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Investigation of the light four-quark states with exotic $J^{PC} = 0^{--}$

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We study the exotic $J^{PC} = 0^{--}$ four-quark states in Laplace sum rules and finite energy sum rules. We use the vector tetraquarklike currents as interpolating currents in the correlator, from which the 1^{+-} states are also studied. In the mass extraction, we use the standard stability criterion with respect to the Borel parameters and the QCD continuum thresholds and consider the effect of the violation of factorization in estimating the high dimensional condensates as a source of uncertainties. The obtained mass prediction 1.76 ± 0.15 GeV is much lower than the previous sum rule predictions obtained using the pseudoscalar currents. Our result favors the four-quark interpretation of the possible $\rho\pi$ dominance in the D^0 decay. We also discuss the possible decay patterns of these exotic four-quark states.

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I. INTRODUCTION

The Dalitz-plot distribution of the $D^0 \to \pi^+\pi^-\pi^0$ process was analyzed by the *BABAR* Collaboration [1,2]. The resonant dominance substructure of this Cabibbo suppressed decay was then studied, and it indicated that an isospin-zero final state may exist [3–5]. The analysis of the Dalitz-plot behavior showed the typical structure of a $\pi^+\pi^-\pi^0$ final state with $I^G J^{PC} = 0^- 0^{--}$ [6], which is exotic and cannot be composed of a quark-antiquark pair in the conventional quark model [7,8]. If such a resonance exists near $M(D^0) \simeq 1865$ MeV, it might be a hybrid or four-quark state [5].

A hybrid meson is composed of a quark-antiquark pair and an excited gluonic field. It provides a good platform to search for exotic quantum numbers that cannot be realized for a $q\bar{q}$ state. The mass of a 0^{--} hybrid was predicted to be about 1.8–2.2 GeV in the constituent gluon model [9], around 2.3 GeV in the QCD Coulomb gauge approach [10], and 2.8–3.3 GeV in QCD sum rules [11,12]. These values and ranges are all higher than or marginally consistent with the mass of the D^0 meson. In the Massachusetts Institute of Technology (MIT) bag model [13,14], the hybrid states in the lightest supermultiplet with quantum numbers $J^{PC} = (0,1,2)^{-+}, 1^{--}$ consist of a S-wave color-octet quark-antiquark pair coupled to an excited gluonic field with $J_g^{P_g C_g} = 1^{+-}$. A higher supermultiplet contains hybrids with $J^{PC} = 0^{+-}$, $(1^{+-})^3$, $(2^{+-})^2$, 3^{+-}, $(0,1,2)^{++}$, which were composed of a P-wave $q\bar{q}$ pair and the same gluonic excitation [15,16]. The hybrid state with $J^{PC} = 0^{--}$ may lie higher than other channels and couple to a different gluonic excitation. Such supermultiplet structures were confirmed in lattice QCD [15], QCD sum rules [17,18], and the P-wave quasigluon approach [19].

In quantum field theory, a hybrid $\bar{q}gq$ operator and a four-quark operator can transform into each other with the same quantum numbers via quark annihilation interactions ($q\bar{q} \to g \to q\bar{q}$) in the $I_{q\bar{q}} = 0$ channel. They tend to mix and couple to the same physical state. In general, there are two types of four-quark operators: tetraquarklike operators $(qq)(\bar{q}\,\bar{q})$ and moleculelike operators $(q\bar{q})(q\bar{q})$. They are related to each other by the Fierz transformation and color rearrangement [20]. The tetraquark formalism was first suggested in the bag model by Jaffe [21,22], then it was extensively investigated and used to study the nature of exotic hadron states [23–28]. In the QCD Coulomb gauge approach, the masses of the 0^{--} molecules and tetraquarks were predicted to be around 1.36 GeV and 2.15 GeV, respectively, in Refs. [29,30].

The four-quark states with $J^{PC} = 0^{--}$ were also systematically studied in the approach of QCD sum rules using the pseudoscalar interpolating currents in Ref. [31], which seem not to support a mass below 2 GeV. However, because the coefficient of the four-quark condensate is zero [31],[1] it is difficult to assess the uncertainties associated with the truncation of the Operator Product Expansion (OPE). Therefore it is worth considering different interpolating currents in the sum rule analysis, which provides a chance

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[1]We have confirmed this result through independent calculation.

to obtain OPE series that have better behaviors. In this work, we shall use the vector currents which can couple to $J^{PC} = 0^{--}$ and 1^{+-} four-quark states. We shall perform the numerical analyses of the mass for both the pseudoscalar and the vector channels. In order to make robust estimates, we shall use Laplace sum rules (LSR) and finite energy sum rules (FESR), and use the standard stability criterion with respect to the Borel parameter τ and the continuum threshold s_0 to extract the masses. We shall conclude the paper by discussing the possible explanation of the $\rho\pi$ dominance in D^0 decay and the decay patterns of the 0^{--} four-quark states.

II. LAPLACE SUM RULES AND FINITE ENERGY SUM RULES

Introduced by Shifman, Vainshtein, and Zakharov (SVZ) in 1979 [32], QCD sum rules have become a powerful method to study the hadronic properties. The basic idea of this approach is to relate the QCD expression of the correlation function (obtained using the well-known operator product expansion) with the phenomenological parametrization by using the standard dispersion relation. The two-point correlation function of the vector current has the following Lorentz structure:

$$\begin{aligned}\Pi_{\mu\nu}(q^2) &= i\int d^4x e^{iqx}\langle 0|T[j_\mu(x)j_\nu^+(0)]|0\rangle \\ &= (q_\mu q_\nu - q^2 g_{\mu\nu})\Pi_v(q^2) + q_\mu q_\nu \Pi_s(q^2), \end{aligned} \tag{1}$$

where $j_\mu(x)$ in this work can be the four-quark currents that couple to both the 1^{+-} and the 0^{--} states, and the invariants $\Pi_v(q^2)$ and $\Pi_s(q^2)$ correspond, respectively, to contributions from the 1^{+-} and 0^{--} states.

The correlation function obeys the dispersion relation, which relates the $\Pi(q^2)$ with its imaginary part $\mathrm{Im}\Pi(q^2)$. For hadrons with light flavor quarks, the dispersion relation reads

$$\Pi_{v/s}(q^2) = \frac{1}{\pi}\int_0^\infty ds \frac{\mathrm{Im}\Pi_{v/s}(s)}{s - q^2 - i\epsilon}. \tag{2}$$

The dispersion relation provides an important connection between QCD and phenomenology as on the theoretical side the correlator can be expanded in terms of QCD vacuum condensates for large Euclidean momentum (i.e., $Q^2 = -q^2$ is much greater than the QCD scale Λ_{QCD}) while on the phenomenological side the spectral function at low energy can be measured and parametrized experimentally. In this work, we adopt the widely used "one single narrow resonance minimal duality ansatz" (which has been tested in $e^+e^- \to$ hadrons and charmonium data [33–35]) to parametrize the spectral function as

$$\begin{aligned}\frac{1}{\pi}\mathrm{Im}\Pi_{v/s}(s) &\simeq \sum_n \delta(s - m_n^2)\langle 0|\eta|n\rangle\langle n|\eta^+|0\rangle \\ &\simeq f_H^2\delta(s - m_H^2) + \text{“QCD continuum”} \\ &\quad \times \theta(s - s_0), \end{aligned} \tag{3}$$

where f_H and s_0, respectively, denote the coupling of the current to the hadron and the QCD continuum threshold.

On the QCD side, the correlation function can be calculated perturbatively using the operator product expansion,

$$\Pi_{v/s}(q^2) \simeq \sum_{d=0,2,4,\ldots} \frac{1}{(q^2)^{d/2}} \sum_{\dim O = d} C(q^2)\langle 0|O|0\rangle, \tag{4}$$

where $\langle 0|O|0\rangle$ are the QCD vacuum condensates of dimension d, $C(q^2)$ are the corresponding Wilson coefficients calculated in the perturbation theory. For the large Euclidian q^2, the OPE reaches good convergence; thus the first few terms (up to condensate dimension eight in this work) are expected to be a good approximation of the correlation function. To further improve the convergence of OPE and also to suppress the continuum contribution in the spectral integral, one can apply the inverse Laplace operator to both sides of the dispersion relation, and then the moment and the ratio of Laplace/Borel sum rules can be derived,

$$M_{v/s}(\tau, s_0) = \int_0^{s_0} ds \exp(-s\tau)\frac{1}{\pi}\mathrm{Im}\Pi_{v/s}(s), \tag{5}$$

$$R_{v/s}(\tau, s_0) = -\frac{d}{d\tau}\log M_{v/s} \simeq M_H^2. \tag{6}$$

Because of the uncertainties induced by the truncation of the OPE series and the simple parametrization of the spectral function, the output results depend on the two external parameters (τ, s_0). SVZ originally suggested the sum rule should be analyzed within a certain range of the Borel parameter τ, which ensures both the validity of the OPE truncation and the suppression of the continuum contribution to the spectral integral. Furthermore, some attempts to determine (τ, s_0) objectively from the sum rules have also been made following either the standard stability criterion with respect to (τ, s_0) [33,36–38][2] or the original concept of the sum rule window [32] (e.g., the Monte Carlo–based weighted-least-squared matching procedure [41–43][3]). One would expect to optimize the output results by demanding that they are insensitive to the variation of (τ, s_0). However, as is well known, LSR for multiquark currents (of high dimension) is more likely than

[2]For recent examples using the stability criterion see [39] for traditional QCD sum rules and [40] for light-cone QCD sum rules.

[3]The Holder inequalities can provide constraints on the sum rule window as discussed in [44–46].

those for ordinary $q\bar{q}$ measons to suffer from the simple parametrization of the spectral function, which could lead to the absence of the s_0 stability. Such cases have occurred in tetraquark and pentaquark sum rules, as has been discussed in [36–38,47–49].

In the cases where the s_0 stability is not reached, FESR has been shown in some other sum rule analyses for multiquark states [36–38,47] to be a useful complement. The moment and ratio of FESR read

$$\mathcal{R}_{v/s}(s_0) = \frac{\int_0^{s_0} dss\frac{1}{\pi}\mathrm{Im}\Pi_{v/s}(s)}{\int_0^{s_0} ds\frac{1}{\pi}\mathrm{Im}\Pi_{v/s}(s)} \simeq M_H^2, \tag{7}$$

which provides a connection between the lowest state mass and the continuum threshold. FESR can be obtained by letting the Borel parameter τ be zero in LSR prior to renormalization-group improvement; thus it is quite natural to expect such an approach can help reduce the effects of high dimensional condensates in the sum rules and provide the possibility to restore the s_0 stability.

III. INTERPOLATING CURRENTS FOR $0^{--}/1^{+-}$ LIGHT FOUR-QUARK STATES

The 0^{--} light four-quark states have been studied in [31] using the pseudoscalar diquark-antidiquark currents, which does not support a mass below 2 GeV. However, as noted earlier, the coefficients of four-quark condensates have been found to be zero in the OPE [31], which raises some doubts on the resulting accuracy of the sum rules. Furthermore, the s_0 stability is not reached in [31], suggesting that the currents used in [31] may not provide sufficiently reliable sum rules. Therefore, here we use diquark-antidiquark vector currents which can couple to both the 1^{+-} and 0^{--} four-quark states.

The Lorentz structures of the $1^{+-}/0^{--}$ diquark–antidiquark vector currents have been systematically studied in [24] for the charmoniumlike states. Here we use the $ud\bar{u}\,\bar{d}$ currents of the same Lorentz structures (under isospin symmetry, $uu\bar{u}\,\bar{u}$ and $dd\bar{d}\,\bar{d}$ share the same sum rules with $ud\bar{u}\,\bar{d}$ at leading order),

$$\begin{aligned}
J_{1\mu} &= u_a^TCd_b(\bar{u}_a\gamma_\mu\gamma_5C\bar{d}_b^T + \bar{u}_b\gamma_\mu\gamma_5C\bar{d}_a^T) - u_a^TC\gamma_\mu\gamma_5d_b(\bar{u}_aC\bar{d}_b^T + \bar{u}_bC\bar{d}_a^T),\\
J_{2\mu} &= u_a^TCd_b(\bar{u}_a\gamma_\mu\gamma_5C\bar{d}_b^T - \bar{u}_b\gamma_\mu\gamma_5C\bar{d}_a^T) - u_a^TC\gamma_\mu\gamma_5d_b(\bar{u}_aC\bar{d}_b^T - \bar{u}_bC\bar{d}_a^T),\\
J_{3\mu} &= u_a^TC\gamma_5d_b(\bar{u}_a\gamma_\mu C\bar{d}_b^T + \bar{u}_b\gamma_\mu C\bar{d}_a^T) - u_a^TC\gamma_\mu d_b(\bar{u}_a\gamma_5C\bar{d}_b^T + \bar{u}_b\gamma_5C\bar{d}_a^T),\\
J_{4\mu} &= u_a^TC\gamma_5d_b(\bar{u}_a\gamma_\mu C\bar{d}_b^T - \bar{u}_b\gamma_\mu C\bar{d}_a^T) - u_a^TC\gamma_\mu d_b(\bar{u}_a\gamma_5C\bar{d}_b^T - \bar{u}_b\gamma_5C\bar{d}_a^T),\\
J_{5\mu} &= u_a^TC\gamma^\nu d_b(\bar{u}_a\sigma_{\mu\nu}\gamma_5C\bar{d}_b^T + \bar{u}_b\sigma_{\mu\nu}\gamma_5C\bar{d}_a^T) - u_a^TC\sigma_{\mu\nu}\gamma_5d_b(\bar{u}_a\gamma^\nu C\bar{d}_b^T + \bar{u}_b\gamma^\nu C\bar{d}_a^T),\\
J_{6\mu} &= u_a^TC\gamma^\nu d_b(\bar{u}_a\sigma_{\mu\nu}\gamma_5C\bar{d}_b^T - \bar{u}_b\sigma_{\mu\nu}\gamma_5C\bar{d}_a^T) - u_a^TC\sigma_{\mu\nu}\gamma_5d_b(\bar{u}_a\gamma^\nu C\bar{d}_b^T - \bar{u}_b\gamma^\nu C\bar{d}_a^T),\\
J_{7\mu} &= u_a^TC\gamma^\nu\gamma_5d_b(\bar{u}_a\sigma_{\mu\nu}C\bar{d}_b^T + \bar{u}_b\sigma_{\mu\nu}C\bar{d}_a^T) - u_a^TC\sigma_{\mu\nu}d_b(\bar{u}_a\gamma^\nu\gamma_5C\bar{d}_b^T + \bar{u}_b\gamma^\nu\gamma_5C\bar{d}_a^T),\\
J_{8\mu} &= u_a^TC\gamma^\nu\gamma_5d_b(\bar{u}_a\sigma_{\mu\nu}C\bar{d}_b^T - \bar{u}_b\sigma_{\mu\nu}C\bar{d}_a^T) - u_a^TC\sigma_{\mu\nu}d_b(\bar{u}_a\gamma^\nu\gamma_5C\bar{d}_b^T - \bar{u}_b\gamma^\nu\gamma_5C\bar{d}_a^T),
\end{aligned} \tag{8}$$

where $J_{1\mu}$, $J_{3\mu}$, $J_{5\mu}$, $J_{7\mu}$ have the color structure $\mathbf{6}\otimes\bar{\mathbf{6}}$ and $J_{2\mu}$, $J_{4\mu}$, $J_{6\mu}$, $J_{8\mu}$ have the color structure $\bar{\mathbf{3}}\otimes\mathbf{3}$. Since the states of different isospin are degenerate in masses at leading order (LO), we do not differentiate the isospin in our calculation.

IV. QCD EXPRESSIONS FOR THE TWO-POINT CORRELATION FUNCTIONS

After performing the SVZ expansion in the chiral limit ($m_u = m_d = 0$) to LO of the perturbation series, we arrive at the following expression for the correlation function (up to dimension-eight condensate contributions) resulting from J_i ($i = 1$–8):

$$\frac{1}{\pi}\mathrm{Im}\Pi_{i;s/v}(s) = a_{i;s/v}\frac{s^3}{\pi^6} + b_{i;s/v}\frac{\langle\alpha_sG^2\rangle s}{\pi^5} + c_{i;s/v}\frac{\langle\bar{q}q\rangle^2}{\pi^2} + d_{i;s/v}\frac{\langle\bar{q}Gq\rangle\langle\bar{q}q\rangle}{\pi^2 s}, \tag{9}$$

where the coefficients $a_{i;v/s} - d_{i;v/s}$ are listed in Table I.[4]

However, a vacuum-factorization violation factor has been noticed for a long time in the process $e^+e^- \to$ hadrons [50–54] and τ decay [55]. Therefore it is necessary to consider the errors induced by the violation of factorization in our

[4]Here we omit the dimension-six and dimension-eight gluon condensate contributions which are suppressed by a loop factor. The complete evaluation of the dimension-eight quark and gluon condensate contributions considering operator mixing under renormalization was done in [43] for the 1^{-+} light hybrid meson, where the contributions from the gluon condensates are comparable to those from the quark condensates.

TABLE I. The coefficients for Eq. (9).

	i							
	1	2	3	4	5	6	7	8
$a_{i;s}$	1/30720	1/61440	1/30720	1/61440	1/10240	1/20480	1/10240	1/20480
$b_{i;s}$	−1/1536	1/1536	−1/1536	1/1536	11/1536	1/1536	11/1536	1/1536
$c_{i;s}$	1/6	1/12	−1/6	−1/12	−5/6	−5/12	1/6	1/12
$d_{i;s}$	1/8	1/16	−1/8	−1/16	−5/8	−5/16	1/8	1/16
$a_{i;v}$	1/18432	1/36864	1/18432	1/36864	1/6144	1/12288	1/6144	1/12288
$b_{i;v}$	−1/4608	1/4608	−1/4608	1/4608	11/4608	1/4608	11/4608	1/4608
$c_{i;v}$	−5/18	−5/36	5/18	5/36	25/18	25/36	−5/18	−5/36
$d_{i;v}$	−1/8	−1/16	1/8	1/16	5/8	5/16	−1/8	−1/16

TABLE II. QCD parameters used in our analysis. The quantity ρ indicates the violation of factorization hypothesis in estimating the four-quark condensates.

	Reference
$\langle\alpha_s G^2\rangle \simeq (7\pm 2)\times 10^{-2}$ GeV^4	Sum rules of $e^+e^- \to$ hadrons [52,54,56] and J/Ψ [57,58]
$g\langle\bar\psi G\psi\rangle \equiv g\langle\bar\psi \frac{\lambda_a}{2}\sigma^{\mu\nu}G^a_{\mu\nu}\psi\rangle \simeq (0.8\pm 0.1)$ GeV$^2\langle\bar\psi\psi\rangle$	Light baryon systems [59,60]
$\rho\alpha_s\langle\bar\psi\psi\rangle^2 \simeq (4.5\pm 0.3)\times 10^{-4}$ GeV^6	$e^+e^- \to$ hadrons [53,54] and τ decay [55]
$\Lambda_{\rm QCD} = (353\pm 15)$ MeV	τ decay [55]

numerical analysis. For the condensates $d \le 6$ and the QCD scale which are under good control from the experiments, we shall use the values given in Table II.

V. THE LSR AND FESR NUMERICAL ANALYSES

In this section, we will present the numerical analysis using Laplace sum rules and finite energy sum rules. We shall optimize the phenomenological predictions following the standard sum rule stability criteria, which provide rigorous constraints on the free parameters (τ and s_0 in LSR and s_0 in FESR). These criteria have been successfully used to study light [11,36,37,40] and heavy [38,39,61] meson spectrum and decay properties and explicitly clarified in [33,62,63]. Here we briefly review these criteria by elaborating on the numerical procedure in this work:

(1) An LSR ratio (at certain s_0) is considered to reach the τ stability if the mass (versus τ) curve presents a platform or an extremum. In the range of s_0 where the τ stability is reached, the mass becomes a single-valued function of s_0. If the mass-s_0 curve presents an extremum, the s_0 stability is considered to be reached.

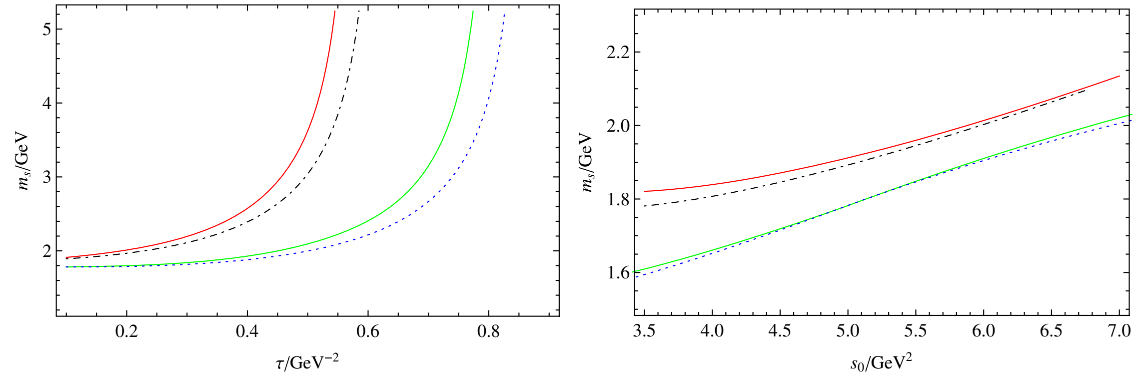
(2) An FESR ratio is considered to reach stability if the mass curve presents an extremum, which can determine the mass and s_0. For the cases where the s_0 stability is absent in LSR, the s_0 can be determined from the FESR ratios.

(3) The LSR/FESR results are considered reliable if the validity of OPE truncation is ensured at the stability points [either (if any) the rigorous LSR/FESR stability points or the LSR τ-stability points obtained at the s_0 deduced from FESR]. The final sum rule predictions are obtained from both the optimized LSR and FESR results.

A. Masses of the 0^{--} four-quark states

As mentioned previously, the sum rules for currents of different color structures at LO are only slightly different (we will see this more obviously in this and the next subsections). What affects the behavior of the sum rules considerably are the Lorentz structures of the currents, by which we will categorize the currents in our analyses. For each category, if stability is reached (with converging OPE for LSR), we will extract the mass considering the effects of violation of factorization as a source of theoretical uncertainty in our analysis.

We first consider the LSR for $J_{1\mu}$ and $J_{2\mu}$. As shown in Fig. 1, the LSR for both J_1 and J_2 reach the τ stability (which is lost after considering the violation of factorization). However, the masses obtained from the τ-stability points increase gradually with s_0, which means the s_0 stability is not reached and thus s_0 cannot be determined from the LSR for J_1 and J_2. We consider the following values (without considering the violation of factorization) obtained at $s_0 = 5.0$ GeV^2 (which can be deduced from the subsequent FESR analyses corresponding to other currents):



(a) (b)

FIG. 1. (a) The 0^{--} four-quark masses versus τ obtained from the LSR for $J_{1\mu}$ (green continuous line), from the LSR for $J_{1\mu}$ with violation of factorization by a factor $\rho = 2$ in estimating the dimension-eight condensates (red continuous line), from the LSR for $J_{2\mu}$ (blue dotted line), and from the LSR for $J_{2\mu}$ with violation of factorization by a factor $\rho = 2$ in estimating the dimension-eight condensates (black dot-dashed line); (b) the same as (a) but for the masses versus s_0.

$$
\begin{aligned}
M_{s1;L} &= 1.78\ \text{GeV} \quad \text{at } s_0 = 5.0\ \text{GeV}^2,\\
M_{s2;L} &= 1.78\ \text{GeV} \quad \text{at } s_0 = 5.0\ \text{GeV}^2. \qquad (10)
\end{aligned}
$$

We have checked that these results are associated with converging OPE (see Table III in Appendix A); therefore they are considered as valid results when making the final estimate.

The FESR for J_1 and J_2 also do not reach stability, which can be seen in Fig. 2. Therefore no results can be obtained from the FESR.

J_3–J_6 belong to two different Lorentz structures, but these four currents have similar sum rule behavior; thus we present their results together. Contrary to J_1 and J_2, J_3–J_6 have worse LSR but better FESR behavior. The LSR ratios for $J_{3\mu}$–$J_{6\mu}$ monotonically increase with the Borel parameter τ; thus no (τ, s_0) stability is reached in LSR, while the FESR ratios reach stability in s_0 as shown in Fig. 3, which allow the following mass predictions:

$$
\begin{aligned}
M_{s3;F} &= 1.67(1.81)\ \text{GeV} \quad \text{at } s_0 = 5.4(5.7)\ \text{GeV}^2,\\
M_{s4;F} &= 1.61(1.73)\ \text{GeV} \quad \text{at } s_0 = 4.6(5.0)\ \text{GeV}^2,\\
M_{s5;F} &= 1.65(1.82)\ \text{GeV} \quad \text{at } s_0 = 5.2(5.6)\ \text{GeV}^2,\\
M_{s6;F} &= 1.71(1.85)\ \text{GeV} \quad \text{at } s_0 = 5.9(6.2)\ \text{GeV}^2, \qquad (11)
\end{aligned}
$$

where we have also presented the mass predictions (in the brackets) obtained considering the deviation of the dimension-eight condensates from their factorized values. From Figs. 10(a)–10(d) in Appendix B, we can observe the FESR curves for different truncations of the OPE. All these FESR ratios start to reach stability after the inclusion of power corrections from the dim-4–6 condensates. In Figs. 10(a) and 10(d), the FESR ratios with the inclusion of the $d \leqq 8$ condensate terms tend to approach those with $d \leqq 6$ condensate terms around the stability points (or the extreme points), justifying the validity of the OPE truncation for the FESR ratios corresponding to J_3 and J_6, while in Figs. 10(b) and 10(c), the inclusion of the $d = 8$ condensate contributions in the OPE still affect the FESR mass at the stability points to a (relatively) large extent, suggesting the OPE truncations for these ratios are still associated with (relatively) large uncertainties. Therefore, we only consider $M_{s3;F}$ and $M_{s6;F}$ in our final mass determination.

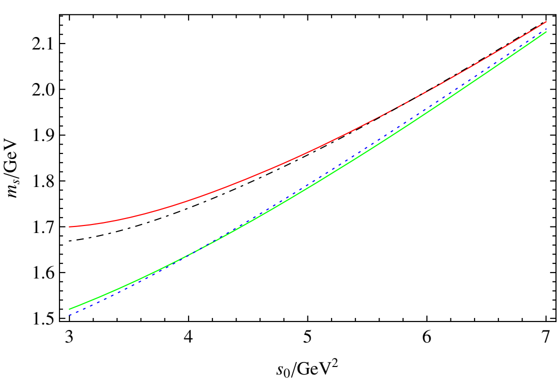


FIG. 2. The 0^{--} four-quark masses versus s_0 obtained from the FESR for $J_{1\mu}$ (green continuous line), from the FESR for $J_{1\mu}$ with violation of factorization by a factor $\rho = 2$ in estimating the dimension-eight condensates (red continuous line), from the FESR for $J_{2\mu}$ (blue dotted line), and from the FESR for $J_{2\mu}$ with violation of factorization by a factor of $\rho = 2$ in estimating the dimension-eight condensates (black dot-dashed line).

For J_7 and J_8, the LSR moment ratios begin to reach the τ stability at $s_0 = 3.8$ and $s_0 = 4.2$, respectively. But again the s_0 stability is absent. If we use $s_0 = 5\ \text{GeV}^2$ (as we did for J_1 and J_2, which are deduced from the FESR ratios corresponding to J_3–J_6), we obtain (from Fig. 4)

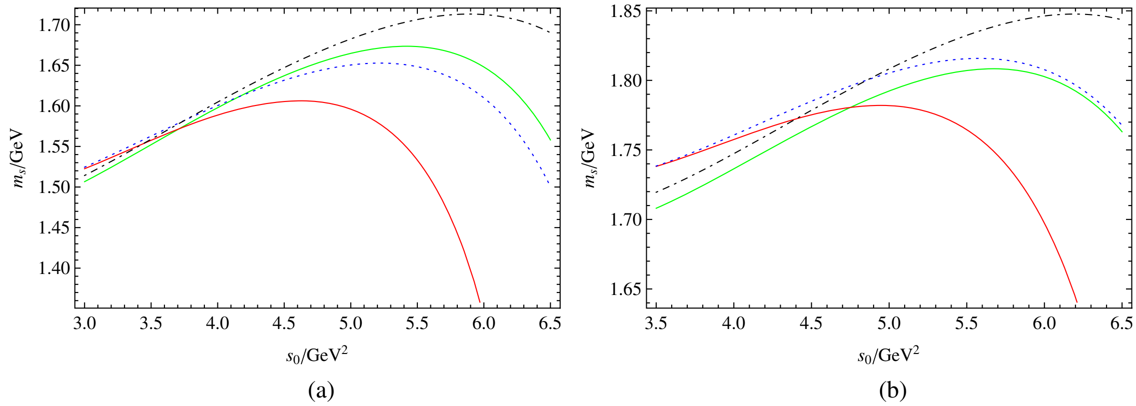


FIG. 3. (a) The 0^{--} four-quark masses versus s_0 obtained from the FESR for $J_{3\mu}$ (green continuous line), from the FESR for $J_{4\mu}$ (red continuous line), from the FESR for $J_{5\mu}$ (blue dotted line), and from the FESR for $J_{6\mu}$ (black dot-dashed line). (b) The same as (a) but for masses versus s_0 from the FESR with violation of factorization by a factor $\rho = 2$ in estimating the dimension-eight condensates.

$$M_{s7;L} = 1.80(1.95)\ \text{GeV} \quad \text{at } s_0 = 5.0(5.5)\ \text{GeV}^2,$$
$$M_{s8;L} = 1.83(1.97)\ \text{GeV} \quad \text{at } s_0 = 5.0(5.5)\ \text{GeV}^2. \quad (12)$$

We have also check that at the stability points corresponding to the above results, the OPE series are converging (see Table III in Appendix A); therefore these results are considered reliable.

Unfortunately, the FESR ratios for both J_7 and J_8 do not reach stability (plots are not shown here for simplicity). On the contrary, they increase with s_0, which means no results can be obtained from these sum rules.

In our analysis of the pseudoscalar channel, the LSR ratios corresponding to $J_{1;2}$ and $J_{7;8}$ reach the τ stability but do not reach the s_0 stability. We fix the s_0 in the LSR analysis with the help of the FESR ratios corresponding to J_3–J_6. From both LSR and FESR results,

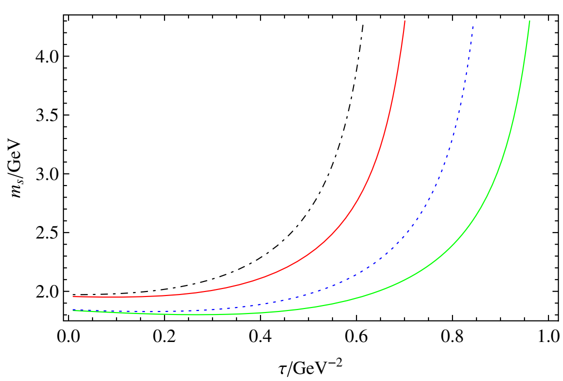


FIG. 4. The 0^{--} four-quark masses versus τ obtained from the LSR for $J_{7\mu}$ (green continuous line), from the LSR for $J_{7\mu}$ with violation of factorization by a factor $\rho = 2$ in estimating the dimension-eight condensates (red continuous line), from the LSR for $J_{8\mu}$ (blue dotted line), and from the LSR for $J_{8\mu}$ with violation of factorization by a factor $\rho = 2$ in estimating the dimension-eight condensates (black dot-dashed line).

our final mass prediction for the 0^{--} four-quark state is 1.76 ± 0.15 GeV.[5]

B. Masses of the 1^{+-} four-quark states

Similarly, we use the stability criterion for extracting the mass of the 1^{+-} four-quark states. For $J_{1\mu}$ and $J_{2\mu}$, LSR reach both the τ and s_0 stabilities as shown in Fig. 5, from which we obtain the following predictions:

$$M_{v1;L} = 1.23(1.33)\ \text{GeV} \quad \text{at } s_0 = 4.8(5.0)\ \text{GeV}^2,$$
$$M_{v2;L} = 1.22(1.32)\ \text{GeV} \quad \text{at } s_0 = 4.8(5.0)\ \text{GeV}^2, \quad (13)$$

where we have presented the results (in the brackets) with the violation of factorization, and we have also checked that the OPE starts to converge at $d = 6$ condensate terms and the $d = 8$ condensate contributions are less than 18% (respectively, 15.6% and 17.1%) of the total QCD expression (see Table IV). Although the proportions of the highest dimensional condensate (HDC) terms nearly fulfill the usual imposed condition of a sum rule window for multiquark states [48,49], the OPE suffer from small contributions of perturbative terms, making it uncertain to make predictions explicitly based on these results. Instead, we shall consider them as a rough check of the FESR results.

We also use the FESR to study the mass of the 1^{+-} four-quark states. For $J_{1\mu}$ and $J_{2\mu}$, the following results can be obtained from the stability points of the FESR ratios (see Fig. 6):

$$M_{v1;F} = 1.42(1.44)\ \text{GeV} \quad \text{at } s_0 = 5.1(5.3)\ \text{GeV}^2,$$
$$M_{v2;F} = 1.40(1.42)\ \text{GeV} \quad \text{at } s_0 = 4.9(5.1)\ \text{GeV}^2. \quad (14)$$

[5]Here we take the arithmetic average of the masses when estimating the central value. We consider the violation of factorization, the use of different currents, and the uncertainties of the input parameters when estimating the errors.

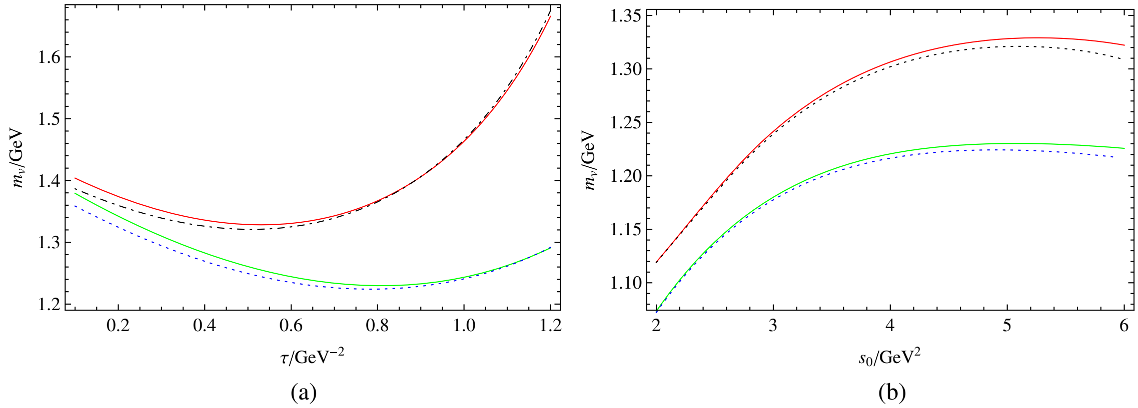


FIG. 5. (a) The 1^{+-} four-quark masses versus τ obtained from the LSR for $J_{1\mu}$ (green continuous line), from the LSR for $J_{1\mu}$ with violation of factorization by a factor $\rho = 2$ in estimating the dimension-eight condensates (red continuous line), from the LSR for $J_{2\mu}$ (blue dotted line), and from the LSR for $J_{2\mu}$ with violation of factorization by a factor $\rho = 2$ in estimating the dimension-eight condensates (black dot-dashed line). (b) The same as (a) but for masses versus s_0.

The behavior of these FESR ratios under different truncations of the OPE are shown in Figs. 9(a) and 9(b), which clearly show a trend of the mass to converge at the stability points when including higher dimensional condensate terms.

As in the pseudoscalar channel, $J_{3\mu}$–$J_{6\mu}$ (belonging to two different Lorentz structures) also have quite similar sum rules. The LSR ratios show τ stability but no s_0 stability. Using the s_0 fixed from the LSR and FESR for $J_{1\mu}$ and $J_{2\mu}$, all these moments give a mass around 1.27 GeV, but the corresponding OPE series do not converge. Therefore we shall not consider this value in our final determination of the mass. The FESR for $J_{3\mu}$–$J_{6\mu}$ do not reach stability in s_0 either, which means no reliable predictions can be obtained from the sum rules for $J_{3\mu}$–$J_{6\mu}$.

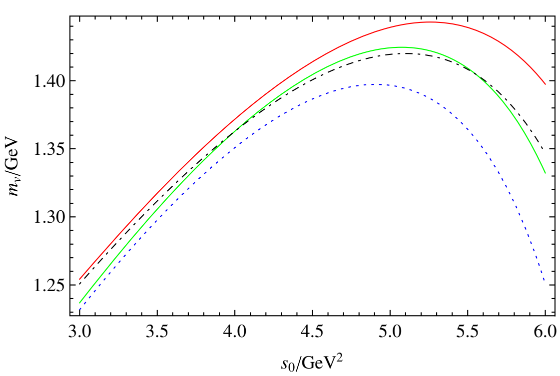


FIG. 6. The 1^{+-} four-quark masses versus s_0 obtained from the FESR for $J_{1\mu}$ (green continuous line), from the FESR for $J_{1\mu}$ with violation of factorization by a factor $\rho = 2$ in estimating the dimension-eight condensates (red continuous line), from the FESR for $J_{2\mu}$ (blue dotted line), and from the FESR for $J_{2\mu}$ with $\rho = 2$ violation of factorization by a factor of 2 in estimating the dimension-eight condensates (black dot-dashed line).

For $J_{7\mu}$ and $J_{8\mu}$, using the stability criterion for (τ, s_0), we obtain from the LSR ratios (see Fig. 7) the following optimal values:

$$M_{v7;L} = 1.15(1.22)\ \text{GeV} \quad \text{at } s_0 = 3.2(3.4)\ \text{GeV}^2,$$
$$M_{v8;L} = 1.17(1.24)\ \text{GeV} \quad \text{at } s_0 = 3.5(3.6)\ \text{GeV}^2. \tag{15}$$

Although the LSR ratios reach the (τ, s_0) stability and also the OPE show obvious trends of convergence, the dimension-eight condensate terms contribute more than half of the total OPE expressions (due to the cancellation between the lower dimensional condensate terms and the perturbative terms; see Table IV), raising some doubts on the validity of the OPE truncation here. Therefore we only consider them as a double-check of other LSR/FESR results.

The FESR for $J_{7\mu}$ and $J_{8\mu}$ also reach stability, which occurs at lower s_0 compared with the sum rules for $J_{1\mu}$ and $J_{2\mu}$. However, the predicted masses are comparable to those obtained using other currents, which read (from Fig. 8):

$$M_{v7;F} = 1.18(1.23)\ \text{GeV} \quad \text{at } s_0 = 3.3(3.5)\ \text{GeV}^2,$$
$$M_{v8;F} = 1.22(1.26)\ \text{GeV} \quad \text{at } s_0 = 3.5(3.7)\ \text{GeV}^2, \tag{16}$$

where the associated OPE truncations are shown to be reliable in Fig. 11(a) and 11(b) because the $d \leqq 6$ masses are already very close to the $d \leqq 8$ masses around the stability points.

From the numerical analysis of the 1^{+-} channel, we can see that the LSR and FESR for $J_{1\mu}$, $J_{2\mu}$, $J_{7\mu}$, and $J_{8\mu}$ reach stability. From these sum rules, we can obtain the optimal result for the 1^{+-} four-quark mass.

Since the OPE truncation at the LSR stability points may associate with relatively large errors, we make our predictions based on the FESR results and consider the LSR

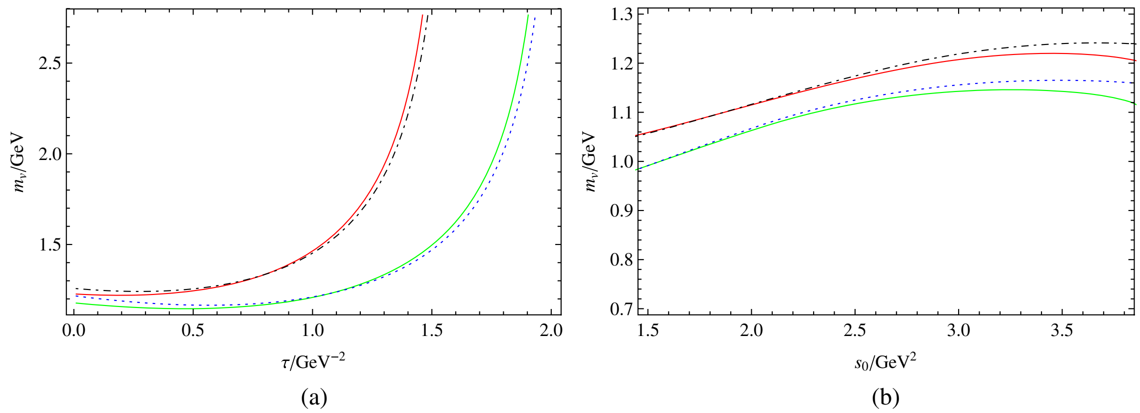


FIG. 7. (a) The 1^{+-} four-quark masses versus τ obtained from the LSR for $J_{7\mu}$ (green continuous line), from the LSR for $J_{7\mu}$ with violation of factorization by a factor $\rho = 2$ in estimating the dimension-eight condensates (red continuous line), from the LSR for $J_{8\mu}$ (blue dotted line), and from the LSR for $J_{8\mu}$ with violation of factorization by a factor $\rho = 2$ in estimating the dimension-eight condensates (black dot-dashed line). (b) The same as (a) but for masses versus s_0.

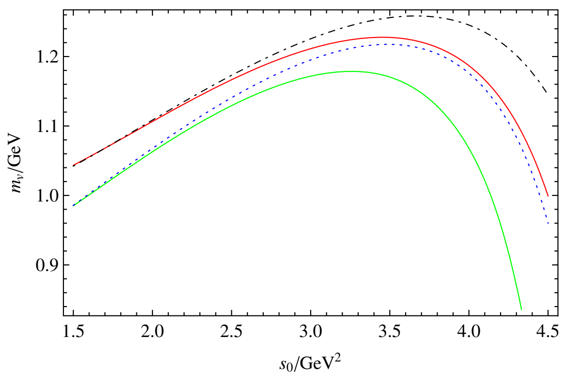


FIG. 8. The 1^{+-} four-quark masses versus τ obtained from the FESR for $J_{7\mu}$ (green continuous line), from the FESR for $J_{7\mu}$ with violation of factorization by a factor $\rho = 2$ in estimating the dimension-eight condensates (red continuous line), from the FESR for $J_{8\mu}$ (blue dotted line), and from the FESR for $J_{8\mu}$ with violation of factorization by a factor $\rho = 2$ in estimating the dimension-eight condensates (black dot-dashed line).

results as double-checks. Because of the lack of valid LSR results, we consider a conservative range instead of extracting a central value from FESR. The obtained mass range is $M_v = 1.18$–1.44 GeV, which is consistent with the rough LSR results within the errors.

VI. DISCUSSION AND CONCLUSIONS

We have studied the 0^{--} and 1^{+-} light four-quark states using Laplace and finite energy sum rules. In the pseudoscalar channel, we have obtained the optimal mass prediction 1.76 ± 0.15 GeV from LSR and FESR following the sum rule stability criteria, where the errors mainly come from the violation of factorization in estimating the dimension-eight condensates and the discrepancy among the results obtained from different sum rule ratios. Our prediction for the 0^{--} four-quark states is significantly lower than the ones obtained in [31] using different interpolating currents, where a mass below 2 GeV is not supported but the results suffer from the absence of four-quark condensates in the OPE. In contrast to the previous work, the results in this work do not exclude the possibility of the $\rho\pi$ dominance in D^0 decay to be a four-quark state. Given that the D^0 mass is much lower than the mass prediction for a 0^{--} hybrid state in the QCD Coulomb gauge approach [10] and QCD sum rules [11] and barely covered by the range predicted in the constituent gluon models [64,65], the four-quark explanation seems to be more reasonable. Moreover, the mixing of the four-quark state and hybrid state is another possible explanation that needs to be considered, which we hope to discuss in the future.

Following our prediction, we can discuss the decay patterns of the 0^{--} four-quark states. Considering the kinematical constraints and the conservation of I, G, J, P, C we find the following two-body hadronic decay modes:

$$
\begin{aligned}
&X_{0^-0^{--}} \to \rho\pi, \omega\eta, f_0 h_1; \\
&X_{1^+0^{--}} \to a_0\pi, \omega\pi, \rho\eta; \\
&X_{2^-0^{--}} \to \rho\pi. \qquad (17)
\end{aligned}
$$

The $\rho\pi$ decay mode of the isoscalar state can be the observed channel (via $\rho \to \pi^+\pi^-$) in *BABAR* [1]. If this 3π resonance exists, it may also be seen in the $\omega\eta$ final states. The isospin partner states are expected to be observed in the above final states, among which the charged $a_0\pi$ final states are worth special attention, for they are the only possible S-wave decay mode; the others are in P wave.

For the vector channel, the LSR ratios which reach the (τ,s_0) stability are not associated with good OPE convergence. We have conservatively estimated the mass to be in the range 1.18–1.44 GeV from FESR, which suggest the 1^{+-} four-quark states lie within the 270 MeV range above the conventional $q\bar{q}$ state $h_1(1170)$.

ACKNOWLEDGMENTS

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APPENDIX A: THE OPE TERMS AT THE LSR STABILITY POINTS

See Tables III and IV.

TABLE III. OPE terms in the Borel transformed 0^{--} invariant correlators at the LSR τ-stability points with the s_0 fixed from FESR.

i	$\frac{1}{\tau}\hat{B}^{a}\Pi_{i;v}^{d=0}/\mathrm{GeV}^4$	$\frac{1}{\tau}\hat{B}\Pi_{i;s}^{d=4}/\mathrm{GeV}^4$	$\frac{1}{\tau}\hat{B}\Pi_{i;s}^{d=6}/\mathrm{GeV}^4$	$\frac{1}{\tau}\hat{B}\Pi_{i;s}^{d=8}/\mathrm{GeV}^4$	τ/GeV^{-2}	s_0/GeV^2
1	1.07067×10^{-2}	-3.41877×10^{-5}	5.67043×10^{-5}	6.68983×10^{-6}	0.066	5.0
2	5.80777×10^{-4}	1.12606×10^{-5}	1.62717×10^{-5}	2.09805×10^{-6}	0.115	5.0
7	1.18144×10^{-4}	2.28077×10^{-5}	1.39645×10^{-5}	3.96477×10^{-7}	0.268	5.0
8	3.91793×10^{-4}	5.33979×10^{-6}	1.12050×10^{-5}	1.26034×10^{-6}	0.167	5.0

[a]The Borel operator is defined as $\hat{B}\equiv\lim_{\substack{Q^2,n\to\infty\\ n/Q^2=\tau}}\frac{(Q^2)^n}{(n-1)!}\left(-\frac{\mathrm{d}}{\mathrm{d}Q^2}\right)^n$.

TABLE IV. OPE terms in the Borel transformed 1^{+-} invariant correlators at the LSR stability points.

i	$\frac{1}{\tau}\hat{B}\Pi_{i;v}^{d=0}/\mathrm{GeV}^4$	$\frac{1}{\tau}\hat{B}\Pi_{i;v}^{d=4}/\mathrm{GeV}^4$	$\frac{1}{\tau}\hat{B}\Pi_{i;v}^{d=6}/\mathrm{GeV}^4$	$\frac{1}{\tau}\hat{B}\Pi_{i;v}^{d=8}/\mathrm{GeV}^4$	τ/GeV^{-2}	s_0/GeV^2
1	7.94391×10^{-7}	-7.60350×10^{-8}	-7.71964×10^{-6}	-1.29002×10^{-6}	0.808	4.8
2	7.48111×10^{-7}	8.07614×10^{-8}	-3.97798×10^{-6}	-7.12718×10^{-7}	0.784	4.8
7	2.13567×10^{-5}	2.50377×10^{-6}	-1.33565×10^{-5}	-3.75212×10^{-6}	0.467	3.2
8	6.73668×10^{-6}	1.80790×10^{-7}	-5.95178×10^{-6}	-1.61746×10^{-6}	0.524	3.5

APPENDIX B: THE FESR MASS CURVES FOR DIFFERENT TRUNCATIONS OF THE OPE

See Figs. 9, 10, and 11.

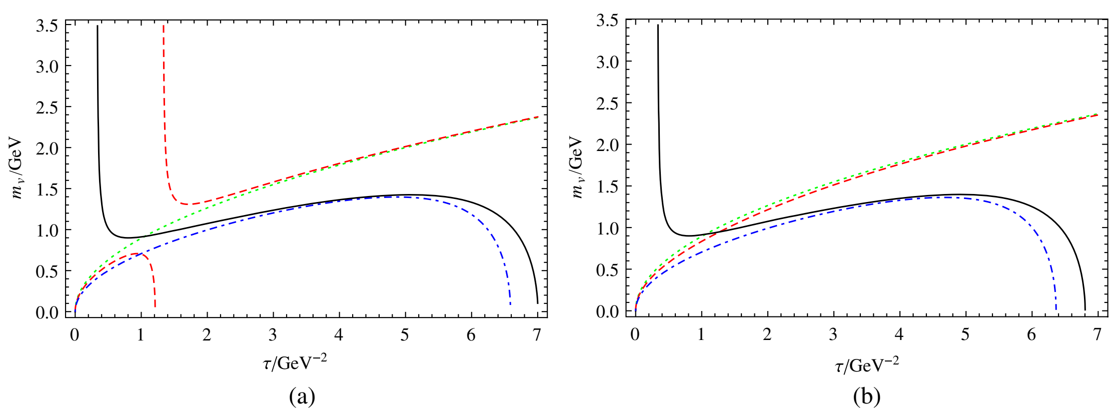

FIG. 9. (a) The 1^{+-} four-quark FESR masses corresponding to J_1, obtained from truncating the OPE at the perturbative terms (green dotted line), at the dim-4 condensate terms (red dashed line), at the dim-6 condensate terms (blue dot-dashed line), and at the dim-8 condensate terms (black continuous line). (b) The same as (a) but for the masses corresponding to J_2.

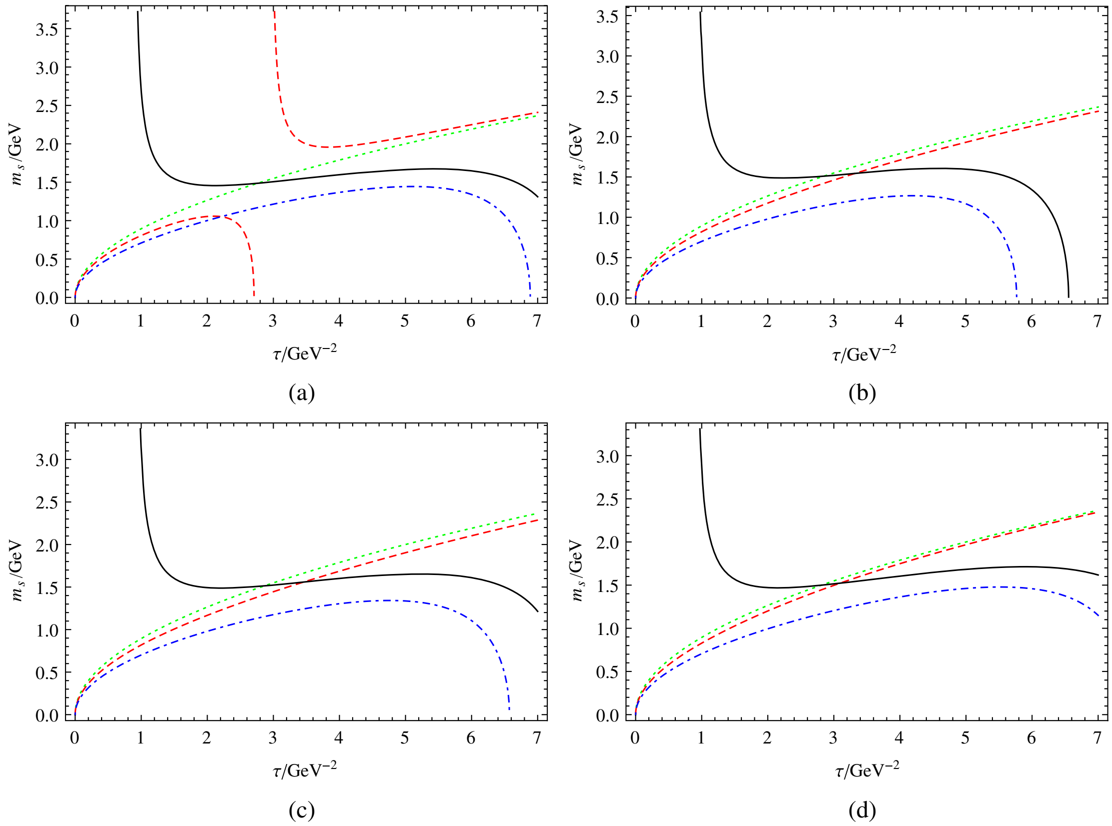

FIG. 10. (a) The 0^{--} four-quark FESR masses corresponding to J_3, obtained from truncating the OPE at the perturbative terms (green dotted line), at the dim-4 condensate terms (red dashed line), at the dim-6 condensate terms (blue dot-dashed line), and at the dim-8 condensate terms (black continuous line). (b) The same as (a) but for the masses corresponding to J_4. (c) The same as (a) but for the masses corresponding to J_5. (d) The same as (a) but for the masses corresponding to J_6.

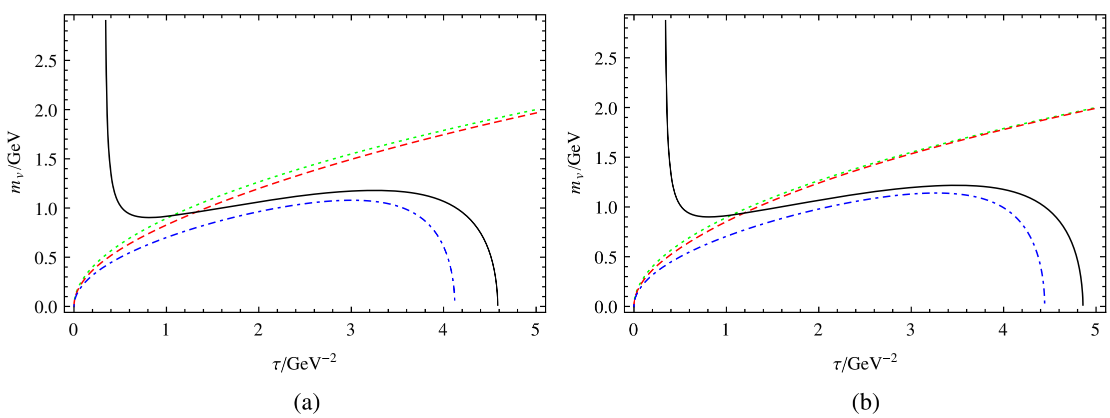

FIG. 11. (a) The 1^{+-} four-quark FESR masses corresponding to J_7, obtained from truncating the OPE at the perturbative terms (green dotted line), at the dim-4 condensate terms (red dashed line), at the dim-6 condensate terms (blue dot-dashed line), and at the dim-8 condensate terms (black continuous line). (b) the same as (a) but for the masses corresponding to J_8.

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