Quantum gravity on foliated spacetimes: Asymptotically safe and sound

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Asymptotic safety provides a mechanism for constructing a consistent and predictive quantum theory of gravity valid on all length scales. Its key ingredient is a non-Gaussian fixed point of the gravitational renormalization group flow which controls the scaling of couplings and correlation functions at high energy. In this work we use a functional renormalization group equation adapted to the Arnowitt-Deser-Misner formalism for evaluating the gravitational renormalization group flow on a cosmological Friedmann-Robertson-Walker background. Besides possessing the non-Gaussian fixed-point characteristic for asymptotic safety the setting exhibits a second family of non-Gaussian fixed points with a positive Newton's constant and real critical exponents. The presence of these new fixed points alters the phase diagram in such a way that all renormalization group trajectories connected to classical general relativity are well defined on all length scales. In particular a positive cosmological constant is dynamically driven to zero in the deep infrared. Moreover, the scaling dimensions associated with the universality classes emerging within the causal setting exhibit qualitative agreement with results found within the ϵ-expansion around two dimensions, Monte Carlo simulations based on lattice quantum gravity, and the discretized Wheeler-DeWitt equation.

I. INTRODUCTION

Obtaining a consistent theory for the gravitational force valid on all length scales is a central challenge in contemporary theoretical high-energy physics. While general relativity provides a highly successful description of gravity from submillimeter up to cosmic scales, it is expected that the theory breaks down for energies around the Planck scale. An interesting mechanism which may provide a short-distance completion of the gravitational force is asymptotic safety [1–7]. Building on results for gravity in $2 + \epsilon$ spacetime dimensions [8,9], it was first suggested by Weinberg [10] that the short-distance behavior of gravity may be controlled by a non-Gaussian fixed point (NGFP) of the underlying renormalization group (RG) flow. Provided that such a fixed point exists and comes with a finite number of relevant parameters the resulting quantum theory of gravity would have the same predictive power as a perturbatively renormalizable quantum field theory. Moreover, the proposal circumvents the pitfalls encountered in the perturbative quantization of the Einstein-Hilbert action [11–13], which basically originate from the fact that the Gaussian fixed point (GFP) repels the flow of a positive Newton's coupling for increasing energy.

One of the main virtues of asymptotic safety is that it is conservative in the sense that it stays within the framework of quantum field theory without relying on the introduction of new physics principles beyond the quantum field theory framework.

Notably, the NGFP underlying the asymptotic safety program is not "put in by hand": its existence must be established or falsified by explicit computations of the gravitational RG flow. Over the years a series of complementary approaches capable of testing the asymptotic safety hypothesis have been developed. Starting from the pioneering work in Ref. [14], functional renormalization group methods have established the existence of a suitable NGFP in a wide range of approximations [15–47], including the demonstration that the NGFP persists in the presence of the perturbative two-loop counterterm [48] and at the level of an infinite number of scale-dependent couplings [49–63]. Moreover, a first step connecting the NGFP to the underlying conformal field theory appeared in Ref. [64], possible completions of the flow at low energy have been discussed in Refs. [17,24,38,40,41] and geometric arguments determining the scaling of Newton's constant at the NGFP have been put forward in Refs. [65,66]. In parallel Monte Carlo approaches to quantum gravity including causal dynamical triangulations [67–75], Euclidean dynamical triangulations [76–80] and lattice quantum gravity [81,82] made vast progress towards constructing phase diagrams at the nonperturbative level.

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While it is conceivable that all of these approaches probe the same universal short-distance physics, a unified picture has yet to emerge.

In this work we report significant progress towards both developing a unified picture from these distinguished programs and constructing complete RG trajectories relevant for describing our world. Building on the functional renormalization group approach [14], we study gravitational RG flows in Arnowitt-Deser-Misner (ADM) variables [83,84]. The ADM formalism imprints spacetime with a foliation structure. The resulting distinguished time direction allows the continuation of the flow equation from Euclidean to Lorentzian signature metrics. Moreover, the phase diagram resulting from projecting the flow onto the (Euclidean) Einstein-Hilbert action evaluated on a Friedmann-Robertson-Walker background exhibits a NGFP suitable for asymptotic safety. The novel fixed-point structure found in $2 + 1$ dimensions furthermore supports the idea [85] that a causal structure may play an important role in obtaining a well-defined long-distance completion of gravity.

II. RG FLOW AND ITS FIXED POINTS

The functional RG provides a powerful tool for investigating the phase structure of a physical system. In its formulation based on the effective average action Γ_k [86], the functional RG equation (FRGE)

$$k\partial_k\Gamma_k = \frac{1}{2}\mathrm{Str}[(\Gamma_k^{(2)} + \mathcal{R}_k)^{-1}k\partial_k\mathcal{R}_k], \qquad (1)$$

implements Wilson's idea that the RG flow of a theory originates from integrating out quantum fluctuations shell by shell in momentum space. The interplay of the scale-dependent regulator $\mathcal{R}_k$ in the numerator and denominator thereby ensures that the change of Γ_k is driven by fluctuations with momenta close to k. The two-point correlator $\Gamma_k^{(2)}$ makes Eq. (1) a formally exact equation with the same information content as the path integral from which it is derived. A central advantage of the FRGE is that it allows for approximations which do not rely on the existence of a small expansion parameter. Moreover, RG flows can be constructed without the need to specify a fundamental action. This makes the framework ideally suited for investigating fixed points of the flow which encode universal scaling behaviors.

In the present investigation we use the formulation of the FRGE where the gravitational degrees of freedom are carried by the ADM fields [87,88]. In this case, the (Euclidean) spacetime metric is decomposed according to

$$ds^2 = N^2 dt^2 + \sigma_{ij}(dx^i + N^i dt)(dx^j + N^j dt) \qquad (2)$$

with the lapse function $N(t, \vec{x})$, the shift vector $N^i(t, \vec{x})$ and a metric on the spatial slices $\sigma_{ij}(t, \vec{x})$ being functions of the

(Euclidean) time t and the spatial coordinates $\vec{x}$. The flow equation is then constructed via the background field method, splitting the component fields into a fixed background and fluctuations $N = \bar{N} + \hat{N}$, $N^i = \bar{N}^i + \hat{N}^i$, and $\sigma_{ij} = \bar{\sigma}_{ij} + \hat{\sigma}_{ij}$. The resulting construction is invariant under foliation-preserving background diffeomorphisms. While the Einstein-Hilbert term of Γ_k realizes the full diffeomorphism group as a background symmetry, the gauge-fixing and ghost terms are invariant with respect to this subgroup only [88].

We study the gravitational RG flow projected on the Einstein-Hilbert action written in terms of the extrinsic curvature $K_{ij} \equiv (2N)^{-1}(\partial_t\sigma_{ij} - D_iN_j - D_jN_i)$, $K \equiv \sigma^{ij}K_{ij}$, and the intrinsic curvature on the d-dimensional spatial slices R

$$\Gamma_k^{\mathrm{EH}} = \frac{1}{16\pi G_k}\int dt d^d x N\sqrt{\sigma}[K_{ij}K^{ij} - K^2 - R + 2\Lambda_k]. \tag{3}$$

The evaluation of the flow is simplified by choosing a flat Friedmann-Robertson-Walker background, $\bar{N} = 1$, $\bar{N}_i = 0$, $\bar{\sigma}_{ij} = a^2(t)\delta_{ij}$, and parametrizing the fluctuations in terms of the field decomposition used in cosmic perturbation theory. Following the standard background field method, the gravitational sector is supplemented by a gauge-fixing term

$$\Gamma_k^{\mathrm{gf}} = \frac{1}{32\pi G_k}\int dt d^d x\sqrt{\bar{\sigma}}[F_i\bar{\sigma}^{ij}F_j + F^2], \qquad (4)$$

accompanied by the standard ghost term exponentiating the Faddeev-Popov determinant. The functionals F and F_i are linear in the fluctuation fields,

$$F = \partial_t\hat{N} + \partial^i\hat{N}_i - \frac{1}{2}\partial_t\hat{\sigma} + \frac{2(d-1)}{d}\bar{K}\hat{N},$$

$$F_i = \partial_t\hat{N}_i - \partial_i\hat{N} - \frac{1}{2}\partial_i\hat{\sigma} + \partial^j\hat{\sigma}_{ji} + (d-2)\bar{K}_{ij}\hat{N}^j, \qquad (5)$$

where $\hat{\sigma} \equiv \bar{\sigma}^{ij}\hat{\sigma}_{ij}$. F and F_i naturally combine in a $(d + 1)$-dimensional vector $F_\mu \equiv (F, F_i)$. The argument of Γ_k^{gf} takes the standard form $F_\mu\bar{g}^{\mu\nu}F_\nu$ with $\bar{g}^{\mu\nu}$ being the (inverse) background spacetime metric. The relations (5) can be understood as first-order differential equations fixing the fluctuations of the lapse function and shift vector in terms of the other fluctuation fields and the background. As its main virtue the gauge choice (5) equips all component fields including the lapse function, the shift vector, and the Faddeev-Popov ghost fields with regular propagators.[1] This condition fixes the gauge uniquely. On a flat Euclidean

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[1]For a related construction in the context of Lorentz-violating field theories see Refs. [89,90].

background all component fields obey the same dispersion relation such that, upon Wick rotation, all fields propagate with the same speed of light. This is the first time that a completely regular off-shell flow equation based on the ADM formalism has been obtained.

The evaluation of the trace utilizes the universal RG machine [91] combined with the heat-kernel methods on foliated spacetimes [92,93] and the off-diagonal heat kernel [94–98]. The flows of Newton's coupling and the cosmological constant are then read off from the coefficients multiplying the extrinsic curvature and volume term, respectively. The result is conveniently expressed in terms of the dimensionless couplings $g_i \equiv \{\lambda, g\}$ and the anomalous dimension η_N of Newton's coupling

$$\lambda_k \equiv \Lambda_k k^{-2}, \qquad g_k \equiv G_k k^{d-1}, \qquad \eta \equiv (G_k)^{-1} k \partial_k G_k,$$

(6)

and encoded in the β functions $k \partial_k g_i \equiv \beta_{g_i}(\lambda, g)$. The details of the derivation will be presented elsewhere [99]. For a Litim-type regulator [100], the β functions for a three-dimensional spatial slice read

$$\beta_g = (2 + \eta) g,$$

$$\beta_\lambda = (\eta - 2)\lambda - \frac{g}{4\pi} \left[8 - \left(1 - \frac{1}{6}\eta\right) \right.$$

$$\left. \times \left(3 + \frac{6}{1 - 2\lambda} + \frac{2}{2 - 3\lambda}\right) \right].$$

(7)

Here $\eta = (g B_1(\lambda))(\pi + g B_2(\lambda))^{-1}$ where

$$B_1(\lambda) \equiv -\frac{1243 - 8164\lambda + 21916\lambda^2 - 26976\lambda^3 + 12240\lambda^4}{72(1 - 2\lambda)^2(2 - 3\lambda)^2},$$

$$B_2(\lambda) \equiv \frac{(1 - \lambda)(87 - 680\lambda + 1298\lambda^2 - 624\lambda^3 - 72\lambda^4)}{144(1 - 2\lambda)^2(2 - 3\lambda)^2}.$$

(8)

The general expression for these beta functions that are valid for general d are provided in Supplemental Material [101] as a MATHEMATICA notebook. They encode the complete RG flow resulting from the ansatz (3).

The universal critical behavior is encoded in the fixed points $g_{i,*}$ of the β functions, satisfying $\beta_{g_i}\big|_{g_{i,*}} = 0$. The flow in the vicinity of a fixed point is governed by the stability matrix $B_{ij} \equiv \partial_{g_j}\beta_{g_i}\big|_{g_*}$. The stability coefficients θ_i, defined as minus the eigenvalues of B_{ij}, indicate whether the corresponding eigendirection of the fixed point attracts (Re $\theta_i > 0$) or repels (Re $\theta_i < 0$) the flow for increasing energy. UV-attractive directions are associated with relevant parameters of the theory which need to be fixed by experiment. Moreover, the stability coefficients contain characteristic information about the underlying universality

class which allows a direct comparison based on different computational approaches.

Remarkably, the $(d + 1)$-dimensional β functions give rise to a rich fixed-point structure. First, there is the GFP located at the origin. Its stability coefficients are given by the classical scaling dimensions. As a consequence trajectories with a positive Newton's coupling are not captured by the GFP at high energy. This reflects the perturbative nonrenormalizability of the Einstein-Hilbert action in the Wilsonian language.

In addition, there are two families of NGFPs whose most important properties are summarized in Fig. 1. In $D = 2 + \epsilon$ dimensions a NGFP with two real, positive stability coefficients (orange line) emerges from the GFP. Its critical exponents agree with the epsilon expansion of perturbative gravity around two dimensions [102] to leading order. This universality class has an upper critical dimension $D = 2.28$. At this point there is a transition to a family of saddle points (SP) characterized by a small UV-attractive and a large UV-repulsive critical exponent (brown line). In $D = 2.37$ these critical exponents swap signs, giving rise to the red line of SP-NGFPs existing for $2.37 \leq D \leq 3.25$. Simultaneously, there is a second family of fixed points (green line) with two real, positive critical exponents. At $D = 3.40$ the two stability coefficients coincide at $\theta_1 = \theta_2 = 1.08$. For $D > 3.40$ the real stability coefficients become complex (blue line) which reflects the typical characteristics of the UV-NGFP seen in the functional RG approach to asymptotic safety. This universality class can be traced up to $D \approx 40$, where g_* becomes
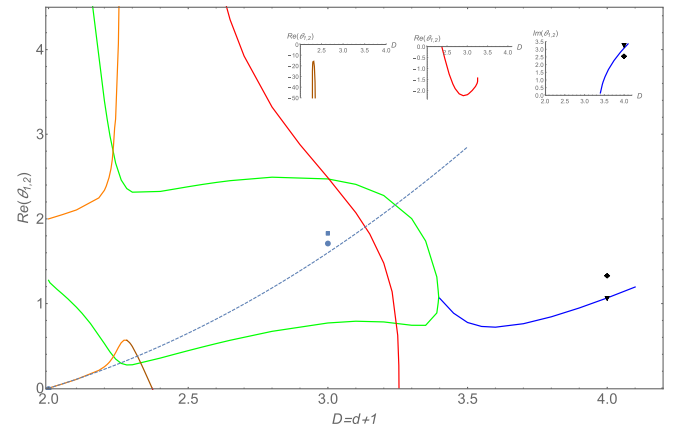

FIG. 1. Stability coefficients of the two families of fixed points emerging from the β functions as a function of the spacetime dimension $D = d + 1$. The dashed line gives the result from the two-loop epsilon expansion [102], the circles indicate the scaling of Newton's coupling found within lattice quantum gravity [82] and the square marks the scaling found from the exact solution of the discretized Wheeler-DeWitt equation [103,104]. In $D = 4$ the down-triangle indicates the critical exponents obtained from foliated spacetimes using the Matsubara formalism [87] while the diamond corresponds to the dynamical fixed point seen in Ref. [24].

exponentially large. The additional information displayed in Fig. 1 encodes the stability coefficients found within related quantum gravity programs. The qualitative agreement of the scaling behavior seen within discrete (Monte Carlo) methods in $D = 2 + 1$ dimensions and the continuum RG is an important indicator for the robustness of the underlying universality classes.

The properties of the NGFPs are summarized as follows. In $D = 2 + 1$, the UV-NGFP and SP-NGFPs are located at

$$\text{UV-NGFP:} \quad g_* = 0.16, \quad \lambda_* = 0.03, \quad g_*^2 \lambda_* = 0.0008,$$
$$\text{SP-NGFP:} \quad g_* = 0.32, \quad \lambda_* = 0.20, \quad g_*^2 \lambda_* = 0.02,$$
$$(9)$$

and come with stability coefficients $\theta_1 = 2.47$, $\theta_2 = 0.77$ (UV-NGFP) and $\theta_1 = 2.49$, $\theta_2 = -2.20$ (SP-NGFP), respectively. In $D = 3 + 1$ there is a unique NGFP for a positive Newton's constant,

$$\text{AS-NGFP:} \quad g_* = 0.90, \quad \lambda_* = 0.24, \quad g_* \lambda_* = 0.21,$$
$$(10)$$

with critical exponents $\theta_{1,2} = 1.06 \pm 3.07i$. The fixed-point structure (9)–(10) is robust under a change of the regulator function to an exponential or sharp cutoff. The AS-NGFP exhibits the typical complex pair of critical exponents familiar from evaluating the flow equation in the metric formulation [15–24]. In particular, there is a very good agreement with the critical exponents obtained for foliated spacetime via the Matsubara formalism [87,88].

The phase diagram resulting from the numerical integration of the β functions in $D = 2 + 1$ and $D = 3 + 1$ is shown in the top and bottom panels of Fig. 2. For $D = 2 + 1$ dimensions, the flow is governed by the interplay of the GFP, the two NGFPs and the two quasifixed points (QFPs) "B" and "C" located at $(\lambda, g) = (-\infty, 0)$ and $(\lambda, g) = (1/2, 0)$, respectively. The transition lines connecting these points are depicted in blue. All RG trajectories located below the transition line $\overline{CDB}$ are complete: they are well behaved for all values $k \in [0, \infty]$. Their high-energy behavior is controlled by the UV-NGFP "A." By lowering the RG scale they cross over to the GFP or the saddle point "D." Trajectories passing sufficiently close to the GFP develop a classical regime where the Newton coupling and cosmological constant are independent of the RG scale. Depending on whether the flow approaches "B," "O," or "C" the classical regime exhibits a negative, zero, or positive cosmological constant.

The picture found in $D = 3 + 1$ dimensions is similar: the UV completion of the flow is controlled by the AS-NGFP and the classical regime emerges from the crossover of the flow to the GFP. As a consequence of the missing SP-NGFP "D" the trajectory $\overline{CD}$ in $D = 2 + 1$ is replaced by a line of singularities where η_N diverges (red line). In this
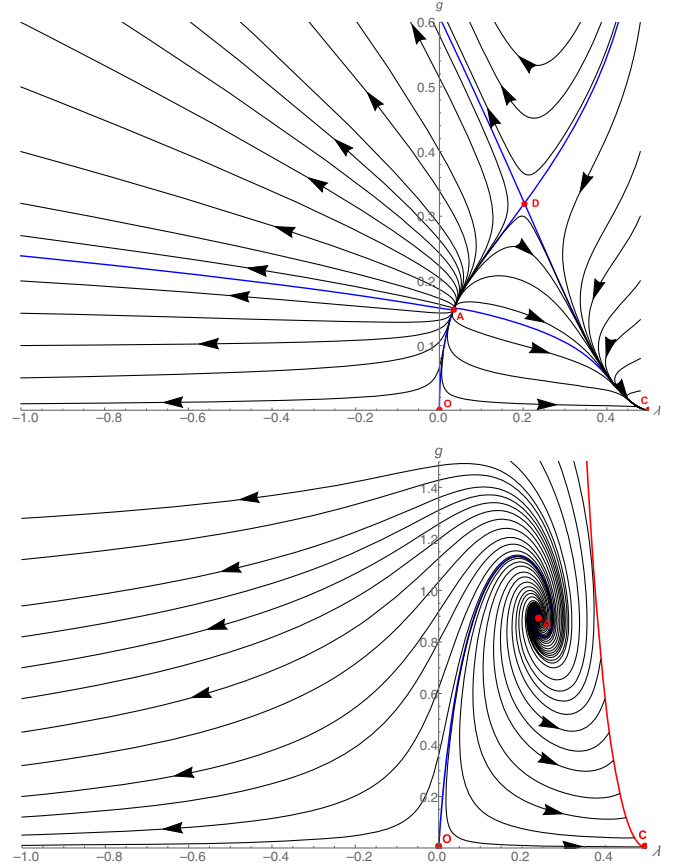

FIG. 2. Phase diagrams describing the RG flow in $D = 2 + 1$ (top) and $D = 3 + 1$ (bottom). The GFP, UV-NGFP and SP-NGFP are marked by the points "O," "A," and "D". In $D = 2 + 1$ the interplay of the fixed points "A" and "D" ensures that the QFP "C" provides the long-distance completion of the RG trajectories with a positive cosmological constant. In $D = 3 + 1$ the corresponding trajectories terminate in a divergence of η (red line).

case solutions with a positive cosmological constant terminate at a finite RG scale.

Since the RG trajectory describing our world should exhibit a classical regime with a positive cosmological constant [105], it is worthwhile to investigate the IR completion occurring in $D = 2 + 1$ dimensions in detail. Figure 3 displays the typical scale dependence of G_k (top) and Λ_k (bottom) for this class of solutions. Flowing from high to low energies the trajectories undergo four phases. The UV-NGFP controls the high-energy regime, ensuring the absence of unphysical UV divergences. Subsequently, there is a semiclassical regime followed by a classical regime where Newton's constant and the cosmological constant exhibit plateaus. These features are independent of the spacetime dimension. In $D = 2 + 1$ the low-energy completion is provided by a novel phase where both G and Λ are dynamically driven to zero. This phase is controlled by the QFP "C." At this point the β functions are ambiguous owed to terms of the form $g/(1 - 2\lambda)^2$. RG trajectories approaching "C" resolve this ambiguity in such
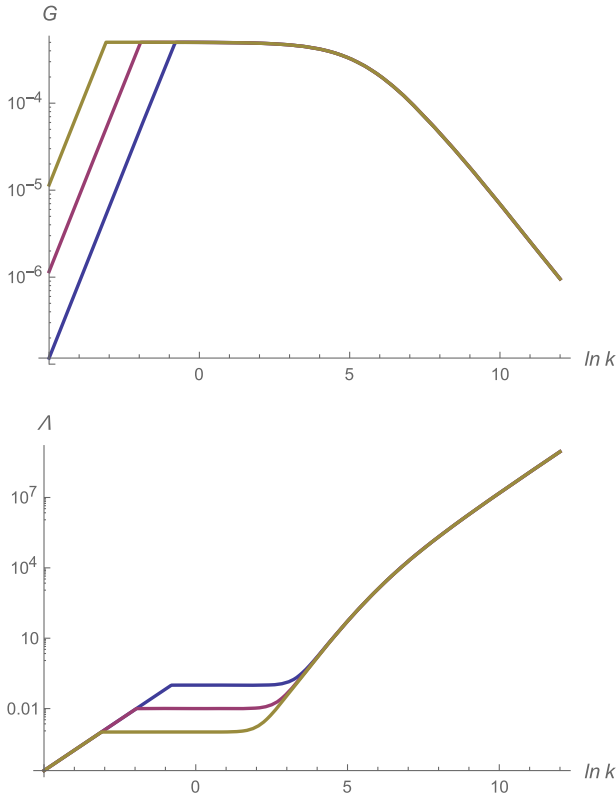
FIG. 3.　Scale dependence of G_k (top) and Λ_k (bottom) for a set of RG trajectories exhibiting a classical regime with a positive cosmological constant for $D = 2 + 1$. The QFP "C" drives the infrared value of Λ to zero dynamically.

a way that $\lim_{k\to 0} g/(1 - 2\lambda)^2 = 5\pi/6$: "C" is turned into a low-energy attractor where $\eta_N = 2$. The mechanism providing the low-energy completion in this sector is essentially the same as the one reported in Ref. [40]. However, in the present case regularity appears at the level of the background couplings.

Figure 3 also illustrates that the value of the cosmological constant in the classical regime is a free parameter and may be set to its experimentally determined value. The transition (respectively, termination) scale k_t from the classical to the low-energy phase can be estimated from the relation $\Lambda = \lambda k^2$. Setting $\lambda = 1/2$ and substituting the measured value of the cosmological constant

$\Lambda_{\rm obs} = 1.19 \times 10^{-52}$ m^2 yields $k_t = 3 \times 10^{-33}$ eV, corresponding to a length scale of $l_t = 6 \times 10^{25}$ m. This value is within the radius of the observable Universe opening the possibility that effects of a new gravitational phase, possibly related to the physics of a QFP similar to "C," may be visible on cosmic scales.

III. CONCLUSIONS

In this work we provided a novel, completely regular off-shell formulation of the functional renormalization group equation (1) on spacetimes carrying a foliation structure. The framework implements the gravitational degrees of freedom in terms of ADM variables and is ideally suited for computing real-time correlation functions, e.g., in a cosmological context. As a first application of the formalism we studied the scale dependence of Newton's coupling and the cosmological constant. The resulting fixed-point structure and universal critical exponents match the leading correction obtained from the perturbative computations in $2 + \epsilon$ spacetime dimensions and show a good agreement with lattice quantum gravity in $D = 2 + 1$ dimensions [82] and the discretized Wheeler-DeWitt equation [103,104]. The real critical exponents of the non-Gaussian fixed points in $D = 2 + 1$ may provide a natural explanation for the apparent mismatch between the real critical exponents seen in Monte Carlo approaches and the complex critical exponents typically obtained from the functional renormalization group. Moreover, the interplay of two non-Gaussian fixed points resolves the IR singularity encountered in the metric formulation. Presupposing that a similar mechanism is also operative in $D = 3 + 1$ dimensions, the framework predicts a distinct modification of general relativity at cosmic distances.

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