Horava-Lifshitz-like Gross-Neveu model

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We describe a Horava-Lifshitz-like reformulated four-fermion Gross-Neveu model describing the dynamics of two-component spinors in $(2+1)$-dimensional space-time. Within our study, we introduce the Lagrange multiplier, study the gap equation (including the finite temperature case) which turns out to display essentially distinct behaviors for even and odd values of the critical exponent z, and show that the dynamical parity breaking occurs only for the odd z. We demonstrate that for any odd z, there exists a critical temperature at which the dynamical parity breaking disappears. Besides of this, we obtain the effective propagator and show that the resulting effective theory is renormalizable within the framework of the $\frac{1}{N}$ expansion for all values of z. As one more application of the dynamical parity breaking, we consider coupling of the vector field to the fermions in the case of a simplified spinor-vector coupling and discuss the generation of the Chern-Simons term.

I. INTRODUCTION

The Horava-Lifshitz (HL) approach [1] has recently received great scientific attention. This approach is characterized by the essential asymmetry between space and time coordinates: the equations of motion of the theory are invariant under the rescaling $x^i \to b x^i$, $t \to b^z t$, where z, the critical exponent, is a number characterizing the ultraviolet behavior of the theory. The main reason for it is that with the HL-like reformulation of the known field theory models with a nontrivial critical exponent $z > 1$ an essential improvement of the renormalization behavior of these models takes place. In particular, the four-dimensional gravity becomes renormalizable at $z = 3$. Different issues related to the HL gravity, including its cosmological aspects [2], exact solutions [3], and black holes [4] were considered in a number of papers.

At the same time, the study of the impacts of the HL extension of other field theory models is a very interesting problem. Some aspects of the HL generalizations for the gauge field theories were presented in Ref. [5]. Renormalizability of the HL-like scalar field theory models has been discussed in detail in Ref. [6], and an explicit study of their renormalization was carried out in Ref. [7]. The Casimir effect for the HL-like scalar field theory has been considered in Ref. [8]. In Ref. [9], the HL modifications of the CP^{N-1} theory were studied. The purely scalar analogue

of the sigma model has been discussed in Ref. [10]. Different examples of calculating the effective potential in HL-like theories involving scalars coupled to gauge and/or spinor fields were presented in Ref. [11].

Many other interesting aspects of HL-like fermionic theories of spinor fields can be studied as well. Indeed, as it is well known, namely different couplings within the four-fermion model give rise to the phenomenon of the emergent dynamics which possibly allows to treat the photon as a Goldstone boson and to develop other phenomenologically interesting concepts [12]. Actually, the original Gross-Neveu model [13] also was introduced in order to study the emergent dynamics, and, to be more precise, to discuss the dynamical parity breaking taking place due to the emergence of the nontrivial mass term for spinors. Therefore, it would be natural to generalize studies of emergent dynamics and dynamical parity breaking for the HL-like theories with a nontrivial z. Some preliminary studies of this issue have been carried in Ref. [14].

Here, we will study the contributions of HL-like spinor fields to an effective dynamics for the Lagrange multiplier, thus establishing the HL-like Gross-Neveu model, and discuss its renormalizability within the framework of the $\frac{1}{N}$ expansion. As one more illustration, we consider the case when the HL-like spinor field is coupled to the vector field. For this coupling, the Maxwell-like terms were earlier generated in Ref. [15], and it remains to study the possibility to carry out the perturbative generation of the Chern-Simons term as well, which we perform in this paper for a some simplified theory.

The structure of the paper is as follows. In Sec. II we present the HL-like Gross-Neveu model. In Sec. III we

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obtain the effective propagators and gap equation for the Lagrange multiplier and prove the renormalizability of the resulting theory. In Sec. IV, we carry out the perturbative generation of the Chern-Simons term. Finally, in Sec. V we discuss our results and perspectives.

II. GROSS-NEVEU MODEL

In this paper, the main object of our study is the Gross-Neveu model [13,16]. Its natural HL-like generalization, for the set of N spinor fields, looks like

$$S = \int dt d^2x \sum_{j=1}^{N} \left[\bar{\psi}_j (i\gamma^0 \partial_0 + (i\gamma^i \partial_i)^z) \psi_j - \frac{g}{2N} (\bar{\psi}_j \psi_j)^2 \right],$$
(1)

where $j = 1, 2, \ldots, N$. The number of spatial derivatives acting on the spinor fields is equal to z, that is, the critical exponent for the spinor fields. We will study the effective dynamics of this model within the framework of the $\frac{1}{N}$ expansion, along the lines developed in Ref. [17] for the Lorentz-invariant case, and using conventions adopted in Ref. [17].

First of all, we introduce the Lagrange multiplier, that is, an auxiliary scalar field σ allowing to avoid the four-fermion coupling, so that the action takes the form

$$S = \int dt d^2x \left[\sum_{j=1}^{N} [\bar{\psi}_j (i\gamma^0 \partial_0 + (i\gamma^i \partial_i)^z - \sigma) \psi_j] + \frac{N}{2g} \sigma^2 \right].$$
(2)

Now, we introduce a mass through the shift $\sigma \to \sigma + m^z$ where m has mass dimension one, so that the vacuum expectation of σ turns out to be equal to zero. We get

$$S = \int dt d^2x \left[\sum_{j=1}^{N} [\bar{\psi}_j (i\gamma^0 \partial_0 + (i\gamma^i \partial_i)^z - m^z - \sigma) \psi_j] \right.$$
$$\left. + \frac{N}{2g} (\sigma^2 + 2\sigma m^z) \right].$$
(3)

Here we disregarded the constant, field-independent additive term. The mass term $m^z \bar{\psi}_j \psi_j$ breaks the parity [17]. Indeed, under the parity transformation, that is, the inversion of the axis x_1, of the form

$$\psi(x_0, x_1, x_2) \to \gamma^1 \psi(x_0, -x_1, x_2)$$
(4)

the mass term changes sign since $(\gamma_1)^2 = -1$ (cf. Ref. [17]); we recall that $\bar{\psi} = \psi^\dagger \gamma^0$. One can note as well that under the parity transformation, the time part of the kinetic term $\bar{\psi} \gamma^0 \partial_0 \psi$ is invariant, and its spatial part $\bar{\psi}(\gamma^i \partial_i)^z \psi$ is invariant for the odd z, including the Lorentz-invariant case $z = 1$, and changes sign for the even z; however, the

case of the odd z, where the kinetic term is completely parity invariant, and the mass term breaks the parity, is the most interesting one for us since just in this case the dynamical mass generation occurs, as we show below.

The propagator of the ψ field is

$$S_{ij}(k) = \langle \bar{\psi}_i(k) \psi_j(-k) \rangle$$
$$= i \frac{\delta_{ij}}{\gamma^0 k_0 + (\gamma^i k_i)^z - m^z} \equiv \delta_{ij} S(k).$$
(5)

We use the signature $(+ - -)$. Now, one should emphasize an important difference between odd and even z. Indeed, for even $z = 2n$, one has $(\gamma^i k_i)^{2n} = (-\vec{k}^2)^n$, and for odd $z = 2n + 1$, one has $(\gamma^i k_i)^{2n+1} = \gamma^i k_i (-\vec{k}^2)^n$. Then, we have

$$z = 2n: \ S(k) = i \frac{\gamma^0 k_0 - [(-\vec{k}^2)^n - m^z]}{k_0^2 - [(-\vec{k}^2)^n - m^z]^2} \equiv i \frac{\gamma^0 k_0 - \omega(\vec{k})}{k_0^2 - \omega^2(\vec{k})},$$

$$z = 2n + 1: \ S(k) = i \frac{\gamma^0 k_0 + \gamma^i k_i (-\vec{k}^2)^n + m^z}{k_0^2 - (\vec{k}^2)^z - m^{2z}}$$

$$\equiv i \frac{\gamma^0 k_0 + \gamma^i k_i (-\vec{k}^2)^n + m^{2n+1}}{k_0^2 - \Omega^2(\vec{k})}.$$
(6)

These propagators will be further used to study quantum corrections in our theory.

III. DYNAMICS OF THE LAGRANGE MULTIPLIER

A. Gap equation in the zero-temperature case

To study the dynamics of the Lagrange multiplier σ, we start with the gap equation, i.e., let us impose a condition that the vacuum expectation $\langle \sigma \rangle = 0$. Graphically, it is represented by the Feynman diagram given in Fig. 1.

Here the sign $\times$ means the constant m^z originated from the action (3), the solid line is for the ψ field, and the dashed one is for the σ field. This equation means that, given the effective potential

$$V_{\text{eff}} = -\frac{\sigma^2}{2g} + i\text{tr} \int \frac{dk_0 d^2\vec{k}}{(2\pi)^3} \ln \left(\gamma^0 k_0 + (\gamma^i k_i)^z - \sigma \right), \quad (7)$$

we are interested in finding the minima, that is, in solutions of the expression

$$- - - - \times \quad + \quad - - - - \bigcirc = 0.$$

FIG. 1. Gap equation.

$$\frac{dV_{\text{eff}}}{d\sigma}\bigg|_{\sigma=m^z} = -\frac{m^z}{g} - i\text{tr}\int\frac{dk_0 d^2\vec{k}}{(2\pi)^3}\frac{1}{\gamma^0 k_0 + (\gamma^i k_i)^z - m^z} = 0. \tag{8}$$

Then, after Wick rotation, $k_0 \to ik_0$, for even $z = 2n$, we have

$$\frac{m^z}{g} = -\text{tr}\int\frac{dk_0 d^2\vec{k}}{(2\pi)^3}\frac{i\gamma^0 k_0 - [(-\vec{k}^2)^n + m^z]}{k_0^2 + [(-\vec{k}^2)^n + m^z]^2}, \tag{9}$$

and for odd $z = 2n + 1$,

$$\frac{m^z}{g} = -\text{tr}\int\frac{dk_0 d^2\vec{k}}{(2\pi)^3}\frac{i\gamma^0 k_0 + \gamma^i k_i(-\vec{k}^2)^n + m^z}{k_0^2 + (\vec{k}^2)^z + m^{2z}}. \tag{10}$$

Calculation of the trace is straightforward, so that for $z = 2n$, we get

$$\frac{m^z}{g} = 2\int\frac{dk_0 d^2\vec{k}}{(2\pi)^3}\frac{[(-\vec{k}^2)^n + m^z]}{k_0^2 + [(-\vec{k}^2)^n + m^z]^2}, \tag{11}$$

and for $z = 2n + 1$,

$$\frac{m^z}{g} = -2\int\frac{dk_0 d^2\vec{k}}{(2\pi)^3}\frac{m^z}{k_0^2 + (\vec{k}^2)^z + m^{2z}}. \tag{12}$$

Then, because of the integral $\int\frac{dk_0}{2\pi}\frac{1}{k_0^2 + A^2} = \frac{1}{2A}$, we find that for the even z, we get the integral of a constant which vanishes within the dimensional regularization. Thus, for $z = 2n$, we have $m^z = 0$, so, there is no consistent vacuum shift, no mass term, and hence, no dynamical parity breaking in this case. At the same time, at $z = 2n + 1$ one has (here, the factor $\frac{1}{2}$ from the integral over k_0 is canceled by the factor 2 from the trace of the 2×2 unit matrix)

$$\frac{1}{g} = -\int\frac{d^2\vec{k}}{(2\pi)^2}\frac{1}{\sqrt{\vec{k}^{2z} + m^{2z}}}, \tag{13}$$

which, for odd z, relates the mass with the coupling g through the equation

$$m^{z-2} = -\frac{g}{4\pi^{3/2}}\Gamma\left(1 + \frac{1}{z}\right)\Gamma\left(\frac{1}{2} - \frac{1}{z}\right). \tag{14}$$

It is clear that for any $z > 1$ this equation is singularity free. Actually, this expression implies that $g < 0$, for any $z > 1$.

Let us now verify that we indeed have a minimum of the effective potential. To do it, we proceed in a manner similar to Ref. [18]. The derivative of the effective potential, with the mass m^z incorporated into the σ field, is given by the expression

$$\frac{dV_{\text{eff}}}{d\sigma} = -\frac{\sigma}{g} - i\text{tr}\int\frac{dk_0 d^2\vec{k}}{(2\pi)^3}\frac{1}{\gamma^0 k_0 + (\gamma^i k_i)^z - \sigma}. \tag{15}$$

Similar calculations have been performed above with the only difference being that m^z instead of σ was used. So, we can directly apply the result given in Ref. (14) for our case, and arrive at

$$\frac{\partial V_{\text{eff}}}{\partial\sigma} = -\frac{\sigma}{g} - \frac{\sigma^{2/z}}{4\pi^{3/2}}\Gamma\left(1 + \frac{1}{z}\right)\Gamma\left(\frac{1}{2} - \frac{1}{z}\right). \tag{16}$$

This expression can be integrated, so that the result is

$$V_{\text{eff}} = -\frac{\sigma^2}{2g} - \frac{\sigma^{1+2/z}}{4\pi^{3/2}(1+2/z)}\Gamma\left(1 + \frac{1}{z}\right)\Gamma\left(\frac{1}{2} - \frac{1}{z}\right). \tag{17}$$

In the above expression, $[g] = z - 2$, $[\sigma] = z$, and, consequently, $[V_{\text{eff}}] = z + 2$. Then, by rescaling g, σ, and consequently, V_{eff}, by the rules $g \to M^{z-1}g$, $\sigma \to M^{z-1}\sigma$, and $V_{\text{eff}} \to M^{z-1}V_{\text{eff}}$, in order to restore the usual dimensions of fields and couplings, where M is some arbitrary mass scale, we obtain

$$V_{\text{eff}} = -\frac{\sigma^2}{2g} - \frac{M^{2(1-1/z)}\sigma^{1+2/z}}{4\pi^{3/2}(1+2/z)}\Gamma\left(1 + \frac{1}{z}\right)\Gamma\left(\frac{1}{2} - \frac{1}{z}\right), \tag{18}$$

where now $[g] = -1$, $[\sigma] = 1$, and $[V_{\text{eff}}] = 3$. It is clear that for any odd $z \geq 3$ this effective potential will possess the minimum for a nonzero σ (recall that $g < 0$).

B. Gap equation in the finite-temperature case

The above consideration can be generalized to the finite temperature case as well. In the finite temperature case the gap equation, in our theory for odd z, is given by Eq. (12). Then, we introduce the finite temperature through the Matsubara formalism. Afterward, Eq. (12) takes the form

$$\frac{1}{g} = -\frac{2}{\beta}\sum_{l=-\infty}^{\infty}\int\frac{d^2\vec{k}}{(2\pi)^2}\frac{1}{\omega_l^2 + (\vec{k}^2)^z + m^{2z}}, \tag{19}$$

where $\omega_l = (2l + 1)\frac{\pi}{\beta}$. Here β is an inverse temperature, i.e., $\beta = 1/T$, with its canonical mass dimension being equal to $-z$.

Let us first evaluate the integral in Eq. (19), by using the solution

$$\int\frac{d^2\vec{k}}{(2\pi)^2}\frac{1}{\omega_l^2 + (\vec{k}^2)^z + m^{2z}}$$
$$= \frac{1}{4\pi}\Gamma\left(1 + \frac{1}{z}\right)\Gamma\left(1 - \frac{1}{z}\right)(\omega_l^2 + m^{2z})^{\frac{1}{z}-1}. \tag{20}$$

Thus, we get the gap equation

$$\frac{1}{g} = -\frac{1}{2\pi\beta}\Gamma\left(1+\frac{1}{z}\right)\Gamma\left(1-\frac{1}{z}\right)\sum_{l=-\infty}^{\infty}(\omega_l^2+m^{2z})^{\frac{1}{z}-1}. \quad (21)$$

Now, in order to carry out the above summation, let us use the expression [19]

$$\sum_{l=-\infty}^{\infty}[(l+b)^2+a^2]^{-\lambda} = \frac{\sqrt{\pi}\Gamma(\lambda-1/2)}{\Gamma(\lambda)(a^2)^{\lambda-1/2}} + 4\sin(\pi\lambda)f_\lambda(a,b),$$
$$(22)$$

valid for $\lambda < 1$, aside from the poles at $\lambda = 1/2, -1/2, -3/2, \cdots$, with

$$f_\lambda(a,b) = \int_{|a|}^{\infty}\frac{dx}{(x^2-a^2)^\lambda}\mathrm{Re}\left(\frac{1}{e^{2\pi(x+ib)}-1}\right). \quad (23)$$

Note that, here, we have $a = \frac{\beta}{2\pi}m^z$, $b = \frac{1}{2}$, and $\lambda = 1 - \frac{1}{z}$. Thus, for our case (odd z) λ always takes values $\lambda = 0, \frac{2}{3}, \frac{4}{5}, \frac{6}{7}, \ldots, \frac{2n}{2n+1}$, i.e., between 0 and 1, so that we can readily use the relation ([19]). Hence, the integral (23) converges and can be rewritten as

$$f_\lambda\left(a,\frac{1}{2}\right) = \int_{|a|}^{\infty}dx\frac{(\tanh(\pi x)-1)}{2(x^2-a^2)^\lambda}. \quad (24)$$

Finally, we have the following expression relating the coupling g with the mass and the temperature:

$$\frac{1}{g} = -\left[\frac{m^{2-z}}{4\pi^{3/2}}\Gamma\left(1+\frac{1}{z}\right)\Gamma\left(\frac{1}{2}-\frac{1}{z}\right)\right.$$
$$\left.+\frac{2\beta^{1-2/z}}{z(4\pi^2)^{1-1/z}}f_{1-1/z}\left(\frac{m^z\beta}{2\pi},\frac{1}{2}\right)\right]. \quad (25)$$

It is evident that the temperature-independent terms of this expression replay the zero-temperature gap equation (14). Therefore, we find that the gap equation is modified in the finite-temperature case. We note that, as the temperature is raised (or, equivalently, β is decreased), the second term in the brackets in Eq. (25), which is always a negative one,
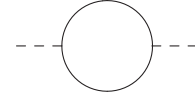

FIG. 2. Contribution to the effective propagator of σ.

will approach the first term, so that they cancel each other at the critical temperature

$$T_c = \left[\frac{2^{-\frac{z+2}{z}}\pi^{\frac{1}{2}-\frac{2}{z}}\Gamma(\frac{1}{2}-\frac{1}{z})\Gamma(\frac{1}{z})}{|f_{1-1/z}|}\right]^{\frac{z}{2-z}}m^z, \quad (26)$$

where we have considered $f_{1-1/z} = -|f_{1-1/z}|$. To find this critical temperature, we must approximate $f_{1-1/z}$ by its asymptotic value in the high-temperature limit, i.e., we must calculate $f_{1-1/z}(a \to 0, \frac{1}{2})$. For example, by taking into account $z = 1$, we get $T_c = \frac{\sigma_0}{2\ln 2}$, with $\sigma_0 = -m$, which is the seminal result discussed in Ref. [17], where we have obtained $f_0 = -\frac{\ln 2}{2\pi}$, in the limit of $a \to 0$.

In the following, for $z = 3$, we have

$$T_c = \frac{16(1-2^{2/3})^3\zeta(\frac{1}{3})^3}{\sqrt{\pi}\Gamma(\frac{1}{6})^3}m^3, \quad (27)$$

where $\zeta(\frac{1}{3})$ is the zeta function. To obtain this result, we have used the fact that, in the high-temperature limit,

$$f_{2/3} = -\frac{(1-2^{2/3})\zeta(\frac{1}{3})\Gamma(\frac{1}{3})}{\sqrt{3}2\pi}. \quad (28)$$

Thus, we see that there exists some critical temperature at which the parity breaking vanishes.

C. Effective propagator and renormalizability

To further study the dynamics, let us take into account the propagator of the σ field. It is characterized by the Feynman diagram given in Fig. 2.

The contribution of this graph, for even $z = 2n$, looks like

$$\Gamma_2(p) = -\frac{N}{2}\mathrm{tr}\int\frac{dk_0 d^2\vec{k}}{(2\pi)^3}\frac{(\gamma^0 k_0 - \omega(\vec{k}))(\gamma^0(k_0+p_0) - \omega(\vec{k}+\vec{p}))}{(k_0^2-\omega^2(\vec{k}))((p_0+k_0)^2-\omega^2(k+p))}\sigma(p)\sigma(-p) \quad (29)$$

and for odd $z = 2n+1$,

$$\Gamma_2(p) = -\frac{N}{2}\mathrm{tr}\int\frac{dk_0 d^2\vec{k}}{(2\pi)^3}\frac{(\gamma^0 k_0 + \gamma^i k_i(-\vec{k}^2)^n + m^z)(\gamma^0(k_0+p_0)+\gamma^j(k_j+p_j)(-(\vec{k}+\vec{p})^2)^n + m^z)}{(k_0^2-\Omega^2(\vec{k}))((k_0+p_0)^2-\Omega^2(k+p))}\times\sigma(p)\sigma(-p). \quad (30)$$

It is easy to calculate a trace in three dimensions since $\mathrm{tr}(\gamma^\mu\gamma^\nu) = 2\eta^{\mu\nu}$. So, for even $z = 2n$, we have (here the mass is introduced for formal reasons, since we note that there is no dynamical mass generation in this case; note nevertheless that either a zero or nonzero mass does not affect the superficial degree of divergence of this theory since the mass never accompanies the UV leading orders of the propagators),

$$\Gamma_2(p) = -N \int \frac{dk_0 d^2\vec{k}}{(2\pi)^3} \frac{k_0(k_0 + p_0) + \omega(\vec{k})\omega(\vec{k} + \vec{p})}{(k_0^2 - \omega^2(\vec{k}))((p_0 + k_0)^2 - \omega^2(\vec{k} + \vec{p}))} \sigma(p)\sigma(-p), \tag{31}$$

and, for odd $z = 2n + 1$,

$$\Gamma_2(p) = -N \int \frac{dk_0 d^2\vec{k}}{(2\pi)^3} \frac{k_0(k_0 + p_0) - \vec{k} \cdot (\vec{k} + \vec{p})(\vec{k}^2(\vec{k} + \vec{p})^2)^n + m^{2z}}{(k_0^2 - \Omega^2(\vec{k}))((k_0 + p_0)^2 - \Omega^2(k + p))} \times \sigma(p)\sigma(-p). \tag{32}$$

It remains to calculate the integrals.

The operator that is the inverse of this effective action is just the propagator. To integrate, we first do the Wick rotation, then use the Feynman representation, where the subsequent change of variables $k_0 \to k_0 + p_0 x$ is carried out for k_0 only. As a result, for even z, we get

$$\Gamma_2^{\text{even}}(p) = -iN \int \frac{dk_{0E} d^2\vec{k}}{(2\pi)^3} \int_0^1 dx \frac{-k_{0E}^2 + p_{0E}^2 x(1 - x) + \omega(\vec{k})\omega(\vec{k} + \vec{p})}{[k_{0E}^2 + p_{0E}^2 x(1 - x) + (1 - x)\omega^2(\vec{k}) + x\omega^2(\vec{k} + \vec{p})]^2} \times \sigma(p)\sigma(-p), \tag{33}$$

and, for odd $z = 2n + 1$,

$$\Gamma_2^{\text{odd}}(p) = iN \int \frac{dk_{0E} d^2\vec{k}}{(2\pi)^3} \int_0^1 dx [k_{0E}^2 - p_{0E}^2 x(1 - x) + \vec{k} \cdot (\vec{k} + \vec{p})(\vec{k}^2(\vec{k} + \vec{p})^2)^n - m^{4n+2}]$$

$$\times [k_{0E}^2 + p_{0E}^2 x(1 - x) + (1 - x)\Omega^2(\vec{k}) + x\Omega^2(\vec{k} + \vec{p})]^{-2} \sigma(p)\sigma(-p). \tag{34}$$

It remains to integrate over k_{0E} and expand numerators and denominators in external momenta $(p_0, \vec{p})$, up to leading orders.

For even $z = 2n$, after integration over k_{0E} and reintroducing the Minkowski zero component through $p_{0E}^2 \to -p_0^2$ we arrive at the following self-energy tensor $T(p)$:

$$T_{\text{even}}(p) = -\frac{iN}{4} \int \frac{d^2\vec{k}}{(2\pi)^2} \int_0^1 dx \frac{x\omega^2(\vec{k} + \vec{p}) + (1 - x)\omega^2(\vec{k}) + \omega(\vec{k})\omega(\vec{k} + \vec{p}) + 2x^2 p_0^2}{[x\omega^2(\vec{k} + \vec{p}) + (1 - x)\omega^2(\vec{k}) - x(1 - x)p_0^2]^{\frac{3}{2}}}, \tag{35}$$

which corresponds to Γ_2 through the relation $\Gamma_2 = \sigma(-p)T(p)\sigma(p)$.

The integral over $d^2\vec{k}$ in the expression above can be calculated only in an approximate manner. In order to consider the UV leading asymptotics of these propagators, we approximate the denominators by higher orders in $\vec{k}$ and $\vec{p}$. For this, let us first use

$$\omega^2(\vec{k}) \approx (\vec{k}^2)^{2n} + m^{4n},$$
$$\omega(\vec{k})\omega(\vec{k} + \vec{p}) \approx (\vec{k}^2)^{2n} + (-1)^n m^{2n}(\vec{p}^2)^n + m^{4n},$$
$$\omega^2(\vec{k} + \vec{p}) \approx (\vec{k}^2)^{2n} + (\vec{p}^2)^{2n} + 2(-1)^n(\vec{p}^2)^n m^{2n} + m^{4n}. \tag{36}$$

With the use of Eq. (36) we can write

$$T_{\text{even}}(p) \approx -\frac{iN}{4} \int_0^1 dx \int \frac{d^2\vec{k}}{(2\pi)^2} \frac{2(\vec{k}^2)^{2n} + x(\vec{p}^2)^{2n} + (1 + 2x)(-1)^n m^{2n}(\vec{p}^2)^n + 2x^2 p_0^2 + 2m^{4n}}{[(\vec{k}^2)^{2n} + x(\vec{p}^2)^{2n} + 2x(-1)^n(\vec{p}^2)^n m^{2n} + m^{4n} - x(1 - x)p_0^2]^{\frac{3}{2}}}. \tag{37}$$

Now, after integrating over $\vec{k}$, we find

$$T_{\text{even}}(p) \approx -\frac{iN}{8\pi} \int_0^1 dx \frac{1}{n\sqrt{\pi}} \{m^{4n} + [(\vec{p}^2)^{2n} + 2(-1)^n(\vec{p}^2)^n m^{2n} - p_0^2(1 - x)]x\}^{\frac{1}{2n} - \frac{3}{2}}$$

$$\times \Gamma\left(\frac{1}{2n} + 1\right) \left\{\Gamma\left(\frac{n - 1}{2n}\right) \{m^{4n} + [(\vec{p}^2)^{2n} + 2(-1)^n(\vec{p}^2)^n m^{2n} - p_0^2(1 - x)]x\}\right.$$

$$\left. + n\Gamma\left(\frac{3}{2} - \frac{1}{2n}\right) [2m^{4n} + (\vec{p}^2)^{2n} + 2p_0^2 x^2 + (\vec{p}^2)^n m^{2n}(1 + 2x)]\right\}. \tag{38}$$

Considering the behavior of this expression in the limit $p_0 \to \infty$, $\vec{p} \to \infty$ in order to obtain the UV asymptotics of the propagator, and approximating the integrals over x as numbers of the order of 1, we see that the behaviour of this self-energy tensor, up to numerical factors of the order of 1 accompanying any monomials (these factors are denoted here by b_1, b_2, b_3), is

$$T_{\text{even}}(p) \simeq N\{m^{4n} + [b_1(\vec{p}^2)^{2n} - b_2 p_0^2 + 2b_3(-1)^n(\vec{p}^2)^n m^{2n} + \cdots]\}^{\frac{1}{2n} - \frac{1}{2}}. \quad (39)$$

This expression is approximate but sufficient to find the superficial degree of divergence. The dots are for subleading orders in momenta.

For the odd z we have the following self-energy tensor:

$$T_{\text{odd}}(p) = -\frac{iN}{4} \int \frac{d^2\vec{k}}{(2\pi)^2} \int_0^1 dx [x\Omega^2(\vec{k}+\vec{p}) + (1-x)\Omega^2(\vec{k}) - \vec{k}\cdot(\vec{k}+\vec{p})(\vec{k}^2)^n(\vec{k}^2 + 2\vec{k}\cdot\vec{p} + \vec{p}^2)^n + m^{4n+2} + 2x^2 p_0^2] \times [x\Omega^2(\vec{k}+\vec{p}) + (1-x)\Omega^2(\vec{k}) - x(1-x)p_0^2]^{-\frac{3}{2}}, \quad (40)$$

where $\Omega^2(\vec{p}) = (\vec{p}^2)^{2n+1} + m^{4n+2}$. Again, we approximate the integral by taking into account only the leading orders in $\vec{k}$ and $\vec{p}$, which implies considering

$$\Omega^2(\vec{k}) = (\vec{k}^2)^{2n+1} + m^{4n+2}, \qquad \Omega^2(\vec{k}+\vec{p}) \approx (\vec{k}^2)^{2n+1} + (\vec{p}^2)^{2n+1} + m^{4n+2}, \qquad (41)$$

and $\vec{k}\cdot(\vec{k}+\vec{p})(\vec{k}^2)^n(\vec{k}^2 + 2\vec{k}\cdot\vec{p} + \vec{p}^2)^n \approx (\vec{k}^2)^{2n+1}$. Therefore, the self-energy tensor is

$$T_{\text{odd}}(p) \approx -\frac{iN}{4} \int \frac{d^2\vec{k}}{(2\pi)^2} \int_0^1 dx \frac{x(\vec{p}^2)^{2n+1} + 2m^{4n+2} + 2x^2 p_0^2}{[x(\vec{p}^2)^{2n+1} + (\vec{k}^2)^{2n+1} + m^{4n+2} - x(1-x)p_0^2]^{\frac{3}{2}}}. \quad (42)$$

Integrating over $\vec{k}$, we arrive at

$$T_{\text{odd}}(p) \approx -\frac{iN}{8\pi^{\frac{3}{2}}} \int_0^1 dx \Gamma\left(\frac{6n+1}{4n+2}\right)\Gamma\left(\frac{2n+2}{2n+1}\right) \frac{x(\vec{p}^2)^{2n+1} + 2m^{4n+2} + 2x^2 p_0^2}{(m^{4n+2} + [(\vec{p}^2)^{2n+1} + p_0^2(x-1)]x)^{\frac{6n+1}{4n+2}}}. \quad (43)$$

Again, we consider the UV limit. We find

$$T_{\text{odd}}(p) \simeq N[(\vec{p}^2)^{2n+1} - a_1 p_0^2 + a_2 m^{4n+2} + \cdots]^{\frac{1-2n}{2+4n}}. \quad (44)$$

Here and below, $a_1, a_2, b_1, b_2, \alpha, \beta$ are numerical factors, and dots are for terms with lower orders in momenta.

These results can be used to find effective propagators, and, finally, to make conclusions about the renormalizability of our theory. For even and odd z respectively we have the propagators of σ of the form $G = T^{-1}$ looking like

$$G_{\text{odd}}(p) = T_{\text{odd}}^{-1} \propto \frac{1}{N} \frac{\alpha}{[(\vec{p}^2)^{2n+1} - a_1 p_0^2 + a_2 m^{4n+2}]^{\frac{1-2n}{2+4n}}},$$

$$G_{\text{even}}(p) = T_{\text{even}}^{-1} \propto \frac{1}{N} \frac{\beta}{[m^{4n} + b_1(\vec{p}^2)^{2n} - b_2 p_0^2]^{\frac{1}{2n} - \frac{1}{2}}}. \quad (45)$$

Here we disregarded subleading orders in momenta. We note that the effective dimension of p_0 is $2n + 1$ for the odd case, and $2n$ for the even case, as it should be. It allows us to calculate the superficial degree of divergence ω of an arbitrary Feynman diagram.

For the odd $z = 2n + 1$, any propagator of ψ contributes to ω with $-(2n + 1)$, and any propagator of σ with $2n - 1$ (the non-negative contribution of propagators to the

superficial degree of divergence is rather typical for the effective dynamics emerging due to quantum corrections, cf. Ref. [16]). Any loop integration yields $2 + z = 2n + 3$. In total, we have in this case

$$\omega = (3 + 2n)L + (2n - 1)P_\sigma - (2n + 1)P_\psi, \quad (46)$$

where P_σ and P_ψ are numbers of corresponding propagators. Using the topological identity $L + V - (P_\sigma + P_\psi) = 1$ together with relations between numbers of vertices and propagators $2P_\psi = 2V - E_\psi, 2P_\sigma = V - E_\sigma$, where $E_{\psi,\sigma}$ are numbers of corresponding external lines, we have

$$\omega = 3 + 2n - E_\psi - (2n + 1)E_\sigma. \quad (47)$$

Thus, the theory is renormalizable in all orders of the $\frac{1}{N}$ expansion, which matches the $z = 1$ ($n = 0$) result found in Ref. [17].

For the even $z = 2n$, any propagator of ψ contributes to ω with $-2n$, and any propagator of σ with $2n - 2$. In this case we have

$$\omega = (2 + 2n)L - 2nP_\psi + (2n - 2)P_\sigma, \quad (48)$$

which yields

$$\omega = 2 + 2n - E_\psi - 2nE_\sigma. \qquad (49)$$

Thus, for the even z the theory is also renormalizable in all orders of the $\frac{1}{N}$ expansion.

IV. GENERATION OF THE CHERN-SIMONS TERM

Now, to illustrate other possible impacts of a HL-like spinor field, let us make the first step in studying its interaction with a vector field, that is, let us couple the spinor field to a gauge field through the vertex

$$\mathcal{L}_{\text{int}} = \frac{e}{\sqrt{N}} \sum_{j=1}^{N} \bar{\psi}_j \gamma_\mu \psi_j A^\mu. \qquad (50)$$

We note that this vertex is really a "toy" coupling which we use only to discuss the possibility for a dynamical parity breaking similarly to the study performed in the relativistic case in Ref. [17]; therefore, the Lagrangian given by the sum of Eqs. (3) and (50) is really not gauge invariant, while to construct the complete model one should use a Lagrangian like $\bar{\psi}(i\gamma^0 D_0 - (i\gamma^i D_i)^z - m^z)\psi$, where
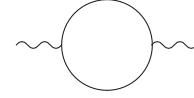

FIG. 3.   Contribution to the Chern-Simons term.

$D_{0,i} = \partial_{0,i} - igA_{0,i}$ are the gauge-covariant derivatives. That model was studied in Ref. [15], and we plan to study the generation of the Chern-Simons (CS) term in it in our next paper, while at this step our aim consists only in discussing the parity breaking for which we use only the sum of Eqs. (3) and (50).

Here, our aim is to find the polarization tensor given by

$$\Pi^{\mu\nu}(p) = e^2 \int \frac{dk_0 d^2\vec{k}}{(2\pi)^3} \text{tr}\gamma^\mu S_F(k+p)\gamma^\nu S_F(k). \qquad (51)$$

The corresponding Feynman diagram is given in Fig. 3.

We consider only the case of odd $z = 2n+1$ since for $z = 2n$ there is no mass generation and hence no CS term can arise.

For odd z we use the propagator (5), so that Eq. (51) yields

$$\Pi^{\mu\nu}(p) = -e^2 \int \frac{dk_0 d^2\vec{k}}{(2\pi)^3} \text{tr}\gamma^\mu \frac{\gamma^0(k_0+p_0) + \gamma^i(k_i+p_i)(-\vec{k}^2 - 2\vec{k}\cdot\vec{p} - \vec{p}^2)^n + m^z}{(k_0+p_0)^2 - (\vec{k}^2 + 2\vec{k}\cdot\vec{p} + \vec{p}^2)^z - m^{2z}} \times \gamma^\nu \frac{\gamma^0 k_0 + \gamma^i k_i(-\vec{k}^2)^n + m^z}{k_0^2 - (\vec{k}^2)^z - m^{2z}}. \qquad (52)$$

Here we look for the CS term. To find it, we concentrate on terms linear in the external momentum p and involving three Dirac matrices, to get the Levi-Civita tensor. The relevant terms are

$$\Pi_{\text{CS}}^{\mu\nu} = -e^2 \int \frac{dk_0 d^2\vec{k}}{(2\pi)^3} \text{tr} \frac{\gamma^\mu\gamma^0 p_0\gamma^\nu m^z + \gamma^\mu\gamma^i p_i\gamma^\nu(-\vec{k}^2)^n m^z}{[(k_0+p_0)^2 - (\vec{k}^2 + 2\vec{k}\cdot\vec{p} + \vec{p}^2)^z - m^{2z}][k_0^2 - (\vec{k}^2)^z - m^{2z}]}. \qquad (53)$$

Now, by calculating the traces, we arrive at

$$\Pi_{\text{CS}}^{\mu\nu} = 2im^z e^2 \int \frac{dk_0 d^2\vec{k}}{(2\pi)^3} \frac{\epsilon^{\mu 0\nu} p_0 + \epsilon^{\mu i\nu} p_i(-\vec{k}^2)^n}{[(k_0+p_0)^2 - (\vec{k}^2 + 2\vec{k}\cdot\vec{p} + \vec{p}^2)^z - m^{2z}][k_0^2 - (\vec{k}^2)^z - m^{2z}]}. \qquad (54)$$

The low-energy leading terms are

$$\Pi_{\text{CS}}^{\mu\nu} = -2m^z e^2 \int \frac{dk_{0E} d^2\vec{k}}{(2\pi)^3} \frac{\epsilon^{\mu 0\nu} p_0 + \epsilon^{\mu i\nu} p_i(-\vec{k}^2)^n}{[k_{0E}^2 + (\vec{k}^2)^z + m^{2z}]^2}, \qquad (55)$$

where we have performed the Wick rotation. After the integration over k_{0E}, we obtain

$$\Pi_{\text{CS}}^{\mu\nu} = -\frac{m^z e^2}{2} \int \frac{d^2\vec{k}}{(2\pi)^2} \frac{\epsilon^{\mu 0\nu} p_0 + \epsilon^{\mu i\nu} p_i(-\vec{k}^2)^n}{[(\vec{k}^2)^z + m^{2z}]^{\frac{3}{2}}}. \qquad (56)$$

Finally, by calculating the integral over $\vec{k}$, we get

$$\Pi_{\text{CS}}^{\mu\nu} = -\frac{e^2}{4\pi^{3/2}} \text{sgn}(m)(c_1\epsilon^{\mu 0\nu} p_0 + c_2\epsilon^{\mu i\nu} p_i), \qquad (57)$$

where

$$c_1 = m^{-4n}\Gamma\left(\frac{2n+2}{2n+1}\right)\Gamma\left(\frac{6n+1}{4n+2}\right),$$

$$c_2 = m^{-2n}\frac{(-1)^n}{n+1}\Gamma\left(\frac{3n+2}{2n+1}\right)\Gamma\left(\frac{4n+1}{4n+2}\right). \qquad (58)$$

Equation (57) is a self-energy tensor describing the generation of the CS term in the Gross-Neveu model at odd z. It is easy to see that for $z \neq 1$ one has $c_1 \neq c_2$ which confirms the space-time anisotropy. To verify the consistency of our result, we note that at $z = 1$, that is, $n = 0$, we get

$$\Pi_{CS}^{\mu\nu} = -\frac{e^2}{4\pi}\text{sgn}(m)\epsilon^{\mu\rho\nu}p_\rho. \tag{59}$$

Equation (59) is just the same as that obtained in Ref. [17], so we see that at $z = 1$ the Lorentz invariance is recovered as it should be.

V. SUMMARY

We considered different issues related with the Horava-Lifshitz-like $(2 + 1)$-dimensional Gross-Neveu model. We showed that within the framework of the $\frac{1}{N}$ expansion, it is renormalizable for any z. Also, we demonstrated (although the vector field was introduced from the beginning as the external one, just as in Ref. [14], and not generated through some Thirring-like interaction) that the anisotropic Chern-Simons term can be generated in it (however, we note that, for the simplified model we considered here, it is not gauge invariant). A consequence of generating the Chern-Simons term consists in the fact that we succeeded in this way to break the parity dynamically. Besides, we succeeded in generalizing some of these results to the finite-temperature case, namely, we considered the gap equation in the finite-temperature case and found that there is some critical temperature at which the parity breaking disappears (in

principle, one could also apply this result to a detailed study of possible phase transitions in this theory extending thus the studies carried out in Ref. [14]).

We showed that our emergent dynamics of the Lagrange multiplier σ is renormalizable for any z. (Unlike in Ref. [10], to prove it we did not use any special identities.) Therefore our HL-like Gross-Neveu model is perturbatively consistent.

The possible continuation of our study could consist in studying the emergent dynamics of vector fields in more detail, that is, the generation of the vector field through a Thirring-like interaction in a manner similar to that used here to introduce the Lagrange multiplier σ, and generating the CS term starting with the full-fledged, gauge-invariant Lagrangian $\bar{\psi}(i\gamma^0 D_0 - (i\gamma^i D_i)^z - m^z)\psi$ [15]. This is a problem we plan to consider in the next paper.

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