Particle production in a gravitational wave background

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We study the possibility that massless particles, such as photons, are produced by a gravitational wave. That such a process should occur is implied by tree-level Feynman diagrams such as two gravitons turning into two photons, i.e., $g + g \rightarrow \gamma + \gamma$. Here we calculate the rate at which a gravitational wave creates a massless scalar field. This is done by placing the scalar field in the background of a plane gravitational wave and calculating the 4-current of the scalar field. Even in the vacuum limit of the scalar field it has a nonzero vacuum expectation value (similar to what occurs in the Higgs mechanism) and a nonzero current. We associate this with the production of scalar field quanta by the gravitational field. This effect has potential consequences for the attenuation of gravitational waves since the massless field is being produced at the expense of the gravitational field. This is related to the time-dependent Schwinger effect, but with the electric field replaced by the gravitational wave background and the electron/positron field quanta replaced by massless scalar "photons." Since the produced scalar quanta are massless there is no exponential suppression, as occurs in the Schwinger effect due to the electron mass.

I. INTRODUCTION

As early as 1855 Faraday recognized the possibility of a relationship between gravity and electricity [1] through his observation, "Such results, if possible, could only be exceedingly small; but, if possible, i.e. if true, no terms could exaggerate the value of the relation they would establish." More recently the potential relationship between gravity and the electromagnetic interactions has been examined for individual quanta in terms of gravitons and photons [2–4] using Feynman diagrams or in terms of electromagnetic waves and gravitational waves [5–7] (i.e., large collections of photons and gravitons). The perturbative, Feynman-diagrammatic calculations of Refs. [2–4] give transitions from gravitons to photons which are consistent with Faraday's expectation that this effect is "exceedingly small." For example, in Ref. [2] it was found that the cross section for two gravitons going to two photons $(g + g \rightarrow \gamma + \gamma)$[1] is of the order $\sigma \sim 10^{-110}$ cm^2 for a wave whose frequency is set by the electron rest mass $\omega \sim m_e$. For such a small cross section this process is not important, even for gravitons traveling cosmological

distances. The frequencies involved in the detection by LIGO of GW150914 [9] where much lower than $\omega \sim m_e$, which would make the cross sections even smaller. The point of these estimates is that one gets a small, but nonzero result for this process. In this paper we want to examine the production of massless quanta from a gravitational wave background. This can be viewed as a gravitational variant of the Schwinger effect where a strong, static electric field can produce electron-positron pairs [10]. In the present case the background field is that of a gravitational wave instead of a static electric field and the particles produced are massless scalar particles (which are stand-ins for photons) instead of electrons-positrons. In the usual Schwinger effect the electron-positron production rate per unit volume is given by

$$\Gamma_{e+e-} = \frac{e^2 E_0^2}{4\pi^3} \exp\left[\frac{-\pi m_e^2}{e E_0}\right]. \qquad (1)$$

This process is exponentially suppressed by the last term in the expression above (E_0 is the magnitude of the electric field, m_e is the electron mass, and e is the electron charge). In the case studied here—a gravitational field creating massless quanta—there will be no exponential suppression since the mass associated with the fields is zero.

A final important point about taking the scalar field to be massless is that it has been shown [11] that a gravitational plane wave cannot create a scalar field *if the scalar field is massive*. The caveat given in Ref. [11] for when it might be possible to create a scalar field from a gravitational plane wave is exactly when the scalar field is massless. This also

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[1]Since here we have in mind to calculate how a gravitational plane wave—which is composed of many gravitons—is converted into a massless field, the gravitons would be taken as going in the forward direction and the massless field created from the gravitational wave would also be going in the forward direction, as expected from energy-momentum conservation [8].

fits in with the particle view point of Ref. [8] where the decay of gravitons into other particles was investigated, and from very simple kinematic arguments it was shown that graviton decay was only possible when the graviton decayed into other massless particles.

The potential significance of the process where electromagnetic radiation is produced from a gravitational wave background is that this would lead to a weakening or attenuation of the gravitational wave, since the creation of the electromagnetic radiation would come at the expense of the gravitational wave. If the production of electromagnetic radiation via gravitational waves is significant, one would need to take this into account when using the detected amplitude of the gravitational wave to determine the characteristics of the event, such as the distance to the source of the gravitational waves. For example, this attenuation would mean that the source of the gravitational wave was closer than implied by the measured amplitude. Another consequence of this process is that one might think to look for the electromagnetic radiation which was produced by the gravitational wave. In fact, there is a claim [12] that the gravitational wave detection by LIGO, GW150914 [9], was potentially accompanied by a γ-ray signal. Our calculations below will show that a gravitational wave might produce electromagnetic radiation, but rather than being in the γ-ray range, the electromagnetic radiation produced would have extremely long wavelengths, on the order of hundreds of kilometers.

Previously, the question of production of electromagnetic radiation from a gravitational background was examined by two of the authors [6] using the formalism of the Unruh-DeWitt detector. The resulting scalar field quanta production rate found in this way was small, but not as small as indicated by the Feynman diagram calculations [2–4] for individual quanta. Based on the Unruh-DeWitt detector calculations of Ref. [6], it was possible that the production of electromagnetic radiation via a gravitational wave background might have an attenuating effect on the gravitational wave. This difference between the Feynman-diagrammatic calculations of Refs. [2–4] and the Unruh-DeWitt detector calculations of Ref. [6] can be compared to the situation that occurs when calculating the decay rate, Γ_{e+e-}, for the Schwinger effect. The expression for Γ_{e+e-} given in Eq. (1) is nonperturbative (this can be seen by the presence of $\exp\left[-\frac{\text{const}}{E_0}\right]$) and cannot be obtained via the perturbative method of Feynman diagrams.

In Minkowski space-time the calculation of vacuum pair production via different methods gives identical results. For example, one can calculate the Schwinger effect via the trace-log method originally used by Schwinger, or via the "scattering/tunneling" of some charged field by the potential due to the background uniform electric field, and the results are the same. (This comparison of different methods of calculating the Schwinger effect can be found in Refs. [13,14] as well as in Ref. [15].) However, in curved space-times, different methods for calculating particle production can give different results for the production rate, as discussed in several papers [16–18]. As pointed out in Ref. [16], the difficulty of studying particle production in the presence of a nonasymptotically flat gravitational background is that the definition of the particle production rate can depend on the method of calculation (e.g., using a creation/annihilation operator versus using a definition of a vacuum state versus using a Feynman Green function). In Ref. [16] the vacuum-to-vacuum amplitude was calculated in the path integral approach for Friedmann-Robertson-Walker (FRW) space-time. The amount by which this amplitude differed from unity was used to obtain the particle production rate and it was found to give a different particle production rate from the usual method of diagonalization of the Hamiltonian. While this warning about different calculation methods yielding different particle production rates may not necessarily apply to the gravitational wave backgrounds, which are the focus of this paper,[2] we nevertheless mention it to point out the subtle issues which surround the definition of particles in curved space-times, and determining if a given space-time will have particle production associated with it or not. Finally, we note that Ref. [18] gave arguments that for general *time-dependent* metrics the issue of the definition of particles and whether or not particle production occurs is still an unresolved problem.

In this paper, we obtain the particle production rate by calculating the conserved 4-current of a massless, scalar field in a gravitational wave background and then comparing this to the massless, scalar field in flat space-time. We take the difference between these two situations—a scalar field in a gravitational wave background versus a scalar field in Minkowski space-time—as a measure of the rate of particle production. This 4-current method for calculating the production rate and/or super radiance is similar to that used in Refs. [18–21]. The 4-current method employed here can also be compared to the work of Gertsenshtein [5], who investigated the production of electromagnetic radiation when a gravitational wave encountered a region of space-time with a uniform magnetic field. There, the interaction between the magnetic field and gravitational wave background produced electromagnetic radiation. Here we replace the magnetic field by a massless scalar field. Recently, Mösta *et al.* [22] carried out a study similar to that of Gertsenshtein, where they studied the electromagnetic radiation produced by a gravitational waves coming from a pair of inspiraling black holes embedded in a constant magnetic field.

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[2]This difference in particle production rates, calculated via different methods, occurs in space-times where it is not clear how to define asymptotic states. This is the case for the FRW space-time considered in Ref. [16], but it is not the case for the gravitational wave background.

In Sec. II we study the solution of a massless scalar field in a gravitational wave background and use this to calculate a particle production rate. In Sec. III we use the results of Sec. II to give a rough estimate of an attenuation length for the gravitational wave due to the production of electromagnetic radiation from the gravitational wave.

II. SCALAR FIELD IN GRAVITATIONAL WAVE BACKGROUND

In this paper we use a massless scalar field as a stand-in for the more physically realistic massless photon. The justification for this is that one can write a vector field as $A_\mu(x_\nu) = \epsilon_\mu \varphi(x_\nu)$, where ϵ_μ is the polarization 4-vector and $\varphi(x_\nu)$ is a scalar function which obeys the massless Klein-Gordon equation. The vector field A_μ certainly has more degrees of freedom (because of ϵ_μ) as compared to a simple scalar field, but this would at most change the production rate by some factor of order unity. At the level of individual quanta one can point to Ref. [3] where the cross sections for gravitons Compton scattering from scalar and vector particles were given, i.e., the processes $S + g \to S + g$ and $\gamma + g \to \gamma + g$. These Compton scattering diagrams can be rotated to give graviton production processes, i.e., $g + g \to S + S$ and $g + g \to \gamma + \gamma$. From Ref. [3] the diagrammatic evaluations of both processes are nonzero, although they do differ by numerical factors of order unity due to the different spins of the scalar versus vector particles.

With this justification we begin by placing a massless scalar field in curved space-time and writing down the Klein-Gordon equation coupled to the space-time described by the metric $g_{\mu\nu}$,

$$\frac{1}{\sqrt{-|g_{\mu\nu}|}} \left(\partial_\mu g^{\mu\nu} \sqrt{-|g_{\mu\nu}|} \partial_\nu \right) \varphi = 0. \tag{2}$$

For our background we take a plane gravitational wave with the $+$ polarization. The metric for this can be written as [23],

$$ds^2 = -dt^2 + dz^2 + f(u)^2 dx^2 + g(u)^2 dy^2. \tag{3}$$

The variables in the metric, $u = z - t$ and $v = z + t$, are light-cone coordinates and the metric components $f(u)$ and $g(u)$ will be required to be oscillatory functions, as expected for a gravitational wave background. The determinant of the metric is $|g_{\mu\nu}| = \det[g_{\mu\nu}]$ and $\sqrt{-|g_{\mu\nu}|} = fg$. Substituting this into Eq. (2),

$$\frac{1}{fg} \left(-\partial_t(fg)\partial_t + \frac{1}{f^2}\partial_x(fg)\partial_x + \frac{1}{g^2}\partial_y(fg)\partial_y + \partial_z(fg)\partial_z \right)$$
$$\times \varphi = 0. \tag{4}$$

Since u is only a function of z and t the expression can be expanded,

$$\left(-\partial_t^2 - \frac{1}{fg}\partial_t(fg)\partial_t + \frac{1}{f^2}\partial_x^2 + \frac{1}{g^2}\partial_y^2 + \partial_z^2 + \frac{1}{fg}\partial_z(fg)\partial_z \right)$$
$$\times \varphi = 0. \tag{5}$$

Applying the chain rule for the t and z derivatives, $\partial_t(fg) = -\partial_u(fg)$, $\partial_z(fg) = \partial_u(fg)$, $(\partial_z^2 - \partial_t^2) = 4\partial_u\partial_v$, $(\partial_t + \partial_z) = 2\partial_v$, $(\partial_z - \partial_t) = 2\partial_u$, and multiplying by $f^2 g^2$,

$$(4f^2 g^2 \partial_u \partial_v + 2fg\partial_u(fg)\partial_v + g^2\partial_x^2 + f^2\partial_y^2)\varphi = 0. \tag{6}$$

At this point we are still looking at the exact solution to the Klein-Gordon equation using the metric of Eq. (3). To evaluate the solution for a weak gravitational wave the linearized gravity approximation will be introduced in the terms of the metric [$f(u) = 1 + \varepsilon(ku)$ and $g(u) = 1 - \varepsilon(ku)$] and substituted into Eq. (6). Also note that the metric of Eq. (3) describes a wave propagating in the z direction and the x and y spatial directions must be physically indistinguishable. Based on the isotropy of space-time and assuming a nonthermal vacuum [6,8,24], we take $(\partial_y^2 - \partial_x^2)\varphi = 0$ as a property of our scalar field solution. Using this and collecting terms together, Eq. (6) can be expressed as

$$[4(1 - 2\varepsilon^2 + \varepsilon^4)\partial_u\partial_v - 4(1 - \varepsilon^2)\varepsilon(\partial_u\varepsilon)\partial_v + (1 + \varepsilon^2)\partial_x^2$$
$$+ (1 + \varepsilon^2)\partial_y^2]\varphi = 0. \tag{7}$$

We now assume that $\varepsilon(ku) = h_+ e^{iku}$, i.e., oscillatory functions typical for gravitational waves in linearized general relativity. In this expression h_+ is the dimensionless amplitude of the gravitational wave. Substituting into Eq. (7), we obtain

$$(4F\partial_u\partial_v - 4ikG\partial_v + H(\partial_x^2 + \partial_y^2))\varphi = 0, \tag{8}$$

where

$$F(ku) \equiv (1 - 2h_+^2 e^{2iku} + h_+^4 e^{4iku}),$$
$$G(ku) \equiv (h_+^2 e^{2iku} - h_+^4 e^{4iku}),$$
$$H(ku) \equiv (1 + h_+^2 e^{2iku}). \tag{9}$$

Equation (8) is separable by taking $\varphi = X(x)Y(y)U(u)V(v)$ and identifying the eigenvalue equations for $X(x)$ and $Y(y)$ as

$$\partial_x^2 X = -k_x^2 X \to X = e^{ik_x x},$$
$$\partial_y^2 Y = -k_y^2 Y \to Y = e^{ik_y y}. \tag{10}$$

Note that the x and y direction eigenfunctions are simply free waves, as is to be expected since the gravitational wave is in the $u = z - t$ direction. Setting $2k_{xy}^2 \equiv k_x^2 + k_y^2$ and using Eq. (10), we find that Eq. (8) becomes

$$F\frac{\partial_u U}{U}\frac{\partial_v V}{V} - ikG\frac{\partial_v V}{V} - H\frac{k_{xy}^2}{2} = 0. \qquad (11)$$

Now since the light-front coordinate v is orthogonal to u and since the gravitational wave only depends on u, one expects that the eigenfunction $V(v)$ is also solved by a free plane wave, as was the case for $X(x)$ and $Y(y)$. This is indeed the case and we find

$$-i\partial_v V = k_v V \rightarrow V = e^{ik_v v}. \qquad (12)$$

Substituting Eq. (12) into Eq. (11) yields

$$ik_v F\frac{\partial_u U}{U} + kk_v G - \frac{k_{xy}^2}{2}H = 0. \qquad (13)$$

Defining the eigenvalue $\lambda \equiv \frac{k_{xy}^2}{2k_v}$, Eq. (13) can be rearranged as

$$i\frac{\partial_u U}{U} = \lambda\frac{H}{F} - k\frac{G}{F}. \qquad (14)$$

This equation can be integrated to give

$$U = Ae^{\frac{\lambda}{k}}e^{\frac{-\lambda}{k(1-h_+^2 e^{2iku})}}(1 - h_+^2 e^{2iku})^{\frac{1}{2}(\frac{\lambda}{k}-1)}e^{-i\lambda u}. \qquad (15)$$

A is a normalization constant which we will fix later. The first term ($e^{\frac{\lambda}{k}}$) is needed to ensure that as $h_+ \rightarrow 0$ (i.e., the gravitational wave is turned off) the eigenfunction for the u direction becomes a free plane wave, $e^{-i\lambda u}$. Collecting together all the terms in x, y, v, and u directions gives the solution of the scalar field in the gravitational wave background,

$$\varphi = Ae^{\frac{\lambda}{k}}e^{-\frac{\lambda}{k(1-h_+^2 e^{2iku})}}(1 - h_+^2 e^{2iku})^{\frac{1}{2}(\frac{\lambda}{k}-1)}e^{-i\lambda u}e^{ik_v v}e^{ik_x x}e^{ik_y y}. \qquad (16)$$

This solution for the scalar field given in Eq. (16) is very similar to the solution found in Ref. [15] for the static electric field pair production evaluated in light-front coordinates. Taking the limit $h_+ \rightarrow 0$ of Eq. (16) returns the expected Minkowski vacuum solution for the scalar field,

$$\varphi_0 = Ae^{-i\lambda u}e^{ik_v v}e^{ik_x x}e^{ik_y y} \rightarrow Ae^{i(k_v+\lambda)t}e^{i(k_v-\lambda)z}e^{ik_x x}e^{ik_y y}. \qquad (17)$$

In the last expression we have reverted from light-front to Cartesian coordinates. It is clear that the scalar field in Eq. (17) is a free wave. By defining an energy $k_0 = k_v + \frac{k_{xy}^2}{2k_v}$ and a momentum in the z direction $k_z = k_v - \frac{k_{xy}^2}{2k_v}$ and using the previously defined $k_x^2 + k_y^2 = 2k_{xy}^2$, one can check that the energy-momentum of the free solution in Eq. (17) satisfies the usual kinematic relationship for a free particle in Minkowski space-time, namely, $k_0^2 = k_x^2 + k_y^2 + k_z^2$.

We now use the result for the scalar field given in Eq. (16) to calculate the associated 4-current density which will then allow us to calculate the rate of pair production of the scalar field from the gravitational wave background. The u component of the scalar field 4-current is given in terms of φ as

$$j_u = -i(\varphi^*\partial_u\varphi - \varphi\partial_u\varphi^*). \qquad (18)$$

Substituting φ from Eq. (16) into Eq. (18), we find that the time-averaged u component of the 4-current is

$$\langle j_u \rangle = -2A^2\lambda - A^2\left(\frac{9}{2}\frac{\lambda^3}{k^2} - \frac{12\lambda^2}{k} + \frac{13}{2}\lambda - k\right)h_+^4. \qquad (19)$$

The brackets represent the time averaging. In obtaining this expression we have taken the light-front coordinate averages for the cosines: $\langle\cos^2(2ku)\rangle = \frac{1}{2}$, $\langle\cos^4(2ku)\rangle = \frac{3}{8}$, and $\langle\cos(2ku)\rangle = \langle\cos(4ku)\rangle = 0$. Also, we have dropped terms higher than h_+^4.

We now examine various limits of Eq. (19). First, in the limit when the gravitational wave vanishes, $h_+ \rightarrow 0$, the current becomes $j_u \rightarrow -2\lambda A^2 \rightarrow -1/V$, where we have fixed the normalization constant $A = \frac{1}{\sqrt{V}}\frac{1}{\sqrt{2\lambda}}$ by requiring that there be one particle per volume V. Another option for A would be to use the condition that there be 2λ particles per volume V, which would give $A = \frac{1}{\sqrt{V}}$. Section 4.3 of Ref. [25] discusses the various normalization conditions for scalar fields. Second, in the presence of both the scalar field ($\lambda \neq 0$) and gravitational wave ($h_+ \neq 0$), Eq. (19) indicates how the current is modified by the potential represented by the gravitational background. For certain values of λ, k, and h_+ the current in Eq. (19) gives a larger outgoing current than incoming. This can be likened to the calculation of the Penrose superradiance process [21] where one "scatters" a real scalar field from a rotating black hole and the outgoing scalar field may have more energy than the incoming field. Finally, one can take the limit $\lambda \rightarrow 0$, $k_v \rightarrow 0$, and $k_{xy} \rightarrow 0$, i.e., the initial scalar field is taken to its vacuum state. In this way one obtains what is called the Minkowski persistence amplitude [26]. Because of the definition $\lambda \equiv \frac{k_{xy}^2}{2k_v}$ the limit $\lambda \rightarrow 0$ also means $k_{xy} \rightarrow 0$. In this limit the scalar field and its 4-current (16) *do not* reduce to the vacuum case (i.e., $\varphi_0 \rightarrow 0$ and $j_u \rightarrow 0$), but rather reduce to

$$\varphi \rightarrow \frac{1}{\sqrt{V}}\frac{1}{\sqrt{2k}}(1 - h_+^2 e^{2iku})^{-\frac{1}{2}} \quad \text{and} \quad j_u \rightarrow \frac{\text{sign}(k)}{2V}h_+^4. \qquad (20)$$

In Eq. (20) we have written out explicitly the normalization constant $A = \frac{1}{\sqrt{V}}\frac{1}{\sqrt{2k}}$. As before V is the volume in which the scalar field is placed. This normalization of φ

(especially the $\frac{1}{\sqrt{2k}}$ factor) is consistent with the normalization found in Ref. [20] via the Wronskian condition. Note that j_u in Eq. (20) is of order h_+^4. There are $h_+^2 e^{\pm 2iku}$ terms in j_u that arise when one substitutes φ from Eq. (20) into Eq. (18). However these terms time average to zero. In Eq. (18) it is only terms that involve products of terms such as, $h_+^2 e^{2iku}$ and $h_+^2 e^{-2iku}$, that give a nonzero value after time averaging. This explains why one is justified in keeping the metric and φ to order h_+^2 while the current derived from φ is of order h_+^4. If one could write out the metric to order h_+^4 then φ should have terms like $h_+^4 e^{4iku}$. However, these terms—when run through the definition of the current in Eq. (18)—would time average to zero unless they were products of terms like $h_+^4 e^{4iku}$ and $h_+^4 e^{-4iku}$. These "direct" product terms would be of order h_+^8 and would contribute to the current j_u, but the highest nonzero terms would still be of order h_+^4 since the h_+^2 terms time average to zero. Terms from Eq. (18) that were "cross" products, such as $h_+^4 e^{4iku}$ and $h_+^2 e^{-2iku}$, or $h_+^4 e^{4iku}$ and 1, time average to zero. There are other cases in general relativity where the metric is of lower order in h_+ as compared to the quantity calculated from the metric. One common example is the energy carried by a linearized gravitational wave where the metric is kept to order h_+ while the energy-momentum calculated from this metric is to order h_+^2 [27]. As a final comment on the order of h_+, we note that if one only kept terms of order h_+^2 in the scalar field (7) then the vacuum limit for the scalar field given in Eq. (20) would become $\varphi \propto (1 - 2h_+^2 e^{2iku})^{-\frac{1}{4}}$, which to order h_+^2 has the same expansion as φ from Eq. (20) and yields the same j_u as in Eq. (20) to order h_+^4.

The result in Eq. (20) can be related to the Higgs mechanism [28] where a scalar field develops a nonzero vacuum expectation value of $\varphi = \sqrt{\frac{m^2}{2\lambda}}$ due to a potential with a quartic *self-interaction* term plus tachyonic mass term (i.e., $-m^2 \varphi^2 + \lambda \varphi^4$). The self-interacting scalar potential in the usual Higgs mechanism is time independent. In the present case the scalar field develops a nonzero vacuum expectation value [the scalar field expression in Eq. (20)] due to the background gravitational wave potential. Because of the space- and time-dependent nature of the background gravitational field the vacuum expectation value from Eq. (20) is also space and time dependent, i.e., the e^{2iku} term in the expression for φ. Since the vacuum expectation value in this case is space and time dependent, one has a nonzero 4-current in the $u = z - t$ direction, $j_u = \frac{1}{2V} h_+^4$, as opposed to the usual Higgs mechanism case where the constant vacuum expectation value of the scalar field gives a zero 4-current associated with $\varphi = \sqrt{\frac{m^2}{2\lambda}}$. Another difference between the present example and the canonical Higgs mechanism is that in the present case the interaction that leads to the vacuum expectation value of φ

in Eq. (20) comes from the interaction between the scalar field and the gravitational field. In the canonical Higgs mechanism the vacuum expectation value is due to the $\lambda \phi^4$ self-interaction of the scalar field. Thus the nonzero scalar vacuum expectation value of the present case can be compared to the version of the Higgs mechanism that occurs in superconductors, where it is the phonons of the background lattice that are responsible for the interaction that binds electrons into Cooper pairs and which leads to superconductivity.

The important point about Eq. (20) is that $j_u \neq 0$ even though we have taken the scalar field to its Minkowski vacuum state. We interpret this nonzero j_u as being connected to a nonzero production rate of the scalar field by the gravitational wave background. (In the next section we draw the exact connection between j_u and the production rate.) That one should get a nonzero result for the process of gravitons converting to these scalar "photons" is supported by the Feynman diagram amplitudes like $g + g \rightarrow \gamma + \gamma$ which are nonzero [2–4].

The calculation of the production of the scalar field via the time-varying gravitational wave background of Eq. (3) can be compared to the similar calculation for de Sitter space-time from Ref. [29]. There, a massive scalar field was placed in the time-dependent de Sitter space-time and the amplitude of the scalar field in the de Sitter background was used to determine the scalar field production rate at the expense of the gravitational field. Unlike the de Sitter space-time metric, there is no horizon in the gravitational wave metric.

In the above discussion, the ansatz functions $f(u)$ and $g(u)$ were not exact solutions to the plane-wave space-time of Eq. (3). We now briefly show that one obtains similar results for an exact plane-wave, oscillatory metric, showing that the results are not an artifact of the approximate metric.

In order for $f(u)$ and $g(u)$ in Eq. (3) to be exact solutions to the Einstein field equations they need to satisfy the condition $\ddot{f}/f + \ddot{g}/g = 0$ [23]. A simple exact, plane-wave solution is given by $f = e^{iku} e^{-ku}$ and $g = e^{iku} e^{ku}$. These ansatz functions have oscillatory wave parts (e^{iku}) but they also have exponentially growing or decaying amplitudes ($e^{\pm ku}$). Near $u = 0$ one has oscillating wave solutions due to the e^{iku} parts of the ansatz function, but as u moves away from $u = 0$ the $e^{\pm ku}$ terms act like growing/decaying amplitudes. Because of this these solutions can only be of use for a restricted range of u near $u = 0$. Asymptotically, as $u \rightarrow \infty$, the functions $f(u)$ and $g(u)$ are not physically acceptable. By substituting $f = e^{iku} e^{-ku}$ and $g = e^{iku} e^{ku}$ into Eq. (6),

$$(4e^{4iku}\partial_u\partial_v + 2e^{2iku}\partial_u(e^{2iku})\partial_v + e^{2iku}e^{2ku}\partial_x^2$$
$$+ e^{2iku}e^{-2ku}\partial_y^2)\varphi = 0, \tag{21}$$

and making the substitution $\varphi = U(u)V(v)X(x)Y(y) = U(u)e^{ik_v v}e^{ik_x x}e^{ik_y y}$,

$$\left(i\frac{\partial_u U}{U} - k - e^{-2iku}e^{2ku}\frac{k_x^2}{4k_v} - e^{-2iku}e^{-2ku}\frac{k_y^2}{4k_v}\right) = 0. \quad (22)$$

In the limit when the gravitational wave is absent (i.e., $k \to 0$) the solution to Eq. (22) is again given by Eq. (17). When $k \neq 0$ the solution is Eq. (22),

$$U = Ae^{\left(\frac{(1-i)}{4k}\lambda_x e^{-2iku}e^{2ku} + \frac{(1+i)}{4k}\lambda_y e^{-2iku}e^{-2ku}\right)}e^{-iku}. \quad (23)$$

As before, A is a constant and $\lambda_x \equiv \frac{k_x^2}{4k_v}$ and $\lambda_y \equiv \frac{k_y^2}{4k_v}$. As before, if we take the limit of the massless scalar field to its vacuum state (i.e., taking the limit $k_x \to 0$, $k_y \to 0$, $\lambda_{x,y} \to 0$, and $k_v \to 0$) one finds $U(u) \to \frac{1}{\sqrt{V}}\frac{1}{\sqrt{2k}}e^{-iku}$, so that as before φ does not go to zero but rather $\varphi \to \frac{1}{\sqrt{V}}\frac{1}{\sqrt{2k}}e^{-iku}$. In this limit we have again written out the normalization constant as $\frac{1}{\sqrt{V}}\frac{1}{\sqrt{2k}}$. As before, we can calculate the current in the u direction in this limit and find that

$$j_u = \lim_{(k_x,k_y)\to 0} -i(U^*\partial_u U - U\partial_u U^*) = \frac{-\text{sign}(k)}{V}. \quad (24)$$

There is no explicit amplitude h_+ in this case since the changing amplitudes of the ansatz functions $f(u)$ and $g(u)$ are given by $e^{\pm ku}$.

There are other exact plane-wave solutions one could examine. The simplest is $f(u) = g(u) = u$ [30] which represents a plane-wave pulse rather than an oscillatory wave. Performing the above analysis with $f(u) = g(u) = u$ leads to $j_u = 0$ in the vacuum limit rather than the nonzero result of Eq. (20) or Eq. (24). Thus the nonzero result for j_u for the oscillatory ansatz functions is nontrivial.

III. ESTIMATED ATTENUATION LENGTH

In this section we use the vacuum current j_u of the last section to estimate the production rate of massless quanta from the gravitational wave background. Other, recent works which connected the particle production rate with currents in curved space-times can be found in Refs. [18–20]. In Ref. [19] the connection between the current and the production rate per unit volume was given by

$$\frac{\Gamma}{V}\Delta T \approx |j_u|, \quad (25)$$

where ΔT is a characteristic time for the problem and V is the volume of the system as before. Using $|j_u| = \frac{h_+^4}{2V}$ from Eq. (20) and taking the characteristic time as $\Delta T \sim \frac{1}{\omega}$, where ω is the frequency of the gravitational wave, we arrive at the production rate

$$\Gamma = \frac{\omega h_+^4}{2}. \quad (26)$$

We now use this production rate to estimate the effect this has on the decay of the amplitude h_+. We ignore the

effect of the usual $\frac{1}{r}$ falloff in h_+ due to the spherical nature of the outgoing gravitational wave. If we take N_g as the number of gravitons, then the standard result for the change of N_g due to Γ reads

$$\frac{dN_g}{dt} = -\Gamma N_g \to c\frac{dN_g}{dz} = -\Gamma N_g. \quad (27)$$

In anticipation that we will be more interested in a decay length than a decay time, we have taken $dt \to dz/c$. As a starting assumption we will take the number of gravitons as $N_g \propto h_+^2$, which is motivated by a similar relationship in QED where the number of photons is related to the square of the vector potential, $N_\gamma \propto A_\mu A^\mu$. In this way, and using the decay rate from Eq. (26), Eq. (27) becomes

$$\frac{d(h_+^2)}{dz} = -\frac{\omega}{2c}h_+^4(h_+^2) \to \frac{dh_+}{dz} = -\frac{1}{4}kh_+^5, \quad (28)$$

where $k = c\omega$. Equation (28) has the solution

$$h_+(z) = (kz + K_0)^{-1/4}, \quad (29)$$

where $K_0 = (h_+^{(0)})^{-4}$ and $h_+^{(0)}$ is the value of h_+ at $z = 0$. What Eq. (29) shows is that for large distances (i.e., large z) h_+ falls off like $\propto z^{-1/4}$, which is slower than the $z^{-1} \sim r^{-1}$ falloff due to the spherical nature of the outgoing gravitational wave. Thus the main factor in determining the falloff of h_+ at *large distances* is just the usual $\frac{1}{r}$ falloff. However, near the source of the gravitational wave, $z = 0$, the falloff in h_+ due to the conversion of the gravitational wave field into the massless field could be important. We can use Eq. (29) to make an estimate of the attenuation length Λ of the gravitational wave due to the conversion into electromagnetic radiation. If we define the decay length Λ as the distance over which $h_+(z = \Lambda) = \frac{1}{2}h_+^{(0)}$ we find that

$$\Lambda = \frac{15}{k(h_+^{(0)})^4} \to \frac{1.5 \times 10^7}{(h_+^{(0)})^4}\text{m}. \quad (30)$$

In the last expression we have assumed a frequency of $\omega \sim 300$ Hz, which yields $k = \frac{\omega}{c} \sim 10^{-6}$ m^{-1}. This estimate of the frequency is a very simple and rough estimate based on the upper range of the "chirp" for GW150914. In Table I we give different Λ's for different initial gravitational wave amplitudes $h_+^{(0)}$.

The size of the observable Universe is approximately 10^{27} meters, so from Table I we see that the estimated decay length will be important only if $h_+^{(0)}$ is fairly large, of the order of 10^{-5} or larger. The take away message from Table I is that the conversion of the gravitational background wave

TABLE I.   Various values of the decay length Λ versus $h_+^{(0)}$ for $\omega \approx 3 \times 10^2$ Hz and $k \approx 10^{-6}$ m^{-1}. We begin with $h_+ \approx 10^{-21}$ which is roughly the measured strain reported for GW150914 [9].

$h_+^{(0)}$	Λ meters
10^{-21}	10^{91}
10^{-15}	10^{67}
10^{-9}	10^{43}
10^{-5}	10^{27}
10^{-3}	10^{19}

into the massless field is significant only close to the source of the gravitational waves where h_+ is large.

IV. DISCUSSION AND CONCLUSIONS

In this paper we have looked at the possibility that a gravitational wave background could create massless scalar particles/fields. The massless scalar field quanta were taken as a simplified model of a photon. This is similar to the Schwinger effect but with the static electric field replaced by a gravitational wave background and the electron/positron replaced by massless scalar "photons." Since the created field was massless, we did not have the exponential suppression of the particle production rate which one finds in the Schwinger effect. Because of this lack of exponential suppression one expects the conversion of gravitational waves into electromagnetic radiation to potentially play a more prominent, physical role. In particular, we suggested that the creation of photons at the expense of the gravitational wave field would lead to an additional falloff of the dimensionless amplitude h_+ with distance from the source, in addition to the usual $\frac{1}{r}$ falloff. Based on the production rate per unit volume given in Eq. (26), we made an estimate of the decay length for various amplitudes h_+, given in Table I. Unless $h_+ > 10^{-5}$, our estimate for the decay length Λ given in Table I is so large that one would not expect this process to weaken the gravitational wave even over cosmological distances. This is in agreement with the conclusions based on Feynman diagram calculations [2]. But close enough to the source one will have $h_+ \geq 10^{-5}$, so in this *near* region one might expect the attenuation to be important. Since we used a gravitational plane wave this ignored the $\frac{1}{r}$ falloff for a more realistic spherical wave. The overall conclusion, both from Table I and from Eq. (29), is that the production of the massless field φ coming from the gravitational wave background and the subsequent attenuation of the gravitational wave background would only be important near the source of the gravitational wave.

One of the main results of this paper is the calculation of the scalar field (16) and associated 4-current (19) in the case when the scalar field is placed in a gravitational wave background. In the vacuum limit (i.e., $k_x \to 0$, $k_y \to 0$, $\lambda_{x,y} \to 0$, and $k_v \to 0$) the scalar field and 4-current take the

nonzero limits $\varphi \to \frac{1}{\sqrt{V}} \frac{1}{\sqrt{2k}} (1 - h_+^2 e^{2iku})^{-\frac{1}{2}}$ and $j_u = \frac{1}{2V} h_=^4$. This can be likened to a time-dependent, Higgs-like mechanism where the scalar field develops a nonzero vacuum expectation value. The difference from the usual Higgs mechanism is that here the effect is driven by the interaction of the scalar field with a gravitational background instead of with a self-interaction (i.e., $\lambda \phi^4$). Also, here the vacuum value of the scalar field is space-time dependent. This Higgs-like mechanism via the gravitational background can be compared to the symmetry breaking that occurs in superconductors, where it is the background lattice and phonons which provide the mechanism leading to a nonzero expectation value for Cooper pairs. This connection to the Higgs mechanism will be discussed further in an upcoming paper [31].

Finally, there are two predictions of physical phenomenon that would occur if the production of photons from the gravitational wave background were significant. First, the amplitude h_+ measured by a detector on Earth would be smaller due to the fact that this amplitude would decrease not only from the $\frac{1}{r}$ falloff for an outward traveling wave, but also the amplitude would decrease as $r^{-1/4}$ due to the production of photons from the gravitational wave background. From the $r^{-1/4}$ dependence of the particle production rate one can see that this effect would only be important relatively close to the source. Second, the gravitational wave would produce electromagnetic radiation/photons traveling in the same direction as the initial gravitational wave. In fact, it has already been suggested [12] that a γ-ray signal which was detected in the same time frame as the gravitational wave signal might be related to the detected gravitational wave. If the production mechanism of electromagnetic radiation from the gravitational wave background proposed here occurs and is significant, then we would predict that the gravitational wave signal should also be accompanied by an electromagnetic signal. However, in our process this electromagnetic signal should have roughly the same frequency as that of the gravitational wave. Thus we would predict that the electromagnetic wave coming from the gravitational wave would have extremely long wavelengths, on the order of hundreds of kilometers, i.e., the associated electromagnetic wave would have very large wavelengths. These wavelengths are of such a length that they could easily have gone undetected up to now.

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