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Is the exotic 0^{--} glueball a pure gluon state?

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We present a new calculation of the mass and width of the exotic 0^{--} glueball in the framework of the QCD sum rules. We next construct a new current which couples to a pure 0^{--} gluon state and derive consistent and stable sum rules. A previously used current in this approach was shown to be inconsistent. We obtain for this state a mass $M_G = 6.3^{+0.8}_{-1.1}$ GeV and an upper limit for the total width $\Gamma_G \leq 235$ MeV. These values can be used as an important guide for the experimental search of this exotic state. We argue that the mixing of this glueball state with the 0^{--} tetraquark is very small. Therefore, the exotic 0^{--} glueball can be considered as a pure gluon state.

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Glueballs carry very important information on the gluonic sector of QCD, and their study is one of the fundamental tasks in strong-interaction physics. While glueballs are predicted by QCD, there has been no clear experimental evidence of their existence, and so they remain as of yet a subject of theoretical and experimental research (see Refs. [1,2]). For this reason, the study of glueball candidates is included in many programs of presently running and future experiments.

One of the main problems of glueball spectroscopy is the mixing of the glueballs with ordinary meson states, which leads to difficulties in disentangling the glueball components in experiments. In this connection, the discovery of the exotic 0^{--} glueball would be extremely useful, because it does not mix with any $q\bar{q}$ states. It is therefore very important to investigate the properties of this glueball within a QCD-based approach. One of the most successful approaches to studying strong-interaction spectroscopy is the QCD sum rules (SRs) method [3].

In this paper, for the first time, a consistent SR for the exotic 0^{--} glueball is obtained. We calculate the operator product expansion (OPE) of the correlator up to dimension 8 with a new interpolating current which couples to this pure gluon state, and show that there is good stability for the SR. From this stable SR, a prediction for the mass and an upper limit of the total width of this state are found.

The QCD SR approach [3] for a bound state consists of two parts. One is the calculation of the OPE of the correlator defined by

$$\Pi(Q^2) = i\int d^4x e^{iqx}\langle J(0)J^\dagger(x)\rangle, \tag{1}$$

where the current couples to the gluonic bound state $|G\rangle$ in our case as

$$\langle 0|J|G\rangle = F_G M_G^{N-2}.$$

Here $Q^2 = -q^2$, N is the dimension of the current J, F_G is the decay constant, and M_G is the mass of the state. To construct the SR, we follow for the second part—usually called the phenomenological part—the pioneering work of Ref. [3] and the recent study of the scalar and pseudoscalar glueballs by Forkel [4]. Putting these pieces together, the corresponding SR for a zero-width resonance model of the spectral density, $[\rho \sim \delta(s - M_G^2) + \text{continuum}]$, has the following form:

$$\frac{1}{\pi}\int_0^{s_0} \frac{\mathrm{Im}\Pi_{(\mathrm{OPE})}(-s)}{s+Q^2}ds = \frac{F_G^2 M_G^{2(N-2)}}{M_G^2+Q^2}, \tag{2}$$

where $\Pi_{(\mathrm{OPE})}(-s)$ is the OPE of the correlator, Eq. (1), and s_0 is the continuum threshold. It is known that the 0^{--} state cannot couple to a three-gluon interpolating current without derivatives [5]. In Ref. [6], a very specific current with derivatives has been constructed to obtain the mass of the three-gluon exotic glueball. However, in Ref. [7], it has been demonstrated that this current leads to the inconsistency of QCD SRs. Here we propose a new gauge invariant current with derivatives which couples to the 0^{--} state. It has the general form

$$J(x) = \frac{2}{3}g_s^3\epsilon^{ijk}\mathrm{Tr}((O_iG_{\mu\nu}(x))(O_jG_{\nu\rho}(x))(O_kG_{\rho\mu}(x))), \tag{3}$$

where $G^a_{\mu\nu}$ is the field-strength tensor, $\tilde{G}^a_{\mu\nu} \equiv G^a_{\alpha\beta}i\epsilon_{\mu\nu\alpha\beta}/2$, and the operators O_i are the products of covariant

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derivatives $O_i = D_{\alpha_1} \cdots D_{\alpha_n}$. The lowest-dimensional current in this form that has a nonzero LO perturbative contribution to the SR corresponds to

$$\begin{aligned} O_1 G_{\mu\nu}(x) &= D_{\alpha_1} D_{\alpha_2} D_{\alpha_3} \tilde{G}_{\mu\nu}(x), \\ O_2 G_{\mu\nu}(x) &= D_{\alpha_1} D_{\alpha_2} G_{\mu\nu}(x), \\ O_3 G_{\mu\nu}(x) &= D_{\alpha_3} G_{\mu\nu}(x). \end{aligned} \tag{4}$$

In general, one might construct other interpolating currents which couple to the 0^{--} state and include four gluons [8], for example. However, the consideration of these states is beyond of the scope of our paper and will be the subject of our future study. The coefficient in the current Eq. (3) was chosen to have the leading term in the following form:

$$J(x) \overset{\text{LO}}{=} g_s^3 d^{abc} \tilde{G}^a_{\mu\nu;\tau_1\tau_2\tau_3}(x) G^b_{\nu\rho;\tau_1\tau_2}(x) G^c_{\rho\mu;\tau_3}(x), \tag{5}$$

where $G^a_{\mu\nu;\tau_1\tau_2\cdots\tau_n} = \partial_{\tau_1}\partial_{\tau_2}\cdots\partial_{\tau_n} G^a_{\mu\nu}$. Using this current with Eqs. (3) and (4), we have calculated the OPE of the correlator up to the dimension-8 operators, given by

$$\begin{aligned} \Pi_{(\text{OPE})}(Q^2) &= \Pi_{(\text{pert})} + \Pi_{(\text{G3})} + \Pi_{(\text{G4})} + \cdots \\ &= \frac{-5\alpha_s^3}{11!4\pi} Q^{20} L \\ &\quad + \frac{-5\pi\alpha_s^3}{3^3 2^5} Q^{14} \left(\langle gG^3 \rangle - \frac{\langle J^2 \rangle}{4}(5 + 2L) \right) \\ &\quad + \frac{205\pi^2\alpha_s^2}{2^6 3^2} Q^{12} L \langle \alpha_s^2 G^4 \rangle + \cdots, \end{aligned} \tag{6}$$

where $\alpha_s = g_s^2/(4\pi)$ is the coupling constant, $L = \ln(Q^2/\mu^2)$, μ^2 is the renormalization scale, the dimension-6 condensates are $\langle gG^3 \rangle = \langle g f^{abc} G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu} \rangle$ and $\langle J^2 \rangle = \langle J^a_\mu J^a_\mu \rangle$ with the quark current $J^a_\mu = \bar{q}\gamma_\mu t^a q$, and the dimension-8 condensate is

$$\langle \alpha_s^2 G^4 \rangle = \langle (\alpha_s f^{abc} G^b_{\mu\nu} G^c_{\alpha\beta})^2 \rangle - 2 \langle (\alpha_s f^{abc} G^b_{\mu\nu} G^c_{\nu\beta})^2 \rangle.$$

We adopt the Mathematica package FEYNCALC [9] to handle the algebraic manipulation.

In contrast with the previous study [6] mentioned above, we have a positive LO imaginary part, and therefore, we expect a consistent SR. We would like to emphasize that the so-called direct instantons, which affect strongly the SRs for the 0^{++} and 0^{-+} two-gluon states [4,10,11], do not contribute in this case due to the symmetric color structure of the current, Eq. (3).

Following the method developed in Ref. [3], we apply the Borel transformation $\hat{B}$,

$$\hat{B}_{Q^2 \to M^2}[\Pi(Q^2)] = \lim_{n\to\infty} \frac{(-Q^2)^n}{\Gamma(n)} \left[\frac{d^n}{dQ^{2n}} \Pi(Q^2) \right]_{Q^2 = nM^2},$$

to both sides of the SR, Eq. (2). Using the Borel transformation allows us to reduce the SR uncertainties by suppression of the contributions from excited resonances and higher-order OPE terms. After the Borel transformation, the new sum rule is

$$\sum_t \mathcal{R}_0^t(M^2, s_0) = \mathcal{R}_0^{(\text{res})}(M^2, s_0), \tag{7}$$

where M^2 is the Borel parameter,

$$\begin{aligned} \mathcal{R}_0^t(M^2, s_0) &= \frac{1}{\pi} \int_0^{s_0} ds \operatorname{Im}\Pi_t(-s) e^{-s/M^2}, \\ \mathcal{R}_0^{(\text{res})}(M^2, s_0) &= M_G^{20} F_G^2 e^{-M_G^2/M^2}, \end{aligned}$$

and Π_t denotes the different contributions to OPE of the correlator: the perturbative term (pert), and the dimension-6 (G3) and dimension-8 (G4) nonperturbative terms. To extract the mass from the SR, we use a family of derivative SRs obtained by differentiation with respect to the Borel parameter M^2:

$$\mathcal{R}_k^t(M^2, s_0) = M^4 \frac{d}{dM^2} \mathcal{R}_{k-1}^t(M^2, s_0).$$

We define the difference of the OPE result and the continuum contribution as

$$\begin{aligned} &\mathcal{R}_k^{(\text{SR})}(M^2, s_0) \\ &= \mathcal{R}_k^{(\text{pert})}(M^2, s_0) + \mathcal{R}_k^{(\text{G3})}(M^2, s_0) + \mathcal{R}_k^{(\text{G4})}(M^2, s_0). \end{aligned}$$

Then the master sum rule ($k = 0$) and the derivative SRs ($k > 0$) can be expressed by the following equations:

$$\mathcal{R}_k^{(\text{SR})}(M^2, s_0) \approx \mathcal{R}_k^{(\text{res})}(M^2, s_0). \tag{8}$$

The fiducial window $M^2 \in [M_-^2, M_+^2]$ is limited by the conditions that insure the reliability of the resonance model and the OPE; i.e.,

$$\begin{aligned} &|\mathcal{R}_k^{(\text{G4})}(M^2, \infty)| / \mathcal{R}_k^{(\text{SR})}(M^2, \infty) < 1/3, \\ &\frac{\mathcal{R}_k^{(\text{res})}(M^2, s_0)|}{\mathcal{R}_k^{(\text{SR})}(M^2, \infty)} \approx \frac{\mathcal{R}_k^{(\text{SR})}(M^2, s_0)|}{\mathcal{R}_k^{(\text{SR})}(M^2, \infty)} > \frac{1}{10}. \end{aligned} \tag{9}$$

Then the QCD SRs for the mass and the decay constant can be presented in the form

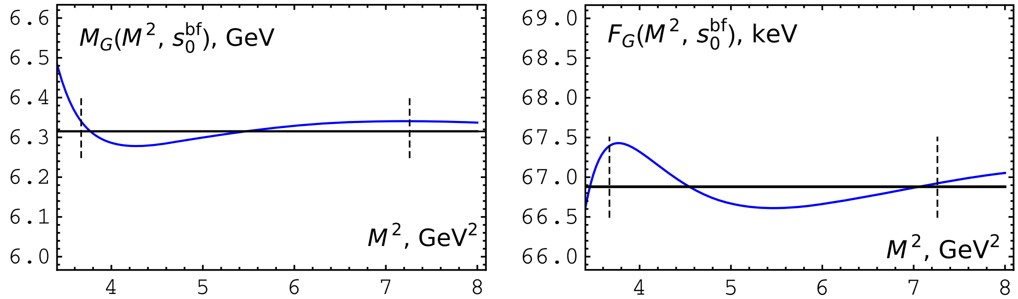

FIG. 1. We show the dependence on the Borel parameter of the mass (left panel) and the decay constant (right panel) for the central value of the gluon condensate and best-fit value of the threshold s_0^{bf}. Both panels are given for the $k = 0$ case. The vertical lines denote the fiducial interval of the Borel parameter where conditions of confidence, Eq. (9), are saturated. The horizontal lines denote average values at fiducial interval.

$$M_G^k(M^2, s_0) = \sqrt{\frac{\mathcal{R}_{k+1}^{(\mathrm{SR})}(M^2, s_0)}{\mathcal{R}_k^{(\mathrm{SR})}(M^2, s_0)}},$$
$$F_G^k(M^2, s_0) = \frac{\sqrt{e^{M_G^2/M^2}\mathcal{R}_k^{(\mathrm{SR})}(M^2, s_0)}}{M_G^{10}}. \qquad (10)$$

We define the mass and decay constant by minimization of the criteria $\delta_k(s_0^{\mathrm{bf}}) = \delta_k^{\min}$ with respect to the threshold s_0 and find the best-fit value s_0^{bf}:

$$\delta_k(s_0) = \frac{\max|M_G^k(M_i^2, s_0) - M_G^k(s_0)|}{M_G^k(s_0)},$$
$$M_G^k(s_0) \equiv \frac{1}{n+1}\sum_{i=0}^{n} M_G^k(M_i^2, s_0),$$

where we consider $n = 20$ points in the fiducial interval $M_i^2 = M_-^2 + (M_+^2 - M_-^2)i/n$. In Fig. 1, we present the $k = 0$ results for the glueball mass and decay constant as a function of the Borel parameter. As one can see, we have a rather good stability plateau for both quantities.

Finally, we define the decay constant and mass as an average in the fiducial interval for the best-fit value of the threshold:

$$M_G = M_G^k(s_0^{\mathrm{bf}}),$$
$$F_G^2 = \frac{1}{n+1}\sum_{i=1}^{n}\frac{e^{M_G^2/M_i^2}}{M_G^{20}}\mathcal{R}_k^{(\mathrm{SR})}(M_i^2, s_0^{\mathrm{bf}}).$$

We next follow the common practice of the renormalization group improvement of the SR: in $\mathrm{Im}\Pi_t(-s)$ all coupling constants are replaced by $\alpha_s \to \alpha_s(M^2)$. We use the strong coupling constant

$$\alpha_s(Q^2) = \frac{4\pi}{b_0 \ln(Q^2/\Lambda_{\mathrm{QCD}}^2)},$$

with $b_0 = 11 - 2N_f/3$ and QCD scale $\Lambda_{\mathrm{QCD}} = 300$ MeV. Since we are working in gluodynamics, we set the number of flavors $N_f = 0$ and eliminate the quark and quark-gluon condensate contributions. The dimension-6 three-gluon condensate $\langle gG^3\rangle$ does not contribute here due to absence of the correspondent $\ln(Q^2)$ terms in the correlator, Eq. (6). For the dimension-8 gluon condensate, the hypothesis of vacuum dominance yields the relation

$$\langle\alpha_s^2 G^4\rangle = \frac{3}{2^4}\langle\alpha_s G^2\rangle^2.$$

In our case, the mass of the exotic glueball is determined by the squared value of the gluon condensate $\langle\alpha_s G^2\rangle = \langle\alpha_s G^a_{\mu\nu}G^a_{\mu\nu}\rangle$. Unfortunately, this value is not well known. Following the analyses carried out in Refs. [12–15], we take

$$\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle = 0.012 \pm 0.006\ \mathrm{GeV}^4.$$

Implementing the QCD SR analysis described above, we obtain for the prediction of the mass and the decay constant from the $k = 0$ SR [see Eqs. (8) and (10)]

$$M_G = 6.3^{+0.8}_{-1.1}\ \mathrm{GeV}, \qquad F_G = 67 \pm 6\ \mathrm{keV}. \qquad (11)$$

The mass and decay constant estimates for the higher values of $k = 1, 2, 3$ are in agreement, within the error bars, with the $k = 0$ case considered. The SR analysis in full QCD (number of flavors $N_f = 3$ and nonzero quark condensate $\langle J^2\rangle$) leads to a reduction of the glueball mass by 0.2 GeV. The mass of the exotic glueball in Eq. (11) is not far away from the recent unquenched lattice result $M_G = 5.166 \pm 1.0$ GeV [16] obtained with a rather large pion mass $m_\pi = 360$ MeV.

Here we would like to note that there are three sources of uncertainties in the above analysis for the mass and decay constant: (i) the variation of the gluon condensate, (ii) the stability of the SR triggering the Borel parameter M^2 dependence in terms of the criteria $\delta_k^{\min}$, and (iii) the roughly estimated SR uncertainty coming from the OPE truncation. The latter uncertainty for the decay constant comes from the definition of the fiducial interval, Eq. (9), in the standard assumption that the contribution from the missing terms is of the order of the last included nonperturbative term squared: $(1/3)^2 \sim 10\%$. The same error for the mass can be expected to be suppressed, since the related errors for $\mathcal{R}_{k+1}^{(\mathrm{SR})}$ and $\mathcal{R}_{k}^{(\mathrm{SR})}$ are correlated. The presumable underestimation of uncertainties related to the OPE truncation is unlikely due to a conservative choice of the gluon condensate uncertainty. The considered three sources of uncertainty can be given in percentages of the final uncertainty for the mass and the decay constant:

$$M_G = 6.3^{+12\%}_{-17\%} \pm 0.5\% \pm 0\%\ \mathrm{GeV},$$
$$F_G = 67^{+2\%}_{-3\%} \pm 0.6\% \pm 5\%\ \mathrm{keV},$$

where the first uncertainty is related to gluon condensate variation, the second is representing the stability of SR, and the third is OPE truncation uncertainty.

The best-fit threshold value is $s_0^{\mathrm{bf}} = 52.4^{+12.6\%}_{-16.2\%}\ \mathrm{GeV}^2$ when only the uncertainty of the gluon condensate is included. Note that the fiducial interval for the central value of the gluon condensate is $M^2 \in [3.7, 7.3]\ \mathrm{GeV}^2$.

The glueball width can be estimated in the QCD SR approach also using the broad resonance distribution. The good stability of the zero-width-resonance-based SR, Eq. (2), shows that we can extract only the upper limit of the glueball width from the QCD SR. The simplest way to introduce the width is by using unit step functions [11]:

$$\mathrm{Im}\Pi^{(\mathrm{res2})}(-s) = \frac{\pi(m^2)^{N-2} f^2}{2m\Gamma}\left(\Theta(s - m^2 + m\Gamma) - \Theta(s - m^2 - m\Gamma)\right).$$

Requiring that the stability of the broad-resonance-based SR be better than the stability of zero-width-resonance-based SR,

$$\max\left|1 - \frac{\mathcal{R}_k^{(\mathrm{res2})}(M_i^2, s_0)}{\mathcal{R}_k^{(\mathrm{SR})}(M_i^2, s_0)}\right| \leq \max\left|1 - \frac{\mathcal{R}_k^{(\mathrm{res})}(M_i^2, s_0)}{\mathcal{R}_k^{(\mathrm{SR})}(M_i^2, s_0)}\right|,$$

we obtain an upper limit for the glueball width, $\Gamma_G \leq 235$ MeV. The used stability test was chosen for the simplicity and transparency of the width estimation, keeping the level of accuracy at the level of SR accuracy for mass and decay constant. In the new SR, we vary only the width value while the values for condensate, mass, and decay constant remain fixed. The Borel parameter value is varied in the interval $M_i^2 \in [3.7, 7.3]\ \mathrm{GeV}^2$. This result indicates that the 0^{--} glueball should be rather narrow. Therefore, it can be seen in the appropriate experiments.

By quantum numbers the exotic glueball could mix with the exotic 0^{--} tetraquark. However, a recent study with QCD SR for this tetraquark has obtained a small mass, $M_{\mathrm{tetra}} = 1.66 \pm 0.14$ GeV [17]. The large mass difference between the two states leads us to expect a very small mixing between them. Thus, we can consider the exotic 0^{--} glueball as a pure gluon state.

Summarizing, we have presented a QCD SR study for the exotic three-gluon glueball state with quantum numbers $J^{PC} = 0^{--}$ using a new interpolating current. We have analyzed the QCD SR consisting of contributions of operators up to dimension 8 and have obtained an estimation of the mass, the decay constant, and an upper limit for the width of the exotic glueball. These results provide a clear guide for the search of this important state in the experiments.

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