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Thermal inflation with flaton chemical potential

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Thermal inflation driven by a scalar field called a "flaton" is a possible scenario to solve the cosmological moduli problem. We study a model of thermal inflation with a flaton chemical potential. In the presence of the chemical potential, a negative mass squared of the flaton—which is necessary to terminate thermal inflation—is naturally induced. We identify the allowed parameter region for the chemical potential (μ) and the flaton self-coupling constant to solve the cosmological moduli problem and satisfy theoretical consistencies. In general, the chemical potential is a free parameter and it can be taken to be much larger than the typical scale of soft supersymmetry-breaking parameters of $\mathcal{O}(1)$ TeV. For $\mu \gtrsim 10^8$ GeV, we find that the reheating temperature after thermal inflation can be high enough for the thermal leptogenesis scenario to be operative. This is in sharp contrast to the standard thermal inflation scenario, in which the reheating temperature is quite low and a special mechanism is necessary for generating a sufficient amount of baryon asymmetry in the Universe after thermal inflation.

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I. INTRODUCTION

The exponentially accelerated expansion of spacetime in the early period of the Universe is well established as the cosmic inflation scenario [1–5]. Primordial inflation solves the flatness and horizon problems in the standard big bang cosmology. On the other hand, supersymmetry (SUSY) is believed to play an important role in the study of elementary particles, especially in the early stages of the Universe. It is known that inflation scenarios in the supersymmetric epoch exhibit various problems. Among other things, the relatively high reheating temperature after primordial inflation causes the overproduction of gravitinos. The late-time decay of gravitinos after big bang nucleosynthesis deconstructs successfully synthesized light elements. This is known as the gravitino problem [6–8]. One resolution to the gravitino problem is achieved by a low reheating temperature, $T_{\rm RH} \lesssim 10^{6\text{–}7}$ GeV [9,10].

There is also a serious cosmological problem in the early Universe known as the cosmological moduli problem [11–13]. Four-dimensional spacetime may be realized in superstring theories, which typically predict massless scalar excitations, i.e., moduli fields. Since the moduli fields only have Planck-suppressed interactions, the energy density of the Universe is dominated by the moduli fields before they decay. If the moduli decay cannot reheat the Universe high enough, $T_{\rm RH} \gtrsim 1$ MeV, the present Universe cannot be realized. This is the cosmological moduli problem. This problem is intractable in the primordial inflation scenario since the moduli particles are produced abundantly even at low reheating temperature.

In order to solve the moduli problem, a short period of secondary inflation with $\mathcal{O}(10)$ e-foldings after primordial inflation has been proposed [14,15]. By this second inflation, the number density of the moduli particles is diluted away and their energy density never dominates the Universe. Since this secondary inflation of spacetime is triggered by the thermal effect, this is called thermal inflation. The realization and phenomenological viability of thermal inflation have been discussed in detail in, for example, Refs. [16–21].

Thermal inflation is driven by a scalar field with an almost flat potential. This field is called the flaton. The typical flaton potential at zero temperature is given by [15]

$$V(\phi) = V_0 - m_{\phi 0}^2|\phi|^2 + \sum_{n=1}^{\infty} \lambda_n \frac{|\phi|^{2n+4}}{\bar{M}_{\rm pl}^{2n}}, \qquad (1)$$

where ϕ is the (complex scalar) flaton field, V_0 is the vacuum energy at the origin, $m_{\phi 0}$ is the mass of the flaton, and λ_n are the coupling constants. The higher-dimensional interactions are suppressed by the reduced Planck mass $\bar{M}_{\rm pl} = 2.4 \times 10^{18}$ GeV. Here the flaton is assumed to interact with a scalar field X which serves as the thermal bath, through which the flaton potential V receives finite-temperature corrections from the thermal bath. At a high temperature T, the effective mass squared $m^2(T)$ of the flaton behaves like $m^2(T) = T^2 - m_{\phi 0}^2 > 0$ and thermal inflation begins at $\phi = 0$. As the temperature decreases, the

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effective mass squared becomes negative, which leads to the violation of the slow-roll condition. Therefore, a tachyonic mass of the flaton is necessary for the end of thermal inflation. It has been discussed that a tachyonic mass is obtained by the renormalization group flow in a supersymmetric model [22]. However, this does not happen in more general situations. After thermal inflation, the flaton rolls down to the true vacuum and then starts to oscillate there. The flaton decays to the Standard Model particles to reheat the Universe. This decay creates entropy, and the moduli problem can be solved. In order to solve the moduli problem, the yield of the moduli field after thermal inflation must be reduced to 10^{-12}–10^{-15} [23] or smaller. However, this mechanism causes another problem: the entropy production from the flaton decay also dilutes the primordial baryon asymmetry produced by some mechanism beforehand.[1] We need a mechanism to produce a sufficient amount of baryon asymmetry before or after thermal inflation. The authors of Refs. [16,19,21] studied whether sufficient baryon number asymmetry is produced with the use of the Affleck-Dine mechanism [25,26] after thermal inflation. However, it was found that the Affleck-Dine mechanism is not phenomenologically viable in this framework. It is normally difficult to resolve the problem since the reheating temperature after the flaton decay is typically not high enough, because of very weak couplings of the flaton to the Standard Model particles.

In this paper we propose a thermal inflation scenario that can solve the problems of the termination of thermal inflation and the generation of a sufficient amount of baryon asymmetry after the flaton decay. For this purpose, we introduce a chemical potential μ for the flaton. We will show that in the thermal effective potential, the chemical potential μ plays the role of the tachyonic mass of the flaton at low temperature. Hence, thermal inflation ends when the chemical potential starts dominating over the thermal mass. Furthermore, μ is a free parameter in any system, which basically has nothing to do with soft SUSY-breaking parameters. This is in contrast with the standard thermal inflation scenario where the tachyonic mass term in Eq. (1) is supposed to be generated through SUSY breaking, and hence we expect $|m_{\phi_0}| \simeq \mathcal{O}(1)$ TeV for the weak-scale SUSY. The mass scale of the flaton is important since it determines the reheating temperature ($T_{\rm RH2}$) after the flaton decay and what mechanism can be implemented for the baryon number generation. In the standard thermal inflation scenario, $T_{\rm RH2}$ is at most $\mathcal{O}(100)$ MeV, as we will discuss below. With such a low reheating temperature, a possible scenario for the baryon number generation is the Affleck-Dine mechanism [25,26]. As mentioned above, although the Affleck-Dine mechanism has been studied in models of thermal inflation, it turns out that a sufficient baryon number cannot be created [16,19,21]. In our model, we can set $\mu \gg 1$ TeV so that the reheating temperature can be much higher and thermal leptogenesis [27] (for a review, see Ref. [28]) can be operative even after the flaton decay.

The organization of this paper is as follows. In the next section, we present a brief review of standard thermal inflation. In Sec. III, we introduce a chemical potential for the flaton field and calculate the thermal effective potential of the flaton. We then evaluate the yields of the moduli after the flaton decay and identify the allowed regions of the chemical potential μ and the flaton coupling constant λ. Section IV is devoted to conclusions and discussions. We give a brief derivation of the thermal effective potential in Appendix A. In Appendix B, we derive the interaction term between the flaton and the Standard Model gauge fields.

II. REVIEW OF STANDARD THERMAL INFLATION

In this section, we review thermal inflation (proposed in Refs. [14,15]) and how the moduli problem is solved. If the flaton field causes thermal inflation, the energy density from the oscillating moduli is diluted and hence the moduli problem can be solved. After thermal inflation, the Universe is thermalized with the reheating temperature $T_{\rm RH2}$ through the flaton decay. If the reheating temperature is high enough to allow big bang nucleosynthesis ($T_{\rm RH2} \gtrsim 1$ MeV), the history of the Universe matches the standard scenario.

We focus on a part of a model which causes thermal inflation, while a part for the primordial inflation is not specified. We assume that the flaton field acquires its mass via SUSY breaking, and hence the mass is naturally of the order of the soft SUSY-breaking mass scale $\sim$1 TeV. Notice that (as we will see below) the negative mass squared for the flaton field is necessary to terminate thermal inflation. For an origin of the negative mass squared, we may consider the renormalization group effect, which drives the running flaton mass squared negative at a certain low scale. For a concrete model, see Ref. [22].

The flaton field ϕ is considered to couple with some light fields (typically the Standard Model particles) that are in thermal equilibrium and yield thermal corrections to the effective potential of the flaton. The high-temperature approximation is valid when the mass scales of the fields are sufficiently small compared to the temperature during thermal inflation. For simplicity, we consider a model with two real scalars ϕ and X for thermal inflation,

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi + \frac{1}{2}\partial_\mu X \partial^\mu X - V(\phi, X), \tag{2}$$

where $\mu = 0, 1, 2, 3$ is the spacetime index, and we use the mostly minus convention of the metric $\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$. The scalar potential V is given by

[1]This problem was pointed out in the early stages of the development of the flaton field [24], before the proposal of the thermal inflation scenario.

$$V_{\rm tree} = V_0 - \frac{m_{\phi 0}^2}{2}\phi^2 + \frac{\lambda}{6\bar{M}_{\rm pl}^2}\phi^6 + \frac{m_{X0}^2}{2}X^2 + \frac{g}{4}\phi^2 X^2, \quad (3)$$

where V_0 is the energy scale at the origin, and $m_{\phi 0}$ and m_{X0} are the masses of the fields ϕ and X, respectively. Here λ and g are coupling constants. We have introduced the higher-dimensional interaction term ϕ^6, and there is no flaton quartic term.[2] This setting realizes an almost flat potential and leads to a large vacuum expectation value (VEV) of the flaton field.[3] Such a large VEV is crucial to solve the moduli problem [15] [see Eq. (30) with Eq. (6)]. The stationary condition for X trivially gives $X = 0$, while the one for ϕ,

$$\left.\frac{\partial V}{\partial \phi}\right|_{X=0} = 0, \quad (4)$$

yields

$$\phi = 0, \qquad \phi = \lambda^{-1/4}\sqrt{m_{\phi 0}\bar{M}_{\rm pl}} \equiv M. \quad (5)$$

The energy scale at the origin is given by

$$V_0 = \frac{1}{3\sqrt{\lambda}} m_{\phi 0}^3 \bar{M}_{\rm pl} = \frac{1}{3} m_{\phi 0}^2 M^2, \quad (6)$$

which guarantees the vanishing cosmological constant at the potential minimum $\phi_c = M$. The VEV of ϕ is denoted by ϕ_c.

The scalar potential (3) receives thermal effects through reheating after the primordial inflation. The thermal effects are introduced by imposing the periodic boundary condition for the fields $\Phi_i = (\phi, X)$ as $\Phi_i(\tau, \vec{x}) = \Phi_i(\tau + \beta, \vec{x})$ in the partition function, where $\tau = ix_0$ is the imaginary time, $\beta = 1/T$ is the inverse temperature, and $\vec{x} = (x_1, x_2, x_3)$. The partition function is given as

$$\begin{aligned} Z &= {\rm Tr} e^{-\beta H} \\ &= \int_{\Phi_i(\tau)=\Phi_i(\tau+\beta)} \\ &\quad \times \prod_i \mathcal{D}\Phi_i \mathcal{D}\Phi_i^\dagger e^{-\int_0^\beta d\tau \int d^3x \sum_i (\frac{1}{2}\partial_0\Phi_i\partial_0\Phi_i + \frac{1}{2}\vec{\nabla}\Phi_i\vec{\nabla}\Phi_i + V(\phi, X))}, \end{aligned} \quad (7)$$

where H is the Hamiltonian and $\vec{\nabla}$ is the derivative with respect to $\vec{x}$. The scalar field X plays the role of the thermal bath and the flaton receives the thermal effects through X-loop corrections. Calculating the thermal one-loop correction of X, we obtain the effective potential for the flaton as [30][4]

$$\begin{aligned} V_{\rm eff}(\phi_c) &= V_0 - \frac{1}{2} m_{\phi 0}^2 \phi_c^2 + \frac{\lambda}{6\bar{M}_{\rm pl}^2}\phi_c^6 + \int \frac{d^3k}{(2\pi)^3}\frac{\omega_k}{2} \\ &\quad + \frac{1}{\beta}\int \frac{d^3k}{(2\pi)^3} \log(1 - e^{-\beta\omega_k}), \end{aligned} \quad (8)$$

where we have defined

$$\omega_k^2 = \vec{k}^2 + m_X^2(\phi), \quad (9)$$

$$m_X^2(\phi) = \left.\frac{\partial^2 V}{\partial X^2}\right|_{\phi=\phi_c} = m_{X0}^2 + \frac{1}{2} g\phi_c^2. \quad (10)$$

The fourth term on the right-hand side of Eq. (8) is the Coleman-Weinberg potential and the fifth term is the thermal effective potential. We consider the situation where the temperature is high enough and the dominant contribution comes from the thermal effective potential. In the subsequent discussions, we therefore neglect the Coleman-Weinberg potential term. Performing the high-temperature expansion, we have

$$\begin{aligned} V_{\rm eff}(\phi_c) &= V_0 - \frac{\pi^2 T^4}{90} + \frac{T^2}{24} m_{X0}^2 + \frac{1}{2} m_\phi^2(T)\phi_c^2 \\ &\quad + \frac{\lambda}{6\bar{M}_{\rm pl}^2}\phi_c^6 + \cdots, \end{aligned} \quad (11)$$

where $m_\phi(T)$ is the flaton mass with the thermal correction

$$m_\phi(T)^2 = -m_{\phi 0}^2 + \frac{g}{24} T^2. \quad (12)$$

For $m_\phi(T)^2 > 0$, the vacuum is located at $\phi_c = 0$, and the potential energy of the flaton dominates over the energy of the Universe. This leads to a second inflation by the flaton, namely, thermal inflation. Thermal inflation ends when the effective mass of the flaton becomes negative, in other words, when the temperature drops below the critical value T_C given by

$$T_C = 2 m_{\phi 0}\sqrt{\frac{6}{g}}. \quad (13)$$

Soon after the temperature becomes less than T_C, the flaton starts rolling down to the vacuum at $\phi_c = M$ and then oscillates around there. The decay of the flaton reheats the Universe, and we roughly estimate the reheating temperature as

[2]Such a form of the potential is found in the low-energy effective theory of superstring theories [29].

[3]When the flat potential includes the flaton quartic coupling, it is necessary to set the coupling constant to be much smaller than λ in Eq. (3) in order to realize the large VEV.

[4]For a derivation, see Appendix A.

$$T_{\mathrm{RH2}} \simeq \left(\frac{90}{\pi^2 g_*}\right)^{1/4} \sqrt{\Gamma \bar{M}_{\mathrm{pl}}}, \tag{14}$$

where $g_*(\simeq 200)$ counts the effective degrees of freedom of the radiation, and Γ is the flaton decay width. Here we simply assume that the flaton decays to the Higgs boson (h) through the effective interaction [18]

$$\mathcal{L}_{\mathrm{int}} \sim \frac{m_\phi^2}{M} \phi hh, \tag{15}$$

where m_ϕ is the flaton mass in the vacuum at $T = 0$ and is given by

$$m_\phi^2 = \left.\frac{\partial^2 V}{\partial \phi^2}\right|_{T=0, \phi_c = M} = 4 m_{\phi 0}^2. \tag{16}$$

The decay width of the process $\phi \to hh$ is obtained as

$$\Gamma \simeq \frac{1}{16\pi} \frac{m_\phi^3}{M^2}, \tag{17}$$

where we have neglected the Higgs boson mass. Substituting Eq. (17) into Eq. (14), we find the reheating temperature as

$$T_{\mathrm{RH2}} \simeq \left(\frac{90}{\pi^2 g_*}\right)^{1/4} \frac{m_\phi}{4M} \sqrt{\frac{m_\phi \bar{M}_{\mathrm{pl}}}{\pi}} = \frac{1}{4\pi} \left(\frac{360\lambda}{g_*}\right)^{\frac{1}{4}} m_\phi. \tag{18}$$

The main role of thermal inflation is to dilute the yield of the moduli field, by which the moduli problem is solved. The dilution is caused by the entropy production from the flaton decay after thermal inflation. Before we discuss the entropy production, we note that there are two relevant scenarios for the moduli oscillation after primordial inflation (see Fig. 1). The first is the one discussed in Ref. [15]. In this scenario, the moduli fields are displaced from the potential minima during primordial inflation. When the Hubble parameter reduces to $H \sim m_\Phi$, the moduli fields start to oscillate around their potential minima. Here m_Φ is the mass of the moduli fields. The Universe enters the matter-dominated era with the oscillating inflaton and moduli fields whose energy densities are comparable. After the moduli oscillation, the first reheating occurs due to the decay of the inflaton and we denote the reheating temperature by T_{RH1}.

The second possibility is that the moduli oscillation takes place after the first reheating. When the Universe cools down to $H \sim m_\Phi$, the moduli fields start to oscillate. As we will see later, in both scenarios the oscillating moduli—which dominate the energy density of the Universe—can be diluted away by thermal inflation.

In the following, we perform a qualitative analysis of the entropy production in these scenarios.

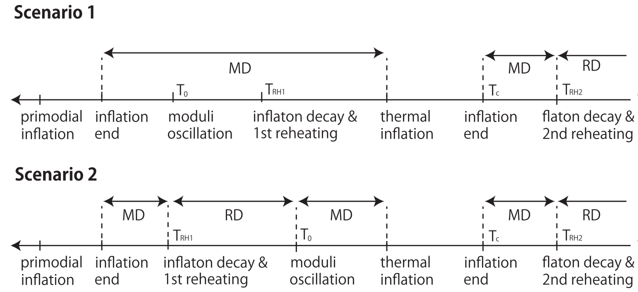

FIG. 1. Two possible scenarios for the moduli oscillation in the thermal history of the Universe. Here "MD" and "RD" mean matter-dominated and radiation-dominated, respectively.

Scenario 1: The increase of the entropy density after the flaton decay is calculated as

$$\Delta = \frac{s(T_{\mathrm{RH2}})}{s(T_C)}, \tag{19}$$

where $s(T)$ is the entropy density at temperature T. In the radiation-dominated era, this is given by

$$s(T) = \frac{2\pi^2}{45} g_* T^3 = \frac{4\rho(T)}{3T}, \tag{20}$$

where we have used the energy density for relativistic particles

$$\rho(T) = \frac{\pi^2}{30} g_* T^4. \tag{21}$$

With the use of Eq. (20), the increase of the entropy (19) is expressed as

$$\Delta = \frac{30 V_0}{\pi^2 g_* T_C^3 T_{\mathrm{RH2}}}, \tag{22}$$

where we have used $V_0 = \rho(T_{\mathrm{RH2}})$.

The yield of the moduli Y_Φ after the flaton decay is given by

$$Y_\Phi = \frac{n_\Phi(T_{\mathrm{RH2}})}{s(T_{\mathrm{RH2}})} = \frac{n_\Phi(T_C)}{s(T_C)\Delta} = \frac{n_\Phi(T_{\mathrm{RH1}})}{s(T_{\mathrm{RH1}})\Delta}, \tag{23}$$

where n_Φ is the number density of the moduli particles, and we have assumed no entropy production before the end of thermal inflation. Since the moduli particles are nonrelativistic in this era, n_Φ at a certain temperature is represented by

$$n_\Phi = \frac{1}{m_\Phi} \rho_\Phi, \tag{24}$$

where ρ_Φ is the energy density of the moduli. The energy density of the moduli at T_{RH2} is produced by moduli oscillation after primordial inflation:

$$\rho_\Phi = \frac{1}{2} m_\Phi^2 \Phi_0^2, \tag{25}$$

where Φ_0 is the amplitude of the moduli fields. During the moduli oscillation, the Universe is in the matter-dominated era and therefore we have

$$\rho_\Phi(T_{\rm RH1}) = \rho_\Phi \left(\frac{a_{\rm osc}}{a(T_{\rm RH1})} \right)^3 = \rho_\Phi \left(\frac{H(T_{\rm RH1})}{H_{\rm osc}} \right)^2, \tag{26}$$

where $a_{\rm osc}$ and $H_{\rm osc}$ are the scale factor and the Hubble parameter when the moduli oscillation starts, and $a(T_{\rm RH1})$ and $H(T_{\rm RH1})$ are the ones at the reheating by primordial inflation. Since the moduli oscillation starts when $H(T_0) \simeq m_\Phi$, we express the moduli number density as

$$n_\Phi(T_{\rm RH1}) = \frac{1}{2 m_\Phi} \Phi_0^2 H(T_{\rm RH1})^2, \tag{27}$$

from Eqs. (24), (25), and (26). The entropy density $s(T_{\rm RH1})$ in the denominator in Eq. (23) is evaluated as

$$s(T_{\rm RH1}) = \frac{4}{T_{\rm RH1}} \bar{M}_{\rm pl}^2 H(T_{\rm RH1})^2, \tag{28}$$

where we have used the relation (21) and the Friedmann equation

$$H^2(T_{\rm RH1}) = \frac{\rho(T_{\rm RH1})}{3\bar{M}_{\rm pl}^2}. \tag{29}$$

Substituting Eqs. (22), (27), and (28) into Eq. (23), we obtain the yield of the moduli:

$$Y_\Phi = \frac{\pi^2 g_*}{240} \frac{T_{\rm RH1} T_{\rm RH2} T_C^3}{m_\Phi V_0} \left(\frac{\Phi_0}{\bar{M}_{\rm pl}} \right)^2. \tag{30}$$

With the use of the specific expressions for $T_{\rm RH2}$, $T_{\rm C}$, and V_0 given in Eqs. (18), (13), and (6) together with the decay width (17), we have

$$\begin{aligned} Y_\Phi &= \frac{9\pi}{g^{3/2}} \left(\frac{g_*}{10} \right)^{3/4} \frac{\lambda^{3/4} T_{\rm RH1} m_\phi}{m_\Phi \bar{M}_{\rm pl}} \\ &\simeq 1.1 \times 10^{-6} \times \lambda^{3/4} \left(\frac{T_{\rm RH1}}{10^{10}\ {\rm GeV}} \right) \left(\frac{1\ {\rm TeV}}{m_\Phi} \right) \\ &\quad \times \left(\frac{m_\phi}{1\ {\rm TeV}} \right) \left(\frac{\Phi_0}{\bar{M}_{\rm pl}} \right)^2, \end{aligned} \tag{31}$$

where we have chosen $g_* = 200$ and $g = 1$. It is natural that the moduli mass is the same order as the soft SUSY-breaking mass and the moduli amplitude is assumed to be the reduced Planck scale. Note that it is not necessary that $T_{\rm RH1} < 10^6$ GeV to solve the gravitino problem since it can be solved after thermal inflation as well. The moduli problem is solved if the yield satisfies the constraint [23]

$$Y_\Phi < 10^{-13}, \tag{32}$$

which leads to an upper bound on λ as

$$\lambda \lesssim 4.0 \times 10^{-10}, \tag{33}$$

for $T_{\rm RH1} = 10^{10}$ GeV, $m_\Phi = m_\phi = 1$ TeV, for example. Taking $\lambda = 10^{-11}$ as a conservative value, the reheating temperature $T_{\rm RH2}$ in Eq. (18) turns out to be

$$T_{\rm RH2} \simeq 164\ {\rm MeV}. \tag{34}$$

Scenario 2: Next we consider the scenario where the moduli oscillation starts after the reheating by the primordial inflation (see Fig. 1). When the moduli oscillation starts at $H \simeq m_\Phi$, the energy density of the radiation becomes of the same order as the energy density of the moduli [Eq. (25)] with $\Phi_0 \simeq \bar{M}_{\rm pl}$. With this observation, we find the temperature when the moduli oscillation starts:

$$T_0 = \left(\frac{15}{\pi^2 g_*} \right)^{1/4} \sqrt{m_\Phi \Phi_0}. \tag{35}$$

Considering that there is no entropy production until the flaton decay, the yield of the moduli after the flaton decay is written as

$$Y_\Phi = \frac{n_\Phi(T_{\rm RH2})}{s(T_{\rm RH2})} = \frac{n_\Phi(T_0)}{s(T_0)\Delta}, \tag{36}$$

where the increase of the entropy density Δ is the same as in Eq. (22) since there is no entropy production during T_0 and T_C.

Substituting Eqs. (20), (22), and (25) into Eq. (36), we have

$$Y_\Phi = \frac{3}{8} \left(\frac{\pi^2 g_*}{15} \right)^{3/4} \frac{\Phi_0^{1/2} T_C^3 T_{\rm RH2}}{m_\Phi^{1/2} V_0}. \tag{37}$$

Combining this result with Eqs. (13) and (18), we obtain

$$\begin{aligned} Y_\Phi &= 27 \times \frac{6^{1/4} g_*^{1/2}}{g^{3/2}} \sqrt{\frac{\pi}{5}} \frac{\lambda^{3/4} m_\phi \Phi_0^{1/2}}{m_\Phi^{1/2} \bar{M}_{\rm pl}} \\ &\simeq 6.2 \times 10^{-6} \lambda^{3/4} \left(\frac{m_\phi}{1\ {\rm TeV}} \right) \left(\frac{1\ {\rm TeV}}{m_\Phi} \right)^{1/2} \left(\frac{\Phi_0}{\bar{M}_{\rm pl}} \right)^{1/2}, \end{aligned} \tag{38}$$

where we have chosen $g_* = 200$ and $g = 1$. The condition (32) leads to

$$\lambda \lesssim 4.0 \times 10^{-11}. \tag{39}$$

The reheating temperature after the flaton decay is given as

$$T_{\rm RH2} \simeq 92\ {\rm MeV}, \tag{40}$$

for a conservative value $\lambda = 10^{-12}$.

In both scenarios, the reheating temperature is sufficiently high to realize big bang nucleosynthesis. However, since thermal inflation dilutes the primordial baryon asymmetry, we need to consider baryogenesis after thermal inflation. A simple baryogenesis—such as thermal leptogenesis [27] or electroweak baryogenesis (for a review, see, e.g., Ref. [31])—cannot be operative at such low reheating temperatures in Eqs. (34) and (40). In order for thermal leptogenesis to work, $T_{\rm RH2} \gtrsim 10^3$ GeV is necessary [32]. On the other hand, the Affleck-Dine mechanism [25,26] could be implemented with Eq. (34), as has been studied in models of thermal inflation [16,21].

We mentioned that the negative mass squared for the flaton field in Eq. (3) can be realized by the renormalization group effect [22]. For instance, assume that the flaton mass squared is positive at a scale where the primordial inflation ends and the flaton couples to a scalar field through the Yukawa interaction in the superpotential. Under certain conditions, the Yukawa interaction drives the flaton mass squared negative. However, in order to realize this, it is likely that a Yukawa coupling beyond the perturbative regime is necessary [22]. In the next section, we propose a simple scenario to terminate thermal inflation. We also show that in a proposed scenario the reheating temperature $T_{\rm RH2}$ can be much larger than 10^3 GeV, which makes it possible to implement thermal leptogenesis.

III. THERMAL INFLATION WITH CHEMICAL POTENTIAL

In this section, we introduce the chemical potential for the flaton in the thermal inflation scenario and study its effect. The existence of the chemical potential means that the flaton is dense at a vacuum realized after the end of thermal inflation. It has been shown that there exists such a vacuum with nonzero chemical potential in $\mathcal{N} = 1$ supersymmetric QCD [33].

Considering the fact that the moduli problem stems from the superstring theories, it is natural to embed a model in the supersymmetry framework. In the following, we consider a supersymmetric model where the flaton field ϕ and a scalar field X, both of which are complex, are realized as the lowest components of $\mathcal{N} = 1$ chiral superfields. We begin with the following tree-level scalar potential of these fields:

$$V = V_0 + \frac{\lambda}{\bar{M}_{\rm pl}^2}|\phi|^6 + m_{X0}^2|X|^2 + g|\phi|^2|X|^2. \tag{41}$$

The potential (41) exhibits $U(1)_c$ global symmetry under the transformation $\phi \to e^{i\alpha}\phi$. Here the constant α is the $U(1)_c$ charge. The chemical potential is introduced by gauging the $U(1)_c$ global symmetry for the flaton [30,33]. The spacetime derivative is replaced with the gauge-covariant derivative $D_\mu = \partial_\mu + i\alpha A_\mu$, where A_μ is a nondynamical gauge field. The gauge field has a vacuum expectation value only in the zeroth component $\langle A_\mu\rangle = (i\mu, \mathbf{0})$. Note that the field X, which is in thermal equilibrium, is neutral under the $U(1)_c$ transformation. We also note that although the complex scalar field ϕ leads to a multiflaton model, we can always rotate away the imaginary (real) part of ϕ during the inflation by the $U(1)_c$ transformation. Therefore the inflation dynamics does not differ from that in single-flaton models.

The partition function with nonzero temperature and the chemical potential is written as

$$Z = {\rm Tr} e^{-\beta(H-\mu\mathcal{N})} = \int_{\Phi_i(\tau)=\Phi_i(\tau+\beta)} \prod_i \mathcal{D}\Phi_i \mathcal{D}\Phi_i^\dagger e^{-\int_0^\beta d\tau \int d^3x (D_0\phi D_0\phi^\dagger + \partial_0 X \partial_0 X^\dagger + \sum_i \vec{\nabla}\Phi_i \vec{\nabla}\Phi_i^\dagger + V)}, \tag{42}$$

where $\mathcal{N}$ is the Noether charge of the $U(1)_c$ symmetry, $\Phi_i = (\phi, X)$, and $D_0 = \frac{\partial}{\partial\tau} - \mu$ with the unit $U(1)_c$ charge $\alpha = 1$. The thermal effective potential for the flaton ϕ after primordial inflation is obtained by calculating the thermal one-loop correction of X:

$$V_{\rm eff} = V_0 - \mu^2|\phi|^2 + \frac{\lambda}{\bar{M}_{\rm pl}^2}|\phi|^6 + \int \frac{d^3k}{(2\pi)^3}\frac{\omega_k}{2} + \frac{1}{\beta}\int \frac{d^3k}{(2\pi)^3}\log(1 - e^{-\beta\omega_k}). \tag{43}$$

Note that the chemical potential yields a negative mass squared for the flaton. This potential has the same form as Eq. (8) when μ is replaced with m_{ϕ_0}. However, it should be emphasized that μ can in general be any value, while $m_{\phi 0} \simeq \mathcal{O}(1)$ TeV in the standard thermal inflation scenario since $m_{\phi 0}$ is considered to be caused by SUSY breaking. The fourth term in Eq. (43) is the Coleman-Weinberg potential, which we will omit in the following discussion. The fifth term is the thermal effective potential with nonzero chemical potential, where ω_k is given in Eqs. (9) with (10).

We study thermal inflation with this potential and how the moduli problem is solved in an analytic way. Performing the high-temperature expansion, we have

$$V_{\rm eff} = V_0 - \frac{\pi^2T^4}{45} + \frac{T^2}{12}m_{X0}^2 + \left(-\mu^2 + \frac{gT^2}{12}\right)|\phi_c|^2 + \frac{\lambda}{\bar{M}_{\rm pl}^2}|\phi_c|^6 + \cdots. \tag{44}$$

When the coefficient of $|\phi_c|^2$ is positive, the potential minimum is at the origin for ϕ_c and thermal inflation takes place. According to the expansion of the Universe, the temperature is decreasing, and thermal inflation eventually ends at the critical temperature given by

$$T_C = 2\mu\sqrt{\frac{3}{g}}. \tag{45}$$

Below this temperature, the flaton rolls down to the vacuum which is determined by the extreme condition

$$\left.\frac{\partial^2 V}{\partial\phi\partial\phi^\dagger}\right|_{X=0,T=0} = 0. \tag{46}$$

From this condition, we have

$$\phi_c = (3\lambda)^{-1/4}\sqrt{\mu\bar{M}_{\rm pl}} \equiv M_c. \tag{47}$$

The flaton mass at the vacuum is given as

$$m_\phi^2 = \left.\frac{\partial^2 V}{\partial\phi\partial\phi^\dagger}\right|_{T=0,\phi_c=M_c} = \mu^2. \tag{48}$$

The potential energy V_0 is determined so that the scalar potential is vanishing at the vacuum:

$$V_0 = \frac{2}{3\sqrt{3\lambda}}\mu^3\bar{M}_{\rm pl} = \frac{2}{3}\mu^2 M_c^2. \tag{49}$$

The flaton oscillates around the vacuum and thermalization occurs. In order to evaluate the reheating temperature, we need to specify the interaction of the flaton with the Standard Model fields. The interaction considered in Eq. (15) cannot be employed since this does not preserve the $U(1)_c$ symmetry related to the chemical potential. Instead, we consider the following $U(1)_c$-preserving interaction (see Appendix B for the derivation):

$$\mathcal{L}_{\rm int} = \sum_{a=1}^{3} c_a \frac{M_c}{\bar{M}_{\rm pl}^2}\chi\left(-\frac{1}{4}F^{a\mu\nu}F^a_{\mu\nu}\right), \tag{50}$$

where $\chi \equiv {\rm Re}(\phi)$, $c_a (a = 1, 2, 3)$ is a constant, and $F^a_{\mu\nu}$ is the gauge field strength. Here the index $a = 1, 2, 3$ corresponds to the Standard Model gauge groups, $SU(3)\times SU(2)_L \times U(1)_Y$. The partial decay widths of χ into the Standard Model gauge bosons are calculated to be [34]

$$\Gamma(\chi\to gg) = \frac{c_3^2}{2\pi}\left(\frac{M_c}{\bar{M}_{\rm pl}^2}\right)^2\mu^3, \tag{51}$$

$$\Gamma(\chi\to\gamma\gamma) = \frac{(c_1\cos^2\theta_W + c_2\sin^2\theta_W)^2}{16\pi}\left(\frac{M_c}{\bar{M}_{\rm pl}^2}\right)^2\mu^3, \tag{52}$$

$$\Gamma(\chi\to ZZ) = \frac{(c_1\cos^2\theta_W + c_2\sin^2\theta_W)^2}{128\pi} \times\left(\frac{M_c}{\bar{M}_{\rm pl}^2}\right)^2\mu^3\beta_Z(3+2\beta_Z^2+3\beta_Z^4), \tag{53}$$

$$\Gamma(\chi\to WW) = \frac{c_2^2}{64\pi}\left(\frac{M_c}{\bar{M}_{\rm pl}^2}\right)^2\mu^3\beta_W(3+2\beta_W^2+3\beta_W^4), \tag{54}$$

$$\Gamma(\chi\to\gamma Z) = \frac{(c_1-c_2)^2\sin^2\theta_W\cos^2\theta_W}{8\pi}\left(\frac{M_c}{\bar{M}_{\rm pl}^2}\right)^2\mu^3 \times\left(1-\frac{m_Z^2}{\mu^2}\right)^3, \tag{55}$$

where θ_W is the weak mixing angle, and $\beta_Z = \sqrt{1-4m_Z^2/\mu^2}$ and $\beta_W = \sqrt{1-4m_W^2/\mu^2}$. Here m_Z and m_W are the masses of the Z and W bosons. With the use of these decay widths, the reheating temperature is

$$T_{\rm RH2} \simeq \left(\frac{90}{\pi^2 g_*}\right)^{\frac{1}{4}}\sqrt{\frac{3}{4\pi}}\frac{\mu^2}{(3\lambda)^{1/4}\bar{M}_{\rm pl}}, \tag{56}$$

where we have chosen $c_1 = c_2 = c_3 = 1$ for simplicity, and have put $\beta_Z \simeq 1$ and $\beta_W \simeq 1$ since we assume $m_Z, m_W \ll \mu$.

We now evaluate the yield (23) through the flaton decay in two scenarios: the moduli starts to oscillate before (Scenario 1) or after (Scenario 2) the reheating by primordial inflation. The chemical potential only plays the role of the flaton mass and does not affect the derivation of the yield from Eqs. (19)–(30) for Scenario 1 and from Eqs. (35)–(37) for Scenario 2 in the previous section. Therefore, we have the same formula for the yield as Eq. (30) for Scenario 1 and Eq. (37) for Scenario 2.

For Scenario 1, substituting Eqs. (45), (49), and (56) into Eq. (30), we find

$$\begin{aligned} Y_\Phi &= \frac{9\pi}{4g^{3/2}}\left(\frac{3g_*}{10}\right)^{3/4}\frac{\lambda^{1/4}\mu^2}{\bar{M}_{\rm pl}^2}\times 10^7\left(\frac{T_{\rm RH1}}{10^{10}\ {\rm GeV}}\right) \\ &\quad\times\left(\frac{1\ {\rm TeV}}{m_\Phi}\right)\left(\frac{\Phi_0}{\bar{M}_{\rm pl}}\right)^2 \\ &\simeq 1.5\times 10^9\times\frac{\lambda^{1/4}\mu^2}{\bar{M}_{\rm pl}^2}\left(\frac{T_{\rm RH1}}{10^{10}\ {\rm GeV}}\right)\left(\frac{1\ {\rm TeV}}{m_\Phi}\right)\left(\frac{\Phi_0}{\bar{M}_{\rm pl}}\right)^2, \end{aligned} \tag{57}$$

where we have taken $g = 1$ and $g_* = 200$. The moduli problem is resolved when the yield (57) satisfies the condition (32). In other words, λ and μ should satisfy the following condition:

$$1.5 \times \frac{\lambda^{1/4}\mu^2}{\bar{M}_{\rm pl}^2} \lesssim 10^{-22}. \tag{58}$$

For Scenario 2, repeating the same derivation for Eq. (37), we obtain

$$Y_\Phi = \frac{81}{2\times 2^{3/4}(10g)^{3/2}}\left(\frac{\pi g_*}{5}\right)^{1/2}\frac{\lambda^{1/4}\mu^2}{\bar{M}_{\rm pl}^{3/2}}\left(\frac{1\ \text{TeV}}{m_\Phi}\right)^{\frac{1}{2}}\left(\frac{\Phi_0}{\bar{M}_{\rm pl}}\right)^{\frac{1}{2}}$$
$$\simeq 2.7\times 10^2 \times \frac{\lambda^{1/4}\mu^2}{\bar{M}_{\rm pl}^{3/2}}\left(\frac{1\ \text{TeV}}{m_\Phi}\right)^{\frac{1}{2}}\left(\frac{\Phi_0}{\bar{M}_{\rm pl}}\right)^{\frac{1}{2}}. \tag{59}$$

This expression with the condition (32) leads to

$$2.7 \times \frac{\lambda^{1/4}\mu^2}{\bar{M}_{\rm pl}^{3/2}} \lesssim 10^{-15}. \tag{60}$$

Allowed values for λ and μ for the conditions (58) and (60) determine the reheating temperature (56). We may require that the reheating temperature be high enough to realize thermal leptogenesis [27], such as $T_{\rm RH2} \gtrsim 1$ TeV. On the other hand, the consistency of our discussion requires $T_C > T_{\rm RH2}$, which leads to

$$\lambda > \frac{1}{3}\left(\frac{90}{\pi^2 g_*}\right)\left(\frac{g}{32\pi}\right)^2 \frac{\mu^2}{\bar{M}_{\rm pl}^2}, \tag{61}$$

where we have used Eqs. (45) and (56). The coupling constant λ and the chemical potential μ are also constrained from the condition that the vacuum expectation value of the flaton should be less than the Planck scale $M_c < \bar{M}_{\rm pl}$. This results in the following condition:

$$\lambda > \frac{\mu^2}{3\bar{M}_{\rm pl}^2}. \tag{62}$$

Figure 2 shows the parameter region for Scenario 1 that satisfies Eqs. (58), (61), and (62) together with

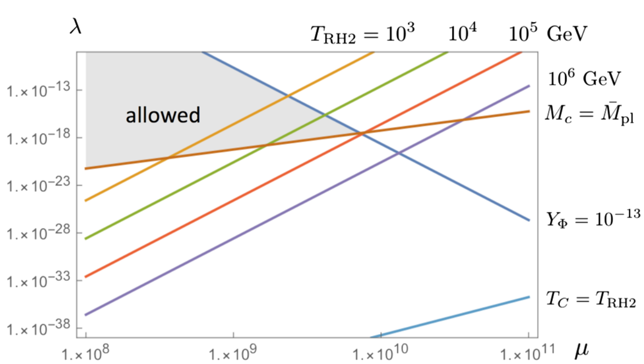

FIG. 2. Allowed parameter region for λ and μ in Scenario 1. Here we have taken $g_* = 200$ and $g = 1$.

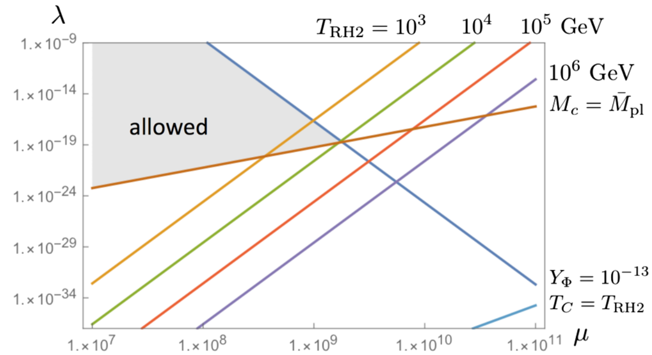

FIG. 3. Same as Fig. 2 but for Scenario 2.

the lines corresponding to the reheating temperatures $T_{\rm RH2} = 10^3, 10^4, 10^5$, and 10^6 GeV. Here we have taken $g_* = 200$ and $g = 1$. We can see that the condition (62) is stronger than Eq. (61). Indeed, Eq. (62) with Eq. (58) sets the upper bound on the chemical potential as $\mu \lesssim 7.2\times 10^9$ GeV. Considering that thermal leptogenesis is operative at least for $T_{\rm RH2} \gtrsim 10^3$ GeV along with Eq. (62), we find that the lower bound on λ is $\lambda \gtrsim 7.5\times 10^{-21}$. It is possible to increase $T_{\rm RH2}$ up to around 9.0×10^4 GeV, beyond which the vacuum expectation value M_c is larger than the Planck scale.

A similar figure for Scenario 2 is shown in Fig. 3. The upper bound on the chemical potential is given as $\mu \lesssim 1.5\times 10^9$ GeV and the lower bound on λ such that thermal leptogenesis is operative is found to be $\lambda \gtrsim 7.5\times 10^{-21}$. The reheating temperature can be taken up to 8.6×10^3 GeV, which is smaller than the one in Scenario 1.

It should be emphasized that in the standard thermal inflation scenario, $T_{\rm RH2}$ cannot be large enough to implement the baryogenesis scenario except for the Affleck-Dine mechanism. The reheating temperature (18) is proportional to $\lambda^{\frac{1}{4}}$ and the flaton mass. Recalling that $m_\phi \simeq 1$ TeV and λ should be small enough to satisfy Eq. (33), we see that $T_{\rm RH2}$ in the standard thermal inflation scenario is at most $\mathcal{O}(100)$ MeV. However, in our scenario the reheating temperature (56) is proportional to $\lambda^{-1/4}$ and μ^2. Since μ is taken to be larger than 1 TeV and in addition λ can be taken to be small to satisfy Eqs. (58) and (60) [but it is constrained by Eq. (62)], one can realize a reheating temperature that is high enough to implement thermal leptogenesis.

IV. CONCLUSION

In this paper, we have studied the models of thermal inflation with the flaton chemical potential which was implemented naturally by using the VEV of the zeroth component of the $U(1)_c$ (nondynamical) gauge field.

This leads to a negative mass squared of the flaton. On the other hand, in standard thermal inflation a negative mass squared of $\mathcal{O}(1)$ TeV—which is the soft SUSY-breaking scale—can be realized by the renormalization group flow with a large coupling constant (most likely in the nonperturbative regime); otherwise, it is introduced by hand. We have evaluated the yield of the moduli in two scenarios: the moduli field starts to oscillate before (Scenario 1) or after (Scenario 2) the reheating by primordial inflation. In both scenarios, the yield depends on λ (the coefficient of the sixth-order term of the flaton potential) and the chemical potential μ. We have found the allowed parameter region in the (λ, μ) plane in which, after thermal inflation, the reheating temperature can be high enough for thermal leptogenesis to be operative. This is in sharp contrast to standard thermal inflation, in which the reheating temperature is at most $\mathcal{O}(100)$ MeV.

In this work we have introduced the flaton chemical potential as a free parameter. It is worth investigating a possible origin of the chemical potential in the framework of superstring theories. It is also interesting to consider a possibility to relate the global $U(1)_c$ to the baryon or lepton numbers in the Standard Model.

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APPENDIX A: THERMAL ONE-LOOP CORRECTION

In this appendix, we give a sketch of the derivations for Eqs. (8) and (43). For details, we refer the reader to Refs. [35,36].

At the one-loop level, the correction of the effective potential from the thermal effect for a real scalar field is given by the determinant

$$\log(\det(\partial^2 + m^2))^{1/2} = \frac{1}{2}\mathrm{tr}\log(\partial^2 + m^2) = \frac{1}{2\beta}\sum_{n=-\infty}^{+\infty}\int\frac{d^3k}{(2\pi)^3} \times \log((2\pi\beta^{-1}n)^2 + \omega_k) \tag{A1}$$

in the absence of the chemical potential. After formally differentiating Eq. (A1) with respect to ω_k, we can sum over the discrete momentum $2\pi\beta^{-1}n$,

$$\begin{aligned}\sum_{n=-\infty}^{+\infty}\frac{\partial}{\partial\omega_k}\log((2\pi\beta^{-1}n)^2 + \omega_k^2) &= \sum_{n=-\infty}^{+\infty}\frac{2\omega_k}{(2\pi\beta^{-1}n)^2 + \omega_k^2} \\ &= \sum_{n=-\infty}^{+\infty}\left(\frac{1}{\omega_k - i(2\pi\beta^{-1}n)} + \frac{1}{\omega_k + i(2\pi\beta^{-1}n)}\right) \\ &= \beta\coth\left(\frac{\beta\omega_k}{2}\right).\end{aligned} \tag{A2}$$

Here we use the partial fraction expansion formula,

$$\pi\coth(\pi x) = \sum_{n=-\infty}^{+\infty}\frac{1}{x + in}. \tag{A3}$$

Integrating Eq. (A2) with ω_k, we obtain

$$\begin{aligned}\log(\det(\partial^2 + m^2))^{1/2} &\simeq \frac{1}{\beta}\int\frac{d^3k}{(2\pi)^3}\log\left|\sinh\left(\frac{\beta\omega_k}{2}\right)\right| \\ &\simeq \int\frac{d^3k}{(2\pi)^3}\left(\frac{\omega_k}{2} + \frac{1}{\beta}\log|1 - e^{-\beta\omega_k}|\right),\end{aligned} \tag{A4}$$

up to an irrelevant constant. In the case of a complex scalar, the correction is twice that of a real scalar.

APPENDIX B: INTERACTION TERMS OF THE FLATON WITH THE STANDARD MODEL SECTOR

We consider the following higher-dimensional term invariant under the $U(1)_c$ transformation for the flaton, $\phi \to e^{i\alpha}\phi$, associated with the chemical potential:

$$\mathcal{L}_{\mathrm{int}} = \frac{1}{4}\int d^4\theta\sum_{a=1}^{3}c_a\frac{\Phi^\dagger\Phi}{\bar{M}_{\mathrm{pl}}^2}(W^{a\alpha}W_\alpha^a\delta^2(\bar{\theta}) + \mathrm{H.c.}), \tag{B1}$$

where Φ is a chiral superfield associated with the flaton and W_α^a is a superfield strength with the index $a = 1$, 2, 3 corresponding to the Standard Model gauge groups $SU(3) \times SU(2)_L \times U(1)_Y$. In order to consider the interaction at the vacuum $\langle\Phi\rangle = M_c$, we substitute the shift

$$\Phi \to M_c + \Phi \tag{B2}$$

into Eq. (B1) and pick up the following three-point vertex part:

$$\mathcal{L}_{\mathrm{int}} \supset \frac{M_c}{4\bar{M}_{\mathrm{pl}}^2}\int d^4\theta\sum_{a=1}^{3}c_a(\Phi + \Phi^\dagger)(W^{a\alpha}W_\alpha^a\delta^2(\bar{\theta}) + \mathrm{H.c.}). \tag{B3}$$

Since we are interested in the flaton decay, we focus on the scalar part of the flaton superfield, $\phi = \Phi|_{\theta=0}$, in Eq. (B3):

$$\mathcal{L}_{\rm int} \supset \frac{M_c}{\bar{M}_{\rm pl}^2}\chi \sum_{a=1}^{3} c_a \left(-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} - i\lambda^a\sigma^\mu\partial_\mu\bar{\lambda}^a\right), \quad \text{(B4)}$$

where $\chi \equiv \mathrm{Re}(\phi)$, and $F^a_{\mu\nu}$ and λ^a are the field strength and the gaugino, respectively. This interaction leads to the decays $\chi \to A_\mu A_\nu$ and $\chi \to \lambda\bar{\lambda}$. The decay widths are obtained as $\Gamma(\chi \to A_\mu A_\nu) \propto (M_c/\bar{M}_{\rm pl}^2)^2\mu^3$ and $\Gamma(\chi \to \lambda\bar{\lambda}) \propto (M_c/\bar{M}_{\rm pl}^2)^2 m_\lambda^2 \mu$, where $m_\lambda \simeq 1$ TeV is the gaugino mass. Since we take $\mu \gg 1$ TeV in our scenario (see Figs. 2 and 3), the flaton mainly decays to the Standard Model gauge bosons.

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