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Precise predictions for Dirac neutrino mixing

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The neutrino mixing parameters are thoroughly studied using renormalization-group evolution of Dirac neutrinos with recently proposed parametrization of the neutrino mixing angles referred to as "high-scale mixing relations." The correlations among all neutrino mixing and CP violating observables are investigated. The predictions for the neutrino mixing angle θ_{23} are precise, and could be easily tested by ongoing and future experiments. We observe that the high-scale mixing unification hypothesis is incompatible with Dirac neutrinos due to updated experimental data.

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I. INTRODUCTION

Neutrino mixing is one of the most fascinating and challenging discoveries. This is starkly different from quark mixing which is small in the standard model (SM). There are a number of ways to explain these two very different phenomena. The quark-lepton unification, which is one of the main attractive features of the grand unified theories (GUT) [1–3], could provide an explanation of the origin of neutrino and quark mixing since quarks and leptons live in a joint representation of the symmetry group. Another interesting approach is to use flavor symmetries [4–8]. These symmetries could also naturally appear in GUT theories [9].

To explain the origin of neutrino and quark mixing, recently a new parametrization of the neutrino mixing angles in terms of quark mixing angles was proposed in Ref. [10]. The various simplified limits of this prameterization are referred as high-scale mixing relations (HSMR). The parametrization is inspired by the high-scale mixing unification (HSMU) hypothesis which states that at certain high scales the neutrino mixing angles are identical to that of the quark mixing angles [11–14]. This hypothesis is studied in detail in Refs. [15–19] .

The HSMR parametrization of the neutrino mixing angles assumes that the neutrino mixing angles are proportional to those of quarks due to some underlying theory which could be a quark-lepton unification or models based on flavor symmetries. In fact, such models are also presented in Ref. [10]. The scale where the HSMR parametrization could be realized is referred to as unification scale. In its most general form, the HSMR parametrization can be written as follows:

$$\theta_{12} = \alpha_1^{k_1}\theta_{12}^q, \qquad \theta_{13} = \alpha_2^{k_2}\theta_{13}^q, \qquad \theta_{23} = \alpha_3^{k_3}\theta_{23}^q, \qquad (1)$$

where θ_{ij} (with $i,\ j = 1,\ 2,\ 3$) denotes leptonic mixing angles and θ_{ij}^q are the quark mixing angles. Exponents k_i with $i = (1, 2, 3)$ are real. Predictions of the HSMR parametrization could be a strong hint of the quark-lepton unification, some flavor symmetry or both.

The HSMR parametrization is studied in the framework of the SM extended by the minimum supersymmetric standard model (MSSM). The beginning point is to run the quark mixing angles from the low scale (mass of the Z boson) to the supersymmetry (SUSY) breaking scale using the renormalization-group (RG) evolution of the SM. The RG equations of the MSSM govern the evolution of quark mixing angles from the SUSY breaking scale to the unification scale. After obtaining quark mixing angles at the unification scale, the HSMR parametrization is used to run neutrino mixing parameters from the unification scale to the SUSY breaking scale via RG evolution of the MSSM. From the SUSY breaking scale to the low scale, the SM RG equations are used to evolve the neutrino mixing parameters. The free parameters controlling the top-down evolution of the neutrino mixing parameters are masses of the three light neutrinos, Dirac CP phase and parameters α_i. Masses of neutrinos must be quasidegenerate and normal hierarchical. Furthermore, the large value of $\tan\beta$ is required [10].

On the other hand, the nature of neutrinos is still unknown. They could be equally Dirac or Majorana in nature. Hence, from the phenomenological point of view, Dirac neutrinos are as important as Majorana neutrinos. There are many ongoing important experiments to test the nature of neutrinos [20–23]. However, for the Dirac mass of neutrinos, the Yukawa couplings for neutrinos seem to be unnaturally small. The elegant way to explain this fine-tuning is the see-saw mechanism which assumes that neutrinos are Majorana in nature [24–28].

The smallness of masses for Dirac neutrinos could be explained in many models using heavy degrees of freedom [29–38]. There are also models based on extra dimensions which explain the smallness of Dirac neutrino mass by a

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small overlapping of zero-mode profiles along extra dimensions [39–41]. Dirac neutrinos seem to be a natural choice in certain orbifold compactifications of the heterotic string where the standard see-saw mechanism is difficult to realize [42]. Cosmological data do not prefer Majorana or Dirac neutrinos either. For instance, the baryon asymmetry of the Universe can also be explained for Dirac neutrinos in various theoretical models [43–49].

Although the RG evolution of Majorana neutrinos is extensively studied in the literature [11–17,50,51], less attention is being paid to the RG evolution of Dirac neutrinos. In fact, as far as we know, it was shown for the first time in Ref. [16] that RG evolution for Dirac neutrinos can explain the large neutrino mixing assuming the HSMU hypothesis. However, as we show later, these results are ruled out by new updated data [52–54] and due to an improved algorithm used in the package REAP [55].

It is established that the HSMR parametrization can explain the observed pattern of the neutrino mixing assuming they are Majorana in nature [10]. In this paper, we investigate the consequences of the HSMR parametrization using the RG evolution of Dirac neutrinos.

This paper is organized in the following way: In Sec. II, we present our results on the RG evolution of the neutrino mixing parameters. In Sec. III we present a model with naturally small Dirac neutrino masses, where the HSMR parametrization discussed in Eq. (1) can be explicitly realized. We summarize our work in Sec. IV.

II. RG EVOLUTION OF THE NEUTRINO MIXING PARAMETERS FOR DIRAC NEUTRINOS

Now we present our results. The RG equations describing the evolution of the neutrino mixing parameters for Dirac neutrinos are derived in Ref. [56]. We have used the *Mathematica*-based package REAP for the computation of the RG evolution at two loops [57]. The first step is to evolve quark mixing angles, gauge couplings, Yukawa couplings of quarks, and charged leptons from the low scale to the SUSY breaking scale. From the SUSY breaking scale to the unification scale, evolution undergoes the MSSM RG equations. The quark mixing angles at the unification scale after evolution are $\theta_{12}^q = 13.02°$, $\theta_{13}^q = 0.17°$ and $\theta_{23}^q = 2.03°$. Now, quark mixing angles are used by the HSMR parametrization at the unification scale and neutrino mixing parameters are evolved down to the SUSY breaking scale using the MSSM RG equations. After this, the evolution of mixing parameters are governed by the SM RG equation. The value of $\tan\beta$ is chosen to be 55. For simplification, we have assumed $k_1 = k_2 = k_3 = 1$ in the HSMR parametrization. The global status of the neutrino mixing parameters is given in Table I.

TABLE I. The global fits for the neutrino mixing parameters [52].

Quantity	Best fit	3σ range
$\Delta m_{21}^2 (10^{-5}\ \mathrm{eV}^2)$	7.60	7.11–8.18
$\Delta m_{31}^2 (10^{-3}\ \mathrm{eV}^2)$	2.48	2.30–2.65
θ_{12}°	34.6	31.8–37.8
θ_{23}°	48.9	38.8–53.3
θ_{13}°	8.6	7.9–9.3

A. Results for the SUSY breaking scale at 2 TeV

In this subsection, we present our results for the SUSY breaking scale at 2 TeV following the direct LHC searches [58]. The unification scale where the HSMR parametrization could be realized is chosen to be GUT scale (2×10^{16} GeV). The free parameters of the analysis are shown in Table II.

In Fig. 1, we show a correlation between mixing angles θ_{13} and θ_{23}. It is obvious that our prediction for θ_{23} is precise. The allowed range of θ_{13} is 7.94°–9.3°. The corresponding range of θ_{23} is 51.5°–52.64°. It is important to note that the predictions for θ_{13} include the best fit value. Another important prediction is that θ_{23} is nonmaximal and lies in the second octant. Being precise, this correlation is easily testable in future and ongoing experiments such as INO, T2K, NOνA, LBNE, Hyper-K, and PINGU [59–64].

In Fig. 2, we show the variation of "averaged electron neutrino mass" m_β [65] with respect to Δm_{31}^2. The allowed range of m_β is 0.4633–0.4690 eV, which is precise. The upper bound on m_β is 2 eV from tritium beta decay [66,67]. The KATRIN experiment is expected to probe m_β as low as 0.2 eV at 90% C.L. [65]. Hence, our prediction for m_β is well within the reach of the KATRIN experiment. The allowed range for Δm_{31}^2 is $(2.30 - 2.37) \times 10^{-3}\ \mathrm{eV}^2$ which is bounded with respect to the 3σ range given by the global fit in Table I. It should be noted that the best fit value of Δm_{31}^2 given in Table I is excluded by our results.

We show in Fig. 3 another important prediction of this work. This is the variation of the CP violating Dirac phase δ_{Dirac} with respect to θ_{13}. The Dirac phase δ_{Dirac} is not known from experiments. Hence, any prediction of this important observable is of great interest. Our prediction for δ_{Dirac} is 80.01° to 287.09° excluding a sufficient part of the allowed parameter space of this quantity. In Fig. 4, we show the

TABLE II. The free parameters of the analysis chosen at the unification scale.

Quantity	Range at the unification scale
α_1	0.7–0.8
α_2	2.12–2.78
α_3	1.002–1.01
m_1 (eV)	0.49227–0.49825
m_2 (eV)	0.494–0.5
m_3 (eV)	0.52898–0.53542
δ_{Dirac}	(−14°, 14°)

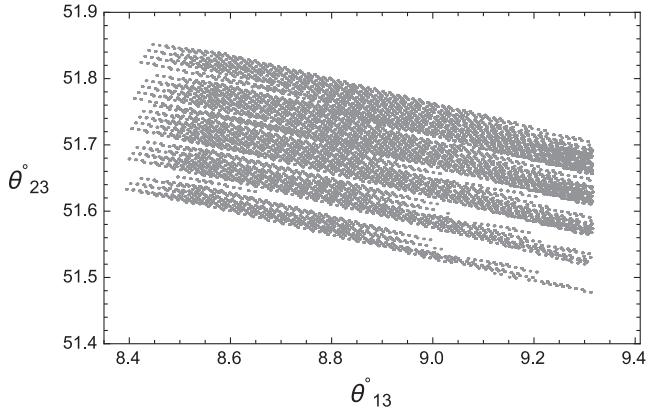

FIG. 1. The variation of θ_{23} with respect to θ_{13}.

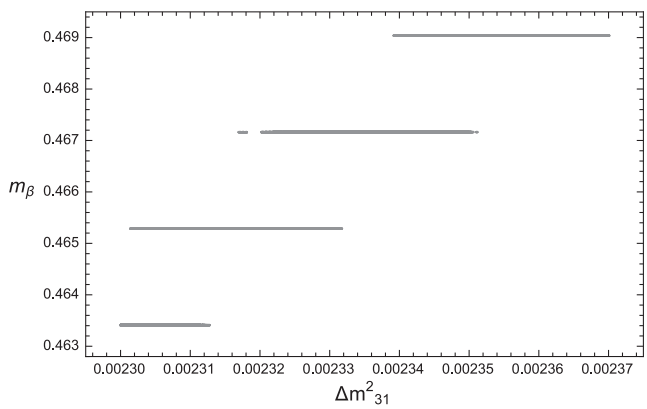

FIG. 2. The variation of m_β with respect to Δm^2_{31}.

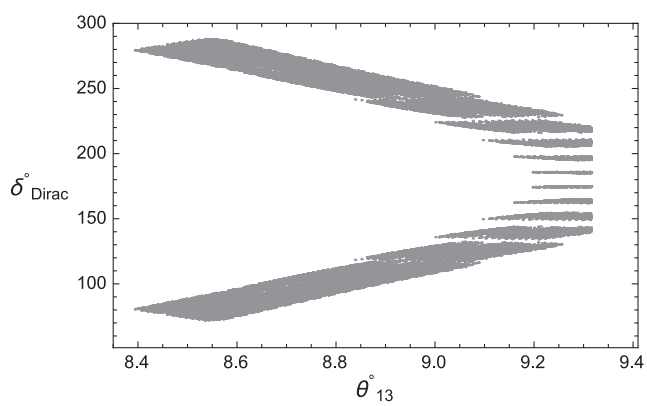

FIG. 3. The variation of $\delta_{\rm Dirac}$ with respect to θ_{13}.

behavior of the Jarlskog invariant J_{CP} with respect to Dirac phase $\delta_{\rm Dirac}$. The allowed range for this observable is -0.266 to 0.266. Thus, a large CP violation is possible in our analysis.

The variation of the sum of three neutrino masses, Σm_i with respect to Δm^2_{31} is shown in Fig. 5. The allowed range of Σm_i is 1.393–1.410 eV, which is precise. We comment that our prediction for Σm_i is a little higher than that provided by the cosmological and astrophysical observations which is

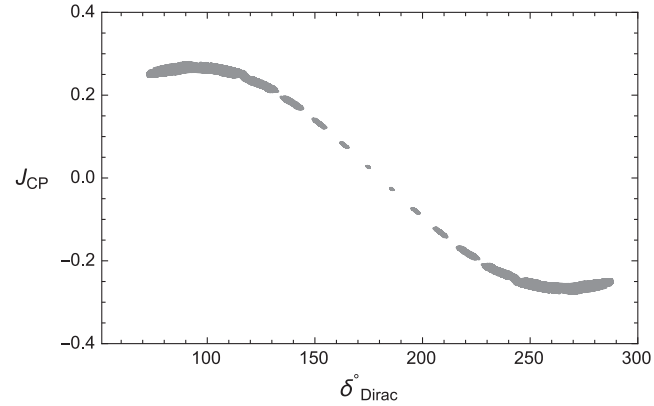

FIG. 4. The variation of J_{CP} with respect to $\delta_{\rm Dirac}$.

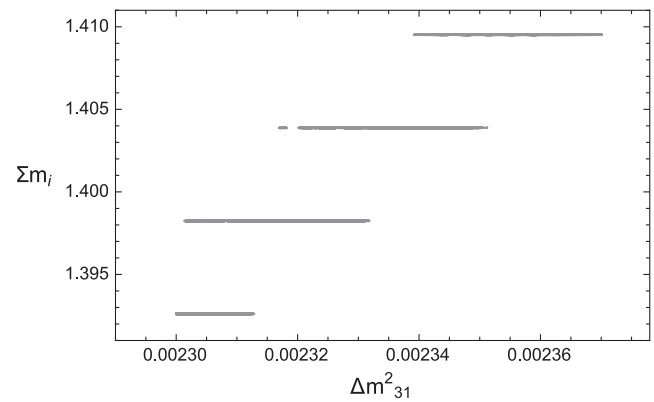

FIG. 5. The variation of Σm_i with respect to Δm^2_{31}.

0.72 eV at 95% C.L. [68]. However, cosmological limit on Σm_i is highly model dependent. For example, as shown in Fig. 29 of Ref. [68] this could be as large as 1.6 eV. Furthermore, Ref. [68] assumes degenerate neutrinos ignoring the observed mass splittings whereas their model (ΛCDM) assumes two massless and one massive neutrino with $\Sigma m_i = 0.06$ eV. Moreover, ΛCDM is facing several challenges in explaining structures on galaxy scales [69]. Hence, our predictions are aimed to be tested in laboratory-based experiments like KATRIN [65].

We do not obtain any constraints on the mixing angle θ_{12} and mass square difference Δm^2_{21}. The whole 3σ ranges of global fit are allowed in this case for these quantities.

B. Variation of the SUSY breaking scale

Now, we discuss the effect of variation of the SUSY breaking scale on our predictions. In this case, we change the SUSY breaking scale to 5 TeV. However, the unification scale is still at the GUT scale. Our results are summarized in Tables III and IV. In Table III, we provide our free parameters, which are chosen at the GUT scale. Our predictions at the low scale are given in Table IV.

We observe that the mixing angle θ_{12} and mass square difference Δm^2_{21} were unconstrained for the SUSY

TABLE III. The free parameters of the analysis chosen at the unification scale for the SUSY breaking scale at 5 TeV.

Quantity	Range at the unification scale
α_1	0.88–1.012
α_2	2.72–2.85
α_3	1.095
m_1 (eV)	0.46878–0.47380
m_2 (eV)	0.47–0.475
m_3 (eV)	0.50321–0.50857
$\delta_{\rm Dirac}$	$(-14°, 14°)$

TABLE IV. Predictions of neutrino mixing parameters and other observables at the low scale for the SUSY breaking scale at 5 TeV.

Quantity	Range at the low scale
θ_{12}	32.85°–37.74°
θ_{13}	7.94°–8.20°
θ_{23}	38.86°–39.45°
m_1 (eV)	0.44458–0.44932
$\Delta m^2_{21}(10^{-5}\ \mathrm{eV}^2)$	7.15–8.15
$\Delta m^2_{31}(10^{-3}\ \mathrm{eV}^2)$	2.30–2.34
m_β (eV)	0.4447–0.4468
Σm_i (eV)	1.337–1.351
$\delta_{\rm Dirac}$	281.28°–355.49° and 0–89.14°
J_{CP}	−0.2511 to 0.2511

breaking scale at 2 TeV in the previous subsection. Now, we observe that these quantities are bounded with respect to the 3σ range given by the global fit. The mixing angle θ_{23}, unlike the investigation for SUSY breaking scale 2 TeV, lies in the first octant and is nonmaximal.

C. Variation of the unification scale

In this subsection, we investigate the variation of the unification scale. In Tables V and VI, we show our results when we choose the unification scale to be 10^{12} GeV which is well below the GUT scale. However, the SUSY breaking scale is kept at 2 TeV. We show in Table V the values of the free parameters chosen at the unification scale. In Table VI, we present our results. The first remarkable prediction is the sum of neutrino masses

TABLE V. The free parameters of the analysis chosen at the unification scale of 10^{12} GeV and SUSY breaking scale of 2 TeV.

Quantity	Range at the unification scale
α_1	0.67–0.85
α_2	19.9–20.92
α_3	7.41–7.42
m_1 (eV)	0.19815–0.20311
m_2 (eV)	0.2–0.205
m_3 (eV)	0.21100–0.21628
$\delta_{\rm Dirac}$	$(-10°, 18°)$

TABLE VI. Predictions of neutrino mixing parameters and other observables for the unification scale of 10^{12} GeV and the SUSY breaking scale at 2 TeV.

Quantity	Range at the low scale
θ_{12}	32.35°–37.34°
θ_{13}	7.94°–8.45°
θ_{23}	38.83°–39.18°
m_1 (eV)	0.18321–0.18801
Δm^2_{21} $(10^{-5}\ \mathrm{eV}^2)$	7.77–8.17
Δm^2_{31} $(10^{-3}\ \mathrm{eV}^2)$	2.30–2.42
m_β (eV)	0.1834–0.1880
Σm_i (eV)	0.556–0.570
$\delta_{\rm Dirac}$	182.66°–203.43° and 0–120°
J_{CP}	−0.1020 to 0.2336

which is well below the cosmological bound. The Dirac CP phase has a precise range. The mixing angle θ_{12} and mass square difference Δm^2_{21} are now relatively constrained. The mixing angle θ_{23} lies in the first octant, and is nonmaximal.

We conclude that there is no parameter space beyond the GUT scale for Dirac neutrinos so that we could recover the experimental data at the low scale using the RG evolution. This is a strong prediction and could be useful in the construction of models (particularly GUT models) where Dirac neutrinos are the natural choice [30–38].

III. MODEL FOR THE HSMR PARAMETRIZATION

We have investigated the HSMR parametrization for Dirac neutrinos in a model independent way. However, for the sake of completeness, in this section we discuss theoretical implementation of the HSMR parametrization in a specific model for Dirac neutrinos. Our model is based on a model presented in Refs. [19,70] which provides Dirac neutrinos with naturally small masses. This model is a type of neutrinophilic SUSY extension of the SM which can easily be embedded in a class of $SU(5)$ models.

To obtain HSMR parametrization in the model given in Ref. [70], we impose a Z_3 discrete symmetry on this model. Under the Z_3 symmetry the first generation of both left- and right-handed quarks and leptons transforms as 1, while the second generation transforms as ω and the third generation transforms as ω^2, where ω denotes cube root of unity with $\omega^3 = 1$. All other fields transform trivially as 1 under the Z_3 symmetry. The Z_3 symmetry ensures that the mass matrices for both up and down quarks as well as for charged leptons and neutrinos are all simultaneously diagonal. This in turn implies that the $V_{\rm CKM}$ as well as $V_{\rm PMNS}$ are both unity and there is no generation mixing in either quark or lepton sectors.

To allow for the mixing, we break Z_3 in a way as done in Ref. [71]. Such corrections can arise from the soft SUSY breaking sector [72–74]. For this purpose, we allow symmetry breaking terms of the form $|y''_i| \ll |y'_i| \ll |y_i|$ where

$|y_i|$ are the terms invariant under Z_3 symmetry, and $|y'_i|$, $|y''_i|$ are the symmetry breaking terms transforming as ω, ω^2 under the Z_3 symmetry. This symmetry breaking pattern is well established and is known to explain the CKM structure of the quark sector [71]. Here, we have imposed this pattern on quarks as well as leptons simultaneously.

Including these symmetry breaking terms, the mass matrices for quarks and leptons become

$$M_{u,d,l} = \begin{pmatrix} y_1 v & y'_2 v & y''_3 v \\ y''_1 v & y_2 v & y'_3 v \\ y'_1 v & y''_2 v & y_3 v \end{pmatrix},$$
$$M_\nu = \begin{pmatrix} y_1 u & y'_2 u & y''_3 u \\ y''_1 u & y_2 u & y'_3 u \\ y'_1 u & y''_2 u & y_3 u \end{pmatrix}, \tag{2}$$

where v stands for the vacuum expectation value (vev) of the usual H_u, H_d doublet scalars of MSSM and u is the vev of the neutrinophilic scalar H_ν as discussed in Ref. [70]. Also, for the sake of brevity we have dropped the sub- and superscripts on the various terms. The mass matrix in (2) is exactly same as the mass matrix obtained in Ref. [71] and can be diagonalized in the same way as done in Ref. [71]. The mass matrices of (2) lead to a "Wolfenstein-like structure" for both CKM and Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrices, thus leading to the HSMR parametrization given in Eq. (1). Since this model is a modification of the model given in Ref. [70] which can be embedded in a class of $SU(5)$ GUT models, therefore, it can also be easily embedded in the $SU(5)$ GUT model in a quite similar way as done in Ref. [70].

IV. SUMMARY

Neutrino mixing is remarkably different from small quark mixing. The aim of the present work is to provide an insight into a common origin of neutrino as well as quark mixing for Dirac neutrinos. Furthermore, we show that smallness of neutrino masses can be explained through the RG evolution of Dirac neutrinos. The HSMR parametrization of neutrino mixing angles is one among many other theoretical frameworks constructed for this purpose. The origin of this parametrization lies in the underlying concept of the quark-lepton unification or flavor symmetries or both. Hence, the confirmation of predictions provided by the HSMR parametrization would be a strong hint of the quark-lepton unification or a grand symmetry operating at the unification scale.

As far as our knowledge is concerned, it was shown for the first time in Ref. [16] that the RG evolution can also explain the large neutrino mixing for Dirac neutrinos. However, as we have shown in this work, these results are no longer valid due to updated experimental data [52–54] and the improved algorithm used in the package REAP [55].

In the present work, we have investigated the RG evolution of Dirac neutrinos in the framework of the HSMR parametrization. To our knowledge, this is the first thorough study on the RG behavior of Dirac neutrinos. The main achievement is that the RG evolution of Dirac neutrinos could explain the large neutrino mixing including the observation of a small and nonzero value of the mixing angle θ_{13}. We obtain strong correlations among different experimental observables. Our predictions for the mixing angles θ_{13}, θ_{23}, averaged electron neutrino mass m_β, Dirac CP phase $\delta_{\rm Dirac}$ and the sum of three neutrino masses, Σm_i, are precise and easily testable at ongoing and future experiments like INO, T2K, NOνA, LBNE, Hyper-K, PINGU and KATRIN [59–65]. The mixing angle θ_{23} is nonmaximal and lies in the second octant for the SUSY breaking scale 2 TeV and unification scale at the GUT scale. For the variation of the SUSY breaking scale and the unification scale, the mixing angle θ_{23} is nonmaximal and lies in the first octant. The predictions for the mass square difference Δm^2_{31} are also well constrained and testable in experiments. Furthermore, the Dirac CP phase is found to be lying in precise ranges in our analysis. The unification scale beyond the GUT scale is ruled out in our investigation. This fact could be useful for the GUT theories having Dirac neutrinos [30–38]. We remark that we have investigated the RG evolution of neutrino mixing parameters at two loops. This is a crucial input since the RG evolution at one loop is insufficient to provide the required enhancement of the mixing angles which in turn cannot yield the results obtained in this work.

One of the main consequences of our investigation is that the HSMU hypothesis is not compatible with Dirac neutrinos due to updated experimental data [52–54] and a better algorithm used in the package REAP [55]. The HSMU hypothesis is a particular realization of the HSMR parametrization when we choose $\alpha_1 = \alpha_2 = \alpha_3 = 1$ for $k_1 = k_2 = k_3 = 1$. As can be observed from Tables II, III and V the allowed range for α_i excludes the $\alpha_1 = \alpha_2 = \alpha_3 = 1$ case. This result is rigorous and robust in the sense that changing the SUSY breaking scale and the unification scale does not change this conclusion. Hence, the HSMR parametrization is one of the preferable frameworks to study the RG evolution of Dirac neutrinos now.

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