Constraints on primordial black holes with extended mass functions

Florian Kühnel[1,2,*] and Katherine Freese[3,1,†]

[1]*The Oskar Klein Centre for Cosmoparticle Physics, Department of Physics, Stockholm University,*
AlbaNova University Center, Roslagstullsbacken 21, SE–10691 Stockholm, Sweden
[2]*Department of Physics, School of Engineering Sciences, KTH Royal Institute of Technology,*
AlbaNova University Center, Roslagstullsbacken 21, SE–10691 Stockholm, Sweden
[3]*Department of Physics, University of Michigan, Ann Arbor, Michigan 48109, USA*
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Constraints on primordial black holes in the range 10^{-18} $M_\odot$ to 10^3 $M_\odot$ are reevaluated for a general class of extended mass functions. Whereas previous work has assumed that PBHs are produced with one single mass, instead there is expected to be a range of masses even in the case of production from a single mechanism; constraints therefore change from previous literature. Although tightly constrained in the majority of cases, it is shown that, even under conservative assumptions, primordial black holes in the mass range 10^{-10} $M_\odot$ to 10^{-8} $M_\odot$ could still constitute the entirety of the dark matter. This stresses both the importance for a comprehensive reevaluation of all respective constraints that have previously been evaluated only for a monochromatic mass function and the need to obtain more constraints in the allowed mass range.

I. INTRODUCTION

Primordial black holes (PBHs) are black holes produced in the early Universe, and have received considerable attraction since they were proposed more than four decades ago [1,2]. In principle, they can span a huge range of mass scales—from as low as the Planck mass to many orders of magnitude above the solar mass. They are unique probes of the amplitude of the density fluctuation on the very small scales of their production.

With the milestone discovery of the LIGO and VIRGO Collaboration of black-hole binary mergers [3,4], the interest in the question of whether PBHs could constitute the dark matter (DM) [5] has recently been revived [6–20]. Depending on the mass scale(s) involved, the black holes potentially cause large observable effects. For the case where all PBHs have one single mass (monochromatic mass function) somewhere in the range 10^{-18} $M_\odot$ to 10^3 $M_\odot$, Fig. 1 summarizes the strongest constraints from previous literature at each value of possible PBH mass. The figure caption lists the physical effects and provides respective references.

Most of the constraints derived in the literature, including those in Fig. 1, are subject to the (at best oversimplifying) assumption that PBH formation occurs monochromatically, i.e., at one particular mass scale only, despite the fact that PBH mass spectra are generically extended due to the nature of the gravitational collapse leading to their formation [30–36]. This incorrect assumption can lead to large errors in the prediction of the PBH abundance (cf. Ref. [36]). It should be stressed that, given the current constraints

displayed in Fig. 1, any monochromatic scenario of 100% PBH dark matter is strictly excluded. This statement crucially relies on the validity of the constraints used in this work. However, some of these have been disputed. For instance, those deriving from neutron-star capture of black holes [24] are challenged due to uncertainties in the amount of dark matter inside globular clusters (cf. Refs. [37,38]). Furthermore, the bounds from PBH accretion [29,39,40] rely on highly model-dependent and relatively insecure assumptions (spherically symmetric Bondi accretion, etc.). Also, they depend on the so-called duty-cycle parameter whose exact value varies significantly in the relevant literature. Hence, monochromatic scenarios of 100% of PBH dark matter might still be allowed. In this article, however, our point is to investigate whether—even if these stringent constraints (to which we will refer as "conservative") are taken at face value—extended mass functions still allow for the possibility that the entirety of the dark matter consists of PBHs.

In this paper we reconsider the bounds on primordial black-hole dark matter in the range 10^{-18} $M_\odot$ to 10^3 $M_\odot$ for a wide class of extended mass functions. All realistic cases with extended mass spectra require a rederivation of the constraints and an integration of these over the whole mass range; this will be done in the present work.

II. PRIMORDIAL BLACK-HOLE FORMATION

There is a plethora of scenarios which lead to the formation of PBHs. All of these require a mechanism to generate large overdensities. In many scenarios, these overdensities are of inflationary origin [41–43]. After reentering the horizon, they collapse if they are larger than a (medium-dependent) threshold, where the case of

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*[*]florian.kuhnel@fysik.su.se
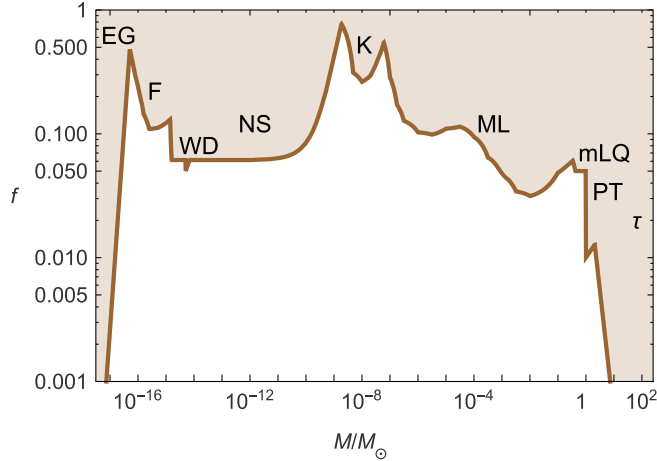[†]ktfreese@umich.edu

FIG. 1. Summary of previous literature for the idealized case in which the entire PBH dark matter consists of PBHs of a single mass M (monochromatic mass function): the constraint "curtain" on the dark-matter fraction $f \equiv \rho_{PBH}/\rho_{DM}$ for a variety of effects associated with PBHs of mass M in units of solar mass $M_\odot$. Only the strongest constraints are included. We show constraints from extragalactic γ rays from evaporation (EG) [21], femtolensing of γ-ray bursts (F) [22], white-dwarf explosions (WD) [23], neutron-star capture (NS) [24], Kepler microlensing of stars (K) [11], MACHO/EROS/OGLE microlensing of stars (ML) [25] and quasar microlensing (ML) [26], millilensing of quasars (mLQ) [27], pulsar timing (PT) (SKA forecast) [28], and accretion effects on the optical thickness (τ) [29]. More details can be found in [17]. Whereas these constraints are only valid for the case of a monochromatic mass function, instead all realistic cases with extended mass spectra require a rederivation of those constraints and an integration of these over the whole mass range; this will be done in the present work.

radiation domination is the one most often considered in the literature. Many more possibilities for PBH formation exist, such as those where the source of the inhomogeneities are first-order phase transitions [6], bubble collisions [44,45], the collapse of cosmic strings [46,47], necklaces [48] or domain walls [49].

As mentioned above, in general, one encounters extended PBH mass spectra rather than PBHs of a single mass. The reason is twofold: On the one hand, the initial spectrum of overdensities is already extended in essentially all of the models mentioned above. On the other hand, even if the initial density spectrum was monochromatic, the phenomenon of critical collapse [7] will inevitably lead to a PBH mass spectrum which is spread out, shifted towards lower masses and lowered, leading to potentially large effects (cf. Ref. [36]).

III. BOUNDS FOR EXTENDED MASS FUNCTIONS

The derivation of bounds for extended mass functions is strictly speaking always subject to a specific model, such as the axion-curvaton model [50–52], the hybrid-inflation model [53,54], or the running-mass model [55–60],

just to mention a few (cf. Ref. [17] for an extensive overview). Therefore, it seems hopeless to draw any model-independent conclusions in this regard. However, it is indeed quite possible at an approximate level. As has been pointed out recently by Green [20], a PBH mass function with derivative

$$\frac{\mathrm{d}n}{\mathrm{d}M} \equiv N \exp\left[-\frac{(\log M/M_f)^2}{2\sigma_f^2}\right], \qquad (1)$$

fits a very large class of inflationary PBH models reasonably well (cf. Refs. [61–63] for the derivation and use of log-normal mass functions) [64]. Above, N is a normalization constant chosen such that the integral of $\mathrm{d}n/\mathrm{d}M$ over all masses is equal to 1. As a function of M, the rhs of Eq. (1) is peaked at M_f; its width is controlled by σ_f. In practice, for each pair of parameters (σ_f, M_f), a given set of constraints needs to be evaluated on the whole mass range, which then yields a limit on the PBH content.

The observational constraints which are included in our analysis are all of those stated in the caption of Fig. 1, adapted for the extended mass function Eq. (1) which we use throughout the rest of our paper.

IV. NEUTRON-STAR CAPTURE

Primordial black holes captured by neutron stars (NS) generically lead to the rapid destruction of the NS. Hence, the observations of neutron stars pose constraints on the PBH abundance. For a monochromatic mass spectrum, it has been argued in Ref. [24] on the basis of a sufficiently large neutron-star survival probability that the constraint can be phrased as

$$1 \geq f t_{NS} F_0, \qquad (2)$$

with the PBH dark-matter fraction $f \equiv \rho_{PBH}/\rho_{DM}$. Here, t_{NS} is the age of the star. The capture rate F_0 inside of a globular cluster (with core dark-matter density $\rho_{DM,GC}$) reads

$$F_0 = \sqrt{6\pi} \frac{\rho_{DM,GC}}{M} \frac{2G_N M_{NS} R_{NS}}{\bar{v}(1 - 2G_N M_{NS}/R_{NS})}$$
$$\times \left[1 - \exp\left(-\frac{3E_{loss}}{M\bar{v}^2}\right)\right], \qquad (3)$$

with G_N being Newton's constant, $\bar{v}$ is the dispersion of the assumed Maxwellian velocity distribution of the PBHs, R_{NS} is the radius of the neutron star, and M_{NS} its mass. Finally, the average energy loss E_{loss} is approximately given by

$$E_{loss} \simeq \frac{58.8 G_N^2 M^2 M_{NS}}{R_{NS}^2}. \qquad (4)$$

In the case of an extended mass distribution, the bound given in Eq. (2) changes. Let $I_{bin} \in \mathbb{N}$ denote the number of

bins $\{[M_i, M_{i+1}], i \leq I_{\text{bin}}\}$. Then, for a given distribution specified by some dn/dM, we have

$$1 \gtrsim \sum_{i=1}^{I_{\text{bin}}} t_{\text{NS}} f F_0^{(i)} \int_{M_i}^{M_{i+1}} dM \frac{dn}{dM}, \qquad (5)$$

where $F_0^{(i)}$ is evaluated on a mass within each bin, and the bin number I_{bin} should be chosen such that, to the precision sought, it does not matter where exactly in each bin the quantity F_0 is evaluated. For the evaluations of the constraint we use the parameter values $M_{\text{NS}} = 1.4\ M_\odot, R_{\text{NS}} = 12$ km, $\rho_{\text{DM,GC}} = 2 \times 10^3$ GeV cm^{-3}, $t_{\text{DM}} = 10^{10}$ yr, and $\bar{v} = 7$ km s^{-1}, as given in Ref. [24].

Of course, in reality, these parameters are subject to variations. Hence, for the most realistic treatment of the neutron-star capture constraints, their respective distributions—if known—would have to be taken into account and convoluted with that of the PBH mass [65]. While the neutron-star mass appears to be narrowly peaked around the used value of $1.4\ M_\odot$, some (e.g. the velocity) have a larger variance. In particular, the dark-matter density inside globular clusters might be an order of magnitude smaller than the assumed value of $\rho_{\text{DM,GC}} = 2 \times 10^3$ GeV cm^{-3}. This would significantly weaken—if not entirely remove—the whole neutron-star capture constraints, leading others (e.g. [38]) to dismiss these bounds altogether. Since our goal is to find the PBH mass ranges that survive even the strongest bounds, we feel it is appropriate to use the neutron-star capture constraints with the standard assumptions about the parameters.

V. PULSAR TIMING

The authors of Ref. [28] argue that the abundance of 1–1000 $M_\odot$ PBHs might be considerably constrained via the nondetection of a third-order Shapiro time delay as the PBHs move around the Galactic halo. They presented the results of a respective Monte Carlo simulation leading to a forecast for the Square Kilometre Array (SKA) which approximately follows an $f \sim M^{1/3}$ scaling. More precisely, and adopted to an extended PBH spectrum, the respective constraint might be written in the form

$$1 \gtrsim \sum_{i=1}^{I_{\text{bin}}} \left(\frac{\tilde{M}_i}{1\ M_\odot}\right)^{1/3} f \int_{M_i}^{M_{i+1}} dM \frac{dn}{dM}, \qquad (6)$$

where $\tilde{M}_i$ is some mass within each bin. As before, I_{bin} should be chosen sufficiently large.

VI. ACCRETION EFFECTS

As first analyzed by Carr [66], in the period after decoupling, PBH accretion and emission of radiation might have a strong effect on the thermal history of the Universe. Ricotti *et al.* [29] have analyzed this possibility in detail.

In particular, it has been discussed that one associated effect of PBH is that they increase the optical thickness $\tau \to \tau + (\Delta\tau)_{\text{PBH}}$ which leads to

$$1 \gtrsim \sum_{i=1}^{I_{\text{bin}}} 17.4 \left(\frac{\tilde{M}_i}{1\ M_\odot}\right)^2 f \int_{M_i}^{M_{i+1}} dM \frac{dn}{dM}, \qquad (7)$$

which is valid in the mass range $[30\ M_\odot, 10^3\ M_\odot]$. In Eq. (7), we utilize the latest Planck constraint $(\Delta\tau)_{\text{PBH}} < 0.012$ (95% C.L.) as in Ref. [67].

VII. OTHER BOUNDS

For the bounds from extragalactic γ rays from evaporation, white-dwarf explosions, and lensing, we use a similar method to the one in Ref. [17]. This allows us to approximately determine, from constraints calculated assuming a delta-function halo fraction, whether an extended mass function is allowed. This method utilizes binning of the relevant mass range. Specifically, a given constraint f_c is first divided into locally monotonic pieces. In each of these pieces one starts with the bin, say i, where the constraint is smallest, and integrates dn/dM in this bin in order to obtain the fraction f_i. Then one goes to the next bin, integrating over $[M_i, M_{i+1}]$ in order to obtain f_{i+1}, and so on. In the original formulation of Ref. [17], each f_j is then compared to the largest value of f_c in this bin (where the constraint is weakest). Recently, Green [20] pointed out that there may be cases where this procedure underestimates the constraint.

In this work, in order to avoid such a possible issue, instead of using the largest value of f_c, we will use the smallest one in each bin (where the constraint is weakest). This may overestimate the constraint, but for the smooth mass functions given by Eq. (1) and the constraint used, by making the bins small enough, the error can be made arbitrarily small, and in particular much smaller than the error of using Eq. (1) instead of the actual mass function of a given model [68].

VIII. RESULTS

Figure 2 shows our main results for the constraints on the amount of PBH dark matter in the (σ_f, M_f)-plane using the distribution given by Eq. (1). Here, the value of M_f, at which this distribution peaks, lies between $10^{-16}\ M_\odot$ and $10^3\ M_\odot$, and its width $\sigma_f \in [0.2, 2]$. The region enclosed by the red dashed contour in the middle of the plot around $10^{-9}\ M_\odot$ indicates the possibility for a PBH dark-matter fraction of 100%. This is not possible outside this narrow region, and excludes a too narrow mass function ($\sigma \lesssim 0.4$), which in particular applies to the monochromatic case. Hence almost all of the parameter space does not allow for 100% PBH dark matter, given the validity of the very stringent bounds in the mass range under investigation.
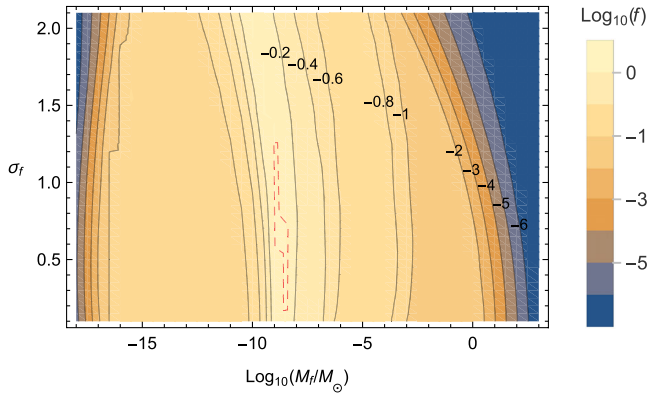
FIG. 2. Maximum PBH dark-matter fraction $f \equiv \rho_{\mathrm{PBH}}/\rho_{\mathrm{DM}}$ as a function of σ_f and M_f for the extended mass function specified in Eq. (1). The region enclosed by the red dashed contour in the middle of the plot indicates the possibility of 100% primordial black-hole dark matter (see the legend). The various contours are at $\log_{10}(f) = 0$ (red dashed line), $-0.2, -0.4, -0.6, -0.8, -1, -2, \ldots, -6$.

As mentioned in the Introduction, it should be stressed that several of these constraints are based on rather uncertain assumptions (such as those deriving from accretion, neutron-star capture, or ultrafaint dwarfs), and might be weakened significantly once a more elaborated treatment of those has been performed. However, even if all the mentioned constraints are taken at face value, in the middle of the M_f axis, there remains a region, and therefore a class of models, with a necessarily extended mass function, which still allows for PBHs to constitute all of the dark matter [69].

IX. SUMMARY AND OUTLOOK

In this paper we have presented the results of our systematic investigation of constraints for a wide class of extended mass functions in the mass range $10^{-18}\, M_\odot$ to $10^3\, M_\odot$. For these results, which are visualized in Fig. 2, very restrictive constraints on and forecasts for the allowed abundance of PBHs (as summarized in Fig. 1) have been used. Hence, Fig. 2 provides an approximate lower bound on the allowed PBH dark-matter fraction.

We confirm the results of Ref. [17] that there still is a window in the mass range $10^{-10}\, M_\odot$ to $10^{-8}\, M_\odot$ which can accommodate for 100% PBH dark matter. Apart from the possibility of Planck-mass relics, to pose new constraints in the mentioned mass window seems to be crucial for providing an answer to the question of whether primordial black holes can constitute the entirety of the dark matter.

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[69] In the process of finalizing this paper, in Ref. [70] additional microlensing constraints, using high-cadence observation of M31 with the Subaru Hyper Suprime-Cam, have been proposed. While these results still need to be investigated carefully, if reliable, they would put the currently most stringent upper bounds on the PBH abundance in the mass range 10^{-14}–$10^{-6}\ M_\odot$. These bounds have very recently been studied in Ref. [38] using a method similar to the one used in this work. However, their Fig. 2 merely indicates whether 100% PBH dark matter is excluded or not in the mentioned limited low-mass range, whereas our Fig. 2 shows results for the whole mass range 10^{-18}–$10^3\ M_\odot$, and furthermore indicates the exact value of the allowed PBH dark-matter fraction.

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