

# Massive vector field on curved background: Nonminimal coupling, quantization, and divergences

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We study the effective action for the massive vector field theory nonminimally coupled to external gravitational field. Such a theory is an interesting model both from the theoretical side and also due to the various phenomenological applications to cosmology and astrophysics. The present work pretends to initiate a systematic study of its properties at the quantum level, by exploring free massive vector coupled to an external symmetric second-rank tensor. Stueckelberg scalar field is used to restore the gauge invariance. After that, by using a special gauge fixing and nonlocal in external fields change of variables, we diagonalize the bilinear form of the action and develop a consistent procedure to study the effective action. As a result we derive a complete nonlinear structure of divergences of the effective action and discuss its properties.

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## I. INTRODUCTION

The vector field models with soft and/or spontaneous violation of gauge invariance is one of objects of modern study in gravity and cosmology (see, e.g., [1,2] and references therein). These models are closely related to the classical and quantum vector Galileons [3–8]. The list of applications of such models includes, in particular, the generation of the initial seeds of magnetic fields in the early universe [9] and vector inflation [10] (see also [11,12] for extensive reviews). Both these subjects attract a great deal of interest, however the considerations are concentrated mainly on the classical theory. It seems natural to extend the study of these theories to the quantum level, especially because we know that quantum effects play an important role in the usual scalar inflaton models. Therefore we can hope that the quantum effects can play some role in the vector inflation.

In the present work, we consider quantum aspects of massive Abelian vector field in curved spacetime. The first observation is that the coupling of such a field to external gravity includes some interesting nontrivial aspects. Indeed, let us start from an arbitrary matter field model in Minkowski space. As we know, a generalization of such

a model to curved spacetime in nonunique (see, e.g., [13]). The first possibility is to apply the procedure of minimal covariant generalization, including interaction with gravity. The last means one should treat the field as a tensor or spinor in curved spacetime, replace partial derivatives by the corresponding covariant derivatives, without changing the canonical dimension of the field and trying to preserve as much as possible global and gauge symmetries of the initial flat-space theory. However, after that there is still a freedom to enrich the Lagrangian by arbitrary local terms containing the matter field and the powers of the curvature tensor with some coupling constants. All these terms describe the nonminimal coupling of matter field to gravity. If we also demand that the matter sector does not contain the new scales in comparison with theory in flat space, then all nonminimal coupling constants must be dimensionless.

Taking into account the above arguments, it is evident that the only possible nonminimal coupling for scalar field  $\phi$  is  $\xi R\phi^2$  where  $R$  is a scalar curvature and  $\xi$  is a dimensionless constant of nonminimal coupling. For the massless vector field, nonminimal interactions with only dimensionless coupling constants is forbidden by gauge invariance. Furthermore, for a fermionic spin-1/2 field the nonminimal interaction to gravity is ruled out by the canonical dimension of the field. Let us now consider the massive vector field model. This model is not gauge invariant in flat space, therefore the arguments based on

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gauge invariance do not work anymore. As a result, the most general action for massive vector field in curved spacetime, without dimensionful coupling constants, has the form

$$S = \int d^4x \sqrt{-g} \left\{ -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2} m^2 A_\mu^2 - \frac{1}{2} X^{\mu\nu}(x) A_\mu A_\nu \right\}, \quad (1)$$

where

$$X_{\mu\nu}(x) = \xi_1 R g_{\mu\nu} + \xi_2 R_{\mu\nu}. \quad (2)$$

Here  $\xi_1$  and  $\xi_2$  are two dimensionless nonminimal coupling constants. All other notations are standard. This is the only admissible nonminimal extension for the theory of a free massive vector field in curved spacetime.

The versions of (1) that are mostly used for existing applications are

$$X_{\mu\nu} = \xi_1 R g_{\mu\nu}, \quad m \neq 0 \quad (3)$$

for vector inflation [10] and

$$X_{\mu\nu} = \xi_1 R g_{\mu\nu} + \xi_2 R_{\mu\nu}, \quad m = 0 \quad (4)$$

in the case of generating magnetic fields [9]. In this case, the curvature-dependent terms are introduced at the phenomenological level to break the local conformal invariance of electromagnetic field. The coefficients  $\xi_1$  and  $\xi_2$  are chosen in each particular case from the phenomenological arguments.

In more general theories, there may be other possibilities for the field  $X_{\mu\nu}$ , besides (2). For example, there can be an external field  $X^{\mu\nu}$  which is not related to the curvature, but can be useful for phenomenological reasons. Another possibility originates if we consider a self-interacting massive vector field model or massive vector field coupled to other dynamical fields like scalars and/or fermions. Then, in the one-loop approximation, one can meet the action (1), where the field  $X^{\mu\nu}$  is constructed from the background vector, scalar, or spinor fields. These circumstances motivate us to consider the model with action (1), where no concrete form for the field  $X^{\mu\nu}$  is assumed.

In the present work, we intend to study the model (1) treating  $X^{\mu\nu}(x)$  as an arbitrary external symmetric tensor field. Our main purpose is to formulate the quantum effective action  $\Gamma[g_{\mu\nu}, X^{\mu\nu}]$  for this model and to calculate its divergences. It is important to note that the naive quantization of the model (1) does not work. Within the standard scheme, the effective action should be defined as a functional integral over the fields  $A_\mu(x)$  of the exponential of the action (1). Hence, the result would have the form of the functional determinant of the operator related to the quadratic part (bilinear form) of the action (1). The reason why this approach is not appropriate is that the kinetic part

of the action (1) is gauge invariant while the mass and nonminimal terms are noninvariant. This yields the constraints on the dynamics of the theory and, as a result, the naive functional integral approach becomes incorrect.

The first study of the model (1) has been undertaken by D. J. Toms [14]. It was shown that the naive approach treating the term with  $X^{\mu\nu}$  as a small perturbation leads to inconsistencies, because the propagator of the zero-order theory, that is the Proca model, is a subject of the mentioned constraint. This observation was confirmed by the canonical analysis. In particular, this means that the standard procedure of dealing with a free Proca model in curved space [15] (see also [16]) leads to an inconsistency.<sup>1</sup> The derivation of the one-loop divergences in [14] was based on the noncovariant approach related to the Faddeev-Jackiw quantization [18]. As a result, the divergences were obtained only for the two special cases with constant external field  $X^{\mu\nu}$ , from which it is difficult to restore the covariant result.

In what follows, we describe the general procedure of consistent quantization of the theory (1) which is manifestly covariant and can be applied for an unconstrained and possibly nonconstant external field  $X_{\mu\nu}(x)$  and for an arbitrary external metric. As a result, we arrive at the covariant expression for the effective action and calculate its divergent part. As was already mentioned, the quantization is not a trivial task because of the degeneracy of the kinetic term and the broken gauge symmetry. In the usual Proca theory, this problem has two standard solutions [19], but both methods meet serious difficulties in the presence of the nonminimal term with an arbitrary field  $X^{\mu\nu}$ . In the rest of the paper, we perform calculations by combining a nonstandard application of the covariant Stueckelberg procedure and different versions of the Schwinger-DeWitt technique [15,20]. The work can be seen as a first step in exploring the quantum properties and loop corrections for a wider class of massive and massless vector theories with a nonminimal interaction.

The paper is organized as follows. In Sec. II, it is shown how the theory of the massive vector field with nonminimal coupling to an external tensor field can be reformulated by introducing a compensating auxiliary scalar and restoring gauge invariance. After that, the gauge fixing is introduced and the basis for quantum theory formulated. In Sec. III, we construct the bilinear form of the action and discuss two alternative ways to make it diagonal. Section IV is devoted to the nonpolynomial change of variables in the scalar sector and to the main quantum calculation. In Sec. V, an independent calculation the first order in  $X^{\mu\nu}$  is performed to provide an independent verification of the main result. Finally, in Sec. VI, we draw our conclusions and discuss possible extensions and generalizations of this work.

<sup>1</sup>Certain aspects of quantum Proca theory were recently discussed in [17].

## II. RESTORING AND FIXING GAUGE SYMMETRY

Our consideration is based on reformulation of the model (1) in dynamical equivalent but gauge invariant form. For this aim we use the Stueckelberg procedure [19] and perform the transformation

$$A_\mu \rightarrow A_\mu - \frac{1}{m} \partial_\mu \varphi. \quad (5)$$

After that the new action, depending now on  $A_\mu$  and  $\varphi$ , becomes invariant under the gauge transformation

$$A_\mu \rightarrow A_\mu + \partial_\mu f, \quad \varphi \rightarrow \varphi + mf. \quad (6)$$

Here  $f$  is a gauge parameter. As a result, we arrive at the action which is dynamically equivalent to (1), but possesses the symmetry under (6),

$$S' = \int d^4x \sqrt{-g} \left\{ -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2} m^2 A_\mu^2 - \frac{1}{2} X^{\mu\nu} A_\mu A_\nu - \frac{1}{2} (\nabla\varphi)^2 - \frac{1}{2m^2} X^{\mu\nu} \nabla_\nu \varphi \nabla_\mu \varphi + mA^\mu \nabla_\mu \varphi + \frac{1}{m} X^{\mu\nu} A_\nu \nabla_\mu \varphi \right\}. \quad (7)$$

In the special gauge  $\varphi = 0$ , the theory comes back to (1). According to the standard consideration (see e.g. [21]), the result of the quantum calculation in this and similar cases does not depend on the gauge fixing condition, and hence one is free to choose another gauge, making quantum theory more transparent and calculations more accessible.

A useful gauge fixing (GF) action is given by

$$S_{gf} = -\frac{1}{2} \int d^4x \sqrt{-g} \chi^2, \quad (8)$$

where

$$\chi = \nabla_\mu A^\mu - m\varphi. \quad (9)$$

The total action with the gauge fixing term has the form

$$S' + S_{gf} = \int d^4x \sqrt{-g} \left\{ -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2} (\nabla_\mu A^\mu)^2 - \frac{1}{2} m^2 A_\mu^2 - \frac{1}{2} X^{\mu\nu} A_\mu A_\nu - \frac{1}{2} (\nabla\varphi)^2 - \frac{1}{2} m^2 \varphi^2 - \frac{1}{2m^2} X^{\mu\nu} \nabla_\mu \varphi \nabla_\nu \varphi + \frac{1}{m} X^{\mu\nu} A_\nu \nabla_\mu \varphi \right\}, \quad (10)$$

which still looks complicated, especially due to the non-minimal second-derivative term in the scalar sector. There is also a linear in derivative term in the mixed  $A\varphi$  sector. Due to the unusual scalar operator, one cannot apply a known technique (see e.g. [13]) of dealing with such a term and it is necessary to look for some other approach.

## III. BILINEAR OPERATOR IN QUANTUM FIELDS

Using the GF condition (9) and the action (10), one can find the following operator of the theory:

$$S' + S_{gf} = \int d^4x \sqrt{-g} \{ \mathcal{L}' + \mathcal{L}_{gf} \} = \frac{1}{2} \int d^4x \sqrt{-g} (A_\mu \quad \varphi) \hat{\mathbf{H}} \begin{pmatrix} A_\nu \\ \varphi \end{pmatrix}, \quad (11)$$

where

$$\hat{\mathbf{H}} = \begin{pmatrix} \hat{H}_{AA} & \hat{H}_{A\varphi} \\ \hat{H}_{\varphi A} & \hat{H}_{\varphi\varphi} \end{pmatrix}. \quad (12)$$

Here and in the following, we use bold notations for the matrix operators only. The blocks of (12) have the form

$$\hat{H}_{AA} = g^{\mu\nu} (\square - m^2) - R^{\mu\nu} - X^{\mu\nu}, \quad (13)$$

$$\hat{H}_{A\varphi} = \frac{1}{m} X^{\mu\nu} \nabla_\mu, \quad (14)$$

$$\hat{H}_{\varphi A} = -\frac{1}{m} (\nabla_\mu X^{\mu\nu}) - \frac{1}{m} X^{\mu\nu} \nabla_\mu, \quad (15)$$

$$\hat{H}_{\varphi\varphi} = \square - m^2 + \frac{1}{m^2} (\nabla_\mu X^{\mu\nu}) \nabla_\nu + \frac{1}{m^2} X^{\mu\nu} \nabla_\mu \nabla_\nu. \quad (16)$$

We emphasize here that no one term in the action (11) is degenerated. It means that the quantization is realized by standard procedure, the corresponding effective action is given by standard functional integral for gauge theory and the effective action will be given by

$$\frac{i}{2} \ln \text{Det} \hat{\mathbf{H}}, \quad (17)$$

plus the corresponding ghost contribution. However, the bilinear form (12) has nonstandard structure for study the effective action. For instance, the operator  $\hat{\mathbf{H}}$  is nonminimal and nondiagonal at the same time. Our next purpose will be to derive the divergent part of the expression (17) for this complicated case. The most standard approach would be to perform this calculation by constructing perturbation theory in  $X_{\mu\nu}$ , similar to what was done by two of us in the theory with broken  $CPT$  or Lorentz symmetry [22]. The main disadvantage of this method is that it works only for a small  $X_{\mu\nu}$ . This is a reasonable assumption in the case of  $CPT$  or Lorentz symmetry violation, but not in the case of vector inflation. Therefore, we shall try to make a non-perturbative in  $X_{\mu\nu}$  calculation. Later on, the perturbative approach will be used for an independent verification of the result in the linear approximation.

In the following, we consider two different ways to eliminate the mixed  $A_\mu\varphi$  term in the action.

### A. Diagonalization by rotational type of transformation

In order to diagonalize the operator (12) let us make the following transformation of the fields:

$$A_\mu = B_\mu + \frac{1}{m} \nabla_\mu \chi, \quad (18)$$

$$\varphi = \chi + \frac{1}{m} \nabla_\nu B^\nu. \quad (19)$$

In the new variables the action (10) becomes

$$\begin{aligned} \mathcal{L}' + \mathcal{L}_{gf} = & \frac{1}{2} B_\mu \hat{H}_{AA} B_\nu + \frac{1}{2m^2} \nabla_\mu B^\mu \hat{H}_{\varphi\varphi} \nabla_\nu B^\nu + \frac{1}{m^2} B_\mu X^{\mu\nu} \nabla_\nu \nabla_\alpha B^\alpha + \frac{1}{2} \chi \hat{H}_{\varphi\varphi} \chi \\ & + \frac{1}{2m^2} \nabla_\mu \chi \hat{H}_{AA} \nabla_\nu \chi + \frac{1}{m^2} \nabla_\mu \chi X^{\mu\nu} \nabla_\nu \chi + \frac{1}{m} B_\mu [g^{\mu\nu} (\square - m^2) - R^{\mu\nu} - X^{\mu\nu}] \nabla_\nu \chi \\ & + \frac{1}{m} \chi \left[ (\square - m^2) + \frac{1}{m^2} (\nabla_\mu X^{\mu\nu}) \nabla_\nu + \frac{1}{m^2} X^{\mu\nu} \nabla_\mu \nabla_\nu \right] \nabla_\alpha B^\alpha + \frac{1}{m} B_\mu X^{\mu\nu} \nabla_\nu \chi \\ & + \frac{1}{m^3} X^{\mu\nu} \nabla_\mu \chi \nabla_\nu \nabla_\alpha B^\alpha. \end{aligned} \quad (20)$$

After some integration by parts one can show that all mixed terms in Eq. (20) cancel and the Lagrangian boils down to

$$\begin{aligned} \mathcal{L}' + \mathcal{L}_{gf} = & \frac{1}{2} B_\mu \hat{H}_{AA} B_\nu + \frac{1}{2m^2} \nabla_\mu B^\mu \hat{H}_{\varphi\varphi} \nabla_\nu B^\nu \\ & + \frac{1}{m^2} B_\mu X^{\mu\nu} \nabla_\nu \nabla_\alpha B^\alpha + \frac{1}{2} \chi \hat{H}_{\varphi\varphi} \chi \\ & + \frac{1}{2m^2} \nabla_\mu \chi \hat{H}_{AA} \nabla_\nu \chi + \frac{1}{m^2} \nabla_\mu \chi X^{\mu\nu} \nabla_\nu \chi. \end{aligned} \quad (21)$$

The last expression provides a diagonal bilinear operator  $\hat{H}$ . However, there is a major problem with the theory (21), because of the nonminimal  $X^{\alpha\beta} \nabla_\alpha \nabla_\beta$  term in the vector field sector and because of the similar nonminimal fourth derivative terms in both scalar and vector sectors. The operator has too complicated form, such that no way to deal with its functional determinant is known. For example, the generalized Schwinger-DeWitt technique of [15] does not look applicable because  $X^{\alpha\beta}$  is a field and hence the nonminimal structure here cannot be parametrized by numerical parameters.

### B. Diagonalization by the shiftlike transformation

Consider another transformation, which involves only vector part,

$$A^\mu = B^\mu + \alpha^{\mu\rho} \partial_\rho \varphi, \quad (22)$$

where  $B_\mu$  is the new quantum variable and  $\alpha^{\mu\rho}$  is an unknown function of background fields. All indices are lowered and raised with the metric  $g_{\alpha\beta}$  and its inverse  $g^{\alpha\beta}$ . Using the transformation (22) in the action (10), we arrive at

$$\begin{aligned} \mathcal{L}' + \mathcal{L}_{gf} = & \frac{1}{2} B_\mu \hat{H}_{AA} B_\nu + \frac{1}{2} \varphi \hat{H}_{\varphi\varphi} \varphi \\ & + \frac{1}{2} \alpha_{\mu\rho} \nabla^\rho \varphi \hat{H}_{AA} \alpha_{\nu\omega} \nabla^\omega \varphi + \frac{1}{m} \alpha_{\nu\rho} \nabla^\rho \varphi X^{\mu\nu} \nabla_\mu \varphi \\ & + B_\mu (H_{AA})^{\mu\nu} \alpha_{\nu\omega} \nabla^\omega \varphi + \frac{1}{m} B_\mu X^{\mu\nu} \nabla_\nu \varphi. \end{aligned} \quad (23)$$

The condition of diagonalization is

$$B_\mu \left[ (H_{AA})^{\mu\nu} \alpha_{\nu\omega} + \frac{1}{m} X_\omega^\mu \right] \nabla^\omega \varphi = 0, \quad (24)$$

which has the nonlocal solution

$$\begin{aligned} \alpha_{\mu\nu} = & -\frac{1}{m} (H_{AA})_{\mu\rho}^{-1} X_\nu^\rho \\ = & -\frac{1}{m} (g^{\alpha\beta} \square - m^2 g^{\alpha\beta} - R^{\alpha\beta} - X^{\alpha\beta})_{\mu\rho}^{-1} X_\nu^\rho. \end{aligned} \quad (25)$$

Let us stress that the Green function in (25) acts only on the field  $X_\nu^\rho$  and not on the quantum and background fields on the right of this expression. Replacing the solution (25) in the Lagrangian (23) we arrive at the expression

$$\begin{aligned} \mathcal{L}' + \mathcal{L}_{gf} = & \frac{1}{2} B_\mu \hat{H}_{AA} B_\nu + \frac{1}{2} \varphi \hat{H}_{\varphi\varphi} \varphi \\ & + \frac{1}{2m} \alpha_{\mu\rho} X^{\mu\nu} \nabla^\rho \varphi \nabla_\nu \varphi, \end{aligned} \quad (26)$$

where  $\hat{H}_{AA}$  and  $\hat{H}_{\varphi\varphi}$  were defined in Eqs. (13) and (17), correspondingly.

The new Lagrangian, Eq. (26), is diagonal, but it is still nonminimal and contains a new nonlocal term in the scalar field sector, with tensor  $X^{\mu\nu}$  being contracted with covariant derivatives. Once again we arrive at the situation when the

standard way of dealing with nonminimal operators [15] is not applicable, because the nonminimality cannot be parametrized by a single real parameter, or even by a finite set of such parameters.

Anyway, the result of the second approach to diagonalization has a serious advantage compared to our first attempt. In the present case the problem corresponds only to the scalar sector, while the form of the vector operator is pretty standard. For instance, the expression (26) is a useful starting point for making calculations perturbatively in  $X^{\mu\nu}$ . At the same time one can do much more than this. In the next section, we describe a new efficient method of dealing with the nonminimal scalar operator.

#### IV. DERIVATION OF ONE-LOOP DIVERGENCES

After applying the procedure described in the subsection III B, the total action of the model consists from the pretty standard minimal vector sector and the complicated nonminimal scalar action. The whole expression can be presented as follows:

$$S' + S_{gf} = S_1 - \int d^4x \sqrt{-g} \left\{ \frac{1}{2} (g^{\mu\nu} + Y^{\mu\nu}) \partial_\mu \varphi \partial_\nu \varphi + \frac{1}{2} m^2 \varphi^2 \right\}, \quad (27)$$

where

$$S_1 = \frac{1}{2} \int d^4x \sqrt{-g} B_\mu (\delta_\nu^\mu \square - R_\nu^\mu - X_\nu^\mu - \delta_\nu^\mu m^2) B^\nu \quad (28)$$

and

$$Y^{\mu\nu} = \frac{1}{m^2} X^{\mu\nu} - \frac{1}{m} \alpha^{\mu\rho} X_\rho^\nu. \quad (29)$$

In order to work out the scalar operator, let us define a new metric,

$$G^{\mu\nu} = g^{\mu\nu} + Y^{\mu\nu}. \quad (30)$$

After that the action (27) becomes

$$S' + S_{gf} = S_1 - \frac{1}{2} \int d^4x \sqrt{-G} f (G^{\mu\nu} D_\mu \varphi D_\nu \varphi + m^2 \varphi^2), \quad (31)$$

where

$$f = \sqrt{\frac{\det G^{\mu\nu}}{\det g^{\mu\nu}}} \quad (32)$$

is the new background scalar field and  $D_\mu \varphi = \partial_\mu \varphi$  is the covariant derivative constructed with the new metric  $G_{\mu\nu}$ ,

defined as an inverse to  $G^{\mu\nu}$  in Eq. (30). In the second term in (31) and related calculations, the indices are lowered and raised with the new metric and with its inverse. It proves useful to introduce the corresponding affine connection,

$$\Upsilon_{\alpha\beta}^\tau = \frac{1}{2} G^{\tau\lambda} (\partial_\alpha G_{\lambda\beta} + \partial_\beta G_{\alpha\lambda} - \partial_\lambda G_{\alpha\beta}), \quad (33)$$

covariant derivative  $D_\mu$ , the curvature tensor

$$[D_\mu, D_\nu] A^\alpha = K_{\beta\mu\nu}^\alpha A^\beta \quad (34)$$

and its contractions  $K_{\alpha\beta} = G^{\mu\nu} K_{\mu\alpha\nu\beta}$  and  $K = G^{\alpha\beta} K_{\alpha\beta}$ . These new curvatures differ from the usual Riemann, Ricci tensors and scalar curvature  $R$  by the terms of first and higher orders in the field  $X_{\mu\nu}$  and in the nonlocal  $X_{\mu\nu}$ -dependent expression  $\alpha_{\mu\nu}$ , defined in Eq. (25). Some useful formulas for the first order expansions can be found in the Appendix.

Starting from this point the derivation of one-loop divergences becomes pretty much standard. The divergences of the one-loop effective action are given by the expression

$$\begin{aligned} \Gamma_{\text{div}}^{(1)} &= \frac{i}{2} \text{Tr} \ln \hat{\mathbf{H}}|_{\text{div}} - i \text{Tr} \ln \hat{H}_{gh}|_{\text{div}} \\ &= \frac{i}{2} \text{Tr} \ln (\delta_\nu^\mu \square - R_\nu^\mu - X_\nu^\mu - \delta_\nu^\mu m^2)|_{\text{div}} \\ &\quad - i \text{Tr} \ln (\square - m^2)|_{\text{div}} \\ &\quad + \frac{i}{2} \text{Tr} \ln (D^2 + 2\hat{h}^\mu D_\mu - m^2)|_{\text{div}}, \end{aligned} \quad (35)$$

where

$$\hat{h}_\mu = \frac{1}{2} D_\mu (\ln f) = \frac{1}{2} \partial_\mu (\ln f) \quad \text{and} \quad D^2 = G^{\mu\nu} D_\mu D_\nu. \quad (36)$$

Each of the terms in Eq. (35) can be calculated separately by means of the standard Schwinger-DeWitt technique [20]. For the first term, one can obtain, in dimensional regularization,

$$\begin{aligned} &\frac{i}{2} \text{Tr} \ln (\delta_\nu^\mu \square - R_\nu^\mu - X_\nu^\mu - \delta_\nu^\mu m^2)|_{\text{div}} \\ &= -\frac{\mu^{n-4}}{\varepsilon} \int d^n x \sqrt{-g} \text{tr} \left[ \frac{\hat{1}}{180} (R_{\mu\nu\alpha\beta}^2 - R_{\alpha\beta}^2 + \square R) \right. \\ &\quad \left. + \frac{1}{2} \hat{P}_1^2 + \frac{1}{6} \square \hat{P}_1 + \frac{1}{12} \hat{R}_{\alpha\beta}^2 \right], \end{aligned} \quad (37)$$

where  $\varepsilon = (4\pi)^2 (n-4)$  is the dimensional regularization parameter,  $\mu$  is the dimensional parameter of renormalization, and

$$\hat{1} = \delta_\nu^\mu, \quad \hat{P}_1 = (P_1)_\nu^\mu = -R_\nu^\mu + \frac{1}{6}\delta_\nu^\mu R - m^2\delta_\nu^\mu - X_\nu^\mu, \\ \hat{\mathcal{R}}_{\alpha\beta} = (\mathcal{R}_{\alpha\beta})_\nu^\mu = R_{\nu\alpha\beta}^\mu. \quad (38)$$

Taking into account the Faddeev-Popov ghost term, after a small algebra we find

$$\frac{i}{2}\text{Tr}\ln(\delta_\nu^\mu\Box - R_\nu^\mu - X_\nu^\mu - \delta_\nu^\mu m^2)|_{\text{div}} - i\text{Tr}\ln(\Box - m^2)|_{\text{div}} \\ = -\frac{\mu^{n-4}}{\varepsilon}\int d^4x\sqrt{-g}\left\{-\frac{13}{180}R_{\alpha\beta\mu\nu}^2 + \frac{22}{45}R_{\alpha\beta}^2 \right. \\ \left. - \frac{5}{36}R^2 - \frac{1}{10}\Box R + \frac{2}{3}m^2R + m^4 + m^2X \right. \\ \left. - \frac{1}{6}XR + X_{\alpha\beta}R^{\alpha\beta} + \frac{1}{2}X_{\alpha\beta}^2 - \frac{1}{6}\Box X\right\}, \quad (39)$$

where  $X = g^{\alpha\beta}X_{\alpha\beta}$ . It is important to remember that the indices here are lowered and raised by the normal space-time metric  $g_{\mu\nu}$  and its inverse.

In the scalar sector, we obtain

$$\frac{i}{2}\text{Tr}\ln(D^2 + 2\hat{h}^\mu D_\mu - m^2)|_{\text{div}} \\ = -\frac{\mu^{n-4}}{\varepsilon}\int d^4x\sqrt{-G}\left[\frac{1}{180}(K_{\mu\nu\alpha\beta}^2 - K_{\alpha\beta}^2 + D^2K) \right. \\ \left. + \frac{1}{2}\hat{P}_0^2 + \frac{1}{6}D^2\hat{P}_0\right], \quad (40)$$

where

$$\hat{P}_0 = -m^2 + \frac{1}{6}K - D_\mu\hat{h}^\mu - \hat{h}_\mu\hat{h}^\mu \quad (41)$$

and

$$\frac{1}{2}\hat{P}_0^2 = \frac{1}{2}m^4 - \frac{1}{6}m^2K + \frac{1}{72}K^2 - m^2F + \frac{1}{6}KF + \frac{1}{2}F^2. \quad (42)$$

In the last expression were introduced the useful notation

$$F = \frac{1}{4f^2}(Df)^2 - \frac{1}{2f}D^2f = -\frac{1}{\sqrt{f}}(D^2\sqrt{f}), \quad (43)$$

where

$$(Df)^2 = G^{\mu\nu}D_\mu f D_\nu f, \quad D^2f = G^{\mu\nu}D_\mu D_\nu f.$$

Thus, the unconventional scalar sector contribution can be written in the form

$$\frac{i}{2}\text{Tr}\ln(D^2 + 2\hat{h}^\mu D_\mu - m^2)|_{\text{div}} \\ = -\frac{\mu^{n-4}}{\varepsilon}\int d^4x\sqrt{-G}\left\{\frac{1}{180}K_{\alpha\beta\mu\nu}^2 - \frac{1}{180}K_{\alpha\beta}^2 \right. \\ \left. + \frac{1}{30}D^2K + \frac{1}{72}K^2 - \frac{1}{6}m^2K + \frac{1}{2}m^4 \right. \\ \left. + \frac{1}{6}KF - m^2F + \frac{1}{2}F^2 + \frac{1}{6}D^2F\right\}. \quad (44)$$

Finally, using the intermediate results (35), (39), and (44), we arrive at the final result for the one-loop divergences in the theory (1),

$$\Gamma_{\text{div}}^{(1)} = -\frac{\mu^{n-4}}{\varepsilon}\int d^4x\sqrt{-g}\left\{-\frac{13}{180}R_{\alpha\beta\mu\nu}^2 + \frac{22}{45}R_{\alpha\beta}^2 \right. \\ \left. - \frac{5}{36}R^2 - \frac{1}{10}\Box R + \frac{2}{3}m^2R + m^4 + m^2X \right. \\ \left. - \frac{1}{6}XR + X_{\alpha\beta}R^{\alpha\beta} + \frac{1}{2}X_{\alpha\beta}^2 - \frac{1}{6}\Box X\right\} \\ - \frac{\mu^{n-4}}{\varepsilon}\int d^4x\sqrt{-G}\left\{\frac{1}{180}K_{\alpha\beta\mu\nu}^2 - \frac{1}{180}K_{\alpha\beta}^2 \right. \\ \left. + \frac{1}{72}K^2 + \frac{1}{30}D^2K - \frac{1}{6}m^2K + \frac{1}{2}m^4 + \frac{1}{6}KF \right. \\ \left. - m^2F + \frac{1}{2}F^2 + \frac{1}{6}D^2F\right\}. \quad (45)$$

The expression (45) is the result of a nonstandard calculational procedure, which involves the change of quantum variables (22) with the nonlocal coefficient defined in (25). It is important to remember that the quantum field theory calculation (Schwinger-DeWitt technique, in our case) is applied, in the scalar sector, to the theory with the background metric  $G^{\mu\nu}$ . In terms of this new metric, the second part of the expression (45) has a rather standard local form. At the same the divergences are given by a nonlocal expression in terms of the original fields  $g_{\mu\nu}$  and  $X_{\mu\nu}$ .

Equation (45) enables one to obtain the one-loop divergences in terms of the original metric  $g^{\mu\nu}$  and in each desired order in  $X^{\mu\nu}$ . It proves useful to obtain an explicit expression in the first order in  $X^{\mu\nu}$ . To this end, we write down the following first-order expansions:

$$G_{\mu\nu} = g_{\mu\nu} - Y_{\mu\nu} + \dots \\ \sqrt{-\det G^{\mu\nu}} = \sqrt{-\det g^{\mu\nu}}\left(1 + \frac{1}{2}Y + \dots\right), \quad (46)$$

where  $Y = g^{\mu\nu}Y_{\mu\nu}$ . Then, for the determinant of the inverse matrix  $G = \det(G_{\mu\nu})$ ,

$$\sqrt{-G} = \sqrt{-g} \left( 1 - \frac{1}{2} Y + \dots \right)$$

$$f = 1 + \frac{1}{2} Y + \dots, \quad F = -\frac{1}{4} \square Y + \dots \quad (47)$$

$$K_{\alpha\beta\mu\nu} = R_{\alpha\beta\mu\nu} + \frac{1}{2} (\nabla_\mu \nabla_\alpha Y_{\beta\nu} - \nabla_\nu \nabla_\alpha Y_{\beta\mu} + \nabla_\nu \nabla_\beta Y_{\alpha\mu} - \nabla_\mu \nabla_\beta Y_{\alpha\nu} + R_{\alpha\mu\nu}^\rho Y_{\rho\beta} - R_{\beta\mu\nu}^\rho Y_{\rho\alpha}) + \dots \quad (48)$$

$$K_{\alpha\beta} = R_{\alpha\beta} + \frac{1}{2} (\square Y_{\alpha\beta} + \nabla_\alpha \nabla_\beta Y - \nabla_\rho \nabla_\alpha Y_\beta^\rho - \nabla_\rho \nabla_\beta Y_\alpha^\rho) + \dots \quad (49)$$

$$K = R + R_{\alpha\beta} Y^{\alpha\beta} + \square Y - \nabla_\alpha \nabla_\beta Y^{\alpha\beta} + \dots \quad (50)$$

$$D^2 = \square + Y^{\alpha\beta} \nabla_\alpha \nabla_\beta + (\nabla_\alpha Y^{\alpha\beta}) \nabla_\beta - \frac{1}{2} (\nabla^\alpha Y) \nabla_\alpha + \dots, \quad (51)$$

where  $D^2$  acts on a scalar field, and

$$Y_{\alpha\beta} = \frac{1}{m^2} X_{\alpha\beta} + \dots \quad (52)$$

Using these expansions, we find the one-loop divergences in the first order in  $X^{\mu\nu}$ , written in terms of the original metric  $g^{\mu\nu}$ ,

$$\Gamma_{\text{div}}^{(1)} = \Gamma_{\text{vac}}^{(1)}[g_{\mu\nu}] - \frac{\mu^{n-4}}{\varepsilon} \int d^4 x \sqrt{-g} \left\{ \frac{3}{4} m^2 X + \frac{5}{6} X_{\mu\nu} R^{\mu\nu} - \frac{1}{12} X R + \frac{X^{\mu\nu}}{180 m^2} (5 R R_{\mu\nu} + 2 R^{\alpha\beta} R_{\alpha\mu\beta\nu} + 2 R_{\alpha\beta\rho\mu} R^{\alpha\beta\rho} - 4 R_{\mu\alpha} R_\nu^\alpha + 3 \square R_{\mu\nu} - 6 \nabla_\mu \nabla_\nu R) - \frac{X}{720 m^2} (2 R_{\mu\nu}^2 + 2 R_{\alpha\beta}^2 + 5 R^2 + 12 \square R) \right\}, \quad (53)$$

where

$$\Gamma_{\text{vac}}^{(1)}[g_{\mu\nu}] = -\frac{\mu^{n-4}}{\varepsilon} \int d^4 x \sqrt{-g} \left\{ \frac{29}{60} R_{\alpha\beta}^2 \right. \quad (54)$$

$$\left. - \frac{1}{15} R_{\alpha\beta\mu\nu}^2 - \frac{1}{8} R^2 + \frac{m^2}{2} R + \frac{3m^4}{2} \right\} \quad (55)$$

is the contribution of Proca field, which depends only on external metric and not on  $X^{\mu\nu}$ . In the case of  $X_{\mu\nu} = 0$ , we come back to the well-known result derived in [15,16,21] by means of different methods. In the formula (53), we omitted the total derivative terms for the sake of brevity. As

one should expect, the linear result is local in terms of the original fields  $g_{\mu\nu}$  and  $X_{\mu\nu}$ .

## V. CALCULATION USING UNIVERSAL TRACES

As we have already mentioned above, the main expression (45) is the result of a nonstandard calculational procedure, hence it would be useful to have its verification. The first order in  $X_{\mu\nu}$  divergences can be obtained independently by means of the universal functional traces method (generalized Schwinger-DeWitt technique) of Barvinsky and Vilkovisky [15]. In this section, we shall perform such a calculation in order to compare the result with Eq. (53), and thus partially verify the main result (45). The calculation described below is analogous to the one developed in Ref. [22] for the complicated gauge model with broken Lorentz and *CPT* background. For this reason, we can skip some technical explanations, which can be found in this reference.

Before the diagonalization procedure, the bilinear form (12) can be written as

$$\hat{\mathbf{H}} = \hat{\mathbf{H}}_m + \hat{\mathbf{H}}_{nm}, \quad (56)$$

where

$$\hat{\mathbf{H}}_m = \hat{\mathbf{I}} \square + 2 \hat{\mathbf{L}}^\mu \nabla_\mu + (\hat{\mathbf{H}}_0 + \hat{\mathbf{M}}) \quad (57)$$

is the minimal part of bilinear operator in quantum fields and

$$\hat{\mathbf{H}}_{nm} = \hat{\mathbf{K}}^{\mu\nu} \nabla_\mu \nabla_\nu \quad (58)$$

is the nonminimal part. The relevant matrices are defined as follows:

$$\hat{\mathbf{I}} = \begin{pmatrix} \delta_\nu^\mu & 0 \\ 0 & 1 \end{pmatrix},$$

$$\hat{\mathbf{L}}^\mu = \begin{pmatrix} 0 & \frac{1}{2m} X^{\mu\nu} \\ -\frac{1}{2m} X^{\mu\nu} & \frac{1}{2m^2} \nabla_\nu X^{\mu\nu} \end{pmatrix},$$

$$\hat{\mathbf{H}}_0 = \begin{pmatrix} -\delta_\nu^\mu m^2 - R_\nu^\mu & 0 \\ 0 & -m^2 \end{pmatrix},$$

$$\hat{\mathbf{M}} = \begin{pmatrix} -X_\nu^\mu & 0 \\ -\frac{1}{m} \nabla_\nu X^{\mu\nu} & 0 \end{pmatrix},$$

$$\hat{\mathbf{K}}^{\mu\nu} = \begin{pmatrix} 0 & 0 \\ 0 & \frac{1}{m^2} X^{\mu\nu} \end{pmatrix}. \quad (59)$$

The one-loop effective action is given by the known formula

$$\Gamma^{(1)} = \frac{i}{2} \text{Tr} \ln (\hat{\mathbf{H}}_m + \hat{\mathbf{H}}_{nm}) - i \text{Tr} \ln \hat{H}_{gh}. \quad (60)$$

Let us make the following transformation:

$$\begin{aligned}\text{Tr ln } \hat{\mathbf{H}} &= \text{Tr ln}(\hat{\mathbf{H}}_m + \hat{\mathbf{H}}_{nm}) \\ &= \text{Tr ln } \hat{\mathbf{H}}_m + \text{Tr ln}(\hat{\mathbf{1}} + \hat{\mathbf{H}}_m^{-1} \cdot \hat{\mathbf{H}}_{nm}) \\ &= \text{Tr ln } \hat{\mathbf{H}}_m + \text{Tr} \hat{H}_{nm} \cdot \hat{H}_0^{-1} + \dots,\end{aligned}\quad (61)$$

where dots stand for the terms of higher orders in  $X^{\mu\nu}$ ,  $\hat{H}_0 = \square - m^2$  and

$$\hat{H}_{nm} = \frac{1}{m^2} X^{\mu\nu} \nabla_\mu \nabla_\nu. \quad (62)$$

In the last line of Eq. (61) we perform the expansion of the logarithm and take into account only terms of the first order in  $X^{\mu\nu}$ . The first term in the last line of Eq. (61) can be directly calculated by the standard Schwinger-DeWitt method [20], while the second term can be calculated by means of the universal functional traces method [15].

For the minimal part it is possible to obtain the one-loop divergences by using the known formula of the Schwinger-DeWitt technique,

$$\begin{aligned}\frac{i}{2} \text{Tr ln } \hat{\mathbf{H}}_m|_{\text{div}} &= -\frac{\mu^{n-4}}{\varepsilon} \int d^4x \sqrt{-g} \text{tr} \left[ \frac{\hat{\mathbf{1}}}{180} (R^2_{\mu\nu\alpha\beta} - R^2_{\alpha\beta}) \right. \\ &\quad \left. + \frac{1}{2} \hat{\mathbf{P}}^2 + \frac{1}{12} \hat{\mathbf{S}}^2_{\alpha\beta} \right],\end{aligned}\quad (63)$$

where

$$\hat{\mathbf{P}} = \hat{\mathbf{P}}_0 + \hat{\mathbf{M}} - \nabla_\mu \hat{\mathbf{L}}^\mu + \dots, \quad (64)$$

$$\hat{\mathbf{S}}_{\alpha\beta} = \hat{\mathbf{R}}_{\alpha\beta} - \nabla_\alpha \hat{\mathbf{L}}_\beta + \nabla_\beta \hat{\mathbf{L}}_\alpha + \dots, \quad (65)$$

with

$$\hat{\mathbf{P}}_0 = \hat{\mathbf{\Pi}}_0 + \frac{1}{6} \hat{\mathbf{1}} R \quad \text{and} \quad \hat{\mathbf{R}}_{\alpha\beta} = \hat{\mathbf{1}} [\nabla_\alpha, \nabla_\beta]. \quad (66)$$

Up to the first order in the new parameters, we obtain

$$\begin{aligned}\frac{1}{2} \text{tr} \hat{\mathbf{P}}^2 &= \frac{1}{2} \text{tr} \hat{\mathbf{P}}_0^2 + \text{tr} \hat{\mathbf{P}}_0 \hat{\mathbf{M}} - \text{tr} \hat{\mathbf{P}}_0 \nabla_\mu \hat{\mathbf{L}}^\mu + \dots, \\ \text{tr} \hat{\mathbf{S}}^2_{\alpha\beta} &= \text{tr} (\hat{\mathbf{R}}^2_{\alpha\beta} + 4 \hat{\mathbf{R}}^{\alpha\beta} \nabla_\beta \hat{\mathbf{L}}_\alpha) + \dots,\end{aligned}\quad (67)$$

while Eq. (63) with the Faddeev-Popov ghost term reduces to

$$\begin{aligned}\frac{i}{2} \text{Tr ln } \hat{\mathbf{H}}_m|_{\text{div}} - i \text{Tr ln } \hat{H}_{gh}|_{\text{div}} \\ = \Gamma_{\text{vac}}^{(1)}[g_{\mu\nu}] - \frac{\mu^{n-4}}{\varepsilon} \int d^4x \sqrt{-g} \text{tr} \left[ \hat{\mathbf{P}}_0 \hat{\mathbf{M}} - \hat{\mathbf{P}}_0 \nabla_\mu \hat{\mathbf{L}}^\mu \right. \\ \left. + \frac{1}{3} \hat{\mathbf{R}}^{\alpha\beta} \nabla_\beta \hat{\mathbf{L}}_\alpha \right] + \dots,\end{aligned}\quad (68)$$

where  $\Gamma_{\text{vac}}^{(1)}[g_{\mu\nu}]$  was defined in Eq. (55). After performing some algebra, one can find

$$\begin{aligned}\text{tr} \hat{\mathbf{P}}_0 \hat{\mathbf{M}} &= m^2 X + X_{\alpha\beta} R^{\alpha\beta} - \frac{1}{6} XR + \dots, \\ \text{tr} \hat{\mathbf{P}}_0 \nabla_\mu \hat{\mathbf{L}}^\mu &= \frac{1}{12m^2} X^{\alpha\beta} \nabla_\alpha \nabla_\beta R + \dots, \\ \text{tr} \hat{\mathbf{R}}^{\alpha\beta} \nabla_\beta \hat{\mathbf{L}}_\alpha &= 0 + \dots.\end{aligned}\quad (69)$$

So, finally

$$\begin{aligned}\frac{i}{2} \text{Tr ln } \hat{\mathbf{H}}_m|_{\text{div}} - i \text{Tr ln } \hat{H}_{gh}|_{\text{div}} \\ = -\frac{\mu^{n-4}}{\varepsilon} \int d^4x \sqrt{-g} \left[ m^2 X + X_{\alpha\beta} R^{\alpha\beta} \right. \\ \left. - \frac{1}{12m^2} X^{\alpha\beta} \nabla_\alpha \nabla_\beta R - \frac{1}{6} XR \right] + \Gamma_{\text{vac}}^{(1)}[g_{\mu\nu}] + \dots.\end{aligned}\quad (70)$$

For calculating the divergent part of the nonminimal piece of Eq. (61), we need the inverse of the operator  $\hat{H}_0$ , which can be expressed as

$$\hat{H}_0^{-1} = \frac{1}{\square} + m^2 \frac{1}{\square^2} + m^4 \frac{1}{\square^3} + \mathcal{O}(l^{-5}). \quad (71)$$

In the last formula,  $1/\square$  is the inverse of the d'Alembert operator and the last term  $\mathcal{O}(l^{-5})$  indicates omitted irrelevant terms of a higher background dimension [15].

Using equation (62), one can obtain the relation

$$\begin{aligned}\text{Tr} \hat{H}_{nm} \cdot \hat{H}_0^{-1} &= X^{\mu\nu} \left( \frac{1}{m^2} \nabla_\mu \nabla_\nu \frac{1}{\square} + \nabla_\mu \nabla_\nu \frac{1}{\square^2} + m^2 \nabla_\mu \nabla_\nu \frac{1}{\square^3} \right) \\ &\quad + \mathcal{O}(l^{-5}).\end{aligned}\quad (72)$$

Equation (72) is already in the form that allows us to apply the tables of universal functional traces of [15]. The calculation is straightforward (albeit not really easy), and the results have the form (note that we use Minkowski signature)



$$\begin{aligned}
 \left. \frac{1}{m^2} X^{\mu\nu} \nabla_\mu \nabla_\nu \frac{1}{\square} \right|_{\text{div}} &= \frac{i\mu^{n-4}}{\varepsilon} \int d^4x \sqrt{-g} \left\{ \frac{1}{m^2} X^{\mu\nu} \left( \frac{1}{45} R_{\alpha\beta\rho\mu} R^{\alpha\beta\rho\nu} + \frac{1}{45} R^{\alpha\beta} R_{\alpha\mu\beta\nu} - \frac{2}{45} R_{\mu\alpha} R_\nu^\alpha + \frac{1}{18} R R_{\mu\nu} \right. \right. \\
 &\quad \left. \left. + \frac{1}{30} \square R_{\mu\nu} + \frac{1}{10} \nabla_\mu \nabla_\nu R \right) - \frac{1}{m^2} X \left( \frac{1}{180} R_{\mu\nu\alpha\beta}^2 - \frac{1}{180} R_{\alpha\beta}^2 + \frac{1}{72} R^2 + \frac{1}{30} \square R \right) \right\}, \\
 X^{\mu\nu} \nabla_\mu \nabla_\nu \frac{1}{\square^2} \Big|_{\text{div}} &= \frac{i\mu^{n-4}}{\varepsilon} \int d^4x \sqrt{-g} \left\{ \frac{1}{6} X R - \frac{1}{3} X_{\alpha\beta} R^{\alpha\beta} \right\}, \\
 m^2 X^{\mu\nu} \nabla_\mu \nabla_\nu \frac{1}{\square^2} \Big|_{\text{div}} &= -\frac{i\mu^{n-4}}{2\varepsilon} \int d^4x \sqrt{-g} m^2 X.
 \end{aligned} \tag{73}$$

By using relations (72) and (74), one can obtain

$$\begin{aligned}
 \left. \frac{i}{2} \text{Tr} \hat{H}_{nm} \cdot \hat{H}_0^{-1} \right|_{\text{div}} &= -\frac{\mu^{n-4}}{\varepsilon} \int d^4x \sqrt{-g} \left\{ \frac{1}{12} X R - \frac{1}{6} X_{\mu\nu} R^{\mu\nu} - \frac{1}{4} m^2 X \right. \\
 &\quad \left. + \frac{1}{m^2} X^{\mu\nu} \left( \frac{1}{90} R_{\alpha\beta\rho\mu} R^{\alpha\beta\rho\nu} + \frac{1}{90} R^{\alpha\beta} R_{\alpha\mu\beta\nu} - \frac{1}{45} R_{\mu\alpha} R_\nu^\alpha + \frac{1}{36} R R_{\mu\nu} + \frac{1}{60} \square R_{\mu\nu} + \frac{1}{20} \nabla_\mu \nabla_\nu R \right) \right. \\
 &\quad \left. - \frac{1}{2m^2} X \left( \frac{1}{180} R_{\mu\nu\alpha\beta}^2 - \frac{1}{180} R_{\alpha\beta}^2 + \frac{1}{72} R^2 + \frac{1}{30} \square R \right) \right\}.
 \end{aligned} \tag{74}$$

Now, from equations (60), (61), (70), and (74), it is not difficult to verify that we arrive exactly at the result for the one-loop divergences derived by the new method, Eq. (53).

## VI. CONCLUSIONS AND DISCUSSIONS

Let us summarize the results. We have developed the generic procedure for consistent formulation of the effective action for the free massive Abelian vector in curved spacetime with nonminimal coupling to an arbitrary symmetric background tensor  $X_{\mu\nu}$ . The result of the calculation for the divergent part of the effective action has the form (45). The remarkable feature of this expression is that it has a standard appearance of a local expression when written in terms of the quantities  $K$ ,  $D$ , and  $F$ , and at the same time, it looks like a rather unusual nonlocal functional when expressed in terms of the original external field  $g_{\mu\nu}$  and  $X_{\mu\nu}$ . The reason for this is the nonlocal change of variable which has been performed in the course of diagonalization of the bilinear form of the action. Nevertheless, this effective action can be expanded in the functional power series in  $X_{\mu\nu}$ . Then we will obtain an infinite number of local terms containing any powers of  $X_{\mu\nu}$  with any number of covariant derivatives action on  $X_{\mu\nu}$ . The number of derivatives will increase with the power of expansion.

It would be quite interesting to explore some extensions and modifications of this result. First of all, it would be interesting to formulate a similar scheme for constructing the effective action and calculation of one-loop divergences for the massless version of the theory, which has an important application to astrophysics [9]. The development of this model would open the way for exploring the

self-interacting massless vector field, with the possibility to analyze the dynamical symmetry breaking in the vector model.

An obvious possible extension of our model is related to introducing the interaction to fermions and other components of the standard model of particle physics. Such an extension would be especially useful for the applications to inflation, because this would opens the gate to explore particle creation on the vector inflation.

On the other hand, it would be certainly interesting to add a self-interaction term, but this is not a simple task. For instance, the most natural form  $(A_\mu A^\mu)^2$  may be quite complicated to deal with, as it was discussed in [13] in relation to the model of conformal invariant axial vector (antisymmetric torsion) field. The theory with such a term, without  $X^{\mu\nu}$  and mass, possesses local conformal invariance, has soft breaking of gauge invariance, and was never explored at both the classical and quantum levels. The massive version looks more accessible and will hopefully be explored in the next publications. We leave this problem for the future, expecting that the technical progress achieved in the present work will be useful then.

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## APPENDIX: FIRST-ORDER RELATIONS FOR CURVATURES

Here we present the useful expansion formulas for the terms in Eq. (53), which are of the fourth order in the extended covariant derivative  $D_\mu$ ,

$$\begin{aligned} K_{\alpha\beta\mu\nu}^2 &= R_{\alpha\beta\mu\nu}^2 + 2R_{\alpha\beta\rho\mu}R^{\alpha\beta\rho}Y^{\mu\nu} + 4R^{\alpha\beta\mu\nu}\nabla_\mu\nabla_\alpha Y_{\beta\nu} + \dots \\ &= R_{\alpha\beta\mu\nu}^2 + 4Y^{\alpha\beta}\square R_{\alpha\beta} - 2Y^{\alpha\beta}\nabla_\alpha\nabla_\beta R + 2R_{\alpha\beta\rho\mu}R^{\alpha\beta\rho}Y^{\mu\nu} + 4R^{\alpha\beta}R_{\alpha\mu\beta\nu}Y^{\mu\nu} \\ &\quad - 4R_{\alpha\beta}R_\mu^\beta Y^{\mu\alpha} + \text{total derivatives}, \end{aligned} \quad (\text{A1})$$

$$\begin{aligned} K_{\alpha\beta}^2 &= R_{\alpha\beta}^2 + R_{\alpha\beta}(2R_\mu^\beta Y^{\mu\alpha} + \square Y^{\alpha\beta} + \nabla^\alpha\nabla^\beta Y - 2\nabla_\rho\nabla^\alpha Y^{\rho\beta}) + \dots \\ &= R_{\alpha\beta}^2 + Y^{\alpha\beta}\square R_{\alpha\beta} + \frac{1}{2}Y\square R - Y^{\alpha\beta}\nabla_\alpha\nabla_\beta R + 2R^{\alpha\beta}R_{\alpha\mu\beta\nu}Y^{\mu\nu} + \text{total derivatives}, \end{aligned} \quad (\text{A2})$$

$$K^2 = R^2 + 2RR_{\alpha\beta}Y^{\alpha\beta} + 2Y\square R - 2Y^{\alpha\beta}\nabla_\alpha\nabla_\beta R + \text{total derivatives}, \quad (\text{A3})$$

$$\begin{aligned} D^2 K &= Y^{\alpha\beta}\nabla_\alpha\nabla_\beta R + (\nabla_\alpha Y^{\alpha\beta})\nabla_\beta R - \frac{1}{2}(\nabla^\alpha Y)\nabla_\alpha R + \text{total derivatives} \\ &= \frac{1}{2}Y\square R + \text{total derivatives}. \end{aligned} \quad (\text{A4})$$

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