t^2}\tilde\Psi_{4;\rho}^\parallel(\underline{\alpha}) - \frac{if_\rho^\parallel m_\rho^3u}{6\pi^2t^2}\Phi_{4;\rho}^\parallel(\underline{\alpha}) - \frac{if_\rho^\parallel m_\rho^3u}{12\pi^2t^2}\Psi_{4;\rho}^\parallel(\underline{\alpha})\Bigg). \tag{C25}$$

5. $[\mathbf{6}_F,1,0,\rho]$

The sum rule for $\Sigma_c^0(\frac{1}{2}^-)$ belonging to $[\mathbf{6}_F,1,0,\rho]$ is

$$G_{\Sigma_c^0[\frac{1}{2}^-]\to\Lambda_c^+\pi^-}(\omega,\omega') = \frac{g_{\Sigma_c^0[\frac{1}{2}^-]\to\Lambda_c^+\pi^-}f_{\Sigma_c^0[\frac{1}{2}^-]}f_{\Lambda_c^+}}{(\bar\Lambda_{\Sigma_c^0[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Lambda_c^+}-\omega)} = 0, \tag{C26}$$

$$G_{\Sigma_c^0[\frac{1}{2}^-]\to\Sigma_c^+\pi^-}(\omega,\omega') = \frac{g_{\Sigma_c^0[\frac{1}{2}^-]\to\Sigma_c^+\pi^-}f_{\Sigma_c^0[\frac{1}{2}^-]}f_{\Sigma_c^+}}{(\bar\Lambda_{\Sigma_c^0[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Sigma_c^+}-\omega)}$$
$$= \int_0^\infty dt\int_0^1 du e^{i(1-u)\omega' t}e^{iu\omega t}\times 8\times\Bigg(\frac{3f_\pi m_\pi^2}{4\pi^2t^4(m_u+m_d)}\phi_{3;\pi}^p(u) + \frac{if_\pi m_\pi^2 v\cdot q}{8\pi^2t^3(m_u+m_d)}\phi_{3;\pi}^\sigma(u)$$
$$-\frac{if_\pi}{16tv\cdot q}\langle\bar qq\rangle\psi_{4;\pi}(u) - \frac{if_\pi t}{256v\cdot q}\langle g_s\bar q\sigma Gq\rangle\psi_{4;\pi}(u)\Bigg). \tag{C27}$$

The sum rule for $\Xi_c^{\prime 0}(\frac{1}{2}^-)$ belonging to $[\mathbf{6}_F,1,0,\rho]$ is

$$G_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Xi_c^+\pi^-}(\omega,\omega') = \frac{g_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Xi_c^+\pi^-}f_{\Xi_c^{\prime 0}[\frac{1}{2}^-]}f_{\Xi_c^+}}{(\bar\Lambda_{\Xi_c^{\prime 0}[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c^+}-\omega)} = 0, \tag{C28}$$

$$G_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Xi_c^{\prime +}\pi^-}(\omega,\omega') = \frac{g_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Xi_c^{\prime +}\pi^-}f_{\Xi_c^{\prime 0}[\frac{1}{2}^-]}f_{\Xi_c^{\prime +}}}{(\bar\Lambda_{\Xi_c^{\prime 0}[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c^{\prime +}}-\omega)}$$
$$= \int_0^\infty dt\int_0^1 du e^{i(1-u)\omega' t}e^{iu\omega t}\times 4\times\Bigg(\frac{3f_\pi m_\pi^2}{4\pi^2t^4(m_u+m_d)}\phi_{3;\pi}^p(u) + \frac{if_\pi m_\pi^2 v\cdot q}{8\pi^2t^3(m_u+m_d)}\phi_{3;\pi}^\sigma(u)$$
$$-\frac{if_\pi}{16tv\cdot q}\langle\bar ss\rangle\psi_{4;\pi}(u) - \frac{if_\pi t}{256v\cdot q}\langle g_s\bar s\sigma Gs\rangle\psi_{4;\pi}(u) - \frac{3if_\pi}{16\pi^2t^3v\cdot q}m_s\psi_{4;\pi}(u)$$
$$+\frac{f_\pi m_\pi^2}{32(m_u+m_d)}m_s\langle\bar ss\rangle\phi_{3;\pi}^p(u) + \frac{if_\pi m_\pi^2 tv\cdot q}{192(m_u+m_d)}m_s\langle\bar ss\rangle\phi_{3;\pi}^\sigma(u)\Bigg), \tag{C29}$$

$$G_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Lambda_c^+K^-}(\omega,\omega') = \frac{g_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Lambda_c^+K^-}f_{\Xi_c^{\prime 0}[\frac{1}{2}^-]}f_{\Lambda_c^+}}{(\bar\Lambda_{\Xi_c^{\prime 0}[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Lambda_c^+}-\omega)} = 0, \tag{C30}$$

$$G_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Sigma_c^+K^-}(\omega,\omega') = \frac{g_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Sigma_c^+K^-} f_{\Xi_c^{\prime 0}[\frac{1}{2}^-]} f_{\Sigma_c^+}}{(\bar\Lambda_{\Xi_c^{\prime 0}[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Sigma_c^+}-\omega)}$$
$$= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \Bigg(\frac{3 f_K m_K^2}{4\pi^2 t^4 (m_u+m_s)} \phi_{3;K}^p(u) + \frac{i f_K m_K^2 v\cdot q}{8\pi^2 t^3 (m_u+m_s)} \phi_{3;K}^\sigma(u)$$
$$- \frac{i f_K}{16 t v\cdot q} \langle \bar q q\rangle \psi_{4;K}(u) - \frac{i f_K t}{256 v\cdot q} \langle g_s \bar q \sigma G q\rangle \psi_{4;K}(u) \Bigg). \tag{C31}$$

The sum rule for $\Omega_c^0(\frac{1}{2}^-)$ belonging to $[\mathbf{6}_F, 1, 0, \rho]$ is

$$G_{\Omega_c^0[\frac{1}{2}^-]\to\Xi_c^+K^-}(\omega,\omega') = \frac{g_{\Omega_c^0[\frac{1}{2}^-]\to\Xi_c^+K^-} f_{\Omega_c^0[\frac{1}{2}^-]} f_{\Xi_c^+}}{(\bar\Lambda_{\Omega_c^0[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c^+}-\omega)} = 0, \tag{C32}$$

$$G_{\Omega_c^0[\frac{1}{2}^-]\to\Xi_c^{\prime +}K^-}(\omega,\omega') = \frac{g_{\Omega_c^0[\frac{1}{2}^-]\to\Xi_c^{\prime +}K^-} f_{\Omega_c^0[\frac{1}{2}^-]} f_{\Xi_c^{\prime +}}}{(\bar\Lambda_{\Omega_c^0[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c^{\prime +}}-\omega)}$$
$$= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 8 \times \Bigg(\frac{3 f_K m_K^2}{4\pi^2 t^4 (m_u+m_s)} \phi_{3;K}^p(u) + \frac{i f_K m_K^2 v\cdot q}{8\pi^2 t^3 (m_u+m_s)} \phi_{3;K}^\sigma(u)$$
$$- \frac{i f_K}{16 t v\cdot q} \langle \bar s s\rangle \psi_{4;K}(u) - \frac{i f_K t}{256 v\cdot q} \langle g_s \bar s \sigma G s\rangle \psi_{4;K}(u) - \frac{3 i f_K}{16\pi^2 t^3 v\cdot q} m_s \psi_{4;K}(u)$$
$$+ \frac{f_K m_K^2}{32 (m_u+m_s)} m_s \langle \bar s s\rangle \phi_{3;K}^p(u) + \frac{i f_K m_K^2 t v\cdot q}{192 (m_u+m_s)} m_s \langle \bar s s\rangle \phi_{3;K}^\sigma(u) \Bigg). \tag{C33}$$

The sum rule for $\Sigma_c^0[\frac{3}{2}^-]$ belonging to $[\mathbf{6}_F, 1, 0, \rho]$ is

$$G_{\Sigma_c^0[\frac{3}{2}^-]\to\Sigma_c^{*+}\pi^-}(\omega,\omega') = \frac{g_{\Sigma_c^0[\frac{3}{2}^-]\to\Sigma_c^{*+}\pi^-} f_{\Sigma_c^0[\frac{3}{2}^-]} f_{\Sigma_c^{*+}}}{(\bar\Lambda_{\Sigma_c^0[\frac{3}{2}^-]}-\omega')(\bar\Lambda_{\Sigma_c^{*+}}-\omega)}$$
$$= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 8 \times \Bigg(- \frac{f_\pi m_\pi^2}{6\pi^2 t^4 (m_u+m_d)} \phi_{3;\pi}^p(u) - \frac{i f_\pi m_\pi^2 v\cdot q}{36\pi^2 t^3 (m_u+m_d)} \phi_{3;\pi}^\sigma(u)$$
$$+ \frac{i f_\pi}{72 t v\cdot q} \langle \bar q q\rangle \psi_{4;\pi}(u) + \frac{i f_\pi t}{1152 v\cdot q} \langle g_s \bar q \sigma G q\rangle \psi_{4;\pi}(u) \Bigg). \tag{C34}$$

The sum rule for $\Xi_c^0(\frac{3}{2}^-)$ belonging to $[\mathbf{6}_F, 1, 0, \rho]$ is

$$G_{\Xi_c^{\prime 0}[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-}(\omega,\omega') = \frac{g_{\Xi_c^{\prime 0}[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-} f_{\Xi_c^{\prime 0}[\frac{3}{2}^-]} f_{\Xi_c^{*+}}}{(\bar\Lambda_{\Xi_c^{\prime 0}[\frac{3}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c^{*+}}-\omega)}$$
$$= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \Bigg(- \frac{f_\pi m_\pi^2}{6\pi^2 t^4 (m_u+m_d)} \phi_{3;\pi}^p(u) - \frac{i f_\pi m_\pi^2 v\cdot q}{36\pi^2 t^3 (m_u+m_d)} \phi_{3;\pi}^\sigma(u)$$
$$+ \frac{i f_\pi}{72 t v\cdot q} \langle \bar s s\rangle \psi_{4;\pi}(u) + \frac{i f_\pi t}{1152 v\cdot q} \langle g_s \bar s \sigma G s\rangle \psi_{4;\pi}(u) + \frac{i f_\pi}{24\pi^2 t^3 v\cdot q} m_s \psi_{4;\pi}(u)$$
$$- \frac{f_\pi m_\pi^2}{144 (m_u+m_d)} m_s \langle \bar s s\rangle \phi_{3;\pi}^p(u) - \frac{i f_\pi m_\pi^2 t v\cdot q}{864 (m_u+m_d)} m_s \langle \bar s s\rangle \phi_{3;\pi}^\sigma(u) \Bigg), \tag{C35}$$

$$G_{\Xi_c^{\prime 0}[\frac{3}{2}^-]\to\Sigma_c^{*+}K^-}(\omega,\omega') = \frac{g_{\Xi_c^{\prime 0}[\frac{3}{2}^-]\to\Sigma_c^{*+}K^-} f_{\Xi_c^{\prime 0}[\frac{3}{2}^-]} f_{\Sigma_c^{*+}}}{(\bar\Lambda_{\Xi_c^{\prime 0}[\frac{3}{2}^-]}-\omega')(\bar\Lambda_{\Sigma_c^{*+}}-\omega)}$$
$$= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \left(-\frac{f_K m_K^2}{6\pi^2 t^4 (m_u+m_s)} \phi_{3;K}^p(u) - \frac{i f_K m_K^2 v\cdot q}{36\pi^2 t^3 (m_u+m_s)} \phi_{3;K}^\sigma(u) + \frac{i f_K}{72 t v\cdot q} \langle \bar q q\rangle \psi_{4;K}(u) + \frac{i f_K t}{1152 v\cdot q} \langle g_s \bar q \sigma G q\rangle \psi_{4;K}(u) \right). \quad \text{(C36)}$$

The sum rule for $\Omega_c^0(\frac{3}{2}^-)$ belonging to $[\mathbf{6}_F, 1, 0, \rho]$ is

$$G_{\Omega_c^0[\frac{3}{2}^-]\to\Xi_c^{*+}K^-}(\omega,\omega') = \frac{g_{\Omega_c^0[\frac{3}{2}^-]\to\Xi_c^{*+}K^-} f_{\Omega_c^0[\frac{3}{2}^-]} f_{\Xi_c^{*+}}}{(\bar\Lambda_{\Omega_c^0[\frac{3}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c^{*+}}-\omega)}$$
$$= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 8 \times \left(-\frac{f_K m_K^2}{6\pi^2 t^4 (m_u+m_s)} \phi_{3;K}^p(u) - \frac{i f_K m_K^2 v\cdot q}{36\pi^2 t^3 (m_u+m_s)} \phi_{3;K}^\sigma(u) + \frac{i f_K}{72 t v\cdot q} \langle \bar s s\rangle \psi_{4;K}(u) + \frac{i f_K t}{1152 v\cdot q} \langle g_s \bar s \sigma G s\rangle \psi_{4;K}(u) + \frac{i f_K}{24\pi^2 t^3 v\cdot q} m_s \psi_{4;K}(u) - \frac{f_K m_K^2}{144(m_u+m_s)} m_s \langle \bar s s\rangle \phi_{3;K}^p(u) - \frac{i f_K m_K^2 t v\cdot q}{864(m_u+m_s)} m_s \langle \bar s s\rangle \phi_{3;K}^\sigma(u) \right). \quad \text{(C37)}$$

6. $[\mathbf{6}_F, \mathbf{0}, \mathbf{1}, \lambda]$

The sum rule for $\Sigma_c^0(\frac{1}{2}^-)$ belonging to $[\mathbf{6}_F, 0, 1, \lambda]$ is

$$G_{\Sigma_c^0[\frac{1}{2}^-]\to\Lambda_c^+\pi^-}(\omega,\omega') = \frac{g_{\Sigma_c^0[\frac{1}{2}^-]\to\Lambda_c^+\pi^-} f_{\Sigma_c^0[\frac{1}{2}^-]} f_{\Lambda_c^+}}{(\bar\Lambda_{\Sigma_c^0[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Lambda_c^+}-\omega)}$$
$$= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 8 \times \left(-\frac{3 f_\pi m_\pi^2}{4\pi^2 t^4 (m_u+m_d)} \phi_{3;\pi}^p(u) - \frac{i f_\pi m_\pi^2 v\cdot q}{8\pi^2 t^3 (m_u+m_d)} \phi_{3;\pi}^\sigma(u) + \frac{i f_\pi}{16 t v\cdot q} \langle \bar q q\rangle \psi_{4;\pi}(u) + \frac{i f_\pi t}{256 v\cdot q} \langle g_s \bar q \sigma G q\rangle \psi_{4;\pi}(u) \right), \quad \text{(C38)}$$

$$G_{\Sigma_c^0[\frac{1}{2}^-]\to\Sigma_c^+\pi^-}(\omega,\omega') = \frac{g_{\Sigma_c^0[\frac{1}{2}^-]\to\Sigma_c^+\pi^-} f_{\Sigma_c^0[\frac{1}{2}^-]} f_{\Sigma_c^+}}{(\bar\Lambda_{\Sigma_c^0[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Sigma_c^+}-\omega)} = 0. \quad \text{(C39)}$$

The sum rule for $\Xi_c^{\prime 0}(\frac{1}{2}^-)$ belonging to $[\mathbf{6}_F, 0, 1, \lambda]$ is

$$G_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Xi_c^+\pi^-}(\omega,\omega') = \frac{g_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Xi_c^+\pi^-} f_{\Xi_c^{\prime 0}[\frac{1}{2}^-]} f_{\Xi_c^+}}{(\bar\Lambda_{\Xi_c^{\prime 0}[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c^+}-\omega)}$$
$$= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \left(-\frac{3 f_\pi m_\pi^2}{4\pi^2 t^4 (m_u+m_d)} \phi_{3;\pi}^p(u) - \frac{i f_\pi m_\pi^2 v\cdot q}{8\pi^2 t^3 (m_u+m_d)} \phi_{3;\pi}^\sigma(u) + \frac{i f_\pi}{16 t v\cdot q} \langle \bar s s\rangle \psi_{4;\pi}(u) + \frac{i f_\pi t}{256 v\cdot q} \langle g_s \bar s \sigma G s\rangle \psi_{4;\pi}(u) + \frac{3 i f_\pi}{16\pi^2 t^3 v\cdot q} m_s \psi_{4;\pi}(u) - \frac{f_\pi m_\pi^2}{32(m_u+m_d)} m_s \langle \bar s s\rangle \phi_{3;\pi}^p(u) - \frac{i f_\pi m_\pi^2 t v\cdot q}{192(m_u+m_d)} m_s \langle \bar s s\rangle \phi_{3;\pi}^\sigma(u) \right), \quad \text{(C40)}$$

$$G_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Xi_c^{\prime +}\pi^-}(\omega,\omega') = \frac{g_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Xi_c^{\prime +}\pi^-} f_{\Xi_c^{\prime 0}[\frac{1}{2}^-]} f_{\Xi_c^{\prime +}}}{(\bar\Lambda_{\Xi_c^{\prime 0}[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c^{\prime +}}-\omega)} = 0, \quad \text{(C41)}$$

$$G_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Lambda_c^+K^-}(\omega,\omega') = \frac{g_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Lambda_c^+K^-} f_{\Xi_c^{\prime 0}[\frac{1}{2}^-]} f_{\Lambda_c^+}}{(\bar\Lambda_{\Xi_c^{\prime 0}[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Lambda_c^+}-\omega)}$$
$$= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \left(-\frac{3 f_K m_K^2}{4\pi^2 t^4 (m_u+m_s)} \phi_{3;K}^p(u) - \frac{i f_K m_K^2 v\cdot q}{8\pi^2 t^3 (m_u+m_s)} \phi_{3;K}^\sigma(u) + \frac{i f_K}{16 t v\cdot q} \langle \bar q q\rangle \psi_{4;K}(u) + \frac{i f_K t}{256 v\cdot q} \langle g_s \bar q \sigma G q\rangle \psi_{4;K}(u) \right), \quad (C42)$$

$$G_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Sigma_c^+K^-}(\omega,\omega') = \frac{g_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Sigma_c^+K^-} f_{\Xi_c^{\prime 0}[\frac{1}{2}^-]} f_{\Sigma_c^+}}{(\bar\Lambda_{\Xi_c^{\prime 0}[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Sigma_c^+}-\omega)} = 0. \quad (C43)$$

The sum rule for $\Omega_c^0(\frac{1}{2}^-)$ belonging to $[\mathbf{6}_F, 0, 1, \lambda]$ is

$$G_{\Omega_c^0[\frac{1}{2}^-]\to\Xi_c^+K^-}(\omega,\omega') = \frac{g_{\Omega_c^0[\frac{1}{2}^-]\to\Xi_c^+K^-} f_{\Omega_c^0[\frac{1}{2}^-]} f_{\Xi_c^+}}{(\bar\Lambda_{\Omega_c^0[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c^+}-\omega)}$$
$$= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 8 \times \left(-\frac{3 f_K m_K^2}{4\pi^2 t^4 (m_u+m_s)} \phi_{3;K}^p(u) - \frac{i f_K m_K^2 v\cdot q}{8\pi^2 t^3 (m_u+m_s)} \phi_{3;K}^\sigma(u) + \frac{i f_K}{16 t v\cdot q} \langle \bar s s\rangle \psi_{4;K}(u) + \frac{i f_K t}{256 v\cdot q} \langle g_s \bar s \sigma G s\rangle \psi_{4;K}(u) + \frac{3 i f_K}{16\pi^2 t^3 v\cdot q} m_s \psi_{4;K}(u) - \frac{f_K m_K^2}{32(m_u+m_s)} m_s \langle \bar s s\rangle \phi_{3;K}^p(u) - \frac{i f_K m_K^2 t v\cdot q}{192(m_u+m_s)} m_s \langle \bar s s\rangle \phi_{3;K}^\sigma(u) \right), \quad (C44)$$

$$G_{\Omega_c^0[\frac{1}{2}^-]\to\Xi_c^{\prime +}K^-}(\omega,\omega') = \frac{g_{\Omega_c^0[\frac{1}{2}^-]\to\Xi_c^{\prime +}K^-} f_{\Omega_c^0[\frac{1}{2}^-]} f_{\Xi_c^{\prime +}}}{(\bar\Lambda_{\Omega_c^0[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c^{\prime +}}-\omega)} = 0. \quad (C45)$$

7. $[\mathbf{6}_F, \mathbf{1}, \mathbf{1}, \lambda]$

The sum rule for $\Sigma_c^0(\frac{1}{2}^-)$ belonging to $[\mathbf{6}_F, 1, 1, \lambda]$ is

$$G_{\Sigma_c^0[\frac{1}{2}^-]\to\Lambda_c^+\pi^-}(\omega,\omega') = \frac{g_{\Sigma_c^0[\frac{1}{2}^-]\to\Lambda_c^+\pi^-} f_{\Sigma_c^0[\frac{1}{2}^-]} f_{\Lambda_c^+}}{(\bar\Lambda_{\Sigma_c^0[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Lambda_c^+}-\omega)} = 0, \quad (C46)$$

$$G_{\Sigma_c^0[\frac{1}{2}^-]\to\Sigma_c^+\pi^-}(\omega,\omega') = \frac{g_{\Sigma_c^0[\frac{1}{2}^-]\to\Sigma_c^+\pi^-} f_{\Sigma_c^0[\frac{1}{2}^-]} f_{\Sigma_c^+}}{(\bar\Lambda_{\Sigma_c^0[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Sigma_c^+}-\omega)}$$
$$= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 8 \times \left(\frac{3 i f_\pi v\cdot q}{2\pi^2 t^4} \phi_{2;\pi}(u) + \frac{3 i f_\pi v\cdot q}{32\pi^2 t^2} \phi_{4;\pi}(u) - \frac{i f_\pi m_\pi^2 v\cdot q}{24(m_u+m_d)} \langle \bar q q\rangle \phi_{3;\pi}^\sigma(u) - \frac{i f_\pi m_\pi^2 t^2 v\cdot q}{384(m_u+m_d)} \langle g_s \bar q \sigma G q\rangle \phi_{3;\pi}^\sigma(u) \right)$$
$$+ \int_0^\infty dt \int_0^1 du \int \mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times 8 \times \left(-\frac{i f_\pi v\cdot q}{8\pi^2 t^2} \Phi_{4;\pi}(\underline{\alpha}) - \frac{i f_\pi v\cdot q}{4\pi^2 t^2} \Psi_{4;\pi}(\underline{\alpha}) - \frac{3 i f_\pi v\cdot q}{8\pi^2 t^2} \tilde\Phi_{4;\pi}(\underline{\alpha}) + \frac{i f_\pi v\cdot q}{4\pi^2 t^2} \tilde\Psi_{4;\pi}(\underline{\alpha}) - \frac{3 i f_\pi u v\cdot q}{4\pi^2 t^2} \Phi_{4;\pi}(\underline{\alpha}) + \frac{i f_\pi u v\cdot q}{2\pi^2 t^2} \Psi_{4;\pi}(\underline{\alpha}) \right). \quad (C47)$$

The sum rule for $\Xi_c^{\prime 0}(\frac{1}{2}^-)$ belonging to $[\mathbf{6}_F, 1, 1, \lambda]$ is

$$G_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Xi_c^+\pi^-}(\omega,\omega') = \frac{g_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Xi_c^+\pi^-} f_{\Xi_c^{\prime 0}[\frac{1}{2}^-]} f_{\Xi_c^+}}{(\bar\Lambda_{\Xi_c^{\prime 0}[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c^+}-\omega)} = 0, \quad (C48)$$

$$\begin{aligned} G_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Xi_c^{\prime +}\pi^-}(\omega,\omega') &= \frac{g_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Xi_c^{\prime +}\pi^-} f_{\Xi_c^{\prime 0}[\frac{1}{2}^-]} f_{\Xi_c^{\prime +}}}{(\bar\Lambda_{\Xi_c^{\prime 0}[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c^{\prime +}}-\omega)} \\ &= \int_0^\infty dt\int_0^1 du e^{i(1-u)\omega' t}e^{iu\omega t}\times 4\times\bigg(\frac{3if_\pi v\cdot q}{2\pi^2 t^4}\phi_{2;\pi}(u)+\frac{3if_\pi v\cdot q}{32\pi^2 t^2}\phi_{4;\pi}(u) \\ &\quad -\frac{if_\pi m_\pi^2 v\cdot q}{24(m_u+m_d)}\langle\bar s s\rangle\phi_{3;\pi}^\sigma(u)-\frac{if_\pi m_\pi^2 t^2 v\cdot q}{384(m_u+m_d)}\langle g_s\bar s\sigma G s\rangle\phi_{3;\pi}^\sigma(u) \\ &\quad -\frac{if_\pi m_\pi^2 v\cdot q}{8\pi^2t^2(m_u+m_d)}m_s\phi_{3;\pi}^\sigma(u)+\frac{if_\pi v\cdot q}{16}m_s\langle\bar s s\rangle\phi_{2;\pi}(u)+\frac{if_\pi t^2 v\cdot q}{256}m_s\langle\bar s s\rangle\phi_{4;\pi}(u)\bigg) \\ &\quad +\int_0^\infty dt\int_0^1 du\int\mathcal{D}\underline{\alpha}e^{i\omega' t(\alpha_2+u\alpha_3)}e^{i\omega t(1-\alpha_2-u\alpha_3)}\times 4\times\bigg(-\frac{if_\pi v\cdot q}{8\pi^2t^2}\Phi_{4;\pi}(\underline{\alpha})-\frac{if_\pi v\cdot q}{4\pi^2t^2}\Psi_{4;\pi}(\underline{\alpha}) \\ &\quad -\frac{3if_\pi v\cdot q}{8\pi^2t^2}\tilde\Phi_{4;\pi}(\underline{\alpha})+\frac{if_\pi v\cdot q}{4\pi^2t^2}\tilde\Psi_{4;\pi}(\underline{\alpha})-\frac{3if_\pi u v\cdot q}{4\pi^2t^2}\Phi_{4;\pi}(\underline{\alpha})+\frac{if_\pi u v\cdot q}{2\pi^2t^2}\Psi_{4;\pi}(\underline{\alpha})\bigg), \end{aligned} \tag{C49}$$

$$G_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Lambda_c^+K^-}(\omega,\omega') = \frac{g_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Lambda_c^+K^-} f_{\Xi_c^{\prime 0}[\frac{1}{2}^-]} f_{\Lambda_c^+}}{(\bar\Lambda_{\Xi_c^{\prime 0}[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Lambda_c^+}-\omega)} = 0, \tag{C50}$$

$$\begin{aligned} G_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Sigma_c^+K^-}(\omega,\omega') &= \frac{g_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Sigma_c^+K^-} f_{\Xi_c^{\prime 0}[\frac{1}{2}^-]} f_{\Sigma_c^+}}{(\bar\Lambda_{\Xi_c^{\prime 0}[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Sigma_c^+}-\omega)} \\ &= \int_0^\infty dt\int_0^1 du e^{i(1-u)\omega' t}e^{iu\omega t}\times 4\times\bigg(\frac{3if_K v\cdot q}{2\pi^2 t^4}\phi_{2;K}(u)+\frac{3if_K v\cdot q}{32\pi^2 t^2}\phi_{4;K}(u) \\ &\quad -\frac{if_K m_K^2 v\cdot q}{24(m_u+m_s)}\langle\bar q q\rangle\phi_{3;K}^\sigma(u)-\frac{if_K m_K^2 t^2 v\cdot q}{384(m_u+m_s)}\langle g_s\bar q\sigma G q\rangle\phi_{3;K}^\sigma(u)\bigg) \\ &\quad +\int_0^\infty dt\int_0^1 du\int\mathcal{D}\underline{\alpha}e^{i\omega' t(\alpha_2+u\alpha_3)}e^{i\omega t(1-\alpha_2-u\alpha_3)}\times 4\times\bigg(-\frac{if_K v\cdot q}{8\pi^2t^2}\Phi_{4;K}(\underline{\alpha})-\frac{if_K v\cdot q}{4\pi^2t^2}\Psi_{4;K}(\underline{\alpha}) \\ &\quad -\frac{3if_K v\cdot q}{8\pi^2t^2}\tilde\Phi_{4;K}(\underline{\alpha})+\frac{if_K v\cdot q}{4\pi^2t^2}\tilde\Psi_{4;K}(\underline{\alpha})-\frac{3if_K u v\cdot q}{4\pi^2t^2}\Phi_{4;K}(\underline{\alpha})+\frac{if_K u v\cdot q}{2\pi^2t^2}\Psi_{4;K}(\underline{\alpha})\bigg). \end{aligned} \tag{C51}$$

The sum rule for $\Omega_c^0(\frac{1}{2}^-)$ belonging to $[\mathbf{6}_F,1,1,\lambda]$ is

$$G_{\Omega_c^0[\frac{1}{2}^-]\to\Xi_c^+K^-}(\omega,\omega') = \frac{g_{\Omega_c^0[\frac{1}{2}^-]\to\Xi_c^+K^-} f_{\Omega_c^0[\frac{1}{2}^-]} f_{\Xi_c^+}}{(\bar\Lambda_{\Omega_c^0[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c^+}-\omega)} = 0, \tag{C52}$$

$$\begin{aligned} G_{\Omega_c^0[\frac{1}{2}^-]\to\Xi_c^{\prime +}K^-}(\omega,\omega') &= \frac{g_{\Omega_c^0[\frac{1}{2}^-]\to\Xi_c^{\prime +}K^-} f_{\Omega_c^0[\frac{1}{2}^-]} f_{\Xi_c^{\prime +}}}{(\bar\Lambda_{\Omega_c^0[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c^{\prime +}}-\omega)} \\ &= \int_0^\infty dt\int_0^1 du e^{i(1-u)\omega' t}e^{iu\omega t}\times 8\times\bigg(\frac{3if_K v\cdot q}{2\pi^2 t^4}\phi_{2;K}(u)+\frac{3if_K v\cdot q}{32\pi^2 t^2}\phi_{4;K}(u) \\ &\quad -\frac{if_K m_K^2 v\cdot q}{24(m_u+m_s)}\langle\bar s s\rangle\phi_{3;K}^\sigma(u)-\frac{if_K m_K^2 t^2 v\cdot q}{384(m_u+m_s)}\langle g_s\bar s\sigma G s\rangle\phi_{3;K}^\sigma(u) \\ &\quad -\frac{if_K m_K^2 v\cdot q}{8\pi^2t^2(m_u+m_s)}m_s\phi_{3;K}^\sigma(u)+\frac{if_K v\cdot q}{16}m_s\langle\bar s s\rangle\phi_{2;K}(u)+\frac{if_K t^2 v\cdot q}{256}m_s\langle\bar s s\rangle\phi_{4;K}(u)\bigg) \\ &\quad +\int_0^\infty dt\int_0^1 du\int\mathcal{D}\underline{\alpha}e^{i\omega' t(\alpha_2+u\alpha_3)}e^{i\omega t(1-\alpha_2-u\alpha_3)}\times 8\times\bigg(-\frac{if_K v\cdot q}{8\pi^2t^2}\Phi_{4;K}(\underline{\alpha})-\frac{if_K v\cdot q}{4\pi^2t^2}\Psi_{4;K}(\underline{\alpha}) \\ &\quad -\frac{3if_K v\cdot q}{8\pi^2t^2}\tilde\Phi_{4;K}(\underline{\alpha})+\frac{if_K v\cdot q}{4\pi^2t^2}\tilde\Psi_{4;K}(\underline{\alpha})-\frac{3if_K u v\cdot q}{4\pi^2t^2}\Phi_{4;K}(\underline{\alpha})+\frac{if_K u v\cdot q}{2\pi^2t^2}\Psi_{4;K}(\underline{\alpha})\bigg). \end{aligned} \tag{C53}$$

The sum rule for $\Sigma_c^0[\frac{3}{2}^-]$ belonging to $[\mathbf{6}_F, 1, 1, \lambda]$ is

$$
\begin{aligned}
G_{\Sigma_c^0[\frac{3}{2}^-]\to\Sigma_c^{*+}\pi^-}(\omega,\omega') &= \frac{g_{\Sigma_c^0[\frac{3}{2}^-]\to\Sigma_c^{*+}\pi^-} f_{\Sigma_c^0[\frac{3}{2}^-]} f_{\Sigma_c^{*+}}}{(\bar\Lambda_{\Sigma_c^0[\frac{3}{2}^-]}-\omega')(\bar\Lambda_{\Sigma_c^{*+}}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 8 \times \bigg(-\frac{f_\pi v\cdot q}{3\pi^2 t^4}\phi_{2;\pi}(u) - \frac{f_\pi v\cdot q}{48\pi^2 t^2}\phi_{4;\pi}(u) \\
&\quad + \frac{f_\pi m_\pi^2 v\cdot q}{108(m_u+m_d)}\langle\bar q q\rangle \phi_{3;\pi}^\sigma(u) + \frac{f_\pi m_\pi^2 t^2 v\cdot q}{1728(m_u+m_d)}\langle g_s\bar q\sigma G q\rangle\phi_{3;\pi}^\sigma(u)\bigg) \\
&\quad + \int_0^\infty dt\int_0^1 du\int \mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times 8 \times \bigg(\frac{f_\pi v\cdot q}{72\pi^2 t^2}\Phi_{4;\pi}(\underline{\alpha}) + \frac{5 f_\pi v\cdot q}{72\pi^2 t^2}\Psi_{4;\pi}(\underline{\alpha}) \\
&\quad + \frac{f_\pi v\cdot q}{24\pi^2 t^2}\tilde\Phi_{4;\pi}(\underline{\alpha}) - \frac{5 f_\pi v\cdot q}{72\pi^2 t^2}\tilde\Psi_{4;\pi}(\underline{\alpha}) + \frac{f_\pi u v\cdot q}{6\pi^2 t^2}\Phi_{4;\pi}(\underline{\alpha}) - \frac{7 f_\pi u v\cdot q}{72\pi^2 t^2}\Psi_{4;\pi}(\underline{\alpha}) + \frac{f_\pi u v\cdot q}{24\pi^2 t^2}\tilde\Psi_{4;\pi}(\underline{\alpha})\bigg).
\end{aligned}
\tag{C54}
$$

The sum rule for $\Xi_c^{\prime 0}(\frac{3}{2}^-)$ belonging to $[\mathbf{6}_F, 1, 1, \lambda]$ is

$$
\begin{aligned}
G_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Xi_c^{*+}\pi^-}(\omega,\omega') &= \frac{g_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Xi_c^{*+}\pi^-} f_{\Xi_c^{\prime 0}[\frac{1}{2}^-]} f_{\Xi_c^{*+}}}{(\bar\Lambda_{\Xi_c^{\prime 0}[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c^{*+}}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \bigg(-\frac{f_\pi v\cdot q}{3\pi^2 t^4}\phi_{2;\pi}(u) - \frac{f_\pi v\cdot q}{48\pi^2 t^2}\phi_{4;\pi}(u) \\
&\quad + \frac{f_\pi m_\pi^2 v\cdot q}{108(m_u+m_d)}\langle\bar s s\rangle \phi_{3;\pi}^\sigma(u) + \frac{f_\pi m_\pi^2 t^2 v\cdot q}{1728(m_u+m_d)}\langle g_s\bar s\sigma G s\rangle\phi_{3;\pi}^\sigma(u) + \frac{f_\pi m_\pi^2 v\cdot q}{36\pi^2 t^2(m_u+m_d)} m_s\phi_{3;\pi}^\sigma(u) \\
&\quad - \frac{f_\pi v\cdot q}{72} m_s\langle\bar s s\rangle\phi_{2;\pi}(u) - \frac{f_\pi t^2 v\cdot q}{1152} m_s\langle\bar s s\rangle\phi_{4;\pi}(u)\bigg) \\
&\quad + \int_0^\infty dt\int_0^1 du\int \mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times 4 \times \bigg(\frac{f_\pi v\cdot q}{72\pi^2 t^2}\Phi_{4;\pi}(\underline{\alpha}) + \frac{5 f_\pi v\cdot q}{72\pi^2 t^2}\Psi_{4;\pi}(\underline{\alpha}) \\
&\quad + \frac{f_\pi v\cdot q}{24\pi^2 t^2}\tilde\Phi_{4;\pi}(\underline{\alpha}) - \frac{5 f_\pi v\cdot q}{72\pi^2 t^2}\tilde\Psi_{4;\pi}(\underline{\alpha}) + \frac{f_\pi u v\cdot q}{6\pi^2 t^2}\Phi_{4;\pi}(\underline{\alpha}) - \frac{7 f_\pi u v\cdot q}{72\pi^2 t^2}\Psi_{4;\pi}(\underline{\alpha}) + \frac{f_\pi u v\cdot q}{24\pi^2 t^2}\tilde\Psi_{4;\pi}(\underline{\alpha})\bigg),
\end{aligned}
\tag{C55}
$$

$$
\begin{aligned}
G_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Sigma_c^{*+}K^-}(\omega,\omega') &= \frac{g_{\Xi_c^{\prime 0}[\frac{1}{2}^-]\to\Sigma_c^{*+}K^-} f_{\Xi_c^{\prime 0}[\frac{1}{2}^-]} f_{\Sigma_c^{*+}}}{(\bar\Lambda_{\Xi_c^{\prime 0}[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Sigma_c^{*+}}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \bigg(-\frac{f_K v\cdot q}{3\pi^2 t^4}\phi_{2;K}(u) - \frac{f_K v\cdot q}{48\pi^2 t^2}\phi_{4;K}(u) \\
&\quad + \frac{f_K m_K^2 v\cdot q}{108(m_u+m_s)}\langle\bar q q\rangle \phi_{3;K}^\sigma(u) + \frac{f_K m_K^2 t^2 v\cdot q}{1728(m_u+m_s)}\langle g_s\bar q\sigma G q\rangle\phi_{3;K}^\sigma(u)\bigg) \\
&\quad + \int_0^\infty dt\int_0^1 du\int \mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times 4 \times \bigg(\frac{f_K v\cdot q}{72\pi^2 t^2}\Phi_{4;K}(\underline{\alpha}) + \frac{5 f_K v\cdot q}{72\pi^2 t^2}\Psi_{4;K}(\underline{\alpha}) \\
&\quad + \frac{f_K v\cdot q}{24\pi^2 t^2}\tilde\Phi_{4;K}(\underline{\alpha}) - \frac{5 f_K v\cdot q}{72\pi^2 t^2}\tilde\Psi_{4;K}(\underline{\alpha}) + \frac{f_K u v\cdot q}{6\pi^2 t^2}\Phi_{4;K}(\underline{\alpha}) \\
&\quad - \frac{7 f_K u v\cdot q}{72\pi^2 t^2}\Psi_{4;K}(\underline{\alpha}) + \frac{f_K u v\cdot q}{24\pi^2 t^2}\tilde\Psi_{4;K}(\underline{\alpha})\bigg).
\end{aligned}
\tag{C56}
$$

The sum rule for $\Omega_c^0(\frac{3}{2}^-)$ belonging to $[\mathbf{6}_F, 1, 1, \lambda]$ is

$$
\begin{aligned}
G_{\Omega_c^0[\frac{1}{2}^-]\to\Xi_c^{*+}K^-}(\omega,\omega') &= \frac{g_{\Omega_c^0[\frac{1}{2}^-]\to\Xi_c^{*+}K^-} f_{\Omega_c^0[\frac{1}{2}^-]} f_{\Xi_c^{*+}}}{(\bar\Lambda_{\Omega_c^0[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c^{*+}}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 8 \times \bigg(-\frac{f_K v\cdot q}{3\pi^2 t^4}\phi_{2;K}(u) - \frac{f_K v\cdot q}{48\pi^2 t^2}\phi_{4;K}(u) \\
&\quad + \frac{f_K m_K^2 v\cdot q}{108(m_u+m_s)}\langle\bar s s\rangle \phi_{3;K}^\sigma(u) + \frac{f_K m_K^2 t^2 v\cdot q}{1728(m_u+m_s)}\langle g_s \bar s\sigma G s\rangle\phi_{3;K}^\sigma(u) \\
&\quad + \frac{f_K m_K^2 v\cdot q}{36\pi^2 t^2(m_u+m_s)} m_s \phi_{3;K}^\sigma(u) - \frac{f_K v\cdot q}{72} m_s\langle\bar s s\rangle\phi_{2;K}(u) - \frac{f_K t^2 v\cdot q}{1152} m_s \langle\bar s s\rangle \phi_{4;K}(u)\bigg) \\
&\quad + \int_0^\infty dt \int_0^1 du \int \mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times 8 \times \bigg(\frac{f_K v\cdot q}{72\pi^2 t^2}\Phi_{4;K}(\underline{\alpha}) + \frac{5 f_K v\cdot q}{72\pi^2 t^2}\Psi_{4;K}(\underline{\alpha}) \\
&\quad + \frac{f_K v\cdot q}{24\pi^2 t^2}\tilde\Phi_{4;K}(\underline{\alpha}) - \frac{5 f_K v\cdot q}{72\pi^2 t^2}\tilde\Psi_{4;K}(\underline{\alpha}) + \frac{f_K u v\cdot q}{6\pi^2 t^2}\Phi_{4;K}(\underline{\alpha}) \\
&\quad - \frac{7 f_K u v\cdot q}{72\pi^2 t^2}\Psi_{4;K}(\underline{\alpha}) + \frac{f_K u v\cdot q}{24\pi^2 t^2}\tilde\Psi_{4;K}(\underline{\alpha})\bigg).
\end{aligned} \tag{C57}
$$

8. $[\mathbf{6}_F, \mathbf{2}, \mathbf{1}, \lambda]$

The sum rule for $\Sigma_c^0[\frac{3}{2}^-]$ belonging to $[\mathbf{6}_F, 2, 1, \lambda]$ is

$$
\begin{aligned}
G_{\Sigma_c^0[\frac{3}{2}^-]\to\Sigma_c^{*+}\pi^-}(\omega,\omega') &= \frac{g_{\Sigma_c^0[\frac{3}{2}^-]\to\Sigma_c^{*+}\pi^-} f_{\Sigma_c^0[\frac{3}{2}^-]} f_{\Sigma_c^{*+}}}{(\bar\Lambda_{\Sigma_c^0[\frac{3}{2}^-]}-\omega')(\bar\Lambda_{\Sigma_c^{*+}}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du \int \mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times 8 \times \bigg(\frac{f_\pi v\cdot q}{24\pi^2 t^2}\Phi_{4;\pi}(\underline{\alpha}) + \frac{f_\pi v\cdot q}{24\pi^2 t^2}\Psi_{4;\pi}(\underline{\alpha}) \\
&\quad - \frac{f_\pi v\cdot q}{24\pi^2 t^2}\tilde\Phi_{4;\pi}(\underline{\alpha}) - \frac{f_\pi v\cdot q}{24\pi^2 t^2}\tilde\Psi_{4;\pi}(\underline{\alpha}) - \frac{f_\pi u v\cdot q}{24\pi^2 t^2}\Psi_{4;\pi}(\underline{\alpha}) + \frac{f_\pi u v\cdot q}{24\pi^2 t^2}\tilde\Psi_{4;\pi}(\underline{\alpha})\bigg).
\end{aligned} \tag{C58}
$$

The sum rule for $\Xi_c^0(\frac{3}{2}^-)$ belonging to $[\mathbf{6}_F, 2, 1, \lambda]$ is

$$
\begin{aligned}
G_{\Xi_c^{\prime 0}[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-}(\omega,\omega') &= \frac{g_{\Xi_c^{\prime 0}[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-} f_{\Xi_c^{\prime 0}[\frac{3}{2}^-]} f_{\Xi_c^{*+}}}{(\bar\Lambda_{\Xi_c^{\prime 0}[\frac{3}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c^{*+}}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du \int \mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times 4 \times \bigg(\frac{f_\pi v\cdot q}{24\pi^2 t^2}\Phi_{4;\pi}(\underline{\alpha}) + \frac{f_\pi v\cdot q}{24\pi^2 t^2}\Psi_{4;\pi}(\underline{\alpha}) \\
&\quad - \frac{f_\pi v\cdot q}{24\pi^2 t^2}\tilde\Phi_{4;\pi}(\underline{\alpha}) - \frac{f_\pi v\cdot q}{24\pi^2 t^2}\tilde\Psi_{4;\pi}(\underline{\alpha}) - \frac{f_\pi u v\cdot q}{24\pi^2 t^2}\Psi_{4;\pi}(\underline{\alpha}) + \frac{f_\pi u v\cdot q}{24\pi^2 t^2}\tilde\Psi_{4;\pi}(\underline{\alpha})\bigg),
\end{aligned} \tag{C59}
$$

$$
\begin{aligned}
G_{\Xi_c^{\prime 0}[\frac{3}{2}^-]\to\Sigma_c^{*+}K^-}(\omega,\omega') &= \frac{g_{\Xi_c^{\prime 0}[\frac{3}{2}^-]\to\Sigma_c^{*+}K^-} f_{\Xi_c^{\prime 0}[\frac{3}{2}^-]} f_{\Sigma_c^{*+}}}{(\bar\Lambda_{\Xi_c^{\prime 0}[\frac{3}{2}^-]}-\omega')(\bar\Lambda_{\Sigma_c^{*+}}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du \int \mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times 4 \times \bigg(\frac{f_K v\cdot q}{24\pi^2 t^2}\Phi_{4;K}(\underline{\alpha}) + \frac{f_K v\cdot q}{24\pi^2 t^2}\Psi_{4;K}(\underline{\alpha}) \\
&\quad - \frac{f_K v\cdot q}{24\pi^2 t^2}\tilde\Phi_{4;K}(\underline{\alpha}) - \frac{f_K v\cdot q}{24\pi^2 t^2}\tilde\Psi_{4;K}(\underline{\alpha}) - \frac{f_K u v\cdot q}{24\pi^2 t^2}\Psi_{4;K}(\underline{\alpha}) + \frac{f_K u v\cdot q}{24\pi^2 t^2}\tilde\Psi_{4;K}(\underline{\alpha})\bigg).
\end{aligned} \tag{C60}
$$

The sum rule for $\Omega_c^0(\frac{3}{2}^-)$ belonging to $[\mathbf{6}_F, 2, 1, \lambda]$ is

$$G_{\Omega_c^0[\frac{3}{2}^-]\to\Xi_c^{*+}K^-}(\omega,\omega') = \frac{g_{\Omega_c^0[\frac{3}{2}^-]\to\Xi_c^{*+}K^-} f_{\Omega_c^0[\frac{3}{2}^-]} f_{\Xi_c^{*+}}}{(\bar\Lambda_{\Omega_c^0[\frac{3}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c^{*+}}-\omega)}$$
$$= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 8 \times \left(\frac{f_K v\cdot q}{24\pi^2 t^2} \Phi_{4;K}(\underline{\alpha}) + \frac{f_K v\cdot q}{24\pi^2 t^2} \Psi_{4;K}(\underline{\alpha}) - \frac{f_K v\cdot q}{24\pi^2 t^2} \tilde\Phi_{4;K}(\underline{\alpha}) \right.$$
$$\left. - \frac{f_K v\cdot q}{24\pi^2 t^2} \tilde\Psi_{4;K}(\underline{\alpha}) - \frac{f_K u v\cdot q}{24\pi^2 t^2} \Psi_{4;K}(\underline{\alpha}) + \frac{f_K u v\cdot q}{24\pi^2 t^2} \tilde\Psi_{4;K}(\underline{\alpha}) \right). \tag{C61}$$

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