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# Effect of magnetic field on dilepton production in a hot plasma

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Noncentral collision of heavy ions can generate a large magnetic field in their neighborhood. We describe a method to calculate the effect of this field on the dilepton emission rate from the colliding region, when it reaches thermal equilibrium. It is calculated in the real time method of the thermal field theory. We find that the rate is affected significantly only for lower momenta of dileptons.

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## I. INTRODUCTION

An important probe into the dynamics of heavy ion collisions is the detection of dilepton production in the process. Accordingly, this topic has been investigated in detail [1–6]. Here we study the change in the production rate due to the magnetic field, which is produced in noncentral collision of individual events [7–15].

According to the present understanding, the two nuclei colliding at ultrarelativistic energies appear as two sheets of color glass condensate [16]. Very shortly after collision a strongly interacting quark gluon system, called glasma, is formed, which is out of thermal equilibrium [17–20]. After thermalization, it gives rise to the quark gluon plasma (QGP) phase. Finally, it evolves into a hadron gas. Dileptons are produced in all these phases. In this work, we address this production in the QGP phase.

The effect of magnetic field in different processes arises through the altered propagation of particles in this field. A nonperturbative, gauge covariant expression for the Dirac propagator in an external electromagnetic field was derived long ago by Schwinger in an elegant way, using a proper-time parameter [21]. It has since been rederived and applied to many processes [22–29]. In the present work, we expand the exact propagator in powers of the magnetic field.

The dilepton production rate is given in terms of the (imaginary part of the) thermal two-point current correlation function. The latter involves the thermal propagator for quarks in the magnetic field. In contrast to the often-used imaginary time formulation of thermal field theory [30–33], we shall use the real time formulation [34–36]. The advantage is that we do not have frequency sums for the propagators, but this is at the cost of dealing with $2\times 2$ matrices for them in the intermediate stage of calculation. The matrices admit spectral representations, just like the vacuum propagators, which we shall use in calculating the thermal correlation function.

In Sec. II, we write the dilepton rate formula and describe our method to evaluate it. In Sec. III, we outline Schwinger's construction of the spinor propagator in magnetic field, leading to its spectral representation. In Sec. IV, we then calculate the thermal two-point correlation function of currents, present in the rate formula. Finally, Sec. V contains the numerical results and discussion.

## II. FORMULATION

The transition amplitude (Fig. 1) from an initial state $I$, composed of quarks and gluons to a final state $F$ of similar composition, along with the emission of a dilepton $l(p,\sigma)$ and $\bar{l}(p',\sigma')$ of momenta $p$ and $p'$ and the $z$ component of spin $\sigma$ and $\sigma'$ is

$$\langle F, l(p,\sigma), \bar{l}(p',\sigma')|S|I\rangle. \tag{2.1}$$

Here the scattering matrix operator $S$ is given by the interaction Lagrangian,

$$\mathcal{L}_{\rm int} = -e(j^\mu(x) + J^\mu(x))A_\mu(x), \tag{2.2}$$

of lepton and quark currents,

$$\begin{aligned} j^\mu(x) &= \bar{\psi}(x)\gamma^\mu\psi(x), \\ J^\mu(x) &= \frac{2}{3}\bar{u}(x)\gamma^\mu u(x) - \frac{1}{3}\bar{d}(x)\gamma^\mu d(x), \end{aligned} \tag{2.3}$$

coupled to the electromagnetic field $A_\mu(x)$. We assume the initial state to be thermal and look for inclusive probability. Then if $N$ is the dilepton emission rate per unit volume, we get, after some calculation [36],

$$\frac{d^4N}{d^4q} = \frac{\alpha^2}{6\pi^3 q^2} e^{-\beta q_0}(-g^{\mu\nu}M^+_{\mu\nu}), \tag{2.4}$$

where $q = p + p'$ is the dilepton momentum and $M^+_{\mu\nu}(q)$ is a thermal two-point function of the quark current,

$$M^+_{\mu\nu}(q) = \int d^4x e^{iq\cdot x}\langle J_\mu(x)J_\nu(0)\rangle. \tag{2.5}$$

Here the symbol $\langle O\rangle$ stands for the ensemble average of the operator $O$ at temperature $1/\beta$,

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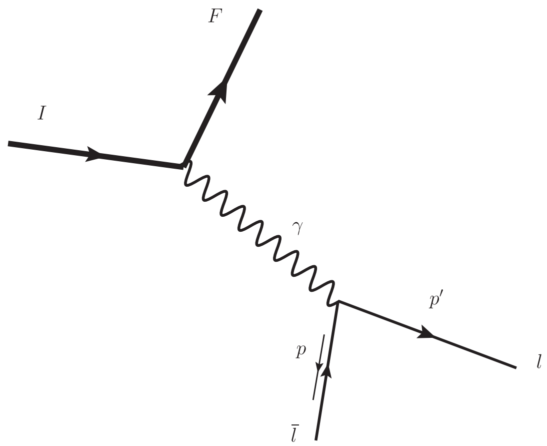


FIG. 1. Dilepton production amplitude in QGP phase. The states I and F consist of quarks and gluons, while $l\bar{l}$ is a dilepton. The weavy line corresponds to a photon.

$$\langle O\rangle = \mathrm{Tr}(e^{-\beta H}O)/\mathrm{Tr}e^{-\beta H}. \tag{2.6}$$

We briefly review how $M^{+}_{\mu\nu}(q)$ may be obtained in the real time thermal field theory [36]. We start with the time contour of Fig. 2 and define the time-ordered two-point function $M_{\mu\nu}(x,x')$ as

$$\begin{aligned} M_{\mu\nu}(x,x') = {} & \Theta_c(\tau-\tau')i\langle J_\mu(x)J_\nu(x')\rangle \\ & + \Theta_c(\tau'-\tau)i\langle J_\nu(x')J_\mu(x)\rangle, \end{aligned} \tag{2.7}$$

where $x=(\tau,\vec{x}), x'=(\tau',\vec{x}')$ with the "times" $\tau$ and $\tau'$ on the contour shown in Fig 2. The subscript $c$ on the $\Theta$ functions refers to contour ordering. Beginning with the spatial Fourier transform, one can show that the vertical segments of the time contour does not contribute. Then the two-point function may be put in the form of a $2\times 2$ matrix, which can be diagonalized with essentially one diagonal element,

$$\bar{M}_{\mu\nu}(q) = \int_{-\infty}^{+\infty}\frac{dq_0'}{2\pi}\frac{\rho_{\mu\nu}(q_0',\vec{q})}{q_0'-q_0-i\eta\epsilon(q_0)}, \tag{2.8}$$

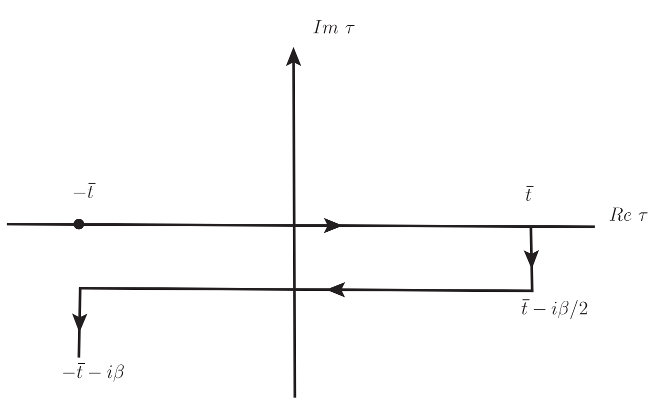


FIG. 2. The time contour in the complex $\tau$ plane with $\bar{t}\to\infty$.

where $\rho_{\mu\nu}(q)$ is the spectral function,

$$\rho_{\mu\nu}(q) = \int d^4x e^{iq\cdot x}\langle[J_\mu(x),J_\nu(0)]\rangle \equiv M^+(q)-M^-(q), \tag{2.9}$$

and Eq. (2.8) gives us

$$\rho_{\mu\nu}(q) = 2\mathrm{Im}\bar{M}_{\mu\nu}(q). \tag{2.10}$$

From the cyclicity of the thermal trace, we get the Kubo-Martin-Schwinger relation:

$$M^+(q) = e^{\beta q_0}M^-(q) \tag{2.11}$$

From Eqs. (2.9)–(2.11), we get

$$M^+(q) = \frac{2e^{\beta q_0}}{e^{\beta q_0}-1}\mathrm{Im}\bar{M}_{\mu\nu}(q), \tag{2.12}$$

giving the dilepton rate (2.4) as

$$\frac{d^4N}{d^4q} = \frac{\alpha^2}{3\pi^3q^2}\frac{W}{e^{\beta q_0}-1}, \qquad W = -g^{\mu\nu}\mathrm{Im}\bar{M}_{\mu\nu}. \tag{2.13}$$

Using the matrix which diagonalizes the $2\times 2$ correlation matrix, we can relate the imaginary part of any one component, say the 11, of the correlation function to that of its diagonal element,

$$\mathrm{Im}\bar{M}_{\mu\nu} = \epsilon(q_0)\tanh(\beta q_0)\mathrm{Im}(M_{\mu\nu})_{11}. \tag{2.14}$$

So far we utilize general properties of two-point functions to relate the problem to $(M_{\mu\nu})_{11}$. Taking $\tau$ and $\tau'$ on the real axis, the contour form (2.7) gives it as

$$M_{\mu\nu}(x,x')_{11} = i\langle TJ_\mu(\vec{x},t)J_\nu(\vec{x}',t')\rangle, \tag{2.15}$$

where $T$, as usual, time orders the operators. It is this quantity which we have to calculate. To leading order in strong interactions, it involves only the thermal quark propagator. The magnetic field enters the problem through this propagator, which we find in the next section.

## III. DIRAC PROPAGATOR IN MAGNETIC FIELD

In deriving the quark propagator, we assume both $u$ and $d$ quarks to have the same absolute electric charge as that of the lepton. (The necessary correction will be included in our formulas at the end of Sec. IV). The Dirac Lagrangian in an external electromagnetic field,

$$\mathcal{L} = \bar{\psi}[i\gamma^\mu(\partial_\mu+ieA_\mu)-m]\psi, \tag{3.1}$$

gives the equation of motion:

$$[i\gamma^\mu(\partial_\mu + ieA_\mu) - m]\psi = 0. \tag{3.2}$$

Then the propagator,

$$S(x,x') = i\langle 0|T\psi(x)\bar\psi(x')|0\rangle, \tag{3.3}$$

satisfies

$$[i\gamma^\mu(\partial_\mu + ieA_\mu) - m]S(x,x') = -\delta^4(x-x'). \tag{3.4}$$

Here $|0\rangle$ is the vacuum state of the Dirac field (in presence of $A_\mu$). Defining states labeled by the spacetime coordinate (suppressing spinor indices), we regard $S(x,x')$ as the matrix element of an operator $S$:

$$S(x,x') = \langle x|S|x'\rangle. \tag{3.5}$$

Then Eq. (3.4) can be written as

$$(\gamma^\mu\pi_\mu - m)S = -1, \qquad \pi_\mu = p_\mu - eA_\mu, \qquad p_\mu = i\partial_\mu, \tag{3.6}$$

which has the formal solution,[1]

$$S = \frac{1}{-\not\pi + m} = (\not\pi + m)\frac{1}{-\not\pi^2 + m^2}. \tag{3.7}$$

Schwinger relates these quantities to the dynamical properties of a "particle" with coordinate $x^\mu$ and canonical and kinematical momenta $p^\mu$ and $\pi^\mu$, respectively. Using their commutation relations, we get $\not\pi^2 = \pi^2 - \frac{e}{2}\sigma F$. Defining $H = -\pi^2 + m^2 + \frac{e}{2}\sigma F$, we can write Eq. (3.7) as

$$S = (\not\pi + m)i\int_0^\infty ds U(s), \qquad U = e^{-iHs}. \tag{3.8}$$

As the notation suggests, $U(s)$ may be regarded as the evolution operator of the particle with Hamiltonian $H$ in time $s$.

We now go to the Heisenberg representation, where the operators $x_\mu$ and $\pi_\mu$ as well as the base ket become time dependent,

$$x_\mu(s) = U^\dagger(s)x_\mu U(s), \qquad \pi_\mu(s) = U^\dagger(s)\pi_\mu U(s),$$
$$|x';s\rangle = U^\dagger(s)|x';0\rangle \tag{3.9}$$

Then the construction of the propagator reduces to the evaluation of

$$\langle x''|U(s)|x'\rangle = \langle x'';s|x';0\rangle, \tag{3.10}$$

which is the transformation function for a state in which the operator $x_\mu(s=0)$ has the value of $x'_\mu$ to a state in which $x_\mu(s)$ has the value $x''_\mu$.

The equation of motion for $x_\mu(s)$ and $\pi_\mu(s)$ following from Eq. (3.9) can be solved to get

$$\pi(s) = -\frac{1}{2}eFe^{-eFs}\sinh^{-1}(eFs)(x(s) - x(0)), \tag{3.11}$$

which may also be put in the reverse order on using the antisymmetry of $F_{\mu\nu}$. The matrix element $\langle x'';s|\pi^2(s)|x';0\rangle$ can now be obtained by using the commutator $[x_\mu(s), x_\nu(0)]$ to reorder the operators $x_\mu(0)$ and $x_\nu(s)$. We then get

$$\langle x'';s|H(x(s),\pi(s))|x';0\rangle = f(x'';x';s)\langle x'';s|x';0\rangle, \tag{3.12}$$

where

$$f = (x''-x')K(x''-x') - \frac{i}{2}\mathrm{tr}[eF\coth(eFs)] - m^2 - \frac{e}{2}\sigma F,$$
$$K = \frac{(eF)^2}{4}\sinh^{-2}(eFs). \tag{3.13}$$

We are now in a position to find the transformation function, which from Eq. (3.10) is found to satisfy

$$i\frac{d}{ds}\langle x'';s|x';0\rangle = \langle x'';s|H|x';s\rangle. \tag{3.14}$$

It can be solved as

$$\langle x'';s|x';0\rangle = \phi(x'',x').\frac{i}{(4\pi)^2 s^2}e^{-L(s)}$$
$$\times \exp\left(-\frac{i}{4}(x''-x')eF\coth(eFs)(x''-x')\right)$$
$$\times \exp\left(-i\left(m^2 + \frac{1}{2}e\sigma F\right)\right), \tag{3.15}$$

where

$$L(s) = \frac{1}{2}\mathrm{tr}\ln[(eFs)^{-1}\sinh(eFs)]. \tag{3.16}$$

Here $\phi(x'',x')$ is a phase factor involving an integral over the potential $A_\mu$ on a straight line connecting $x'$ and $x''$. It will cancel out in our calculation. The spinor propagator is now given by

$$S(x'',x') = i\int_0^\infty ds\langle x''|(\not\pi + m)U(s)|x'\rangle$$
$$= i\int_0^\infty ds[\gamma^\mu\langle x'';s|\pi_\mu(s)|x';0\rangle + m\langle x'';s|x';0\rangle] \tag{3.17}$$

with $\pi_\mu(s)$ and $\langle x'';s|x';0\rangle$ given by Eqs. (3.11) and (3.15).

[1] Another equivalent form follows by writing $(\not\pi + m)$ on the right in Eq. (3.7) [21], but we shall not use it.

We now specialize the external electromagnetic field to magnetic field $B$ in the $z$ direction, $F^{12} = -F^{21} = B$. It is convenient to diagonalize the antisymmetric $2 \times 2$ matrix $F^{ij}$ with eigenvalues $\pm iB$. Going over to the spatial metric, we get[2]

$$S(x) = \frac{i}{(4\pi)^2} \int \frac{ds}{s} \frac{eB}{\sin(eBs)} \exp\left[\frac{i}{4} x_\perp^2 eB \cot(eBs) - \frac{i}{4s^2} x_\parallel^2 - i\left(m^2 + \frac{1}{2} e\sigma F\right) s\right] \times \left[\left(\frac{1}{2s}(x\cdot\gamma)_\parallel + m\right)(\cos(eBs) - \gamma^1\gamma^2 \sin(eBs)) - \frac{eB}{2\sin(eBs)}(x\cdot\gamma)_\perp\right], \tag{3.18}$$

which can be Fourier transformed to

$$S(p) = i\int_0^\infty ds e^{is(p^2-m^2+i\epsilon)} e^{-isp_\perp^2(\frac{\tan(eBs)}{eBs}-1)} \times [(\not{p}_\parallel + m)(1 - \gamma^1\gamma^2 \tan(eBs)) - \not{p}_\perp(1 + \tan^2(eBs))]. \tag{3.19}$$

Expanding the exponential and tangent functions, we immediately get $S(p)$ as a series in powers of $eB$. To order $(eB)^2$, it is

$$S(p) = \frac{-(\not{p} + m)}{p^2 - m^2 + i\eta} + eB \frac{i(\not{p}_\parallel + m)\gamma^1\gamma^2}{(p^2 - m^2)^2} - (eB)^2 \left[\frac{2\not{p}_\perp}{(p^2 - m^2)^3} - \frac{2p_\perp^2(\not{p} + m)}{(p^2 - m^2)^4}\right]. \tag{3.20}$$

To put the propagator (3.20) in the form of a spectral representation, we introduce a variable mass $m_1$ to replace $1/(p^2 - m^2)$ by $1/(p^2 - m_1^2)$, keeping the physical mass $m$ unaltered at other places. The higher powers of the scalar propagator can then be expressed as derivatives of the propagator with respect to $m_1^2$. We, thus, get

$$S(p) = -F(p, m, m_1) \frac{1}{p^2 - m_1^2}\bigg|_{m_1=m}, \tag{3.21}$$

where

$$F = (\not{p} + m) + ai(\not{p}_\parallel + m)\gamma^1\gamma^2 + b\not{p}_\perp + c p_\perp^2(\not{p} + m), \tag{3.22}$$

with coefficients $a$, $b$, and $c$ carrying the derivative operators,

$$a = -eB \frac{\partial}{\partial m_1^2}; \qquad b = (eB)^2 \frac{\partial^2}{\partial (m_1^2)^2};$$
$$c = -\frac{1}{3}(eB)^2 \frac{\partial^3}{\partial (m_1^2)^3}. \tag{3.23}$$

From Eq. (3.21), the spectral function for $S(p)$ will be recognized as[3]

$$\sigma(p) = F(p, m, m_1)\rho(p, m_1)|_{m_1=m}, \tag{3.24}$$

with $\rho$ being the spectral function for the scalar propagator of mass $m_1$:

$$\rho(p, m_1) = 2\pi\epsilon(p_0)\delta(p^2 - m_1^2). \tag{3.25}$$

The desired spectral representation for the spinor propagator in vacuum (in presence of magnetic field) can be written in the form

$$S(p) = \int_{-\infty}^{+\infty} \frac{dp_0'}{2\pi} \frac{\sigma(p_0', \vec{p})}{p_0' - p_0 - i\eta\epsilon(p_0)}, \tag{3.26}$$

as can be readily verified by doing the $p_0'$ integral.

## IV. THERMAL CURRENT CORRELATION FUNCTION

Like the thermal correlation function of currents, the thermal quark propagator can also be analyzed in the same way. In particular, its $2 \times 2$ matrix form can be diagonalized, again with essentially a single diagonal element, which turns out to be the vacuum propagator (in magnetic field) derived above. But in our calculation below, we need the 11-element of the original matrix, which is conveniently written as [36]

$$S_{11}(p) = \int_{-\infty}^{+\infty} \frac{dp_0'}{2\pi} \sigma(p_0', \vec{p}) \left\{\frac{1 - \tilde{f}(p_0')}{p_0' - p_0 - i\eta} + \frac{\tilde{f}(p_0')}{p_0' - p_0 + i\eta}\right\},$$
$$\tilde{f}(p_0) = \frac{1}{e^{\beta p_0} + 1}, \tag{4.1}$$

where the spectral function $\sigma$ is given by (3.24).

[2] For any two vectors $a^\mu$ and $b^\mu$, we write $(ab)_\parallel = a^0b^0 - a^3b^3$ and $(ab)_\perp = a^1b^1 + a^2b^2$. Note that the longitudinal and transverse directions are defined with respect to the direction of the magnetic field, not the collision axis of ions.

[3] We have three different spectral functions in this problem. The $\rho_{\mu\nu}$ introduced in Sec. II is the spectral function of the current correlation function, while $\rho$ and $\sigma$ are spectral functions for the scalar and Dirac propagators.

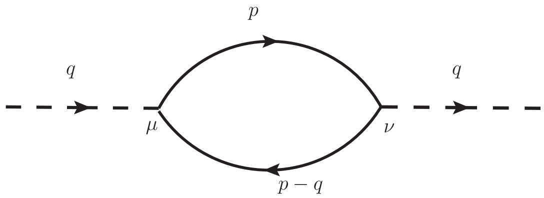


FIG. 3. Current correlation function to one loop. The dashed and solid lines represent currents and quarks.

The graph of Fig. 3 gives two terms involving the $u$ and $d$ quark propagators. Assuming these to be equal (which is true only for $eB = 0$),[4] we combine them to give

$$(M_{\mu\nu}(q))_{11} = \frac{5i}{3}\int \frac{d^4p}{(2\pi)^4}\mathrm{tr}[S_{11}(p)\gamma_\nu S_{11}(p-q)\gamma_\mu], \tag{4.2}$$

where the prefactor includes a factor of 3 for the color of the quarks. Inserting the propagator (4.1) in it, we want to work out the $p_0$ integral. For this purpose, we write it as

$$M_{\mu\nu}(q)_{11} = \int\frac{d^3p}{(2\pi)^3}\int\frac{dp_0'}{2\pi}\rho(p_0',\vec{p}) \times\int\frac{dp_0''}{2\pi}\rho(p_0'',\vec{p}-\vec{q})K_{\mu\nu}(q). \tag{4.3}$$

where

$$K_{\mu\nu}(q) = i\int_{-\infty}^{+\infty}\frac{dp_0}{2\pi}N_{\mu\nu}(q) \times\left(\frac{1-\tilde{f}'}{p_0'-p_0-i\eta}+\frac{\tilde{f}'}{p_0'-p_0+i\eta}\right) \times\left(\frac{1-\tilde{f}''}{p_0''-(p_0-q_0)-i\eta}+\frac{\tilde{f}''}{p_0''-(p_0-q_0)+i\eta}\right) \tag{4.4}$$

with $\tilde{f}' = \tilde{f}(p_0')$, $\tilde{f}'' = \tilde{f}(p_0'')$ and

$$N_{\mu\nu}(q) = \frac{5}{3}\mathrm{tr}\{\overleftarrow{F}(p,m,m_1)\gamma_\nu\overleftarrow{F}(p-q,m,m_2)\gamma_\mu\}. \tag{4.5}$$

Here, the masses $m_1$ and $m_2$ are variables on which the mass derivatives act in the two propagators. The left arrow on $F$ indicates the derivatives in it to be put farthest to the left (outside the integrals). As we are interested in the imaginary part of $K_{\mu\nu}$, we can put $p_0 = p_0'$, $p_0 - q_0 = p_0''$ and bring $N_{\mu\nu}$ outside the $p_0$ integral. Then it is simple to evaluate $K_{\mu\nu}$, from which we get its imaginary part. Extracting a factor $\coth(\beta q_0)$, it becomes linear in $\tilde{f}'$ and $\tilde{f}''$,

$$\mathrm{Im}K_{\mu\nu}(q) = N_{\mu\nu}(p_0',p_0'')\pi(\tilde{f}''-\tilde{f}') \times\coth(\beta q_0)\delta(p_0''-p_0'+q_0). \tag{4.6}$$

[The hyperbolic function will cancel out in (2.14)]. Next, the $p_0'$ and $p_0''$ integrals in Eq. (4.3) can be removed, using the delta functions present in the spectral functions, namely $\delta(p_0'\pm\omega_1)$ and $\delta(p_0''\pm\omega_2)$ with $\omega_1 = \sqrt{|\vec{p}|^2+m_1^2}$ and $\omega_2 = \sqrt{(\vec{p}-\vec{q})^2+m_2^2}$. We need only the imaginary part in the physical region, $q_0 > (\omega_1+\omega_2)$. From (2.14), (4.3), and (4.6), we then get

$$W = \pi\int\frac{d^3p}{(2\pi)^3}\frac{N^\mu_\mu(\omega_1,-\omega_2)}{4\omega_1\omega_2}\{1-\tilde{n}(\omega_1)-\tilde{n}(\omega_2)\} \times\delta(q_0-\omega_1-\omega_2), \tag{4.7}$$

where we convert $\tilde{f}$'s to distribution functions, $\tilde{n}(\omega) = 1/(e^{\beta\omega}+1)$.

Working out the trace over $\gamma$ matrices in $N^\mu_\mu$, we get

$$N^\mu_\mu = -\frac{40}{3}[(1-a_1a_2)p\cdot(p-q) - (b_1+b_2+a_1a_2)[p\cdot(p-q)]_\perp + p\cdot(p-q)\{c_1p_\perp^2+c_2(p-q)_\perp^2\}]. \tag{4.8}$$

Let us now consider collision events in which the transverse components of momenta are small compared to the longitudinal ones, when we can omit the last two terms and calculate the dilepton rate analytically. Neglecting quark mass, we thus get

$$W = \frac{20\pi}{3}q^2(1-a_1a_2)J, \tag{4.9}$$

where

$$J = \int\frac{d^3p}{(2\pi)^3 4\omega_1\omega_2}\{1-\tilde{n}(\omega_1)-\tilde{n}(\omega_2)\}\delta(q_0-\omega_1-\omega_2). \tag{4.10}$$

After working out this integral analytically, we shall apply the mass derivatives contained in $a_1$ and $a_2$.

If $\theta$ is the angle between $\vec{q}$ and $\vec{p}$, we can carry out the $\theta$ integral by the delta function in Eq. (4.10). However, a constraint remains to ensure that $\cos\theta$ remains in the physical region, as we integrate over the angle. We get

$$J = \frac{1}{16\pi^2|\vec{q}|}\int d\omega_1\Theta(1-|\cos\theta|)\{1-\tilde{n}(\omega_1)-\tilde{n}(\omega_2)\}. \tag{4.11}$$

[4]For $eB \neq 0$, we include the necessary correction at the end of this section.

The $\Theta$-function constraint gives a quadratic expression in $\omega_1$,

$$(\omega_1-\omega_+)(\omega_1-\omega_-)\leq 0, \tag{4.12}$$

where

$$\omega_\pm=\frac{q_0R\pm|\vec{q}|\sqrt{R^2-4q^2m_1^2}}{2q^2}, \qquad R=q^2+m_1^2-m_2^2. \tag{4.13}$$

With the corresponding limits on $\omega_1$, we get [37]

$$\begin{aligned}J&=\frac{1}{16\pi^2|\vec{q}|}\int_{\omega_-}^{\omega_+}d\omega_1\left(1-\frac{1}{e^{\beta\omega_1}+1}-\frac{1}{e^{\beta(q_0-\omega_1)}+1}\right)\\&=\frac{1}{16\pi^2|\vec{q}|\beta}\left[\ln\left(\frac{\cosh(\beta\omega_+/2)}{\cosh(\beta\omega_-/2)}\right)\right.\\&\quad\left.-\ln\left(\frac{\cosh(\beta(q_0-\omega_+)/2)}{\cosh(\beta(q_0-\omega_-)/2)}\right)\right].\end{aligned} \tag{4.14}$$

We now recall that the $u$ and $d$ quark charges were included correctly only in the currents but not in the propagators. The resulting correction will effect only $e^2$, contained in $a_1$, $a_2$ in the expression for $W$. We can readily find that we need to multiply $e^2$ by $17/45$ to restore the actual charges of the quarks in their propagators. Carrying out the mass derivatives in Eq. (4.9) and going to the limit of zero quark masses, we finally get

$$W=\frac{5q^2}{12\pi|\vec{q}|\beta}\left[2\ln\left(\frac{\cosh\alpha_+}{\cosh\alpha_-}\right)-\frac{17}{45}(eB)^2\mathcal{M}\right], \tag{4.15}$$

where $\mathcal{M}$ gives the effect of the magnetic field to the leading-order result,

$$\mathcal{M}=\frac{\beta^2}{8q^2}(\mathrm{sech}^2\alpha_+-\mathrm{sech}^2\alpha_-)+\frac{\beta|\vec{q}|}{q^4}(\tanh\alpha_++\tanh\alpha_-). \tag{4.16}$$

Here we use the abbreviation $\alpha_\pm=\beta(q_0\pm|\vec{q}|)/4$. Note that $W$ is finite as $|\vec{q}|\to 0$.

## V. NUMERICAL RESULTS AND DISCUSSION

Some earlier works estimate the magnetic contribution to the dilepton rate in the QGP phase in heavy ion collisions. Reference [11] uses the Weizsäcker-Williams equivalent photon approximation, in which the two vertices of Fig. 1 become independent amplitudes involving the photon, whose probabilities are calculated in the magnetic field. In Ref. [14], the quark propagator is calculated using the method of eigenfunction expansion [22]. Here the anisotropy induced by the (constant) direction of the magnetic field is investigated in detail. In Ref. [15], the result for the very high magnetic field is reported, taking the lowest Landau level into account.

Here we propose a different method to include the effect of the magnetic field on the dilepton production rate. Assuming thermal equilibrium in the *QGP* phase, there results the correlation function of quark currents. This is evaluated with the quark propagator in the magnetic field after expanding it up to $(eB)^2$. The calculation is carried out in the real time method of thermal field theory.

The plots of $\mathcal{M}$, the coefficient of $(eB)^2$ in $W$, as functions of the invariant dilepton mass $m_{l\bar{l}}=\sqrt{q^2}$ and temperature $T$ are shown in Fig. 4 for typical values of parameters. If the second-order term in (4.16) provides any indication of the behavior of the series, the expansion parameters are $eB/q^2$ and $eB/T^2$. In Fig. 5, we plot $W/W_{B=0}$ as a function of $m_{l\bar{l}}$ for a few values of $eB$.

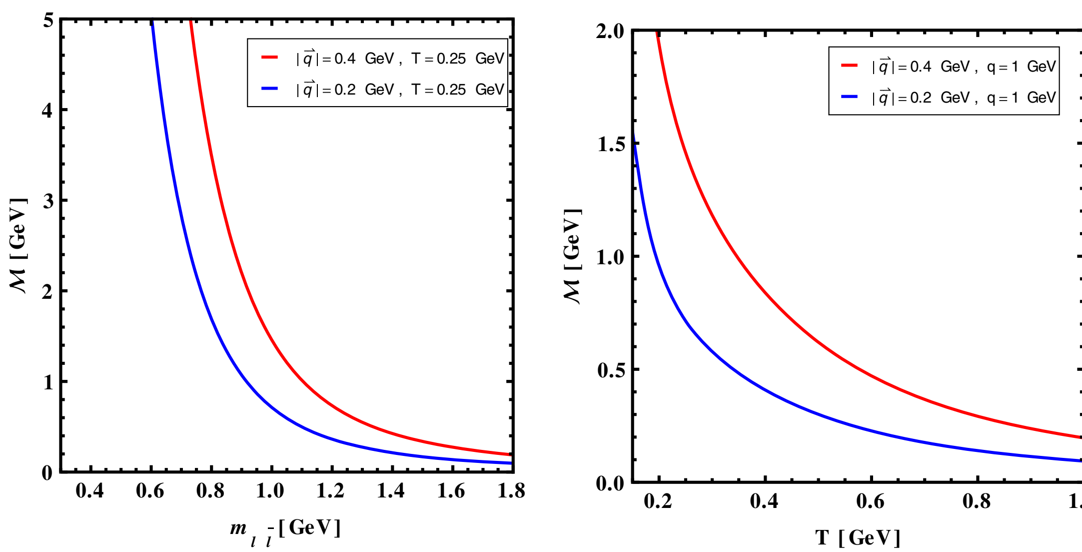


FIG. 4. Variation of $\mathcal{M}$ as a function of invariant mass $m_{l\bar{l}}$ (left) and temperature $T$ (right).

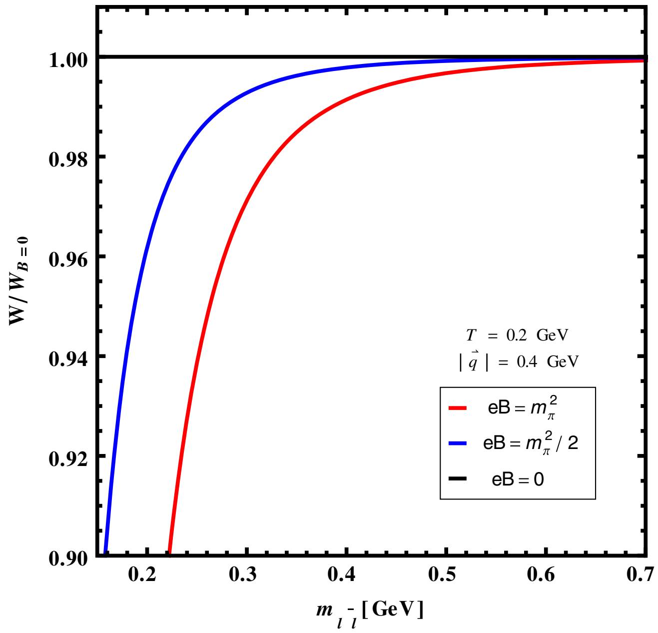


FIG. 5. Plot of the ratio of the dilepton rate with and without the presence of the magnetic field as a function of $m_{l\bar{l}}$.

Figure 5 shows that the magnetic field changes the dilepton rate only at lower $q^2$, reflecting the behavior of the first and second term of $\mathcal{M}$ [Eq. (4.16)] as $1/q^2$ and $1/q^4$ at low $q^2$. We also note that earlier theoretical calculations without magnetic field disagree with experiment at low $q^2$ [38–40]. It would, therefore, be tempting to speculate if the effect of the magnetic field can bring the agreement, at least in part. However, to verify this speculation, we have to improve our calculation in a number of ways. First, we should include the terms in (IV.9) that are left out in our calculation. Then we need to replace the constant magnetic field by one with its magnitude having (adiabatic) time dependence, as realized in noncentral collisions. One can also include the first-order QCD correction to the current correlation function [41].

The time dependence of the magnetic field mentioned above has to be included in the spacetime evolution of dilepton production, which is needed to determine its spectrum. Without going into the details of this evolution, we may estimate roughly the effect of the time dependence as follows. The magnetic field realized in the core may be approximated as [7,10,11]

$$eB(t) = \frac{8\alpha}{\gamma}\frac{Z}{t^2 + (2R/\gamma)^2}, \tag{5.1}$$

where $\alpha$ is the fine structure constant ($= 1/137$), $Z$ and $R$ are the atomic number and radius of the colliding nuclei, and $\gamma$ is the Lorentz contraction factor. This expression excludes large magnetic fields generated immediately after collisions, so it may represent the magnetic field during the QGP phase. Consider Au-Au collision at RHIC, for which $Z=79$, $R=6.5$ fm, and $\gamma=100$, giving $eB(t=0)=m_\pi^2/15$. However, considering Pb-Pb collision at LHC, where $Z=82$, $R=7.1$ fm and $\gamma=2800$, we get $eB(t=0)=1.4m_\pi^2$. In conclusion, we find that the effect of magnetic field in the dilepton spectrum is confined to low invariant masses.


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