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Possibility of searching for $B_c^* \to B_{u,d,s}V$, $B_{u,d,s}P$ decays

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The $B_c^* \to B_{u,d,s}V$, $B_{u,d,s}P$ decays are investigated with the QCD factorization approach, where V and P denote the ground $SU(3)$ vector and pseudoscalar mesons, respectively. The $B_c^* \to B_{u,d,s}$ transition form factors are calculated with the Wirbel-Stech-Bauer model. It is found that branching ratios for the color-favored and Cabibbo-favored $B_c^* \to B_s\rho$, $B_s\pi$ decays can reach up to $\mathcal{O}(10^{-7})$, which might be measurable in the future LHC experiments.

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I. INTRODUCTION

The vector B_c^* meson, a spin-triplet ground state, consists of two heavy quarks with different flavor numbers $B = C = \pm 1$, i.e., $\bar{b}c$ for B_c^{*+} meson and $b\bar{c}$ for B_c^{*-} meson. With nonzero bottom and charm numbers, the bottom and charm quarks of the B_c^* meson cannot annihilate into gluons and photons via the strong and electromagnetic interactions, respectively, unlike the decay modes of the unflavored $J/\psi(1S)$ and $\Upsilon(1S)$ mesons. The B_c^* meson serves as a unique object in studying the heavy quark dynamics, which is inaccessible through both charmonium and bottomonium.

The B_c^* meson lies below the B_qD_q ($q = u$, d, s) meson pair threshold. And the mass splitting $m_{B_c^*} - m_{B_c} \approx 50$ MeV [1] is less than the pion mass. Hence, the B_c^* meson decays via the strong interaction are strictly forbidden. The electromagnetic transition process, $B_c^* \to B_c\gamma$, dominates the B_c^* meson decays, but suffers seriously from a compact phase space suppression, which results in a lifetime of $\tau_{B_c^*} \sim \mathcal{O}(10^{-17}\ \text{s})$ [2]. Besides, the B_c^* meson decays via the weak interaction, although with very small decay rates, are allowable within the standard model.

The B_c^* meson has a relatively large mass. In addition, both constituent quarks b and c of the B_c^* meson can decay individually. Therefore, the B_c^* meson has rich weak decay channels. The B_c^* meson weak decays, similar to the pseudoscalar B_c meson weak decays [3–9], can be divided into three classes: (1) the c quark decay with the spectator b quark, (2) the b quark decay with the c quark as a spectator, and (3) the b and c quarks annihilation into a virtual $W^\pm$ boson. This property makes the B_c^* meson another potentially fruitful laboratory for studying the weak decay mechanism of heavy flavor hadrons.

The study of B_c^* weak decays might be interesting, but has not really started yet. One of the major reasons is the extraordinary difficulty of producing the B_c^* meson. The production cross section for the B_c^* meson in hadronic collisions via the dominant process of $g + g \to B_c^* + b + \bar{c}$ [9–14] is at least at the order of α_s^4. The nature of QCD's asymptotic freedom implies a much small possibility of creating two heavy quark pairs ($b\bar{b}$ and $c\bar{c}$) from the vacuum at the ultrahigh energy. Fortunately, the high luminosities of the running LHC and the future *Super proton proton Collider* (SppC, which is still under discussion today) will promisingly improve this situation. It is expected that a huge amount of the B_c^* data samples would be accumulated, and offer a valuable opportunity to investigate the B_c^* weak decays.

As is well known, there exist some hierarchical structures among the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. The CKM coupling strength for the bottom quark weak decay is proportional to $|V_{cb}| \sim \mathcal{O}(\lambda^2)$ or $|V_{ub}| \sim \mathcal{O}(\lambda^3)$, while the CKM coupling strength for the charm quark weak decay is proportional to $|V_{cs}| \sim \mathcal{O}(1)$ or $|V_{cd}| \sim \mathcal{O}(\lambda)$, with the Wolfenstein parameter $\lambda \approx 0.2$ [15]. The B_q ($q = u$, d, s) weak decays are induced dominantly by the bottom quark decay with the phenomenological spectator scheme. The $B_c^* \to B_qV$, B_qP decays are actually induced by the charm quark weak decay, where V and P denote respectively the lightest 9-pelts $SU(3)$ vector and pseudoscalar mesons. With respect to the B_q weak decays, the $B_c^* \to B_qV$, B_qP decays are favored by the CKM matrix elements. In this paper, we will study the $B_c^* \to B_{u,d,s}V$, $B_{u,d,s}P$ weak decays with the QCD factorization (QCDF) approach [16–24], in order to provide an available reference for the future experimental investigation. There is a more than 2.0σ discrepancy between the value for CKM matrix element $|V_{cs}|$ obtained from semileptonic D decays and that from leptonic D_s decays[1] [15]. The $B_c^{(*)} \to B_sV$, B_sP decays, together with semileptonic D decays and leptonic D_s decays, will provide $|V_{cs}|$ with more stringent constraints. In addition, some of the B_c

[1]The value for CKM matrix element $|V_{cs}|$ is $|V_{cs}| = 0.953 \pm 0.008 \pm 0.024$ from semileptonic D decays, and $|V_{cs}| = 1.008 \pm 0.021$ from leptonic D_s decays [15].

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weak decays, for example, the $B_c \to B_s \pi$ decay [25], have been measured now. One possible background might come from the B_c^* decays, due to a slightly larger production cross section $\sigma(B_c^*)$ than $\sigma(B_c)$ in hadronic collisions [11–14], and the nearly equal mass $m_{B_c^*} \simeq m_{B_c}$ [1]. Hence, the study of the $B_c^* \to B_{u,d,s}V$, $B_{u,d,s}P$ decays will be helpful to the experimental analysis on the $B_c \to B_{u,d,s}V$, $B_{u,d,s}P$ decays.

This paper is organized as follows. The theoretical framework and decay amplitudes will be presented in Sec. II. Section III is the numerical results and discussion. The last section is a summary.

II. THEORETICAL FRAMEWORK

A. The effective Hamiltonian

Using the operator product expansion and the renormalization group (RG) method, the low-energy effective weak Hamiltonian describing the $B_c^* \to B_{u,d,s}V$, $B_{u,d,s}P$ decays has the following general structure [26],

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} \sum_{q,q'=s,d} V_{cq}^* V_{uq'} \{C_1(\mu)Q_1(\mu) + C_2(\mu)Q_1(\mu)\} + \mathrm{H.c.}, \tag{1}$$

where the Fermi coupling constant $G_F \simeq 1.166 \times 10^{-5}$ GeV^{-2} [15]; $V_{cq}^* V_{uq'}$ is a product of the CKM matrix elements. Using the Wolfenstein parametrization, there are [15]

$$V_{cs}^* V_{ud} = 1 - \lambda^2 - \frac{1}{2}A^2\lambda^4 + \frac{1}{2}A^2\lambda^6\{1 - \rho^2 - \eta^2 - 2(\rho - i\eta)\} + \mathcal{O}(\lambda^8), \tag{2}$$

$$V_{cs}^* V_{us} = \lambda - \frac{\lambda^3}{2} - \frac{\lambda^5}{8} - \frac{\lambda^7}{16} - \frac{1}{2}A^2\lambda^5 + \frac{1}{2}A^2\lambda^7\left\{\frac{1}{2} - \rho^2 - \eta^2 - 2(\rho - i\eta)\right\} + \mathcal{O}(\lambda^8), \tag{3}$$

$$V_{cd}^* V_{ud} = -V_{cs}^* V_{us} - A^2\lambda^5(\rho - i\eta) + \mathcal{O}(\lambda^8), \tag{4}$$

$$V_{cd}^* V_{us} = -\lambda^2 + \frac{1}{2}A^2\lambda^6\{1 - 2(\rho - i\eta)\} + \mathcal{O}(\lambda^8), \tag{5}$$

where the values for these Wolfenstein parameters A, λ, ρ, and η are given in Table III.

The renormalization scale μ separates the physical contributions into two parts. The hard contributions above the scale μ are summarized into the Wilson coefficients $C_i(\mu)$. With the RG equation for $C_i(\mu)$, the Wilson coefficients at an appropriate scale $\mu_c \sim \mathcal{O}(m_c)$ for the charm quark decay are given by [26]

$$\vec{C}(\mu_c) = U_4(\mu_c, m_b)U_5(m_b, m_W)\vec{C}(m_W), \tag{6}$$

where m_W, m_b, and m_c are the mass of the W boson, b quark, and c quark, respectively. Here $U_f(m_2, m_1)$ denotes the RG evolution matrix for f active flavors. The initial values for the Wilson coefficients $\vec{C}(m_W)$ at scale $\mu_W = m_W$ to a desired order in α_s can be calculated with perturbation theory. The expressions for the RG evolution matrix $U_f(m_2, m_1)$ and Wilson coefficients $\vec{C}(m_W)$, including both leading order (LO) and next-to-leading order (NLO) corrections, have been presented in Ref. [26]. The contributions below the scale μ are included in the hadronic matrix elements (HME) where the local four-quark operators Q_i are sandwiched between the initial and final states. The expressions for the four-quark operators in question are

$$Q_1 = [\bar{q}_\alpha\gamma_\mu(1-\gamma_5)c_\alpha][\bar{u}_\beta\gamma^\mu(1-\gamma_5)q'_\beta], \tag{7}$$

$$Q_2 = [\bar{q}_\alpha\gamma_\mu(1-\gamma_5)c_\beta][\bar{u}_\beta\gamma^\mu(1-\gamma_5)q'_\alpha], \tag{8}$$

where the subscripts α and β are color indices. It should be pointed out that (1) because the contributions from the penguin operators and annihilation topologies are proportional to the CKM factor $V_{cb}^* V_{ub} \sim \mathcal{O}(\lambda^5)$ and therefore negligible in the actual calculation of branching ratio [7], only the contributions of tree operators are considered here. (2) The participation of the strong interaction, especially, the nonperturbative QCD effects, makes the theoretical treatment of HME very complicated. The main problem at this stage is how to effectively factorize HME into hard and soft parts, and how to evaluate HME properly.

B. Hadronic matrix elements

Hadronic matrix elements might be the most intricate part in the calculation of heavy flavor weak decay, due to the entanglement of perturbative and nonperturbative contributions. Phenomenologically, one has to turn to some approximation and assumption, which bring uncertainties and model dependence to theoretical predictions. A simple approximation is the naive factorization ansatz (NF) according to Bjorken's color transparency argument, which says that the colorless energetic hadron has flown away from the weak interaction point during the formation time of the emission hadron [27]. With the NF approach, HME is parametrized as a product of decay constants and hadron transition form factors [28–31]. A major flaw of the NF approach is the disappearance of scale dependence and strong phases from HME, which results directly in a scale-sensitive nonphysical prediction and none of CP violation for nonleptonic meson weak decays. In order to overcome these shortcomings of the NF approach, nonfactorizable contributions to HME should be carefully considered, as commonly recognized. Some QCD-inspired models, such as, the QCDF approach [16–24], the soft and collinear effective theory [32–39], the perturbative QCD approach [40–42], and so on, have been developed recently, based on the Lepage-Brodsky treatment on exclusive processes [43] and some power counting rules in the expansion in α_s and $\Lambda_{\rm QCD}/m_Q$, where α_s is the strong coupling, $\Lambda_{\rm QCD}$ is the QCD characteristic scale, and m_Q is the mass of a heavy quark. In these QCD-inspired models, HME is generally written as a convolution integral of hadron's distribution amplitudes (DAs) and hard rescattering kernels. A virtue of the QCDF approach is that the NF's result can be reproduced, if both the nonfactorizable contributions and the power suppressed contributions are neglected [16–21].

For the $B_c^* \to B_q V$, $B_q P$ decays ($q = u$, d, s), the spectator quark is a heavy quark—the bottom quark. It is generally assumed that the bottom quark in both the B_c^* and B_q mesons is nearly on shell, and that the gluon exchanged between the heavy spectator quark and other quarks is soft. The virtuality of emission gluon from the spectator quark is of order $\Lambda_{\rm QCD}^2$. The contributions of spectator scattering are power suppressed relative to the leading order contributions [17]. In addition, it is supposed that the recoiled B_q meson should move slowly in the rest frame of the B_c^* meson. There should be a large overlap between the B_c^* and B_q mesons. The recoiled B_q meson cannot be clearly factorized from the $B_c^* B_q$ system due to the soft and nonperturbative contributions. The $B_c^* B_q$ system should be parametrized by some physical from factors. Hence, with the QCDF approach, up to leading power corrections of order $\Lambda_{\rm QCD}/m_Q$, hadronic matrix elements have the following structure [17],

$$\begin{aligned}\langle B_q M|Q_i|B_c^*\rangle &= f_M \sum_j F_j^{B_c^*\to B_q} \int dx \mathcal{H}_{ij}(x)\phi(x) \\ &= f_M \sum_j F_j^{B_c^*\to B_q}\{1+\alpha_s r_j + \cdots\}, \end{aligned} \tag{9}$$

where f_M is the decay constant for the light final M ($\equiv V$ and P) meson; $F_j^{B_c^*\to B_q}$ is a transition form factor; $\mathcal{H}_{ij}(x)$ is a hard rescattering kernel; $\phi(x)$ is a DA of parton momentum fraction x. For the light pseudoscalar P and longitudinally polarized vector V mesons, the leading twist DAs are expanded in terms of the Gegenbauer polynomials [44,45]

$$\phi_P(x) = 6x\bar{x}\left\{1+\sum_{n=1} a_n^P C_n^{3/2}(x-\bar{x})\right\}, \tag{10}$$

$$\phi_V(x) = 6x\bar{x}\left\{1+\sum_{n=1} a_n^V C_n^{3/2}(x-\bar{x})\right\}, \tag{11}$$

where $\bar{x} = 1-x$; $a_n^{P,V}$ is a nonperturbative parameter, also called the Gegenbauer moment. The expressions for the Gegenbauer polynomials $C_n^{3/2}(z)$ are

$$C_1^{3/2}(z) = 3z, \qquad C_2^{3/2}(z) = \frac{3}{2}(5z^2-1), \qquad \cdots \tag{12}$$

C. Decay amplitudes

The typical Feynman diagrams for the $B_c^* \to B_s \pi$ decay within the QCDF framework are shown in Fig. 1, where no hard gluons are exchanged between the spectator quark and other partons. There is no gluon exchange in factorizable topology of Fig. 1(a), so the emitted hadron matrix element is entirely separated from that of the $B_c^* B_s$ system. In this approximation, the hard rescattering kernel $\mathcal{H}_{ij} = 1$ and the integral in Eq. (9) reduces to the normalization condition for distribution amplitude. According to the QCDF power counting rules, the leading order contributions come from the factorizable topology of Fig. 1(a), and recover the NF's results at the order of α_s^0. For the radiative correction diagrams in Fig. 1(b–e), hard gluons are exchanged between the emission meson and the $B_c^* B_s$ system. The hard rescattering kernel $\mathcal{H}_{ij}$ and x-integral in Eq. (9) are nontrivial. It has already been shown [17–21] that although both collinear and soft divergences exist for each of diagrams in Fig. 1(b–e), infrared divergences cancel after summing up the vertex corrections. The strong phases could then come from HME. The renormalization scale μ dependence of HME is recuperated from the nonfactorizable contributions, which will reduce partly the μ-dependence of Wilson coefficients.

After a straightforward calculation using the QCDF master formula Eq. (9), the amplitudes for the $B_c^* \to B_q M$ decays ($q = u$, d, s) are written as

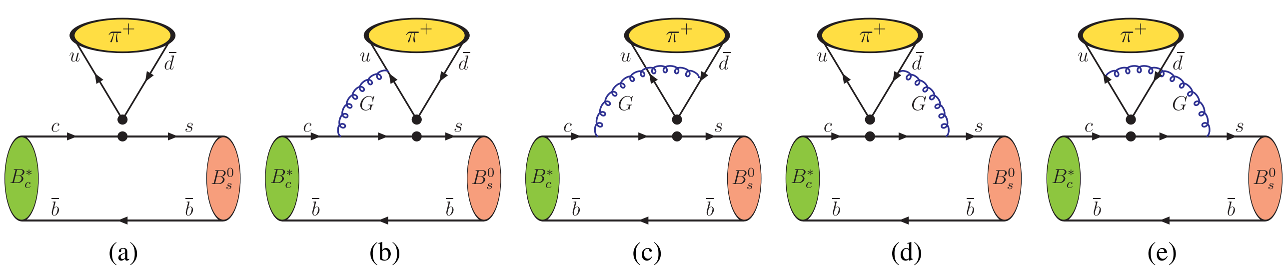

FIG. 1. Feynman diagrams for $B_c^* \to B_s \pi$ decay within the QCDF framework, where (a) denotes the factorizable contributions, and (b, c,d,e) correspond to the nonfactorizable vertex corrections at the order of α_s.

TABLE I. The numerical values for the Wilson coefficients and $a_{1,2}$ for the $B_c^* \to B_q \pi$ decay.

Scale	LO		NLO		NF		QCDF	
μ	C_1	C_2	C_1	C_2	a_1	a_2	a_1	a_2
$0.8m_c$	1.310	−0.553	1.253	−0.473	1.096	−0.055	$1.235 + i0.081$	$-0.384 - i0.192$
m_c	1.259	−0.479	1.209	−0.404	1.074	−0.001	$1.193 + i0.060$	$-0.313 - i0.157$
$1.2m_c$	1.227	−0.430	1.180	−0.358	1.061	0.035	$1.167 + i0.048$	$-0.269 - i0.137$

$$\mathcal{A}(B_c^* \to B_q M) = \langle B_q M|\mathcal{H}_{\mathrm{eff}}|B_c^*\rangle = \frac{G_F}{\sqrt{2}} V_{cq}^* V_{uq'} a_i \langle M|j^\mu|0\rangle \langle B_q|j_\mu|B_c^*\rangle. \quad (13)$$

With the naive dimensional regularization scheme, the effective coefficients are [17–21]

$$a_1 = C_1^{\mathrm{NLO}} + \frac{1}{N_c} C_2^{\mathrm{NLO}} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} C_2^{\mathrm{LO}} \mathcal{V}, \quad (14)$$

$$a_2 = C_2^{\mathrm{NLO}} + \frac{1}{N_c} C_1^{\mathrm{NLO}} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} C_1^{\mathrm{LO}} \mathcal{V}, \quad (15)$$

$$\mathcal{V} = 6 \log\left(\frac{m_c^2}{\mu^2}\right) - 18 - \left(\frac{1}{2} + i3\pi\right) + \left(\frac{11}{2} - i3\pi\right) a_1^M - \frac{21}{20} a_2^M + \cdots, \quad (16)$$

where $N_c = 3$ and $C_F = 4/3$; $C_{1,2}^{\mathrm{NLO,LO}}$ are Wilson coefficients containing NLO or LO contributions; a_i^M is a Gegenbauer moment. For the transversely polarized vector meson, the vertex factor $\mathcal{V} = 0$ beyond the leading twist DAs. For convenience, the numerical values for $a_{1,2}$ of the $B_c^* \to B_q \pi$ decay are listed in Table I.

There are some comments on the coefficients $a_{1,2}$. (1) The first two terms on the right-hand side of Eq. (14) and Eq. (15) correspond to the leading order contributions. The third terms correspond to nonfactorizable contributions. The NF scenario follows when one neglects the nonfactorizable contributions, i.e., $\mathcal{V} = 0$. (2) Nonfactorizable vertex corrections to HME are of order α_s. They include the dependence on the renormalization scale. It is shown [21] that with the RG equations for the Wilson coefficients at leading order logarithm approximation, one can obtain $\mu \frac{d}{d\mu} a_{1,2} = 0$. In principle, the residual scale dependence could be compensated by higher order corrections to HME. (3) Compared with the LO contributions, nonfactorizable contributions are generally suppressed by α_s and the factor $1/N_c$ [see Eq. (14) and Eq. (15)]. Because the LO contributions of a_2 are color-suppressed, vertex corrections multiplied by the large Wilson coefficient C_1^{LO} could be sizable to branching rates of the a_2-dominated heavy flavor decays. The coefficients $a_{1,2}$ contain strong phases via the imaginary parts of vertex corrections. Correspondingly, strong scattering phase of a_1 (a_2) is small (large). This argument is also confirmed by the numerical results for $a_{1,2}$ in Table I. (4) With the QCDF approach, nonfactorizable radiative corrections to HME occur first at order α_s as well as the leading strong phases at order α_s. In addition, it should be pointed out that nonfactorizable power corrections beyond leading order are neglected here. For the charm quark decay, power $\Lambda_{\mathrm{QCD}}/m_c$ is comparable to α_s. The strong phases due to soft (hard) interactions are of order $\Lambda_{\mathrm{QCD}}/m_c$ (α_s). One should not expect these phases to have great precision, as stated in Ref. [17]. (5) With the QCDF approach, the values for $a_{1,2}$ are close to those for the charm quark decay [46–50], $|a_{1,2}| \approx |C_{1,2}|$, and basically consistent with those of the large-N_c approach [46].

The hadronic matrix elements of diquark current operators are defined as [30]:

$$\langle V(\epsilon, p)|\bar{q}_1 \gamma^\mu (1-\gamma_5) q_2|0\rangle = f_V m_V \epsilon^{*\mu}, \quad (17)$$

$$\langle P(p)|\bar{q}_1 \gamma^\mu (1-\gamma_5) q_2|0\rangle = -i f_P p^\mu, \quad (18)$$

$$\begin{aligned}
&\langle B_q(p_2)|\bar{q}\gamma_\mu(1-\gamma_5)c|B_c^*(p_1,\epsilon)\rangle \\
&= -\varepsilon_{\mu\nu\alpha\beta}\epsilon^\nu q^\alpha (p_1+p_2)^\beta \frac{V(q^2)}{m_{B_c^*}+m_{B_q}} - i2m_{B_c^*}\frac{\epsilon\cdot q}{q^2} q_\mu A_0(q^2) \\
&\quad - i\epsilon_\mu (m_{B_c^*}+m_{B_q}) A_1(q^2) - i\frac{\epsilon\cdot q}{m_{B_c^*}+m_{B_q}} (p_1+p_2)_\mu A_2(q^2) \\
&\quad + i2m_{B_c^*}\frac{\epsilon\cdot q}{q^2} q_\mu A_3(q^2), \quad (19)
\end{aligned}$$

where f_V and f_P are the decay constants of vector V and pseudoscalar P mesons, respectively; $q = p_1 - p_2$; ϵ is the polarization vector of vector mesons; $V(q^2)$ and $A_{0,1,2,3}(q^2)$ are the $B_c^* \to B_q$ transition form factors. To eliminate singularities at the pole of $q^2 = 0$, a relation, $A_0(0) = A_3(0)$, is required, with $A_3(q^2)$ given by [30]:

$$2m_{B_c^*} A_3(q^2) = (m_{B_c^*} + m_{B_q}) A_1(q^2) + (m_{B_c^*} - m_{B_q}) A_2(q^2). \quad (20)$$

In the bottom conservation transition $B_c^* \to B_q$, both the initial and final mesons contain a heavy bottom quark. After a sudden kick, the B_q meson would move slowly, even remain nearly intact, with respect to the B_c^* meson. Therefore, the zero-recoil configuration ($q^2 = 0$) would be a good approximation. Simultaneously, the emission meson would take up most of the energy available and fly rapidly away from the interaction point. This fact not only reproduces the NF scenario [Fig. 1(a)] but also requires the exchanged gluon in vertex corrections [Fig. 1(b–e)] to be hard. Due to the large virtuality of gluon exchanged between the emitted light meson and the $B_c^* B_q$ system, perturbative calculation of nonfactorizable vertex corrections with the QCDF approach should be applicable and reliable.

With the form factors given above, the decay amplitudes are expressed as

$$\begin{aligned}\mathcal{A}(B_c^* \to B_q V) = {}& -i\frac{G_F}{\sqrt{2}} m_V f_V V_{cq}^* V_{uq'} \{a_1 \delta_{B_q,B_{d,s}} + a_2 \delta_{B_q,B_u}\} \\ & \times \bigg\{ (\epsilon_{B_c^*} \cdot \epsilon_V^*)(m_{B_c^*} + m_{B_q}) A_1 \\ & + (\epsilon_{B_c^*} \cdot p_V)(p_{B_c^*} \cdot \epsilon_V^*) \frac{2A_2}{m_{B_c^*} + m_{B_q}} \\ & + i\epsilon_{\mu\nu\alpha\beta} \epsilon_{B_c^*}^{\mu} \epsilon_V^{*\nu} p_{B_c^*}^{\alpha} p_V^{\beta} \frac{2V}{m_{B_c^*} + m_{B_q}} \bigg\}, \end{aligned} \tag{21}$$

$$\begin{aligned}\mathcal{A}(B_c^* \to B_q P) = {}& \sqrt{2} G_F m_{B_c^*} (\epsilon_{B_c^*} \cdot p_{B_q}) f_P A_0 V_{cq}^* V_{uq'} \\ & \times \{a_1 \delta_{B_q,B_{d,s}} + a_2 \delta_{B_q,B_u}\}. \end{aligned} \tag{22}$$

The $B_c^* \to B_q V$ decay amplitude is a sum of S-, P-, D-wave amplitudes [51,52], i.e.,

$$\begin{aligned}\mathcal{A}(B_c^* \to B_q V) = {}& a(\epsilon_{B_c^*} \cdot \epsilon_V^*) + \frac{b}{m_{B_c^*} m_V} (\epsilon_{B_c^*} \cdot p_V)(p_{B_c^*} \cdot \epsilon_V^*) \\ & + \frac{ic}{m_{B_c^*} m_V} \epsilon_{\mu\nu\alpha\beta} \epsilon_{B_c^*}^{\mu} \epsilon_V^{*\nu} p_{B_c^*}^{\alpha} p_V^{\beta}, \end{aligned} \tag{23}$$

with a, b, c, the S-, D-, and P-wave amplitudes respectively, in the notation of [52],

$$a = \mathcal{F}(m_{B_c^*} + m_{B_q}) A_1, \tag{24}$$

$$b = \mathcal{F} \frac{2 m_{B_c^*} m_V}{m_{B_c^*} + m_{B_q}} A_2, \tag{25}$$

$$c = \mathcal{F} \frac{2 m_{B_c^*} m_V}{m_{B_c^*} + m_{B_q}} V, \tag{26}$$

$$\mathcal{F} = -i\frac{G_F}{\sqrt{2}} m_V f_V V_{cq}^* V_{uq'} \{a_1 \delta_{B_q,B_{d,s}} + a_2 \delta_{B_q,B_u}\}. \tag{27}$$

From the above expressions, one can find that the P- and D-wave amplitudes are suppressed by a factor of $\frac{2 m_{B_c^*} m_V}{(m_{B_c^*} + m_{B_q})^2}$ relative to the S-wave amplitude. The relations among the helicity amplitudes and the S-, P-, D-wave amplitudes are [52]

$$H_{\pm} = a \pm c\sqrt{y^2 - 1}, \tag{28}$$

$$H_0 = -ay - b(y^2 - 1), \tag{29}$$

$$y = \frac{p_{B_c^*} \cdot p_V}{m_{B_c^*} m_V} = \frac{m_{B_c^*}^2 - m_{B_q}^2 + m_V^2}{2 m_{B_c^*} m_V}, \tag{30}$$

$$p_{\rm cm} = \frac{\sqrt{[m_{B_c^*}^2 - (m_{B_q} + m_V)^2][m_{B_c^*}^2 - (m_{B_q} - m_V)^2]}}{2 m_{B_c^*}}, \tag{31}$$

$$p_{\rm cm}^2 = m_V^2 (y^2 - 1), \tag{32}$$

where $p_{\rm cm}$ is the common momentum of final states in the rest frame of the B_c^* meson.

We assume that the vector mesons are ideally mixed in the singlet-octet basis, i.e., $\phi = s\bar{s}$ and $\omega = (u\bar{u} + d\bar{d})/\sqrt{2}$. As for the pseudoscalar η and η' mesons, they are usually written as a linear superposition of states in either flavor basis or the singlet-octet basis. Here, we adopt the quark flavor basis description proposed in Ref. [53], i.e.,

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}, \tag{33}$$

where $\eta_q = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\eta_s = s\bar{s}$; the mixing angle $\phi \approx (39.3 \pm 1.0)^\circ$ [53]. Due to the symmetric flavor configurations of both η_q and η_s states, we assume that DAs for η_q and η_s states are similar to DAs for pion. It should be pointed out that the contributions from possible $c\bar{c}$ and gluonium compositions are not considered in our calculation for the moment, because (1) the final states with B_q meson and $c\bar{c}$ or gluonium states lie above the B_c^* meson mass; (2) the fraction of gluonium components in η and η' is rather tiny [54]. Thus, the amplitudes for the $B_c^* \to B_u \eta$, $B_u \eta'$ decays are written as

$$\begin{aligned}\mathcal{A}(B_c^* \to B_u \eta) = {}& \cos\phi \mathcal{A}(B_c^* \to B_u \eta_q) \\ & - \sin\phi \mathcal{A}(B_c^* \to B_u \eta_s), \end{aligned} \tag{34}$$

$$\begin{aligned}\mathcal{A}(B_c^* \to B_u \eta') = {}& \sin\phi \mathcal{A}(B_c^* \to B_u \eta_q) \\ & + \cos\phi \mathcal{A}(B_c^* \to B_u \eta_s). \end{aligned} \tag{35}$$

D. Form factors

The hadron transition form factors are the basic input parameters for decay amplitudes [see Eq. (21)

and Eq. (22)]. It is assumed [17] that form factors come mainly from soft contributions, and form factors are generally regarded as nonperturbative parameters in the QCDF master formula of Eq. (9). Fortunately, form factors are universal. Form factors determined by other means or extracted from data can be employed here to make predictions. Phenomenologically, form factors are written as overlap integrals of wave functions.

Here, we will employ the Wirbel-Stech-Bauer model [30] for evaluating the form factors. With a factorization of spin and spatial motion, wave function is written as

$$\phi^{(j,j_z)}(\vec{k}_\perp, x) = \phi(\vec{k}_\perp, x)|s, s_z; s_1, s_2\rangle, \tag{36}$$

where $\vec{k}_\perp$ and x are the transverse momentum and longitudinal momentum fraction, respectively; j (s) is the total angular momentum (spin); j_z (s_z) is the magnetic quantum number; s_1 and s_2 are spins of valence quarks. $j = s = 1$ for the ground vector B_c^* meson, and 0 for the ground pseudoscalar $B_{u,d,s}$ meson. The spatial wave function of a relativistic scalar harmonic oscillator potential is given by [30]

$$\phi(\vec{k}_\perp, x) = N_m \sqrt{x(1-x)} \exp\left\{-\frac{\vec{k}_\perp^2}{2\omega^2}\right\} \times \exp\left\{-\frac{m^2}{2\omega^2}\left(x - \frac{1}{2} - \frac{m_{q_1}^2 - m_{q_2}^2}{2m^2}\right)^2\right\}, \tag{37}$$

where parameter ω determines the average transverse momentum of partons, i.e., $\langle \vec{k}_\perp^2 \rangle = \omega^2$; m is the mass of the concerned meson; m_{q_1} (m_{q_2}) is the constituent mass of the decaying (spectator) quark carrying a gluon cloud; N_m is a normalization factor determined by

$$\int d^2k_\perp \int_0^1 dx |\phi(\vec{k}_\perp, x)|^2 = 1. \tag{38}$$

The form factors at zero momentum transfer are given by [30]

$$A_0(0) = A_3(0) = \int d^2k_\perp \int_0^1 dx \phi_{B_c^*}^{(1,0)}(\vec{k}_\perp, x) \sigma_z^{(1)} \phi_B(\vec{k}_\perp, x), \tag{39}$$

$$J = \sqrt{2} \int d^2k_\perp \int_0^1 \frac{dx}{x} \phi_{B_c^*}^{(1,-1)}(\vec{k}_\perp, x) i\sigma_y^{(1)} \phi_B(\vec{k}_\perp, x), \tag{40}$$

$$V(0) = \frac{m_c - m_q}{m_{B_c^*} - m_{B_q}} J, \tag{41}$$

$$A_1(0) = \frac{m_c + m_q}{m_{B_c^*} + m_{B_q}} J, \tag{42}$$

where $\sigma_{z,y}^{(1)}$ are Pauli matrixes acting on the spin indices of the decaying quark q_1.

It has been shown [30] that the form factors are sensitive to the choice of parameter ω. And it is argued [30] that parameter ω is not expected to be largely different for various mesons due to the flavor independence of the QCD interactions. Thus the same ω might be applied to all mesons with the same spectator quark. The motion of the spectator (bottom) quark is nearly nonrelativistic in the $B_c^* \to B_q$ transition. Thus, nonrelativistic QCD (NRQCD) effective theory [55–57] could be used to deal with both B_c^* and B_q mesons. According to the NRQCD power counting rules, the average transverse momentum is the order of $\omega \approx m\alpha_s$. In order to see the parameter ω effects on the form factors, we explore two scenarios. One is the same parameter ω for both the B_c^* and B_q mesons, and the other is $\omega = m\alpha_s$, i.e., $\omega \approx 1.24$ GeV for the B_c^* meson, 1.10 GeV for the B_s meson, and 1.09 GeV for the $B_{u,d}$ mesons. The numerical results for form factors are shown in Table II.

There are some comments on the form factors. (1) From the expressions in Eq. (41) and Eq. (42), it is seen that due to the factor $\frac{m_c - m_q}{m_{B_c^*} - m_{B_q}} \approx 1$ and $\frac{m_c + m_q}{m_{B_c^*} + m_{B_q}} \ll 1$, one can obtain a relation, $A_1(0) < V(0)$. (2) Compared with the integrand in Eq. (39), there is a factor $1/x$ for the integrand in Eq. (40) with longitudinal momentum fraction $0 < x < 1$. Thus, it is expected to have in general $A_{0,3}(0) < V(0)$. (3) With the

TABLE II. The numerical values for the form factors in the $B_c^* \to B_q$ transition, where the uncertainties come from both m_c and m_b.

Transition	ω	$A_0(0)$	$A_1(0)$	$A_2(0)$	$V(0)$
$B_c^* \to B_u$	0.4 GeV	$0.540^{+0.015}_{-0.015}$	$0.291^{+0.002}_{-0.002}$	$3.286^{+0.210}_{-0.209}$	$1.953^{+0.038}_{-0.040}$
	0.6 GeV	$0.784^{+0.008}_{-0.008}$	$0.429^{+0.011}_{-0.011}$	$4.694^{+0.219}_{-0.222}$	$2.877^{+0.111}_{-0.109}$
	$m\alpha_s$	$0.944^{+0.002}_{-0.002}$	$0.539^{+0.017}_{-0.017}$	$5.403^{+0.208}_{-0.213}$	$3.613^{+0.160}_{-0.156}$
$B_c^* \to B_d$	0.4 GeV	$0.540^{+0.015}_{-0.015}$	$0.291^{+0.002}_{-0.002}$	$3.288^{+0.210}_{-0.209}$	$1.954^{+0.038}_{-0.040}$
	0.6 GeV	$0.784^{+0.008}_{-0.008}$	$0.429^{+0.011}_{-0.011}$	$4.696^{+0.219}_{-0.222}$	$2.878^{+0.111}_{-0.109}$
	$m\alpha_s$	$0.944^{+0.002}_{-0.002}$	$0.539^{+0.017}_{-0.017}$	$5.405^{+0.208}_{-0.213}$	$3.614^{+0.160}_{-0.156}$
$B_c^* \to B_s$	0.4 GeV	$0.609^{+0.015}_{-0.015}$	$0.361^{+0.003}_{-0.003}$	$3.618^{+0.234}_{-0.234}$	$1.867^{+0.059}_{-0.061}$
	0.6 GeV	$0.821^{+0.007}_{-0.007}$	$0.494^{+0.012}_{-0.012}$	$4.785^{+0.242}_{-0.247}$	$2.554^{+0.122}_{-0.120}$
	$m\alpha_s$	$0.954^{+0.002}_{-0.002}$	$0.598^{+0.018}_{-0.017}$	$5.268^{+0.232}_{-0.238}$	$3.097^{+0.163}_{-0.159}$

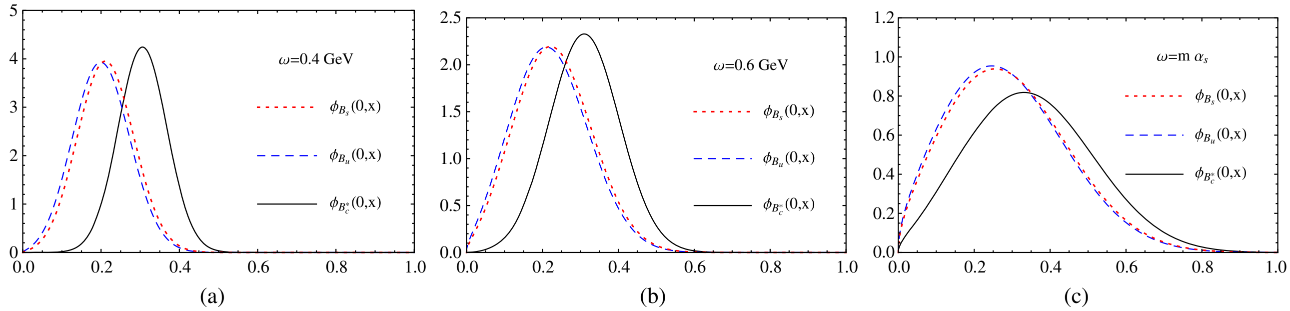

FIG. 2. The shapes of the normalized wave functions for the B_c^* and $B_{u,s}$ meson, with parameter $\omega = 0.4$ GeV (a), 0.6 GeV (b), and $m\alpha_s$ (c), respectively.

relation of form factors in Eq. (20), $A_2(0)$ is significantly enhanced by a factor of $\frac{2m_{B_c^*}}{m_{B_c^*}-m_{B_q}}$ (or $\frac{m_{B_c^*}+m_{B_q}}{m_{B_c^*}-m_{B_q}}$) relative to $A_3(0)$ [or $A_1(0)$]. These relations are comprehensively verified by the numerical results for form factors in Table II.

In addition, from the numbers in Table II, it is seen that (1) the form factors increase as parameter ω increases, due to the fact that the overlap between wave functions of B_c^* and B_q mesons increases as parameter ω increases, as shown in Fig. 2. (2) The flavor symmetry breaking effects on form factors are small. (3) The values for $A_2(0)$ ($V(0)$) are about ten (five) times as large as those for $A_1(0)$, as explained above. The large values for A_2 and V would enhance the contributions from the D- and P-wave amplitudes [see Eq. (25) and Eq. (26)].

III. NUMERICAL RESULTS AND DISCUSSION

In the rest frame of the B_c^* meson, branching ratios are defined as

$$\mathcal{B}r(B_c^* \to BV) = \frac{1}{24\pi} \frac{p_{\rm cm}}{m_{B_c^*}^2 \Gamma_{B_c^*}} \{|H_+|^2 + |H_0|^2 + |H_-|^2\}, \tag{43}$$

$$\mathcal{B}r(B_c^* \to BP) = \frac{1}{24\pi} \frac{p_{\rm cm}}{m_{B_c^*}^2 \Gamma_{B_c^*}} |\mathcal{A}(B_c^* \to BP)|^2, \tag{44}$$

where $\Gamma_{B_c^*}$ is the full width of the B_c^* meson.

Because the electromagnetic radiation process $B_c^* \to B_c\gamma$ dominates the B_c^* meson decay, to a good approximation, $\Gamma_{B_c^*} \simeq \Gamma(B_c^* \to B_c\gamma)$. However, there is still no experimental information about the partial width $\Gamma(B_c^* \to B_c\gamma)$ now, because the photon from the $B_c^* \to B_c\gamma$ process is too soft to be easily identified. The information on $\Gamma(B_c^* \to B_c\gamma)$ comes mainly from theoretical estimation on the magnetic dipole (M1) transition, i.e., [2]

$$\Gamma(B_c^* \to B_c\gamma) = \frac{4}{3}\alpha_{\rm em} k_\gamma^3 \mu_h^2, \tag{45}$$

where $\alpha_{\rm em}$ is the fine-structure constant of electromagnetic interaction; k_γ is the photon momentum in the rest frame of initial state; μ_h is the M1 moment of B_c^* meson. There are plenty of theoretical predictions on $\Gamma(B_c^* \to B_c\gamma)$, for example, the numbers in Tables 3 and 6 in Ref. [2]. However, these estimations still suffer from large uncertainties due to our lack of a precise value for μ_h. To give a quantitative evaluation, $\Gamma_{B_c^*} = 50$ eV will be fixed in our calculation for the moment. The value of 50 eV seems reasonable since it is close to the value given by the potential model (PM) which produces good agreement with experiment for the measured $J/\psi \to \eta_c\gamma$ decay rate. The value for the charm quark magnetic moment μ_c obtained from the charmonium M1 decay width can now be used to predict the $B_c^* \to B_c\gamma$ decay width, with a very small b quark magnetic moment $\mu_b = -0.06\mu_N$ given in Ref. [2].

The numerical values for other input parameters are listed in Table III. Unless otherwise stated, their central values will be fixed as the default inputs. Our numerical results are presented in Table IV. The following are some comments.

(1) According to the relative sizes of coefficients $a_{1,2}$ and CKM factors, the $B_c^* \to B_qV$, B_qP decays could be classified into six cases (see Table IV). There is a clear hierarchical relation among branching ratios, i.e., $\mathcal{B}r(\text{case 1-I}) \sim \mathcal{O}(10^{-7})$, $\mathcal{B}r(\text{case 1-II}) \sim \mathcal{O}(10^{-8})$, $\mathcal{B}r(\text{case 1-III}) \sim \mathcal{O}(10^{-9})$, and $\mathcal{B}r(\text{case 2-I}) \sim \mathcal{O}(10^{-8})$, $\mathcal{B}r(\text{case 2-II}) \sim \mathcal{O}(10^{-9})$, $\mathcal{B}r(\text{case 2-III}) \sim \mathcal{O}(10^{-10})$.

(2) Branching ratios for the $B_c^* \to B_qV$ decays are generally larger than those for the $B_c^* \to B_qP$ decays with the same final B_q meson, where V and P have the same quark components. There are two reasons for this. One is the decay constant relation $f_V > f_P$, and the other is three partial wave contributions to the $B_c^* \to B_qV$ decays rather than only the P-wave contributions to the $B_c^* \to B_qP$ decays.

It should be pointed out that although the P- and D-wave amplitudes for the $B_c^* \to B_qV$ decays are enhanced by large values for the form factors V

TABLE III. The numerical values for input parameters.

Wolfenstein parameters [15]			
$\lambda = 0.22506 \pm 0.00050$,	$A = 0.811 \pm 0.026$,	$\bar{\rho} = 0.124^{+0.019}_{-0.018}$,	$\bar{\eta} = 0.356 \pm 0.011$;
Mass of particles and QCD characteristic scale			
$m_{B_c^*} = 6332 \pm 9$ MeV[a] [1],	$m_{\pi^+} = 139.57$ MeV [15],		$m_{\pi^0} = 134.98$ MeV [15],
$m_{B_u} = 5279.31 \pm 0.15$ MeV [15],	$m_{K^+} = 493.677 \pm 0.016$ MeV [15],		$m_{K^0} = 497.611 \pm 0.013$ MeV [15],
$m_{B_d} = 5279.62 \pm 0.15$ MeV [15],	$m_{\eta} = 547.862 \pm 0.017$ MeV [15],		$m_{\eta'} = 957.78 \pm 0.06$ MeV [15],
$m_{B_s} = 5366.82 \pm 0.22$ MeV [15],	$m_{K^{*+}} = 891.66 \pm 0.26$ MeV [15],		$m_{K^{*0}} = 895.81 \pm 0.19$ MeV [15],
$m_{\rho} = 775.26 \pm 0.25$ MeV [15],	$m_{\omega} = 782.62 \pm 0.12$ MeV [15],		$m_{\phi} = 1019.461 \pm 0.019$ MeV [15],
$m_b = 4.18^{+0.04}_{-0.03}$ GeV [15],	$m_c = 1.27 \pm 0.03$ GeV [15],		$\Lambda_{\mathrm{QCD}}^{(5)} = 210 \pm 14$ MeV [15],
$m_s = 0.51$ GeV [58],	$m_{u,d} = 0.31$ GeV [58],		$\Lambda_{\mathrm{QCD}}^{(4)} = 292 \pm 16$ MeV [15],
Decay constants			
$f_{\pi} = 130.2 \pm 1.7$ MeV [15],	$f_K = 155.6 \pm 0.4$ MeV [15],		$f_{K^*} = 220 \pm 5$ MeV [45],
$f_{\rho} = 216 \pm 3$ MeV [45],	$f_{\omega} = 187 \pm 5$ MeV [45],		$f_{\phi} = 215 \pm 5$ MeV [45],
$f_q = (1.07 \pm 0.02) f_{\pi}$ [53],	$f_s = (1.34 \pm 0.06) f_{\pi}$ [53],		
Gegenbauer moments at the scale of $\mu = 1$ GeV			
$a_1^{\pi} = a_1^{\eta_q} = a_1^{\eta_s} = 0$ [44],	$a_1^K = 0.06 \pm 0.03$ [44],		$a_1^{\rho} = a_1^{\omega} = a_1^{\phi} = 0$ [45],
$a_2^{\pi} = a_2^{\eta_q} = a_2^{\eta_s} = 0.25 \pm 0.15$ [44],	$a_2^K = 0.25 \pm 0.15$ [44],		$a_2^{\rho} = a_2^{\omega} = 0.15 \pm 0.07$ [45],
$a_1^{K^*} = 0.03 \pm 0.02$ [45],	$a_2^{K^*} = 0.11 \pm 0.09$ [45],		$a_2^{\phi} = 0.18 \pm 0.08$ [45].

[a]Other predictions of the B_c^* meson mass with different models can be found in Table II of Ref. [59].

and A_2, they are suppressed by a factor of $\frac{2m_{B_c^*}m_V}{(m_{B_c^*}+m_{B_q})^2}$ relative to the S-wave amplitude, as discussed above. In addition, the P- and D-wave contributions to helicity amplitudes $H_{\pm}$ in Eq. (28) and H_0 in Eq. (29) are future suppressed respectively by factors of $\sqrt{y^2-1}$ and $(y^2-1)/y$ relative to the S-wave contribution. Take the $B_c^* \to B_s \rho$ decay for example, $\frac{2m_{B_c^*}m_{\rho}}{(m_{B_c^*}+m_{B_s})^2} \approx 7\%$, $\sqrt{y^2-1} \approx 0.7$

TABLE IV. Branching ratios for the $B_c^* \to B_q V, B_q P$ decays calculated with the scale of $\mu = m_c$, where the parameters in the "CKM" ("a_i") column give the CKM factors (coefficients) of the decay amplitude; the uncertainties come from mass m_c and m_b.

Final	Parameters			Branching ratio			
state	CKM	a_i	Case	$\omega = 0.4$ GeV	$\omega = 0.6$ GeV	$\omega = m\alpha_s$	Unit
$B_s^0\rho^+$	$V_{cs}^*V_{ud} \sim \mathcal{O}(1)$	a_1	1-I	$6.29^{+0.02}_{-0.02}$	$11.67^{+0.36}_{-0.35}$	$16.77^{+0.72}_{-0.68}$	10^{-7}
$B_s^0\pi^+$	$V_{cs}^*V_{ud} \sim \mathcal{O}(1)$	a_1	1-I	$4.00^{+0.21}_{-0.20}$	$7.26^{+0.15}_{-0.15}$	$9.82^{+0.06}_{-0.06}$	10^{-7}
$B_s^0K^{*+}$	$V_{cs}^*V_{us} \sim \mathcal{O}(\lambda)$	a_1	1-II	$2.21^{+0.02}_{-0.03}$	$4.12^{+0.18}_{-0.17}$	$6.01^{+0.32}_{-0.30}$	10^{-8}
$B_s^0K^+$	$V_{cs}^*V_{us} \sim \mathcal{O}(\lambda)$	a_1	1-II	$1.98^{+0.10}_{-0.10}$	$3.60^{+0.08}_{-0.08}$	$4.87^{+0.03}_{-0.03}$	10^{-8}
$B_d^0\rho^+$	$V_{cd}^*V_{ud} \sim \mathcal{O}(\lambda)$	a_1	1-II	$3.51^{+0.05}_{-0.05}$	$7.51^{+0.17}_{-0.17}$	$11.47^{+0.44}_{-0.41}$	10^{-8}
$B_d^0\pi^+$	$V_{cd}^*V_{ud} \sim \mathcal{O}(\lambda)$	a_1	1-II	$2.14^{+0.13}_{-0.13}$	$4.50^{+0.11}_{-0.11}$	$6.53^{+0.05}_{-0.05}$	10^{-8}
$B_d^0K^{*+}$	$V_{cd}^*V_{us} \sim \mathcal{O}(\lambda^2)$	a_1	1-III	$1.45^{+0.01}_{-0.01}$	$3.13^{+0.11}_{-0.11}$	$4.84^{+0.24}_{-0.23}$	10^{-9}
$B_d^0K^+$	$V_{cd}^*V_{us} \sim \mathcal{O}(\lambda^2)$	a_1	1-III	$1.15^{+0.07}_{-0.07}$	$2.41^{+0.06}_{-0.06}$	$3.50^{+0.02}_{-0.02}$	10^{-9}
$B_u^+\bar{K}^{*0}$	$V_{cs}^*V_{ud} \sim \mathcal{O}(1)$	a_2	2-I	$5.59^{+0.03}_{-0.03}$	$12.08^{+0.39}_{-0.36}$	$18.80^{+0.85}_{-0.80}$	10^{-8}
$B_u^+\bar{K}^0$	$V_{cs}^*V_{ud} \sim \mathcal{O}(1)$	a_2	2-I	$3.48^{+0.29}_{-0.27}$	$7.32^{+0.34}_{-0.32}$	$10.60^{+0.31}_{-0.30}$	10^{-8}
$B_u^+K^{*0}$	$V_{cd}^*V_{us} \sim \mathcal{O}(\lambda^2)$	a_2	2-III	$1.59^{+0.01}_{-0.01}$	$3.44^{+0.11}_{-0.10}$	$5.34^{+0.24}_{-0.23}$	10^{-10}
$B_u^+K^0$	$V_{cd}^*V_{us} \sim \mathcal{O}(\lambda^2)$	a_2	2-III	$9.82^{+0.81}_{-0.77}$	$20.67^{+0.94}_{-0.90}$	$29.95^{+0.85}_{-0.82}$	10^{-11}
$B_u^+\rho^0$	$V_{cd}^*V_{ud} \sim \mathcal{O}(\lambda)$	a_2	2-II	$1.84^{+0.04}_{-0.03}$	$3.95^{+0.07}_{-0.07}$	$6.08^{+0.20}_{-0.19}$	10^{-9}
$B_u^+\omega$	$V_{cd}^*V_{ud} \sim \mathcal{O}(\lambda)$	a_2	2-II	$1.36^{+0.02}_{-0.02}$	$2.93^{+0.06}_{-0.05}$	$4.51^{+0.15}_{-0.14}$	10^{-9}
$B_u^+\phi$	$V_{cs}^*V_{us} \sim \mathcal{O}(\lambda)$	a_2	2-II	$1.36^{+0.01}_{-0.01}$	$2.95^{+0.13}_{-0.12}$	$4.64^{+0.26}_{-0.25}$	10^{-9}
$B_u^+\pi^0$	$V_{cd}^*V_{ud} \sim \mathcal{O}(\lambda)$	a_2	2-II	$9.24^{+0.77}_{-0.72}$	$19.44^{+0.89}_{-0.85}$	$28.17^{+0.81}_{-0.78}$	10^{-10}
$B_u^+\eta$	$V_{cd}^*V_{ud}$, $V_{cs}^*V_{us}$	a_2	2-II	$2.37^{+0.20}_{-0.19}$	$4.98^{+0.23}_{-0.22}$	$7.22^{+0.21}_{-0.20}$	10^{-9}
$B_u^+\eta'$	$V_{cd}^*V_{ud}$, $V_{cs}^*V_{us}$	a_2		$6.62^{+0.55}_{-0.52}$	$13.93^{+0.64}_{-0.61}$	$20.18^{+0.58}_{-0.56}$	10^{-11}

and $(y^2-1)/y \approx 0.4$, resulting in the polarization fractions $f_0 \approx 60\%$, $f_+ \approx 30\%$, and $f_- \approx 10\%$ with $f_{0,+,-} \equiv \frac{|H_{0,+,-}|^2}{|H_0|^2+|H_+|^2+|H_-|^2}$.

(3) The branching ratios for the $B_c^* \to B_s\rho$, $B_s\pi$ decays can reach up to $\mathcal{O}(10^{-7})$. With the estimated production cross section of the B_c^* meson $\sim 30nb$ at LHC [14], it is expected to have more than 10^{10} B_c^* mesons per ab^{-1} data at LHC, corresponding to more than 10^3 events of the $B_c^* \to B_s\rho$, $B_s\pi$ decays. Therefore, even with the identification efficiency, the $B_c^* \to B_s\rho$, $B_s\pi$ decays might be measurable in the future.

(4) Branching ratios for the $B_c^* \to B_qV$, B_qP decays are several orders of magnitude smaller, especially for the a_1 dominant decays, than those for the $B_c \to B_qP$, B_qV decays [8]. This fact might imply that possible background from the $B_c^* \to BV$, BP decays could be safely neglected for an analysis of the $B_c \to B_qP$, B_qV decays, but not vice versa, i.e., one of main pollution for the $B_c^* \to B_qV$, B_qP decays would likely come from the B_c decays.

(5) It is seen clearly that the numbers in Table IV are very sensitive to the choice of the parameter ω. In addition, with a different value for $\Gamma_{B_c^*}$, branching ratios in Table IV should be multiplied by a factor of $50\ \mathrm{eV}/\Gamma_{B_c^*}$. Of course, many factors, such as the choice of scale μ, higher order corrections to HME, q^2-dependence of form factors, final state interactions, etc., are not carefully considered in detail here, but have effects on the estimation and deserve more dedicated study in the future.

IV. SUMMARY

With the running and upgrading of the LHC, there are certainly huge amounts of the B_c^* mesons. This would provide us with a possibility of searching for the B_c^* weak decays in the future. In this paper, the $B_c^* \to B_qV$, B_qP decays ($q = u$, d and s), induced by the charm quark weak decay, are studied phenomenologically with the QCDF approach. The form factors for the $B_c^* \to B$ transitions are calculated using the Wirbel-Stech-Bauer model. The nonfactorizable contributions from the vertex radiative corrections are considered at the order of α_s. It is found that (1) form factors and branching ratios are sensitive to models of wave functions; (2) the color-favored and CKM-allowed $B_c^* \to B_s\rho$, $B_s\pi$ decays have large branching ratios of $\mathcal{O}(10^{-7})$, and might be accessible in the future LHC experiments.

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