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Large-scale tidal effect on redshift-space power spectrum in a finite-volume survey

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Long-wavelength matter inhomogeneities contain cleaner information on the nature of primordial perturbations as well as the physics of the early Universe. The large-scale coherent overdensity and tidal force, not directly observable for a finite-volume galaxy survey, are both related to the Hessian of large-scale gravitational potential and therefore are of equal importance. We show that the coherent tidal force causes a homogeneous anisotropic distortion of the observed distribution of galaxies in all three directions, perpendicular and parallel to the line-of-sight direction. This effect mimics the redshift-space distortion signal of galaxy peculiar velocities, as well as a distortion by the Alcock-Paczynski effect. We quantify its impact on the redshift-space power spectrum to the leading order, and discuss its importance for ongoing and upcoming galaxy surveys.

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I. INTRODUCTION

Observations of large-scale structure in the Universe through a wide-area spectroscopic survey of galaxies are a very powerful probe of fundamental physics, e.g., to test the nature of dark energy via the baryon acoustic oscillation (BAO) measurements of cosmological distances [1–4], to test the gravity theory on cosmological scales [5], to weigh the neutrino mass [6–8], to extract the physics of the early Universe [9–11], to constrain the spatial curvature [12], and to constrain the abundance of light relics such as axions [13]. The current-generation galaxy surveys such as the SDSS Baryon Oscillation Spectroscopic Survey (BOSS) have provided stringent cosmological constraints that are yet complementary to constraints from the cosmic microwave background [14,15]. There are upcoming wide-area galaxy surveys probing the three-dimensional distribution of galaxies at higher redshifts: the Subaru Prime Focus Spectrograph [16], the Dark Energy Spectrograph Instrument [17], the ESA Euclid [18], the NASA SPHEREx [19], and the NASA WFIRST-AFTA [20].

To attain the full potential of wide-area galaxy surveys, it is crucial to understand the statistical properties of large-scale structure probes. Even though the initial density field is nearly Gaussian, the subsequent nonlinear evolution of structure formation causes substantial non-Gaussian features in the observed distribution of galaxies and matter [21]. Most of the useful cosmological information lies in the weakly or deeply nonlinear regime, where different Fourier modes are no longer independent but tightly coupled.

The fact that any galaxy survey has to be done within a finite volume also causes an unavoidable uncertainty in the actual cosmological analysis. Matter density perturbations with very long wavelengths outside a survey volume, hereafter called supersurvey modes, should be present, but are not directly observable. In the nonlinear regime of structure formation, the supersurvey modes become coupled to short-wavelength modes inside the survey volume. Consequently cosmological probes measured from a given survey region are modulated coherently by the supersurvey modes, and the effects need to be taken into account in the analysis in order not to have any bias in cosmological parameter estimation. In addition the supersurvey modes are tricky to consider, because their effects vanish for N-body simulations with periodic boundary conditions that have no contribution of modes outside the simulation box.

Various works have studied the supersurvey effects for cosmological observables such as the weak lensing correlation functions [22–37]. Most of them focused on the effects of the large-scale coherent overdensity, denoted by $\delta_{\rm b}$ (see [29] for a unified formulation of the effect). The effect of $\delta_{\rm b}$ on subsurvey modes for a cold dark matter model with the cosmological constant (ΛCDM) can be absorbed into an apparent curvature parameter of the local volume—a separate universe picture [32,36,38–42]. This approach allows one to include the fully nonlinear mode-coupling of $\delta_{\rm b}$ with all short-wavelength modes, by performing N-body simulations on a perturbed background correctly capturing the local expansion.

However, the effects of a long-wavelength and coherent gravitational tidal force on short-wavelength modes have yet to be fully studied. The coherent overdensity and the coherent tidal force are both related to the Hessian of the gravitational potential (or more generally the metric perturbations), and have comparable amplitudes in each realization. Since the long-wavelength tidal field could have a direct link to the physics of the early Universe (e.g., [11,43–45]), it would be interesting to explore the effects from the observed galaxy distribution. Recently Ip and Schmidt [46] developed a formulation to describe effects of the coherent tidal force on nonlinear structure formation in a local volume within the framework of general relativity (see also [47–49]). In this paper we study how the supersurvey coherent tidal force causes an apparent anisotropic clustering in the galaxy distribution. We will show that the effects appear to look like the redshift-space distortion (RSD) due to peculiar motions of galaxies as well as the Alcock-Paczynski (AP) effect.

The structure of this paper is as follows. In Sec. II we derive a formula to describe an effect of the large-scale coherent gravitational tidal force on the redshift-space galaxy power spectrum measured in a given realization of a finite-volume survey, followed by its contribution to the covariance matrix of the quadrupole power spectrum. In Sec. III, we assess its impact on the quadrupole power spectrum measurement for a hypothetical galaxy survey. Section IV is devoted to discussion. In the Appendix we derive the response of the power spectrum to the large-scale tide, based on the perturbation theory.

II. SUPERSURVEY TIDAL EFFECT

A. Supersurvey modes

For the purpose of the following discussion let us consider the gravitational potential field smoothed with a survey window function,

$$\Phi_W(\mathbf{x}) \equiv \frac{1}{V_W}\int \mathrm{d}^3\mathbf{y}\Phi(\mathbf{y})W(\mathbf{y}-\mathbf{x}), \tag{1}$$

where $V_W = \int \mathrm{d}^3\mathbf{y}W(\mathbf{y}-\mathbf{x})$. For simplicity throughout the paper we assume a connected survey geometry, which does not have any hole or masked region. The survey window thus defines the boundary of a survey region around the fiducial point $\mathbf{x}$; $W(\mathbf{y}-\mathbf{x}) = 1$ if the vector $\mathbf{y}-\mathbf{x}$ is inside a survey region, otherwise $W(\mathbf{y}-\mathbf{x}) = 0$. In this way we can consider $\Phi_W(\mathbf{x})$ as the smoothed gravitational field as a function of the position $\mathbf{x}$. If a typical length scale of the survey window is L, the above integration smooths out all fluctuations with scales smaller than L around the position $\mathbf{x}$. $\Phi_W(\mathbf{x})$ only varies significantly on scales comparable to or greater than L.

Now suppose that a hypothetical survey region is located at the position $\mathbf{x}_0$. Then consider Taylor-expanding the smoothed gravitational field around the position $\mathbf{x}_0$ as

$$\begin{aligned}\Phi_W(\mathbf{x}) &= \Phi_W(\mathbf{x}_0) + \nabla_i\Phi_W|_{\mathbf{x}_0}x^i + \frac{1}{2}\nabla_i\nabla_j\Phi_W|_{\mathbf{x}_0}x^ix^j \\ &\quad + \mathcal{O}(\nabla^3\Phi_W|_{\mathbf{x}_0}x^3) \\ &= \Phi_W(\mathbf{x}_0) + \nabla_i\Phi_W|_{\mathbf{x}_0}x^i + \frac{2}{3}\pi G\bar{\rho}_\mathrm{m}a^2\delta_\mathrm{b}|_{\mathbf{x}_0}x^2 \\ &\quad + 2\pi G\bar{\rho}_\mathrm{m}a^2\tau_{Wij}|_{\mathbf{x}_0}x^ix^j + \mathcal{O}(\nabla^3\Phi_W|_{\mathbf{x}_0}x^3),\end{aligned} \tag{2}$$

where the comoving displacement $x^i \equiv (\mathbf{x}-\mathbf{x}_0)^i$, $\nabla_i \equiv \partial/\partial x^i$, $a(t)$ is the scale factor of the global universe, and δ_b is the smoothed overdensity in the survey window (see below). We have used the Poisson equation, $\Delta\Phi(\mathbf{x}) = 4\pi G\bar{\rho}_\mathrm{m}a^2\delta(\mathbf{x})$. τ_{Wij} is the smoothed tidal field defined as the traceless Hessian matrix of the smoothed gravitational field

$$\tau_{Wij} \equiv \frac{1}{4\pi G\bar{\rho}_\mathrm{m}a^2}\left(\Phi_{W,ij} - \frac{1}{3}\delta^K_{ij}\Delta\Phi_W\right), \tag{3}$$

and δ^K_{ij} is the Kronecker delta function. We introduced the prefactor $(1/4\pi G\bar{\rho}_\mathrm{m}a^2)$ in the definition of τ_{Wij} to make it dimensionless. By using the properties of survey window as well as the partial integral, we can rewrite the partial derivatives of the smoothed gravitational field, for example, as

$$\begin{aligned}\nabla_i\Phi_W|_{\mathbf{x}_0} &\equiv \frac{\partial}{\partial x^i}\left[\frac{1}{V_W}\int \mathrm{d}^3\mathbf{y}\Phi(\mathbf{y})W(\mathbf{y}-\mathbf{x})\right]_{\mathbf{x}_0} \\ &= \frac{1}{V_W}\int \mathrm{d}^3\mathbf{y}\Phi(\mathbf{y})\frac{\partial W(\mathbf{y}-\mathbf{x})}{\partial x^i}\bigg|_{\mathbf{x}_0} \\ &= \frac{1}{V_W}\int \mathrm{d}^3\mathbf{y}\Phi(\mathbf{y})(-1)\frac{\partial W(\mathbf{y}-\mathbf{x})}{\partial y^i}\bigg|_{\mathbf{x}_0} \\ &= \frac{1}{V_W}\left[\int \mathrm{d}^3\mathbf{y}\frac{\partial}{\partial y^i}\{\Phi(\mathbf{y})(-1)W(\mathbf{y}-\mathbf{x})\}\right. \\ &\quad \left. - \int \mathrm{d}^3\mathbf{y}\frac{\partial\Phi(\mathbf{y})}{\partial y^i}(-1)W(\mathbf{y}-\mathbf{x})\right]_{\mathbf{x}_0} \\ &= \frac{1}{V_W}\int \mathrm{d}^3\mathbf{y}\frac{\partial\Phi(\mathbf{y})}{\partial y^i}W(\mathbf{y}-\mathbf{x})\bigg|_{\mathbf{x}_0} = \Phi_{W,i}|_{\mathbf{x}_0}.\end{aligned} \tag{4}$$

That is, the derivatives of the smoothed field in Eq. (2) are equivalent to the survey window average of the derivatives of the gravitational potential field. With this equality, we rewrote the Laplacian of the smoothed field in the third line on the right-hand side of Eq. (2) as

$$\begin{aligned}\Delta\Phi_W|_{\mathbf{x}_0} &= \frac{1}{V_W}\int \mathrm{d}^3\mathbf{y}\Delta\Phi(\mathbf{y})W(|\mathbf{y}-\mathbf{x}|)\bigg|_{\mathbf{x}_0} \\ &= \frac{1}{V_W}\int \mathrm{d}^3\mathbf{y}4\pi G\bar{\rho}_\mathrm{m}a^2\delta(\mathbf{y})W(|\mathbf{y}-\mathbf{x}|)\bigg|_{\mathbf{x}_0} \\ &= 4\pi G\bar{\rho}_\mathrm{m}a^2\delta_\mathrm{b}|_{\mathbf{x}_0},\end{aligned} \tag{5}$$

where $\delta_{\rm b}(\mathbf{x}_0) \equiv (1/V_W)\int d^3\mathbf{x} W(\mathbf{y}-\mathbf{x}_0)\delta(\mathbf{y})$. All the coefficients of x^n on the right-hand side of Eq. (2) are evaluated at the position $\mathbf{x}_0$, at a given time, $\delta_{\rm b} = \delta_{\rm b}(t)$ and $\tau_{Wij} = \tau_{Wij}(t)$. Hereafter we will often omit the dependence of $\mathbf{x}_0$ in the supersurvey modes when considering a fixed position of the survey region. As long as the survey window is sufficiently large, the supersurvey modes evolve linearly; i.e., $\delta_{\rm b} \propto D(t)$ and $\tau_{Wij} \propto \Phi_W/(\bar{\rho}_{\rm m}a^2) \propto D(t)$, where $D(t)$ is the linear growth function [50].

The ensemble averages of the supersurvey modes, which are equivalent to the average when varying the position $\mathbf{x}_0$ for a fixed survey window function, can be estimated based on the linear theory for an assumed ΛCDM model. For a general survey window $\langle\Phi_W\rangle = \langle\tau_{Wij}\rangle = \langle\delta_{\rm b}\rangle = 0$ and their variances are expressed as

$$\sigma_{\rm b}^2 \equiv \langle\delta_{\rm b}^2\rangle = \frac{1}{V_W^2}\int\frac{d^3\mathbf{q}}{(2\pi)^3}P^L(q)|\tilde{W}(\mathbf{q})|^2,$$

$$(\sigma_{\tau_{ij}\tau_{lm}})^2 \equiv \langle\tau_{Wij}\tau_{Wlm}\rangle = \frac{1}{V_W^2}\int\frac{d^3\mathbf{q}}{(2\pi)^3}\left(\hat{q}_i\hat{q}_j - \frac{\delta^K_{ij}}{3}\right) \times \left(\hat{q}_l\hat{q}_m - \frac{\delta^K_{lm}}{3}\right)P^L(q)|\tilde{W}(\mathbf{q})|^2, \quad (6)$$

where $\hat{q}_i \equiv q_i/q$, we have used $\langle\tilde{\delta}_{\mathbf{k}}\tilde{\delta}_{\mathbf{k}'}\rangle \equiv (2\pi)^3P^L(k)\delta_D^3(\mathbf{k}+\mathbf{k}')$ as well as the Poisson equation in the Fourier space, $-k^2\tilde{\Phi}_{\mathbf{k}} = (4\pi G\bar{\rho}_{\rm m}a^2)\tilde{\delta}_{\mathbf{k}}$, and $P^L(k)$ is the linear mass power spectrum. For a general survey window, $\langle\delta_{\rm b}\tau_{Wij}\rangle \neq 0$. The linear variance $\sigma_{\rm b}$ and $\langle\tau_{Wij}\tau_{Wlm}\rangle$ can be easily computed for any survey geometry, either by evaluating Eq. (6) directly or using Gaussian realizations of the linear density field. Note that, even for a fixed survey volume, different components of the linear tidal variance $\langle\tau_{Wij}\tau_{Wlm}\rangle$ generally have different amplitudes for an irregular-shaped window; for example, if a survey window has a collapsed shape rather than an isotropic shape, $\langle\tau_{Wij}\tau_{Wlm}\rangle$ has a greater amplitude for the components corresponding to the smaller window size (see below for further discussion).

For an isotropic window, $\tilde{W}(\mathbf{q}) = \tilde{W}(q)$, the components of the linear tidal variance are simplified as

$$\langle\delta_{\rm b}\tau_{Wij}\rangle = 0,$$

$$\sigma_\tau^2 \equiv \langle(\tau_{W11})^2\rangle = \langle(\tau_{W22})^2\rangle = \langle(\tau_{W33})^2\rangle = \frac{3}{4}\langle(\tau_{Wij})^2\rangle_{i\neq j} = \frac{4}{45V_W^2}\int\frac{q^2dq}{2\pi^2}P_\delta^L(q)|\tilde{W}(q)|^2 = \frac{4}{45}\sigma_{\rm b}^2, \quad (7)$$

and other variances are vanishing: $\langle\tau_{Wij}\tau_{Wlm}\rangle = 0$. Thus the large-scale overdensity and tidal variances have comparable amplitudes, $\sigma_\tau \sim \sigma_{\rm b}/3$, because both are related to the Hessian of the gravitational field. No correlation between $\delta_{\rm b}$ and τ_{Wij} means that the two carry independent information of the supersurvey modes. The higher-order derivatives than $\Phi_{W,ijl}$ in Eq. (2) are more sensitive to smaller-scale modes (larger-k modes), and are suppressed by a factor of $(x/L)^n$, where x is the length scale of subsurvey modes we are interested in and L is the survey size. Hence $\delta_{\rm b}$ and τ_{Wij} give leading-order contributions to the supersurvey effects. In fact Li *et al.* [32] showed that the higher-order contributions with $n \geq 3$ seem negligible for the matter power spectrum, using the separate universe simulations.

B. The redshift-space power spectrum

Let us consider structure formation in a finite-volume survey window in the Universe. To do this we employ a "separate universe picture" [32,36,41,42]: we consider time evolution of motions of particles comoving with this finite volume region in a Lagrangian picture, which is separated from the global universe. As can be found from Eq. (2), the same coherent force arising from the large-scale gravitational field, $\nabla_{\mathbf{x}}\Phi_W(\mathbf{x})$, acts on all particles inside the survey region. The first term of Eq. (2), $\Phi_W|_{\mathbf{x}_0}$, is vanishing, and is thus irrelevant for $\nabla_{\mathbf{x}}\Phi_W(\mathbf{x})$. The force from the second term, $\Phi_{W,i}|_{\mathbf{x}_0}$, causes a parallel translation of all the particles by the same amount, and does not cause any additional clustering inside the survey region. The force arising from the third and fourth terms ($\delta_{\rm b}$ and τ_{Wij}) causes the leading-order effect on which we focus in this paper. If we consider particles that were initially comoving with the global comoving coordinates at a sufficiently high redshift (where $|\delta_{\rm b}|, |\tau_{Wij}| \ll 1$), their subsequent trajectories deviate from the global comoving coordinates as time goes by, due to the large-scale gravitational force. That is, their equation of motion in this "separate" survey region is given as

$$\ddot{X}^i = -\frac{4}{3}\pi G\bar{\rho}_{\rm m}(1+\delta_{\rm b})X^i + \frac{\Lambda}{3}X^i - 4\pi G\bar{\rho}_{\rm m}\tau_{Wij}X^j, \quad (8)$$

where X^i is the displacement vector between the two particles (the initially comoving particles) in the physical coordinates, and we have taken into account the gravitational force for a background universe, including the effect of the cosmological constant [50]. The above Eq. (8) can be realized as a modified Friedmann-Robertson-Walker (FRW) equation that describes an effective expansion of the local survey region due to the presence of supersurvey modes. The term involving $\delta_{\rm b}$ causes a greater or smaller gravitational force relative to the FRW background, if the survey region is embedded into a coherent over- or underdensity region ($\delta_{\rm b} > 0$ or < 0), respectively. This effect can be absorbed by a redefinition of the background density, $\bar{\rho}_{W{\rm m}} = \bar{\rho}_{\rm m}(1+\delta_{\rm b})$, as can be found from the above equation. The effect on the growth of subsurvey modes, through the nonlinear mode coupling, can be described by introducing an apparent curvature parameter of the order of δ_b in the effective FRW equation of the local universe, in the separate universe picture [32] (see also [33,39,42]). The term involving the supersurvey tidal tensor τ_{Wij} is a novel effect, and causes a homogeneous anisotropic expansion due to its tensor nature. By "homogeneous" we mean here that the expansion rate between two points inside the local volume is the same, or homogeneous, independently of where the two points are placed inside the volume, as long as the two points are taken along the same direction, as in the Hubble law. This homogeneity is guaranteed by the assumption that here we considered only up to the second order of the Taylor expansion of the gravitational potential, which is the leading-order effects of the supersurvey modes as we discussed above.

Using the Zel'dovich approximation [51] or the linearized Lagrangian perturbation theory (e.g., [52]), the effect of supersurvey modes on the local expansion can be described by a temporal perturbation of the comoving coordinates of the local survey region as

$$q_{Wi} = q_i + \Psi_{Wij}(t) q_j, \tag{9}$$

where

$$\Psi_{Wij}(t) = \frac{\delta^K_{ij}}{3}\delta_\mathrm{b}(t) + \tau_{Wij}(t). \tag{10}$$

Here q_{Wi} are the perturbed comoving coordinates in the local survey region, and q_i is the comoving coordinate of the global background. In the following, quantities with or without subscript "W" denote the quantities in the local survey volume or the global background, respectively. For a sufficiently high redshift, $|\Psi_{Wij}| \ll 1$, $q_{Wi} \simeq q_i$. Hence, the Lagrangian coordinates in the local volume can be defined by the global comoving coordinates at sufficiently high redshift. These effects can be also described by a modification of the scale factor of the local background. Note that, in the separate universe picture, the physical length scale should be kept the same in the local volume and the global background, as discussed in Li *et al.* [32],

$$a_W \lambda_W = a\lambda, \tag{11}$$

where λ_W and λ are in the comoving wavelength scales. Hence, the effect of δ_b can also be realized as a modification of the scale factor: $a_W(t) \simeq a(t)[1 - \delta_\mathrm{b}(t)/3]$ up to the linear order of δ_b, which reproduces the results around Eq. (35) in Li *et al.* [32]. On the other hand, the coherent tidal force τ_{Wij} causes a homogeneous anisotropic expansion effect on the local comoving coordinates. If we take the axes of local comoving coordinates along the principal axes of the coherent tidal force, which can be done without loss of generality, the tensor τ_{Wij} becomes diagonal: $\tau_{Wij} = \tau_{Wi}\delta^K_{ij}$. Then the deformation of the comoving coordinates can be realized as a homogeneous anisotropic deformation of the scale factor along each axis up to the linear order of τ_{Wi}: $a_{Wi}(t) \simeq a(t)[1 - \tau_{Wi}(t)]$, satisfying the trace condition $\mathrm{Tr}(a_{Wi}) = 3a(t)$ (see also [46,48,49] for similar discussion).

As discussed in Sherwin and Zaldarriaga [27] (see also [29,32]), the supersurvey modes affect the clustering correlation function measured in the local survey volume. Extending the method in Sherwin and Zaldarriaga [27] to include the coherent tidal force, we can deduce that the clustering correlation function of total matter, $\xi_W(\mathbf{r})$, in the local volume is modified, up to the linear order of the supersurvey modes, as

$$\begin{aligned}\xi_W(\mathbf{r}) &\equiv \langle \delta(\mathbf{q}_{W1})\delta(\mathbf{q}_{W2})\rangle_{\mathbf{r}=\mathbf{q}_{W1}-\mathbf{q}_{W2}} \\ &= \left(1+\frac{68}{21}\delta_\mathrm{b}\right)\xi\left(r^i + \frac{\delta_\mathrm{b}}{3}r^i + \tau_{Wij}r^j\right) \\ &\simeq \xi(r) + \left[\frac{68}{21}\xi(r) + \frac{1}{3}r^i\frac{\partial\xi(r)}{\partial r^i}\right]\delta_\mathrm{b} + \frac{\partial\xi(r)}{\partial r^i}r^j\tau_{Wij}.\end{aligned} \tag{12}$$

Note that the above correlation function is from the ensemble average of subsurvey modes on a realization basis of the local volume that has the fixed supersurvey modes δ_b and τ_{Wij}. Equation (12) shows that, even if the real-space clustering is isotropic, the correlation function measured in the local volume generally becomes two-dimensional due to the coherent tidal force. It causes an apparent anisotropic clustering in the local volume, and the amount of the anisotropic clustering depends on angles between the directions of τ_{Wij} and the separation vector $\mathbf{r}$.

Fourier-transforming Eq. (12), we can find that the power spectrum measured in the local volume is modified as

$$\begin{aligned}P_W(\mathbf{k}) &\simeq P(k) + \delta_\mathrm{b}\left[\frac{47}{21} - \frac{1}{3}\frac{\partial \ln P(k)}{\partial \ln k}\right]P(k) \\ &\quad - \tau_{Wij}\hat{k}_i\hat{k}_j\frac{\partial P(k)}{\partial \ln k},\end{aligned} \tag{13}$$

where $\hat{k}_i \equiv k_i/k$. Furthermore, in the Appendix, we use the formulation in Takada and Hu [29] to derive the full expression for the responses of the power spectrum to the supersurvey modes in the weakly nonlinear regime. We show that the large-scale tide also causes a change in the amplitude of the power spectrum. Thus the full expression is given as

$$\begin{aligned}P_W(\mathbf{k}) &\simeq P(k) + \delta_\mathrm{b}\left[\frac{47}{21} - \frac{1}{3}\frac{\partial \ln P(k)}{\partial \ln k}\right]P(k) \\ &\quad + \hat{k}_i\hat{k}_j\tau_{Wij}\left[\frac{8}{7} - \frac{\partial \ln P(k)}{\partial \ln k}\right]P(k).\end{aligned} \tag{14}$$

The term with prefactor $8/7$ gives the effect of the large-scale tide on the power spectrum amplitude. For an arbitrary line-of-sight direction that an observer takes, the anisotropic power spectrum in the above equation appears exactly similar to the Alcock-Paczynski distortion effect [53] (see also [1,2,7]) as well as the RSD effect, the Kaiser effect [54]. The large-scale overdensity δ_b alters the power spectrum amplitude as well as causes an isotropic dilation effect that is given by the term involving $\partial P(k)/\partial k$. Note that the terms involving δ_b reproduce the 2-halo term of Eq. (27) in Li *et al.* [32] (see also [29]). On the other hand, the coherent tidal force causes a homogeneous anisotropic dilation in all three directions, perpendicular and parallel to the line-of-sight direction, while the RSD effect causes a distortion of the clustering along the line-of-sight direction. In particular, the terms involving the power spectrum derivative, $\partial P/\partial k$, cause a shift of the BAO peak location compared to what the BAO location should be in the global background (see also [27] for the effect of δ_b on the BAO peak location). Due to the tensor nature of τ_{Wij}, the directional dependence of $k^i k^j$ causes a quadratic anisotropy in the power spectrum. Thus the coherent tidal force causes a systematic error when estimating the Hubble expansion rate and the angular diameter distance from the anisotropic clustering via the AP effect.

Next we consider effects of supersurvey modes on the redshift-space power spectrum of galaxies. Galaxies are biased tracers of the underlying matter distribution in the large-scale structure. In this paper, we assume that the number density fluctuation field of galaxies is locally related to the matter density fluctuation field at the same position via a linear bias parameter b: $\delta_g(\mathbf{x}) = b\delta_m(\mathbf{x})$. As shown in Hu and Kravtsov [22], the mean number density of galaxies in a finite-volume survey is modulated from the global mean by δ_b as

$$\bar{n}_{gW} \simeq \bar{n}_g[1 + b\delta_b]. \tag{15}$$

Then the two-point correlation function of the galaxies in a local volume is estimated relative to the local mean density, $\xi_{gW}(\mathbf{r}) = \langle n_g(\mathbf{x}) n_g(\mathbf{x}+\mathbf{r})\rangle/\bar{n}_{gW}^2 - 1$. As discussed in Li *et al.* [32] (see also [55]), the real-space power spectrum of galaxies is modified by supersurvey modes as

$$P_{gW}(k) \simeq b^2(1 - 2b\delta_b)P_W(k). \tag{16}$$

Combining this with the supersurvey effects [Eq. (13)] and the Kaiser RSD effect, we can find that the redshift-space power spectrum of galaxies is given as

$$P^S_{gW}(\mathbf{k}) = [1+\beta\mu^2]^2 \times \left[P_g(k) + \delta_b\left\{\frac{47}{21} - 2b - \frac{1}{3}\frac{\partial \ln P_g(k)}{\partial \ln k}\right\}P_g(k) + \hat{k}_i\hat{k}_j\tau_{Wij}\left\{\frac{8}{7} - \frac{\partial \ln P_g(k)}{\partial \ln k}\right\}P_g(k)\right], \tag{17}$$

where $P_g(k)$ is the real-space power spectrum in the global background, μ is the cosine angle between the line-of-sight direction and the wave vector $\mathbf{k}$, and $\beta \equiv (1/b)\mathrm{d}\ln D/\mathrm{d}\ln a$. In the above equation we simply assumed that the Kaiser RSD effect causes an additional distortion of the galaxy distribution, and treated the effect as a multiplicative factor to the real-space power spectrum (see below for further discussion). Thus the redshift-space power spectrum in the presence of the supersurvey effects have redshift-space distortions up to μ^6, in the weakly nonlinear regime. In the following we focus on the effect of τ_{Wij} on the redshift-space spectrum, and ignore the effect of δ_b (i.e., set $\delta_b = 0$).

Now we consider the multipole power spectra that are useful spectra to quantify the RSD effects. Without loss of generality, we can assume that the z-axis direction in the local coordinates is along the line-of-sight direction of an observer. Taking into account the fact that the coherent tidal force also causes an anisotropic dilation even in the xy plane perpendicular to the line-of-sight direction, where the redshift distortion effect is absent, we can define the multipole power spectrum as

$$P^S_{g\ell W}(k) = (2\ell+1)\int_{-1}^{1}\frac{\mathrm{d}\mu}{2}\int_0^{2\pi}\frac{\mathrm{d}\varphi}{2\pi}P^S_{gW}(\mathbf{k})\mathcal{L}_\ell(\mu), \tag{18}$$

where φ and μ are the angle and cosine angle between the coordinate axes and the wave vector $\mathbf{k}$, i.e., $\mathbf{k} \equiv k(\sqrt{1-\mu^2}\cos\varphi, \sqrt{1-\mu^2}\sin\varphi, \mu)$, and $\mathcal{L}_\ell(\mu)$ is the ℓth order Legendre polynomial; $\mathcal{L}_0(\mu) = 1$ and $\mathcal{L}_2(\mu) = (3\mu^2-1)/2$, which are relevant for the following calculation.

The monopole power spectrum is found to be

$$\begin{aligned} P^S_{g0W}(k) &= \left(1 + \frac{2\beta}{3} + \frac{\beta^2}{5}\right)P_g(k) + \left(\frac{1}{3} + \frac{2\beta}{5} + \frac{\beta^2}{7}\right) \\ &\quad \times (\tau_{W11} + \tau_{W22} + \tau_{W33})\left[\frac{8}{7} - \frac{\partial \ln P_g(k)}{\partial \ln k}\right]P_g(k) \\ &= \left(1 + \frac{2\beta}{3} + \frac{\beta^2}{5}\right)P_g(k), \end{aligned} \tag{19}$$

where we used $\mathrm{Tr}(\tau_{Wij}) = 0$. Thus the coherent tidal force does not affect the monopole power spectrum because of the trace-free nature of τ_{Wij}.

On the other hand, the supersurvey tide causes a modulation in the quadrupole power spectrum,

$$\begin{aligned} P^S_{g2W}(k) &= \left(\frac{4\beta}{3} + \frac{4\beta^2}{7}\right)P_g(k) + \left(1 + \frac{22\beta}{21} + \frac{3\beta^2}{7}\right)\tau_{W33} \\ &\quad \times \left[\frac{8}{7} - \frac{\partial \ln P_g(k)}{\partial \ln k}\right]P_g(k), \end{aligned} \tag{20}$$

where we used the fact $\tau_{W11} + \tau_{W22} = -\tau_{W33}$ in deriving the above equation. Since the quadrupole power spectrum amplitude depends on β, or in other words there is no contribution from the monopole power spectrum, it is a useful probe of the growth rate. However, the coherent tidal force causes an extra contribution to the quadrupole power spectrum (the second term on the rhs). As we emphasized above, the tidal effect varies with a position of the survey region, and in this sense τ_{Wij} is a statistical variable. Note that even if the coherent density mode $\delta_{\rm b}$ exists in the survey region, it only affects the amplitude of the quadrupole power spectrum, and the effect is therefore perfectly degenerate with the bias parameter.

Similarly one can compute the extra contribution to the higher-order multipole power spectra,

$$P^S_{{\rm g}4W}(k) = \frac{8\beta^2}{35}P_{\rm g}(k) + \left(\frac{24\beta}{35}+\frac{136\beta^2}{385}\right)\tau_{W33} \times\left[\frac{8}{7}-\frac{\partial\ln P_{\rm g}(k)}{\partial \ln k}\right]P_{\rm g}(k),$$
$$P^S_{{\rm g}6W}(k) = \frac{8\beta^2}{77}\tau_{W33}\left[\frac{8}{7}-\frac{\partial\ln P_{\rm g}(k)}{\partial \ln k}\right]P_{\rm g}(k) \quad (21)$$

and $P^S_{\ell W}(k)=0$ for $\ell \geq 8$. Thus the coherent tidal force generally induces a nonvanishing P_6 power spectrum, which is absent in the Kaiser formula.

In the following, we consider $P^S_{{\rm g}2W}(k)$, the leading-order anisotropic power spectrum, to study the impact of the coherent tidal force.

C. Supersample covariance

We have so far shown that the supersurvey tidal force affects a measurement of the redshift-space power spectrum, and here estimate how the effect is important compared to a statistical precision of the power spectrum measurement.

Extending the formulation for the real-space power spectrum in Scoccimarro *et al.* [56] (see also [24,29]), we can write down an estimator for the quadrupole power spectrum in a given survey region,

$$\hat{P}^S_{{\rm g}2}(k_i) \equiv \frac{5}{V_W}\int_{|\mathbf{k}|\in k_i}\frac{{\rm d}^3\mathbf{k}}{V_{k_i}}\tilde{\delta}_{{\rm g}W}(\mathbf{k})\tilde{\delta}_{{\rm g}W}(-\mathbf{k})\mathcal{L}_2(\mu), \quad (22)$$

where $\tilde{\delta}_{{\rm g}W}(\mathbf{k})$ is the density fluctuation field of galaxies convolved with the survey window, the prefactor 5 is from the definition of multipole power spectrum [Eq. (18)], $(2l+1)$, the integral is over a shell in k-space of width Δk, and volume $V_{k_i}\simeq 4\pi k_i^2\Delta k$ for $\Delta k/k_i \ll 1$. We have here employed the continuous limit of discrete Fourier transforms under the approximation that the total volume for the Fourier transform is much greater than the survey region (see Refs. [24,28] for a pedagogical derivation of power spectrum estimator and the covariance based on the discrete Fourier decomposition).

Similarly to the formulation in Schaan *et al.* [31], we introduce the ensemble average of subsurvey modes for a fixed coherent tidal force, τ_{Wij}, denoted as $\langle\rangle_{\tau_W}$. When we focus on wave number modes satisfying $k \gg 1/L$, the average of the estimator (22) is computed as

$$\begin{aligned}\langle \hat{P}^S_{{\rm g}2}(k_i)\rangle_{\tau_W} &\equiv \frac{5}{V_W}\int_{|\mathbf{k}|\in k_i}\frac{{\rm d}^3\mathbf{k}}{V_{k_i}}\langle\tilde{\delta}_{{\rm g}W}(\mathbf{k})\tilde{\delta}_{{\rm g}W}(-\mathbf{k})\rangle_{\tau_W}\mathcal{L}_2(\mu)\\
&\simeq \frac{5}{V_W}\int_{|\mathbf{k}|\in k_i}\frac{{\rm d}^3\mathbf{k}}{V_{k_i}}P^S_{{\rm g}W}(\mathbf{k};\tau_W)\mathcal{L}_2(\mu)\int\frac{{\rm d}^3\mathbf{q}}{(2\pi)^3}|\tilde{W}(\mathbf{q})|^2\\
&= 5\int_{|\mathbf{k}|\in k_i}\frac{4\pi k^2{\rm d}k}{V_{k_i}}\int_{-1}^{1}\frac{{\rm d}\mu}{2}P^S_{{\rm g}W}(\mathbf{k};\tau_W)\mathcal{L}_2(\mu)\\
&= \int_{|\mathbf{k}|\in k_i}\frac{4\pi k^2{\rm d}k}{V_{k_i}}\left[\left(\frac{4\beta}{3}+\frac{4\beta^2}{7}\right)P_{\rm g}(k)+\left(1+\frac{22\beta}{21}+\frac{3\beta^2}{7}\right)\tau_{W33}\left\{\frac{8}{7}-\frac{\partial\ln P_{\rm g}(k)}{\partial\ln k}\right\}P_{\rm g}(k)\right]\\
&\simeq \left(\frac{4\beta}{3}+\frac{4\beta^2}{7}\right)P_{\rm g}(k_i)+\left(1+\frac{22\beta}{21}+\frac{3\beta^2}{7}\right)\tau_{W33}\left[\frac{8}{7}-\frac{\partial\ln P_{\rm g}(k)}{\partial\ln k}\right]_{k_i}P_{\rm g}(k_i),\end{aligned} \quad (23)$$

where $P^S_{{\rm g}W}(\mathbf{k};\tau_W)$ is the power spectrum obtained by setting $\delta_{\rm b}=0$ in Eq. (17). Furthermore, because the convolution changes the power spectrum only around $k \lesssim 1/L$ due to the nature of the window function while here we are interested in the spectra of modes satisfying $k \gg 1/L$. That is, we have used $P^S_W(\mathbf{k}-\mathbf{q};\tau_W)\simeq P^S_W(\mathbf{k};\tau_W)$ over the integral range of ${\rm d}^3\mathbf{q}$, and assumed that the power spectrum $P_{\rm g}(k)$ is not a rapidly varying function within k-bin. Thus the average of the estimator [Eq. (22)] for a fixed τ_{Wij} recovers Eq. (20).

Now we introduce the ensemble average that is the average of the estimator with varying positions of the survey regions, denoted as $\langle\rangle$,

$$\langle \hat{P}^S_{g2}(k_i)\rangle \simeq \left(\frac{4\beta}{3}+\frac{4\beta^2}{7}\right)P_g(k_i)+\left(1+\frac{22\beta}{21}+\frac{3\beta^2}{7}\right) \times \langle\tau_{W33}\rangle\left[\frac{8}{7}-\frac{\partial \ln P_g(k)}{\partial \ln k}\right]_{k_i}P_g(k_i) \simeq \left(\frac{4\beta}{3}+\frac{4\beta^2}{7}\right)P_g(k_i), \quad (24)$$

where we assumed $\langle\hat{\tau}_{W33}\rangle = 0$, i.e., the average of the coherent tidal force is vanishing in the ensemble average sense. Thus the ensemble average of the estimator (22) recovers the quadrupole power spectrum in the Kaiser formula.

Now we consider the covariance of the quadrupole power spectrum, defined in terms of the estimator as

$$C_{ij} \equiv \langle \hat{P}_2(k_i)\hat{P}_2(k_j)\rangle - \langle\hat{P}_2(k_i)\rangle\langle\hat{P}_2(k_j)\rangle. \quad (25)$$

Similarly to Takada and Hu [29], we find that the covariance is decomposed into two contributions

$$\mathbf{C} \simeq \mathbf{C}^{\rm G} + \mathbf{C}^{\rm SSC}. \quad (26)$$

The first term is a Gaussian term, and the second term is the non-Gaussian error arising from the coherent tidal force on which we focus in this paper. Here we ignored the trispectrum contribution of subsurvey modes to the sample variance for simplicity.

Following method in Guzik *et al.* [57] and Takada and Hu [29], we can compute the Gaussian term as

$$C^{\rm G}_{ij} \simeq \delta^K_{ij}\frac{25}{V_W}\frac{(2\pi)^3}{V_{k_i}}\int_{|\mathbf{k}|\in k_i}\frac{d^3\mathbf{k}}{V_{k_i}}2[1+\beta\mu^2]^2\left[P_g(k)+\frac{1}{\bar{n}_g}\right]^2[\mathcal{L}_2(\mu)]^2 = \delta^K_{ij}\frac{50}{V_W}\frac{(2\pi)^3}{V_{k_i}}\int_{|\mathbf{k}|\in k_i}\frac{4\pi k^2 dk}{V_{k_i}}\int_{-1}^{1}\frac{d\mu}{2}[1+\beta\mu^2]^2\left[P_g(k)+\frac{1}{\bar{n}_g}\right]^2[\mathcal{L}_2(\mu)]^2 \simeq \delta^K_{ij}\frac{(2\pi)^3}{V_W V_{k_i}}10\left[1+\frac{44}{21}\beta+\frac{18}{7}\beta^2+\frac{340}{231}\beta^2+\frac{415}{1287}\beta^4\right]\left[P_g(k)+\frac{1}{\bar{n}_g}\right]^2, \quad (27)$$

where we have included the shot noise term arising from a finite number of sampled galaxies, given by the terms including $1/\bar{n}_g$. The Gaussian covariance scales as $1/V_W$. More exactly speaking, it scales as the number of independent k-modes in the shell as

$$N_{\rm mode}(k_i) = \frac{V_{k_i}V_W}{(2\pi)^3} \simeq \frac{4\pi k_i^2 \Delta k V_W}{(2\pi)^3}. \quad (28)$$

The Gaussian covariance matrix is diagonal, and, in other words, its off-diagonal components are vanishing.

On the other hand, the supersample covariance (SSC) term is given as

$$C^{\rm SSC}_{ij} = \left(1+\frac{22\beta}{21}+\frac{3\beta^2}{7}\right)^2\left[\frac{8}{7}-\frac{\partial \ln P_g(k)}{\partial \ln k}\right]_{k_i} \times \left[\frac{8}{7}-\frac{\partial \ln P_g(k)}{\partial \ln k}\right]_{k_j}P_g(k_i)P_g(k_j)\sigma^2_{\tau 33}, \quad (29)$$

where $\sigma^2_{\tau 33}$ can be calculated using Eq. (6) for a given cosmological model and survey window, and we have assumed $\langle\delta_b\tau_{W33}\rangle \ll \sigma^2_{\tau 33}$ for a reasonable window. The SSC covariance has off-diagonal components.

In the following we will use Eqs. (27) and (29) to compute the covariance for a measurement of the quadrupole power spectrum for a hypothetical galaxy survey.

III. RESULTS

Throughout this paper, we employ cosmological parameters that are consistent with the nine-year WMAP results [58]: $\Omega_{c0}h^2 = 0.1165$, $\Omega_{b0}h^2 = 0.02248$, and $\Omega_\Lambda = 0.7055$ for the density parameters of CDM, baryon, and the cosmological constant, $A_s = 2.455\times 10^{-9}$ for the amplitude of the primordial curvature perturbation, $n_s = 0.967$ for the tilt of primordial power spectrum, and $h = 0.687$ for the Hubble constant. In this model $\sigma_8 = 0.815$, which is the variance of present-day, linear matter fluctuations within a sphere of radius 8 Mpc$/h$.

The key quantity to characterize the effect of coherent tidal force is the variance of linear tidal field averaged over the survey window, σ_τ [Eq. (6)]. The left panel of Fig. 1 shows the variance for a ΛCDM model, for a spherical window as a function of survey volume V_W. Other covariance term scales with $1/V_W$, so the curve shows the relative contribution of the coherent tidal force to the sample variance. Likewise the effect of supersurvey overdensity σ_b [29], the supersurvey covariance has a weak dependence on the volume. For a sufficiently large cosmological volume such as $V_W \gtrsim 1\ ({\rm Gpc}/h)^3$, $\sigma_\tau \sim 10^{-3}$.

As can be found from Eq. (6), the different components of the linear variance of supersurvey tidal tensor, $\sigma_{\tau ij}$, depends on the shape of survey window. The right panel of Fig. 1 studies this for a cylinder window as a function of the different shape, for a fixed survey volume of

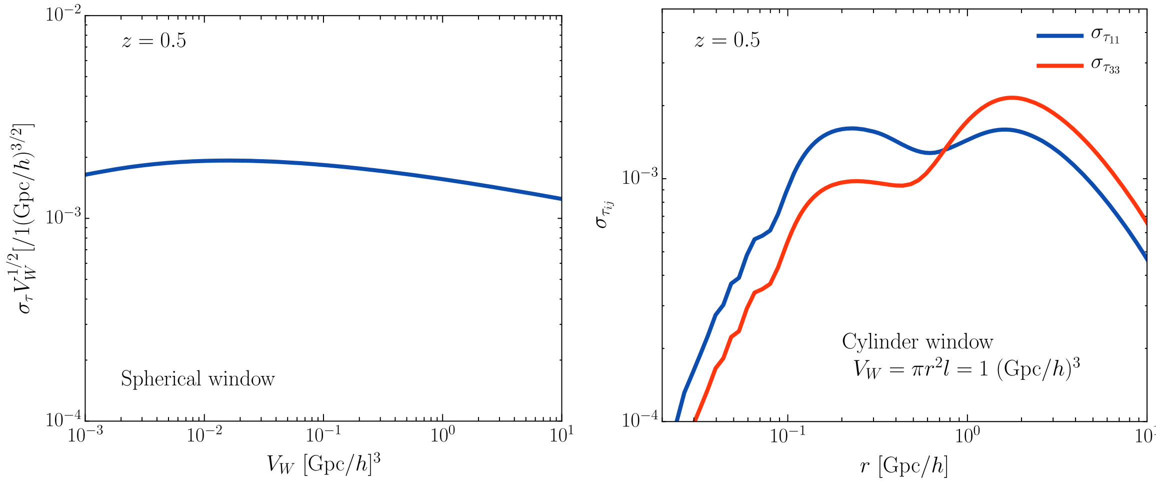

FIG. 1. The rms of linear gravitational tidal field convolved with the survey window, $\sigma_{\tau ij}$ for a ΛCDM model and $z = 0.5$ [see Eqs. (6) and (7)]. Left panel: the rms as a function of survey volume for spherical window, $V_W = 4\pi r^3/3$. In this case, the tidal tensor becomes diagonal: $\sigma_\tau \equiv \sigma_{\tau 11} = \sigma_{\tau 22} = \sigma_{\tau 33}$. Right panel: the rms for cylinder windows of fixed volume $V_W = \pi r^2 \ell = 1\ (\mathrm{Gpc}/h)^3$, as a function of the radius of base circle, r. Here we assume that the height of cylinder is along the 3-axis (the light-of-sight) direction, and the base circle is in the plane perpendicular to the line-of-sight direction (therefore $\sigma_{\tau 11} = \sigma_{\tau 22}$). For an elongated cylinder window, i.e., a tubelike shaped survey, $\sigma_{\tau 11}$ has a greater amplitude, while $\sigma_{\tau 33}$ has a greater amplitude for a pill-like shape. When $r \sim \ell$, $\sigma_{\tau 11} \simeq \sigma_{\tau 33}$, and the linear variance has a largest amplitude.

$V_W = \pi r^2 \ell = 1\ (\mathrm{Gpc}/h)^3$. When $r \ll \ell$, a survey window corresponds to a survey being "narrow" in area coverage on the sky, but deep in redshift direction—a "tube-shaped" survey. A survey with $\ell \ll r$ corresponds to a survey being "wide" in area, but shallow in redshift—a "pill-shaped" survey. The linear variance components have different amplitudes depending on angles between the coordinate axes and the principal axes of tidal tensor. Here we consider the line-of-sight direction to lie along the third-axis direction of an observer coordinate's system and the height direction of the cylinder window (ℓ direction); in this case $\sigma_{\tau 11} = \sigma_{\tau 22}$. The variance components $\sigma_{\tau 33} \simeq \sigma_{\tau 11}$ when $r \simeq 0.7\ \mathrm{Gpc}/h$ or equivalently $\ell \simeq r$. For either case of extreme "tube" or "pill" shape, one component $\sigma_{\tau 11}$ or $\sigma_{\tau 33}$ has a greater amplitude than the other. However, the variance amplitude gets smaller due to the cancellation effect of the linear variances [also see [29]]. However, the extreme cases are not desirable, because one length scale of the volume can be in the nonlinear regime, and the linear-order approximation of the supersurvey modes breaks down.

Figure 2 compares the Gaussian and supersurvey covariance terms in the covariance matrix of the quadrupole power spectrum, for a spherical window of $V_W = 1\ (\mathrm{Gpc}/h)^3$ [see Eqs. (27) and (29)]. Here we assume a survey probing the three-dimensional distribution of galaxies at $z = 0.5$ and with linear bias parameter $b = 2$, which roughly resemble SDSS CMASS-type galaxies [14]. Here we ignored the effect of a finite number density of the galaxies. Since the Gaussian covariance depends on the bin width of wave number, we employ $\Delta \log k = 0.1$. Note that, in order to show the effect of the coherent tidal force on the BAO features, we employed a much finer k-binning to plot the curve, but used $\Delta \log k = 0.1$ to compute the Gaussian term at each k-bin. The figure shows that the supersurvey effect gives a dominant contribution to the sample variance in the weakly nonlinear regime, $k \gtrsim 0.7\ h/\mathrm{Mpc}$.

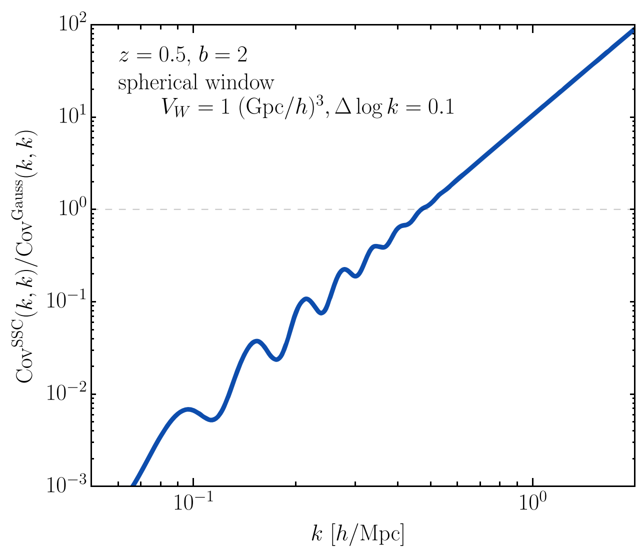

FIG. 2. Shown is how the supersurvey tidal force causes an increase in the sample variance in a measurement of the quadrupole power spectrum of redshift-space galaxy distribution. The curve shows the SSC contribution to the sample variance relative to the Gaussian variance, for a survey with volume $1\ (\mathrm{Gpc}/h)^3$, at $z = 0.5$ and galaxies with linear bias $b = 2$ [Eqs. (27) and (29)]. Here we ignored the shot noise contribution due to a finite number density of galaxies. Since the Gaussian term depends on the bin width of wave number, we assumed a binning of $\Delta \log k = 0.1$ (10 bins in one decade of wave number). The SSC effects cause a significant sample variance at $k \gtrsim 0.5\ h/\mathrm{Mpc}$.

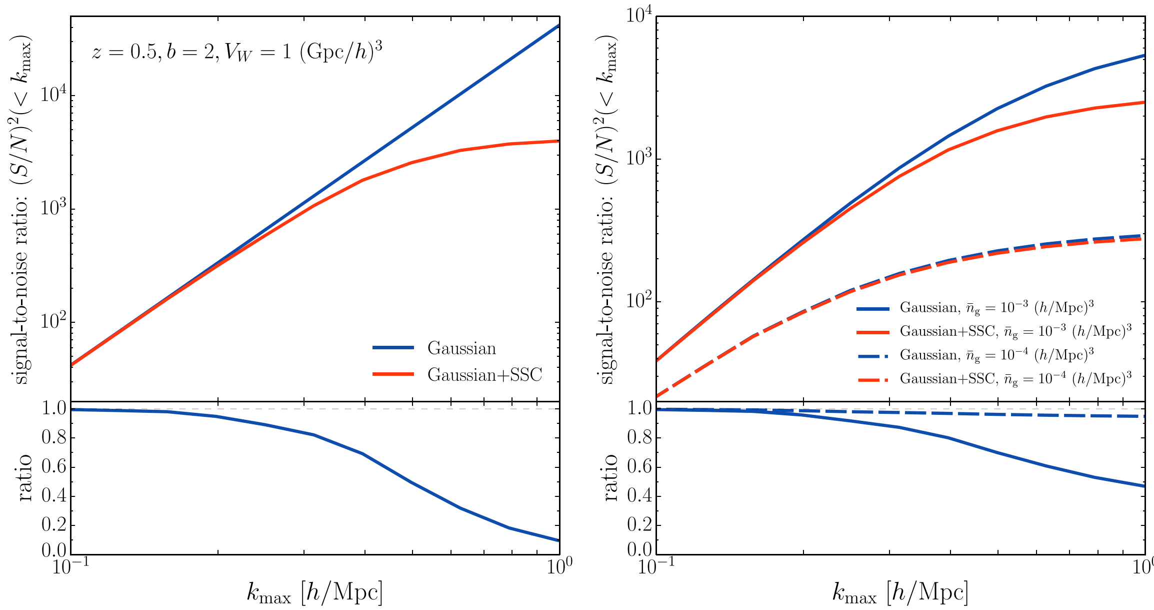

FIG. 3. Cumulative S/N ratio for a measurement of the quadrupole power spectrum, as a function of maximum wave number $k_{\rm max}$. We here assumed $V_W = 1\ (\mathrm{Gpc}/h)^3$, $z = 0.5$, and $b = 2$ as in the previous figure. S/N ratio does not depend on the bin width of k. Left panel: The results for an infinite number density of galaxies; we ignored the shot noise contribution. The upper or lower curves in the upper plot are the results when assuming the Gaussian covariance or including the SSC contribution, respectively. The lower panel shows the ratio. Right panel: A similar plot, but for a finite number density of galaxies: $\bar{n}_{\rm g} = 10^{-3}$ or 10^{-4} $(h/\mathrm{Mpc})^3$, respectively.

As one demonstration of the impact of the coherent tidal force on a measurement of the quadrupole power spectrum in redshift space, we study a cumulative signal-to-noise $(\frac{S}{N})$ ratio, defined as

$$\left(\frac{S}{N}\right)^2_{\le k_{\rm max}} \equiv \sum_{k_i k_j \in k_{\rm max}} P^S_{{\rm g}2W}(k_i)[\mathbf{C}^{-1}]_{ij} P^S_{{\rm g}2W}(k_j), \tag{30}$$

where $\mathbf{C}^{-1}$ is the inverse of the covariance matrix, and the summation runs over all wave number bins up to a given maximum wave number $k_{\rm max}$. This quantity does not depend on the bin width. The inverse of (S/N) gives a statistical precision of measuring the overall amplitude of the power spectrum, if the shape is completely known. Figure 3 shows the results. The supersurvey tidal force causes a degradation in the power spectrum measurement, at $k_{\rm max} \gtrsim$ a few h/Mpc, and for a galaxy survey with a high number density such as $\bar{n}_{\rm g} \simeq 10^{-3} (h/\mathrm{Mpc})^3$, which is the case for the WFIRST-AFTA survey [20].

IV. DISCUSSION

We have derived a formula to describe the effect of supersurvey, coherent tidal force on the redshift-space power spectrum measured in a finite-volume survey. The large-scale coherent overdensity and tidal field both arise from the Hessian of the long-wavelength gravitational potential, and are of equal importance. Since the supersurvey modes are not direct observables, the effects on cosmological observables need to be theoretically modeled. The supersurvey tide causes a characteristic, anisotropic clustering pattern in the distribution of the tracers, in all three directions perpendicular and parallel to the line-of-sight direction [see Eq. (17)]. This effect appears to be exactly similar to the geometrical AP distortion as well as the redshift-space distortion effect of peculiar velocities. We then derived a formula to model the contribution of the coherent tidal force to the sample variance in a measurement of the quadrupole redshift-space power spectrum. We showed that the supersample variance is not negligible if including the power spectrum information up to the weakly nonlinear regime, $k \gtrsim$ a few h/Mpc, or for a galaxy survey with a high number density such as $\bar{n}_{\rm g} \simeq 10^{-3}$ $(h/\mathrm{Mpc})^3$. In our derivation, we have not yet properly included the supersurvey effects on the nonlinear Kaiser factor. This can be done by using the perturbation theory, and will be our future work.

For a galaxy survey with a volume coverage greater than $\sim(\mathrm{Gpc/h})^3$ and $z = 0.5$ as in the SDSS survey, the linear variance of super-survey tidal force $\sigma_\tau \sim 10^{-3}$ for a ΛCDM model. This implies that the supersurvey tide causes about 0.1% anisotropy in the clustering distribution. However, the expectation value of the variance is after t he angle average, compared to the variance of coherent density contrast: $\sigma_\tau \simeq \sqrt{4/45}\sigma_{\rm b} \simeq \sigma_{\rm b}/3.4$ [see Eq. (7)]. If the principal axes of the supersurvey tidal tensor have an alignment to directions parallel and/or perpendicular to the line-of-sight direction, the tide could have a similar amplitude as δ_b in a particular realization: $\tau \sim \delta_b$ corresponding to ~0.3% anisotropy. Since the current state-of-the-art SDSS BOSS survey already achieved about 1% accuracy for the BAO distance measurements [15], the supersurvey tide could cause a bias by an amount of the 1σ statistical error, if the SDSS survey volume is embedded into a particular region if the aligned tide has a 3σ value. Hence, it would be more important to study how the coherent tidal force could cause a bias in measurements of the cosmological distances via the AP test as well as the growth rate via the RSD effect. This requires us to propagate the expected statistical accuracy of the redshift-space power spectrum measurement into parameter estimation for a given survey geometry, including marginalization over other parameters. This will be our future work, and will be presented elsewhere.

The redshift-space clustering of galaxies is anisotropic by nature, and the coherent tidal force causes a similar anisotropic clustering pattern in the observed distribution. For the monopole power spectrum such as the weak lensing power spectrum, the effect disappears at the first order of τ_{Wij} due to the traceless nature $\mathrm{Tr}(\tau_{Wij}) = 0$. There are other effects of the coherent tidal force that can be observed in principle from upcoming wide-area galaxy surveys. First, it is shown that the coherent tidal force causes a modification of dark matter halo formation via a coupling of the inertia of mass distribution in a protohalo region with the coherent tidal force, leaving a nonlocal bias effect relative to the underlying matter distribution at the second order: $\partial\delta_h/\partial(\tau_W^2) = b_\tau$ [59,60]. The nonlocal bias can be measured by combining measurements of the power spectrum (two-point) and bispectrum (three-point) of large-scale structure tracers. Another observable is the correlation of the large-scale tidal force with shapes of galaxies at much smaller scales, the so-called intrinsic alignments [61,62]. The intrinsic alignments are one of the major systematic effects for ongoing and upcoming weak lensing surveys. Conversely, the intrinsic alignments can be regarded as a "signal," rather than a contaminating systematic error, and can be measured from these wide-area galaxy surveys in order to constrain the large-scale tidal force [63,64]. Furthermore, a better understanding of the nonlinear mode coupling allows one to use a combination of the observed subsurvey modes to estimate the large-scale tidal field [65–67].

In order to realize the effect of coherent tidal force on structure formation in the deeply nonlinear regime, such as halo formation, we need to use N-body simulations. For this purpose, a separate universe simulation technique would be powerful; since the large-scale tidal force can be absorbed into the perturbed scale factors along each coordinate axis, $a_W(t) \simeq a(t)[1 - \tau_{Wi}]$, we can follow the full nonlinear mode coupling of the large-scale tide with sub-box modes by running N-body simulations in the perturbed background. For the coherent overdensity δ_b, the effect for a ΛCDM model can be absorbed as an apparent curvature, even if the global background is flat. Several works have developed the separate universe simulation technique to study the mode coupling effect of δ_b with sub-box modes [32,33,42,55,68,69]. The separate universe simulation allows for a better calibration of various effects such as the supersample covariance and the local halo bias, without running a large number of huge box simulations. In a very similar way we believe that the separate universe simulation technique can be applied to the large-scale tidal effect. Recently Ip and Schmidt [46] developed a unified formula to model the effect of the coherent tidal force on the evolution of subsurvey modes within the framework of general relativity. However, there are in general two contributions to the large-scale tidal field: the internal tidal force arising from the anisotropic matter distribution within a finite-volume boundary and the external tidal force that is not specified by the internal boundary conditions (see also [48,49]). Nevertheless, as long as we are interested in the effects of the linear tidal force, it would be possible to develop a separate universe simulation technique to include the large-scale tide in the simulation as well as to study the effect on nonlinear structure formation. If this is true, the separate universe simulations would give us a better way to calibrate the large-scale tidal effects on various cosmological observables. This is in progress and will be presented elsewhere.

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APPENDIX: TAKADA AND HU DERIVATION OF POWER SPECTRUM RESPONSE TO SUPERSURVEY MODES

In this appendix we derive the responses of the real-space power spectrum to the long-wavelength tidal force, based on the formulation in Takada and Hu [29].

Taking into account the survey window, the observed field of the matter fluctuation field can be defined as

$$\delta_W(\mathbf{x}) = \delta(\mathbf{x})W(\mathbf{x}), \tag{A1}$$

whose Fourier transform is a convolution

$$\tilde{\delta}_W(\mathbf{k}) = \int \frac{d^3\mathbf{q}}{(2\pi)^3}\tilde{W}(\mathbf{q})\tilde{\delta}(\mathbf{k}-\mathbf{q}). \tag{A2}$$

In order to study how the large-scale tide causes an anisotropic modulation in the measured power spectrum, let us define an estimator of the two-dimensional power spectrum of wave vector $\mathbf{k}$ as

$$\hat{P}(\mathbf{k}) = \frac{1}{V_W}\tilde{\delta}_W(\mathbf{k})\tilde{\delta}_W(-\mathbf{k}). \tag{A3}$$

Note that the wave vector bin $\mathbf{k}$ can be finite, compared to the fundamental mode of a survey, $k_f \simeq 2\pi/L$, and in that case the above estimator is defined from a sum of the modes within the bin width. The power spectrum estimator satisfies a parity invariance,

$$\hat{P}(\mathbf{k}) = \hat{P}(-\mathbf{k}). \tag{A4}$$

The ensemble average of the estimator is found to recover the underlying true power spectrum

$$\begin{aligned}\langle\hat{P}(\mathbf{k})\rangle &= \frac{1}{V_W}\int\frac{\mathrm{d}^3\mathbf{q}}{(2\pi)^3}|\tilde{W}(\mathbf{q})|^2 P(\mathbf{k}-\mathbf{q}) \\ &\simeq P(\mathbf{k})\frac{1}{V_W}\int\frac{\mathrm{d}^3\mathbf{q}}{(2\pi)^3}|\tilde{W}(\mathbf{q})|^2 = P(\mathbf{k}). \end{aligned}\tag{A5}$$

Here we have used that $P(\mathbf{k}-\mathbf{q}) \simeq P(\mathbf{k})$ over the integration range of $d^3\mathbf{q}$ which the window function supports and also assumed that $P(\mathbf{k})$ is not a rapidly varying function within the k-bin. In the third equality on the rhs, we have used the general identity for the window function,

$$\begin{aligned}V_W &= \int d^3\mathbf{x}W^n(\mathbf{x}) \\ &= \int\left[\prod_{a=1}^{n}\frac{d^3\mathbf{q}_a}{(2\pi)^3}\tilde{W}(\mathbf{q}_a)\right](2\pi)^3\delta_D^3(\mathbf{q}_{1\ldots n}),\end{aligned}\tag{A6}$$

where $\mathbf{q}_{1\ldots n} = \mathbf{q}_1 + \ldots\mathbf{q}_n$ here and below. For $n=2$, $V_W = \int|\tilde{W}(\mathbf{q})|^2 d^3\mathbf{q}/(2\pi)^3$.

As we have discussed, the supersurvey modes affect the power spectrum measured in a finite-volume survey region. Hence, when a given survey volume has supersurvey modes of δ_b and τ_{Wij}, the effects on power spectrum measured in the survey realization are expressed as

$$P(\mathbf{k};\delta_\mathrm{b},\tau_W) \simeq P(k) + \frac{\partial P(k)}{\partial\delta_\mathrm{b}}\delta_\mathrm{b} + \frac{\partial P(k)}{\partial\tau_{Wij}}\tau_{Wij}. \tag{A7}$$

Here $\partial P(k)/\partial\delta_\mathrm{b}$ and $\partial P(k)/\partial\tau_{Wij}$ are the responses of the power spectrum to the supersurvey modes, δ_b and τ_{Wij}, respectively. Here we consider the power spectrum responses at the leading order of the supersurvey modes, or in other words we ignored the responses at the higher orders of $O(\delta_\mathrm{b}^2, \tau_W^2)$. The ensemble average of the above power spectrum, which is equivalent to the average of the power spectrum estimator for different survey regions, is

$$\begin{aligned}\langle P(\mathbf{k};\delta_\mathrm{b},\tau_W)\rangle &= P(k) + \frac{\partial P(k)}{\partial\delta_\mathrm{b}}\langle\delta_\mathrm{b}\rangle + \frac{\partial P(k)}{\partial\tau_{Wij}}\langle\tau_{Wij}\rangle \\ &= P(\mathbf{k}),\end{aligned}\tag{A8}$$

where we used $\langle\delta_\mathrm{b}\rangle = \langle\tau_{Wij}\rangle = 0$. Thus the ensemble average of the power spectrum estimator recovers the true power spectrum in the global universe.

Now let us consider the covariance matrix of the power spectrum estimator [Eq. (A3)],

$$C(\mathbf{k},\mathbf{k}') = \langle\hat{P}(\mathbf{k})\hat{P}(\mathbf{k}')\rangle - P(\mathbf{k})P(\mathbf{k}'). \tag{A9}$$

Inserting Eq. (A7) into the above equation leads us to find a formal expression of the supersample covariance due to δ_b and τ_W,

$$\begin{aligned}C^{\mathrm{SSC}}(\mathbf{k},\mathbf{k}') &= \sigma_\mathrm{b}^2\frac{\partial P(k)}{\partial\delta_\mathrm{b}}\frac{\partial P(k')}{\partial\delta_\mathrm{b}} \\ &\quad + \langle\tau_{Wij}\tau_{Wlm}\rangle\frac{\partial P(k)}{\partial\tau_{Wij}}\frac{\partial P(k')}{\partial\tau_{Wlm}},\end{aligned}\tag{A10}$$

where we have assumed $\langle\delta_\mathrm{b}\tau_{Wij}\rangle \simeq 0$ for a reasonably symmetric survey window.

Following the formulation in Takada and Hu [29,33], we advocate that the squeezed trispectrum can be characterized by the responses of the power spectrum to the supersurvey modes as

$$\begin{aligned}&\lim_{q\to 0}[T^{\mathrm{PT}}(\mathbf{k},-\mathbf{k}+\mathbf{q},\mathbf{k}',-\mathbf{k}-\mathbf{q}) - T^{\mathrm{PT}}(\mathbf{k},-\mathbf{k},\mathbf{k}',-\mathbf{k}')] \\ &\simeq P^L(q)\left[\frac{\partial P(k)}{\partial\delta_\mathrm{b}} + \tau_{Wij}\frac{\partial P(k)}{\partial\tau_{Wij}}\right]\left[\frac{\partial P(k')}{\partial\delta_\mathrm{b}} + \tau_{Wlm}\frac{\partial P(k')}{\partial\tau_{Wlm}}\right],\end{aligned}\tag{A11}$$

where the Fourier modes $\mathbf{q}$ are supersurvey modes satisfying $k, k' \gg q$. Using the perturbation theory [21], we can compute the squeezed trispectrum contribution,

$$\begin{aligned}C^{\mathrm{SSC}}(\mathbf{k},\mathbf{k}') &= \frac{1}{V_W^2}\int\frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^3}|\tilde{W}(\mathbf{q})|^2 \\ &\quad\times[T^{\mathrm{PT}}(\mathbf{k},-\mathbf{k}+\mathbf{q},\mathbf{k}',-\mathbf{k}'-\mathbf{q}) \\ &\quad - T^{\mathrm{PT}}(\mathbf{k},-\mathbf{k},\mathbf{k}',-\mathbf{k}')].\end{aligned}\tag{A12}$$

$T^{\rm PT}$ is the tree-level trispectrum, defined as

$$\langle\delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta(\mathbf{k}_3)\delta(\mathbf{k}_4)\rangle_c = (2\pi)^3\delta_D^3(\mathbf{k}_{1234})T^{\rm PT}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4), \tag{A13}$$

where

$$T^{\rm PT}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4) = 4[F_2(\mathbf{k}_{13},-\mathbf{k}_1)F_2(\mathbf{k}_{13},\mathbf{k}_2)P^{\rm L}(k_{13}) \times P^{\rm L}(k_1)P^{\rm L}(k_2) + 11\text{ perm}] + 6[F_3(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)P^{\rm L}(k_1) \times P^{\rm L}(k_2)P^{\rm L}(k_3) + 3\text{ perm}], \tag{A14}$$

with the Fourier kernels defined as

$$F_2(\mathbf{k}_1,\mathbf{k}_2) = \frac{5}{7} + \frac{1}{2}\left(\frac{1}{k_1^2}+\frac{1}{k_2^2}\right)(\mathbf{k}_1\cdot\mathbf{k}_2) + \frac{2}{7}\frac{(\mathbf{k}_1\cdot\mathbf{k}_2)^2}{k_1^2k_2^2}, \tag{A15}$$

and the definition of F_3 is given by Eq. (31) in Takada and Hu [29], but the term involving F_3 is not relevant for the following calculation.

Inserting Eqs. (A14) and (A15) into Eq. (A12) leads to

$$C^{\rm SSC}(\mathbf{k},\mathbf{k}') \simeq \frac{1}{V_W^2}\int\frac{d^3\mathbf{q}}{(2\pi)^3}|\tilde{W}(\mathbf{q})|^2 4P^L(q)[P^L(k)F_2 \times(\mathbf{q},-\mathbf{k}) + P^L(|\mathbf{k}-\mathbf{q}|)F_2(\mathbf{q},\mathbf{k}-\mathbf{q})] \times[P^L(k')F_2(\mathbf{q},\mathbf{k}') + P^L(|\mathbf{k}'+\mathbf{q}|) \times F_2(-\mathbf{q},\mathbf{k}'+\mathbf{q})]. \tag{A16}$$

To further proceed with the calculation, we need to care about the fact that the mode coupling kernel F_2 has a pole. More especially, under the fact $k, k' \gg q$, we need to use an expansion such as the following:

$$P^L(|\mathbf{k}-\mathbf{q}|)F_2(\mathbf{q},\mathbf{k}-\mathbf{q}) \simeq \left[P(k) - \frac{\partial P(k)}{\partial k}(\mathbf{k}\cdot\mathbf{q})\right] \times\left[\frac{5}{7}+\frac{1}{2}\left(\frac{1}{q^2}+\frac{1}{k^2}\right)(\mathbf{k}\cdot\mathbf{q}-q^2) + \frac{(\mathbf{k}\cdot\mathbf{q}-q^2)^2}{q^2k^2}\right]. \tag{A17}$$

Then we can find that the supersample covariance can be computed as

$$C^{\rm SSC}(\mathbf{k},\mathbf{k}') \simeq \sigma_{\rm b}^2\left[\frac{47}{21}-\frac{1}{3}\frac{\partial\ln P(k)}{\partial\ln k}\right] \times\left[\frac{47}{21}-\frac{1}{3}\frac{\partial\ln P(k')}{\partial\ln k'}\right]P^L(k)P^L(k') + \langle\tau_{Wij}\tau_{Wlm}\rangle\hat{k}_i\hat{k}_j\hat{k}'_l\hat{k}'_m\left[\frac{8}{7}-\frac{\partial\ln P(k)}{\partial\ln k}\right] \times\left[\frac{8}{7}-\frac{\partial\ln P(k')}{\partial\ln k'}\right]P^L(k)P^L(k'), \tag{A18}$$

where $\hat{\mathbf{k}} = \mathbf{k}/k$. To arrive at this equation, we used the following identities for the $\mathbf{q}$-integration:

$$\int\frac{d^3\mathbf{q}}{(2\pi)^3}|\tilde{W}(\mathbf{q})|^2P^L(q) = \sigma_{\rm b}^2 \tag{A19}$$

$$\int\frac{d^3\mathbf{q}}{(2\pi)^3}|\tilde{W}(\mathbf{q})|^2P^L(q)q_iq_j = \int\frac{d^3\mathbf{q}}{(2\pi)^3}|\tilde{W}(\mathbf{q})|^2P^L(q)\left[\left(q_iq_j-\frac{\delta_{ij}^K}{3}\right)+\frac{\delta_{ij}^K}{3}\right] = \langle\delta_{\rm b}\tau_{Wij}\rangle + \frac{\delta_{ij}^K}{3}\sigma_{\rm b}^2 \simeq \frac{\delta_{ij}^K}{3}\sigma_{\rm b}^2 \tag{A20}$$

$$\int\frac{d^3\mathbf{q}}{(2\pi)^3}|\tilde{W}(\mathbf{q})|^2P^L(q)q_iq_jq_lq_m \simeq \int\frac{d^3\mathbf{q}}{(2\pi)^3}|\tilde{W}(\mathbf{q})|^2P^L(q) \times\left[\left(q_iq_j-\frac{\delta_{ij}^K}{3}\right)\left(q_lq_m-\frac{\delta_{lm}^K}{3}\right)+\frac{\delta_{ij}^K\delta_{lm}^K}{9}\right] = \langle\tau_{Wij}\tau_{Wlm}\rangle + \frac{\delta_{ij}^K\delta_{lm}^K}{9}\sigma_{\rm b}^2. \tag{A21}$$

Note that we also used the fact that terms involving the moments with an odd power of q_i, or equivalently an odd power of k_i, are vanishing under the parity invariance conditions of $\mathbf{k}\leftrightarrow-\mathbf{k}$ and $\mathbf{k}'\leftrightarrow-\mathbf{k}'$.

Comparing Eqs. (A10) and (A18) leads us to find that the power spectrum response can be given as

$$P(\mathbf{k};\delta_{\rm b},\tau_W) \simeq P(k) + \delta_{\rm b}\left[\frac{47}{21}-\frac{1}{3}\frac{\partial\ln P(k)}{\partial\ln k}\right]P(k) + \hat{k}_i\hat{k}_j\tau_{Wij}\left[\frac{8}{7}-\frac{\partial\ln P(k)}{\partial\ln k}\right]P(k). \tag{A22}$$

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