Anomalous magnetic moment of an electron in a magnetized plasma of topologically massive two-dimensional electrodynamics

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The electron self-energy and anomalous magnetic moment in $(2+1)$ QED with a Chern-Simons term are investigated at finite temperature and density in an external magnetic field. In the limiting case of a relatively weak magnetic field, the exact expression for the vacuum anomalous magnetic moment (AMM) has been found at zero temperature and density of the medium. The energy shift and AMM of an electron are analyzed as a function of the temperature and Chern-Simons parameter in the charge-symmetric case. We obtained the new asymptotic expression for the AMM in the high-temperature region. The electron AMM has been calculated also in the case of a completely degenerate magnetized electron gas.

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I. INTRODUCTION

The study of quantum processes in $(2+1)$-dimensional theories, causing great interest, is also associated with practical applications in condensed matter physics [1–4], and with unusual properties of topologically massive two-dimensional models of quantum field theory [5–10]. The further study of radiation effects in $(2+1)$-dimensional quantum electrodynamics (QED_{2+1}) in the presence of external conditions, such as an external field, a finite temperature, and a density of matter is of current interest in many areas of physics.

In the free case, when there is no external field, the one-loop thermodynamic potential, polarization, and mass operators were calculated in QED_{2+1} without the Chern-Simons term at finite temperature in charge-symmetric plasma [11], as well as in a degenerate electron gas [12].

The polarization operator of QED_{2+1} has been analyzed in the papers [13,14] in a constant magnetic field based on the results previously obtained within the framework of $(3+1)$-dimensional electrodynamics (QED_{3+1}) [15]. It has been shown that, as opposed to QED_{3+1}, in QED_{2+1} along with a symmetric part, a polarization operator contains an antisymmetric part, which determines the induced Chern-Simons mass.

The polarization operator in QED_{2+1} with a nonzero fermion density has been analyzed in the papers [16,17] at zero temperature in a constant magnetic field. Radiation shift of electron energy in QED_{2+1} has been studied in the papers [18,19] at a finite temperature and density. The results of the performed calculations show that a finite electron mass is being induced in the original massless theory at a finite temperature due to the energy of electron interaction with charge-symmetric plasma.

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The first calculations of the electron AMM based on the vertex function were performed in the limiting case of so-called pure Chern-Simons theory [20–23], being a limiting case of QED_{2+1} with the Chern-Simons term, when

$$e^2, \qquad \theta \to \infty, \qquad \frac{e^2}{\theta} = \mathrm{const},$$

where θ is the value of the Chern-Simons coefficient. We note also that e^2 has the dimensions of mass in QED_{2+1} and is dimensionless in QED_{3+1} (in the system of units, where $\hbar = c = 1$).

The study of electron motion in a constant magnetic field with due consideration of radiation effects is of fundamental interest in a quantum electrodynamics. This problem has a direct application to the dynamic theory on an electron anomalous magnetic moment, i.e. AMM depends on the field strength and electron energy in a constant magnetic field [24–26].

In the paper [27] within the framework of QED_{2+1}, whose Lagrangian includes a Chern-Simons term of a topological nature, the radiation-induced shift of the ground state energy of an electron in a magnetic field has been analyzed. An asymptotical behavior of the energy shift has been studied and it was shown for the first time that the presence of the Chern-Simons term leads to finiteness of the effective magnetic susceptibility at zero external magnetic field.

This result was later indirectly confirmed in the paper [28] via AMM calculation based on the computation of the vertex function of QED_{2+1} with the Chern-Simons term. The asymptotics of temperature contribution to the electron AMM in the limiting case of high temperatures ($T \gg m, \theta$) in the case of charge-symmetric plasma is presented in this paper along with the exact result for vacuum anomalous magnetic moment. The vertex function calculation has been performed in the paper [28] in the approximation of small transferred momentum of an electron. In quantum electrodynamics this asymptotics corresponds to the calculation of electron AMM in a relatively weak magnetic field without regard to dynamic nature of electron AMM in a topologically massive QED_{2+1} [24–26].

Another method of developing a theory of electron AMM different from the one used in [28] enables one to obtain complete physical information about the dynamic nature of AMM and is based on the calculation of the fermion mass operator in magnetized electron-positron plasma. It was proposed in the paper [29] and applied, for example, when studying the dynamic nature of the energy shift and AMM electron in QED_{3+1} [29,30]. The energy shift and radiative decay of a massive Dirac neutrino in magnetized electron-positron plasma was investigated in the paper [31–33].

The objective of the present work is to study the total radiative mass shift and AMM electron in magnetized electron-positron plasma at a finite temperature and density within the framework of two-dimensional electrodynamics with the Chern-Simons term.

The exact formulas describing the radiative shift of the ground state energy of an electron is obtained in Sec. II. Consideration of electron spin properties has been performed based on the operator proposed in the paper, which is the two-dimensional analogue of a projections operator of a three-dimensional spin unit vector in QED_{3+1} on the direction of a magnetic field. The electron ground-state energy shift has been calculated in Sec. III in a relatively weak magnetic field at zero temperature and medium density and an exact expression for vacuum electron AMM in QED_{2+1} with the Chern-Simons term having been obtained. In Sec. IV, the dependence of the energy shift and AMM electron on the temperature of charge-symmetric plasma has been analyzed in a weak magnetic field. The contribution of the finite density effects at zero temperature to the AMM of the electron has been considered as well. A discussion of work results is held in the conclusions in Sec. V.

II. GENERAL CASE

Topologically massive QED_{2+1} is described by the Lagrangian [6]

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}[(\hat{p} + e\hat{A}) - m]\Psi + \frac{1}{4}\theta\epsilon^{\mu\nu\alpha}F_{\mu\nu}A_\alpha - \frac{1}{2\xi}(\partial_\mu A^\mu)^2, \tag{2.1}$$

where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ is the field tensor, ξ is the gauge parameter, m is the electron mass, and the electron charge is $-e < 0$. Adding the Chern-Simons term

$$\mathcal{L}_{\mathrm{CS}} = \frac{1}{4}\theta\epsilon^{\mu\nu\alpha}F_{\mu\nu}A_\alpha \tag{2.2}$$

to the Lagrangian of the gauge field A^μ leads to the fact that the gauge field obtains a mass equal to the parameter θ, but the theory gauge invariance is not violated. This mass generation mechanism is independent of the famous Higgs mechanism, and both mechanisms can operate simultaneously [9]. The finite mass of a gauge field leads, for example, to screening of both electric and magnetic fields, and the attraction between electrons becomes possible [34,35].

We consider the four-component fermions in QED_{2+1}, connected with a four-dimensional reducible representation of Dirac's matrices [11,36]

$$\gamma^0 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}, \qquad \gamma^{1,2} = \begin{pmatrix} i\sigma_{1,2} & 0 \\ 0 & -i\sigma_{1,2} \end{pmatrix}, \tag{2.3}$$

where $\sigma_{1,2,3}$ are the Pauli matrices. First, we shall present in real time formalism a finite-temperature photon propagator in QED_{2+1} with the Chern-Simons term in the Landau gauge [5,18,19]:

$$D^\beta_{\mu,\nu}(p) = -\left[\frac{i}{p^2 - \theta^2 + i0} + 2\pi\delta(p^2 - \theta^2)N_B(|p_0|)\right] \times \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 + i0} + i\theta\varepsilon_{\mu\nu\lambda}\frac{p^\lambda}{p^2 + i0}\right). \tag{2.4}$$

Here

$$N_B(x) = \frac{1}{\exp(\beta x) - 1} \tag{2.5}$$

is the Bose-Einstein distribution function, $g_{\mu\nu} = (+,-,-)$, $\varepsilon^{\mu\nu\lambda}$ is the completely antisymmetric third rank unit pseudotensor, and $\beta = \frac{1}{T}$ is the reciprocal temperature. The dynamic nature of the electron energy shift and AMM in a constant magnetic field is described using the mass operator constructed from the temperature Green's functions in the real time formalism [26]

$$\Sigma_\beta(x,x') = -ie^2\gamma^\mu G(H,T,\mu;x,x')\gamma^\nu D^\beta_{\mu\nu}(x-x'), \tag{2.6}$$

where $D^\beta_{\mu\nu}(x-x')$ is the photon Green's function at finite temperature, H is the magnetic field strength, and μ is the chemical potential. For the Green's function of an ideal electron-positron gas in a constant magnetic field, we shall use the following representation [26,29]:

$$G(H,T,\mu;x,x') = S_c(H;x,x') + S_\beta(H,T,\mu;x,x'), \tag{2.7}$$

where

$$S_c(H;x,x') = -\frac{1}{2\pi i}\int_{-\infty}^{\infty} d\omega \exp[i\omega(t-t')] \times \sum_{s,\varepsilon=\pm1}\frac{\varepsilon\Psi^\varepsilon_s(\vec{x})\bar{\Psi}^\varepsilon_s(\vec{x'})}{\omega + \varepsilon E_s(1-i\delta)} \tag{2.8}$$

is an ordinary causal Green's function of an electron in a constant magnetic field, and a temperature-dependent part of the time Green's function equals

$$S_\beta(H,T,\mu;x,x') = i\sum_{s,\varepsilon=\pm1}\frac{\varepsilon\Psi_s^\varepsilon(\vec{x})\bar{\Psi}_s^\varepsilon(\vec{x'})}{\exp[\beta(E_s-\varepsilon\mu)+1]} \times \exp[-i\varepsilon E_s(t-t')]. \tag{2.9}$$

The summation in the formulas (2.8) and (2.9) is carried out over all quantum numbers $\{s\}$ of the positive ($\varepsilon=+1$) and negative ($\varepsilon=-1$) frequency states; $\Psi_s^\varepsilon(\vec{x})$ is the coordinate part of the Dirac equation solution in a static magnetic field in QED_{2+1} and E_s is the energy of the electron stationary states. Using the Dirac-Schwinger equation and perturbation theory, we find further that a radiative shift of electron energy is determined by the diagonal matrix element of mass operator (2.6), i.e.

$$E_q^{\zeta,\zeta'}(H,T,\mu) = -ie^2\iint d^3x d^3x'\bar{\Psi}_{q\zeta'}(x) \times\gamma^\mu G(H,T,\mu;x,x') \times\gamma^\nu D_{\mu\nu}^\beta(x-x')\Psi_{q\zeta}(x'), \tag{2.10}$$

where the functions $G(H,T,\mu;x,x')$ and $D_{\mu\nu}^\beta(x-x')$ are defined by the formulas (2.4) and (2.7)–(2.9), $\Psi_{q\zeta}(\vec{x})$ is the wave function of a stationary state $(s)=(q,\zeta)$, of which the radiation energy shift is to be found, the quantum numbers ζ and $\zeta'=\pm1$ characterize the dependence of the energy shift on the spin initial and final states, and $\{q\}$ is the set of quantum numbers of stationary state, except the spin. The case with $\zeta=1$ corresponds to the electron spin oriented along the direction of the magnetic field, while $\zeta=-1$ is opposite to the latter.

Thus, in order to calculate the value (2.10) and develop a theory of the electron AMM in a topologically massive QED_{2+1}, it is necessary to find an exact solution of the stationary Dirac equation, including the electron spin properties. Choosing the vector potential A_μ of an external magnetic field in the Landau gauge ($A_0=A_1=0, A_2=xH$), the Hamiltonian of the Dirac equation

$$i\frac{\partial\Psi}{\partial t} = \hat{H}\Psi \tag{2.11}$$

in a static magnetic field can be represented as

$$\hat{H} = \alpha_1\hat{p}_x + \alpha_2(\hat{p}_y+exH) + m\gamma^0, \tag{2.12}$$

where the matrices $\alpha_{1,2}=\gamma^0\gamma^{1,2}$, $\hat{p}_x$ and $\hat{p}_y$ are the projections of a momentum operator, and H is the magnetic field strength. The Hamiltonian (2.12) commutes with the operator $\hat{p}_y$ and the Dirac equation solution we represent as

$$\Psi = Ne^{-i\varepsilon E_n t+iyp_y}\begin{pmatrix} C_1u_{n-1}(\eta) \\ C_2u_n(\eta) \\ C_3u_{n-1}(\eta) \\ C_4u_n(\eta)\end{pmatrix}. \tag{2.13}$$

Here, N is the normalization factor, $C_k(k=1,2,3,4)$ are the constant coefficients, argument of the Hermite functions $u_n(\eta)$

$$\eta = \sqrt{eH}\left(x+\frac{p_y}{eH}\right). \tag{2.14}$$

Eigenvalues of the Hamiltonian (2.12), meeting the condition of square-law integrability of the eigenfunction, are defined by the

$$E_n = \sqrt{m^2+2eHn}, \quad n=0,1,2,\ldots, \tag{2.15}$$

and the constants C_k satisfy the system of equations

$$C_1(\varepsilon E_n - m) - C_2p_\perp = 0, \quad C_3(\varepsilon E_n + m) - C_4p_\perp = 0, \tag{2.16}$$

where $p_\perp=\sqrt{2eHm}$ is the magnitude of the electron momentum in a magnetic field.

As we can see, the electron energy levels in a magnetic field in $(2+1)$-dimensional electrodynamics are discrete ones and do not depend on the quantum number p_y, as well as they are determined by the main quantum number n only. Along with p_y and n, in order to define a fermion quantum state completely, it is necessary to introduce a third quantum number, which makes it possible to divide solutions of the Dirac equation taking into account spin states.

We assume that the wave function (2.13) is also the eigenfunction of an operator

$$\hat{A} = i\varepsilon\gamma^0\gamma^1\gamma^2, \tag{2.17}$$

which commutes with the Hamiltonian (2.12) and is an integral of motion. If we subordinate the wave function (2.13) to the additional condition

$$\hat{A}\Psi = \zeta\Psi, \tag{2.18}$$

where $\zeta=\pm1$, then the Dirac normalized positive- and negative-frequency solutions shall be determined from the following formulas:

$$\Psi_{\varepsilon=+1} = \frac{(eH)^{\frac{1}{4}}}{\sqrt{2E_n}}\exp[-iE_nt+iyp_y]\left[\begin{pmatrix}\sqrt{E_n+m}u_{n-1} \\ \sqrt{E_n-m}u_n \\ 0 \\ 0\end{pmatrix}D_1 + \begin{pmatrix} 0 \\ 0 \\ \sqrt{E_n-m}u_{n-1} \\ \sqrt{E_n+m}u_n\end{pmatrix}D_{-1}\right], \tag{2.19}$$

$$\Psi_{\varepsilon=-1} = \frac{(eH)^{\frac{1}{4}}}{\sqrt{2E_n}}\exp[iE_nt + iyp_y]\left[\begin{pmatrix} 0 \\ 0 \\ -\sqrt{E_n+m}\,u_{n-1} \\ \sqrt{E_n-m}\,u_n \end{pmatrix} D_1 + \begin{pmatrix} -\sqrt{E_n-m}\,u_{n-1} \\ \sqrt{E_n+m}\,u_n \\ 0 \\ 0 \end{pmatrix} D_{-1}\right], \tag{2.20}$$

where with $\zeta = +1$ it is necessary to set $D_1 = 1$, $D_{-1} = 0$ (the spin is directed along the field), and with $\zeta = -1$, vice versa, $D_1 = 0$, $D_{-1} = 1$ (the spin is directed opposite the field). The coefficients D_1 and D_{-1} satisfy the normalization condition

$$D_1^2 + D_{-1}^2 = 1.$$

In Sec. III we show that the operator $\hat{A}$ is a $(2+1)$-dimensional analogue of the projection of the operator three-dimensional spin vector on the direction of the magnetic field in QED_{3+1}. In the absence of the longitudinal component of momentum it is proportional to the operator transverse polarization [24] and the quantum number $\zeta = \pm 1$ does have meaning projection of electron spin on the direction of the magnetic field. It follows from (2.19) that in the ground state ($n = 0$) the electron spin can only be directed opposite the direction of the magnetic field ($D_1 = 0$, $D_{-1} = 1$).

Using (2.3)–(2.4), (2.6)–(2.10), (2.19)–(2.20), we present a radiative shift of the ground state energy of an electron in magnetized electron-positron plasma of QED_{2+1} with the Chern-Simons term in the following form, suitable for further analysis:

$$\Delta E = \Delta E(H) + \Delta E^B(H,T) + \Delta E^F(H,T,\mu), \tag{2.21}$$

$$\begin{pmatrix} \Delta E(H) \\ \Delta E^B(H,T) \end{pmatrix} = -\frac{e^2}{4\pi}\int_{-\infty}^{\infty} dp_0 \int_0^{\infty} p\,dp\,ds\,G(p_0,p,s,\theta) \times \exp[is(p_0^2 - 2mp_0) - \delta s] \times \begin{pmatrix} \frac{1}{p_0^2-p^2-\theta^2+i0} \\ (-2\pi i)\frac{\delta(p_0^2-p^2-\theta^2)}{\exp[\frac{|p_0|}{T}-1]} \end{pmatrix}, \tag{2.22}$$

$$\Delta E_0^F(H,T,\mu) = \frac{e^2}{4\pi^2}\sum_{\varepsilon=\pm1}\int_{-\infty}^{\infty} dx\,d\lambda \exp(i\lambda x)\int_0^{\infty} p\,dp \times G(p_0 = m - \varepsilon\sqrt{m^2+x}, p, \lambda, \theta) \times \frac{1}{(m-\varepsilon\sqrt{m^2+x})^2 - p^2 + i0} \times \frac{1}{\exp[(\sqrt{m^2+x} - \varepsilon\mu)\beta] + 1}, \tag{2.23}$$

where the following notations are take: $\Delta E(H)$ is the nonrenormalized radiative energy shift in an external magnetic field at $T = \mu = 0$, $\Delta E^B(H,T)$ is the electron energy shift through its interaction with equilibrium radiation, $\Delta E^F(H,T,\mu)$ is the temperature electron energy shift owing to its exchange interaction with plasma electrons ($\varepsilon = +1$) and positrons ($\varepsilon = -1$), and $G(p_0,p,s,\theta)$ is defined by the formula

$$G(p_0,p,s,\theta) = \cos(eHs)\exp(-iseH) \times \exp\left[-\frac{p^2}{eH}(1-\cos(eHs))\exp(-iseH)\right] \times \left[F_0 + \frac{F_2 - F_1}{p_0^2 - p^2 + i0}\right]. \tag{2.24}$$

Here, we have identified the contributions corresponding to three terms of sum, determining the structure of the photon propagator in QED_{2+1} with the Chern-Simons term (2.4). The value F_0 describes the contribution corresponding to the term of the sum proportional to $g^{\mu\nu}$, the $p^\mu p^\nu$ term in the photon propagator contributes only to F_1, and the value F_2 arises due to the term of sum proportional to the gauge field mass θ. They have the following form:

$$F_0 = 2m + p_0 + i\,tg(eHs)(2m - 3p_0),$$
$$F_1 = (2mp_0^2 - p_0^3)\frac{\exp(iseH)}{\cos(eHs)} + p_0p^2\frac{\exp(-iseH)}{\cos(eHs)},$$
$$F_2 = -2\theta(p_0^2 - p^2)\frac{\exp(-iseH)}{\cos(eHs)}. \tag{2.25}$$

Note that the temperature part of the energy shift in QED_{2+1} with the Chern-Simons term has no need of renormalization and is a finite one in the whole range of external field variation, and the renormalization of a vacuum radiative energy shift in a constant magnetic field, as in QED_{3+1}, shall be conducted by subtracting from the unrenormalized quantity its value in the absence of an external field.

III. ELECTRON MASS SHIFT AND AMM IN A CONSTANT MAGNETIC FIELD

Let us perform the calculation of vacuum radiative mass shift, using the Schwinger parametrization

$$\frac{1}{p_0^2 - p^2 - \theta^2 + i0} = -i\int_0^{\infty} dt \exp[it(p_0^2 - p^2 - \theta^2 + i0)]$$

as well as substitution of variables s and t with u and y according to the formulas

$$u = \frac{s}{s+t}, \qquad y = u(s+t) = s,$$
$$0 \le u \le 1, \qquad 0 \le y < \infty.$$

Separating different contributions to the shift of the electron energy due to the individual terms in the photon propagator proportional to $g_{\mu\nu}$, $p_\mu p_\nu$, and θ, respectively, after integration over the variables p_0 and p we obtain the following result:

$$\Delta E(H) = \Delta E(F_0) + \Delta E(F_1) + \Delta E(F_2), \tag{3.1}$$

where

$$\begin{pmatrix} \Delta E(F_0) \\ \Delta E(F_1) \\ \Delta E(F_2) \end{pmatrix} = \frac{e^2}{8\pi^{\frac{3}{2}}} \int_0^1 \frac{du}{\sqrt{u}} \times \int_0^\infty \frac{du}{\sqrt{y}} \exp(-m^2 uy) \begin{pmatrix} A_0 \\ A_1 \\ A_2 \end{pmatrix}, \tag{3.2}$$

$$A_0 = \frac{e^{-\nu}[2 - u + 2u\exp(-2eHy)]}{\Phi} - u - 2, \tag{3.3}$$

$$A_1 = \frac{e^{-\nu} - 1}{\nu}(1-u)\left[\frac{1}{\Phi}\left[1 - \frac{3}{2}u - \frac{u\exp(-2eHy)}{\Phi} - u(2-u)m^2 uy\right] - \left(1 - \frac{5}{2}u - u(2-u)m^2 uy\right)\right], \tag{3.4}$$

$$A_2 = \frac{e^{-\nu} - 1}{\nu}(1-u)\frac{\theta}{m}[\exp(-2eHy) \times \left(1 + \frac{2}{\Phi} - 2m^2 uy\right) - (3 - 2m^2 uy)] \tag{3.5}$$

and the notations are taken

$$\nu = \frac{y(1-u)\theta^2}{u},$$
$$\Phi = 1 - u + u\frac{\sin(eHy)}{eHy}\exp(-eHy). \tag{3.6}$$

The results (3.1)–(3.6) are exact in the one-loop approximation and are consistent with the corresponding result obtained by another method in the paper [27]. But systematically studying the energy shift and AMM of the electron in the $(2+1)$-dimensional Chern-Simons theory, except for the analysis of the asymptotics described by formulas (10) and (11) of this paper, is not considered in this paper. Such investigation is fulfilled in the present paper. Below we carried our calculation from Eqs. (3.1)–(3.6) of the anomalous magnetic moment of the electron at zero temperature and density in QED_{2+1} with the Chern-Simons term, previously obtained in [28] as a result of the vertex function calculation.

Let us consider the limiting case of a relatively weak magnetic field, when the conditions

$$eH \ll m^2, m\theta \tag{3.7}$$

are fulfilled and the parameter $k = \frac{\theta}{m}$ can take any values. In this case, the main contribution to the integral (3.2) provides the domain $t = eHy \ll 1$, and in the approximation linear in the magnetic field, the following asymptotics take place from the formulas (3.3)–(3.6):

$$\Delta E(F_0) \simeq \frac{e^2}{16\pi}\frac{eH}{m^2}\int_0^1 \frac{u^2(u-2)du}{[u^2 + k^2(1-u)]^{\frac{3}{2}}} = \frac{e^2}{16\pi}\frac{eH}{m^2}\left[(3 - 3k) - \left(2 - \frac{3}{2}k^2\right)\ln\frac{k+2}{k}\right], \tag{3.8}$$

$$\Delta E(F_2) \simeq \frac{e^2}{4\pi k}\frac{eH}{m^2}\int_0^1 udu\left[\frac{5u-6}{[u^2 + k^2(1-u)]^{\frac{1}{2}}} + \frac{u^2(2-u)}{[u^2 + k^2(1-u)]^{\frac{3}{2}}}\right] = \frac{e^2}{4\pi}\frac{eH}{m^2}\left[2 - k\ln\frac{k+2}{k}\right], \tag{3.9}$$

$$\Delta E(F_1) \simeq \frac{e^2}{16\pi k}\frac{eH}{m^2}\int_0^1 du\left[\frac{u^2(6-7u)}{[u^2 + k^2(1-u)]^{\frac{1}{2}}} - \frac{u^4(2-u)}{[u^2 + k^2(1-u)]^{\frac{3}{2}}}\right] = 0. \tag{3.10}$$

The integral over the variable u in the formula (3.10) is exactly equal to zero. Thus, the $p^\mu p^\nu$ term in the photon propagator does not contribute to the mass shift and vacuum electron AMM accordingly. This finding is consistent with the similar result of the paper [28], where, in consequence of the vertex function calculation, it has been found that the said term of the photon propagator does not contribute to the magnetic form factor of an electron also at finite temperature.

As is known, that part of the energy shift of the electron excited states in a magnetic field, which is proportional to bilinear combination $(D_{-1}D'_{-1} - D_1 D'_1)$ [24–26] is directly associated with the presence of electron AMM. In the ground state, when the electron spin can only be directed opposite the magnetic field orientation ($D_1 = 0$, $D_{-1} = 1$), the whole amount of the energy shift is equal to the energy of interaction of the anomalous magnetic moment with the magnetic field [24]

$$\Delta\mu = \frac{\Delta E(H)}{H}. \tag{3.11}$$

In view of (3.8)–(3.10), we shall find the exact expression for the vacuum electron AMM in a topologically massive QED_{2+1}:

$$\Delta\mu = \frac{\Delta E(F_0) + \Delta E(F_2)}{H} = \left(-\frac{e}{2m}\right)\frac{e^2}{8\pi m}\left[3k - 7 + \left(2 + 2k - \frac{3}{2}k^2\right)\ln\frac{k+2}{k}\right], \tag{3.12}$$

which coincides with the result obtained in the paper [28] based on the electron AMM calculation using the vertex function. This compliance provides an opportunity to consider the quantum number ζ as a projection of the electron spin onto the "orientation" of the magnetic field in two-dimensional electrodynamics, and the spin operator (2.17) as a two-dimensional analogue of transverse polarization operator μ_3 in QED_{3+1} [24]. In the limiting case

$$\frac{eH}{m^2} \ll k = \frac{\theta}{m} \ll 1, \tag{3.13}$$

we shall find from the formulas (3.1), (3.8)–(3.10)

$$\Delta E(H) = \left(\frac{e^2}{8\pi}\right)\frac{eH}{m^2}\ln k. \tag{3.14}$$

This result coincides with the asymptotics of the ground electronic state radiation energy shift in QED_{2+1} with the Chern-Simons term obtained for the first time in the paper [27] [formula (11)]. In the case of the so-called pure Chern-Simons theory, from Eq. (3.12) we shall find the results

$$\Delta\mu = \left(\frac{e}{2m}\right)\left(\frac{e^2}{2\pi\theta}\right), \tag{3.15}$$

which is also in line with the first AMM calculations based on the vertex function [20–23].

IV. SHIFT OF ELECTRON ENERGY AND AMM IN MAGNETIZED PLASMA

First, let' us consider the case of charge-symmetric plasma, when the chemical potential is $\mu = 0$. Carrying out integration with respect to the variable p_0 in the formula (2.22) for the Bose contribution by means of the delta function, the exact result, as in the zero temperature case, is presented in the form of a double integral. In the limiting case of a relatively weak magnetic field, when

$$b = \frac{eH}{mT} \ll 1, \tag{4.1}$$

for the Bose contribution to the energy shift $\Delta E(H,T)$ we shall obtain the following presentation:

$$\Delta E^B(H,T) = \Delta E(F_0) + \Delta E(F_1) + \Delta E(F_2), \tag{4.2}$$

$$\begin{pmatrix} \Delta E(F_0) \\ \Delta E(F_2) \\ \Delta E(F_1) \end{pmatrix} = \frac{e^2}{4\pi}\int_0^\infty \frac{p\,dp}{\sqrt{p^2+\theta^2}}\frac{1}{\exp[\frac{\sqrt{p^2+\theta^2}}{T}] - 1} \times \begin{pmatrix} G_0(p_0) + G_0(-p_0) \\ -2\theta[G_2(p_0) + G_2(-p_0)] \\ 0 \end{pmatrix}, \tag{4.3}$$

where

$$p_0 = \sqrt{p^2+\theta^2},$$
$$G_0 = \frac{2m + p_0}{2mp_o - \theta^2} - \frac{4eHp_0}{(2mp_0 - \theta^2)^2} + \frac{2eH(2m+p_0)p^2}{(2mp_0-\theta^2)^3},$$
$$G_2 = \frac{1}{2mp_o - \theta^2} - \frac{2eH}{(2mp_0-\theta^2)^2} + \frac{2eHp^2}{(2mp_0-\theta^2)^3}. \tag{4.4}$$

In charge-symmetrical plasma in the range of low temperatures, which are small compared to the electron mass ($T \ll m$), the Fermi contribution to be defined by the formula (2.23) is exponentially small compared to the Bose contributions (2.22) and the electron energy temperature shift shall be determined by Eqs. (4.3)–(4.4).

In the limiting case (4.1), the Fermi contribution to the energy shift of the electron ground state is determined by the following expression:

$$\Delta E^F(H,T,\mu) = \Delta E(F_0) + \Delta E(F_1) + \Delta E(F_2), \tag{4.5}$$

$$\begin{pmatrix} \Delta E^F(F_0) \\ \Delta E^F(F_2) \\ \Delta E^F(F_1) \end{pmatrix} = \frac{e^2}{4\pi}\int_1^\infty dv \times \begin{pmatrix} D_0(v) + D_0(-v) \\ -2\frac{\theta}{m}[D_2(v) + D_2(-v)] \\ 0 \end{pmatrix}, \tag{4.6}$$

where

$$D_0(v) = \frac{v}{g(v)}\left[\frac{n_F(3-v)}{2v} + \left(\frac{eH}{m^2}\right)\frac{1}{g(v)}\left[\frac{1-v}{v^2}n'_F - n_F\left(\frac{1}{v^3} - \frac{(v-1)^2}{v^2 g(v)}\right)\right] + \frac{1}{8}\left(\frac{eH}{m^2}\right)\frac{v^2-1}{v^2 g(v)}\left[\left(\frac{3}{v}-1\right)\left(n''_F - \frac{n'_F}{v}\right) - n'_F\left(\frac{6}{v^2} + 2\frac{(3-v)(v-1)}{vg(v)}\right) + n_F\left(\frac{9}{v^3} + \frac{7v-9}{v^2 g(v)} + \left(\frac{6}{v}-2\right)\frac{(v-1)^2}{g^2(v)}\right)\right]\right], \tag{4.7}$$

$$D_2(v) = \frac{v}{g(v)}\left[\frac{n_F}{2v} + \left(\frac{eH}{m^2}\right)\frac{1}{2v^2 g(v)}\left[n'_F - n_F\left(\frac{1}{v} + \frac{v-1}{g(v)}\right)\right] + \left(\frac{eH}{m^2}\right)\frac{v^2-1}{8v^3 g(v)}\left[\left(n''_F - \frac{n'_F}{v}\right) + \frac{2n'_F}{v}\left(-1 + \frac{(1-v)v}{g(v)}\right) + n_F\left(\frac{3}{v^2} + 2\frac{(v-1)^2}{g^2(v)} + \frac{2v-3}{vg(v)}\right)\right]\right], \tag{4.8}$$

and the following notations are taken:

$$g(v) = 1 - v - \frac{\theta^2}{2m^2}, \qquad n_F(v) = \frac{1}{\exp[\frac{mv}{T} + 1]},$$

$$n'_F = \frac{\partial n_F}{\partial v}.$$

Within the brackets of Eq. (4.6), we explicitly identified the terms of the sum describing the contributions to electron mass shift due to the processes of its scattering by electrons [positive frequency states, of which the contribution is described via the values $D_0(v)$ and $D_2(v)$] as well as positrons [negative-frequency states, of which the contribution is described via the values $D_0(-v)$ and $D_2(-v)$] of plasma.

We also note that values $\Delta E^B(F_1)$ of Bose and $\Delta E^F(F_1)$ of Fermi contributions are equal to zero in the approximation linear in the magnetic field for all values of temperature and chemical potential.

From formulas (4.4)–(4.8), in the limiting case, when $H = 0, \mu = 0$, for the temperature correction to the electron mass we shall obtain the result which agrees with that obtained in the paper [19]. Let us present two asymptotics of the electron mass temperature shift:

$$\Delta m = \begin{cases} \frac{e^2}{8\pi}[\ln\frac{\theta}{2m} + \frac{T}{m}\ln 2 - \frac{T}{m}\ln\frac{\theta}{Te}], & \theta \ll 2m \ll T, \\ \frac{e^2}{2\pi}[\frac{T}{\theta}\ln(1-e^{-\beta\theta}) - \frac{T}{\theta}\ln(1+e^{-\beta m})], & 2m \ll \theta, \theta^2 \gg 2mT. \end{cases} \tag{4.9}$$

Result (4.9) allows us to consider the limiting case of the massless electrodynamics at relatively low temperatures ($\theta \gg m, \theta^2 \gg 2mT$)

$$\Delta m = \frac{e^2}{2\pi}\left[-\frac{T}{\theta}\ln 2 + \frac{T}{\theta}\ln(1-e^{-\beta\theta})\right]. \tag{4.10}$$

The result (4.10) is the induced electron mass in the massless QED_{2+1} with the Chern-Simon's term at finite temperature [18,19].

In the limiting case

$$T \gg m, \theta, \tag{4.11}$$

it follows from (4.3)–(4.4), that the leading order thermal corrections to the electron energy shift, which depend on the field and coming from the terms proportional to $g^{\mu\nu}$ in the photon propagator, is defined by the formula

$$\Delta E(F_0) \simeq \frac{e^2}{8\pi m}\frac{eH}{m^2}\int_0^\infty \frac{pdp}{\sqrt{p^2+\theta^2}}\frac{1}{\exp[\beta\sqrt{p^2+\theta^2}] - 1} \simeq \frac{e^2}{8\pi}\frac{eH}{m^2}\frac{T}{m}\ln\frac{T}{\theta}. \tag{4.12}$$

We note that the ratio $\frac{\theta}{m}$ is arbitrarily in the condition (4.11). In the limiting case, when

$$T \gg m \gg \theta, \tag{4.13}$$

the leading contribution to the value $\Delta E(F_2)$, coming from the $\varepsilon^{\mu\nu\lambda}$ term, has an asymptotic

$$\Delta E(F_2) \simeq \frac{e^2}{2\pi}\theta\frac{eH}{m^2}\int_0^\infty \frac{pdp}{(p^2+\theta^2)^{\frac{3}{2}}}\frac{1}{\exp[\beta\sqrt{p^2+\theta^2}] - 1} \simeq \frac{e^2}{8\pi}\frac{eH}{m^2}\frac{T}{\theta}. \tag{4.14}$$

It is interesting to compare (4.12) and (4.14) with the results (42) and (44) of the paper [28], describing the contribution to the magnetic form factor of the electron the terms in the photon propagator proportional to $g^{\mu\nu}$ and θ under the condition (4.11)

$$F_2^{(\beta)(\eta)}(q^2=0) = \frac{e^2}{4\pi m}\frac{T}{m}\ln\frac{\theta}{T} + O(\beta^{-1}), \tag{4.15}$$

$$F_2^{(\beta)(\varepsilon)}(q^2=0) = O(\beta^{-1}). \tag{4.16}$$

The comparison shows that the asymptotics (4.12) of the present paper coincides with the result (4.15) of the paper [28]. With regard to the result (4.16), one might assume that the authors of [28] mean $O(\frac{T}{m})$ instead of $O(\beta^{-1})$. Regardless, it is essential that the contribution (4.16), coming from the $\varepsilon^{\mu\nu\lambda}$ term in the photon propagator, is small compared to (4.15) including also the case (4.13). This conclusion [28] seems incorrect to us, since the contribution represented by the formula (4.14) is proportional to the parameter $\frac{T}{\theta}$, rather than $\frac{T}{m}$. Therefore, the ratio of the contributions (4.14) and (4.12)

$$\frac{\Delta E(F_2)}{\Delta E(F_0)} = \frac{\frac{m}{\theta}}{\ln(\frac{T}{\theta})} \tag{4.17}$$

under fulfillment of the conditions (4.13) may also be the value much greater than the unit. In the limiting case of high temperatures in QED_{3+1} Bose and Fermi contributions to electron AMM are proportional to $(\frac{T}{m})^2$ [29,37,38]. However, when passing from QED_{3+1} to QED_{2+1}, the magnetic properties of photons and electrons are fundamentally changing and the Fermi contribution to the electron mass shift and AMM in the case of high temperatures is small compared to the Bose contribution (see [28] as well). Thus, in the limiting case (4.13), the temperature contribution to the electron AMM in topologically massive QED_{2+1} shall be determined from the following formula:

$$\Delta\mu \simeq \left(\frac{e}{2m}\right)\left(\frac{e^2}{4\pi m}\right)\frac{T}{m}\left[\ln\frac{T}{\theta} + \frac{m}{\theta}\right]. \tag{4.18}$$

In the case of a degenerated electron gas, when there are no positrons in the plasma, the contribution of the electron states should only be left in the formula (4.6) and the Fermi distribution function shall be reduced to step θ-function:

$$n_F(v) = \left[\exp\left[\frac{mv-\mu}{T}+1\right]\right]^{-1} \to \theta(\mu - mv),$$
$$\frac{\partial n_F(v)}{\partial v} \to -\delta\left(v - \frac{\mu}{m}\right), \tag{4.19}$$

where the chemical potential μ is related with the number density of the completely degenerate two-dimensional electron gas by the relation

$$\mu = \sqrt{2\pi n + m^2}. \tag{4.20}$$

As a result, in the main logarithmic approximation by the parameter $\frac{\mu}{m}$, in the limiting case

$$\frac{\mu}{m} \gg 1, \qquad \frac{\theta}{m}. \tag{4.21}$$

In the linear approximation with respect to the magnetic field, we shall obtain the following result from the formulas (4.6)–(4.8) for the contribution of the finite density effects to the electron AMM in topologically massive QED_{2+1}:

$$\Delta\mu \simeq \left(\frac{e}{2m}\right)\frac{e^2}{8\pi m}\ln\frac{\mu}{m}. \tag{4.22}$$

It is interesting to compare this expression with the corresponding result obtained in QED_{3+1} [30,39]:

$$\delta a_e^{\mu} = \frac{\Delta\mu}{\mu_B} \simeq \frac{\alpha}{3\pi}\left(\frac{\mu}{m}\right)^2, \tag{4.23}$$

where α is the fine-structure constant, and $\mu_B = \frac{e}{2m}$ is the Born magneton. In the field-free case the electron mass shift in a degenerated electron gas is described by the exact formula

$$\Delta m(\mu) = \left(\frac{e^2}{8\pi}\right)\left[\frac{\mu}{m} - 1 - 2\left(1+\frac{\theta}{2m}\right)^2 \ln\left|\frac{\frac{\mu}{m} - 1 + \frac{\theta^2}{2m^2}}{\frac{\theta^2}{2m^2}}\right|\right]. \tag{4.24}$$

The latter result is consistent with the result (9) of the paper [19], where $\vec{p} = 0$ should be set.

V. CONCLUSION

A complete description of the electron stationary states in a magnetic field has been conducted in two-dimensional electrodynamics based on the proposed spin operator. The calculation of radiation energy shift of the electron ground state and AMM in magnetized plasma of topologically massive QED_{2+1} has been performed for the first time.

In the real-time presentation, the electron energy radiation shift is represented as a sum of vacuum electron energy shift in an external magnetic field, shift of the electron energy due to interaction with the equilibrium radiation in a constant magnetic field and the electron energy temperature shift due to exchange interaction with the electrons and positrons of magnetized plasma.

In the limiting case of a relatively weak magnetic field, the dependence of the found values on the task characteristic parameters, i.e. temperature, chemical potential, and parameter θ, acting as a gauge field mass, has been studied. It is shown that the part of the electron ground state energy shift, depending on the strength of a weak magnetic field, coincides with the interaction energy of a vacuum electron AMM in QED_{2+1} with the Chern-Simons term with the external magnetic field. It was found that the contribution to total shift of an electron mass in the magnetized plasma, arising due to the term in the photon propagator, proportional to the product $p^\mu p^\nu$, is identically zero. The exact value of the vacuum electron AMM in QED_{2+1} with the Chern-Simons term, which coincides with the result previously obtained based on the analysis of the vertex function [28], has been calculated on the basis of the value of vacuum electron mass shift found in a weak magnetic field. The detailed analysis of the energy temperature shift in charge-symmetric magnetized plasma shows that the high-temperature asymptotic of the AMM presented in the paper [28] is incorrect. The result for the electron AMM in a constant magnetic field at a nonzero chemical potential and zero temperature has been obtained for the first time.

Under fulfillment of conditions (4.13), in the charge-symmetric plasma, electron AMM in a two-dimensional QED with the Chern-Simons term is determined by the sum of vacuum (3.12) and temperature (4.18) contributions

$$\Delta\mu = \left(\frac{e}{2m}\right)\frac{e^2}{4\pi m}\left[-\ln\frac{m}{\theta} + \frac{T}{m}\left(\ln\frac{T}{\theta} + \frac{m}{\theta}\right)\right]. \tag{5.1}$$

In the other limiting case of a completely degenerated electron gas, an AMM electron has the following asymptotics under fulfillment of the conditions (4.21) and $\theta \ll 2m$:

$$\Delta\mu = \left(\frac{e}{2m}\right)\frac{e^2}{4\pi m}\left[-\ln\frac{m}{\theta} + \frac{1}{2}\ln\frac{\mu}{m}\right]. \tag{5.2}$$

Note that both vacuum contribution and temperature contribution to electron AMM in charge-symmetric plasma exhibit infrared divergence at $\theta \to 0$. At the same time, the contribution of nonzero fermion density effects to electron AMM at zero temperature is finite.

It is possible to draw a general conclusion that the effects of finite temperature and medium density make a substantial contribution to the electron AMM in a two-dimensional topologically massive electrodynamics and this contribution may be decisive in high temperature and density of medium. The case of a strong magnetic field, when $eH \gg mT$, will be considered elsewhere.

The proposed method of description electron spin may be used for investigation of radiation effects also and in other two-dimensional models of quantum theory, including, while studying the spin effects in graphene in the presence of an external magnetic field.

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