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Twist-4 contributions to semi-inclusive deeply inelastic scatterings with polarized beam and target

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We present for the first time the complete twist-4 result for the semi-inclusive deeply inelastic scattering $e^-N \to e^-qX$ with polarized electron and proton beams at the tree level of perturbative QCD. The calculations have been carried out using the formalism obtained after collinear expansion where the multiple gluon scatterings are taken into account and gauge links are obtained automatically in a systematical way. The results show in particular that there are twist-4 contributions to all the eight twist-2 structure functions for $e^-N \to e^-hX$ that correspond to the eight twist-2 transverse-momentum-dependent parton distribution functions. Such higher-twist effects could be very significant and therefore have important impacts on extracting these three-dimensional parton distribution functions from the asymmetry data on $e^-N \to e^-hX$. We suggest also an approximate way to obtain a rough estimation of such higher-twist contributions.

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I. INTRODUCTION

Three-dimensional or transverse-momentum-dependent (TMD) parton distribution functions (PDFs) are one of the frontiers in hadron physics in particular in the study of hadron structure and properties of QCD [1,2]. When the transverse momentum of the parton is concerned, the sensitive measurable quantities in high-energy reactions are often different azimuthal angle asymmetries. Higher-twist contributions could be very significant and thus play an important role when going from the one-dimensional to the three-dimensional case.

In contrast to twist-3 contributions that often lead to azimuthal asymmetries that are missing when only twist-2 contributions are considered (see e.g. Refs. [3–5]), in many cases, twist-4 contributions are just addenda to twist-2 asymmetries. Since the asymmetries themselves are usually not very large, twist-4 contributions can be relatively very significant and have large influences on determining the twist-2 PDFs from experimental data. This is particularly the case in light of the fact that most of the data currently available are from experiments at not very high energies (see e.g. Refs. [6–11], or Ref. [12] for a recent review). It is therefore necessary and important to make systematic studies including higher-twist contributions. However, such a systematic study up to twist-4 is considered as very complicated and might be even impossible in particular because we need to deal with quark-two-gluon-quark correlators with three independent parton momenta and related complicated problems.

Higher-twist effects in inclusive deeply inelastic lepton-nucleon scattering (DIS) and Drell-Yan processes have been studied already in the 1980s to 1990s [13–15]. It has been shown that the collinear expansion is a necessary procedure for obtaining the hadronic tensor or the cross section in terms of gauge-invariant one-dimensional PDFs. More recently, it has been shown that [16] collinear expansion can be extended to the semi-inclusive DIS process $e^-N \to e^-qX$, where q denotes a quark that corresponds to a jet of hadrons in experiments. In the formalism obtained, the multiple gluon scattering are taken into account and gauge links are obtained automatically and systematically. Moreover, the expressions for the hadronic tensor obtained after the collinear expansion are simple and elegant in the sense that they are given in terms of PDFs and hard parts. The hard parts are not only calculable but also simplified to a form independent of the parton momentum, and correspondingly the involved PDFs are not only gauge invariant but also all defined via quark-quark or quark-j-gluon-quark correlators with only one independent parton momentum left. This makes the expressions much simpler and higher-twist calculations much more feasible. Based on this formalism, the complete twist-3 results for $e^-N \to e^-qX$ and $e^+e^- \to hqX$ have been obtained and were presented in Refs. [16–18].

Although there are still large differences between $e^-N \to e^-qX$ and $e^-N \to e^-hX$, the study of the former can provide useful references at least qualitatively for the latter. In this paper, we present for the first time the complete twist-4 result for $e^-N \to e^-qX$ with polarized electron and nucleon beams. After this introduction, in Sec. II, we make a brief summary of the general formalism including the involved TMD PDFs defined via the corresponding quark-j-gluon-quark correlators and the relationships between them. In Sec. III, we present the results for the structure functions at the tree level of pQCD up to twist-4. We present also the results for azimuthal asymmetries and suggest an approximate method for a rough estimation of twist-4 contributions. In Sec. IV, we present a short summary and discussion.

II. THE FORMALISM

To be explicit, we consider the semi-inclusive DIS (SIDIS) $e^-N \to e^-qX$. The cross section is given in terms of the well-known leptonic tensor $L^{\mu\nu}$ and the hadronic tensor $W_{\mu\nu}$ as

$$d\sigma = \frac{\alpha_{\rm em}^2 e_q^2}{sQ^4} L^{\mu\nu}(l,\lambda_l,l') W_{\mu\nu}^{(si)}(q,p,S,k') \frac{d^3l' d^3k'}{(2\pi)^3 2E_{l'} 2E_{k'}}, \tag{2.1}$$

where l, l', p and k' are the 4-momenta of the incident electron, the outgoing electron, the incident nucleon and the outgoing quark q respectively; $q = l - l'$ and $Q^2 = -q^2$; λ_l and S are the helicity of the electron and the spin of the nucleon. The leptonic tensor is given by

$$L^{\mu\nu}(l,\lambda_l,l') = 2(l^\mu l'^\nu + l^\nu l'^\mu - g^{\mu\nu} l\cdot l') + 2i\lambda_l \varepsilon^{\mu\nu\rho\sigma} l_\rho l'_\sigma, \tag{2.2}$$

which consists of an unpolarized symmetric part and a polarized antisymmetric part. We also note that the leptonic tensor is space-reflection even, i.e., it satisfies $L^{\mu\nu}(l^{\mathcal{P}},\lambda_l^{\mathcal{P}},l'^{\mathcal{P}}) = L_{\mu\nu}(l,\lambda_l,l')$, where $l^{\mathcal{P}}$ denotes l under space reflection. The hadronic tensor $W_{\mu\nu}^{(si)}$ is defined as

$$W_{\mu\nu}^{(si)}(q,p,S,k') = \frac{1}{2\pi}\sum_X \langle p,S|J_\mu(0)|k',X\rangle\langle k',X|J_\nu(0)|p,S\rangle \times (2\pi)^4\delta^4(p+q-k'-p_X), \tag{2.3}$$

where $J_\mu = \bar\psi\gamma_\mu\psi$ is the electromagnetic current. In addition, the hadronic tensor is also space-reflection even in this case, i.e., it satisfies $W_{\mu\nu}^{(si)}(q^{\mathcal{P}},p^{\mathcal{P}},S^{\mathcal{P}},k'^{\mathcal{P}}) = W^{(si)\mu\nu}(q,p,S,k')$.

We note that $W_{\mu\nu}^{(si)}$ is related to the hadronic tensor $W_{\mu\nu}^{(in)}(q,p,S)$ for the inclusive process $e^-N \to e^-X$ by

$$W_{\mu\nu}^{(in)}(q,p,S) = \frac{1}{(2\pi)^3}\int W_{\mu\nu}^{(si)}(q,p,S,k') \frac{d^3k'}{2E_{k'}}. \tag{2.4}$$

If we consider only the $k'_\perp$ dependence of the cross section, we have

$$d\sigma = \frac{\alpha_{\rm em}^2 e_q^2}{sQ^4} L^{\mu\nu}(l,\lambda_l,l') W_{\mu\nu}(q,p,S,k'_\perp) \frac{d^3l' d^2k'_\perp}{2E_{l'}}, \tag{2.5}$$

where $W_{\mu\nu}(q,p,S,k'_\perp)$ is the TMD semi-inclusive hadronic tensor that is related to $W_{\mu\nu}^{(si)}(q,p,S,k')$ by

$$W_{\mu\nu}(q,p,S,k'_\perp) = \frac{1}{(2\pi)^3}\int \frac{dk'_z}{2E_{k'}} W_{\mu\nu}^{(si)}(q,p,S,k'). \tag{2.6}$$

A. The general form of the cross section in terms of structure functions

The general form of the cross section in terms of structure functions is obtained from the general form of the hadronic tensor expressed as a sum of basic Lorentz tensors multiplied by Lorentz scalar coefficients. This can be found e.g. in Refs. [4,19–21] for $e^-N \to e^-hX$ or in Refs. [22,23] for $e^+e^- \to h_1h_2X$. It has in particular been shown in Ref. [23] that the spin-dependent tensor set can be given by spin-dependent Lorentz scalar(s) and/or pseudoscalar(s) times the unpolarized set.

For $e^-N \to e^-qX$, the hadronic tensor takes exactly the same form as that for $e^-N \to e^-hX$ or $e^+e^- \to h_1h_2X$ for spin-1/2 h_1 and spin-zero h_2. The spin-independent (or unpolarized) part is given by

$$h_{Ui}^{S\mu\nu} = \left\{ g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}, p_q^\mu p_q^\nu, p_q^{\{\mu} k_q'^{\nu\}}, k_q'^\mu k_q'^\nu \right\}, \tag{2.7}$$

$$\tilde h_{Ui}^{S\mu\nu} = \{\varepsilon^{\{\mu qpk'}(p_q,k'_q)^{\nu\}}\}, \tag{2.8}$$

$$h_U^{A\mu\nu} = p_q^{[\mu} k_q'^{\nu]}, \tag{2.9}$$

$$\tilde h_{Ui}^{A\mu\nu} = \{\varepsilon^{\mu\nu qp}, \varepsilon^{\mu\nu qk'}\}, \tag{2.10}$$

where h represents the space-reflection-even tensor while $\tilde h$ represents the space-reflection-odd one; the superscript S or A denotes symmetric or antisymmetric under exchange of $(\mu \leftrightarrow \nu)$, and the subscript U denotes the unpolarized (spin-independent) part. A 4-momentum p with a subscript q denotes $p_q \equiv p - q(p\cdot q)/q^2$ satisfying $p_q \cdot q = 0$. We use, as in e.g. Ref. [23], the shorthand notations to make the expressions more concise such as $\varepsilon^{\mu qpk'} \equiv \varepsilon^{\mu\alpha\rho\sigma} q_\alpha p_\rho k'_\sigma$, $A^{\{\mu}B^{\nu\}} = A^\mu B^\nu + A^\nu B^\mu$, $A^{[\mu}B^{\nu]} = A^\mu B^\nu - A^\nu B^\mu$ and so on. We see that there are in total nine such basic tensors in the unpolarized case.

For the spin-dependent part, in the γ^*N center-of-mass frame, the basic Lorentz tensors can be given by

$$h_{Vi}^{S\mu\nu} = \{[\lambda_h, (k'_\perp\cdot S)]\tilde h_{Ui}^{S\mu\nu}, \varepsilon_\perp^{k'S} h_{Uj}^{S\mu\nu}\}, \tag{2.11}$$

$$\tilde h_{Vi}^{S\mu\nu} = \{[\lambda_h, (k'_\perp\cdot S)] h_{Ui}^{S\mu\nu}, \varepsilon_\perp^{k'S} \tilde h_{Uj}^{S\mu\nu}\}, \tag{2.12}$$

$$h_{Vi}^{A\mu\nu} = \{[\lambda_h, (k'_\perp\cdot S)]\tilde h_{Ui}^{A\mu\nu}, \varepsilon_\perp^{k'S} h_U^{A\mu\nu}\}, \tag{2.13}$$

$$\tilde h_{Vi}^{A\mu\nu} = \{[\lambda_h, (k'_\perp\cdot S)] h_U^{A\mu\nu}, \varepsilon_\perp^{k'S} \tilde h_{Uj}^{A\mu\nu}\}, \tag{2.14}$$

where $\varepsilon_\perp^{k'S} = \varepsilon_\perp^{\rho\sigma} k'_\rho S_\sigma$ and $\varepsilon_\perp^{\rho\sigma} = \varepsilon^{\alpha\beta\rho\sigma}\bar{n}_\alpha n_\beta$. There are in total 27 such S-dependent basic tensors, three times as many as those for the unpolarized part, corresponding to three independent polarization modes.

Under the one-photon exchange approximation, parity is conserved. We need only the space-reflection-even parts $h^{S\mu\nu}$ and $h^{A\mu\nu}$. In this case, we have five unpolarized, four λ_h-dependent and nine S_T-dependent Lorentz tensors left. Time reversal invariance does not give any restriction here due to the final-state interaction [24–26]. There are finally 18 independent Lorentz tensors that contribute and each corresponds to an independent structure function.

After making the Lorentz contraction with the leptonic tensor, we obtain the general form for the cross section. In the $\gamma^* N$ c.m. frame, we have

$$\frac{d\sigma}{dxdyd\phi_S d^2k'_\perp} = \frac{\alpha_{\rm em}^2}{xyQ^2}\mathcal{K}(\mathcal{W}_{UU} + \lambda_l \mathcal{W}_{LU} + \lambda_h \mathcal{W}_{UL} + \lambda_l\lambda_h \mathcal{W}_{LL} + |\vec{S}_T|\mathcal{W}_{UT} + \lambda_l|\vec{S}_T|\mathcal{W}_{LT}), \quad (2.15)$$

$$\mathcal{W}_{UU} = W_{UU,T} + \varepsilon W_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)}W_{UU}^{\cos\phi}\cos\phi + \varepsilon W_{UU}^{\cos 2\phi}\cos 2\phi, \quad (2.16)$$

$$\mathcal{W}_{LU} = \sqrt{2\varepsilon(1-\varepsilon)}W_{LU}^{\sin\phi}\sin\phi, \quad (2.17)$$

$$\mathcal{W}_{UL} = \sqrt{2\varepsilon(1+\varepsilon)}W_{UL}^{\sin\phi}\sin\phi + \varepsilon W_{UL}^{\sin 2\phi}\sin 2\phi, \quad (2.18)$$

$$\mathcal{W}_{LL} = \sqrt{1-\varepsilon^2}W_{LL} + \sqrt{2\varepsilon(1-\varepsilon)}W_{LL}^{\cos\phi}\cos\phi, \quad (2.19)$$

$$\begin{aligned}\mathcal{W}_{UT} = {}& \sqrt{2\varepsilon(1+\varepsilon)}W_{UT}^{\sin\phi_S}\sin\phi_S \\ &+ (W_{UT,T}^{\sin(\phi-\phi_S)} + \varepsilon W_{UT,L}^{\sin(\phi-\phi_S)})\sin(\phi-\phi_S) \\ &+ \varepsilon W_{UT}^{\sin(\phi+\phi_S)}\sin(\phi+\phi_S) \\ &+ \sqrt{2\varepsilon(1+\varepsilon)}W_{UT}^{\sin(2\phi-\phi_S)}\sin(2\phi-\phi_S) \\ &+ \varepsilon W_{UT}^{\sin(3\phi-\phi_S)}\sin(3\phi-\phi_S),\end{aligned} \quad (2.20)$$

$$\begin{aligned}\mathcal{W}_{LT} = {}& \sqrt{2\varepsilon(1-\varepsilon)}W_{LT}^{\cos\phi_S}\cos\phi_S \\ &+ \sqrt{1-\varepsilon^2}W_{LT}^{\cos(\phi-\phi_S)}\cos(\phi-\phi_S) \\ &+ \sqrt{2\varepsilon(1-\varepsilon)}W_{LT}^{\cos(2\phi-\phi_S)}\cos(2\phi-\phi_S),\end{aligned} \quad (2.21)$$

where $x = Q^2/2p\cdot q$, $y = p\cdot q/p\cdot l$, $\varepsilon = (1-y-\frac{1}{4}\gamma^2y^2)/(1-y+\frac{1}{2}y^2+\frac{1}{4}\gamma^2y^2)$, $\gamma = 2Mx/Q$; $\mathcal{K}$ is a kinematic factor, $\mathcal{K} = (1+\gamma^2/2x)y^2/(1-\varepsilon)$; ϕ is the azimuthal angle of $\vec{k}'$ defined with respect to the leptonic plane and ϕ_S is that for $\vec{S}_T$. Here, we suggest unifying the notations in the following way: the $\mathcal{W}$'s denote the sum of the contributions in each polarized case specified by the subscripts; the W's are called structure functions, and they are Lorentz scalars and are functions of x, Q and $k'^2_\perp$, where the superscripts denote azimuthal angle dependences and the subscripts denote the polarizations. We use $\mathcal{W}$'s and W's to denote those for $e^-N \to e^-qX$ and $\mathcal{F}$ and F's are for $e^-N \to e^-hX$ where the expressions have exactly the same form with the replacements of $\mathcal{W}$ by $\mathcal{F}$, W by F, $k'_\perp$ by $p_{h\perp}$, and ϕ by ϕ_h.

All measurable quantities can be expressed in terms of these 18 structure functions. In the QCD parton model, these structure functions are given in terms of gauge-invariant PDFs. The results up to twist-3 can e.g. be found in Ref. [17]. In the remainder of this paper, we present the results up to twist-4.

If we integrate over the azimuthal angle ϕ, we obtain

$$\frac{d\sigma}{dxdyd\phi_S dk'^2_\perp} = \frac{\pi\alpha_{\rm em}^2}{xyQ^2}\mathcal{K}(\bar{\mathcal{W}}_{UU} + \lambda_l\bar{\mathcal{W}}_{LU} + \lambda_h\bar{\mathcal{W}}_{UL} + \lambda_l\lambda_h\bar{\mathcal{W}}_{LL} + |\vec{S}_T|\bar{\mathcal{W}}_{UT} + \lambda_l|\vec{S}_T|\bar{\mathcal{W}}_{LT}), \quad (2.22)$$

$$\bar{\mathcal{W}}_{UU} = W_{UU,T} + \varepsilon W_{UU,L}, \quad (2.23)$$

$$\bar{\mathcal{W}}_{LU} = \bar{\mathcal{W}}_{UL} = 0, \quad (2.24)$$

$$\bar{\mathcal{W}}_{LL} = \sqrt{1-\varepsilon^2}W_{LL}, \quad (2.25)$$

$$\bar{\mathcal{W}}_{UT} = \sqrt{2\varepsilon(1+\varepsilon)}W_{UT}^{\sin\phi_S}\sin\phi_S, \quad (2.26)$$

$$\bar{\mathcal{W}}_{LT} = \sqrt{2\varepsilon(1-\varepsilon)}W_{LT}^{\cos\phi_S}\cos\phi_S. \quad (2.27)$$

We see that only 5 of the 18 structure functions contribute in this case. We can use this to study these five structure functions more intensively. We see in particular that two transverse spin asymmetries could exist: one is the $\sin\phi_S$ single-spin asymmetry that has to vanish in the inclusive DIS $e^-N \to e^-X$ as demanded by the time reversal invariance. For the SIDIS $e^-N \to e^-qX$ as discussed here, final-state interactions between the struck q and the rest of the hadronic system could give rise to such asymmetry.

If we further integrate over $k'_\perp$, we obtain the well-known result for the inclusive process $e^-N \to e^-X$. We note that the single-spin asymmetry vanishes in this case because of the time-reversal invariance, so we obtain

$$\frac{d\sigma^{\rm in}}{dxdyd\phi_S} = \frac{\alpha_{\rm em}^2}{xyQ^2}\mathcal{K}(\mathcal{F}_{UU}^{\rm in} + \lambda_l\mathcal{F}_{LU}^{\rm in} + \lambda_h\mathcal{F}_{UL}^{\rm in} + \lambda_l\lambda_h\mathcal{F}_{LL}^{\rm in} + |\vec{S}_T|\mathcal{F}_{UT}^{\rm in} + \lambda_l|\vec{S}_T|\mathcal{F}_{LT}^{\rm in}), \quad (2.28)$$

$$\mathcal{F}_{UU}^{\rm in} = F_{UU,T}^{\rm in} + \varepsilon F_{UU,L}^{\rm in}, \quad (2.29)$$

$$\mathcal{F}_{LL}^{\rm in} = \sqrt{1-\varepsilon^2} F_{LL}^{\rm in}, \tag{2.30}$$

$$\mathcal{F}_{LT}^{\rm in} = \sqrt{2\varepsilon(1-\varepsilon)} F_{LT}^{\rm in, \cos\phi_S} \cos\phi_S, \tag{2.31}$$

$$\mathcal{F}_{LU}^{\rm in} = \mathcal{F}_{UL}^{\rm in} = \mathcal{F}_{UT}^{\rm in} = 0. \tag{2.32}$$

As is well known there are four independent structure functions in this case. Their relationships to the well-known F_1, F_2, g_1 and g_2 are given by [4]

$$F_{UU,L}^{\rm in} + F_{UU,T}^{\rm in} = (1+\gamma^2) F_2, \tag{2.33}$$

$$F_{UU,T}^{\rm in} = 2xF_1, \tag{2.34}$$

$$F_{LL}^{\rm in} = 2x(g_1 - \gamma^2 g_2), \tag{2.35}$$

$$F_{LT}^{\rm in, \cos\phi_S} = -2x\gamma(g_1 + g_2). \tag{2.36}$$

B. The collinear expansion and the hadronic tensor in the QCD parton model

At the tree level of the QCD parton model, to obtain the hadronic tensor in terms of gauge-invariant PDFs, we need to consider the contributions from the series of diagrams given by Fig. 1, i.e., to include contributions from the multiple gluon scattering. It has been shown [16] that the collinear expansion [13,14] can be applied also to the semi-inclusive DIS process $e^-N \to e^- qX$ and, after collinear expansion, the hadronic tensor is expressed in terms of the gauge-invariant quark-quark and quark-j-gluon-quark correlators and calculable hard parts [16,17]

$$W_{\mu\nu}^{(si)}(q,p,S,k') = \sum_{j,c} \tilde{W}_{\mu\nu}^{(si;j,c)}(q,p,S,k'), \tag{2.37}$$

where j denotes the number of gluons exchanged and c denotes eventually different cuts. After integration over k'_z, these $\tilde{W}_{\mu\nu}^{(si;j,c)}$'s are simplified to [17]

$$\tilde{W}_{\mu\nu}^{(0)} = {\rm Tr}[\hat{h}_{\mu\nu}^{(0)} \hat{\Phi}^{(0)}]/2, \tag{2.38}$$

$$\tilde{W}_{\mu\nu}^{(1,L)} = {\rm Tr}[\hat{h}_{\mu\nu}^{(1)\rho} \hat{\varphi}_\rho^{(1)}]/4q\cdot p, \tag{2.39}$$

$$\tilde{W}_{\mu\nu}^{(2,L)} = {\rm Tr}[\hat{h}_{\mu\nu}^{(1)\rho} \hat{\phi}_\rho^{(2)} + \hat{N}_{\mu\nu}^{(2)\rho\sigma} \hat{\varphi}_{\rho\sigma}^{(2)}]/(2q\cdot p)^2, \tag{2.40}$$

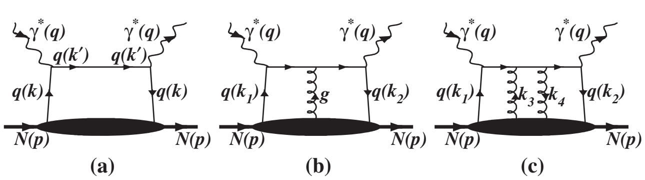

FIG. 1. The first three of the Feynman diagram series where (a) $j=0$, (b) $j=1$ and (c) $j=2$ gluon(s) are exchanged.

$$\tilde{W}_{\mu\nu}^{(2,M)} = {\rm Tr}[\hat{h}_{\mu\nu}^{(2)\rho\sigma} \hat{\varphi}_{\rho\sigma}^{(2,M)}]/(2q\cdot p)^2, \tag{2.41}$$

and the hard parts are reduced to the simple forms independent of the parton momentum, i.e.,

$$\hat{h}_{\mu\nu}^{(0)} = \gamma_\mu \not{n} \gamma_\nu / p^+, \tag{2.42}$$

$$\hat{h}_{\mu\nu}^{(1)\rho} = \gamma_\mu \not{\bar{n}} \gamma_\perp^\rho \not{n} \gamma_\nu, \tag{2.43}$$

$$\hat{N}_{\mu\nu}^{(2)\rho\sigma} = q^- \gamma_\mu \gamma_\perp^\rho \not{n} \gamma_\perp^\sigma \gamma_\nu, \tag{2.44}$$

$$\hat{h}_{\mu\nu}^{(2)\rho\sigma} = \gamma_\mu \gamma_\perp^\rho \not{p} \gamma_\perp^\sigma \gamma_\nu, \tag{2.45}$$

where $\gamma_\perp^\rho \equiv g_\perp^{\rho\sigma}\gamma_\sigma$ and $g_\perp^{\rho\sigma} \equiv g^{\rho\sigma} - \bar{n}^\rho n^\sigma - \bar{n}^\sigma n^\rho$. The involved quark-quark and quark-j-gluon-quark correlators are given by

$$\hat{\Phi}^{(0)}(x,k_\perp) = \int \frac{p^+ dy^- d^2 y_\perp}{(2\pi)^3} e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp} \times \langle N|\bar{\psi}(0)\mathcal{L}(0,y)\psi(y)|N\rangle, \tag{2.46}$$

$$\hat{\varphi}_\rho^{(1)}(x,k_\perp) = \int \frac{p^+ dy^- d^2 y_\perp}{(2\pi)^3} e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp} \times \langle N|\bar{\psi}(0)D_{\perp\rho}(0)\mathcal{L}(0,y)\psi(y)|N\rangle, \tag{2.47}$$

$$\hat{\varphi}_{\rho\sigma}^{(2)}(x,k_\perp) = \int \frac{p^+ dy^- d^2 y_\perp}{(2\pi)^3} \int_0^\infty ip^+ dz^- e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp} \times \langle N|\bar{\psi}(0)\mathcal{L}(0,z)D_{\perp\rho}(z)D_{\perp\sigma}(z) \times \mathcal{L}(z,y)\psi(y)|N\rangle, \tag{2.48}$$

$$\hat{\phi}_\rho^{(2)}(x,k_\perp) = \bar{n}^\sigma \int \frac{p^+ dy^- d^2 y_\perp}{(2\pi)^3} e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp} \times \langle N|\bar{\psi}(0)D_\sigma(0)D_{\perp\rho}(0)\mathcal{L}(0,y)\psi(y)|N\rangle, \tag{2.49}$$

$$\hat{\varphi}_{\rho\sigma}^{(2,M)}(x,k_\perp) = \int \frac{p^+ dy^- d^2 y_\perp}{(2\pi)^3} e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp} \times \langle N|\bar{\psi}(0)D_{\perp\rho}(0)\mathcal{L}(0,y)D_{\perp\sigma}(y)\psi(y)|N\rangle, \tag{2.50}$$

where $D_\rho = -i\partial_\rho + gA_\rho$, and $\mathcal{L}(0,y)$ is the well-known gauge link. We see that the involved $\hat{\varphi}^{(j)}$ are all D-type and are simplified to depend only on one independent parton momentum. We note in particular that the leading power contribution of $\tilde{W}_{\mu\nu}^{(j)}$ is twist-$(j+2)$. For $\tilde{W}_{\mu\nu}^{(2,L)}$, the leading power contribution of the first term on the rhs of Eq. (2.40) is at twist-5 because of the presence of the factor $\bar{n}^\sigma$ in the definition of $\hat{\phi}^{(2)}$ given by Eq. (2.49). It has no contribution up to twist-4.

In $e^-N \to e^-qX$, where the fragmentation is not considered, only chiral-even PDFs are involved. We need only the γ^α and $\gamma_5\gamma^\alpha$ terms in the expansion of the correlator in terms of the Γ matrices, e.g., $\hat{\Phi}^{(0)} = (\Phi^{(0)}_\alpha \gamma^\alpha - \tilde{\Phi}^{(0)}_\alpha \gamma_5\gamma^\alpha + \cdots)/2$.

It is useful to note that

$$\hat{\Phi}^{(0)\dagger}(x,k_\perp) = \gamma_0 \hat{\Phi}^{(0)}(x,k_\perp)\gamma_0, \tag{2.51}$$

$$\hat{\varphi}^{(2,M)\dagger}_{\rho\sigma}(x,k_\perp) = \gamma_0 \hat{\varphi}^{(2,M)}_{\sigma\rho}(x,k_\perp)\gamma_0, \tag{2.52}$$

while no such simple relation for $\hat{\varphi}^{(1)}_\rho$ or $\hat{\varphi}^{(2)}_{\rho\sigma}$ holds. This implies that $\Phi^{(0)*}_\alpha = \Phi^{(0)}_\alpha$, $\tilde{\Phi}^{(0)*}_\alpha = \tilde{\Phi}^{(0)}_\alpha$; $\varphi^{(2,M)*}_{\rho\sigma\alpha} = \varphi^{(2,M)}_{\sigma\rho\alpha}$ and $\tilde{\varphi}^{(2,M)*}_{\rho\sigma\alpha} = \tilde{\varphi}^{(2,M)}_{\sigma\rho\alpha}$. We also emphasize that due to the QCD equation of motion, these quark-j-gluon-quark correlators are not fully independent of each other. We will come back to this point in the next section.

C. Lorentz decompositions of quark-quark and quark-gluon-quark correlators

1. The involved decompositions and TMD PDFs

Up to twist-4, we need the complete Lorentz decompositions of $\Phi^{(0)}_\alpha$ and $\tilde{\Phi}^{(0)}_\alpha$. The chiral-even parts are given by [27]

$$\begin{aligned}\Phi^{(0)}_\alpha = {}& p^+\bar{n}_\alpha\left(f_1 - \frac{\varepsilon^{kS}_\perp}{M} f^\perp_{1T}\right) \\ & + k_{\perp\alpha} f^\perp - M\tilde{S}_{T\alpha} f_T - \lambda_h \tilde{k}_{\perp\alpha} f^\perp_L - \frac{k_{\perp\langle\alpha}k_{\perp\beta\rangle}}{M}\tilde{S}^\beta_T f^\perp_T \\ & + \frac{M^2}{p^+} n_\alpha\left(f_3 - \frac{\varepsilon^{kS}_\perp}{M} f^\perp_{3T}\right),\end{aligned} \tag{2.53}$$

$$\begin{aligned}\tilde{\Phi}^{(0)}_\alpha = {}& -p^+\bar{n}_\alpha\left(\lambda_h g_{1L} - \frac{k_\perp\cdot S_T}{M} g^\perp_{1T}\right) \\ & - \tilde{k}_{\perp\alpha} g^\perp - MS_{T\alpha} g_T - \lambda_h k_{\perp\alpha} g^\perp_L + \frac{k_{\perp\langle\alpha}k_{\perp\beta\rangle}}{M} S^\beta_T g^\perp_T \\ & - \frac{M^2}{p^+} n_\alpha\left(\lambda_h g_{3L} - \frac{k_\perp\cdot S_T}{M} g^\perp_{3T}\right),\end{aligned} \tag{2.54}$$

where $\tilde{k}_{\perp\alpha} \equiv \varepsilon_{\perp\alpha\beta} k^\beta_\perp$, and $k_{\perp\langle\alpha}k_{\perp\beta\rangle} \equiv k_{\perp\alpha}k_{\perp\beta} - g_{\perp\alpha\beta}k^2_\perp/2$. Here, we use the same naming system as that used in Refs. [4,17,18,23,27], where f's and g's are defined from the γ_α or $\gamma_5\gamma_\alpha$ term respectively; a digit j in the subscript stands for twist-$(j+1)$, and those without a j are for twist-3; the subscript T or L denotes hadron polarization, and those without T or L denote unpolarized hadrons. We see that there are four chiral-even twist-4 TMD PDFs defined via $\hat{\Phi}^{(0)}$. For $\hat{\varphi}^{(1)}$, the chiral-even parts are

$$\begin{aligned}\varphi^{(1)}_{\rho\alpha} = {}& p^+\bar{n}_\alpha\bigg(k_{\perp\rho} f^\perp_d - M\tilde{S}_{T\rho} f_{dT} - \lambda_h \tilde{k}_{\perp\rho} f^\perp_{dL} \\ & - \frac{k_{\perp\langle\rho}k_{\perp\beta\rangle}}{M}\tilde{S}^\beta_T f^\perp_{dT}\bigg) + M^2 g_{\perp\rho\alpha}\left(f_{3d} - \frac{\varepsilon^{kS}_\perp}{M} f^\perp_{3dT}\right) \\ & + k_{\perp\langle\rho}k_{\perp\alpha\rangle}\left(f^\perp_{3d} + \frac{\varepsilon^{kS}_\perp}{M} f^{\perp 2}_{3dT}\right) \\ & + iM^2\varepsilon_{\perp\rho\alpha}\left(\lambda_h f_{3dL} - \frac{k_\perp\cdot S_T}{M} f^{\perp 3}_{3dT}\right) \\ & + \frac{1}{2}k_{\perp\{\rho}\tilde{k}_{\perp\alpha\}}\left(\lambda_h f^\perp_{3dL} + \frac{k_\perp\cdot S_T}{M} f^{\perp 4}_{3dT}\right),\end{aligned} \tag{2.55}$$

$$\begin{aligned}\tilde{\varphi}^{(1)}_{\rho\alpha} = {}& ip^+\bar{n}_\alpha\bigg(\tilde{k}_{\perp\rho} g^\perp_d + MS_{T\rho} g_{dT} + \lambda_h k_{\perp\rho} g^\perp_{dL} \\ & - \frac{k_{\perp\langle\rho}k_{\perp\beta\rangle}}{M} S^\beta_T g^\perp_{dT}\bigg) + iM^2\varepsilon_{\perp\rho\alpha}\left(g_{3d} - \frac{\varepsilon^{kS}_\perp}{M} g^\perp_{3dT}\right) \\ & + i\frac{1}{2}k_{\perp\{\rho}\tilde{k}_{\perp\alpha\}}\left(g^\perp_{3d} + \frac{\varepsilon^{kS}_\perp}{M} g^{\perp 2}_{3dT}\right) \\ & + M^2 g_{\perp\rho\alpha}\left(\lambda_h g_{3dL} - \frac{k_\perp\cdot S_T}{M} g^{\perp 3}_{3dT}\right) \\ & + ik_{\perp\langle\rho}k_{\perp\alpha\rangle}\left(\lambda_h g^\perp_{3dL} + \frac{k_\perp\cdot S_T}{M} g^{\perp 4}_{3dT}\right),\end{aligned} \tag{2.56}$$

where we, as in ref. [23], add in the subscript a lowercase letter d to denote TMDs defined via the D-type quark-gluon-quark correlator.

Up to twist-4, we need only the leading power contributions from $\hat{\varphi}^{(2)}$. For the chiral-even part, we need only the $\bar{n}_\alpha$ terms. They are given by

$$\begin{aligned}\varphi^{(2)}_{\rho\sigma\alpha} = {}& p^+\bar{n}_\alpha\bigg[M^2 g_{\perp\rho\sigma}\left(f_{3dd} - \frac{\varepsilon^{kS}_\perp}{M} f^\perp_{3ddT}\right) \\ & + k_{\perp\langle\rho}k_{\perp\sigma\rangle}\left(f^\perp_{3dd} + \frac{\varepsilon^{kS}_\perp}{M} f^{\perp 2}_{3ddT}\right) \\ & + iM^2\varepsilon_{\perp\rho\sigma}\left(\lambda_h f_{3ddL} - \frac{k_\perp\cdot S_T}{M} f^{\perp 3}_{3ddT}\right) \\ & + \frac{1}{2}k_{\perp\{\rho}\tilde{k}_{\perp\sigma\}}\left(\lambda_h f^\perp_{3ddL} + \frac{k_\perp\cdot S_T}{M} f^{\perp 4}_{3ddT}\right)\bigg],\end{aligned} \tag{2.57}$$

$$\begin{aligned}\tilde{\varphi}^{(2)}_{\rho\sigma\alpha} = {}& p^+\bar{n}_\alpha\bigg[iM^2\varepsilon_{\perp\rho\sigma}\left(g_{3dd} - \frac{\varepsilon^{kS}_\perp}{M} g^\perp_{3ddT}\right) \\ & + \frac{1}{2}k_{\perp\{\rho}\tilde{k}_{\perp\sigma\}}\left(g^\perp_{3dd} + \frac{\varepsilon^{kS}_\perp}{M} g^{\perp 2}_{3ddT}\right) \\ & + M^2 g_{\perp\rho\sigma}\left(\lambda_h g_{3ddL} - \frac{k_\perp\cdot S_T}{M} g^{\perp 3}_{3ddT}\right) \\ & + k_{\perp\langle\rho}k_{\perp\sigma\rangle}\left(\lambda_h g^\perp_{3ddL} + \frac{k_\perp\cdot S_T}{M} g^{\perp 4}_{3ddT}\right)\bigg],\end{aligned} \tag{2.58}$$

where the subscript dd denotes that they are defined via the D-type quark-two-gluon-quark correlator. Besides the superscript M, those for $\hat{\varphi}^{(2,M)}$ are exactly the same as $\hat{\varphi}^{(2)}$.

Since there are more than one $f_{3dT}^{\perp}$ and $g_{3dT}^{\perp}$ according to the naming rules, we introduce an additional digit in the superscript to distinguish them from each other. We meet the same problem in decompositions of $\hat{\varphi}^{(1)}$ and $\hat{\varphi}^{(2)}$ and we use also similar notations. In total, we have four $f_{3dT}^{\perp}$'s associated with the four independent Lorentz tensors $g_{\perp\rho\alpha}$, $k_{\perp\langle\rho}k_{\perp\alpha\rangle}$, $\varepsilon_{\perp\rho\alpha}$ and $k_{\perp\{\rho}\tilde{k}_{\perp\alpha\}}$ respectively; while $g_{3dT}^{\perp}$ to $g_{3dT}^{\perp 4}$ are associated with $\varepsilon_{\perp\rho\alpha}$, $k_{\perp\{\rho}\tilde{k}_{\perp\alpha\}}$, $g_{\perp\rho\alpha}$, $k_{\perp\langle\rho}k_{\perp\alpha\rangle}$ respectively. The four Lorentz tensors are orthogonal to each other.

We see that for the twist-4 parts, the decompositions of φ and $\tilde{\varphi}$ have an exact one-to-one correspondence. For each f_3, there is correspondingly a g_3. They always appear in pairs. We have in total eight such pairs from $\hat{\varphi}^{(1)}$, $\hat{\varphi}^{(2)}$ and $\hat{\varphi}^{(2,M)}$ respectively. Due to the Hermiticity of $\hat{\Phi}^{(0)}$ and $\hat{\varphi}^{(2,M)}$, PDFs defined via these two correlators are real. However, those defined via $\hat{\varphi}^{(1)}$ and $\hat{\varphi}^{(2)}$ are complex, which contain both real and imaginary parts.

The operator expressions of these twist-4 TMD PDFs can be obtained by reversing the corresponding equations for Lorentz decompositions. This can easily be done and we will not present them here.

2. Relationships derived from the QCD equation of motion

From the QCD equation of motion, $\gamma\cdot D\psi = 0$, we obtain immediately that, for the two transverse components of $\Phi_\rho^{(0)}$ or $\tilde{\Phi}_\rho^{(0)}$, we have

$$xp^+\Phi^{(0)\rho} = -g_\perp^{\rho\sigma}\mathrm{Re}\varphi_{\sigma+}^{(1)} - \varepsilon_\perp^{\rho\sigma}\mathrm{Im}\tilde{\varphi}_{\sigma+}^{(1)}, \quad (2.59)$$

$$xp^+\tilde{\Phi}^{(0)\rho} = -g_\perp^{\rho\sigma}\mathrm{Re}\tilde{\varphi}_{\sigma+}^{(1)} - \varepsilon_\perp^{\rho\sigma}\mathrm{Im}\varphi_{\sigma+}^{(1)}, \quad (2.60)$$

where $\varphi_{\sigma+} \equiv n^\alpha\varphi_{\sigma\alpha}$ and similarly for others. By inserting the corresponding Lorentz decompositions of these correlators into Eqs. (2.59)–(2.60), we obtain a set of relationships between the twist-3 TMD PDFs defined via the quark-gluon-quark correlator $\hat{\varphi}^{(1)}$ and those defined via the quark-quark correlator $\hat{\Phi}^{(0)}$. For the chiral-even part, we obtain eight real equations by inserting Eqs. (2.53)–(2.56) into Eqs. (2.59)–(2.60), and they can be given in the unified form [23]

$$f_{dS}^K - g_{dS}^K = -x(f_S^K - ig_S^K), \quad (2.61)$$

where $S =$ null, L or T and $K =$ null or $\perp$ whenever applicable. Using the relations given by Eq. (2.61), we can replace all the TMD PDFs defined via $\hat{\varphi}^{(1)}$ by those defined via $\hat{\Phi}^{(0)}$ in the final twist-3 results for the hadronic tensor in SIDIS [4,17], and similarly for e^+e^- annihilations [18,23].

Similarly, for the minus components of $\Phi_\rho^{(0)}$ and $\tilde{\Phi}_\rho^{(0)}$, we have

$$(xp^+)^2\Phi_-^{(0)} = -(g_\perp^{\rho\sigma}\varphi_{\rho\sigma+}^{(2,M)} + i\varepsilon_\perp^{\rho\sigma}\tilde{\varphi}_{\rho\sigma+}^{(2,M)})/2, \quad (2.62)$$

$$(xp^+)^2\tilde{\Phi}_-^{(0)} = -(g_\perp^{\rho\sigma}\tilde{\varphi}_{\rho\sigma+}^{(2,M)} + i\varepsilon_\perp^{\rho\sigma}\varphi_{\rho\sigma+}^{(2,M)})/2, \quad (2.63)$$

and for the transverse components of $\varphi_{\rho\sigma}$ and $\tilde{\varphi}_{\rho\sigma}$, we have

$$xp^+(g_\perp^{\rho\sigma}\varphi_{\rho\sigma}^{(1)} + i\varepsilon_\perp^{\rho\sigma}\tilde{\varphi}_{\rho\sigma}^{(1)}) = -g_\perp^{\rho\sigma}\varphi_{\rho\sigma+}^{(2,M)} - i\varepsilon_\perp^{\rho\sigma}\tilde{\varphi}_{\rho\sigma+}^{(2,M)}, \quad (2.64)$$

$$xp^+(g_\perp^{\rho\sigma}\tilde{\varphi}_{\rho\sigma}^{(1)} + i\varepsilon_\perp^{\rho\sigma}\varphi_{\rho\sigma}^{(1)}) = -g_\perp^{\rho\sigma}\tilde{\varphi}_{\rho\sigma+}^{(2,M)} - i\varepsilon_\perp^{\rho\sigma}\varphi_{\rho\sigma+}^{(2,M)}. \quad (2.65)$$

By inserting Eqs. (2.53)–(2.58) into Eqs. (2.62)–(2.65), we obtain a set of relationships between the twist-4 PDFs defined via $\hat{\varphi}^{(2,M)}$, $\hat{\varphi}^{(1)}$, and $\hat{\Phi}^{(0)}$. They are given by

$$x^2 f_3 = xf_{-3d} = -f_{-3dd}^M, \quad (2.66)$$

$$x^2 f_{3T}^\perp = xf_{-3dT}^\perp = -f_{-3ddT}^{M\perp}, \quad (2.67)$$

$$x^2 g_{3L} = xf_{-3dL} = -f_{-3ddL}^M, \quad (2.68)$$

$$x^2 g_{3T}^\perp = xf_{-3dT}^{\perp 3} = -f_{-3ddT}^{M\perp 3}, \quad (2.69)$$

where $f_\pm \equiv f \pm g$ such as $f_{-3d} \equiv f_{3d} - g_{3d}$ and so on. We note that Eqs. (2.66)–(2.69) represent 12 real equations and can be used to replace those independent twist-4 TMD PDFs in parton model results for the cross section.

D. Relationships between twist-4 and twist-2 PDFs at $g = 0$

When the multiple gluon scattering is taken into account, these higher-twist TMD PDFs are all new and quite involved. They reflect not only the parton distributions but also quantum interference effects in the scattering. There is little data available that gives direct insights into them. However, if we neglect the multiple gluon scattering, i.e., put $g = 0$, we obtain a set of simple equations relating them to the twist-2 counterparts. They could be helpful in understanding the significance of these higher-twist PDFs in particular at the present stage.

By putting $g = 0$ into Eqs. (2.46)–(2.50), we relate $\hat{\varphi}^{(j)}$ to $\hat{\Phi}^{(0)}$, i.e., $\hat{\varphi}_\rho^{(1)} = -k_{\perp\rho}\hat{\Phi}^{(0)}$, $\hat{\varphi}_{\rho\sigma}^{(2,M)} = k_{\perp\rho}k_{\perp\sigma}\hat{\Phi}^{(0)}$, and $\hat{\varphi}_{\rho\sigma}^{(2)} + \gamma^0\hat{\varphi}_{\sigma\rho}^{(2)\dagger}\gamma^0 = k_{\perp\rho}k_{\perp\sigma}\partial\hat{\Phi}^{(0)}/\partial x$. Together with the equation of motion, these relations relate all higher-twist PDFs to leading-twist ones. For those defined via $\hat{\varphi}^{(1)}$, we have

$$xf_{3d} = \frac{k_\perp^2}{2M^2}xf_{3d}^\perp = x^2 f_3 = -\frac{k_\perp^2}{2M^2}f_1, \quad (2.70)$$

$$xg_{3dL} = i\frac{k_\perp^2}{2M^2}xg_{3dL}^\perp = -x^2 g_{3L} = \frac{k_\perp^2}{2M^2}g_{1L}, \quad (2.71)$$

$$xf_{3dT}^\perp = -\frac{k_\perp^2}{2M^2}xf_{3dT}^{\perp 2} = x^2 f_{3T}^\perp = -\frac{k_\perp^2}{2M^2}f_{1T}^\perp, \quad (2.72)$$

$$xg_{3dT}^{\perp 3} = -i\frac{k_\perp^2}{2M^2}xg_{3dT}^{\perp 4} = -x^2 g_{3T}^{\perp} = \frac{k_\perp^2}{2M^2}g_{1T}^{\perp}, \quad (2.73)$$

and all the others vanish. For those defined via $\hat{\varphi}^{(2)}$, we have

$$2\text{Re}f_{3dd} = 2\text{Re}\frac{k_\perp^2}{2M^2}f_{3dd}^{\perp} = \frac{k_\perp^2}{2M^2}\frac{\partial}{\partial x}f_1, \quad (2.74)$$

$$2\text{Re}g_{3ddL} = 2\text{Re}\frac{k_\perp^2}{2M^2}g_{3ddL}^{\perp} = -\frac{k_\perp^2}{2M^2}\frac{\partial}{\partial x}g_{1L}, \quad (2.75)$$

$$2\text{Re}f_{3ddT}^{\perp} = -2\text{Re}\frac{k_\perp^2}{2M^2}f_{3ddT}^{\perp 2} = \frac{k_\perp^2}{2M^2}\frac{\partial}{\partial x}f_{1T}^{\perp}, \quad (2.76)$$

$$2\text{Re}g_{3ddT}^{\perp 3} = -2\text{Re}\frac{k_\perp^2}{2M^2}g_{3ddT}^{\perp 4} = -\frac{k_\perp^2}{2M^2}\frac{\partial}{\partial x}g_{1T}^{\perp}, \quad (2.77)$$

and all the others vanish. Time-reversal invariance demands $f_{1T}^{\perp} = 0$ in this case [24].

III. THE COMPLETE TWIST-4 RESULT

A. The hadronic tensor

We substitute the Lorentz decompositions of the quark-quark and quark-j-gluon-quark correlators given by Eqs. (2.53)–(2.58) into Eqs. (2.38)–(2.41), carry out the calculations, and obtain the hadronic tensor and cross section up to twist-4. Those up to twist-3 can be found in Ref. [17]. We present the results—in particular, those at twist-4—in the following.

To get the contributions from $\tilde{W}_{\mu\nu}^{(0)}$, we need to carry out the two traces in connection with the hard parts. They are

$$\text{Tr}[\hat{h}_{\mu\nu}^{(0)}\not{n}] = 8n_\mu n_\nu/p^+, \quad (3.1)$$

$$\text{Tr}[\hat{h}_{\mu\nu}^{(0)}\gamma_5\not{n}] = 0. \quad (3.2)$$

We see in particular that the trace with $\gamma_5\not{n}$ is zero. This implies that there is no contribution from $\tilde{\Phi}_\alpha^{(0)}$ or g_{3S} to the hadronic tensor at twist-4. The only contributions are from $\Phi_\alpha^{(0)}$ or f_{3S} and they are given by

$$\tilde{W}_{t4\mu\nu}^{(0)} = \frac{2M^2}{(q\cdot p)^2}(q+x_B p)_\mu (q+x_B p)_\nu\left(f_3 - \frac{1}{M}\varepsilon_\perp^{kS}f_{3T}^{\perp}\right), \quad (3.3)$$

where we use a subscript $t4$ to denote the twist-4 part only, and the superscript (0) denotes that it is from $\tilde{W}_{\mu\nu}^{(0)}$.

To get the contributions from $\tilde{W}_{\mu\nu}^{(1)}$, we need

$$\text{Tr}[\hat{h}_{\mu\nu}^{(1)\rho}\gamma_\perp^\alpha] = 4(2\bar{n}_\mu n_\nu g_\perp^{\rho\alpha} + g_{\perp\mu\nu}g_\perp^{\rho\alpha} - g_{\perp\mu}{}^{\{\rho}g_{\perp\nu}{}^{\alpha\}}), \quad (3.4)$$

$$\text{Tr}[\hat{h}_{\mu\nu}^{(1)\rho}\gamma_5\gamma_\perp^\alpha] = -4i(2\bar{n}_\mu n_\nu \varepsilon_\perp^{\rho\alpha} + g_{\perp\mu}{}^\rho\varepsilon_{\perp\nu}{}^\alpha + g_{\perp\nu}{}^\alpha\varepsilon_{\perp\mu}{}^\rho), \quad (3.5)$$

and the results for the contribution from the diagram with the left cut are given by

$$\begin{aligned}\tilde{W}_{t4\mu\nu}^{(1,L)} = \frac{1}{2q\cdot p}\Big[&4M^2\bar{n}_\mu n_\nu\left(f_{-3d} - \frac{\varepsilon_\perp^{kS}}{M}f_{-3dT}^{\perp}\right)\\ &- k_{\perp\langle\mu}k_{\perp\nu\rangle}\left(f_{-3d}^{\perp} + \frac{\varepsilon_\perp^{kS}}{M}f_{-3dT}^{\perp 2}\right)\\ &- k_{\perp\{\mu}\tilde{k}_{\perp\nu\}}\left(\lambda_h f_{+3dL}^{\perp} + \frac{k_\perp\cdot S_T}{M}f_{+3dT}^{\perp 4}\right)\Big]. \quad (3.6)\end{aligned}$$

For the contributions from $\tilde{W}_{\mu\nu}^{(2,L)}$, we need

$$\text{Tr}[\hat{N}_{\mu\nu}^{(2)\rho\sigma}\not{p}] = 4p\cdot q(g_{\perp\mu\nu}g_\perp^{\rho\sigma} + g_{\perp[\mu}{}^\rho g_{\perp\nu]}{}^\sigma), \quad (3.7)$$

$$\text{Tr}[\hat{N}_{\mu\nu}^{(2)\rho\sigma}\gamma_5\not{p}] = 4ip\cdot q(g_{\perp\mu}{}^\rho\varepsilon_{\perp\nu}{}^\sigma - g_{\perp\nu}{}^\sigma\varepsilon_{\perp\mu}{}^\rho), \quad (3.8)$$

and obtain,

$$\begin{aligned}\tilde{W}_{t4\mu\nu}^{(2,L)} = \frac{M^2}{p\cdot q}\Big[&g_{\perp\mu\nu}\left(f_{+3dd} - \frac{\varepsilon_\perp^{kS}}{M}f_{+3ddT}^{\perp}\right)\\ &+ i\varepsilon_{\perp\mu\nu}\left(\lambda_h f_{+3ddL} - \frac{k_\perp\cdot S_T}{M}f_{+3ddT}^{\perp 3}\right)\Big]. \quad (3.9)\end{aligned}$$

For that from $\tilde{W}_{\mu\nu}^{(2,M)}$, we have

$$\text{Tr}[\hat{h}_{\mu\nu}^{(2)\rho\sigma}\not{p}] = -8p_\mu p_\nu g_\perp^{\rho\sigma}, \quad (3.10)$$

$$\text{Tr}[\hat{h}_{\mu\nu}^{(2)\rho\sigma}\gamma_5\not{p}] = 8ip_\mu p_\nu\varepsilon_\perp^{\rho\sigma}, \quad (3.11)$$

and the result at twist-4 is given by

$$\tilde{W}_{t4\mu\nu}^{(2,M)} = -\frac{2M^2}{(p\cdot q)^2}p_\mu p_\nu\left(f_{-3dd}^{M} - \frac{\varepsilon_\perp^{kS}}{M}f_{-3ddT}^{M\perp}\right). \quad (3.12)$$

Adding the contributions from $\tilde{W}_{\mu\nu}^{(0)}$, $\tilde{W}_{\mu\nu}^{(1)}$ and $\tilde{W}_{\mu\nu}^{(2)}$ together, and using the relations between the PDFs defined via the quark-j-gluon-quark correlator and those defined via the quark-quark correlator given by Eqs. (2.66)–(2.69) to eliminate the PDFs that are not independent of others, we obtain the complete result for the hadronic tensor up to twist-4. It is given by

$$\begin{aligned} W_{t4\mu\nu} = {} & \frac{2M^2}{(q\cdot p)^2}(q+2x_Bp)_\mu(q+2x_Bp)_\nu\left(f_3-\frac{\varepsilon_\perp^{kS}}{M}f_{3T}^\perp\right) \\ & -\frac{1}{q\cdot p}\left[k_{\perp\langle\mu}k_{\perp\nu\rangle}\left(f_{-3d}^\perp+\frac{\varepsilon_\perp^{kS}}{M}f_{-3dT}^{\perp 2}\right)\right. \\ & -k_{\perp\{\mu}\tilde{k}_{\perp\nu\}}\left(\lambda_h f_{+3dL}^\perp+\frac{k_\perp\cdot S_T}{M}f_{+3dT}^{\perp 4}\right) \\ & +2M^2 g_{\perp\mu\nu}\left(f_{+3dd}-\frac{\varepsilon_\perp^{kS}}{M}f_{+3ddT}^\perp\right) \\ & \left.+2iM^2\varepsilon_{\perp\mu\nu}\left(\lambda_h f_{+3ddL}-\frac{k_\perp\cdot S_T}{M}f_{+3ddT}^{\perp 3}\right)\right]. \end{aligned} \quad (3.13)$$

Here, only the real part of the TMD PDF contributes and this is true for all the twist-4 PDFs involved in Eq. (3.13). We therefore just omit the symbol Re for clarity of the equation as well as in the remainder of this paper.

In addition, we see from Eq. (3.13) that, just as with the twist-2 or twist-3 part [17], the complete twist-4 hadronic tensor also satisfies the requirement of current conservation, i.e., $q^\mu W_{t4\mu\nu} = q^\nu W_{t4\mu\nu} = 0$. It contains both a real symmetric part and a pure imaginary antisymmetric part. This implies that we will obtain twist-4 contributions to the cross section in reactions with polarized or unpolarized leptons.

B. The structure functions

Performing the Lorentz contraction of the hadronic tensor with the leptonic tenor given by Eq. (2.2), we obtain the differential cross section in terms of the gauge-invariant TMD PDFs. By comparing the results with the general form given by Eq. (2.15), we obtain the expressions for the structure functions in terms of PDFs. Among the 18 structure functions, ten have twist-4 contributions. They are given by

$$W_{UU,T} = xf_1 + 4x^2\kappa_M^2 f_{+3dd}, \quad (3.14)$$

$$W_{UU,L} = 8x^3\kappa_M^2 f_3, \quad (3.15)$$

$$W_{UU}^{\cos 2\phi} = -2x^2\kappa_M^2\frac{|\vec{k}_\perp|^2}{M^2}f_{-3d}^\perp, \quad (3.16)$$

$$W_{UL}^{\sin 2\phi} = 2x^2\kappa_M^2\frac{|\vec{k}_\perp|^2}{M^2}f_{+3dL}^\perp, \quad (3.17)$$

$$W_{LL} = xg_{1L} + 4x^2\kappa_M^2 f_{+3ddL}, \quad (3.18)$$

$$W_{UT,T}^{\sin(\phi-\phi_S)} = \frac{|\vec{k}_\perp|}{M}\left(xf_{1T}^\perp + 4x^2\kappa_M^2 f_{+3ddT}^\perp\right), \quad (3.19)$$

$$W_{UT,L}^{\sin(\phi-\phi_S)} = 8x^3\kappa_M^2\frac{|\vec{k}_\perp|}{M}f_{3T}^\perp, \quad (3.20)$$

$$W_{UT}^{\sin(\phi+\phi_S)} = -x^2\kappa_M^2\frac{|\vec{k}_\perp|^3}{M^3}(f_{+3dT}^{\perp 4} + f_{-3dT}^{\perp 2}), \quad (3.21)$$

$$W_{UT}^{\sin(3\phi-\phi_S)} = -x^2\kappa_M^2\frac{|\vec{k}_\perp|^3}{M^3}(f_{+3dT}^{\perp 4} - f_{-3dT}^{\perp 2}), \quad (3.22)$$

$$W_{LT}^{\cos(\phi-\phi_S)} = \frac{|\vec{k}_\perp|}{M}(xg_{1T}^\perp + 4x^2\kappa_M^2 f_{+3ddT}^{\perp 3}). \quad (3.23)$$

Here, we omit the overall factor e_q^2 and a sum over flavor is also implicit. We introduce $\kappa_M \equiv M/Q$ that is the typical suppression factor for higher-twist contributions. κ_M^2 symbolizes twist-4 whereas κ_M symbolizes twist-3.

The other eight structure functions have only twist-3 contributions up to twist-4. They are given in e.g. Ref. [17]. For completeness and comparison, we include them in the following:

$$W_{UU}^{\cos\phi} = -2x^2\kappa_M\frac{|\vec{k}_\perp|}{M}f^\perp, \quad (3.24)$$

$$W_{UL}^{\sin\phi} = -2x^2\kappa_M\frac{|\vec{k}_\perp|}{M}f_L^\perp, \quad (3.25)$$

$$W_{LU}^{\sin\phi} = 2x^2\kappa_M\frac{|\vec{k}_\perp|}{M}g^\perp, \quad (3.26)$$

$$W_{LL}^{\cos\phi} = -2x^2\kappa_M\frac{|\vec{k}_\perp|}{M}g_L^\perp, \quad (3.27)$$

$$W_{UT}^{\sin\phi_S} = -2x^2\kappa_M f_T, \quad (3.28)$$

$$W_{UT}^{\sin(2\phi-\phi_S)} = -x^2\kappa_M\frac{|\vec{k}_\perp|^2}{M^2}f_T^\perp, \quad (3.29)$$

$$W_{LT}^{\cos\phi_S} = -2x^2\kappa_M g_T, \quad (3.30)$$

$$W_{LT}^{\cos(2\phi-\phi_S)} = -x^2\kappa_M\frac{|\vec{k}_\perp|^2}{M^2}g_T^\perp. \quad (3.31)$$

From the results given by Eqs. (3.14)–(3.31), we see clearly the following distinct features:

(1) Up to twist-4, all 18 structure functions are nonzero. Besides those eight that have twist-3 as leading power contributions, the remaining ten have twist-4 contributions. For $e^-N \to e^-qX$ that we consider here, four of them have twist-2 and the other six have twist-4 as leading power contributions. And all four twist-2 structure functions have twist-4 addenda to them. It is also very interesting and important to note that the twist-3 part contributes to azimuthal asymmetries that are all missing at either twist-2 or twist-4 and hence can be studied separately. However the twist-4 and twist-2 contributions may mix with each

other and give rise to the same asymmetry and hence are difficult to separate from each other.

(2) The structure functions that describe the azimuthal asymmetries given by either the cosine or sine of a single ϕ or ϕ_S or $2\phi - \phi_S$, i.e., $\cos\phi$, $\sin\phi$, $\cos\phi_S$, $\sin\phi_S$, $\cos(2\phi - \phi_S)$, and $\sin(2\phi - \phi_S)$, have only twist-3 contributions. For the structure functions that describe the azimuthal-angle-independent part, or the azimuthal angle dependences given by the cosine or sine of even number of ϕ or ϕ_S such as 2ϕ, $\phi - \phi_S$, $\phi + \phi_S$ and $3\phi - \phi_S$, we have twist-2 and/or twist-4 contributions.

(3) We recall that for $e^-N \to e^-hX$ where fragmentation is considered, there are eight twist-2 structure functions F that correspond to the eight twist-2 TMD PDFs. We see that all the W's corresponding to them have twist-4 contributions. This means that we have to consider twist-4 contributions if we use data on $e^-N \to e^-hX$ to extract the corresponding twist-2 TMD PDFs. Since $e^-N \to e^-hX$ is one of the major sources for the data [12] now available for extracting TMD PDFs, it is thus important to study these twist-4 contributions to get correct and precise knowledge even on twist-2 TMDs.

(4) If we consider $e^-N \to e^-hX$, besides $W_{UU,L}$ and $W_{UT,L}^{\sin(\phi-\phi_S)}$, all the twist-4 contributions are addenda to twist-2 structure functions. Since $W_{UU,L}$ is added to $W_{UU,T}$ and $W_{UT,L}^{\sin(\phi-\phi_S)}$ to $W_{UT,T}^{\sin(\phi-\phi_S)}$ to give the final observable effects, this means that all the twist-4 contributions are addenda to twist-2 contributions in $e^-N \to e^-hX$. This makes it very difficult to separate them from each other. A clean and perhaps practical way to study twist-4 effects is to study $e^-N \to e^-qX$, i.e., by measuring the jet production. In this case we have six structure functions that have twist-4 as the leading power contributions and four of them correspond to separate azimuthal asymmetries.

C. Azimuthal asymmetries

There are two twist-2 azimuthal asymmetries for $e^-N \to e^-qX$ and, up to twist-4, they are given by,

$$\langle\sin(\phi - \phi_S)\rangle_{UT} = \frac{|\vec{k}_\perp|}{2M}\frac{f_{1T}^\perp}{f_1}(1 - \alpha_{UT}\kappa_M^2), \tag{3.32}$$

$$\langle\cos(\phi - \phi_S)\rangle_{LT} = \frac{|\vec{k}_\perp|}{2M}\frac{C(y)}{A(y)}\frac{g_{1T}^\perp}{f_1}(1 - \alpha_{LT}\kappa_M^2), \tag{3.33}$$

where the twist-4 modification factors are given by,

$$\alpha_{UT} = \alpha_{UU} - 8x^2\frac{E(y)}{A(y)}\frac{f_{3T}^\perp}{f_{1T}^\perp} - 4x\frac{f_{+3ddT}^\perp}{f_{1T}^\perp}, \tag{3.34}$$

$$\alpha_{LT} = \alpha_{UU} - 4x\frac{f_{+3ddT}^{\perp 3}}{g_{1T}^\perp}, \tag{3.35}$$

where α_{UU} is due to twist-4 contributions to W_{UU}. It is the ratio of the twist-4 to twist-2 contributions in unit of κ_M^2, i.e.,

$$\alpha_{UU} = 8x^2\frac{E(y)}{A(y)}\frac{f_3}{f_1} + 4x\frac{f_{+3dd}}{f_1}. \tag{3.36}$$

Here, $A(y) = 1 + (1-y)^2$, $C(y) = y(2-y)$, and $E(y) = 2(1-y)$. There are four twist-4 azimuthal asymmetries given by,

$$\langle\cos 2\phi\rangle_{UU} = -\kappa_M^2\frac{|\vec{k}_\perp|^2}{M^2}\frac{E(y)}{A(y)}\frac{xf_{-3d}^\perp}{f_1}, \tag{3.37}$$

$$\langle\sin 2\phi\rangle_{UL} = \kappa_M^2\frac{|\vec{k}_\perp|^2}{M^2}\frac{E(y)}{A(y)}\frac{xf_{+3dL}^\perp}{f_1}, \tag{3.38}$$

$$\langle\sin(\phi + \phi_S)\rangle_{UT} = -x\kappa_M^2\frac{|\vec{k}_\perp|^3}{2M^3}\frac{E(y)}{A(y)}\frac{f_{+3dT}^{\perp 4} + f_{-3dT}^{\perp 2}}{f_1}, \tag{3.39}$$

$$\langle\sin(3\phi - \phi_S)\rangle_{UT} = -x\kappa_M^2\frac{|\vec{k}_\perp|^3}{2M^3}\frac{E(y)}{A(y)}\frac{f_{+3dT}^{\perp 4} - f_{-3dT}^{\perp 2}}{f_1}. \tag{3.40}$$

They have only twist-4 contributions up to this level and can therefore serve as good places to study such twist-4 effects.

Corresponding to the eight structure functions that have twist-3 contributions, we have eight twist-3 azimuthal asymmetries. The expressions were given in Ref. [17]. For completeness, we repeat them here:

$$\langle\cos\phi\rangle_{UU} = -\kappa_M\frac{|\vec{k}_\perp|}{M}\frac{B(y)}{A(y)}\frac{xf^\perp}{f_1}, \tag{3.41}$$

$$\langle\sin\phi\rangle_{UL} = -\kappa_M\frac{|\vec{k}_\perp|}{M}\frac{B(y)}{A(y)}\frac{xf_L^\perp}{f_1}, \tag{3.42}$$

$$\langle\sin\phi\rangle_{LU} = \kappa_M\frac{|\vec{k}_\perp|}{M}\frac{D(y)}{A(y)}\frac{xg^\perp}{f_1}, \tag{3.43}$$

$$\langle\cos\phi\rangle_{LL} = -\kappa_M\frac{|\vec{k}_\perp|}{M}\frac{B(y)xf^\perp + \lambda_l\lambda_h D(y)xg_L^\perp}{A(y)f_1 + \lambda_l\lambda_h C(y)g_{1L}}, \tag{3.44}$$

$$\langle\sin\phi_S\rangle_{UT} = -\kappa_M\frac{B(y)}{A(y)}\frac{xf_T}{f_1}, \tag{3.45}$$

$$\langle\sin(2\phi - \phi_S)\rangle_{UT} = -\kappa_M|\frac{|\vec{k}_\perp|^2}{2M^2}\frac{B(y)}{A(y)}\frac{xf_T^\perp}{f_1}, \tag{3.46}$$

$$\langle \cos\phi_S\rangle_{LT} = -\kappa_M \frac{D(y)}{A(y)}\frac{xg_T}{f_1}, \quad (3.47)$$

$$\langle \cos(2\phi-\phi_S)\rangle_{LT} = -\kappa_M \frac{|\vec{k}_\perp|^2}{2M^2}\frac{D(y)}{A(y)}\frac{xg_T^\perp}{f_1}, \quad (3.48)$$

where $B(y)=2(2-y)\sqrt{1-y}$, and $D(y)=2y\sqrt{1-y}$. It is clear that if we insert the relations given by Eqs. (2.70)–(2.77) into Eqs. (3.32)–(3.40), we obtain results for $g=0$ such as those obtained in Refs. [28,29]. The deviations from them reflect the effects of multiple gluon scattering.

We note in particular that by replacing ϕ by ϕ_h, the six azimuthal asymmetries given by Eqs. (3.32)–(3.33) and (3.37)–(3.40) are just the six twist-2 asymmetries in $e^-N\to e^-hX$. Measurements of them are one of the major tools that we use to extract twist-2 TMDs. Here we see clearly that, even if the fragmentation part is not considered, there are twist-4 contributions to all of them. We emphasize that the factor $(1-\alpha_{UU}\kappa_M^2)$ is due to the twist-4 contributions to W_{UU}. It exists for all azimuthal asymmetries that have twist-2 contributions. This means that this is the least modification factor that we have for all six twist-2 azimuthal asymmetries for $e^-N\to e^-hX$.

In light of the fact that Q^2 in the experiments such as HERMES or JLab (see e.g. Refs. [7,9]) are usually from 1 to 10 GeV^2 so κ_M^2 takes values from 0.1 to 1, the twist-4 modifications can be quite large depending on the coefficient of κ_M^2 in the equations given above. A reliable estimation of these twist-4 contributions depends on the unknown twist-4 PDFs involved. We note that there are in total 18 independent twist-4 TMD PDFs involved in the final results: two from $\hat{\Phi}^{(0)}$, four pairs from $\hat{\varphi}^{(1)}$, and four pairs from $\hat{\varphi}^{(2)}$. These twist-4 TMDs contain information on the intrinsic parton distribution in the nucleon and the effects of multiple gluon scattering contained in the gauge link. They contain in particular quantum interference effects in the multiple gluon scattering and thus there are no simple probability interpretations.

Clearly, it is still a long way to go to make precise measurements of all of the twist-4 TMD PDFs involved here. Presently, lacking knowledge about these twist-4 TMD PDFs, we suggest to use relationships between higher-twist TMDs and corresponding twist-2 ones obtained at $g=0$ given by Eqs. (2.70)–(2.77) as a first approximation to make rough estimates of twist-4 effects. More precisely, we use the relationships given by Eqs. (2.70)–(2.77) to replace the twist-4 PDFs by the corresponding twist-2 ones, and make estimations of their contributions to the cross section and/or azimuthal asymmetries. Though this is very crude, it might be helpful at this stage to get a feeling of the magnitudes of these twist-4 contributions. A more reliable model calculation will be left for a future study.

Under such approximations, we obtain that $\langle\sin 2\phi\rangle_{UL}\approx 0$, the other three twist-4 asymmetries become

$$\langle\cos 2\phi\rangle_{UU} \approx \kappa_M^2 \frac{|\vec{k}_\perp|^2}{M^2}\frac{E(y)}{A(y)}, \quad (3.49)$$

$$\langle\sin(\phi+\phi_S)\rangle_{UT} \approx -\kappa_M^2 \frac{|\vec{k}_\perp|^3}{2M^3}\frac{E(y)}{A(y)}\frac{f_{1T}^\perp}{f_1}, \quad (3.50)$$

$$\langle\sin(3\phi-\phi_S)\rangle_{UT} \approx \kappa_M^2 \frac{|\vec{k}_\perp|^3}{2M^3}\frac{E(y)}{A(y)}\frac{f_{1T}^\perp}{f_1}, \quad (3.51)$$

and the modification factors for the two twist-2 asymmetries given by Eqs. (3.32)–(3.33) become

$$\alpha_{UT} \approx \frac{|\vec{k}_\perp|^2}{M^2}\left[-\frac{\partial\ln f_1}{\partial\ln x}+\frac{\partial\ln f_{1T}^\perp}{\partial\ln x}\right], \quad (3.52)$$

$$\alpha_{LT} \approx \frac{|\vec{k}_\perp|^2}{M^2}\left[\frac{4E(y)}{A(y)}-\frac{\partial\ln f_1}{\partial\ln x}-\frac{\partial\ln g_{1T}^\perp}{\partial\ln x}\right], \quad (3.53)$$

$$\alpha_{UU} \approx \frac{|\vec{k}_\perp|^2}{M^2}\left[\frac{4E(y)}{A(y)}-\frac{\partial\ln f_1}{\partial\ln x}\right]. \quad (3.54)$$

For the twist-4 azimuthal asymmetries, by comparing Eqs. (3.50)–(3.51) with the twist-2 part of the Sivers asymmetry given by Eq. (3.32), we see that the asymmetries are suppressed by a factor $|\vec{k}_\perp|^2/Q^2$ and might be significant in the energy regions currently available to experiments [6–12].

For the modification factors α given by Eqs. (3.52)–(3.54), we see that the first term in the square bracket for α_{LT} or α_{UU} can already reach 4. To get a feeling of how large their magnitudes could be, in Fig. 2, we plot $\tilde{\alpha}_{UU}=M^2\alpha_{UU}/|\vec{k}_\perp|^2$ and $\tilde{\alpha}_{UT}=M^2\alpha_{UT}/|\vec{k}_\perp|^2$ as functions of x given by Eqs. (3.52) and (3.54) using the parametrizations

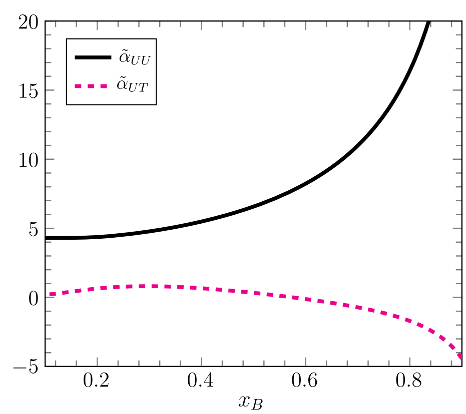

FIG. 2. Estimation of the modification factor $\tilde{\alpha}_{UU}$ or $\tilde{\alpha}_{UT}$ as a function of x at $y=0.5$.

available for $f_{1T}^{\perp}$ given in Ref. [30]. The parametrization of f_1 is taken from Ref. [31]. In obtaining the figure, only light flavors were taken into account. We see from Fig. 2 that these modifications could be quite significant.

D. Contributions from the four-quark correlator

The calculations presented so far are made for $e^-N \to e^-qX$ where quark-j-gluon-quark correlators ($j = 0, 1, 2, \cdots$) are included. At the twist-4 level, there are also contributions from processes with four-quark correlator defined as

$$\hat{\varphi}^{(0)}_{(4q)}(k_1, k, k_2) = \frac{g^2}{16}\int \frac{d^4y}{(2\pi)^4}\frac{d^4y_2}{(2\pi)^4}\frac{d^4y_1}{(2\pi)^4} e^{i(k_2-k)y_1}e^{i(k-k_1)y_2}e^{ik_1y} \times \langle N|[\bar{\psi}(0)\mathcal{L}(0,y_1)\psi(y_1)][\bar{\psi}(y_2)\mathcal{L}(y_2,y)\psi(y)]|N\rangle. \tag{3.55}$$

This comes from the four-quark diagrams (Fig. 3) widely studied for the inclusive reaction $e^-N \to e^-X$ [13,15]. They contribute to $e^-N \to e^-gX$ if the cut is at the middle and to $e^-N \to e^-qX$ if the cut is at the left or right. In experiments, it is difficult to differentiate between $e^-N \to e^-qX$ and $e^-N \to e^-gX$, as both give rise to $e^-N \to e^- + \text{jet} + X$.

It can be shown that the collinear expansion is also applicable in this case and the gauge links included in Eq. (3.55) are obtained during the expansion by taking the multiple gluon scattering into account. Up to twist-4, we need only to consider the leading power contribution of $\hat{\varphi}^{(0)}_{(4q)}$. The calculations are essentially the same as those for the inclusive process [13,15]. The only difference is that a $k_\perp$ dependence in the correlation functions is not integrated. We present the contributions $e^-N \to e^-gX$ and $e^-N \to e^-qX$ separately. They both take the form

$$\tilde{W}^{(g/q)}_{(4q)\mu\nu} = \frac{2}{p\cdot q}\int dx_1dx_2dx h^{g/q}_{4q}(g_{\perp\mu\nu}C_s - i\epsilon_{\perp\mu\nu}C_{ps}), \tag{3.56}$$

where C_s and C_{ps} are two TMD four-quark correlation functions. They depend on, besides p and S of the nucleon, the longitudinal variables x_1, x, and x_2, and $k_\perp$ of the gluon. $C_s = C_{vv} + C_{aa}$ and $C_{ps} = C_{va} + C_{av}$ are scalar and pseudoscalar respectively, and C_{ij} is given by

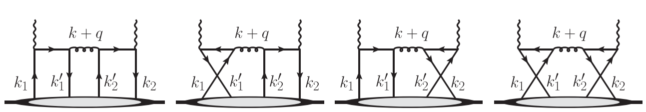

FIG. 3. Four-quark diagrams without multiple gluon scattering. The quark 4-momenta are k_1, k_2, $k_1' = k_1 - k$ and $k_2' = k_2 - k$ respectively.

$$C_{ij} = \int d^4k_1d^4kd^4k_2\delta(k_1^+ - x_1p^+)\delta(k_2^+ - x_2p^+) \times \delta(x - k^+/p^+)\delta^2(k_\perp - k'_\perp)\varphi^{(0)}_{(4q)ij}(k_1, k, k_2; p, S). \tag{3.57}$$

The unintegrated correlation function $\varphi^{(0)}_{(4q)ij}$ is defined as

$$\varphi^{(0)}_{(4q)ij} = \frac{g^2}{16}\int \frac{d^4y}{(2\pi)^4}\frac{d^4y_2}{(2\pi)^4}\frac{d^4y_1}{(2\pi)^4} e^{i(k_2-k)y_1}e^{-i(k_1-k)y_2}e^{ik_1y} \times \langle N|[\bar{\psi}(0)\Gamma_i^+\mathcal{L}(0,y_1)\psi(y_1)] \times [\bar{\psi}(y_2)\Gamma_j^+\mathcal{L}(y_2,y)\psi(y)]|N\rangle, \tag{3.58}$$

and $\Gamma_V^\mu = \gamma^\mu$, $\Gamma_A^\mu = \gamma^\mu\gamma_5$.

The coefficient $h^{g/q}_{4q}$ is determined by the hard part after collinear expansion and is a function of the longitudinal variables x_1, x and x_2. It is a sum of four terms corresponding to the four graphs in Fig. 3. For $e^-N \to e^-gX$, we have

$$h^g_{4q} = \frac{\delta(x - x_B)}{(x_2 - x_B - i\epsilon)(x_1 - x_B + i\epsilon)} + \frac{\delta(x - x_B)}{(x_2 + i\epsilon)(x_1 - i\epsilon)} + \frac{\delta(x - x_B)}{(x_2 - x_B - i\epsilon)(x_1 - i\epsilon)} + (1 \leftrightarrow 2)^*, \tag{3.59}$$

where $(1 \leftrightarrow 2)$ denotes the left-neighboring term after exchange of 1 and 2. For $e^-N \to e^-qX$, $h^q_{4q} = h^{qL}_{4q} + h^{qR}_{4q}$,

$$h^{qL}_{4q} = \frac{\delta(x_1 - x_B)}{(x_2 - x_B - i\epsilon)(x - x_B - i\epsilon)} - (x_2 \to x - x_2) - \frac{\delta(x - x_1 - x_B)}{(x_2 - x_B - i\epsilon)(x - x_B - i\epsilon)} + (x_2 \to x - x_2), \tag{3.60}$$

and $h^{qR}_{4q}(x_1, x, x_2) = h^{qL*}_{4q}(x_2, x, x_1)$. Adding all of them together, we obtain $h_{4q} = h^q_{4q} + h^g_{4q}$ that is exactly the result given by Eqs. (65)–(66) in Ref. [15] for the inclusive reaction.

After the integration over x_1, x, and x_2, C_s and C_{ps} reduce to functions of x_B, $k_\perp$, p and S. They are decomposed as

$$\int dx_1dxdx_2h_{4q}C_s = M^2\left(f_{4q} - \frac{\varepsilon_\perp^{kS}}{M}f^\perp_{4qT}\right), \tag{3.61}$$

$$\int dx_1dxdx_2h_{4q}C_{ps} = M^2\left(\lambda_h f_{4qL} - \frac{k_\perp\cdot S_\perp}{M}f^{\perp 2}_{4qT}\right). \tag{3.62}$$

We obtain their contributions to structure functions as

$$W_{UU,T;4q} = 4x^2\kappa_M^2 f_{4q}, \tag{3.63}$$

$$W_{LL;4q} = -4x^2\kappa_M^2 f_{4qL}, \tag{3.64}$$

$$W_{UT,T;4q}^{\sin(\phi-\phi_S)} = 4x^2\kappa_M^2 \frac{|\vec{k}_\perp|}{M} f_{4qT}^\perp, \tag{3.65}$$

$$W_{LT;4q}^{\cos(\phi-\phi_S)} = -4x^2\kappa_M^2 \frac{|\vec{k}_\perp|}{M} f_{4qT}^{\perp 2}. \tag{3.66}$$

We see that they contribute to the unpolarized and double-longitudinally polarized structure functions, the Sivers asymmetry and so on. They behave as addenda to f_{+3dd}, f_{+3ddL}, $f_{+3ddT}^\perp$, and $f_{+3ddT}^{\perp 3}$ defined via the quark-two-gluon-quark correlator. They all vanish at $g = 0$ and bring no change to the discussions given in the last subsections.

E. Reducing to inclusive DIS

After we integrate over $k_\perp$ and apply the constraints from time-reversal invariance, we obtain the cross section and structure functions for the inclusive DIS process. The results are given as follows:

$$F_{UU,T}^{\rm in} = xf_1 + 4x^2\kappa_M^2(f_{+3dd} + f_{4q}), \tag{3.67}$$

$$F_{UU,L}^{\rm in} = 8x^3\kappa_M^2 f_3, \tag{3.68}$$

$$F_{LL}^{\rm in} = xg_{1L} + 4x^2\kappa_M^2(f_{+3ddL} - f_{4qL}), \tag{3.69}$$

$$F_{LT}^{{\rm in},\cos\phi_S} = -2x^2\kappa_M g_T, \tag{3.70}$$

where the PDFs are the $k_\perp$-integrated ones and are functions of x only. Comparing with Eqs. (2.33)–(2.36), we obtain that,

$$xF_1 = [xf_1 + \gamma^2(f_{+3dd} + f_{4q})]/2, \tag{3.71}$$

$$F_2 = 2x(F_1 + \gamma^2 f_3)/(1+\gamma^2), \tag{3.72}$$

$$xg_1 = [xg_{1L} + \gamma^2(f_{+3ddL} - f_{4qL}) + \gamma^2 xg_T]/2(1+\gamma^2), \tag{3.73}$$

$$g_1 + g_2 = g_T/2. \tag{3.74}$$

From these results, we see twist-4 contributions to the violation of Callan-Gross relations and so on.

IV. SUMMARY

In summary, benefitting from the collinear expansion, we carried out the calculations up to twist-4 and presented for the first time the complete twist-4 result for $e^-N \to e^-qX$ with a polarized beam and target. The results show that, among the 18 structure functions, besides the eight that have only twist-3 contributions, the other ten have twist-4 contributions. We showed in particular that among these twist-4 contributions, four correspond to azimuthal asymmetries where twist-4 are the leading power contributions in $e^-N \to e^-qX$ and can serve as good places to study these twist-4 effects. We also showed that for all eight twist-2 structure functions for $e^-N \to e^-hX$ that correspond to the eight twist-2 TMD PDFs, there are twist-4 addenda to them. These twist-4 contributions could be quite significant and have a strong impact on the study of TMD PDFs in particular in the energy regions of existing DIS experiments such as HERMES and those in JLab. We suggested an approximate way to obtain rough estimations of twist-4 contributions using corresponding twist-2 PDFs.

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