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Analysis of the wave equations for the near horizon static isotropic metric

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We find that, for the near horizon static isotropic metric, all of the massless field equations of the spin ≤ 2 have the same characteristic: they can be reduced to the Fuchsian-type equation with three regular singular points. Three general solutions, corresponding to different parameter values, are obtained. Two of the solutions have discrete imaginary frequencies. Based on the results above, we obtain exact formulas for the quasinormal modes of the Rindler-type spacetime. We also derive an elegant wave equation, which is the fundamental formula of the perturbation theory for general static spherically symmetric spacetime.

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I. INTRODUCTION

Black hole perturbation theory is one of the most important fields of black hole physics. It began to develop in 1957 under the influence of Regge and Wheeler's work [1]. Now the study of black hole perturbation involves the quasinormal modes [2–4], stability [5,6], scattering [7,8], gravitational waves [9,10], the Hawking effect [11,12], statistical entropy [13–16], etc. An exhaustive review of the methods and results of this stage of the theory can be found in the well-known literature by Chandrasekhar [17], Nollert [18], Kokkotas and Schmidt [19], Berti *et al.* [20], and Konoplya and Zhidenko [21].

As already mentioned, the content of black hole perturbation theory is very rich. No matter what people want to do, they have to start with wave equations. Hence, wave equations are fundamental for perturbations. In 1973, Teukolsky [22] provided an elegant and very useful master equation for the massless scalar, the Weyl neutrino, electromagnetic, and gravitational fields in the Kerr spacetime. The discovery of Teukolsky is a milestone in black hole perturbation theory; since, not only can the equation separate variables, but it can also have very close relations with special equations. The method of Teukolsky has been extended to various spacetime backgrounds [17,21] through the years.

On the other hand, quasinormal modes have played a central role in black hole perturbation theory for over 50 years. Various arguments were given that a black hole has the characteristic oscillations. However, despite extensive discussion, the experimental investigation of the phenomenon would seem to be virtually impossible. There is a window of observation for analogue black holes in laboratories, such as acoustic, or "dumb," black holes [23,24] and condensed-matter analogue black holes [25,26]. Therefore, a fundamental physical issue is how to establish a relation between real black holes and analogue black holes.

It is obvious that some properties of the acoustic black hole and Rindler spacetime may be observable in laboratories on Earth. Of special interest is that the metrics effectively have the same form as that near the event horizon of static spherically symmetric black holes. This means that the properties of the physics near the event horizon of the black hole may be simulated via acoustic black holes or Rindler spacetime. Hence, the wave equation and its solutions near the horizon are of foundational interest in both black hole perturbations and experimental observation. In this paper we investigate these equations and solutions using analytical methods.

The paper is organized as follows. In Sec. II we derive the wave equation governing massless fields of all spins ($s = 0, 1, 1/2, 1, 3/2$, and 2) in a general static spherically symmetric spacetime. In Sec. III we find the solutions of the wave equation near the event horizon, and discuss the asymptotic behavior of the wave function. In Sec. IV we show that the results can be applied to Rindler-type spacetime, and obtain exact formulas for the frequencies of the quasinormal modes. Finally, Sec. V contains a further discussion and some concluding remarks.

II. GENERAL WAVE EQUATION

A general ansatz for static isotropic metric is given by

$$ds^2 = B(r)dt^2 - A(r)dr^2 - C(r)(d\theta^2 + \sin^2\theta d\varphi^2). \quad (2.1)$$

Which can describe various static spherically symmetric spacetimes, for instance, the Schwarzschild, Reissner-Nordström, Barriola-Vilenkin [27], quintessence [28], Rindler [29], and (anti-) de Sitter spacetime backgrounds or any combination of these.

To study the nature of spacetime, for the metric (2.1), we introduce a null tetrad of basis vectors, l_μ, n_μ, m_μ, and $\bar{m}_\mu$, as follows:

$$\begin{aligned} l_\mu &= (AB, -A\sqrt{AB}, 0, 0), \\ n_\mu &= \left(\frac{1}{2A}, \frac{1}{2\sqrt{AB}}, 0, 0\right), \\ m_\mu &= \left(0, 0, -\sqrt{\frac{C}{2}}, -i\sqrt{\frac{C}{2}}\sin\theta\right), \\ \bar{m}_\mu &= \left(0, 0, -\sqrt{\frac{C}{2}}, i\sqrt{\frac{C}{2}}\sin\theta\right). \end{aligned} \quad (2.2)$$

The tetrad consists of two real null vectors, l_μ and n_μ, and a pair of complex null vectors, m_μ and $\bar{m}_\mu$, which satisfies the orthonormal conditions, $l_\mu n^\mu = -m_\mu \bar{m}^\mu = 1$, and $l_\mu l^\mu = n_\mu n^\mu = m_\mu m^\mu = \bar{m}_\mu \bar{m}^\mu = 0$. Using the metric (2.1) and null tetrad (2.2), the spin coefficients [for definition see Eq. (A1)] can be written as

$$\kappa = \sigma = \nu = \lambda = \pi = \tau = 0,$$
$$\rho = -\frac{\sqrt{AB}}{2}\frac{C'}{C}, \qquad \mu = -\frac{1}{4A\sqrt{AB}}\frac{C'}{C},$$
$$\varepsilon = \frac{\sqrt{AB}}{2}\left(\frac{A'}{A} + \frac{B'}{B}\right), \qquad \gamma = -\frac{1}{4A\sqrt{AB}}\frac{A'}{A},$$
$$\alpha = -\beta = -\frac{1}{2\sqrt{2C}}\cot\theta, \tag{2.3}$$

where the prime denotes the derivative with respect to r.

According to the geometrical interpretation of spin coefficients [17], the vanishing of κ is the condition for the integral curves of l^μ to be geodesic, while, if σ is also zero, this congruence of geodesics is shear free. The same role is played by ν and λ for the n^μ-congruence. From the shear-free character of these congruences, we can conclude on the basis of the Goldberg-Sachs theorem [30] that the general static spherically symmetric spacetime is of Petrov type D. The Weyl scalars [for a definition see Eq. (A2)], ψ_0, ψ_1, ψ_3, and ψ_4 must, therefore, vanish in the chosen basis. The Weyl scalar ψ_2 does not, however, vanish. From which, by satisfying the spin-coefficient equations [31], we calculate the Weyl scalar ψ_2 and the Ricci scalar R as follows:

$$\psi_2 = -\frac{1}{24A}\left[\frac{A'B'}{AB} - \frac{A'C'}{AC} + \frac{B'C'}{BC} + \left(\frac{B'}{B}\right)^2 - 2\left(\frac{C'}{C}\right)^2 - 2\frac{B''}{B} + 2\frac{C''}{C}\right] - \frac{1}{6C}. \tag{2.4}$$

$$R = \frac{1}{A}\left[\frac{1}{2}\left(\frac{B'}{B}\right)^2 + \frac{1}{2}\left(\frac{A'B'}{AB}\right) + \frac{1}{2}\left(\frac{C'}{C}\right)^2 + \frac{A'C'}{AC} - \frac{B'C'}{BC} - \frac{B''}{B} - 2\frac{C''}{C}\right] + \frac{2}{C}. \tag{2.5}$$

Now we will consider the perturbation equation for fields of various spin for $s \le 2$ on the general static spherically symmetric background. In general, the field equations of spin $1/2$, 1, $3/2$, and 2 are not accurately separable, but in all of the type-D metrics, the massless field equations of spin $1/2$, 1, $3/2$, and 2 can be decoupled in the case of perturbations [32–34], and the equations for the source free case can also be combined into [14,33] (see Appendix B)

$$\begin{aligned}
&\{[D - (2s-1)\varepsilon + \bar{\varepsilon} - 2s\rho - \bar{\rho}](\Delta - 2s\gamma + \mu)\\
&\quad - [\delta + \bar{\pi} - \bar{\alpha} - (2s-1)\beta - 2s\tau](\bar{\delta} + \pi - 2s\alpha)\\
&\quad - (2s-1)(s-1)\psi_2\}\Phi_{+s} = 0,\\
&\{[\Delta + (2s-1)\gamma - \bar{\gamma} + 2s\mu + \bar{\mu}](D + 2s\varepsilon - \rho)\\
&\quad - [\bar{\delta} - \bar{\tau} + \bar{\beta} + (2s-1)\alpha + 2s\pi](\delta - \tau + 2s\beta)\\
&\quad - (2s-1)(s-1)\psi_2\}\Phi_{-s} = 0,
\end{aligned} \tag{2.6}$$

where D, Δ, and δ are the directional derivatives defined by

$$D = l^\mu \partial_\mu, \qquad \Delta = n^\mu \partial_\mu, \qquad \delta = m^\mu \partial_\mu. \tag{2.7}$$

In Eq. (2.6) the first equation is for the spin states of $p = s$, while the other one is for $p = -s$.

A straightforward computation, using the spin coefficients, Weyl scalar, Ricci scalar, and directional derivatives is written down in Eqs. (2.3)–(2.5) and (2.7), and makes the transformations

$$\Phi_p = C^{(p-s)/2}\Psi_p. \tag{2.8}$$

Eq. (2.6) has the compact form

$$\left[(\nabla^\mu + p\Gamma^\mu)(\nabla_\mu + p\Gamma_\mu) - 4p^2\psi_2 + \frac{1}{6}R\right]\Psi_p = 0, \tag{2.9}$$

where

$$\Gamma^t = -\frac{1}{2\sqrt{AB}}\left(\frac{B'}{B} - \frac{C'}{C}\right),$$
$$\Gamma^r = \frac{1}{2A}\left(2\frac{A'}{A} + \frac{B'}{B} - \frac{C'}{C}\right),$$
$$\Gamma^\theta = 0,$$
$$\Gamma^\varphi = -\frac{1}{C}\frac{i\cos\theta}{\sin^2\theta}. \tag{2.10}$$

This general result [Eq. (2.9)] is consistent with the conventional theory of Dirac particles, as of course it had to be. In particular, when the metric of Eq. (2.1) respectively describes the Schwarzschild, Reissner-Nordström, and Reissner-Nordström-de Sitter spacetime, Eq. (2.9) reduces to the results of Refs. [14,16,22] in the case of $a = 0$. Evidently, when $p = 0$, Eq. (2.9) is just the (conformally invariant) massless scalar field equation. Therefore, Eq. (2.9) governs not only the massless fields of spin $1/2$, 1, $3/2$, and 2, but also the scalar field. Equation (2.9) is the fundamental formula of the perturbation theory for arbitrary static spherically symmetric spacetime.

III. ANALYTICAL SOLUTIONS OF THE GENERAL WAVE EQUATION NEAR THE HORIZON

The exact solution of Eq. (2.9) has the form

$$\Psi_p = e^{-i\omega t} S(\theta,\varphi) R(r). \tag{3.1}$$

Notice that the angular part of the wave function, $S(\theta,\varphi)$, is the same for all spherically symmetric metrics, which is a spin-weighted spherical harmonic [35]; the actual shape of the metric functions affects only the radial part of the wave function $R(r)$.

If only considering a nonextremal case, in the vicinity of the horizon, the metric functions should have the form

$$\begin{aligned} B(r) &\approx (B')_{r_H}(r-r_H), \\ A(r) &\approx \frac{1}{(\frac{1}{A})'_{r_H}(r-r_H)}, \\ C(r) &\approx C_0 r^2, \end{aligned} \tag{3.2}$$

where r_H is the horizon radius, and C_0 is constant. If we introduce the dimensionless variable

$$z = 1 - \frac{r_H}{r}, \tag{3.3}$$

in terms of z, the radial wave equation in the vicinity of the horizon takes the form

$$\left[\frac{d^2}{dz^2} + P(z)\frac{d}{dz} + Q(z)\right] R(z) = 0, \tag{3.4}$$

where

$$\begin{aligned} P(z) &= \frac{A_1}{z} + \frac{A_2}{z-1}, \\ Q(z) &= \frac{B_1}{z^2} + \frac{B_2}{(z-1)^2} + \frac{C_1}{z} + \frac{C_2}{z-1}, \end{aligned} \tag{3.5}$$

with

$$\begin{aligned} &A_1 = p+1, \qquad A_2 = -(3p+1), \\ &B_1 = \left(\frac{\omega}{2\kappa}\right)^2 - ip\left(\frac{\omega}{2\kappa}\right), \\ &B_2 = (2p+1)(p+1)\left(\frac{\omega}{2\kappa}\right)^2 + ip\left(\frac{\omega}{2\kappa}\right), \\ &C_1 = -C_2 = 2\left(\frac{\omega}{2\kappa}\right)^2 + (p+1)(2p+1)\left(\frac{2}{3} - \frac{c}{6\kappa r_H}\right) \\ &\qquad - \frac{c\lambda^2}{2\kappa r_H}. \end{aligned} \tag{3.6}$$

Here, c is a constant, κ (in Secs. III and IV) is the surface gravity of the horizon, λ (in Secs. III and IV) is the separation constant, given by

$$\begin{aligned} c &= \frac{1}{C_0}\sqrt{\frac{(B')_{r_H}}{(\frac{1}{A})'_{r_H}}}, \\ \kappa &= \frac{1}{2}\sqrt{(B')_{r_H}\left(\frac{1}{A}\right)'_{r_H}}, \\ \lambda &= \sqrt{(l-p)(l+p+1)}, \end{aligned} \tag{3.7}$$

where l is a positive integer satisfying the inequalities of $l \geq s$.

Equation (3.4) is of the Fuchsian type, which has three regular singular points, located at $z = 0$, 1, and ∞. Note that $z = \infty$ is unphysical. The local exponents at these points are respectively

$$\alpha_1 = i\frac{\omega}{2\kappa}, \qquad \beta_1 = -p - i\frac{\omega}{2\kappa}; \tag{3.8}$$

$$\alpha_2 = 2p + 1 - i\frac{\omega}{2\kappa}, \qquad \beta_2 = p + 1 + i\frac{\omega}{2\kappa}; \tag{3.9}$$

$$\begin{aligned} \alpha_3 = &-\left(p+\frac{1}{2}\right) + \left[\left(p+\frac{1}{2}\right)^2 \right. \\ &\left. -(p+1)(2p+1)\left(\frac{1}{3}+\frac{c}{6\kappa r_H}\right) - \frac{c(l-p)(l+p+1)}{2\kappa r_H}\right]^{\frac{1}{2}}, \\ \beta_3 = &-\left(p+\frac{1}{2}\right) - \left[\left(p+\frac{1}{2}\right)^2 \right. \\ &\left. -(p+1)(2p+1)\left(\frac{1}{3}+\frac{c}{6\kappa r_H}\right) - \frac{c(l-p)(l+p+1)}{2\kappa r_H}\right]^{\frac{1}{2}}. \end{aligned} \tag{3.10}$$

Note that the sum of the exponents is equal to the number of regular singular points minus 2, namely, $(\alpha_1+\beta_1)+(\alpha_2+\beta_2)+(\alpha_3+\beta_3) = 1$.

The set of all solutions of Eq. (3.4) is denoted

$$R(z) \in P\left\{\begin{matrix} 0 & 1 & \infty & \\ \alpha_1 & \alpha_2 & \alpha_3, & z \\ \beta_1 & \beta_2 & \beta_3 & \end{matrix}\right\}. \tag{3.11}$$

This notion is called the Riemann's P symbol.

In a neighbourhood of $z = 0$, the general solution has the form

$$\begin{aligned} R(z) = {} & D_1 z^{\alpha_1}(1-z)^{\alpha_2} F(\alpha_1+\alpha_2+\alpha_3, \alpha_1+\alpha_2+\beta_3, 1 \\ & + \alpha_1 - \beta_1, z) + D_2 z^{\beta_1}(1-z)^{\alpha_2} F(\alpha_2+\alpha_3+\beta_1, \alpha_2 \\ & + \beta_1 + \beta_3, 1 - \alpha_1 + \beta_1, z), \end{aligned} \tag{3.12}$$

where $F(\alpha,\beta,\gamma,z)$ is the hypergeometric function and D_1 and D_2 are arbitrary constants. It is assumed that the $1+\alpha_1-\beta_1$ is not an integer. If in which case it is a positive integer, the general solution takes the form

$$R(z) = z^{\alpha_1}(1-z)^{\alpha_2}[D_1 F(\alpha_1+\alpha_2+\alpha_3, \alpha_1+\alpha_2+\beta_3, 1 + \alpha_1-\beta_1, z) + D_2 G(\alpha_1+\alpha_2+\alpha_3, \alpha_1+\alpha_2+\beta_3, 1 + \alpha_1-\beta_1, z)], \tag{3.13}$$

while when the $1+\alpha_1-\beta_1$ is zero or a negative integer, it becomes

$$R(z) = z^{\beta_1}(1-z)^{\alpha_2}[D_1 F(\alpha_2+\alpha_3+\beta_1, \alpha_2+\beta_1+\beta_3, 1 - \alpha_1+\beta_1, z) + D_2 G(\alpha_2+\alpha_3+\beta_1, \alpha_2+\beta_1+\beta_3, 1 - \alpha_1+\beta_1, z)]. \tag{3.14}$$

Here the function G is defined by

$$G(\alpha,\beta,\gamma,z) = F(\alpha,\beta,\gamma,z)\ln z - \sum_{k=0}^{\gamma-2}\frac{k!(1-\gamma)_{k+1}}{(1-\alpha)_{k+1}(1-\beta)_{k+1}}z^{-k-1} + \sum_{k=0}^{\infty}\frac{(\alpha)_k(\beta)_k}{k!(\gamma)_k}[\psi(\alpha+k)+\psi(\beta+k) - \psi(\gamma+k)-\psi(1+k)]z^k, \tag{3.15}$$

where $\psi(\alpha)$ is the Euler psi function.

It should be noted that the integer values of $1+\alpha_1-\beta_1$ lead directly to an exact determination of the frequencies as follows

$$\omega = i(1+p-m)\kappa, \qquad m = 0, \pm 1, \pm 2, \ldots, \tag{3.16}$$

which shows that, in this case, ω takes only discrete imaginary values.

When the value of $1+\alpha_1-\beta_1$ is not an integer, adopting the Eddington-Finkelstein null coordinates v, u is convenient in discussing the asymptotic behavior of the wave functions, which take the form

$$v = t + r_*, \qquad u = t - r_* \tag{3.17}$$

and satisfy

$$\partial^\mu v \partial_\mu v = 0, \qquad \partial^\mu u \partial_\mu u = 0. \tag{3.18}$$

Here r_* is called the tortoise coordinate. Its exact form depends on the metric functions near the horizon, but with suitable choices for the integration constants, we find that

$$r_* = \frac{1}{2\kappa}\ln\left|\frac{r-r_H}{r_H}\right|. \tag{3.19}$$

The solution of Eq. (3.12) can easily be seen in terms of the tortoise coordinate r_* when $z\to 0$ takes the form

$$R(r_*) = D_1 e^{i\omega r_*} + D_2 e^{-(2\kappa p + i\omega)r_*}. \tag{3.20}$$

Therefore, in the Eddington-Finkelstein null coordinates, the time-dependent radial wave function can be written

$$R(v, r_*) = D_1 e^{2i\omega r_*}e^{-i\omega v} + D_2 e^{-2p\kappa r_*}e^{-i\omega v}. \tag{3.21}$$

In Eq. (3.21), the first term is the outgoing wave, and the second term is the ingoing wave. From the outgoing wave, one can easily obtain the Hawking radiation spectrum [12]. Of special interest is that the amplitude of the ingoing wave is obviously dependent on the spin state.

IV. EXACT QUASINORMAL MODES FOR RINDLER-TYPE SPACETIMES

In terms of the metric functions in the vicinity of the horizon, given by Eq. (3.2), the metric is of the form

$$ds^2 = \Omega\left[2\kappa(r-r_H)dt^2 - \frac{dr^2}{2\kappa(r-r_H)} - \frac{r^2}{c}(d\theta^2 + \sin^2\theta d\varphi^2)\right], \tag{4.1}$$

where $\Omega = \sqrt{(B')_{r_H}/(\frac{1}{A})'_{r_H}}$, which does not affect wave equation.

When the metric of Eq. (4.1) describes the near horizon various static spherically symmetric spacetimes, we have tacitly assumed that $r_H > 0$. If r_H is an arbitrary real constant (except $r_H = 0$), the metric not only describes the special quintessence [28], conformal gravity [36], and Rindler [29] spacetimes, but also the acoustic black hole for the scalar field. In this paper, they are collectively called Rindler-type spacetimes.

Note that, for Rindler-type spacetimes, the solutions of Eqs. (3.12)–(3.14) are exact without any approximation at all. In other words, the wave functions, described by Eqs. (3.12)–(3.14), are valid in the total space. From which one can discuss various problems of the spin fields on the backgrounds. However, here, we focus our attention on the quasinormal modes.

Quasinormal modes are solutions of the wave equation, satisfying specific boundary conditions at the horizon and at spatial infinity. Similar to asymptotically flat and de Sitter spacetimes, for Rindler-type spacetimes we naturally suppose that

$$R(r_*) \sim e^{\pm i\omega r_*} \quad (r_* \to \pm\infty), \tag{4.2}$$

Therefore, we must select a solution that is ingoing near the event horizon $z = 0$. This corresponds to

$$R(z) = D_2 z^{\beta_1}(1-z)^{\alpha_2}F(\alpha_2+\alpha_3+\beta_1, \alpha_2+\beta_1 + \beta_3, 1-\alpha_1+\beta_1, z). \tag{4.3}$$

At a large r, the solution behaves like:

$$R(z) \sim (1-z)^{\alpha_2} \frac{\Gamma(1-\alpha_1+\beta_1)\Gamma(\beta_2-\alpha_2)}{\Gamma(1-\alpha_1-\alpha_2-\alpha_3)\Gamma(1-\alpha_1-\alpha_2-\beta_3)} + (1-z)^{\beta_2} \frac{\Gamma(1-\alpha_1+\beta_1)\Gamma(\alpha_2-\beta_2)}{\Gamma(\alpha_2+\alpha_3+\beta_1)\Gamma(\alpha_2+\beta_1+\beta_3)}. \tag{4.4}$$

If requiring $R(z)$ to match the boundary at infinity, we must set

$$\alpha_2+\alpha_3+\beta_1=-n, \quad \text{or} \quad \alpha_2+\beta_1+\beta_3=-n, \qquad n=0,1,2,\ldots, \tag{4.5}$$

which leads directly to the frequencies of quasinormal modes as

$$\omega = -i\kappa\left(n+\frac{1}{2}\right) \pm \kappa\left[(p+1)(2p+1)\left(\frac{1}{3}+\frac{c}{6\kappa r_H}\right) + \frac{c(l-p)(l+p+1)}{2\kappa r_H} - \left(p+\frac{1}{2}\right)^2\right]^{\frac{1}{2}}. \tag{4.6}$$

The wave function is therefore not a pure oscillation, for ω is complex. It is seen from Eq. (4.6) that the imaginary part of ω leads to an exponentially decreasing function of time. The damping factor is only the function of the surface gravity. The real part of Eq. (4.6) corresponds to the oscillatory factor in the wave function. The oscillation frequency not only depends on the characteristics of the metric, but also the characteristics of the particle.

V. DISCUSSION AND CONCLUSION

We have derived the wave equation governing massless fields of arbitrary spin of $s \leq 2$, from which, one can investigate the perturbations of all static spherically symmetric spacetimes.

We have found that, in the vicinity of the horizon, the wave equation can be reduced to the Fuchsian-type equation with three regular singular points. There are three different solutions that correspond to the three kinds of $1+\alpha_1-\beta_1$: the non-integer, positive integer, and zero or negative integer. The integer values of $1+\alpha_1-\beta_1$ lead directly to the discrete imaginary frequencies of Eq. (3.16). This is a very interesting result, because in general, the discrete values of the frequency emerged as a rather technical consequence of the boundary conditions on the solutions to the wave equation. However, in this special case it is not necessary to introduce the boundary conditions. Remembering that $\Psi_p \sim e^{-i\omega t}$, hence negative m means an instability.

Using the solutions near the horizon, we have obtained exact frequencies of the quasinormal modes for the Rindler-type spacetimes, which are two complex conjugate values. The real part is the oscillation frequency of the mode, and the imaginary part is proportional to its damping rate. In Eq. (4.6), the negative imaginary part means that the wave function is damped.

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APPENDIX A: SPIN COEFFICIENTS AND WEYL SCALARS

The 12 spin coefficients are defined by [17,37]

$$\begin{aligned}
&\kappa = \nabla_\nu l_\mu m^\mu l^\nu, \qquad \lambda = -\nabla_\nu n_\mu \bar{m}^\mu \bar{m}^\nu,\\
&\sigma = \nabla_\nu l_\mu m^\mu m^\nu, \qquad \nu = -\nabla_\nu n_\mu \bar{m}^\mu n^\nu,\\
&\rho = \nabla_\nu l_\mu m^\mu \bar{m}^\nu, \qquad \tau = \nabla_\nu l_\mu m^\mu n^\nu,\\
&\mu = -\nabla_\nu n_\mu \bar{m}^\mu m^\nu, \qquad \pi = -\nabla_\nu n_\mu \bar{m}^\mu l^\nu,\\
&\alpha = \frac{1}{2}(\nabla_\nu l_\mu n^\mu \bar{m}^\nu - \nabla_\nu m_\mu \bar{m}^\mu \bar{m}^\nu),\\
&\beta = \frac{1}{2}(\nabla_\nu l_\mu n^\mu m^\nu - \nabla_\nu m_\mu \bar{m}^\mu m^\nu),\\
&\gamma = \frac{1}{2}(\nabla_\nu l_\mu n^\mu n^\nu - \nabla_\nu m_\mu \bar{m}^\mu n^\nu),\\
&\varepsilon = \frac{1}{2}(\nabla l_\mu n^\mu l^\nu - \nabla_\nu m_\mu \bar{m}^\mu l^\nu).
\end{aligned} \tag{A1}$$

The five Weyl scalars are defined by [17,37]

$$\begin{aligned}
&\psi_0 = -C_{\mu\nu\rho\sigma} l^\mu m^\nu l^\rho m^\sigma,\\
&\psi_1 = -C_{\mu\nu\rho\sigma} l^\mu n^\nu l^\rho m^\sigma,\\
&\psi_2 = -\frac{1}{2} C_{\mu\nu\rho\sigma}(l^\mu n^\nu l^\rho n^\sigma - l^\mu n^\nu m^\rho \bar{m}^\sigma),\\
&\psi_3 = -C_{\mu\nu\rho\sigma} \bar{m}^\mu n^\nu l^\rho n^\sigma,\\
&\psi_3 = -C_{\mu\nu\rho\sigma} \bar{m}^\mu n^\nu \bar{m}^\rho n^\sigma,
\end{aligned} \tag{A2}$$

where $C_{\mu\nu\rho\sigma}$ is the Weyl tensor, which satisfies

$$R_{\mu\nu\rho\sigma} = C_{\mu\nu\rho\sigma} + \frac{1}{2}(g_{\mu\rho}R_{\nu\sigma} - g_{\mu\sigma}R_{\nu\rho} - g_{\nu\rho}R_{\mu\sigma} + g_{\nu\sigma}R_{\mu\rho}) + \frac{1}{6}(g_{\mu\sigma}g_{\nu\rho} - g_{\mu\rho}g_{\nu\sigma})R. \tag{A3}$$

APPENDIX B: SPIN FIELD EQUATIONS

The Weyl neutrino satisfies the wave equation [38]

$$\nabla_{AA'}P^A = 0, \tag{B1}$$

where P^A is the two-component spinor, $\nabla_{AA'}$ is the symbol for the covariant spinor differentiation. Equation (B1) can be written in the Newman-Penrose formalism in the form

$$(D+\varepsilon-\rho)P^0+(\bar{\delta}+\pi-\alpha)P^1=0,$$
$$(\delta+\beta-\tau)P^0+(\Delta+\mu-\gamma)P^1=0. \tag{B2}$$

In type-D spacetime, the equation can be decoupled into [37]

$$[(D+\bar{\varepsilon}-\rho-\bar{\rho})(\Delta-\gamma+\mu)$$
$$-(\delta-\bar{\alpha}-\tau+\bar{\pi})(\bar{\delta}-\alpha+\pi)]\Phi_{+1/2}=0,$$
$$[(\Delta-\bar{\gamma}+\mu+\bar{\mu})(D+\varepsilon-\rho)$$
$$-(\bar{\delta}+\bar{\beta}+\pi-\bar{\tau})(\delta+\beta-\tau)]\Phi_{-1/2}=0, \tag{B3}$$

with $\Phi_{+1/2}=P^1$ and $\Phi_{-1/2}=-P^0$.

Similarly, the equations of the electromagnetic ($s=1$), the massless Rarita-Schwinger ($s=3/2$), and the gravitational ($s=2$) fields on any type-D spacetime background can also be decoupled. For the source free case, they are given by [34,37]

$$[(D-\varepsilon+\bar{\varepsilon}-2\rho-\bar{\rho})(\Delta-2\gamma+\mu)$$
$$-(\delta+\bar{\pi}-\bar{\alpha}-\beta-2\tau)(\bar{\delta}+\pi-2\alpha)]\Phi_{+1}=0,$$
$$[(\Delta+\gamma-\bar{\gamma}+2\mu+\bar{\mu})(D+2\varepsilon-\rho)$$
$$-[\bar{\delta}-\bar{\tau}+\bar{\beta}+\alpha+2\pi)(\delta-\tau+2\beta)\Phi_{-1}=0. \tag{B4}$$

$$[D-2\varepsilon+\bar{\varepsilon}-3\rho-\bar{\rho})(\Delta-3\gamma+\mu)$$
$$-(\delta+\bar{\pi}-\bar{\alpha}-2\beta-3\tau)(\bar{\delta}+\pi-3\alpha)-\psi_2]\Phi_{+3/2}=0,$$
$$[(\Delta+2\gamma-\bar{\gamma}+3\mu+\bar{\mu})(D+3\varepsilon-\rho)$$
$$-(\bar{\delta}-\bar{\tau}+\bar{\beta}+2\alpha+3\pi)(\delta-\tau+3\beta)-\psi_2]\Phi_{-3/2}=0. \tag{B5}$$

$$[(D-3\varepsilon+\bar{\varepsilon}-4\rho-\bar{\rho})(\Delta-4\gamma+\mu)$$
$$-(\delta+\bar{\pi}-\bar{\alpha}-3\beta-4\tau)(\bar{\delta}+\pi-4\alpha)-3\psi_2]\Phi_{+2}=0,$$
$$[(\Delta+3\gamma-\bar{\gamma}+4\mu+\bar{\mu})(D+4\varepsilon-\rho)$$
$$-[\bar{\delta}-\bar{\tau}+\bar{\beta}+3\alpha+4\pi](\delta-\tau+4\beta)-3\psi_2]\Phi_{-2}=0. \tag{B6}$$

In fact, we can combine Eqs. (B3)–(B6) into a single statement: Eq. (2.6).

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