More on the bending of light in quantum gravity

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We reconsider the long-range effects of the scattering of massless scalars and photons from a massive scalar object in quantum gravity. At the one-loop level, the relevant quantum mechanical corrections could be sorted into the graviton double-cut contributions, massless-scalar double-cut contributions and photon double-cut contributions. In Reference [N. E. J. Bjerrum-Bohr, J. F. Donoghue, B. R. Holstein, L. Planté, and P. Vanhove, Phys. Rev. Lett. **114**, 061301 (2015), J. High Energy Phys. 11 (2016) 117] N. E. J. Bjerrum-Bohr *et al.* have considered explicitly the implications of the graviton double-cut contributions on the gravitational bending of light and some classical formulations of the equivalence principle, using the modern double-copy constructions and on-shell unitarity techniques. In this article, instead we consider all three contributions and redo the analysis using the traditional Feynman diagrammatic approach. Our results on the graviton double-cut contributions agree with the aforementioned references, acting as a nontrivial check of previous computations. Furthermore, it turns out that the massless-scalar double-cut contributions and the photon double-cut contributions do leave nonvanishing quantum effects on the scattering amplitudes and the gravitational bending of light. Yet, we find that the general structure of the gravitational amplitudes and the quantum discrepancy of the equivalence principle suggested in the aforementioned references remain intact.

I. INTRODUCTION

The reconciliation of quantum mechanics and general relativity has been a long-standing open question in theoretical physics since the beginning of the last century, and different proposals have been put forward. For instance, recently a possible unification framework based on spin and scaling gauge invariance has been proposed in Refs. [1,2]. At present, there is no general consensus on the ultimate solution yet. But still, the long-range effects of the underlying quantum gravity can now be calculated reliably by treating general relativity as a low-energy effective field theory [3,4] (see also Refs. [5–20] for later developments), even though general relativity is nonrenormalizable and requires an infinite number of counterterms to absorb all the ultraviolet divergences. For a recent review, we recommend Ref. [21]. It is shown in Refs. [10,15,22] that the spin-independent part of the quantum Newtonian potential between a small mass and a large mass is

$$V(r) = -\frac{GMm}{r}\left(1 + \frac{3G(M+m)}{r} + \frac{41G\hbar}{10r^2}\right), \quad (1.1)$$

where M is a large scalar object, say, the Sun; m is the small test mass whose spin could be 0, 1/2 or 1; and r is the

relative distance between these two objects. G and $\hbar$ denote Newton's constant and the Planck constant, respectively.

It is of physical interest to study other long-range effects of quantum gravity, among which, the leading quantum corrections to the gravitational bending of light[1] around the Sun are a perfect target. In order to handle this problem in the framework of quantum field theory, the Sun is mimicked by a heavy scalar field. As shown later, this approximation gives the correct classical bending angles. With this in mind, one could consider the Einstein-Hilbert action coupled to photons and two neutral scalars, one massless and the other massive,

$$S = \int d^4x \sqrt{-g}\left[-\frac{2}{\kappa^2}R - \frac{1}{4}(\nabla_\mu A_\nu - \nabla_\nu A_\mu)^2 + \frac{1}{2}(\partial_\mu \varphi)^2 + \frac{1}{2}(\partial_\mu \Phi)^2 - \frac{1}{2}M^2\Phi^2\right]. \quad (1.2)$$

Here, A_μ is the photon field, and φ and Φ are the massless and massive scalar fields, respectively. In this article, we adopt the mostly minus metric signature $(+,-,-,-)$ and $\kappa^2 = 32\pi G$. Note that ∇_μ is the usual covariant derivative

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[1]In this article, by "the gravitational bending of light," what we really mean is the gravitational bending of massless particles, including not only the photon but also the hypothetical massless scalar particle, etc.

with $\nabla_\mu A^\nu = \partial_\mu A^\nu + \Gamma^\nu{}_{\mu\lambda} A^\lambda$. These fields are denoted in the rest of this article as follows: graviton h, photon γ, massless scalar φ, massive scalar Φ, and graviton Faddeev-Popov (FP) ghost c (not shown explicitly in the above action). Although there might not be a massless scalar particle in the real nature, the massless scalar φ is introduced for the sake of studying the impact of quantum corrections on some classical formulations of the equivalence principle (see Sec. IV).

To calculate the leading quantum corrections to the bending angles of the massless scalar φ and photon γ in the gravitational field of the massive field Φ, one needs to first calculate the relevant scattering amplitudes up to one loop. As microscopically it is the scattering process $\varphi(\gamma)\Phi \to \varphi(\gamma)\Phi$ that is under consideration, there are only t-channel contributions. The s-channel and u-channel Feynman diagrams simply do not exist. Generally, for the scattering of a test particle in the gravitational field of a large mass, a typical scattering amplitude up to one loop is

$$\mathcal{M} \sim A + Bq^2 + \cdots + \alpha\kappa^4 \frac{1}{q^2} + \beta_1 \kappa^4 \ln(-q^2)$$

$$+ \beta_2 \kappa^4 \frac{1}{\sqrt{-q^2}} + \gamma\kappa^4 \ln\left(1 + \frac{q^2}{m^2}\right) + \cdots, \quad (1.3)$$

where q is the momentum transfer. Among these terms, only the *gapless* nonanalytic contributions, whose branch cuts extend to the origin of the complex plane, would lead to long-range effects after Fourier transformations. The analytic and gapped nonanalytic corrections will yield short-range $\delta^3(r)$ and $\exp(-mr)$ effects in the potential. Therefore, in order to work out the long-range effects of the one-loop quantum corrections, one only needs to consider diagrams with t-channel massless double cuts [9,10,22–24].

For scattering processes between $\varphi(\gamma)$ and Φ, the relevant Feynman diagrams with t-channel massless double cuts could be sorted into three classes: the graviton double-cut contributions, the massless-scalar double-cut contributions, and the photon double-cut contributions, whose meanings will be clear later. In Refs. [23,24], the graviton double-cut contributions have been calculated using the modern on-shell unitarity techniques and the double-copy constructions (see also Refs. [25–27] for relevant discussions). In this article, we consider the implications of all three contributions. Compared to Refs. [23,24], instead we adopt the traditional Feynman diagrammatic approach. As shown in later sections, our Feynman diagrammatic calculations of the graviton double-cut diagrams match the results of Refs. [23,24] exactly, which could be viewed as a nontrivial check of previous calculations. On the other hand, our results on the net effects of all three contributions are new and could be useful for further studies in this direction.

This article is organized as follows: In Sec. II, we present the tree-level amplitudes for the massless scalar and photon and provide a concise description of how to extract the Newtonian potential from scattering amplitudes. In Sec. III, we calculate the relevant one-loop amplitudes using Feynman diagrams. The main results are presented in terms of a set of parameters inspired by the modern one-loop calculation techniques. Then, we derive the low-energy limit of the one-loop amplitudes obtained. In Sec. IV, we calculate the bending of light in quantum gravity and comment on the validity of quantum violation of some classical formulations of the equivalence principle. In Sec. V, we make brief remarks on possible further directions. In particular, we find it physically interesting to calculate quantum corrections to other classical observables of general relativity, such as the Shapiro time delay effect, the so-called fourth test of general relativity. We also attach two appendixes at the end of this article to provide extra details of our computations. The MATHEMATICA codes for the one-loop quantum gravity calculations carried out in this article can be found in Ref. [28].

II. TREE-LEVEL AMPLITUDES

In this section, we extract the Newtonian potential from the tree-level scattering amplitudes. The model under investigation is given by Eq. (1.2). To do the perturbative calculations, one could write the metric as $\eta_{\mu\nu} + \kappa h_{\mu\nu}$ and expand all terms in $h_{\mu\nu}$. After quantization, $h_{\mu\nu}$ gives rise to the massless spin-2 graviton.

With the above action, the tree-level scattering amplitudes of the massless scalar φ and the photon γ in the gravitational field of the massive scalar Φ could be worked out straightforwardly, and they are presented as follows:

(i) Massless scalar φ:

$$\mathcal{M}^{(0)}(\phi(p_1)\Phi(p_2)\phi(p_3)\Phi(p_4))$$
$$= \frac{\kappa^2}{4} \frac{(s-M^2)(u-M^2)}{t}. \quad (2.1)$$

(ii) Photon γ:

$$\mathcal{M}^{(0)}(\gamma(p_1^+)\Phi(p_2)\gamma(p_3^+)\Phi(p_4)) = 0,$$
$$\mathcal{M}^{(0)}(\gamma(p_1^-)\Phi(p_2)\gamma(p_3^-)\Phi(p_4)) = 0, \quad (2.2)$$

$$\mathcal{M}^{(0)}(\gamma(p_1^+)\Phi(p_2)\gamma(p_3^-)\Phi(p_4)) = \frac{\kappa^2}{4} \frac{\langle p_3|p_2|p_1]^2}{t},$$
$$\quad (2.3)$$

$$\mathcal{M}^{(0)}(\gamma(p_1^-)\Phi(p_2)\gamma(p_3^+)\Phi(p_4)) = \frac{\kappa^2}{4} \frac{\langle p_1|p_2|p_3]^2}{t}.$$
$$\quad (2.4)$$

In the above, we have adopted the in-in formalism with all momenta defined as incoming, as well as the spinor-helicity variables. The Mandelstam variables s, t, and u are given by $s = (p_1 + p_2)^2$, $t = (p_1 + p_3)^2$, and $u = (p_1 + p_4)^2$, respectively, and they satisfy the identity $s + t + u = 2M^2$.

There are various definitions of the quantum gravity potential in the literature, depending on the physical situations to be handled, the Feynman diagrams involved, etc. A physical plausible definition should certainly be gauge independent [8]. In this article, we take the definition of the (quantum) Newtonian potential given by Eqs. (A15)–(A17), which is used by various authors [6,9–11,23,24]. Such construction relates the potential directly to the full scattering amplitudes and therefore enjoys an explicit gauge independence. In the low-energy limit, $t \to -\mathbf{q}^2$, $s \to M^2 + 2M\omega$, we have the following:

(i) Massless scalar φ:

$$\mathcal{M}^{(0)}(\phi(p_1)\Phi(p_2)\phi(p_3)\Phi(p_4))$$
$$= \frac{32\pi GM^2\omega^2}{\mathbf{q}^2} \Rightarrow \tilde{V}_\varphi^{(0)}(\mathbf{q})$$
$$= -\frac{8\pi GM\omega}{\mathbf{q}^2}. \tag{2.5}$$

When Fourier transformed back to the coordinate space, we get the Newtonian potential

$$V_\varphi^{(0)}(r) = \int \frac{d^3\mathbf{q}}{(2\pi)^3} \exp(i\mathbf{q} \cdot \mathbf{r}) \tilde{V}_\varphi^{(0)}(\mathbf{q}) = -\frac{2GM\omega}{r}. \tag{2.6}$$

(ii) Photon γ: The tree-level amplitudes for the photon shown above, especially the spinor chains, are displayed in terms of the in-in formalism. When translated into the standard in-out formalism (with $p_3 \to -p_3$), one has

$$\mathcal{M}^{(0)}(\gamma(p_1^+)\Phi(p_2)\gamma(p_3^-)\Phi(p_4)) = -\frac{\kappa^2}{4}\frac{\langle p_3|p_2|p_1]^2}{t}, \tag{2.7}$$

$$\mathcal{M}^{(0)}(\gamma(p_1^-)\Phi(p_2)\gamma(p_3^+)\Phi(p_4)) = -\frac{\kappa^2}{4}\frac{\langle p_1|p_2|p_3]^2}{t}, \tag{2.8}$$

and the Mandelstam variable $t = (p_1 - p_3)^2$. In the low-energy limit, besides $t \to -\mathbf{q}^2$, $s \to M^2 + 2M\omega$, one has $\langle p_3|p_2|p_1]^2(\langle p_1|p_2|p_3]^2) \to 4M^2\omega^2$. As a result, the low-energy limits of the relevant scattering amplitudes are given by

$$\mathcal{M}^{(0)}(\gamma(p_1^+)\Phi(p_2)\gamma(p_3^-)\Phi(p_4)) = \frac{32\pi GM^2\omega^2}{\mathbf{q}^2}, \tag{2.9}$$

$$\mathcal{M}^{(0)}(\gamma(p_1^-)\Phi(p_2)\gamma(p_3^+)\Phi(p_4)) = \frac{32\pi GM^2\omega^2}{\mathbf{q}^2}, \tag{2.10}$$

$$\Rightarrow \tilde{V}_\gamma^{(0)}(\mathbf{q}) = -\frac{8\pi GM\omega}{\mathbf{q}^2}. \tag{2.11}$$

The Newtonian potential in coordinate space is then given by $V_\gamma^{(0)}(r) = -2GM\omega/r$, which is symbolically the same as that of the massless scalar φ. This is nothing but a direct manifestation of the classical equivalence principle.

III. ONE-LOOP AMPLITUDES

In this section, we calculate the relevant one-loop amplitudes. As mentioned in the Introduction, the one-loop scatterings of φ and γ in the gravitational field of the massive object Φ could be sorted into three classes: the graviton double-cut diagrams, the massless-scalar double-cut diagrams, and the photon double-cut diagrams. Let us take the photon as an example to show our classifications. The relevant Feynman diagrams could be found in Figs. 1 and 2. Figure 1 is characterized by the feature that one could find a horizontal cut for every diagram such that two graviton propagators are cut down. Here we have also included the graviton FP ghost c in the graviton double-cut diagrams. As a result, we call diagrams in Fig. 1 the graviton double-cut diagrams. Similarly, diagrams in Fig. 2 are named the photon double-cut diagrams, as for these diagrams one could find a horizontal cut such that two photon propagators are cut down. Here we have omitted the massless-scalar double-cut diagrams for the photon scattering, i.e., the Feynman diagram with the massless scalar φ running in the vacuum polarization loop. This is because, unlike the photon, the massless scalar φ is an "optional" ingredient for our real world. Similarly, for the massless-scalar scattering, we omit the Feynman diagram with the photon γ in the vacuum polarization loop and consider only the graviton double-cut diagrams and the massless-scalar double-cut diagrams (Fig. 3).[2]

Unlike Refs. [23,24] the one-loop calculations are done using the traditional Feynman diagrammatic approach rather than the modern on-shell unitarity techniques.
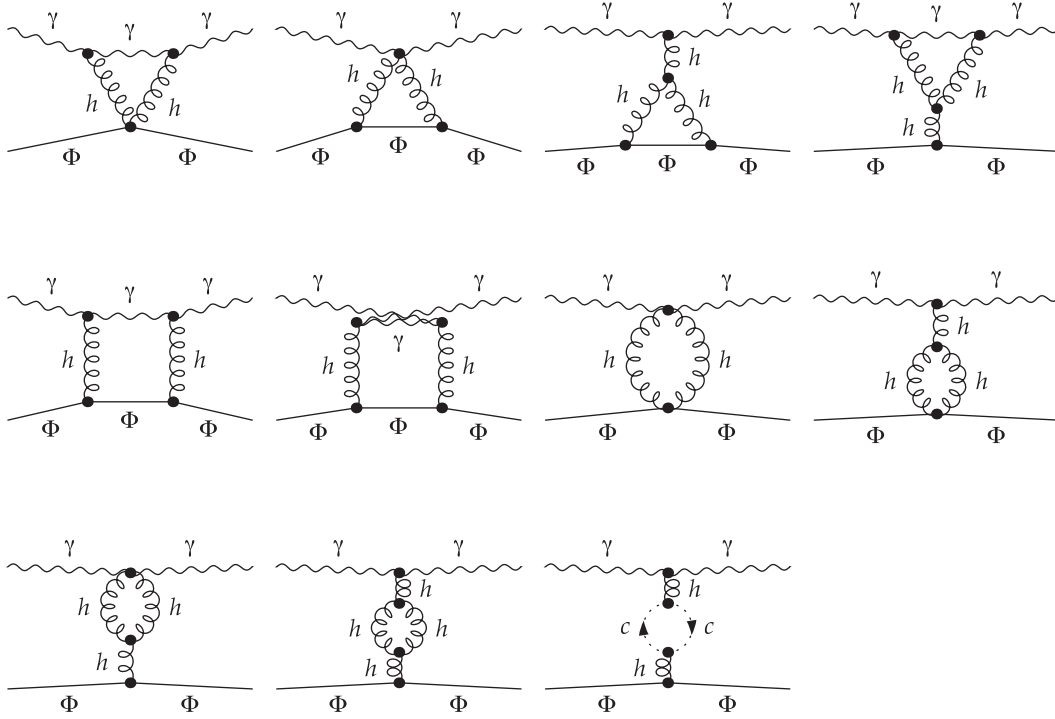
FIG. 1. Graviton double-cut diagrams for the one-loop scattering between the photon γ and the massive scalar Φ. Unlike Refs. [23,24], in this article we take the time direction to be horizontal. In the last diagram, we have the graviton FP ghost running in the loop.
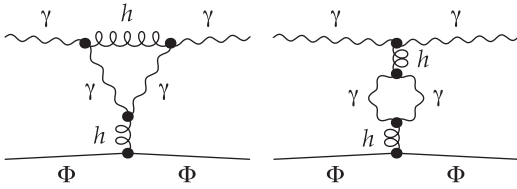

FIG. 2. Photon double-cut diagrams for the one-loop scattering between the photon γ and the massive scalar Φ.

There are several reasons for this choice. First, this allows us to carry out independent calculations of bending of light in quantum gravity and could be viewed as a useful complement to the existing literature. Second, there are various well-developed public MATHEMATICA codes, such as FeynRules [29], FeynArts [30], FormCalc [31], etc., which semi-automatize the one-loop Feynman diagrammatic calculations.

Concerning the representation of our results, we utilize the fact that a generic one-loop integral could be represented as $\mathcal{M} = \sum_i c_i I_i$, where c_i are rational functions of various kinematic invariants and I_i are some known scalar integral functions representing simple one-loop diagrammatic contributions such as box, triangle, and bubble diagrams. This result goes back to Passarino and Veltman [32]. As a result, for the problem of the gravitational bending of light, the gapless nonanalytic parts of the one-loop amplitudes, the only parts that

contribute to the long-range quantum gravity effects, could be parametrized in the following way:

$$
\begin{aligned}
\mathcal{M}_\eta^{(1)}&(p_1, p_2, p_3, p_4)|_{\text{nonanalytic}} \\
&\sim \text{Bo}^\eta \times I_4(t, s) + \text{Bo}'^\eta \times I_4(t, u) + \text{T}^\eta \times I_3(t, 0) \\
&\quad + \text{T}'^\eta \times I_3(t, M^2) + \text{Bu}^\eta \times I_2(t, 0).
\end{aligned}
\tag{3.1}
$$

Note that $I_4(t, s)(I_4(t, u))$, $I_3(t, 0)$, $I_3(t, M^2)$, and $I_2(t, 0)$ are the standard scalar integrals, whose explicit expressions are found in Appendix B. The Bo^η, T^η, T'^η, and Bu^η parameters are meromorphic functions of kinematic invariants, such as the Mandelstam variables s, t, and u.

For the massless scalar φ, the one-loop coefficients are given by the following:

(i) The box coefficients:

$$
\text{Bo}^\varphi = \frac{\kappa^4}{256\pi^2} (M^2 - s)^4,
\tag{3.2}
$$

$$
\text{Bo}'^\varphi = \frac{\kappa^4}{256\pi^2} (-M^2 + s + t)^4.
\tag{3.3}
$$

(ii) The triangle coefficients:

$$
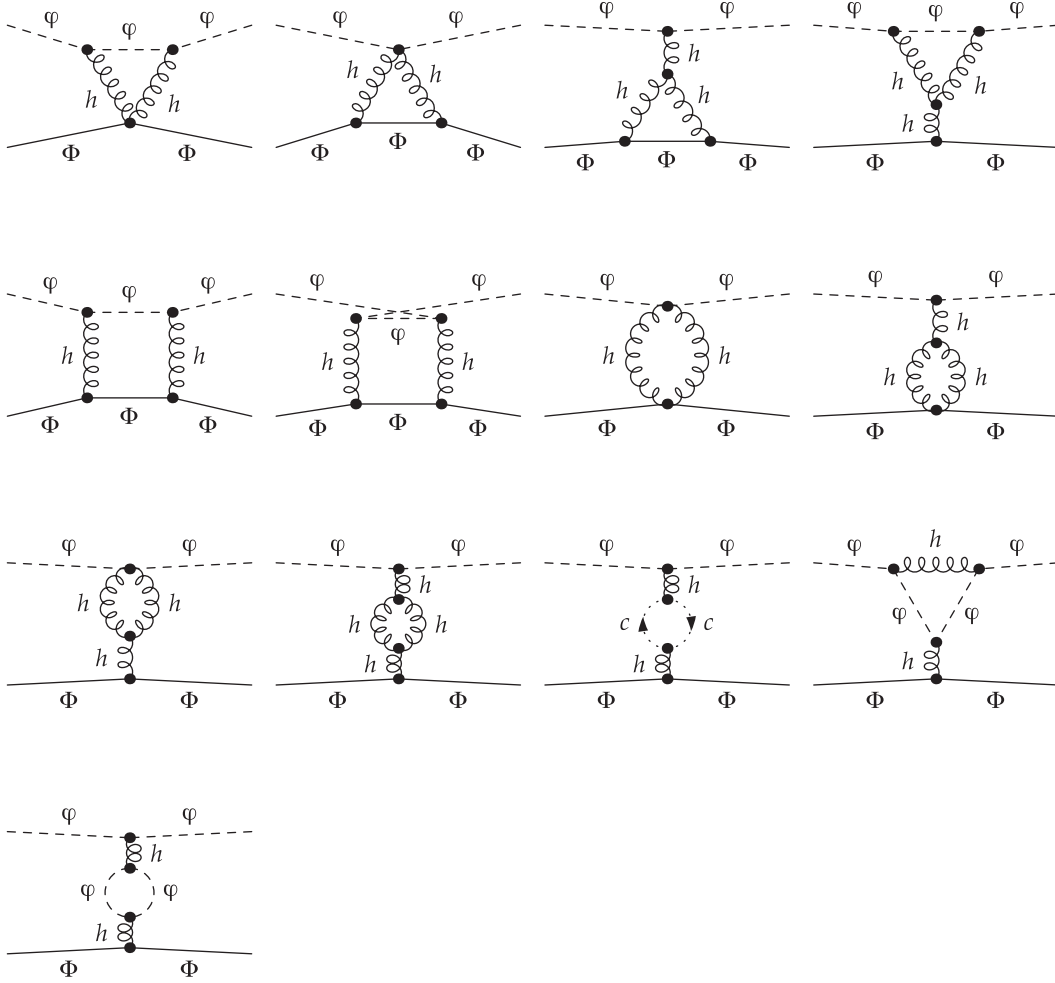\text{T}^\varphi = \frac{\kappa^4}{256\pi^2} t(-2M^2 + 2s + t)^2,
\tag{3.4}
$$

FIG. 3. The one-loop Feynman diagrams for the scattering of the massless scalar φ from the massive scalar Φ. The first eleven diagrams constitute the graviton double-cut contributions, while the last two diagrams constitute the massless-scalar double-cut contributions.

$$T'^{\varphi} = \frac{\kappa^4}{256\pi^2(-4M^2 + t)^2}$$
$$\times [-60M^{10} + 2M^8(60s + 73t)$$
$$+ t^3(3s^2 + 3st + t^2)$$
$$- 20M^6(3s^2 + 12st + 7t^2)$$
$$- M^2t^2(30s^2 + 36st + 13t^2)$$
$$+ 3M^4t(30s^2 + 50st + 21t^2)]. \qquad (3.5)$$

(iii) The bubble coefficient:

$$\mathrm{Bu}^{\varphi} = \frac{\kappa^4}{15360\pi^2(-4M^2 + t)^2}$$
$$\times [2968M^8 - 424M^6(14s + 9t)$$
$$+ t^2(103s^2 + 103st + 23t^2)$$
$$- M^2t(1064s^2 + 1270st + 341t^2)$$
$$+ M^4(2968s^2 + 5096st + 1787t^2)]. \qquad (3.6)$$

The one-loop coefficients for the photon γ with the $(-+)$ helicity are given by the following:

(i) The box coefficients:

$$\mathrm{Bo}^{\gamma} = \frac{\kappa^4 \langle p_1|p_2|p_3]^2}{512\pi^2(M^4 - 2M^2s + s(s + t))^2}$$
$$\times (M^2 - s)^2[2M^8 - 8M^6s - 4M^2s^2(2s + t)$$
$$+ 2M^4s(6s + t) + s^2(2s^2 + 2st + t^2)],$$
$$\qquad (3.7)$$

$$\mathrm{Bo}'^{\gamma} = \frac{\kappa^4 \langle p_1|p_2|p_3]^2}{512\pi^2(M^4 - 2M^2s + s(s + t))^2}$$
$$\times (-M^2 + s + t)^2[t^2(-2M^2 + s + t)^2$$
$$+ 2(-M^2 + s + t)^2$$
$$\times (M^4 - 2M^2s + s(s + t))]. \qquad (3.8)$$

(ii) The triangle coefficients:

$$
\mathrm{T}^{\gamma} = \frac{\kappa^4 \langle p_1 | p_2 | p_3]^2}{512\pi^2 (M^4 - 2M^2 s + s(s+t))^2} t[8M^8 + 8s^4 + 16s^3 t + 13s^2 t^2 + 5st^3 + t^4 - 2M^6(16s + 5t)
$$
$$
+ 2M^4(24s^2 + 18st + 5t^2) - M^2(32s^3 + 42s^2 t + 20st^2 + 5t^3)], \tag{3.9}
$$

$$
\mathrm{T}'^{\gamma} = -\frac{\kappa^4 \langle p_1 | p_2 | p_3]^2}{512\pi^2 (-4M^2 + t)^2 (M^4 - 2M^2 s + s(s+t))^2}
$$
$$
\times [120M^{14} - 20M^{12}(24s + 17t) + 4M^{10}(180s^2 + 320st + 131t^2)
$$
$$
- 8M^8(60s^3 + 215s^2 t + 195st^2 + 62t^3) - t^3(6s^4 + 12s^3 t + 11s^2 t^2 + 5st^3 + t^4)
$$
$$
+ M^2 t^2(60s^4 + 144s^3 t + 144s^2 t^2 + 70st^3 + 15t^4)
$$
$$
- 2M^4 t(90s^4 + 300s^3 t + 353s^2 t^2 + 191st^3 + 45t^4)
$$
$$
+ 4M^6(30s^4 + 240s^3 t + 400s^2 t^2 + 261st^3 + 70t^4)]. \tag{3.10}
$$

(iii) The bubble coefficient:

$$
\mathrm{Bu}^{\gamma} = \frac{\kappa^4 \langle p_1 | p_2 | p_3]^2}{7680\pi^2 (-4M^2 + t)^2 (M^4 - 2M^2 s + s(s+t))}
$$
$$
\times [452M^8 + M^6(-904s + 944t) + M^4(452s^2 + 484st - 973t^2)
$$
$$
- t^2(13s^2 + 13st + 30t^2) + 2M^2 t(-8s^2 + 5st + 150t^2)]. \tag{3.11}
$$

For the photon case, the one-loop coefficients for the $(+-)$ helicity configuration could be obtained simply by replacing $\langle p_1 | p_2 | p_3]^2$ with $\langle p_3 | p_2 | p_1]^2$. What is potentially confusing in the above results is that while the photon helicity configurations [e.g., the helicity configuration $(-+)$, $(+-)$, etc.] are written in terms of the in-in formalism, which means that all momenta flow in and it is much more common in the modern scattering amplitude community, we have adopted the standard in-out formalism to present the explicit expressions of various one-loop coefficients, in which we have two particles incoming and two particles outgoing. This choice introduces an extra minus sign in various places involving the spinor chain $\langle p_1 | p_2 | p_3]^2$, when compared to expressions of the in-in formalism.[3]

This hybrid convention would be helpful when comparing our results with those in previous literature.

When compared with the results of Refs. [23,24], one could find that the inclusion of the massless-scalar double-cut diagrams and photon double-cut diagrams changes both the triangle coefficient T^{η} and the bubble coefficient Bu^{η}. It is also interesting to note that the Bern-Carrasco-Johansson (BCJ)-inspired relation

$$
\frac{\mathrm{Bo}^{\eta}}{M^2 - s} + \frac{\mathrm{Bo}'^{\eta}}{M^2 - u} = \mathrm{T}^{\eta}, \quad \text{for } \eta = \varphi, \gamma, \tag{3.12}
$$

proposed in Refs. [23,24] no longer holds. But we have checked explicitly that the above relation still holds if only the graviton double-cut diagrams are taken into consideration.

A. Low-energy limit

In the previous part of Sec. III, we have provided all the relevant one-loop coefficients of gravitational scattering of the massless scalar and photon from a massive scalar. To figure out the long-range effects of scattering processes of quantum gravity, we need to first compute the low-energy limit of the scattering amplitude, in which we have $s \to M^2 + 2M\omega$ and $t \to -\mathbf{q}^2$ for the kinematic invariants, and $\langle p_1 | p_2 | p_3]^2 (\langle p_3 | p_2 | p_1]^2) \to 4M^2 \omega^2$ for the spinor chains. The low-energy limits of various one-loop coefficients are given as follows:

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[3]One way to see this extra minus sign is through the explicit realization of the dotted and undotted Weyl spinors. Following Ref. [33], we have

$$
\lambda^{\alpha} = \frac{1}{\sqrt{p^0 + p^3}} \begin{pmatrix} p^0 + p^3 \\ p^1 + ip^2 \end{pmatrix},
$$
$$
\tilde{\lambda}^{\dot{\alpha}} = \frac{1}{\sqrt{p^0 + p^3}} \begin{pmatrix} p^0 + p^3 \\ p^1 - ip^2 \end{pmatrix},
$$

with p^0, p^1, p^2, and p^3 the four components of the four-momentum p^{μ}. When reverting the direction of the momentum flow by making the replacement $p^{\mu} \to -p^{\mu}$, it is straightforward to see that

$$
\lambda^{\alpha} \to i\lambda^{\alpha}, \qquad \tilde{\lambda}^{\dot{\alpha}} \to i\tilde{\lambda}^{\dot{\alpha}}.
$$

The extra minus sign in various spinor chains squared then follows straightforwardly.

$$\text{Bo}^\varphi = \text{Bo}'^\varphi = \text{Bo}^\gamma = \text{Bo}'^\gamma = \frac{\kappa^4}{16\pi^2} M^4 \omega^4,$$

$$\text{T}^\varphi = \text{T}^\gamma = -\frac{\kappa^4}{16\pi^2} M^2 \omega^2 \mathbf{q}^2,$$

$$\text{T}'^\varphi = \text{T}'^\gamma = -\frac{15\kappa^4}{256\pi^2} M^4 \omega^2,$$

$$\text{Bu}^\varphi = \frac{371\kappa^4}{7680\pi^2} M^2 \omega^2,$$

$$\text{Bu}^\gamma = \frac{113\kappa^4}{7680\pi^2} M^2 \omega^2. \tag{3.13}$$

It is straightforward to show that the above results for the photon one-loop coefficients hold regardless of whether the photon helicity configuration is chosen to be $(-+)$ or $(+-)$.

On the other hand, the low-energy limits of the scalar integrals are found to be (see Appendix B for more details)

$$I_4(t,s) + I_4(t,u) = -i\frac{1}{\mathbf{q}^2} \frac{2\pi}{M\omega} \log\left(\frac{\mathbf{q}^2}{M^2}\right), \tag{3.14}$$

$$I_3(t,0) = \frac{1}{2\mathbf{q}^2} \log^2\left(\frac{\mathbf{q}^2}{\mu^2}\right), \tag{3.15}$$

$$I_3(t,M^2) = -\frac{1}{2M^2}\left(\log\left(\frac{\mathbf{q}^2}{M^2}\right) + \frac{\pi^2 M}{|\mathbf{q}|}\right), \tag{3.16}$$

$$I_2(t,0) = 2 - \log\left(\frac{\mathbf{q}^2}{\mu^2}\right). \tag{3.17}$$

Here μ^2 is the mass scale used in dimensional regularization. It is worthwhile to note that the dimensional regularization is adopted to handle not only ultraviolet divergences but also infrared divergences. Temporarily, we label μ^2 resulting from ultraviolet divergences and infrared divergences as μ_{UV}^2 and μ_{IR}^2, respectively. Certainly, physical observables should not depend on μ_{UV}^2, but they can depend on μ_{IR}^2. For instance, in the calculation of inclusive cross sections including massless particles, μ_{IR} is the detector threshold, representing the resolution of the measurement. We have not done an explicit calculation by taking the detector resolution into consideration at this stage. It was pointed out by Refs. [23,24] that a full analysis of the impact of detector resolution is complicated. In the absence of such calculations, we simply replace all μ's in the logarithms by an infrared scale $1/r_0$, which is also the option taken by Refs. [23,24].

Provided with the above results, the total gravitational scattering amplitude

$$\mathcal{M}_\eta = \mathcal{M}_\eta^{(0)} + \mathcal{M}_\eta^{(1)} \tag{3.18}$$

then has the low-energy expansion

$$\mathcal{M}_\eta = \kappa^2 \frac{M^2\omega^2}{\mathbf{q}^2} + \kappa^4 \frac{15M^3\omega^2}{512|\mathbf{q}|} + \kappa^4 \frac{15M^2\omega^2}{512\pi^2} \log\left(\frac{\mathbf{q}^2}{M^2}\right)$$

$$\quad - \kappa^4 \frac{M^2\omega^2}{32\pi^2} \log^2\left(\frac{\mathbf{q}^2}{\mu^2}\right) - \kappa^4 \frac{\text{bu}^\eta}{(8\pi)^2} M^2\omega^2 \log\left(\frac{\mathbf{q}^2}{\mu^2}\right)$$

$$\quad - \kappa^4 \frac{M^3\omega^3}{8\pi} \frac{i}{\mathbf{q}^2} \log\left(\frac{\mathbf{q}^2}{M^2}\right), \tag{3.19}$$

where bu^η is related to Bu^η by

$$\text{bu}^\eta = \frac{(8\pi)^2}{\kappa^4 M^2\omega^2} \text{Bu}^\eta. \tag{3.20}$$

For the massless scalar φ and the photon γ, bu^η's are given explicitly by

$$\text{bu}^\varphi = \frac{371}{120}, \qquad \text{bu}^\gamma = \frac{113}{120}. \tag{3.21}$$

As mentioned before, the mass scale μ^2 is introduced by dimensional regularization. In the following discussions of quantum corrections to the Newtonian potential in the coordinate space and bending of light, we simply replace the scale μ by the infrared scale $1/r_0$.

Before moving on, we would like to remark on the logarithm-squared part $\log^2(\mathbf{q}^2/\mu^2)$. It comes from the massless triangle integral $I_3(t,0)$ and is related to infrared divergences. It is noteworthy that it is not canceled by the extra contributions from the massless-scalar and photon double-cut diagrams. In fact, the general structure of the one-loop amplitudes suggested in Refs. [23,24] remains intact even with the extra massless-scalar and photon double-cut contributions. None of them is canceled.

IV. BENDING OF LIGHT

Now, we are ready to calculate quantum corrections to gravitational bending of light, which is perhaps the most famous experimental verification of Einstein's general relativity. To relate the microscopic scattering amplitude data to the macroscopic bending angle, one could use either the eikonal approximation or the semiclassical potential method. We recommend Ref. [24] for a recent discussion on this issue. In this article, we simply use the semiclassical potential method.

Let us first calculate the quantum Newtonian potential $V_\eta(r)$. The classical Newtonian potential $V_\eta^{(0)}(r)$ is given in Sec. II, while the quantum corrections $V_\eta^{(1)}(r)$ could be induced easily from Sec. III A. The total quantum Newtonian potential $V(r) = V_\eta^{(0)}(r) + V_\eta^{(1)}(r)$ is then given by

$$\tilde{V}_\eta(\mathbf{q}) = -\frac{\kappa^2}{4}\frac{M\omega}{\mathbf{q}^2} - \kappa^4\frac{15M^2\omega}{2048|\mathbf{q}|} - \kappa^4\frac{15M\omega}{2048\pi^2}\log\left(\frac{\mathbf{q}^2}{M^2}\right)$$
$$+ \kappa^4\frac{M\omega}{128\pi^2}\log^2\left(\frac{\mathbf{q}^2}{\mu^2}\right) + \kappa^4\frac{bu^\eta}{256\pi^2}M\omega\log\left(\frac{\mathbf{q}^2}{\mu^2}\right),$$

$$\tag{4.1}$$

$$\Longrightarrow V_\eta(r) = \int\frac{d^3\mathbf{q}}{(2\pi)^3}\exp(i\mathbf{q}\cdot\mathbf{r})\tilde{V}_\eta(\mathbf{q})$$
$$= -\frac{2GM\omega}{r} - \frac{15}{4}\frac{G^2M^2\omega}{r^2}$$
$$+ \frac{-8bu^\eta + 15 + 64\log(r/r_0)}{4\pi}\frac{\hbar G^2M\omega}{r^3},$$

$$\tag{4.2}$$

where r_0 is an undetermined infrared mass scale.

Provided with the above semiclassical potential, the bending angles of the massless scalar and the photon could be derived as follows:

$$\theta_\eta = \frac{b}{\omega}\int_{-\infty}^{\infty}du\frac{V_\eta'(b\sqrt{1+u^2})}{\sqrt{1+u^2}}$$
$$= \frac{4GM}{b} + \frac{15}{4}\frac{G^2M^2\pi}{b^2}$$
$$+ \frac{8bu^\eta - 47 + 64\log(2r_0/b)}{\pi}\frac{G^2\hbar M}{b^3}. \tag{4.3}$$

The above result is expressed in terms of the gauge-invariant impact parameter b. See Ref. [34] for a nice discussion on the gauge-invariant parametrization of the gravitational bending angle. The first two terms give the correct classical bending angle, including the first post-Newtonian correction, while the last term gives the quantum gravity effect. The difference between bending angles of the photon and the massless scalar is then given by

$$\theta_\gamma - \theta_\varphi = \frac{8(bu^\gamma - bu^\varphi)}{\pi}\frac{G^2\hbar M}{b^3}, \tag{4.4}$$

with the $bu^\gamma - bu^\varphi$ coefficient given by

$$bu^\gamma - bu^\varphi = -\frac{43}{20}. \tag{4.5}$$

As a result, the quantum violation of some classical formulations of the equivalence principle suggested in Refs. [23,24] remains valid after including the massless-scalar and photon double-cut contributions.

Last but not least, it is noted in Refs. [23,24] that their calculations could reproduce the classical first post-Newtonian correction exactly. It is interesting to see how this comes about even without including the massless-scalar and photon double-cut contributions. Given the general structure of the gravitational scattering amplitudes and the explicit expressions of the standard scalar integrals, it is straightforward to see that only the massive triangle integral $I_3(t, M^2)$ contributes to the post-Newtonian correction. As the massless-scalar and photon double-cut contributions do not contain any contribution from $I_3(t, M^2)$, they do not contribute to the post-Newtonian correction.

V. FURTHER DIRECTIONS

In this article, we have derived the one-loop amplitudes for the massless scalar and the photon scattering gravitationally from a massive scalar object, and we have extracted the corresponding gravitational bending angles. We consider not only the graviton double-cut contributions, which have been studied in Refs. [23,24], but also the extra contributions from the massless-scalar and photon double-cut diagrams. The final results for the gravitational bending angles are given by

$$\theta_\eta = \frac{4GM}{b} + \frac{15}{4}\frac{G^2M^2\pi}{b^2}$$
$$+ \frac{8bu^\eta - 47 + 64\log(2r_0/b)}{\pi}\frac{G^2\hbar M}{b^3}, \tag{5.1}$$

with

$$bu^\varphi = \frac{371}{120}, \qquad bu^\gamma = \frac{113}{120}, \tag{5.2}$$

for the massless scalar φ and photon γ, respectively. In spite of the quantitative differences in various one-loop coefficients, quantum Newtonian potentials, and gravitational bending angles, we find that the general structures of one-loop amplitudes suggested in Refs. [23,24] remain the same, as does the quantum discrepancy of some classical formulations of the equivalence principle.

There are several directions to extend our analysis:

(1) It is interesting to compute quantum corrections to other general relativity observables, such as the gravitational redshift, the Shapiro time delay, etc. In particular, it was shown recently that the Shapiro time delay puts extra constraints on the graviton three-point couplings [35] and the popular de Rham-Gabadadze-Tolley massive gravity [36]. It is theoretically interesting to study its possible quantum corrections.

(2) It is interesting to compute the one-loop gravitational amplitudes for massless spin-1/2, spin-3/2, and spin-2 particles. There has been some progress in this direction. For instance, the graviton double-cut contributions of the massless spin-1/2 particle have been derived in Ref. [24] using the on-shell unitarity method.

(3) It is recognized that the first post-Newtonian correction appears in the one-loop calculations. A natural guess is that the post-Newtonian corrections of higher orders should correspond to the multiloop amplitudes. The second post-Newtonian correction could be calculated using the method of Ref. [34] from the general-relativity side. To validate the above conjecture, one then has to carry out a two-loop calculation from the quantum-field-theory side. Hopefully, we will return to these issues in future publications.

APPENDIX A: NOTATIONS AND CONVENTIONS

This appendix gives a comprehensive description of the notations and conventions used in this paper, and we hope that it helps the readers reproduce our results by themselves. The following notations and conventions could also be easily implemented in various MATHEMATICA packages like xAct [37], FeynRules [29], FeynArts [30], FormCalc [31], and S@M [38], which (semi-)automatize the symbolic calculations.

(i) Units:

$$\hbar = c = 1. \tag{A1}$$

(ii) Metric signature:

$$g_{\mu\nu} = \text{diag}(+, -, -, -). \tag{A2}$$

In other words, we adopt the mostly minus metric signature, which is commonly used in perturbative calculations of particle physics.

(iii) Levi-Civita tensor:

$$\epsilon^{\mu\nu\rho\sigma} = \begin{cases} 1 & \text{if } \{\mu, \nu, \rho, \sigma\} \text{ is an even permutation of } \{0, 1, 2, 3\}, \\ -1 & \text{if } \{\mu, \nu, \rho, \sigma\} \text{ is an odd permutation of } \{0, 1, 2, 3\}, \\ 0 & \text{others.} \end{cases} \tag{A3}$$

(iv) Fourier transformations: In four dimensions, the Fourier transformations are defined as

$$f(x) = \int \frac{d^4 k}{(2\pi)^4} \exp(-ik \cdot x) \tilde{f}(k), \tag{A4}$$

$$\tilde{f}(k) = \int d^4 x \exp(ik \cdot x) f(x). \tag{A5}$$

In three dimensions, the Fourier transformations are defined as

$$f(x) = \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \exp(i\mathbf{k} \cdot \mathbf{x}) \tilde{f}(\mathbf{k}), \tag{A6}$$

$$\tilde{f}(\mathbf{k}) = \int d^3 \mathbf{k} \exp(-i\mathbf{k} \cdot \mathbf{x}) f(\mathbf{x}). \tag{A7}$$

In this article, the following relations are useful in deriving the classical and quantum Newtonian potentials in coordinate space:

$$\int \frac{d^3 \mathbf{q}}{(2\pi)^3} \exp(i\mathbf{q} \cdot \mathbf{r}) \frac{1}{\mathbf{q}^2} = \frac{1}{4\pi r}, \tag{A8}$$

$$\int \frac{d^3 \mathbf{q}}{(2\pi)^3} \exp(i\mathbf{q} \cdot \mathbf{r}) \frac{1}{|\mathbf{q}|} = \frac{1}{2\pi^2 r^2}, \tag{A9}$$

$$\int \frac{d^3 \mathbf{q}}{(2\pi)^3} \exp(i\mathbf{q} \cdot \mathbf{r}) \log(\mathbf{q}^2) = -\frac{1}{2\pi r^3}, \tag{A10}$$

$$\int \frac{d^3 \mathbf{q}}{(2\pi)^3} \exp(i\mathbf{q} \cdot \mathbf{r}) \log^2\left(\frac{\mathbf{q}^2}{\mu^2}\right) = \frac{2 \log \frac{r}{r_0}}{\pi r^3},$$
$$\text{with} \quad r_0 = e^{1-\gamma_E} \mu^{-1}. \tag{A11}$$

Here γ_E is the Euler constant. When deriving the above three-dimensional Fourier transformation relations, it is convenient to first do the calculations in the general D dimensions using, say, MATHEMATICA, and then Taylor expand the obtained results around $d = 3$. This is how the Euler constant γ_E comes about in Eq. (A11).

(v) Christoffel symbols:

$$\Gamma^{\mu}_{\alpha\beta} = \frac{1}{2} g^{\mu\nu}(\partial_{\alpha}g_{\nu\beta} + \partial_{\beta}g_{\nu\alpha} - \partial_{\nu}g_{\alpha\beta}). \quad (A12)$$

(vi) Riemann tensor:

$$R_{\delta\gamma\beta}{}^{\alpha} = \partial_{\gamma}\Gamma^{\alpha}_{\beta\delta} - \partial_{\delta}\Gamma^{\alpha}_{\beta\gamma} + \Gamma^{\alpha}_{\mu\gamma}\Gamma^{\mu}_{\beta\delta} - \Gamma^{\alpha}_{\mu\delta}\Gamma^{\mu}_{\beta\gamma}. \quad (A13)$$

This is the convention adopted by Wald's textbook [39] and the MATHEMATICA package xAct [37].

(vii) Einstein-Hilbert action:

$$\mathcal{S}_{\text{EH}} = \int d^4x \sqrt{-g}\left(-\frac{2}{\kappa^2}R\right), \quad (A14)$$

with $\kappa^2 = 32\pi G$, where G is Newton's constant. The Einstein-Hilbert action here is different from that in Wald's textbook by a minus sign. This is due to the fact that we adopt the mostly minus metric signature, rather than the mostly plus metric signature employed by Wald. More explicitly, $R = g^{\mu\nu}R_{\mu\nu} = g^{\mu\nu}R_{\mu\alpha\nu}{}^{\alpha}$. Given the above definitions of Christoffel symbols and the Riemann tensor, one only needs to replace $g_{\mu\nu}$ with $-g_{\mu\nu}$ in order to convert expressions in the mostly plus metric signature to those in the mostly minus metric signature. That gives the minus sign in our Einstein-Hilbert action.

(viii) S matrix:

$$S = \mathbf{1} + iT,$$

$$\times \langle \mathbf{k}_1\mathbf{k}_2\cdots |iT|\mathbf{p}_1\mathbf{p}_2\rangle \cdots$$

$$\equiv (2\pi)^4\delta^{(4)}\left(\sum p_i - \sum k_f\right)$$

$$\cdot i\mathcal{M}(\mathbf{p}_1\mathbf{p}_2\cdots \to \mathbf{k}_1\mathbf{k}_2\cdots).$$

Here the four-momentum is denoted by normal letters, while the three-momentum is denoted by boldface letters. For the nonrelativistic scattering in the external field of a massive object, the semi-classical potential function $V(\mathbf{r})$ is given by

$$\langle \mathbf{p}_f|iT|\mathbf{p}_i\rangle \equiv (2\pi)^4\delta^{(4)}(p_i - p_f) \cdot i\mathcal{M}(q)$$

$$= -(2\pi)\delta(E_i - E_f) \cdot i\tilde{V}(\mathbf{q}), \quad (A15)$$

with p_i and p_f being the incoming and outgoing four-momentum, and $q = p_f - p_i$. One could further see that

$$\tilde{V}(\mathbf{q}) = -\frac{1}{2M}\frac{1}{2\omega}\mathcal{M}, \quad (A16)$$

where ω is the energy of the massless particle (e.g., the photon γ) and M is the mass of the massive object (e.g., the massive scalar Φ). As a result,

$$V(r) = -\frac{1}{2M}\frac{1}{2\omega}\int \frac{d^3\mathbf{q}}{(2\pi)^3}\exp(i\mathbf{q}\cdot\mathbf{r})\mathcal{M}. \quad (A17)$$

(ix) Spinor-helicity formalism: We follow the convention of Henn and Plefka's textbook [33].

(x) Feynman propagators: In this article, we quantize gravity using the background-field method following 't Hooft and Veltman [40]. In the background-field method, fields are expanded with respect to arbitrary classical background fields, with only the quantized fields treated as dynamical. The gauge invariances of the quantized fields are then broken by a choice of quantum gauge in such a way that the resulting action is still invariant under *background* gauge transformations. Explicitly, we adopt the standard de Donder gauge and Feynman gauge to fix the quantum gauge invariances of the graviton and photon, respectively. For a detailed description of the quantization of the Einstein-Maxwell system, we recommend Ref. [41]. The relevant propagators in the Minkowski spacetime could then be found as follows:

- Scalar propagator $\quad \dfrac{i}{q^2 - m^2 + i\epsilon}, \quad (A18)$

- Photon propagator $\quad \dfrac{-i\eta_{\mu\nu}}{q^2 + i\epsilon}, \quad (A19)$

- Graviton propagator $\quad \dfrac{1}{2}$

$$\times [\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2} + \eta_{\mu_1\nu_2}\eta_{\nu_1\mu_2} - \eta_{\mu_1\nu_1}\eta_{\mu_2\nu_2}]\frac{i}{q^2 + i\epsilon}, \quad (A20)$$

- Graviton-ghost propagator $\quad \dfrac{-i\eta_{\mu\nu}}{q^2 + i\epsilon}. \quad (A21)$

See Ref. [28] for a FeynArts model file containing all the above propagators as well as interacting vertices up to four points. This acts as the starting point for our automatic calculation routines for the one-loop scattering amplitudes.

APPENDIX B: USEFUL INTEGRALS

In this appendix, we summarize the explicit expressions of various scalar integrals that appear in our calculations. We adopt the notations of Ref. [42]:

$$I_2^D(p_1^2; m_1^2, m_2^2) = \frac{\mu^{4-D}}{i\pi^{\frac{D}{2}} r_\Gamma} \int \mathrm{d}^D l \frac{1}{(l^2 - m_1^2 + i\epsilon)((l+q_1)^2 - m_2^2 + i\epsilon)}, \tag{B1}$$

$$I_3^D(p_1^2, p_2^2, p_3^2; m_1^2, m_2^2, m_3^2) = \frac{\mu^{4-D}}{i\pi^{\frac{D}{2}} r_\Gamma} \int \mathrm{d}^D l \frac{1}{(l^2 - m_1^2 + i\epsilon)((l+q_1)^2 - m_2^2 + i\epsilon)((l+q_2)^2 - m_3^2 + i\epsilon)}, \tag{B2}$$

$$I_4^D(p_1^2, p_2^2, p_3^2, p_4^2; s_{12}, s_{23}; m_1^2, m_2^2, m_3^2, m_4^2)$$
$$= \frac{\mu^{4-D}}{i\pi^{\frac{D}{2}} r_\Gamma} \int \mathrm{d}^D l \frac{1}{(l^2 - m_1^2 + i\epsilon)((l+q_1)^2 - m_2^2 + i\epsilon)((l+q_2)^2 - m_3^2 + i\epsilon)((l+q_3)^2 - m_4^4 + i\epsilon)}, \tag{B3}$$

where $r_\Gamma = \Gamma(1-\epsilon)^2 \Gamma(1+\epsilon)/\Gamma(1-2\epsilon)$, $q_n = \sum_{i=1}^n p_i$, $q_0 = 0$, and $s_{ij} = (p_i + p_j)^2$.

The scalar integrals $I_4(t, s)$, $I_4(t, u)$, $I_3(t, 0)$, $I_3(t, M^2)$, and $I_2(t, 0)$ in this article are then given as follows:

$$I_4(t, s) = I_4^D(t, 0, s, M^2; 0, M^2; 0, 0, 0, M^2)$$
$$= -\frac{1}{t(M^2 - s)} \left(\frac{\mu^2}{M^2}\right)^\epsilon \left[\frac{2}{\epsilon^2} - \frac{1}{\epsilon}\left(2\log\frac{M^2 - s}{M^2} + \log\frac{-t}{M^2}\right)\right.$$
$$\left. + 2\log\frac{M^2 - s}{M^2}\log\frac{-t}{M^2} - \frac{\pi^2}{2} + O(\epsilon)\right], \tag{B4}$$

$$I_3(t, 0) = I_3^D(t, 0, 0; 0, 0, 0) = -\frac{1}{t\epsilon^2} - \frac{\log(-t/\mu^2)}{t\epsilon} - \frac{\log^2(-t/\mu^2)}{2t} + O(\epsilon), \tag{B5}$$

$$I_3(t, M^2) = I_3^D(t, M^2, M^2; 0, 0, M^2)$$
$$= \frac{1}{t\beta}\left[\frac{2\pi^2}{3} + 2\mathrm{Li}_2\left(\frac{\beta - 1}{\beta + 1} + \frac{1}{2}\log^2\left(\frac{\beta - 1}{\beta + 1}\right)\right)\right], \quad \text{with} \quad \beta^2 = 1 - \frac{4M^2}{t}, \tag{B6}$$

$$I_2(t, 0) = I_2^D(t, 0, 0) = \frac{1}{\epsilon} + 2 - \log\left(\frac{-t}{\mu^2}\right) + O(\epsilon). \tag{B7}$$

The scalar integral $I_4(t, u)$ could be obtained by replacing s with u in Eq. (B4).

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