PHYSICAL REVIEW D **95,** 084035 (2017)

Recovering a redshift-extended varying speed of light signal from galaxy surveys

Vincenzo Salzano[*]

Institute of Physics, University of Szczecin, Wielkopolska 15, 70-451 Szczecin, Poland

(Received 30 June 2016; revised manuscript received 28 February 2017; published 21 April 2017)

We investigate a new method to recover (if any) a possible varying speed of light (VSL) signal from cosmological data. It comes as an upgrade by Salzano, Dąbrowski, and Lazkoz [Phys. Rev. Lett.**114**, 101304 (2015); Phys. Rev. D **93**, 063521 (2016)], where it was argued that such a signal could be detected at a single redshift location only. Here, we show how it is possible to extract information on a VSL signal on an extended redshift range. We use mock cosmological data from future galaxy surveys (BOSS, DESI, *WFirst-2.4* and SKA): the sound horizon at decoupling imprinted in the clustering of galaxies (baryon acoustic oscillations) as an angular diameter distance, and the expansion rate derived from those galaxies recognized as cosmic chronometers. We find that, given the forecast sensitivities of such surveys, a $\sim 1\%$ VSL signal can be detected at 3σ confidence level in the redshift interval $z \in [0., 1.55]$. Smaller signals ($\sim 0.1\%$) will be hardly detected (even if some lower possibility for a 1σ detection is still possible). Finally, we discuss the degeneration between a VSL signal and a non-null spatial curvature; we show that, given present bounds on curvature, any signal, if detected, can be attributed to a VSL signal with a very high confidence. On the other hand, our method turns out to be useful even in the classical scenario of a constant speed of light: in this case, the signal we reconstruct can be totally ascribed to spatial curvature and, thus, we might have a method to detect a 0.01-order curvature in the same redshift range with a very high confidence.

DOI: 10.1103/PhysRevD.95.084035

I. INTRODUCTION

The idea that fundamental constants of physics were not properly *constant*, but could instead vary with time (and possibly in space) is not a new one [1–6], but a fruitful revival has been possible only quite recently, stimulated by the progress achieved in observational cosmology (for a review, see [7]). On one side, the standard big bang scenario suffers some theoretical shortcomings, as the horizon and flatness problems, which are at the base of the introduction of cosmological inflation [8–13]. On the other, we have attended the discovery of the accelerated expansion of our Universe [14,15] and the detection of a possible variation of the fine structure constant from quasar absorption lines [16–34].

We have focused our attention on the possibility that the speed of light might change in time during the evolution of the Universe; such a scenario is generally called a varying speed of light (VSL) theory. A serious theoretical approach to define in the correct way a valid VSL theory is recent, and aimed exactly at solving horizon, flatness and the acceleration problems, in a "more natural" way, without relying on inflationary scenarios and the cosmological constant (the main successful candidate to lead Universe accelerated expansion, but also herald of many theoretical problems, see [35] for a review). The most exemplificative literature on this topic includes [36–51]. In the very own words of some of its pioneers, VSL foundations are still far from fixed, and a lot of debate is around them [52–54]. We want to stress here an important point, in order to make our work judged with the right perspective: we do not want to make any claim about the VSL theoretical background. This is not the purpose of this paper and will be postponed to future works. Here, instead, we will study whether, if there is a VSL signal and whatever the way it can be explained, it can be detected or not, by present or future observations.

In this context, recently, we have proposed a method to *measure* the speed of light *on cosmological scales* and *at relatively high redshift* [55,56] using observations from galaxy surveys. This method should overcome some of the criticisms related to the fact that the speed of light is, actually, a dimensional quantity: we can measure *here and now* such speed in the laboratory; we have *relocated* this laboratory in the outer Universe, where observations provide us a (cosmological) ruler and a (cosmological) clock, which both can be employed to measure the speed of light.

Given that we will use these rulers and clocks in this present work, we briefly review what they are. Both of them can be measured by a galaxy survey. The ruler is the sound horizon measured at late times as it is imprinted in the clustering of galaxies at cosmological scales or, equivalently, in the baryon acoustic oscillations (BAO) [57–71]. The sound horizon has some very important properties: it is generally considered as a standard ruler,

[*]enzo.salzano@wmf.univ.szczecin.pl

because its size in comoving coordinates is constant in time and thus can be used to calibrate/measure cosmological distances (for alternatives, see [72]); its length can be exactly calculated from theory [$\approx$150 Mpc in physical units; the best-precision value, measured by Planck, is $r_s(z_{\rm rec}) = 144.81 \pm 0.24$ Mpc for the baseline model [73]]. Due to the strong correlation between photons/radiation and gas in the early times (prior to the recombination epoch), by analyzing the galaxy correlation function now, it is possible to infer a correlation length which, expressed in comoving units, corresponds exactly to the sound horizon. Generally, from galaxy distribution we can measure the tangential and radial scales [74], respectively defined as

$$y_t(z) = \frac{D_A(z)}{r_s(z_{\rm rec})} \quad \text{and} \quad y_r(z) = \frac{c_0}{H(z) r_s(z_{\rm rec})}, \tag{1}$$

where c_0 is the speed of light (generally assumed constant); z is the cosmological redshift; D_A is the angular diameter distance; H is the Hubble function (expansion rate); and $r_s(z_{\rm dec})$ is the sound horizon, evaluated at recombination (or dragging epoch). Actually, with the present data we do not have a strong enough signal to measure the two directions separately, or, at least, not at the level of accuracy which should be theoretically possible [75–80]. This will eventually be possible with future surveys, when a larger number of galaxies is available; see, for example, forecast analysis for the Square Kilometer Array (SKA),[1] *Euclid*[2] [81–84], *WFIRST-2.4*[3] [85], the Baryon Oscillation Spectroscopic Survey (BOSS) [77,79,80,86], the extended BOSS survey (eBOSS) [87–89], the Dark Energy Spectroscopic Instrument (DESI)[4] [90] and the Hobby-Eberly Telescope Dark Energy Experiment (HETDEX).[5]

Galaxy surveys are beneficial for our purposes also because they can provide us the cosmological clocks we need for our method to be implemented: a sample of the observed galaxies can be targeted as *cosmic chronometers*. The key idea [91] is to find a "cosmological clock," which is able to give the variation of the age of the Universe with redshift. If one has this clock, then, one simply has to measure the age difference Δt between two redshifts separated by Δz, and calculate the derivative $dz/dt \approx \Delta z/\Delta t$. Then, this quantity can be directly related to the expansion rate (Hubble function), defined as

$$H(z) = -\frac{1}{1+z}\frac{dz}{dt}. \tag{2}$$

It is important to note here that Eq. (2) is derived from the more general and independent of the specific model form of the $a(t)$ definition of the expansion rate H,

$$H(t) = \frac{\dot{a}(t)}{a(t)}, \tag{3}$$

when the redshift-scale factor relation

$$1 + z = \frac{1}{a} \tag{4}$$

is assumed. But Eq. (4) holds in a homogeneous and isotropic universe described by a Friedmann-Robertson-Walker metric. But this choice is not the most general. For example, an inhomogeneous universe would lead to a different relation between the redshift and the scale factor as, for example, the model described in [92–95], or to a position-dependent expansion rate, as in the Lemaître-Tolman-Bondi model [96,97]. Thus, to properly use cosmic chronometers data, we will start from this same general assumption (and confirmed by data nowadays) for our cosmological background, i.e. homogeneity, isotropy and Friedmann-Robertson-Walker metric.

Actually, passively evolving early-type galaxies (ETG) turned out to be reliable candidates to play the role of such clocks. Since the first proposal [91], stellar population models have been improved; a much larger number of galaxies has been observed and collected, up to a redshift $z \sim 2$; and more precise tools to calibrate the clocks have been introduced (e.g., the 4000 Å break in ETG spectra) [98–103]. This scenario can still be improved using future galaxy surveys in the optical, as *Euclid* and *WFIRST-2.4*, which should observe at least 10 times more galaxies compared to the present (and ETG, eventually).

Our works [55,56] have been recently questioned by [104], where the authors point out two possible drawbacks of our method: first, that the speed of light is measured only at one single redshift z_M; second, that we ignore the spatial curvature contribution, which is degenerate with VSL. For what concerns the first point, it is true, but we were interested in the intrinsic novelty of the method. It is well known that the angular diameter distance has a maximum at some high redshift value, which we called *maximum redshift*, z_M; we found that at the maximum, the relation

$$D_A(z_M) \cdot H(z_M) = c(z_M) \tag{5}$$

holds, e.g. only at the maximum redshift, the combination of the angular diameter distance and the expansion rate is *exactly equal* to the value of the speed of light at that epoch, with a minimal number of theoretical assumptions on the cosmological background, and with no need of any information at all about how the speed of light should vary or not. It is a *direct* measurement, albeit local. About the second point, we have always been aware that, among the minimal number of theoretical assumptions we needed in order to derive Eq. (5), we have to assume a null spatial curvature. However, we have also discussed how a non-vanishing curvature may impact our results.

[1] https://www.skatelescope.org/.
[2] http://sci.esa.int/euclid/.
[3] http://wfirst.gsfc.nasa.gov/.
[4] http://desi.lbl.gov/.
[5] http://hetdex.org/.

Thus, in this present work, we move a step forward: using the same cosmological rulers and clocks from [55,56], we build a new method (different from [104]) which can be employed to recover a *redshift-extended* VSL signal (no more limited to z_M). We also report a detailed discussion about how a nonzero curvature can influence the application of such a method in a VSL theory context. Finally, we show how the method can be applied also in a classical context, where c_0 is constant in order to measure the spatial curvature itself.

In Sec. III we describe the theoretical apparatus at the base of our method; in Sec. IV we describe all the steps required for our method to be built in more detail; in Sec. V we discuss the results obtained from the application of our method to some mock data from future galaxy surveys; and, finally, in Sec. VI we sum up our results.

II. PRELIMINARIES ABOUT VSL THEORIES

As we have anticipated in the Introduction, VSL theories are quite debated today. Thus, it is noteworthy to explain further and state clearly some points. Most of the debate concerns breaking of the Lorentz invariance, which VSL theories intrinsically produce, and the correctness of discussing variation of dimensional quantities, and in particular c, which has been fixed to a well-defined and constant value in the international system of units (SI). While the former question has some possible reliable solutions [105], which can be freely debated theoretically, the latter one has to be set clearly.

Actually, this problem might be circumvented in two steps. On one hand, if we start from the beginning, i.e. from the Lagrangian, we can introduce the speed of light as a new scalar field, and take it into account properly in the derivation of all the cosmologically useful equations (as it is done in [36,51], for example). The fields always have units (dimensions). This is a general solution for all dimensional quantities.

On the other hand, we can fix a new system of units in which c can be safely considered varying. In order to avoid confusion and misunderstanding, we will define this new system as the varying speed of light unit system (VSL-US), to be compared with the standard SI. Moreover, we will make a clear distinction among *fundamental units*, and *experimental units*. We will show below that in the VSL-US, we can properly define the fundamental units so that c can be safely considered as a varying quantity, while the VSL-US experimental units are completely equivalent to the SI ones.

As pointed out in [106], there is no unique way to introduce a VSL theory, basically because different choices of units can lead to different varying quantities but the same effective theory. The theoretical approach we will follow in the next sections is derived from [38,42,45], which are based on the assumption that the quantity $Q \equiv \hbar/c$ is constant, together with the electron mass, m_e, and the electron charge, e. Given such constants, one can easily derive the corresponding unique set of constant *fundamental units* of mass (M), length (L) and time (T), namely, we can defined the VSL-US based on

$$
\begin{aligned}
u_M &= m_e,\\
u_L &= \frac{Q}{m_e},\\
u_T &= \frac{Q^{3/2}}{m_e e'},
\end{aligned}
\tag{6}
$$

where $e' = e/\sqrt{4\pi\epsilon_0}$, with ϵ_0 the vacuum permittivity.

It is important to notice that in this VSL-US, the length unit is no more entangled to the constancy of c; c can be varying, but the length unit is still constant. Thus, within the VSL-US, we can safely consider c as a varying quantity (and the variation of the dimensionless fine structure constant, α, will depend on it too). Note that these newly defined fundamental units u_X, in terms of the *experimental units* of the SI, would correspond to

$$
\begin{aligned}
u_M &\sim 10^{-31}\ \text{kg},\\
u_L &\sim 10^{-13}\ \text{m},\\
u_T &\sim 10^{-20}\ \text{s}.
\end{aligned}
\tag{7}
$$

Thus, at least in principle, one could redefine the experimental SI units (kg, m and s) in terms of the new VSL-US fundamental ones. More clearly: this means that we can still go on using, for example, the meter as a length unit, but, in the VSL-US, the meter is no more defined through the second and the (constant) speed of light, but in terms of u_M. It should be clear now why we have chosen to describe units as fundamental and experimental. The fundamental units definition can be completely free and dictated by theoretical requirements; experimental units are defined responding to some criteria of *reproducibility* and *practicality.*

At this point, it is not useless to show that such a way to proceed is by no means new, but quite customary, even though mostly implicit. For example, let us focus on the second, as it is defined now in the SI. From the same official page of the Bureau International des Poids et Mesures (BIPM), the second *is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom.*[6] Such a duration was officially defined in 1967/68, by matching the atomic clock measurement (from microscopic scales) with the ephemeris second (from macroscopic scales). In this case, the frequency of the atoms is the *fundamental unit*; the ephemeris second is the *experimental unit*. Of course, there was an improvement in that the newly define "time

[6] http://www.bipm.org/en/publications/si-brochure/second.html.

stick/ruler," i.e. the atomic clock, was much more stable than the previous astronomical definition. But what has to be taken home from this discussion is that the second, as time interval, was the same before and after this definition was made official.

The same reasoning applies to the meter. Since 1983 the meter is officially defined as *the length of the path travelled by light in vacuum during a time interval of* $1/299,792,458$ *of a second.*[7] Thus, as said above, in the SI, we need to fix the second and assume the speed of light is constant. But in the VSL-US, we can simply state that the meter is some multiple of the *fundamental unit*, u_M. At first sight, one might claim that such a unit violates any reasonable criterion of *reproducibility* and *practicality* which we have invoked above. But, as we have said, at least theoretically, this new system of units sounds correct and, within it, the assumption of a varying speed of light is correct.

Another consequence of such reasoning is that the speed of light can be still considered *numerically* equal to the value 299792.458 km/s that it has in SI.[8] The difference is in the implicit definition of the *fundamental units*, but not in the *experimental ones*, i.e. the meter, which is unchanged. The note also that, after 1973 and before 1983, the value of the speed of light was measured as *the product of the frequency and wavelength of an electromagnetic wave is the speed of propagation of that wave*, see [107]. Avoiding technicalities, what was practically done was to measure the frequency (proportional to time) and the wavelength (a length) of a well-defined laser, and from them the value of the speed of light in vacuum was derived. Thus, a velocity was derived from a ruler and a clock. In order to do that, of course, you needed to define the *experimental units* of the second and of the meter.

Finally, note also that even in the standard case where c is constant (SI units) cosmological distances are calculated indirectly by parallax, for example, as there would be no ruler working properly both locally in our laboratories and at such large scales. Moreover, cosmological distances are calculated theoretically by multiplying the speed of light with some integral of the scale factor. Then, using the conversion factor between the parsec and the meter, which is intrinsic to the definition of the former, one can find the numerical value of cosmological distances in units of parsec. The main assumption underlying cosmological-scale measurement processes is that units (once chosen) are invariant in time and space; if not, then, only local physics would be correct, while all the entire cosmology would be based on fallacies. In the c-constant scenario, the meter is fixed by c itself; in the VSL-US we have a new length *fundamental unit*, but always the same *experimental unit*. Thus, we are only assuming that the conversion is between the parsec and the meter, with the latter being defined with respect to new fundamental units. The same consideration holds true for cosmological times: in $H(z)$ data, they are given in terms of $\mathrm{km\,Mpc^{-1}\,s^{-1}}$. The second is still the experimental unit of time, also at cosmological scales, but defined in the new fundamental units of the VSL-US.

[7] http://www.bipm.org/en/CGPM/db/17/1/.

[8] http://www.bipm.org/en/CGPM/db/15/2/; http://www.bipm.org/en/CGPM/db/17/1/.

III. METHODOLOGY

To start, we need the observational data which is available from future BAO galaxy surveys: the angular diameter distance (D_A) and the expansion rate (H). We define as D_A^{real} and H^{real} the results of such observation, i.e. the *numbers* that outcome the measurement processes. Then, we need to fix the theoretical background underlying the implementation of our method. Our work is based on two main and general assumptions: we assume a Friedmann-Robertson-Walker metric and no spatial curvature. The former is the simplest and most general assumption which agrees with data and, up to some statistical accuracy, one of the main ingredients of the nowadays accepted consensus model. The latter assumption can hide a possible degeneracy between a VSL signal and the curvature; we will discuss this point in a later section and we show that, sticking to the present observational status, it is of limited concern.

Now, starting from the theoretical definition of the angular diameter distance in a classical context, we assume[9] [108,109] that in a VSL it can be generalized as

$$D_A \doteq \frac{1}{1+z}\int_0^z \frac{c(z')}{H(z')}dz', \tag{8}$$

where $H(z)$ is the theoretical Hubble function; and $c(z)$ is the speed of light expressed as any possible function of redshift. In a standard scenario, the speed of light is constant and $c(z) = c_0$; in the more extended context of a VSL approach, it can be any function, unknown to us until we do not recover it from the data. For what concerns $H(z)$ instead, in principle it can be derived from the first Friedmann equation (in combination with a continuity equation) once a cosmological model is given and so contains any possible information on the cosmological background. Then, we proceed assuming that

$$D_A \equiv D_A^{\mathrm{real}} \doteq \frac{1}{1+z}\int_0^z \frac{c(z')}{H^{\mathrm{real}}(z')}dz', \tag{9}$$

i.e., that the theoretical D_A function [ignoring what is on the right-hand side of Eq. (9)] is *explicitly* equal to the function that can be directly obtained by observations. On the other hand, we can also assume that the *unknown* theoretical $H(z)$ is *explicitly* equal to the function that can be obtained

[9] We are also assuming that the redshift is defined in VSL in the same way of the standard scenario [38,45]. For the role of c in the metric, see [50].

by observation, $H^{\rm real}$. Actually, this is much more than just an assumption: observations always bring signatures of the real underlying cosmological model, whose ignorance we parametrize in many ways. For example, by introducing the energy-matter dimensionless parameters (e.g. Ω_m and $\Omega_{\rm DE}$), or the dark energy equation of state ($w_{\rm DE}$), and so on. We want to highlight a very important point of our approach: we do not need any cosmological assumption (apart from the two we have stated above) for $H(z)$, because we will directly use the output from the observations, i.e. $D_A^{\rm real}(z)$ and $H^{\rm real}(z)$, in order to calculate all the quantities we will define. This also means that, strictly speaking, our method is quite useless from a cosmological point of view. Generally, all the information we need is hidden in $H(z)$: one proposes a cosmological model, which leads to $H(z)$ as a function of some parameters; and finally one tries a fit of this model with observational data, in order to recover some information about it. Here we use directly the numbers coming out of observations (i.e. $H^{\rm real}$), without any underlying theoretical background, so that we are losing any possibility to recover the information on the cosmological model. But, if we change our perspective, and we strictly look at VSL theories, then they reveal their benefit.

The main point here is that we do not know, *a priori*, if the speed of light appearing in Eq. (9) is a constant or not. In fact we have a question: what if we have a *real* VSL signal to be detected? In this case, on one side we will have the *direct* observational data,[10] obtained from the derivative of the real observed $D_A^{\rm real}$ as

$$y_r^{\rm real}(z) \doteq \frac{\partial}{\partial z}[(1+z)D_A^{\rm real}(z)] \equiv \frac{c(z)}{H^{\rm real}(z)}, \qquad (10)$$

where, again, we have identified the unknown theoretical $H(z)$ function in Eq. (8) with the observed $H^{\rm real}(z)$, as we did in Eq. (9). On the other, we will have a *reconstructed* set of

$$y_r^{\rm rec}(z) \doteq \frac{c_0}{H^{\rm real}(z)}, \qquad (11)$$

where we need an explicit assumption of a constant speed of light in order to convert time observations (H) into distances (y_r). Again, if we find that

$$y_r^{\rm real}(z) = y_r^{\rm rec}(z), \qquad (12)$$

then the assumption that the speed of light is constant will reveal to be well based. On the contrary, if

$$y_r^{\rm real}(z) \neq y_r^{\rm rec}(z), \qquad (13)$$

then $c(z) \neq c_0$. What is important to stress is that, by working with y_r, we can directly obtain (or reconstruct) the redshift function $c(z)$, through the ratio:

$$\frac{y_r^{\rm real}}{y_r^{\rm rec}} = \frac{c(z)}{c_0}. \qquad (14)$$

In this way we are also circumventing the "dimensionless-dimensional measurement" debate, because we are going to reconstruct a (dimensionless) relative variation of the speed of light, not an absolute (dimensional) quantity.

IV. APPLICATION

Given the methodological basis of our model, we will now describe how to apply it in the best possible way and, in particular, we will focus on the limits of the accessible information we should expect from future surveys. Our method will basically consist of four steps:

(1) fiducial H and D_A are obtained for a ΛCDM cosmology with an ansatz for $c(z)$;
(2) mock survey data are created from the fiducial values and a noise covariance matrix;
(3) some interpolating functions for H and D_A are fitted to the mock data;
(4) derivatives of the interpolating functions are used to find the reconstructed $c(z)$ and its covariance for each of the redshift bins.

Among all these steps, one could argue if point 3 is necessary or not. One alternative could be to use a bin approach, but there are many caveats which make it not feasible. The main one is that in order to calculate $c(z)$, we need to calculate the derivatives of D_A. In the bin approach, there would be no way to detect anything at all: we would have too few discrete points; one could use, for example, the finite differences methods to calculate such derivatives, but the errors would be too large, making all the efforts useless. Starting from this, the leading idea has been how to improve (if possible) the use of such mock measurements, and the best choice has been found in proposing some functions which were as general as possible and as close as possible to the real physical behavior of D_A and H. Actually, when ones compares a given cosmological model expectation for D_A and H with the chosen functions, one realizes that they work quite well.

A. Mock data

The first point to be addressed is: what kind of data are we going to use? As stated in Sec. I, our purpose is to show how to employ future galaxy surveys for a nonstandard cosmological analysis. From such surveys we will expect to obtain separate information on D_A (from BAO) and H (from cosmic chronometers). At the present stage, we do

[10]Note that from now on we will omit the sound horizon in our expressions for y_r [compare with Eq. (1)] just for sake of legibility, because the final quantity we are going to use [i.e. next Eq. (14)] is basically independent of its value, and because its contribution to the error budget is negligible.

not have yet independent measurements on D_A and H, so we will need to produce mock data for our analysis, in the style of [55,56]. As we have pointed out in Sec. I, there is no uniformity in approaching a VSL scenario; but we also want to stress that the exact choice of the VSL approach is meaningless in our case. We are not going to have any fit, or any test of any particular model; we only need to produce some mock observational data with a VSL signal included. The only requirement we will ask for is that, at least, such mock data were compatible with the present observations and with the present consensus model, at least in the redshift range now covered. In such a case, clearly, the VSL model would be indistinguishable from the standard scenario, but still compatible with observations. This sounds like a quite reasonable requirement: "observations are observations," what we measure and see is independent of our understanding of the underlying theory. Of course, a VSL signal can imply a different physical evolution in/of some processes, but the measured outcomes cannot be different from what we see now. For example, the sound horizon can be obviously influenced by a VSL. But it can also be measured with a very high confidence in a cosmic microwave background experiment. Theory has to adjust to this measurement, not vice versa. Thus, if we create a mock VSL data set which is compatible with present observations, we are just implying that the VSL signal has to be consistent with them.

Following [42], if a VSL is introduced with a minimal coupling with gravity, then we have modified versions of the first Friedmann equation and of the continuity equation which are, respectively,

$$H^2(t) = \frac{8\pi G}{3}\rho(t) - \frac{k}{a^2(t)}c^2(t) \tag{15}$$

and

$$\dot{\rho}(t) + 3H(t)\left(\rho(t) + \frac{p(t)}{c^2(t)}\right) = \frac{3k}{4\pi G a^2(t)}c(t)\dot{c}(t), \tag{16}$$

where ρ and p are, respectively, the mass density and the pressure of any fluid in the Universe; $a(t)$ is the scale factor; G is the universal gravitational constant; and the speed of light is expressed as a general function of time (or redshift), $c(t)$. As we have anticipated before, a degeneracy between VSL and geometry is possible: indeed, any change produced by a VSL is connected with the spatial curvature. We will discuss this later in more detail; for now, we will assume that Universe is spatially flat, e.g. $k = 0$, which implies that no effective change is effective in the continuity equation and, consequently, in the first Friedmann equation (at least, in terms of the energy-mass equations of state). On the other hand, in the calculation of D_A^{real}, the VSL also operates through the $c(z)$ function which enters the integral.

It is thus clear that, in order to produce our mock data, we need to assume an ansatz for $c(z)$; we follow [50] and consider the ansatz:

$$c(a) \propto c_0(1 + a/a_c)^n, \tag{17}$$

where $a \equiv 1/(1+z)$ is the scale factor, and a_c sets the transition epoch from some $c(a) \neq c_0$ (at early times) to $c(a) \to c_0$ (now).

The fiducial cosmological model used to produce the mock data in this work is a slightly modified version of the baseline ΛCDM model from the Planck 2015 release,[11] `base_plikHM_TTTEEE_lowTEB_lensing_post_BAO`. This choice seems to be the most conservative at this stage: if we have to add new elements to our theoretical building, it is better to start, at least, from a statistical and mutually agreed consensus base, exactly what the ΛCDM model is, nowadays, given our present knowledge. This model is characterized by a dimensionless matter density today equal to $\Omega_m = 0.31$. We have to modify slightly this parameter when introducing a VSL because a VSL can mimic an accelerated expansion (this was the original motivation for starting to study VSL theories) and, thus, can be seen as a contribution to the dark energy sector. More precisely, in a VSL context, the acceleration would not be given by a real cosmological fluid, but would be an implicit effect due to a varying speed of light. In any way, this means that, when adding a VSL, the contribution from a dark energy fluid diminishes and, consequently, in a spatially flat Universe, Ω_m might grow. In this work we have considered two different VSL scenarios: one, given by the parameters $a_c = 0.05$ and $n = -0.001$, corresponds to a redshift-increasing speed of light, with an average variation ∼0.1% at redshift 1.5–1.6; the other, given by the parameters $a_c = 0.05$ and $n = -0.01$, corresponds to a redshift-increasing speed of light, with an average variation ∼ 1% at redshift 1.5–1.6. This redshift range is used as a reference, following the nomenclature used in [55,56]. In order to match such VSL signals with present observations, we have thus needed to change the Ω_m: in Table I of [56] we show how both of the models can be made consistent with present observations and with the fiducial cosmological model from Planck, if we assume for them, respectively, $\Omega_m = 0.314$ and $\Omega_m = 0.348$.[12]

Once we have defined our input cosmological model, we can produce the mock H^{fid} and D_A^{fid}; but in order to produce realistic mock data, i.e., the previously defined H^{real} and D_A^{real}, we still need the observational errors on these quantities. In [110], many ongoing and future surveys

[11] http://wiki.cosmos.esa.int/planckpla/index.php/Cosmological_Parameters.

[12] Remember this is not a proper fit, which was out of the purpose of such work. It has to be considered more as a "by-eye" match.

are analyzed; the authors analyze the errors we should expect on such quantities by each one of these surveys, assuming redshift bins of 0.1 width. Among them, we will focus on BOSS, DESI, and *WFIRST-2.4* because, in their respective redshift ranges, they show the best performances. For BOSS, we will consider $z = 0.05$; for DESI, $z \in [0.15, 0.55]$; for *WFIRST-2.4*, $z \in [1.95, 2.75]$. For the intermediate range $z \in [0.65, 1.85]$ we use the SKA results from [111]; the performance of SKA will outperform the others by at least 1 order of magnitude, thus, it will be quite natural to expect the best results of our approach from this redshift range.

With $H^{\rm fid}$ and $D_A^{\rm fid}$ and the corresponding errors, we can now simulate realistic data: we randomly generate our values of $H^{\rm real}$ and $D_A^{\rm real}$ from a multivariate Gaussian centered on the fiducial values, and with a total covariance matrix built up from the errors defined for each survey. We also additionally assume a correlation factor between $H^{\rm fid}$ and $D_A^{\rm fid} \sim 0.4$, as derived in [112].

We want to stress here two points. First, the errors from [110] are not given directly in terms for H and D_A but, instead, for $H \cdot r_s(z_*)$ and $D_A/r_s(z_*)$, where $r_s(z_*)$ is the sound horizon at the decoupling/dragging epoch. Thus, if we want to work with H and D_A derived from a BAO survey, we need to multiply the previous defined combination by the sound horizon. Examining all the cases covered by the Planck mission and collected in the Planck Legacy Archive,[13] it is possible to check that the dependence of the sound horizon on the cosmological model is very weak (on the other hand, if this were not the case, it could not be considered as a standard ruler). Its dispersion is much smaller than the observational error, which is $\approx 0.15\%$. Thus, the contribution of the sound horizon to the total error budget, whatever is the value used for it, is quite negligible.

Second, we do not have this problem for H, because we assume it is derived from cosmic chronometers; but we lack a forecast analysis for the errors expected on H using this probe for future surveys. In [102] we have some estimations from *Euclid*, with a minimum statistical error $\sim 5\%$ and a total systematic contribution up to $\sim$10%. For our analysis we will use the H errors estimated by a BAO survey; we have to think about them as a possible precision goal for future surveys (next to stage IV), but they will always give us precise indication of how feasible and applicable is our approach.

Anyway, one could ask how a VSL theory might influence ETG use as a cosmic chronometer *ab initio*. To give an answer, we have to look at all the steps involved in the ETG data processing, see [98] for more details. In particular:

(1) the 4000 Å break is represented by the $D4000_n$ index (see [98] for its definition), which is strongly sensitive to metallicity, star formation history and age of a stellar population. Thus, [98] creates libraries based on two different stellar population synthesis models, and varying both star formation rates and metallicities;
(2) from such libraries a linear relation between the $D4000_n$ index and the age of the stellar population is found. The slope A of this relation is the most fundamental parameter, because

$$H(z) = -\frac{1}{1+z}\frac{dz}{dt} = -\frac{A}{1+z}\frac{dz}{dD4000_n}; \qquad (18)$$

(3) the value of A depends on the wavelength ranges of $D4000_n$, and on the metallicity of the sample. First, A is determined from the libraries, for some characteristic metallicity values and for different ranges of $D4000_n$. Then, these values are interpolated, as a function of the metallicity. Finally, the A to be used in Eq. (18) is an average $\langle A \rangle$ obtained by using a median metallicity from the interpolated function (more details in [98]).

This entire theoretical stuff is then applied to data. From [98] to [103] the main improvement has been on the statistical side, using larger galaxy data sets. The $D4000_n$-redshift relation, as derived from previous points, is thus applied to observed galaxies, see Fig. 6 in [101]. The most notable point to be stressed is that a relation involving an average value for A is used to obtain the $D4000_n$ for each galaxy. Then, the full sample is divided in redshift bins (regular trends are also found out in both mass and velocity dispersion bins); and, finally, in each redshift bin a median value for the $D4000_n$-redshift relation is obtained, i.e., both a median $D4000_n$ and a median redshift are calculated in order to have $H(z)$ from Eq. (18).

In [102] errors expected from future *Euclid* survey are estimated, in the redshift range [1.5;2.2]. The statistical error, due to the larger number of galaxies which will be observed, goes down to $\sim$5%. The systematic contribution is only slightly improved, and still can raise the error bar up to a total $\sim$15%. The nature of such a systematic contribution is ascribed to some assumptions which enter each of the previous steps as, for example, stellar formation history dependence, the stellar population synthesis models used, a not perfect estimation of the metallicity, and departures from passive evolution. In terms of impact onto cosmological parameters estimation, passive evolution uncertainty has the largest impact; followed by metallicity and stellar population models [98].

It seems to be clear from the above list that there are a lot of *average procedures* taken both at a preliminary theoretical stage and at the moment of analyzing data, and leading to a quite large systematic uncertainty. Where

[13] http://wiki.cosmos.esa.int/planckpla/index.php/Cosmological_Parameters.

might a VSL signal have any influence? The ETG method does not depend on the absolute age of the galaxies [98]; even if there is any influence of VSL on the time life of the stellar populations, it will have no consequence on the final $H(z)$ value and error budget.

A VSL signal might have some influence on the spectra, through variations in the fine structure constant, α. For example, the determination of the redshift should take into account the different response of different elements to light interaction, through a change in their energy levels ($\propto \alpha^2$). That is what exactly happens with quasar absorption lines [16,17]. In this case, the effect of the VSL should be detectable by comparing different elements with a different order of magnitude dependence on α. This would help to recover the true redshift and would serve as a direct way to constrain VSL theories. Of course, also the determination of the 4000 Å break would be affected: the $D4000_n$ index is determined by the ratio of fluxes in two well determined wavelength bands. Also in this case, the comparison of different stellar populations at different redshifts with synthesis models could help to constrain VSL theories.

Anyway, the final data $H(z)$, and their errors σ_H, are determined by applying average relations from theory, binning data, and taking into account full systematic effects. Considering all these elements, it is very unlikely that any influence of VSL might result to be detectable (at least, for the orders of magnitude we are considering). Actually, it might be even possible that a VSL is already present and, in some way, it is mimicked by some of the systematic uncertainty sources described above. But this is not a problem for the application of our method which, as we have explained in previous sections, points to a constraint VSL signal through the ratio $c(z)/H$, not directly through H only.

Thus, the real question should be: are 1% or 0.1% VSL signals able to produce a variation in $H(z)$ larger than the 15% uncertainty claimed today? A full quantitative answer can be given only by performing *ab initio* (i.e. starting from stellar models) a full detailed analysis; but this is out of the purposes of this work. It is surely interesting and necessary, anyway, if one wants to validate VSL theories, and we will try to cope with it in the future. For what concerns this work, qualitatively, we feel confident that the variations we have considered in this work are so small that they cannot bias $H(z)$ numbers (or the $D4000_n$-redshift relation slope) in such a considerable way to make this analysis wrong.

B. Fitting quantities

Before using Eq. (14) in order to reconstruct a possible VSL signal, many questions have to be addressed and problems solved. The first one is intrinsic to our definition of $y_r^{\rm real}$ given by Eq. (10): it is the derivative with respect to redshift of a quantity ($D_A^{\rm real}$) which is represented by a discrete set of points (observations) which have an intrinsic dispersion around the underlying fiducial cosmological model. The problems related to the dispersion cannot be avoided: the dispersion is intrinsic to the measurement process, and we can only hope to have, in the future, better measurements which can reduce it (but its nature is not of statistical origin only). Thus, we will always have an intrinsic systematic error in the derivation of $y_r^{\rm real}$; moreover, the dispersion alters the derivative calculation and thus, as it is known and expected, the errors on the derivated quantity tend to explode.

Having assumed that this problem cannot be avoided, we can rely on another property of our approach: given that we are not interested in the explicit form of H, because we will directly use observations to infer a function which *interpolates* them, we are not forced to fit our quantities following some cosmological-model-based requirements. Thus, we can try a fit based on the best analytic functions which can work in this situation.

In our case, we need analytic functions for fitting both $H^{\rm real}$ and $D_A^{\rm real}$, and they have different requirements. For $H^{\rm real}$ we have found that a simple sixth-order redshift polynomial gives an optimal fit to $H^{\rm real}$ in the redshift range we are covering, i.e. $z \in [0.05, 2.75]$; higher-order polynomials do not improve the fit. As only general prior, we only ask that $H(z) > 0$ all over the redshift range $z \in [0, \infty)$.

For $D_A^{\rm real}$ a polynomial fit is unsatisfactory to describe the peculiar property of the angular diameter distance to have a maximum at relatively low redshift values. A better and more flexible fit is given by the Padéapproximant:

$$D_A^{\rm real}(z) = \frac{d_1^t z}{1 + d_1^b z + d_2^b z^2}, \tag{19}$$

which clearly satisfies the expected conditions: $D_A^{\rm real} = 0$ for $z \to 0$ and $z \to \infty$; moreover, we require that $D_A^{\rm real} > 0$ for $z \in [0, \infty)$.

Once the fits are run for both $H^{\rm real}$ and $D_A^{\rm real}$, we have a set of parameters (the parameters of the polynomial and of the Padéapproximant), respectively, with their covariance matrix and errors bars; after the correct propagation error rules are applied, we end up with a set of polynomial-reconstructed $y_r^{\rm real}$ and $y_r^{\rm rec}$, with related errors, from which we can derive the $c(z)/c_0$ ratio through Eq. (14). Finally, this last quantity can also be fitted (or reconstructed); the function we have been working with is the Padéapproximant given by

$$\frac{c(z)}{c_0} = \frac{1 + c_1^t z}{1 + c_1^b z + c_2^b z^2}, \tag{20}$$

imposing the conditions $c(z=0)/c_0 = 1$, and that $c(z)$ is always positive for $z \in [0, \infty)$.

We have verified that the functions we have finally chosen to fit $H^{\rm real}$ and $D_A^{\rm real}$ are really good approximations to the fiducial model all over the entire redshift range $z \in [0, \infty)$, and not only in the redshift interval covered by next galaxy surveys. On the other hand, the function chosen for $c(z)$ has some degree of arbitrariness: it describes very well our input VSL in the galaxy surveys redshift range, but not at very high redshifts. But it is very general, and with such a high level of flexibility it can be used as a testing function to detect if a VSL signal is working or not in any case (but any suggestion can be considered).

V. RESULTS

The first point to be examined is how good is the reconstruction of the hidden VSL signal; then, we will move to a much detailed analysis of the possible degeneracy between the VSL signal and a nonzero spatial curvature.

A. Pure VSL signal

In our case, we know the behavior of $c(z)$, so we can easily check if the final reconstructed $c(z)$ gives a reliable description of this known input, thus testing if our algorithm works well or not. In Fig. 1 we show both cases of a 1% and of a 0.1% VSL signal, joining results from all the 10^3 simulations we have realized. In black, we show the 1σ confidence level span by all the simulations for the VSL signal in Eq. (14), i.e. the quantity $c(z)/c_0$, as it comes out from $y_r^{\rm real}$ and $y_r^{\rm rec}$ after fitting $H^{\rm real}$ with a sixth-order polynomial and $D_A^{\rm real}$ with the Padéapproximant given in Eq. (19). In red, we plot the 1σ confidence level span by all the simulations for the VSL signal in Eq. (14) but after fitting the ratio $y_r^{\rm real}/y_r^{\rm rec}$ with the Padéapproximant given in Eq. (20). If we make a parallel with, for example, dark energy studies, we might claim that black points are "raw" data, while red ones are obtained from a fit procedure. Thus, the latter can be used to infer some statistical significance.

From a simple visual inspection, it can be seen that, at least at the 1σ level, a 1% VSL signal can in principle be detected in the redshift range [0.85, 1.35], where the angular diameter distance from BAO and the Hubble function from cosmic chronometers have been assumed to have the precision actually forecast from SKA. On the other hand, it is also clear that a 0.1% VSL signal will be hardly detected with the same prescriptions.

A more detailed inspection of the possibility to detect a VSL signal is given in Fig. 2: for each simulation, and in each redshift bin, we calculate the residuals with respect to a constant speed of light, i.e. $c(z)/c_0 = 1$; then, we plot the normalized number of simulations for which such residuals are positive, implying a clear detection of a nonconstant $c(z)/c_0$. In blue, we show results when the residuals with respect to constant speed of light are calculated using the best fit Padéapproximant, Eq. (20); in red, the residuals are calculated using the lower 1σ limit derived from the same best fit function, thus indicating a detection of the VSL signal at 1σ confidence level; in green and yellow, respectively, the residuals are calculated using the lower 2σ and 3σ limits. We focused on the lower limits because our fiducial input VSL corresponds to a speed of light higher than c_0 at higher redshift; in a more realistic case, one should check for both positive and negative residuals and, eventually, fix a significance threshold (e.g. $\mathcal{P}_{\lesseqgtr 1} >$ 0.95–0.99%) for a possible detection to test if there is any statistically clear evidence for one trend over the other.

Looking at Fig. 2 we can now have a more clear and precise prediction of what could happen in the next future. The probability to detect a 1% VSL at a 3σ level is higher than the 95% (e.g., in 95% of our simulations we are able to detect a 1% VSL signal at a 3σ level) approximately in the redshift range [0.75, 1.45]. For higher redshifts, using the precision actually forecasted for next future galaxy surveys, we see that the signal degrades very rapidly. This point is interesting: given our input $c(z)$, the deviation from the $c(z)/c_0 = 1$ limit grows with redshift. Thus, higher redshifts imply larger deviations. But this does not automatically convert in a clearer or easier detection: if the survey precision degrades too fast, we are going to lose any possibility to detect the signal at high redshift. At the same time, this is also encouraging: given the possibly larger deviation from the constancy of the speed of light at high redshifts (if the VSL

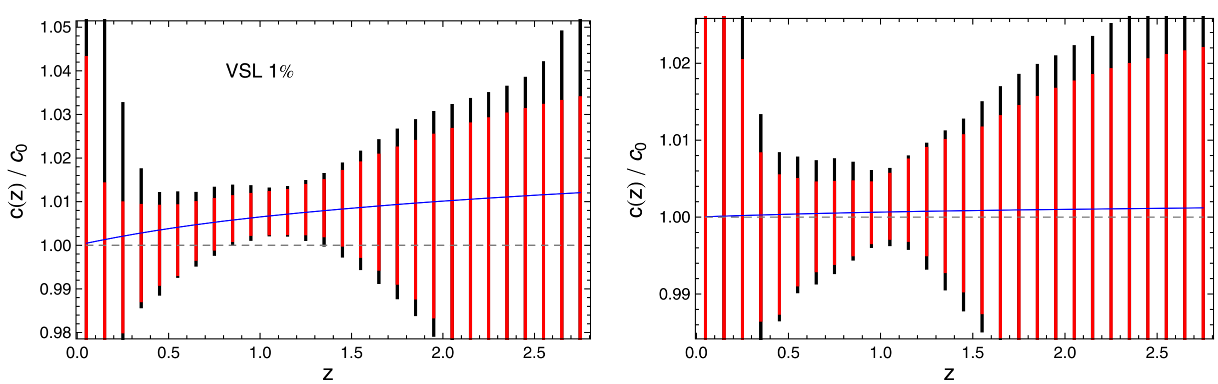

FIG. 1. Reconstruction of $c(z)$ from mock data: results. Black: 1σ confidence level from the total 10^3 simulations for Eq. (14); red: 1σ confidence level from the total 10^3 simulations for Eq. (14) after the ratio $c(z)/c_0$ is fit with Eq. (14); blue: VSL $c(z)/c_0$ used as input from Eq. (17); dashed grey line: standard constant $c(z)/c_0 = 1$. Different ranges are shown on the vertical axis.

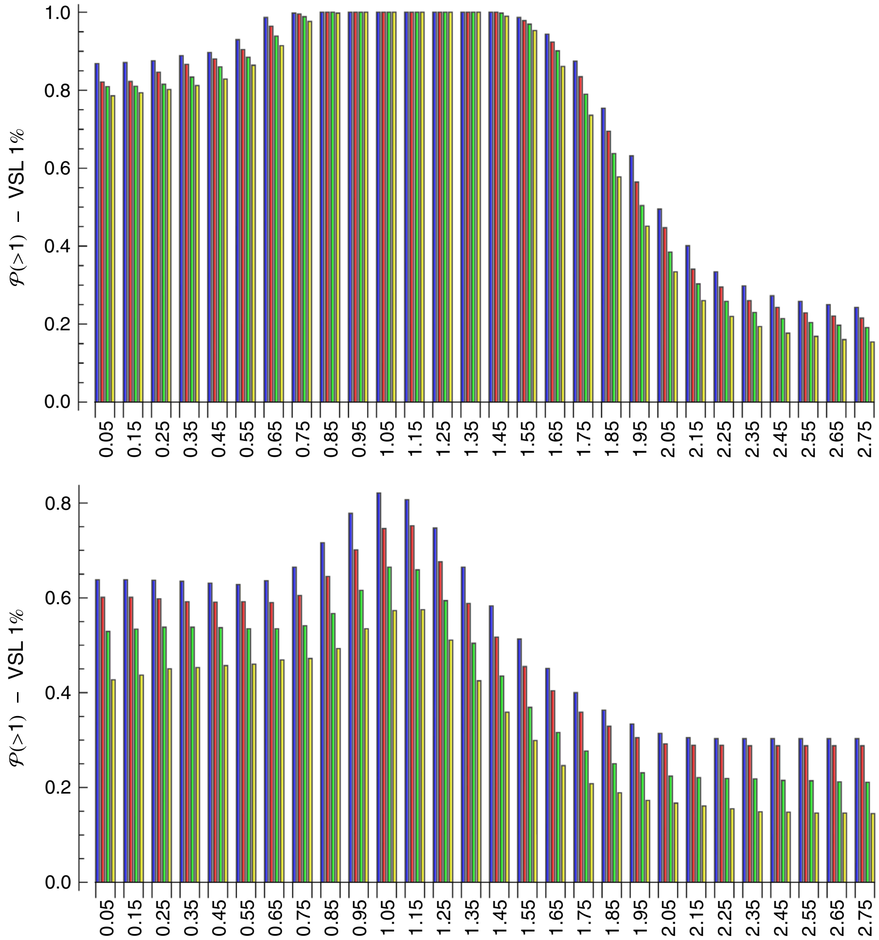

FIG. 2. Probability to detect positive residuals of post-fitting reconstructed $c(z)$ vs $c(z) = c_0$. Blue: residuals calculated from the best fit reconstructed values; red: residuals calculated from the 1σ lower confidence level from the reconstructed values; green: residuals calculated from the 2σ lower confidence level from the reconstructed values; yellow: residuals calculated from the 3σ lower confidence level from the reconstructed values.

signal is a monotonical function, of course), we have a lot of room to improve its detection in this range, because the precision update required for future surveys at higher redshifts is well inside our technological possibilities.

Another interesting point to stress is that in [55,56] we have shown how SKA will be able to put a 3σ limit on a 1% VSL signal at the maximum redshift in the angular diameter distance, which should locate at $z \sim 1.55-1.65$ (at least, basing this claim on our present knowledge of the cosmological background model). In the method exposed in this work, the detection at 3σ for a redshift ≈ 1.65 is possible in 80% of our simulations; still a high probability, even if quite lower than the 95% limit assessed above. It is not surprising that the two methods have different sensitivity at this redshift, because they rely on different algorithms; in particular, the maximum detection method described in [55,56] can be pushed to a better precision, while the present method is mainly limited by the not-perfect correspondence of the derivatives calculated from real data with those intrinsic to the unknown cosmological background. But, still, the two methods are complementary, helping to extend the final redshift range of VSL detection.

Unfortunately, from Fig. 2 it is also clear that a 0.1% signal will be hardly detected: at the 1σ level, the probability detection of a VSL signal of such magnitude is ~80% in the redshift range [0.95, 1.15]; a 3σ detection in the same range is achieved only in 55% of our simulations, thus making it difficult to statistically state if it can be really reached or not.

Another important question to be stated is what level of goodness our method has to reconstruct the real VSL background. In order to assess this question, we calculate the quantity:

$$\Delta_i = \sum_{j=1}^{\mathcal{N}_{\rm sim}} \frac{(c_{\rm theo}(z_i) - c_{\rm ans}(z_i))^2}{c^2_{\rm ans}(z_i)}, \tag{21}$$

where $\mathcal{N}_{\rm sim} = 10^3$ is the total number of simulations we have run; $c_{\rm theo}(z_i)$ is the varying speed of light given by the resulting best Padéapproximant, Eq. (20), evaluated at each

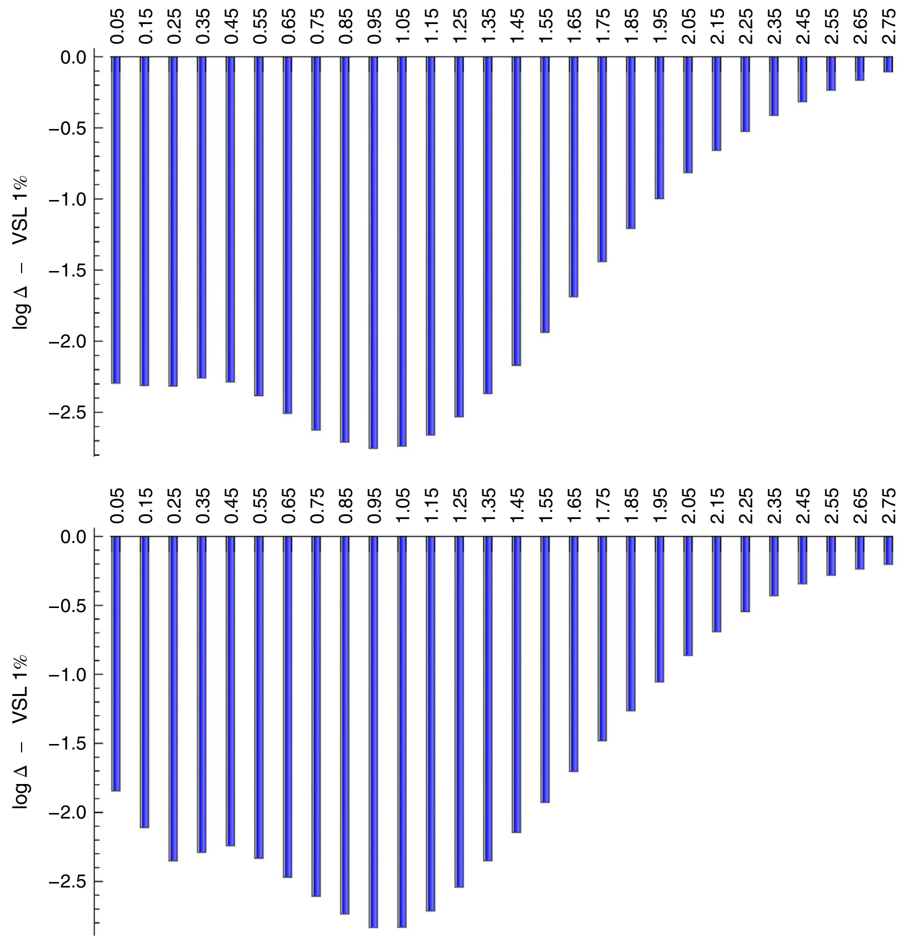

FIG. 3. Reconstruction goodness criterium (residuals sum parameter) for the reconstruction of the underlying VSL signal.

redshift z_i; and $c_{\rm ans}(z_i)$ is the fiducial speed of light given by Eq. (17). Thus, the quantity Δ_i is the sum, at each redshift and all over our simulations, of the relative squared residuals between our final reconstructed VSL ($c_{\rm theo}$) and the fiducial one ($c_{\rm ans}$). Smaller is its value, better is the agreement between our reconstruction and the true underlying model. In this way, we have a criterium to establish if our reconstruction is accurate or not. In Fig. 3 we plot the logarithm of this quantity; conclusions are the same we have derived above: in the case of a 1% VSL, it is clear that the agreement is quite good and very similar in the redshift range [0.75, 1.35]; then it starts to decrease, with a Δ that, at $z \sim 1.65$, is 1 order of magnitude larger than the minimum value achieved; and things go even worse for larger redshift values. The same is more or less valid for the 0.1% case.

B. Curvature degeneracy

As pointed out in previous sections, in the Friedmann and continuity equations the VSL-based terms, $c(t)$ and $\dot{c}(t)$, come coupled with the spatial curvature parameter (k). Thus, it is natural to expect some degree of degeneracy between a possible VSL signal and what could instead be interpreted as a geometric effect. Our main equations are derived, as said previously, assuming that the Universe if spatially flat, i.e. $k = 0$. All of the most updated observations confirm such an assumption [73]; but we want to show here that, even if we take into account curvature, still there is a wide range of validity for our equations and, even, our method might be generalized and used, in the standard context of constant speed of light, as an alternative way to measure the spatial curvature.

When taking into account spatial curvature, the main change is in the determination of what we have defined $y^{\rm real}$, defined in Eq. (10) as the derivative of observational $D_A^{\rm real}$ with respect to redshift. If the curvature is allowed to vary, then, the most general definition for the angular diameter distance is

$$D_A(z) = \begin{cases} \frac{D_H}{\sqrt{\Omega_k}(1+z)} \sinh\left(\frac{\sqrt{\Omega_k} D_C(z)}{D_H}\right) & \text{for } \Omega_k > 0 \\ \frac{D_C(z)}{1+z} & \text{for } \Omega_k = 0 \\ \frac{D_H}{\sqrt{|\Omega_k|}(1+z)} \sin\left(\frac{\sqrt{|\Omega_k|} D_C(z)}{D_H}\right) & \text{for } \Omega_k < 0, \end{cases} \quad (22)$$

where $\Omega_k \equiv kc_0^2/H_0^2$ is the dimensionless curvature density parameter today; $D_H = c_0/H_0$ is the Hubble distance; and the line-of-sight comoving distance is defined as $D_C(z) = D_H \int_0^z \mathcal{F}_c(z')/E(z')dz'$, where we have made use of the general ansatz $c(z) \equiv c_0\mathcal{F}_c(z)$, with $\mathcal{F}_c(z) = 1$ for $z = 0$. We are assuming here the most general case of a varying speed of light $c(z)$; but the standard scenario can be easily recovered simply replacing $c(z)$ with c_0 any time it appears. If we now calculate $y^{\rm real}$ through the same Eq. (10), we have

$$y_r^{\rm real}(z) \equiv \begin{cases} \frac{c(z)}{H(z)}\cosh\left(\frac{\sqrt{\Omega_k}D_C(z)}{D_H}\right) & \text{for } \Omega_k > 0 \\ \frac{c(z)}{H(z)} & \text{for } \Omega_k = 0 \\ \frac{c(z)}{H(z)}\cos\left(\frac{\sqrt{|\Omega_k|}D_C(z)}{D_H}\right) & \text{for } \Omega_k < 0. \end{cases} \qquad (23)$$

It is clear that even if we assume $c(z) = c_0$, we would still have some contribution from the $\Omega_k \neq 0$ term; thus the case "VSL + spatial flatness" would be equivalent to "constant $c(z)$ + curvature." We can easily quantify how much information we might derive, and which we might erroneously attribute to a VSL signal only, should instead be shared with a non-null curvature signal. From the Planck Legacy Archive, the extension of the baseline model with a free curvature parameter, named `base_plikHM_TTTEEE_lowTEB_lensing_post_BAO`, gives the value of $\Omega_k = 0.0008 \pm 0.002$ at the 68% confidence level (and ± 0.004 at the 95%). We can thus compare the curvature-correction terms in Eq. (23) with the null curvature hypothesis, using our ansatz for the VSL, Eq. (17). Results are shown in Fig. 4. In order to be as complete as possible, we have also analyzed the lower and upper limits for the curvature parameters, i.e. $\Omega_k = -0.0012$ and $\Omega_k = 0.0028$.

The first possible conclusion is that a realistic contribution from the spatial curvature to our method would be $\lesssim 0.06\%$ at the maximum in D_A (for a more direct and straightforward comparison, we use the same maximum criterium we have used to define the 1% and the 0.1% VSL models) for both $\Omega_k = 0.0008$ (solid red line) and $\Omega_k = -0.0012$ (solid blue line). Thus, it would be even smaller than the 0.1% VSL signal (black dashed line) we have considered so far, and would thus result, finally, undetectable. This result, obtained in an independent and alternative way, is also consistent with a recent attempt described in [113]. On the other hand, the upper limit $\Omega_k = 0.0028$, would give a $\sim 0.15\%$ contribution; a slightly larger value, but still out of possible detection with SKA.

For the sake of precision, we have to stress again that, anyway, in general, a pure VSL and a pure curvature signal are degenerate. We can detect a total signal, without being able to ascribe it to one or another. What we can estimate is that, given present bounds on curvature, a 1% signal (solid black line) could be attributed with no doubt to VSL only,

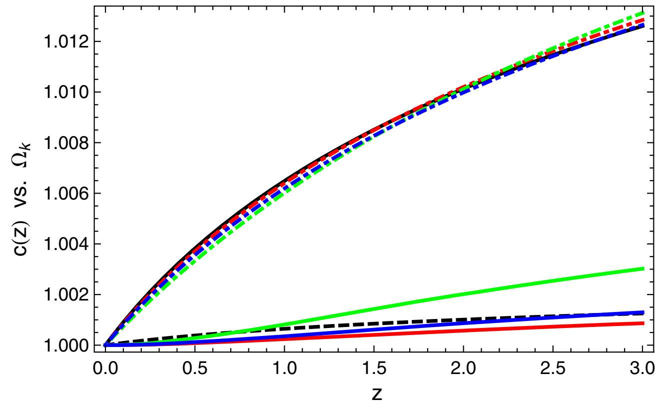

FIG. 4. VSL vs spatial curvature degeneracy displayed using Eq. (23). Black lines: solid—1% VSL signal from Eq. (17) plus null curvature ($\Omega_k = 0$ case in Eq. (23)); dashed—0.1% VSL from Eq. (17) plus null curvature ($\Omega_k = 0$ case in Eq. (23)). Red lines: solid—correction from curvature term in Eq. (23) when $\Omega_k = 0.0008$ and assuming $c(z) = c_0$; dot-dashed—correction from curvature term in Eq. (23) when $\Omega_k = 0.0008$ and assuming a 0.95% VSL signal. Blue lines: solid—correction from curvature term in Eq. (23) when $\Omega_k = -0.0012$ and assuming $c(z) = c_0$; dot-dashed—correction from curvature term in Eq. (23) when $\Omega_k = 0.0008$ and assuming a 0.95% VSL signal. Green lines: solid—correction from curvature term in Eq. (23) when $\Omega_k = 0.0028$ and assuming $c(z) = c_0$; dot-dashed—correction from curvature term in Eq. (23) when $\Omega_k = 0.0008$ and assuming a 0.85% VSL signal.

rather than to any curvature contribution. Even in the case of assuming both a VSL and non-null curvature, given the actual constraints on the latter one, the VSL signal might be $\sim 0.95\%$ for $\Omega_k = 0.0008$ (dot-dashed red line) and $\Omega_k = -0.0012$ (dot-dashed blue line), and $\sim 0.85\%$ for $\Omega_k = 0.0028$ (dot-dashed blue line), in order to have a final total 1% detection. Thus, at least at the scales which we have shown to be directly testable in the near future, curvature might play a negligible role. But if the total signal should result to be less than 1%, then we could have problems and would not be able to discriminate between them.

VI. CONCLUSIONS

In this work we have extended the method previously described in [55,56]: while the latter made it possible to measure the speed of light (and, incidentally, detect any possible variation of the same quantity) only in one well-located point (the maximum redshift), here we show how it is possible to recover a *redhisft-extended* VSL signal on a much larger redshift range. We have used the cosmological observations which will be available in the near future from galaxy surveys, i.e.: estimations of the sound horizon at decoupling/dragging epoch, imprinted as an angular diameter distance D_A in the clustering of the galaxies; and the expansion rate data H inferred from ETG galaxies designated as cosmological clocks. We have employed quite a various number of future galaxy surveys, BOSS, DESI, *WFirst-2.4* and SKA, which result to have the best performances in different (nonoverlapping) redshift windows.

As we have discussed in Sec. IV, and as it is shown in Fig. 2, given the sensitivities forecast for the previous surveys, there is a quite high probability (>95%) to detect a 1% VSL signal (if any) at the 3σ confidence level in the redshift range $z \in [0., 1.55]$. Smaller signals, of the order of 0.1%, will be hardly detected by the same surveys.

We have also given a more detailed discussion about the impact that a possible non-null spatial curvature might have on the detection of the VSL signal. In particular, we have shown that values of the curvature compatible with the present bounds given by Planck are absolutely negligible with respect to a 1% VSL signal. We emphasize here that, even if we were not considering a VSL signal, but the classical constancy of the speed of light, then our method would result to be useful to detect *curvature-only* contributions. In particular, if such a contribution should result to be ≈0.01, then it would be equivalent to a 1% VSL signal, and all the discussion we have spent for the VSL theory might be equivalently exported to spatial curvature measurements only.

More problematic would be to disentangle smaller VSL signals, which would result to be of the same order of the geometrical contribution; but, as we have shown here, such small signals are out of the detection possibilities of currently forecast galaxy surveys for the next 15 years. In the meantime, we may work to improve our method, and/or find alternative ones.

ACKNOWLEDGMENTS

The author warmly thanks the anonymous referees for their insightful reports, which have helped to improve the general quality of the paper and have also enlightened possible future research topics. This work is financed by the Polish National Science Center Grant No. DEC-2012/06/A/ST2/00395.

---

[1] H. Weyl, Ann. Phys. (Berlin) **59**, 129 (1919).
[2] A. S. Eddington, *The Mathematical Theory of Relativity* (Cambridge University Press, Cambridge, England, 1923).
[3] A. S. Eddington, *New Pathways in Science* (Cambridge University Press, Cambridge, England, 1934).
[4] H. Weyl, Naturwissenschaften **22**, 145 (1934).
[5] P. A. M. Dirac, Nature (London) **139**, 323 (1937).
[6] P. A. M. Dirac, Proc. R. Soc. A **165**, 199 (1938).
[7] J.-P. Uzan, Living Rev. Gen. Relativity **14**, 2 (2011).
[8] A. A. Starobinsky, JETP Lett. **30**, 682 (1979).
[9] A. A. Starobinsky, Phys. Lett. **91B**, 99 (1980).
[10] A. H. Guth, Phys. Rev. D **23**, 347 (1981).
[11] A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982).
[12] A. D. Linde, Phys. Lett. **108B**, 389 (1982).
[13] A. D. Linde, Phys. Lett. **129B**, 177 (1983).
[14] A. G. Riess, A. V. Filippenko, P. Challis *et al.*, Astrophys. J. **116**, 1009 (1998).
[15] S. Perlmutter, G. Aldering, G. Goldhaber *et al.*, Astrophys. J. **517**, 565 (1999).
[16] J. K. Webb, V. V. Flambaum, C. W. Churchill, M. J. Drinkwater, and J. D. Barrow, Phys. Rev. Lett. **82**, 884 (1999).
[17] V. A. Dzuba, V. V. Flambaum, and J. K. Webb, Phys. Rev. Lett. **82**, 888 (1999).
[18] M. T. Murphy, J. K. Webb, V. V. Flambaum, J. X. Prochaska, and A. M. Wolfe, Mon. Not. R. Astron. Soc. **327**, 1237 (2001).
[19] M. T. Murphy, J. K. Webb, V. V. Flambaum, M. J. Drinkwater, F. Combes, and T. Wiklind, Mon. Not. R. Astron. Soc. **327**, 1244 (2001).
[20] J. K. Webb, M. T. Murphy, V. V. Flambaum, V. A. Dzuba, J. D. Barrow, C. W. Churchill, J. X. Prochaska, and A. M. Wolfe, Phys. Rev. Lett. **87**, 091301 (2001).
[21] M. T. Murphy, J. K. Webb, and V. V. Flambaum, Mon. Not. R. Astron. Soc. **345**, 609 (2003).
[22] J. N. Bahcall, C. L. Steinhardt, and D. Schlegel, Astrophys. J. **600**, 520 (2004).
[23] H. Chand, R. Srianand, P. Petitjean, and B. Aracil, Astron. Astrophys. **417**, 853 (2004).
[24] R. Srianand, H. Chand, P. Petitjean, and B. Aracil, Phys. Rev. Lett. **92**, 121302 (2004).
[25] S. A. Levshakov, M. Centurión, P. Molaro, and M. V. Kostina, Astron. Astrophys. **447**, L21 (2006).
[26] M. T. Murphy, J. K. Webb, and V. V. Flambaum, Phys. Rev. Lett. **99**, 239001 (2007).
[27] R. Srianand, H. Chand, P. Petitjean, and B. Aracil, Phys. Rev. Lett. **99**, 239002 (2007).
[28] P. Molaro, D. Reimers, I. I. Agafonova, and S. A. Levshakov, Eur. Phys. J. Spec. Top. **163**, 173 (2008).
[29] M. T. Murphy, J. K. Webb, and V. V. Flambaum, Mon. Not. R. Astron. Soc. **384**, 1053 (2008).
[30] I. I. Agafonova, P. Molaro, S. A. Levshakov, and J. L. Hou, Astron. Astrophys. **529**, A28 (2011).
[31] J. C. Berengut, V. V. Flambaum, J. A. King, S. J. Curran, and J. K. Webb, Phys. Rev. D **83**, 123506 (2011).
[32] J. K. Webb, J. A. King, M. T. Murphy, V. V. Flambaum, R. F. Carswell, and M. B. Bainbridge, Phys. Rev. Lett. **107**, 191101 (2011).
[33] P. Molaro, M. Centurión, J. B. Whitmore *et al.*, Astron. Astrophys. **555**, A68 (2013).
[34] H. Rahmani, N. Maheshwari, and R. Srianand, Mon. Not. R. Astron. Soc. **439**, L70 (2014).
[35] P. Bull, Y. Akrami, J. Adamek *et al.*, Phys. Dark Univ. **12**, 56 (2016).
[36] J. Moffat, Int. J. Mod. Phys. D **02**, 351 (1993).
[37] J. Moffat, Found. Phys. **23**, 411 (1993).

[38] A. Albrecht and J. Magueijo, Phys. Rev. D **59**, 043516 (1999).
[39] J. D. Barrow, Phys. Rev. D **59**, 043515 (1999).
[40] P. P. Avelino and C. J. A. P. Martins, Phys. Lett. B **459**, 468 (1999).
[41] J. D. Barrow and J. Magueijo, Phys. Lett. B **443**, 104 (1998).
[42] J. D. Barrow and J. Magueijo, Phys. Lett. B **447**, 246 (1999).
[43] J. D. Barrow and J. Magueijo, Classical Quantum Gravity **16**, 1435 (1999).
[44] M. A. Clayton and J. W. Moffat, Phys. Lett. B **460**, 263 (1999).
[45] J. D. Barrow and J. Magueijo, Astrophys. J. Lett. **532**, L87 (2000).
[46] M. A. Clayton and J. W. Moffat, Phys. Lett. B **477**, 269 (2000).
[47] B. A. Bassett, S. Liberati, C. Molina-Paris, and M. Visser, Phys. Rev. D **62**, 103518 (2000).
[48] M. A. Clayton and J. W. Moffat, Phys. Lett. B **506**, 177 (2001).
[49] M. A. Clayton and J. W. Moffat, Int. J. Mod. Phys. D **11**, 187 (2002).
[50] J. Magueijo, Rep. Prog. Phys. **66**, 2025 (2003).
[51] J. W. Moffat, Eur. Phys. J. C **76**, 130 (2016).
[52] G. F. R. Ellis and J.-P. Uzan, Am. J. Phys. **73**, 240 (2005).
[53] G. F. R. Ellis, Gen. Relativ. Gravit. **39**, 511 (2007).
[54] J. Magueijo and J. W. Moffat, Gen. Relativ. Gravit. **40**, 1797 (2008).
[55] V. Salzano, M. P. Dąbrowski, and R. Lazkoz, Phys. Rev. Lett. **114**, 101304 (2015).
[56] V. Salzano, M. P. Dąbrowski, and R. Lazkoz, Phys. Rev. D **93**, 063521 (2016).
[57] P. J. E. Peebles and J. T. Yu, Astrophys. J. **162**, 815 (1970).
[58] R. A. Sunyaev and Ya. B. Zeldovich, Astrophys. Space Sci. **7**, 3 (1970).
[59] A. G. Doroshkevich, Ya. B. Zeldovich, and R. A. Sunyaev, Sov. Astron. **22**, 523 (1978).
[60] D. J. Eisenstein and W. Hu, Astrophys. J. **496**, 605 (1998).
[61] D. J. Eisenstein, W. Hu, and M. Tegmark, Astrophys. J. **504**, L57 (1998).
[62] A. Cooray, W. Hu, D. Huterer, and M. Joffre, Astrophys. J. **557**, L7 (2001).
[63] C. Blake and K. Glazebrook, Astrophys. J. **594**, 665 (2003).
[64] W. Hu and Z. Haiman, Phys. Rev. D **68**, 063004 (2003).
[65] H.-J. Seo and D. J. Eisenstein, Astrophys. J. **598**, 720 (2003).
[66] D. J. Eisenstein, New Astron. Rev. **49**, 360 (2005).
[67] D. J. Eisenstein, I. Zehavi, D. W. Hogg *et al.*, Astrophys. J. **633**, 560 (2005).
[68] H.-J. Seo and D. J. Eisenstein, Astrophys. J. **633**, 575 (2005).
[69] D. J. Eisenstein, H.-J. Seo, and M. White, Astrophys. J. **664**, 660 (2007).
[70] B. Bassett and R. Hlozek, *Dark Energy: Observational and Theoretical Approaches*, edited by P. Ruiz-Lapuente (Cambridge University Press, Cambridge, England, 2010).
[71] D. H. Weinberg, M. J. Mortonson, D. J. Eisenstein, C. Hirata, A. G. Riess, and E. Rozo, Phys. Rep. **530**, 87 (2013).
[72] S. Anselmi, G. D. Starkman, and R. K. Sheth, Mon. Not. R. Astron. Soc. **455**, 2474 (2016).
[73] P. A. R. Ade, N. Aghanim, M. Arnaud *et al.* (Planck Collaboration), Astron. Astrophys. **594**, A13 (2016).
[74] C. Blake, D. Parkinson, B. Bassett, K. Glazebrook, M. Kunz, and R. C. Nichol, Mon. Not. R. Astron. Soc. **365**, 255 (2006).
[75] C. Blake, E. A. Kazin, F. Beutler *et al.*, Mon. Not. R. Astron. Soc. **418**, 1707 (2011).
[76] C. Blake, S. Brough, M. Colless *et al.*, Mon. Not. R. Astron. Soc. **425**, 405 (2012).
[77] L. Anderson, E. Aubourg, S. Bailey *et al.*, Mon. Not. R. Astron. Soc. **441**, 24 (2014).
[78] E. A. Kazin, J. Koda, C. Blake *et al.*, Mon. Not. R. Astron. Soc. **441**, 3524 (2014).
[79] L. Samushia, B. A. Reid, M. White *et al.*, Mon. Not. R. Astron. Soc. **439**, 3504 (2014).
[80] T. Delubac, J. E. Bautista, N. G. Busca *et al.*, Astron. Astrophys. **574**, A59 (2015).
[81] L. Amendola, S. Appleby, D. Bacon *et al.*, Living Rev. Relativ. **16**, 6 (2013).
[82] R. Laureijs *et al.* (Euclid Collaboration), arXiv:0912.0914.
[83] A. Refregier *et al.*, arXiv:1001.0061.
[84] R. Laureijs *et al.* (Euclid Collaboration), arXiv:1110.3193.
[85] D. Spergel, N. Gehrels, J. Breckinridge *et al.*, arXiv:1305.5422.
[86] K. S. Dawson, D. J. Schegel, C. P. Ahn *et al.*, Astron. J. **145**, 10 (2013).
[87] J. Comparat, T. Delubac, S. Jouvel *et al.*, Astron. Astrophys. **592**, A121 (2016).
[88] A. Prakash and J. Newman (SDSS-IV/eBOSS Collaboration) (2015).
[89] J. Tinker (SDSS-IV Collaboration), *AAS Meeting 225, Seattle, Washington, 2015* (American Astronomical Society, 2015), 125:06.
[90] M. Levi, C. Bebek, T. Beers *et al.*, arXiv:1308.0847.
[91] R. Jimenez and A. Loeb, Astrophys. J. **573**, 37 (2002).
[92] H. Stephani, Commun. Math. Phys. **4**, 137 (1967).
[93] M. P. Dąbrowski, J. Math. Phys. (N.Y.) **34**, 1447 (1993).
[94] M. P. Dąbrowski, Astrophys. J. **447**, 43 (1995).
[95] A. Balcerzak, M. P. Dąbrowski, T. Denkiewicz, D. Polarski, and D. Puy, Phys. Rev. D **91**, 083506 (2015).
[96] K. Enqvist, Gen. Relativ. Gravit. **40**, 451 (2008).
[97] J. Garcia-Bellido and T. Haugboelle, J. Cosmol. Astropart. Phys. 04 (2008) 003.
[98] M. Moresco, R. Jimenez, A. Cimatti, and L. Pozzetti, J. Cosmol. Astropart. Phys. 03 (**2011**) 045.
[99] R. Jimenez, P. Talavera, L. Verde, M. Moresco, A. Cimatti, and L. Pozzetti, J. Cosmol. Astropart. Phys. 03 (2012) 014.
[100] M. Moresco, L. Verde, L. Pozzetti, R. Jimenez, and A. Cimatti, J. Cosmol. Astropart. Phys. 07 (2012) 053.
[101] M. Moresco, R. Jimenez, A. Cimatti *et al.*, J. Cosmol. Astropart. Phys. 08 (2012) 006.
[102] M. Moresco, Mon. Not. R. Astron. Soc. **450**, L16 (2015).
[103] M. Moresco, L. Pozzetti, A. Cimatti, R. Jimenez, C. Maraston, L. Verde, D. Thomas, A. Citro, R. Tojeiro,

and D. Wilkinson, J. Cosmol. Astropart. Phys. 05 (2016) 014.
[104] R.-G. Cai, Z.-K. Guo, and T. Yang, J. Cosmol. Astropart. Phys. 08 (2016) 016.
[105] J. Magueijo, Phys. Rev. D **62**, 103521 (2000).
[106] P. P. Avelino, C. J. A. P. Martins, and G. Rocha, Phys. Lett. B **483**, 210 (2000).
[107] K. M. Evenson, J. S. Wells, F. R. Petersen, B. L. Danielson, G. W. Day, R. L. Barger, and J. L. Hall, Phys. Rev. Lett. **29**, 1346 (1972).
[108] A. Balcerzak and M. P. Dąbrowski, Phys. Lett. B **728**, 15 (2014).
[109] A. Balcerzak and M. P. Dąbrowski, J. Cosmol. Astropart. Phys. 06 (2014) 035.
[110] A. Font-Ribera, P. McDonald, N. Mostek, B. A. Reid, H.-J. Seo, and A. Slosar, J. Cosmol. Astropart. Phys. 05 (2014) 023.
[111] S. Yahya, P. Bull, M. G. Santos, M. Silva, R. Maartens, P. Okouma, and B. Bassett, Mon. Not. R. Astron. Soc. **450**, 2251 (2015).
[112] H.-J. Seo and D. J. Eisenstein, Astrophys. J. **665**, 14 (2007).
[113] M. Takada and O. Doré, Phys. Rev. D **92**, 123518 (2015).