

Revisiting the factorization theorem for $\rho\gamma^* \rightarrow \pi(\rho)$ at twist 3Shan Cheng,^{1,*} Ya-lan Zhang,² Jun Hua,² Hsiang-nan Li,^{3,†} and Zhen-jun Xiao^{2,‡}¹*Theoretische Elementarteilchenphysik, Naturwissenschaftlich Technische Fakultät, Universität Siegen, 57068 Siegen, Germany*²*Department of Physics and Institute of Theoretical Physics, Nanjing Normal University, Nanjing, Jiangsu 210023, People's Republic of China*³*Institute of Physics, Academia Sinica, Taipei, Taiwan 115, Republic of China*

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We revisit the proof of the perturbative QCD factorization for the exclusive processes $\rho\gamma^* \rightarrow \pi(\rho)$ at the two-parton twist-3 level. It is pointed out that the residual collinear divergences observed in the literature, which break the factorization of the above processes at the considered accuracy, are attributed to the improper insertion of the Fierz identity for factorizing the fermion flow. We show that the factorization theorem indeed holds at the two-parton twist-3 level after the mishandling is corrected.

DOI: [10.1103/PhysRevD.95.076005](https://doi.org/10.1103/PhysRevD.95.076005)**I. INTRODUCTION**

The factorization theorem is the foundation of the perturbative QCD formalism [1–6], which states that non-perturbative dynamics in a hard QCD process can be factorized into universal nonlocal hadronic matrix elements defined in an infinite momentum frame. Recently, some of us have attempted to extend the proof of the collinear factorization for exclusive processes involving only pseudoscalar mesons at the subleading-power (twist) accuracy [7,8] to those involving also vector mesons: it was examined whether the collinear divergences in the light-to-light scattering $\rho\gamma^* \rightarrow \pi(\rho)$ are factorized into the two-parton twist-3 ρ meson light-cone distribution amplitudes [9,10]. Their next-to-leading-order (NLO) analysis indicated that the triple-gluon vertex gives residual collinear contributions, which cannot be absorbed into the meson distribution amplitudes. Namely, the universality of the meson distribution amplitudes, and thus the collinear factorization for $\rho\gamma^* \rightarrow \pi(\rho)$, was violated at the twist-3 level.

In this paper we revisit the factorization theorem for the above processes, pointing out that the residual collinear divergences at the two-parton twist-3 level observed in [9,10] are attributed to the improper insertion of the Fierz identity: the twist-3 spin projectors were inserted into the scattering amplitudes first to factorize the fermion flow between the hadronic matrix elements and the hard kernels; radiative gluons were added to the hard kernels and the resultant infrared divergences were investigated subsequently. A hard kernel is not a physical quantity, based on which the matching between the full QCD and the effective theory for infrared physics cannot be performed correctly. For instance, missing the power-law behavior of

hadronic matrix elements would lead to wrong power counting for scattering amplitudes. This is the reason why some collinear divergences survive the matching, and break the factorization theorem. We claim that the appropriate procedure to examine the factorization theorem at a subleading level follows the one proposed in [8], which starts with an analysis of infrared divergences in quark-level scattering amplitudes. The Fierz identity is inserted to factorize the fermion flow between the hadronic matrix elements and the hard kernels, after the collinear divergences have been absorbed into the former.

We first derive the power counting for the products of various gamma matrices with a valence quark spinor by means of the equation of motion without the three-parton terms. It is then demonstrated that the residual collinear divergences in radiative corrections to the $\rho\gamma^* \rightarrow \pi(\rho)$ scattering amplitudes are actually power suppressed and negligible up to twist 3. As a result, the soft divergences cancel, and the collinear divergences can be absorbed completely into the hadronic matrix elements. The twist-3 spin projectors are inserted into the scattering amplitudes at this stage to factorize the fermion flow between the hadronic matrix elements and the hard kernels [7,8], with the former defining the two-parton twist-3 meson distribution amplitudes. After proving the collinear factorization, we allow valence quarks to be off their mass shell by including quark transverse momenta k_T . The collinear divergences, regularized into $\ln k_T$ in this formalism, can be collected into the transverse-momentum-dependent (TMD) two-parton twist-3 meson wave functions in a similar way. It is concluded, contrary to the observation in [9,10], that the factorization theorem indeed holds for the processes $\rho\gamma^* \rightarrow \pi(\rho)$ up to the two-parton twist-3 level.

The plan of this paper is as follows. In Sec. II we explain the source of the factorization violation in [9,10], taking the process $\rho\gamma^* \rightarrow \pi$ as an example. The correct collinear factorization is presented in Sec. III with the help of the

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equation of motion for a valence quark. The above approach applies to another considered process $\rho\gamma^* \rightarrow \rho$ apparently, and can be extended to the more complicated k_T factorization. Section IV contains the conclusion.

II. FACTORIZATION VIOLATION AT TWIST 3?

We choose the kinematic variables for the process $\rho\gamma^* \rightarrow \pi$ in the light-cone coordinates,

$$p_1 = \frac{Q}{\sqrt{2}}(1, 0, \mathbf{0}), \quad p_2 = \frac{Q}{\sqrt{2}}(0, 1, \mathbf{0}),$$

$$k_1 = x_1 p_1, \quad k_2 = x_2 p_2, \quad (1)$$

where p_1 (p_2) is the 4-momentum of the ρ meson (pion), and k_1 (k_2) is the parton momentum carried by the antiquark in the ρ meson (pion), x_1 and x_2 being the momentum fractions. We consider the region with a large momentum transfer squared $Q^2 = -(p_1 - p_2)^2$, where perturbative QCD is applicable. To define the direction of the gauge links for the ρ meson light-cone distribution amplitudes, we introduce the dimensionless vector $v = (0, 1, \mathbf{0})$.

The LO, namely, $\mathcal{O}(\alpha_s)$ diagrams for the process $\rho\gamma^* \rightarrow \pi$ are displayed in Fig. 1. For the purpose of explaining the factorization violation found in [9,10], it is enough to focus on radiative corrections to Fig. 1(a). Figure 1(a) yields the partial $\rho \rightarrow \pi$ scattering amplitude

$$G^{(0)}(x_1, x_2) = ie_u g_s^2 C_F N_C \text{Tr}$$

$$\times \left[\frac{\gamma_\nu d(k_2) \bar{u}(\bar{k}_2) \gamma_\mu (\not{p}_1 - \not{k}_2) \gamma^\nu u(\bar{k}_1) \bar{d}(k_1)}{(p_1 - k_2)^2 (k_1 - k_2)^2} \right], \quad (2)$$

with the electric charge e_u of the u quark, the strong coupling g_s , the color factor C_F , the number of colors N_C , the quark spinors u and d , the momenta $\bar{k}_1 = p_1 - k_1$ and $\bar{k}_2 = p_2 - k_2$, and the gamma matrix γ_μ from the virtual

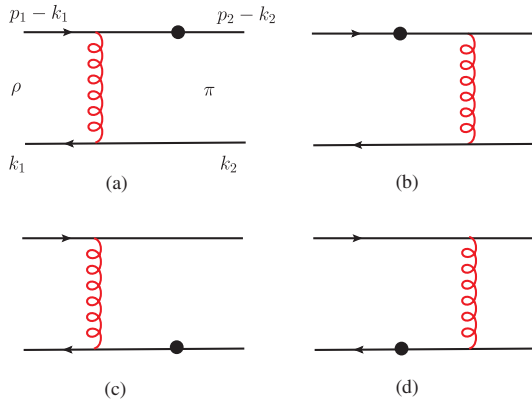


FIG. 1. Leading-order (LO) diagrams for the scattering $\rho\gamma^* \rightarrow \pi(\rho)$. The symbol \bullet represents the virtual photon vertex.

photon vertex. We factorize the fermion flow by inserting the Fierz identity

$$I_{ij} I_{lk} = \frac{1}{4} I_{ik} I_{lj} + \frac{1}{4} (\gamma_5)_{ik} (\gamma_5)_{lj} + \frac{1}{4} (\gamma^\alpha)_{ik} (\gamma_\alpha)_{lj}$$

$$+ \frac{1}{4} (\gamma_5 \gamma^\alpha)_{ik} (\gamma_\alpha \gamma_5)_{lj} + \frac{1}{8} (\sigma^{\alpha\beta})_{ik} (\sigma_{\alpha\beta})_{lj} \quad (3)$$

into Eq. (2), where $\sigma^{\alpha\beta} = i[\gamma^\alpha, \gamma^\beta]/2$, and different terms on the right-hand side lead to contributions characterized by different powers in $1/Q$. The color flow is factorized by inserting the identity

$$I_{ij} I_{lk} = \frac{1}{N_C} I_{ik} I_{lj} + 2 \sum_c T_{ik}^c T_{lj}^c, \quad (4)$$

with T^c being a color matrix. The first term on the right-hand side is associated with a two-parton (quark-antiquark) Fock state, which we concentrate on, and the second term is associated with a three-parton (quark-antiquark-gluon) Fock state.

We then arrive at the LO factorization formula in terms of the convolution in the parton momentum fraction $\xi_{1,2}$,

$$G^{(0)}(x_1, x_2) = \int d\xi_1 d\xi_2 \sum_{i=1,2} \Phi^{\pi(0)}(x_2, \xi_2) H_i^{(0)}$$

$$\times (\xi_1, \xi_2) \Phi_i^{\rho(0)}(x_1, \xi_1), \quad (5)$$

between the LO meson distribution amplitudes $\Phi^{(0)}$ and the LO hard kernels $H^{(0)}$,

$$\Phi_{1,2}^{\rho(0)}(x_1, \xi_1) = \frac{1}{4} \bar{d}(k_1) (\gamma_\perp, \gamma_5 \gamma_\perp) u(\bar{k}_1) \delta(\xi_1 - x_1), \quad (6)$$

$$\Phi^{\pi(0)}(x_2, \xi_2) = \frac{1}{4} \bar{u}(\bar{k}_2) \gamma^- \gamma_5 d(k_2) \delta(\xi_2 - x_2), \quad (7)$$

$$H_{1,2}^{(0)}(\xi_1, \xi_2) = ie_u g_s^2 C_F \text{Tr}$$

$$\times \left[\frac{\gamma_5 \gamma^+ \gamma_\mu (\not{p}_1 - \not{\xi}_2 \not{p}_2) \gamma^\nu (\gamma_\perp, \gamma_\perp \gamma_5) \gamma_\nu}{(p_1 - \xi_2 p_2)^2 (\xi_1 p_1 - \xi_2 p_2)^2} \right]. \quad (8)$$

The spin projectors γ_\perp , $\gamma_\perp \gamma_5$, and $\gamma_5 \gamma^+$, which give the nonvanishing hard kernels $H_{1,2}^{(0)}(\xi_1, \xi_2)$, come from Eq. (3). Since only a transversely polarized ρ meson can transit into a pseudoscalar pion, the spin projectors involving γ_\perp are relevant. Both Φ_1^ρ and Φ_2^ρ , defined by γ_\perp and $\gamma_5 \gamma_\perp$, respectively, represent the two-parton twist-3 ρ meson distribution amplitudes. The pion distribution amplitude Φ^π defined by $\gamma^- \gamma_5$ is of twist 2. Without collinear gluon exchanges at LO, the modified momentum fractions are equal to the initial fractions in the external mesons, as indicated by the delta functions $\delta(\xi - x)$ in Eqs. (6) and (7).

A crucial step to prove the factorization theorem is to explore infrared divergences in radiative corrections, and to examine whether soft divergences cancel and collinear divergences are absorbed completely into hadronic matrix

elements. The former are generated when the momentum of a radiative gluon, exchanged between two on-shell particles, vanishes like $l \sim (\Lambda, \Lambda, \Lambda)$, with Λ denoting a small scale. The latter appear when a radiative gluon is collimated to on-shell massless particles. As a gluon is aligned with the initial ρ meson, its momentum scales like $l \sim (Q, \Lambda^2/Q, \Lambda)$. The factorization of collinear divergences is first verified at NLO, and then generalized to all orders by the induction [7,8]. As stated in the introduction, the NLO analysis of the infrared divergences in the scattering $\rho\gamma^* \rightarrow \pi$ was performed by adding radiative gluons to the hard kernels in Eq. (8) [9,10], instead of to the scattering amplitude in Eq. (2). Since a hard kernel is not a physical quantity, the matching between the full QCD and the effective theory for infrared physics may not be implemented correctly due to wrong power counting for collinear divergences.

The NLO corrections to Fig. 1(a) with the additional gluon radiated from the initial ρ meson are displayed in Fig. 2. Take Fig. 2(d) with a triple-gluon vertex, sandwiched by γ_\perp or $\gamma_\perp\gamma_5$ from the ρ meson side and $\gamma_5\gamma^+$ from the pion side, as an example. It contains the Feynman rule

$$F_{\alpha\beta\gamma} = g_{\alpha\beta}(2k_2 - 2k_1 + l)_\gamma + g_{\gamma\alpha}(k_1 - k_2 + l)_\beta + g_{\beta\gamma}(k_1 - k_2 - 2l)_\alpha, \quad (9)$$

where the indices α, β and γ are labeled in the diagram. In the collinear region with the loop momentum l being almost parallel to p_1 , the leading contribution arises from the gamma matrix $\gamma^\gamma = \gamma^+$, because the adjacent quark propagator is proportional to $\not{p}_1 - k_1 + l \propto \gamma^-$. The first term in Eq. (9) yields the expected factorization of a

collinear gluon: the metric tensor $g_{\alpha\beta}$ is identified as the one in the LO hard kernel; the term $2k_{2\gamma} = 2k_2^-$, picked up by the vertex $\gamma^\gamma = \gamma^+$, facilitates the splitting of the two gluon propagators,

$$\frac{2k_{2\gamma}}{(k_1 - k_2 - l)^2(k_1 - k_2)^2} \approx \left[\frac{1}{(k_1 - k_2)^2} - \frac{1}{(k_1 - k_2 - l)^2} \right] \frac{v_\gamma}{v \cdot l}. \quad (10)$$

The first (second) term in the above square brackets corresponds to the LO hard kernel without (with) the loop momentum flowing through it. The eikonal vertex v_γ and the eikonal propagator $1/v \cdot l$ are the Feynman rules associated with the gauge links along the direction v , which are necessary for defining gauge-invariant nonlocal hadronic matrix elements. Hence, the collinear divergence from the first term in Eq. (9) contributes to the NLO two-parton twist-3 ρ meson distribution amplitudes $\Phi_{1,2}^{\rho(1)}$.

Nevertheless, the second term in Eq. (9) also gives rise to a collinear divergence as l is parallel to p_1 , which does not respect the factorization. The gamma matrix γ^α can be set to γ^- owing to the spin projectors γ_\perp or $\gamma_\perp\gamma_5$ from the ρ meson side, and $\gamma_5\gamma^+$ from the pion side. Then $\gamma^\alpha = \gamma^-$ and $\gamma^\gamma = \gamma^+$ for the collinear gluon contract to the tensor $g_{\gamma\alpha}$. The gamma matrix γ^β is chosen as γ^+ , also because of the adjacent quark propagator proportional to $\not{p}_1 - k_1 + l \propto \gamma^-$, which picks up the nonvanishing component $-k_{2\beta} = -k_2^-$. The above configuration produces the residual collinear divergence, which cannot be absorbed into $\Phi_{1,2}^{\rho(1)}$ having been defined by

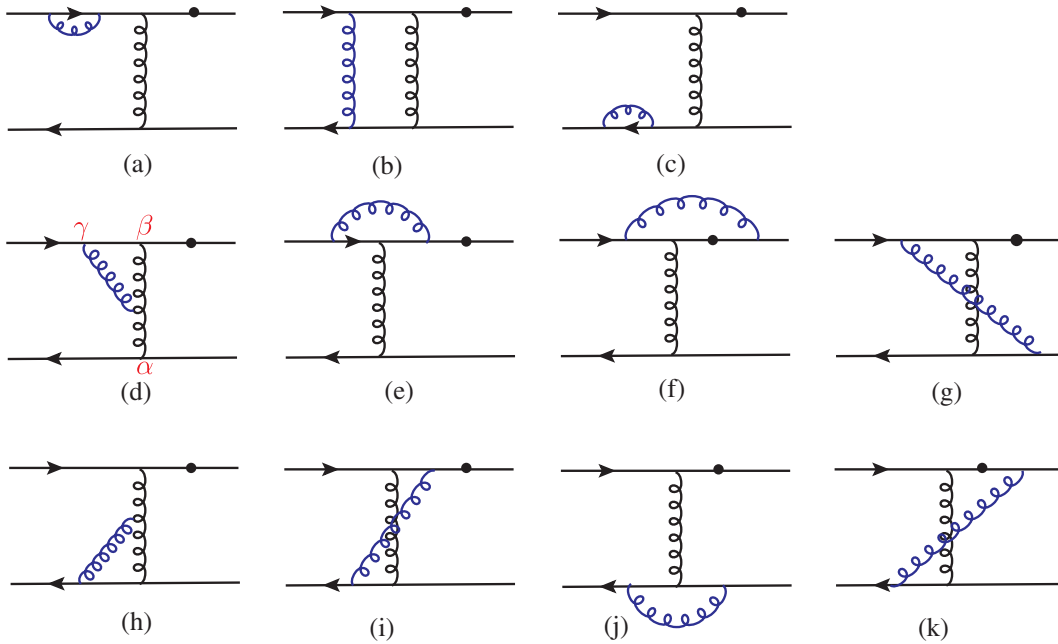


FIG. 2. NLO corrections to Fig. 1(a) with the gluon radiated from the initial ρ meson.

the contribution from the first term in Eq. (9). The third term in Eq. (9) does not produce a collinear divergence, since $\gamma^\gamma = \gamma^+$ requires $\gamma^\beta = \gamma^-$ through $g_{\beta\gamma}$, which then suppresses the adjacent quark propagator proportional to $\not{p}_1 - \not{k}_1 + \not{l} \propto \gamma^-$. As elaborated comprehensively in Ref. [10], the other triple-gluon diagrams, sandwiched by the various twist-3 spin projectors, generate the similar residual collinear divergences. Note that this source of factorization violation does not exist, as the NLO diagrams are sandwiched only by the twist-2 spin projectors: the twist-2 spin projector γ^- from the initial ρ meson would suppress the violation source from $\gamma^\alpha = \gamma^-$.

III. PROOF OF THE COLLINEAR FACTORIZATION

As postulated in the introduction, the correct procedure to examine the factorization starts with analyzing infrared structures of higher-order scattering amplitudes, for which the equations of motion obeyed by valence quarks can be applied [7,8]. In this section we demonstrate, with the help of the equations of motion, the factorization of the collinear divergences in the scattering $\rho\gamma^* \rightarrow \pi$ at NLO, focusing on those associated with the initial ρ meson. The approach in [9,10] does not cause a problem at the leading-twist accuracy, because the effect of the twist-2 spin projectors, as mentioned at the end of the previous section, is equivalent to the equations of motion for valence quarks.

An energetic valence quark of momentum $k = (k^+, k^-, \mathbf{k}_T)$, as an on-shell parton with $k^- = k_T^2/(2k^+)$, satisfies the equation of motion

$$ku(k) = (k^+\gamma^- + k^-\gamma^+ - \mathbf{k}_T \cdot \boldsymbol{\gamma}_\perp)u(k) = 0, \quad (11)$$

when the three-parton terms are neglected. The above decomposition leads to the power counting

$$\gamma^- u(k) \sim \mathcal{O}\left(\frac{\Lambda}{Q}\right)\gamma_\perp u(k), \quad (12)$$

for $k^+ \sim \mathcal{O}(Q)$ and $k_T \sim \mathcal{O}(\Lambda)$. That is, the product $\gamma^- u(k)$ is suppressed by a power of $1/Q$ relative to $\gamma_\perp u(k)$.

For the first term in Eq. (9), γ^α can be set to γ_\perp (γ^β also takes γ_\perp due to the tensor $g_{\alpha\beta}$). As explained in the previous section, the source of the factorization violations comes from $\gamma^\alpha = \gamma^-$. By considering the infrared structure of the scattering amplitude, instead of the hard kernels, the above γ_\perp and γ^- attach to the spinor of the valence antiquark. The power counting in Eq. (12) then implies that the contribution from the latter is down by a power of $1/Q$ compared to the contribution from the former, which respects the factorization at twist 3. Therefore, the violation source should be dropped at the twist-3 accuracy, and the proof of the factorization theorem at twist 3 based on the equations of motion in [7,8] is verified. After absorbing the collinear

divergences into the two-parton twist-3 ρ meson distribution amplitudes, we insert the Fierz identity to factorize the fermion flow between the NLO distribution amplitudes and the remaining LO hard kernels. The NLO factorization of the collinear gluons emitted by the outgoing valence quark and antiquark of the pion at the twist-2 level is the same as in [7], so we do not touch it here.

As to the soft divergences, it is straightforward to show that they cancel each other among the reducible diagrams Figs. 2(a)–2(c), between the irreducible diagrams Figs. 2(f) and 2(g), as well as between Figs. 2(j) and 2(k) [7,8]. This soft cancellation can be understood via the color-transparency argument for an energetic ρ meson. In other words, a soft gluon extends over a huge space-time, so it cannot resolve the color structure of the ρ meson. Figures 2(d), 2(e), 2(h), and 2(i) do not contain soft divergences, since the radiative gluons attach to the LO hard particles.

The proof of the collinear factorization to all orders in the strong coupling follows the induction method based on the Ward identity [7,8]: the eikonal approximation holds for every internal particle line which a collinear gluon attaches to (in the absence of the Glauber divergences); the summation over all possible attachments of a collinear gluon to internal lines leads to its factorization in color space; at last, the induction extends the factorization from the assumed order to the next higher order. The above steps complete the all-order proof of the collinear factorization for $\rho\gamma^* \rightarrow \pi$ up to the two-parton twist-3 accuracy, and we arrive at the definition of the two-parton twist-3 ρ meson distribution amplitudes

$$\begin{aligned} \Phi_{1,2}^\rho(\xi_1, x_1) &= \int \frac{dy^-}{2\pi} e^{i\xi_1 p_1^+ y^-} \langle 0 | \bar{d}(y^-) (\gamma_\perp, \gamma_5 \gamma_\perp) W_v \\ &\quad \times (y^-) u(0) | u(\bar{k}_1) \bar{d}(k_1) \rangle, \end{aligned} \quad (13)$$

where W_v is a path-ordered exponential function

$$W_v(y^-) = \mathcal{P}\text{Exp} \left[-ig_s \int_0^{y^-} dz v \cdot A(zv) \right]. \quad (14)$$

It is easy to see that the above gauge links produce the Feynman rules described by the eikonal vertex and propagator in Eq. (10). The discussion of the scattering $\rho\gamma^* \rightarrow \rho$, i.e., the ρ meson electromagnetic form factor, is basically the same as of $\rho\gamma^* \rightarrow \pi$ before inserting the appropriate spin projectors associated with the final-state meson, so we do not repeat it in this work. To derive the physical two-parton twist-3 ρ meson distribution amplitudes $\Phi_{1,2}^\rho(\xi_1)$, we simply replace the Fock state $|u(\bar{k}_1) \bar{d}(k_1)\rangle$ in Eq. (13) by the ρ meson state $|\rho(p_1, \epsilon_1)\rangle$, with ϵ_1 being the polarization vector.

The k_T factorization is more apposite to QCD processes dominated by contributions from small parton momenta. For its proof as an extension of the collinear factorization, we retain the dependence of parton transverse momenta k_T in hard kernels. The factorization of collinear divergences

in radiative corrections, which are then regularized by the small parton off-shellness into $\ln k_T^2$, gives TMD hadron wave functions. A TMD wave function, collecting the collinear logarithm $\ln k_T^2$ to all orders, describes parton distributions in both longitudinal and transverse momenta inside a hadron. Note that the k_T terms appearing in numerators of internal particle propagators are still dropped [11], whose contribution is supposed to be combined with that from the three-parton Fock state [12] to form a gauge-invariant three-parton wave function. Moreover, neglecting k_T in numerators allows the eikonal approximation to hold for collinear gluons, so that the gauge links, which guarantee the gauge invariance of a TMD wave function, can be constructed. Following the similar procedure in [11], the k_T factorization for a high-energy QCD process can be proved, despite the above distinct features, to the two-parton twist-3 accuracy.

It should be stressed that we can no longer drop the product $\gamma^- u(k)$, i.e., the residual collinear divergences at the twist-4 accuracy. Hence, we speculate that the factorization of the $\rho\gamma^* \rightarrow \pi$ scattering amplitude into a convolution involving the twist-2 and twist-4 meson distribution amplitudes for the initial and final states separately may break down. We do not pursue either the factorization for the cases with both the initial and final states taking the twist-3 meson distribution amplitudes, which exhibit a power-law behavior the same as in the combination of the twist-2 and twist-4 meson distribution amplitudes.

IV. CONCLUSION

In this paper we have elaborated how the residual collinear divergences, which violate the factorization of the exclusive processes $\rho\gamma^* \rightarrow \pi(\rho)$ at the two-parton

twist-3 level, appear in the proof of [9,10]. They are attributed to the improper factorization of the fermion flow in the scattering amplitudes before absorbing the collinear divergences into the hadronic matrix elements. Namely, the NLO analysis of the infrared divergences was performed by adding radiative gluons to the unphysical hard kernels, instead of to the scattering amplitudes. This is the reason why the residual collinear divergences survive under the wrong power counting, and break the factorization theorem. We have proposed to study the infrared structure of the higher-order scattering amplitudes first, such that the equations of motion for valence quarks can be applied. With the correct power counting based on the equations of motion, it has been shown that the residual collinear divergences are in fact power suppressed, and negligible at the twist-3 accuracy. The Fierz identity is inserted into the scattering amplitudes to factorize the fermion flow, after the collinear divergence has been absorbed into the meson distribution amplitudes completely. The rest of the proof follows the steps outlined in [7,8], which is then extended to all orders by the induction. The above procedure works for the more complicated k_T factorization, and for the factorization of other high-energy exclusive processes, including B meson transition form factors.

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- [1] G. P. Lepage and S. J. Brodsky, *Phys. Lett.* **87B**, 359 (1979).
 - [2] G. P. Lepage and S. J. Brodsky, *Phys. Rev. Lett.* **43**, 545 (1979); **43**, 1625(E) (1979).
 - [3] S. J. Brodsky, Y. Frishman, G. P. Lepage, and C. T. Sachrajda, *Phys. Lett.* **91B**, 239 (1980).
 - [4] A. V. Efremov and A. V. Radyushkin, *Phys. Lett.* **94B**, 245 (1980).
 - [5] I. V. Musatov and A. V. Radyushkin, *Phys. Rev. D* **56**, 2713 (1997).
 - [6] A. Duncan and A. H. Mueller, *Phys. Rev. D* **21**, 1636 (1980).
 - [7] H. n. Li, *Phys. Rev. D* **64**, 014019 (2001).
 - [8] M. Nagashima and H. n. Li, *Eur. Phys. J. C* **40**, 395 (2005).
 - [9] S. Cheng and Z. J. Xiao, *Phys. Rev. D* **90**, 076001 (2014).
 - [10] Y. L. Zhang, S. Cheng, J. Hua, and Z. J. Xiao, *Phys. Rev. D* **93**, 036002 (2016).
 - [11] M. Nagashima and H. n. Li, *Phys. Rev. D* **67**, 034001 (2003).
 - [12] Y. C. Chen and H. n. Li, *Phys. Rev. D* **84**, 034018 (2011).