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Kerr-Newman-AdS black hole in quintessential dark energy

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Quintessential dark energy with pressure p and density ρ is related by equation of state $p = \omega\rho$ with the state parameter $-1 < \omega < -1/3$. The cosmological dark energy influence on black hole spacetime is interesting and important. In this paper, we study the Kerr-Newman-AdS solutions of the Einstein-Maxwell equation in quintessence field around a black hole by Newman-Janis algorithm and complex computations. From the horizon structure equation, we obtain the expression between quintessence parameter α and cosmological constant Λ if the black hole exists two cosmological horizon r_q and r_c when $\omega = -2/3$, the result is different from rotational black hole in quintessence matter situation. Through analysis we find that the black hole charge cannot change the value of α. But the black hole spin and cosmological constant are opposite. The black hole spin and cosmological constant make the maximum value of α small. The existence of four horizon leads seven types of extremal black holes to constrain the parameter α. With the state parameter ω ranging from -1 to $-1/3$, the maximum value of α changes from Λ to 1. When $\omega \to -1$, the quintessential dark energy likes cosmological constant. The singularity of the black holes is the same with that of Kerr black hole. We also discuss the rotation velocity of the black holes on the equatorial plane for $\omega = -2/3$, $-1/2$ and $-1/3$. For small value of α, the rotation velocity on the equatorial plane is asymptotically flat and it can explain the rotation curves in spiral galaxies.

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I. INTRODUCTION

In recent years, cosmological observations found that the universe is accelerating expansion, demanding the existence of dark energy [1–3]. The recent measurements of cosmic microwave background (CMB) anisotropy by PLANCK also confirmed this results [4]. Cosmological tests indicate that the dark energy accounts for 70% of energy content in the universe. The state equation of the dark energy is very close to the cosmological constant or vacuum energy. Besides the cosmological constant, an important dark energy model is called quintessence [1].

The dark energy content such as the cosmological constant or quintessence changes the spacetime structure of black hole. For the case of the cosmological constant, the asymptotic structure of black hole becomes the asymptotical de Sitter spacetime [5,6], in which a cosmological horizon exists. For the black hole in quintessence field, the cosmological horizon also exists [7].

The importance of cosmological constant in high energy astrophysical objects, such as active galactic nuclei and supermassive black holes, has been discussed ([8]). The spherically symmetric spacetime influenced by Λ term is described by the vacuum Schwarzschild-de Sitter spacetime (SdS) [5]. When the spacetime metric satisfies the axially symmetric case, the vacuum spacetime is described by Kerr-de Sitter spacetime (KdS) [9]. In these spacetimes, the motion of test particles or photons have been discussed by many authors [10–19]. For the spherically symmetric black hole in quintessence field, its spacetime solution has been discussed by [7]. The universe accelerating expansion demands the state parameter to be in range $-1 < \omega < -1/3$. The recent works generalized this result to Kerr black hole by the Janis-Newman algorithm [20,21], and the spacetime metric was studied [22–24]. Following these works, we generalize Kerr black hole solutions to Kerr-Newman black hole solutions in quintessential dark energy. Following we extend the Kerr-Newman solution to the cosmological constant presented case of quintessential dark energy.

In this paper, we want to seek for Kerr-Newman-AdS solution in the quintessence by Janis-Newman algorithm and complex computations, we also discuss the properties of a black hole solution. The outline of the paper is as follows. In Sec. II, we introduce the Reissner-Nordstrom black hole in quintessence matter and derive the Kerr-Newman solution through the Janis-Newman algorithm. Later we extend quintessence Kerr-Newman black hole to

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the case of existing cosmological constant. In Sec. III, we study the horizon structure, stationary limit surfaces and singularity of the black hole in Boyer-Lindquist coordinates. In Sec. IV, we calculate the circular geodesics on the equatorial plane. Summaries are drawn in Sec. V.

II. KERR-NEWMAN-ADS BLACK HOLE SOLUTION IN QUINTESSENCE

From spherically symmetric Reissner-Nordstrom black hole metric in the quintessence matter, we use the Newman-Janis algorithm to get the Kerr-Newman black hole metric around by quintessential dark energy. Because the Newman-Janis algorithm does not include a cosmological constant, we obtain the Kerr-Newman-AdS solution around by quintessential dark energy through direct computations.

A. Reissner-Nordstrom black hole in the quintessence

For the Reissner-Nordstrom black hole in the quintessence, the line element is expressed by

$$ds^2 = -f(r)dt^2 + \frac{1}{g(r)}dr^2 + r^2 d\Omega^2, \tag{1}$$

where $f(r)$ and $g(r)$ are given by [7]

$$f(r) = g(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\alpha}{r^{3\omega+1}}. \tag{2}$$

In this spacetime formalism, M is the black hole mass and α is the quintessence parameter that represents the intensity of the quintessence field related to the black hole. The parameter ω describes the equation of state with $\omega = p/\rho$, where p and ρ are the pressure and energy density of the quintessence respectively, in which ω will not equal $0, 1/3, -1$ if $-1 < \omega < -1/3$ can explain the universe accelerating expansion. Thus we have a general form of the Reissner-Nordstrom spacetime metric for the Einstein-Maxwell equation representing charge black hole in quintessential field. The parameter ω determines the property of spacetime metric. If $-1/3 < \omega < 0$, the spacetime has the asymptotically flat solution. If $-1 < \omega < -1/3$, the spacetime has de Sitter horizon, causing the universe acceleration, and reduces to the Reissner-Nordstrom black hole for the $\alpha = 0$.

B. Newman-Janis algorithm and Kerr-Newman solution in quintessence matter

Now we derive a Kerr-Newman black hole solution in a quintessential field via the Newman-Janis algorithm. Following the Newman-Janis algorithm [25–27] and more general discussion [28], we get the coordinate transformation as

$$du = dt - \frac{dr}{1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\alpha}{r^{3\omega+1}}}, \tag{3}$$

and Eq. (1) is written as

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\alpha}{r^{3\omega+1}}\right)du^2 - 2dudr + r^2 d\Omega^2. \tag{4}$$

Using the null tetrad, we write the metric matrix as

$$g^{\mu\nu} = -l^\mu n^\nu - l^\nu n^\mu + m^\mu \bar{m}^\nu + m^\nu \bar{m}^\mu, \tag{5}$$

where the corresponding components are

$$\begin{aligned} l^\mu &= \delta^\mu_r, \\ n^\mu &= \delta^\mu_0 - \frac{1}{2}\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\alpha}{r^{3\omega+1}}\right)\delta^\mu_r, \\ m^\mu &= \frac{1}{\sqrt{2}r}\delta^\mu_\theta + \frac{i}{\sqrt{2}r\sin\theta}\delta^\mu_\phi, \\ \bar{m}^\mu &= \frac{1}{\sqrt{2}r}\delta^\mu_\theta - \frac{i}{\sqrt{2}r\sin\theta}\delta^\mu_\phi. \end{aligned} \tag{6}$$

For any point in the spacetime, we choose the tetrad in the following manner: l^μ is the outward null vector tangent to the light cone, and n^μ is the inward null vector. l^μ and n^μ are real vectors. $\bar{m}^\mu$ indicates the complex conjugate of m^μ, and m^μ is a complex vector. In the null tetrad, they satisfy $l_\mu l^\mu = n_\mu n^\mu = m_\mu m^\mu = 0$, $l_\mu n^\mu = -m_\mu \bar{m}^\mu = 1$, $l_\mu m^\mu = n_\mu m^\mu = 0$. Making the complex coordinate transformations on the (u, r) plane as $u \to u - ia\cos\theta$, $r \to r - ia\cos\theta$, and following the changes of $f(r) \to F(r, a, \theta)$, $g(r) \to G(r, a, \theta)$ and $\Sigma^2 = r^2 + a^2\cos^2\theta$, we write the null tetrad in a new coordinate system as

$$\begin{aligned} l^\mu &= \delta^\mu_r, \\ n^\mu &= \sqrt{\frac{G}{F}}\delta^\mu_0 - \frac{1}{2}F\delta^\mu_r, \\ m^\mu &= \frac{1}{\sqrt{2}\Sigma}\left(\delta^\mu_\theta + ia\sin\theta(\delta^\mu_0 - \delta^\mu_r) + \frac{i}{\sin\theta}\delta^\mu_\phi\right), \\ \bar{m}^\mu &= \frac{1}{\sqrt{2}\Sigma}\left(\delta^\mu_\theta - ia\sin\theta(\delta^\mu_0 - \delta^\mu_r) - \frac{i}{\sin\theta}\delta^\mu_\phi\right). \end{aligned} \tag{7}$$

Using Eq. (5), we can get the metric tensor $g^{\mu\nu}$ in Eddington-Finkelstein coordinates. The covariant components of the metric tenser are given by

$$
\begin{aligned}
g_{uu} &= -F, \\
g_{\theta\theta} &= \Sigma^2, \\
g_{ur} &= g_{ru} = -\sqrt{\frac{G}{F}}, \\
g_{\phi\phi} &= \sin^2\theta\left(\Sigma^2 + a^2\left(2\sqrt{\frac{F}{G}} - F\right)\sin^2\theta\right), \\
g_{u\phi} &= g_{\phi u} = a\left(F - \sqrt{\frac{F}{G}}\right)\sin^2\theta, \\
g_{r\phi} &= g_{\phi r} = a\sin^2\theta\sqrt{\frac{F}{G}}. \qquad (8)
\end{aligned}
$$

Finally, we make the coordinate transformations from the Eddington-Finkelstein coordinates (u, r, θ, ϕ) to the Boyer-Lindquist coordinates (t, r, θ, ϕ) as

$$
\begin{aligned}
du &= dt + \lambda(r)dr, \\
d\phi &= d\phi + h(r)dr, \qquad (9)
\end{aligned}
$$

where

$$
\begin{aligned}
\lambda(r) &= -\frac{r^2 + a^2}{r^2 g(r) + a^2}, \\
h(r) &= -\frac{a}{r^2 g(r) + a^2}, \\
F(r,\theta) &= G(r,\theta) = \frac{r^2 g(r) + a^2\cos^2\theta}{\Sigma^2}. \qquad (10)
\end{aligned}
$$

In the Boyer-Lindquist coordinates (t, r, θ, ϕ), the Kerr-Newman metric in the Kiselev quintessence is

$$
\begin{aligned}
ds^2 = &-\left(1 - \frac{2Mr - Q^2 + \alpha r^{1-3\omega}}{\Sigma^2}\right)dt^2 \\
&+ \frac{\Sigma^2}{\Delta_r}dr^2 - \frac{2a\sin^2\theta(2Mr - Q^2 + \alpha r^{1-3\omega})}{\Sigma^2}d\phi dt + \Sigma^2 d\theta^2 \\
&+ \sin^2\theta\left(r^2 + a^2 + a^2\sin^2\theta\frac{2Mr - Q^2 + \alpha r^{1-3\omega}}{\Sigma^2}\right)d\phi^2, \qquad (11)
\end{aligned}
$$

where

$$
\Delta_r = r^2 - 2Mr + a^2 + Q^2 - \alpha r^{1-3\omega}. \qquad (12)
$$

Through calculating $R_{\mu\nu}$ and $T_{\mu\nu}$, Azreg-Ainou [28] found that this spacetime metric satisfies the Einstein equation. When the quintessence does not exist or $\alpha = 0$, the spacetime metric reduces to the Kerr-Newman black hole [26]. If $Q = 0$, the spacetime metric reduces to the rotational situation in the Kiselev quintessence black hole [21].

C. Kerr-Newman-AdS solution in quintessence matter

Now we extend the Kerr-Newman solution to the Kerr-Newman-AdS case of quintessential dark energy. First we rewrite the Kerr-Newman metric in quintessence matter as

$$
\begin{aligned}
ds^2 = &\frac{\Sigma^2}{\Delta_r}dr^2 + \Sigma^2 d\theta^2 + \frac{\sin^2\theta}{\Sigma^2}(adt - (r^2 + a^2)d\phi)^2 \\
&- \frac{\Delta_r}{\Sigma^2}(dt - a\sin^2 d\phi)^2. \qquad (13)
\end{aligned}
$$

Using the formula $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$, we derive the Einstein tenser by *Mathematica* package RGTC as

$$
\begin{aligned}
G_{tt} &= \frac{2[r^4 - 2r^3\rho + a^2r^2 - a^4\sin^2\theta\cos^2\theta]\rho'}{\Sigma^6} - \frac{ra^2\sin^2\theta\rho''}{\Sigma^4}, \qquad G_{rr} = -\frac{2r^2\rho'}{\Sigma^2\Delta_r} \\
G_{\theta\theta} &= -\frac{2a^2\cos^2\theta\rho'}{\Sigma^2} - r\rho'', \qquad G_{t\phi} = \frac{2a\sin^2\theta[(r^2 + a^2)(a^2\cos^2\theta - r^2)]\rho'}{\Sigma^6} - \frac{ra^2\sin^2\theta(r^2 + a^2)\rho''}{\Sigma^4}, \\
G_{\phi\phi} &= -\frac{a^2\sin^2\theta[(r^2 + a^2)(a^2 + (2r^2 + a^2)\cos 2\theta) + 2r^3\sin^2\theta\rho)]\rho'}{\Sigma^6} - \frac{r\sin^2\theta(r^2 + a^2)^2\rho''}{\Sigma^4}, \qquad (14)
\end{aligned}
$$

where $2\rho = \alpha r^{-3\omega} + 2M - \frac{Q^2}{r}$. For $Q = 0$, these Einstein tensors have been obtained [21]. Using Einstein equation with a cosmological constant and Maxwell equation

$$
G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu}, \qquad (15)
$$

$$
F^{\mu\nu}_{;\nu} = 0; \qquad F^{\mu\nu;\alpha} + F^{\nu\alpha;\mu} + F^{\alpha\mu;\nu} = 0, \qquad (16)
$$

where $F^{\mu\nu}$ is the Faraday tensor, we obtain the Kerr-Newman-AdS solution in quintessential dark energy.

Considering the cosmological constant, we guess the solution of Einstein-Maxwell equation in quintessence matter given by

$$
\begin{aligned}
ds^2 = &\frac{\Sigma^2}{\Delta_r}dr^2 + \frac{\Sigma^2}{\Delta_\theta}d\theta^2 + \frac{\Delta_\theta\sin^2\theta}{\Sigma^2}\left(a\frac{dt}{\Xi} - (r^2 + a^2)\frac{d\phi}{\Xi}\right)^2 \\
&- \frac{\Delta_r}{\Sigma^2}\left(\frac{dt}{\Xi} - a\sin^2\frac{d\phi}{\Xi}\right)^2, \qquad (17)
\end{aligned}
$$

where

$$\Delta_r = r^2 - 2Mr + a^2 + Q^2 - \frac{\Lambda}{3} r^2 (r^2 + a^2) - \alpha r^{1-3\omega},$$
$$\Delta_\theta = 1 + \frac{\Lambda}{3} a^2 \cos^2\theta \qquad \Xi = 1 + \frac{\Lambda}{3} a^2. \tag{18}$$

Calculating by *Mathematica* package RGTC, we also get the Einstein tenser as

$$\begin{aligned}
G_{tt} &= \frac{2[r^4 - 2r^3\rho + a^2r^2 - a^4\sin^2\theta\cos^2\theta]\rho'}{\Sigma^6} - \frac{ra^2\sin^2\theta\rho''}{\Sigma^4} + \Lambda\frac{a^2\sin^2\theta - \Delta_r}{\Sigma^2},\\
G_{rr} &= -\frac{2r^2\rho'}{\Sigma^2\Delta_r} + \Lambda\frac{\Sigma^2}{\Delta_r},\\
G_{\theta\theta} &= -\frac{2a^2\cos^2\theta\rho'}{\Sigma^2} - r\rho'' + \Lambda\Sigma^2 G_{t\phi}\\
&= \frac{2a\sin^2\theta[(r^2+a^2)(a^2cos^2\theta - r^2)]\rho'}{\Sigma^6}\\
&\quad - \frac{ra^2\sin^2\theta(r^2+a^2)\rho''}{\Sigma^4} + \Lambda\frac{a\sin^2\theta[\Delta_r - r^2 - a^2]}{\Sigma^2},\\
G_{\phi\phi} &= -\frac{a^2\sin^2\theta[(r^2+a^2)(a^2 + (2r^2+a^2)\cos 2\theta) + 2r^3\sin^2\theta\rho)]\rho'}{\Sigma^6} - \frac{r\sin^2\theta(r^2+a^2)^2\rho''}{\Sigma^4} + \Lambda\frac{\sin^2\theta[(r^2+a^2)^2 - a^2\Delta_r]}{\Sigma^2}.
\end{aligned} \tag{19}$$

By calculation, we find that the above metric satisfies the Einstein-Maxwell equation in quintessence matter including the cosmological constant.

III. KERR-NEWMAN-ADS BLACK HOLE IN QUINTESSENCE

A. Horizon structures

In order to know the properties of the black hole, we calculate the horizon structure of the black hole. From the definition of the horizon

$$g^{rr} = 0, \tag{20}$$

we find that the horizon satisfies the following equation

$$\Delta_r = r^2 - 2Mr + a^2 + Q^2 - \frac{\Lambda}{3} r^2(r^2 + a^2) - \alpha r^{1-3\omega} = 0, \tag{21}$$

which depends on α, a, Q, Λ and ω. It is very interesting and different from the Kerr black hole. α, a, Q, Λ and ω will determine the horizon number.

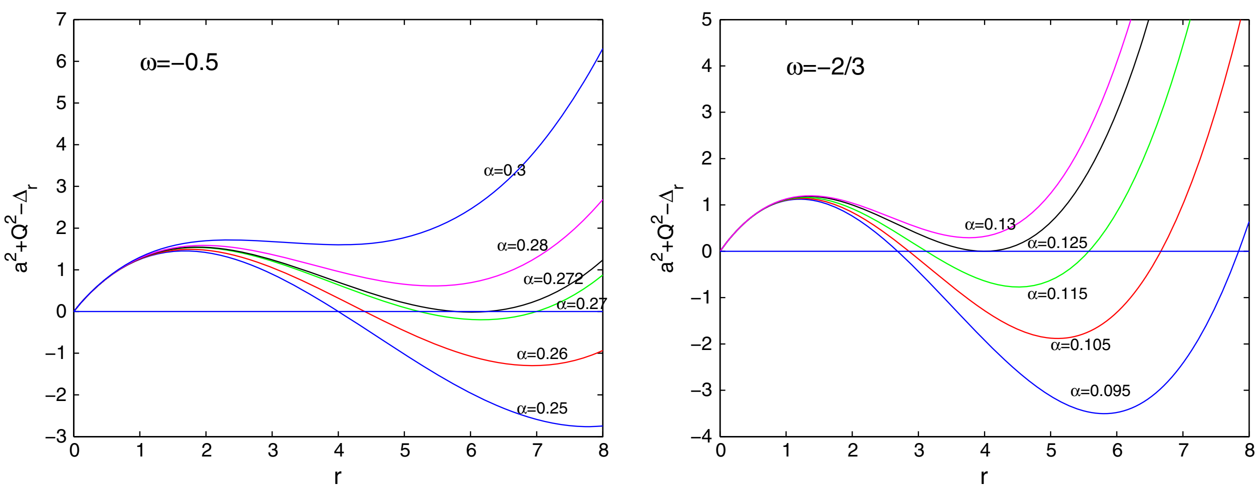

FIG. 1. Two pictures show the behavior of $a^2 + Q^2 - \Delta_r$ with r for fixed $M = 1$, in which for different ω, α will satisfy different value when the cosmological horizon exists. Due to the small value of the cosmological constant, there always exists the cosmological horizon r_c.

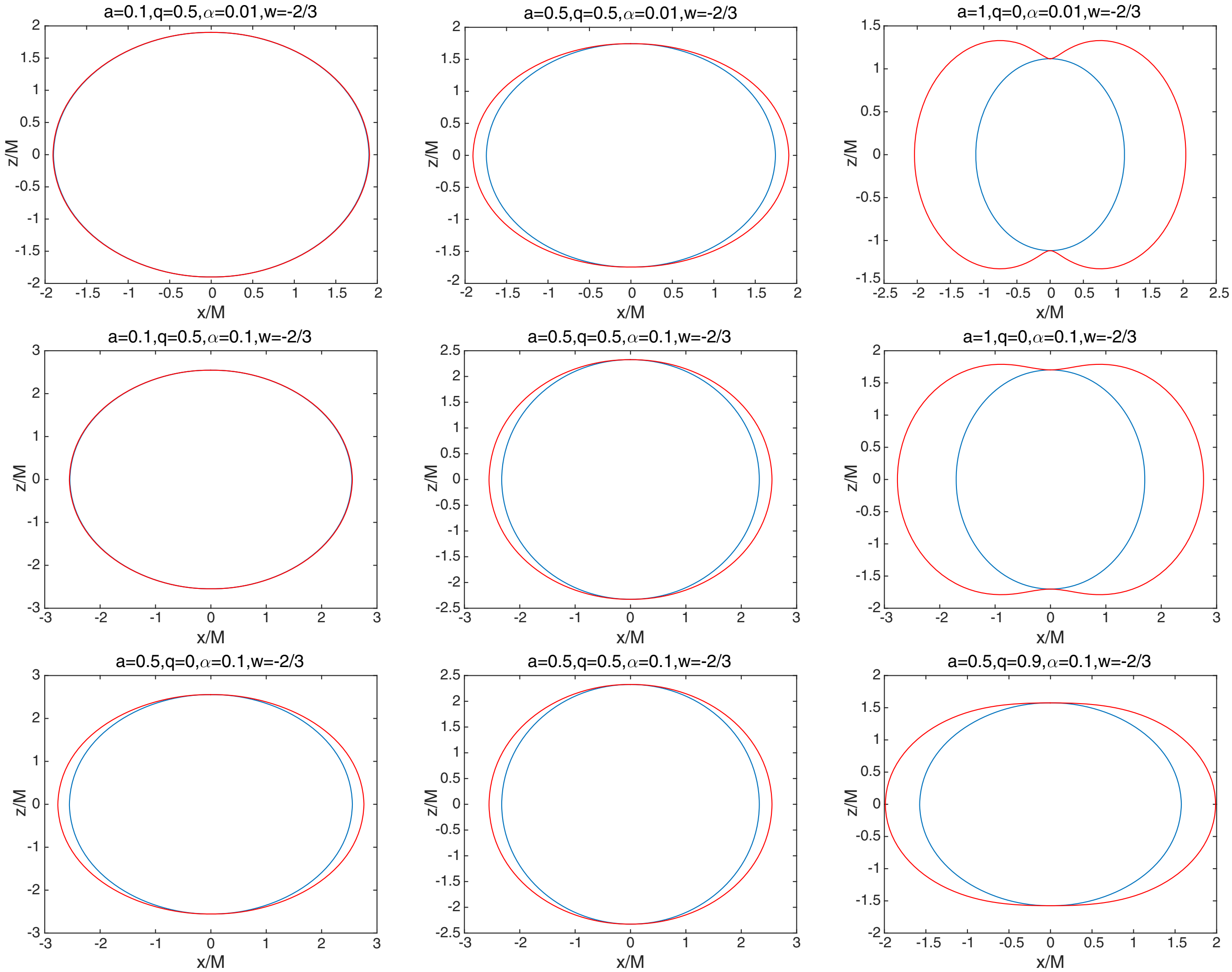

FIG. 2. The shape of ergosphere of the Kerr-Newman-AdS black hole in quintessence for different a, $Q(=q)$, α and $\omega = -2/3$. The blue lines represent event horizons and red lines represent stationary limit surfaces. The region between event horizon and stationary limit surface is the ergosphere. Because the cosmological constant is small, its influence can be ignored. Here $\Lambda = 1.3 \times 10^{-56}$ cm^{-2}.

It is very convenient to analyze the properties of the black hole if we make (21) to become the following form

$$a^2 + Q^2 = -r^2 + 2Mr + \alpha r^{1-3\omega} + \frac{\Lambda}{3} r^2 (r^2 + a^2). \quad (22)$$

For general $\omega(-1 < \omega < -1/3)$ situation, four horizons exist, including Cauchy horizon $r_{\rm in}$ (r_-), event horizon $r_{\rm out}(r_+)$ and two cosmological horizon r_q and r_c, where r_q is the cosmological horizon determined by quintessential dark energy and r_c is the cosmological horizon determined by the cosmological constant. When the cosmological constant is zero, using the method of [21], if the cosmological horizon r_q exists, we find that the parameter α will satisfy

$$\alpha \le \frac{2}{(1-3\omega)} 8^{\omega}. \quad (23)$$

For $\omega = -2/3$, we get $\alpha \le 1/6$ from Eq. (23), which is the same with the rotational black hole in quintessence matter. For $\omega = -1/2$, we obtain $\alpha \le \sqrt{2}/5$. These results imply that black hole charge cannot change the value of parameter α.

In Fig. 1, we show the behavior of $a^2 + Q^2 - \Delta_r$ with r for fixed $M = 1$, in which α satisfy different values for different ω when the cosmological horizon r_q exists. Far away from black hole such as cosmological scale, another cosmological horizon r_c exists.

For $\Lambda \neq 0$, the equation exists four roots. If we consider the case of $\omega = -2/3$, the horizon equation becomes

$$r^4 + \frac{3\alpha}{\Lambda} r^3 + \left(a^2 - \frac{3}{\Lambda}\right) r^2 + \frac{6M}{\Lambda} r - \frac{3}{\Lambda}(a^2 + Q^2) = 0, \quad (24)$$

this fourth order algebra equation can be expressed as

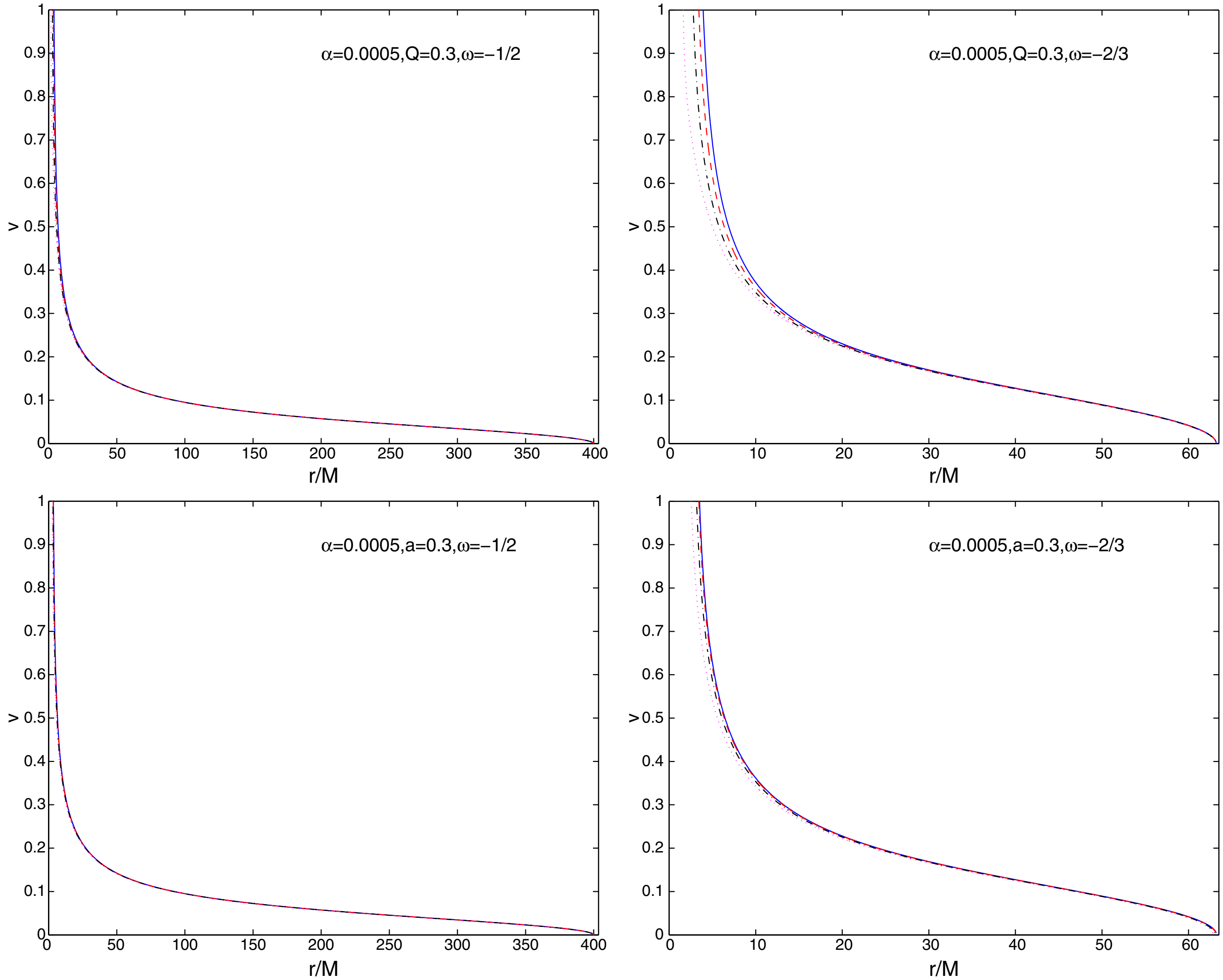

FIG. 3. The behavior of rotation velocity v with r in the equatorial plane of the Kerr-Newman-AdS black hole in quintessential dark energy for two examples $\omega = -1/2$ and $\omega = -2/3$. The top panels show the curves for different parameter a: solid line $a = 0$, dashed line $a = 0.3$, dot-dashed line $a = 0.6$ and dotted line $a = 0.9$. The bottom panels present the curves for different parameter Q: solid line $Q = 0$, dashed line $Q = 0.3$, dot-dashed line $Q = 0.6$ and dotted line $Q = 0.9$. Here $\Lambda = 10^{-56} cm^{-2}$ and $\alpha = 0.0005$.

$$(r - r_{\rm in})(r - r_{\rm out})(r - r_q)(r - r_c) = 0. \tag{25}$$

The existence of cosmological horizon r_q will change the parameter α. Through analyzing the equation, we find that α satisfies

$$\left(\frac{27\alpha^3}{64\Lambda^3} - \frac{3\alpha}{8\Lambda}\left(\frac{a^2}{2} - \frac{3}{2\Lambda}\right) + \frac{3M}{4\Lambda}\right)^2 + \left(\frac{a^2}{6} - \frac{1}{2\Lambda} - \frac{9\alpha^2}{16\Lambda^2}\right)^3 < 0. \tag{26}$$

From the above equation, we find that the cosmological constant make the value of α to become small.

The extremal black holes have seven types. For the first type, the inner horizon r_- and the outer horizon r_+ are equal. For the second type, the outer horizon r_+ and the cosmological horizon r_q are equal. For the third type, the cosmological horizon r_q equals to the cosmological horizon r_c. For the fourth type, $r_{\rm in} = r_{\rm out} = r_q$. For the fifth type, $r_{\rm out} = r_q = r_c$. For the sixth type, $r_{\rm in} = r_{\rm out} = r_q = r_c$. For the seventh type, $r_{\rm in} = r_{\rm out}$ and $r_q = r_c$. For the different types of extremal black hole, the maximum value of α is also different.

Through analyzing these extremal black holes, we find that when $\omega \to -1$, the quintessential dark energy will be like the cosmological constant and α is close to the cosmological constant. For $\omega \to -1/3$, the cosmological horizon determined by quintessence will be close to outer horizon and α satisfies $\alpha < 1$. From these analyses, we find that the black hole spin and cosmological constant will lead the value of α to become small. With the state parameter ω

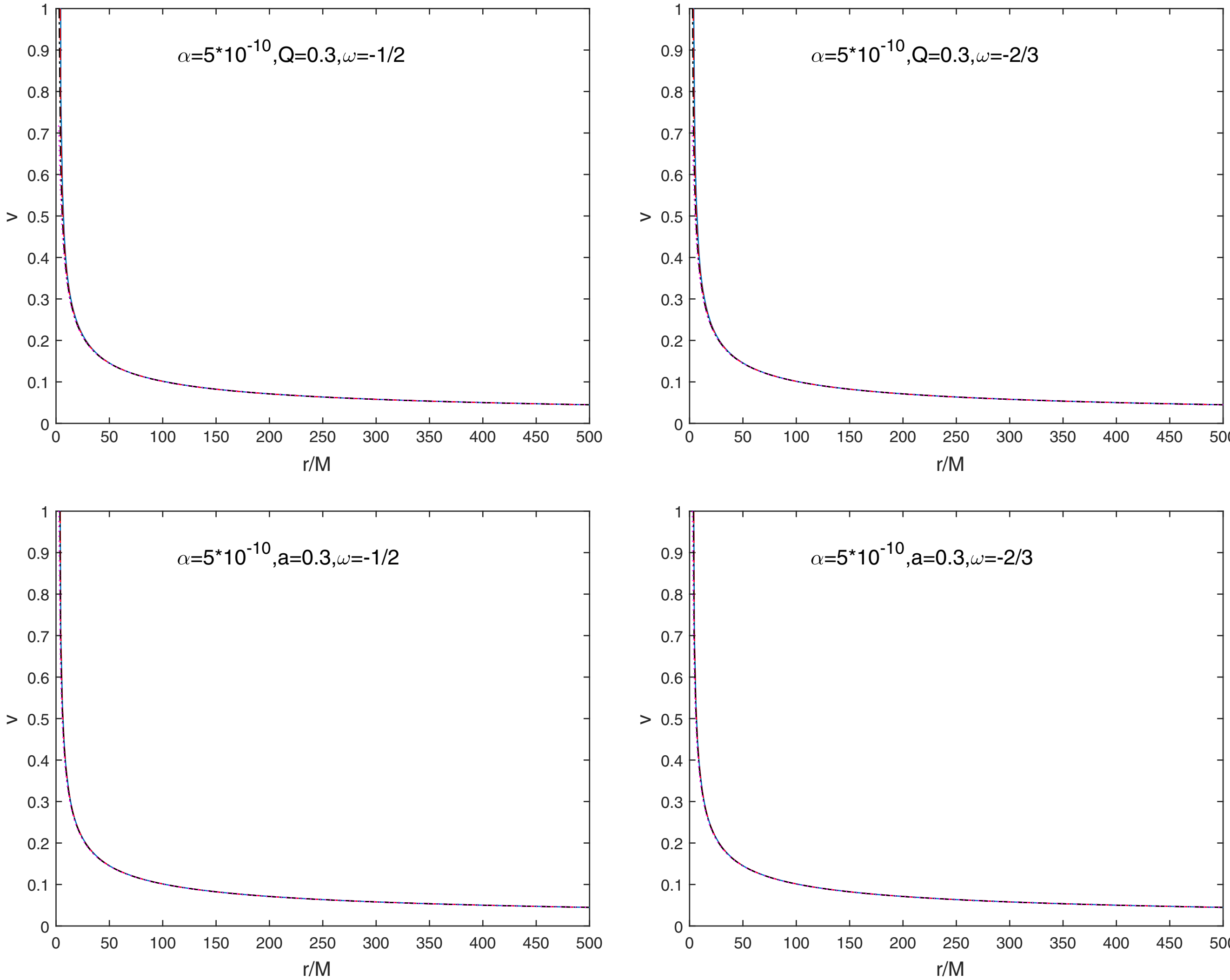

FIG. 4. The behavior of rotation velocity v with r in the equatorial plane of the Kerr-Newman-AdS black hole in quintessential dark energy for two examples $\omega = -1/2$ and $\omega = -2/3$. The top panels show the curves for different parameter a: solid line $a = 0$, dashed line $a = 0.3$, dot-dashed line $a = 0.6$ and dotted line $a = 0.9$. The bottom panels present the curves for different parameter Q: solid line $Q = 0$, dashed line $Q = 0.3$, dot-dashed line $Q = 0.6$ and dotted line $Q = 0.9$. Here $\Lambda = 10^{-56}\ \mathrm{cm}^{-2}$ and $\alpha = 5 * 10^{-10}$.

ranging from -1 to $-1/3$, the maximum value of α changes from Λ to 1.

B. Stationary limit surfaces

The stationary limit surfaces of the Kerr black hole have interesting properties and are defined by $g_{tt} = 0$. From the metric (11), $g_{tt} = 0$ becomes the following equation

$$g_{tt} = \frac{1}{\Sigma^2\Xi^2}(a^2\sin^2\theta\Delta_\theta - \Delta_r) = 0. \tag{27}$$

Following the similar equation (22), we make this equation to become

$$Q^2 + a^2\cos^2\theta = -r^2 + 2Mr + \alpha r^{1-3\omega} + \frac{\Lambda}{3}a^4\sin^2\theta\cos^2\theta + \frac{\Lambda}{3}r^2(r^2 + a^2). \tag{28}$$

There are two surfaces, e.g., out event horizon and static limit surface. They meet the poles and exist a region between horizon and static limit surface, called the ergosphere. The shape of the ergosphere is determined by the parameters a, ω, q, α, Λ and θ, and is shown in Fig. 2.

C. Singularities

It is interesting to study the singularity of the black hole. By calculating the scale curvature R in the metric (11) given by $R = R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$, we can study the singularity of the black hole. The black hole is determined by ω, for general ω we obtain

$$R = R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} = \frac{4H(r,\theta,a,\alpha,Q^2)}{\Sigma^{12}}, \tag{29}$$

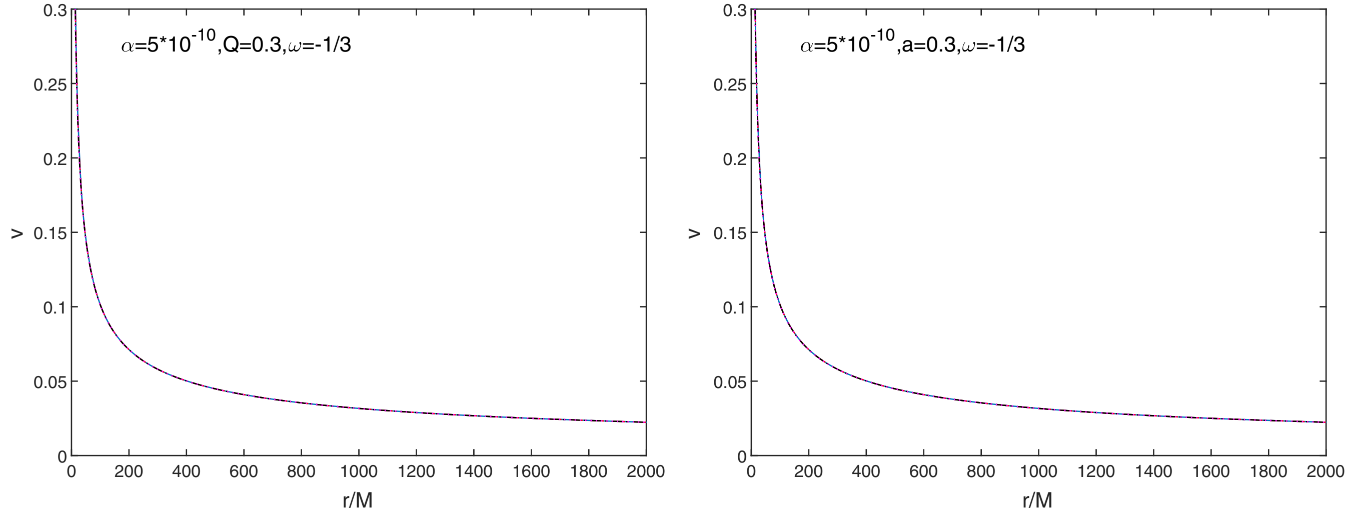

FIG. 5. The behavior of rotation velocity v with r in the equatorial plane of the Kerr-Newman-AdS black hole in quintessential dark energy. The left picture show the curves for different parameter a: solid line $a=0$, dashed line $a=0.3$, dot-dashed line $a=0.6$ and dotted line $a=0.9$. The right picture present the curves for different parameter Q: solid line $Q=0$, dashed line $Q=0.3$, dot-dashed line $Q=0.6$ and dotted line $Q=0.9$. Here $\Lambda=10^{-56}$ cm^{-2} and $\omega=-1/3$.

where H are polynomial function about r, θ and a. The function also includes α, Q, ω and Λ.

We find that only $\Sigma^2=r^2+a^2\cos^2\theta=0$, the real singularity exists and is given by

$$r=0 \quad \text{and} \quad \theta=\frac{\pi}{2}. \tag{30}$$

Here, we calculate the scale curvature R in Boyer-Lindquist coordinates, that $\Sigma^2=r^2+a^2\cos^2\theta=0$ represent a ring at the equatorial plane with the radius a, centered on the symmetry axis of this black Hole. It is the same with one in Kerr black hole [29].

IV. ROTATION VELOCITY IN THE EQUATORIAL PLANE APPLICATION TO DARK MATTER

We derive the relation between the space-time metric components and the rotation velocity. For simplicity, we focus on the rotation motion near the equatorial plane with $\theta=\pi/2$ and $\frac{d\theta}{dt}=0$.

We describe the rotation curves in four-dimensional space-time formalism. The observer is in the ZAMO (zero angular momentum observers), the four-velocity satisfies the normalized condition

$$g_{\mu\nu}u^\mu u^\nu=-1, \tag{31}$$

and here we consider the space-time with rotational symmetry. There are two conserved quantities as

$$P_\mu\xi^\mu=L,E. \tag{32}$$

Using the expressions of u^μ and u^ν, we rewrite the normalized condition equation as

$$g_{tt}\left(\frac{dt}{d\tau}\right)^2+2g_{t\phi}\frac{dt}{d\tau}\frac{d\phi}{d\tau}+g_{\phi\phi}\left(\frac{d\phi}{d\tau}\right)^2+g_{rr}\left(\frac{dr}{d\tau}\right)^2=-1. \tag{33}$$

Using the Eqs. (32) and (33), we get the following equation

$$-E\frac{dt}{d\tau}+L\frac{d\phi}{d\tau}+g_{rr}\left(\frac{dr}{d\tau}\right)^2=-1. \tag{34}$$

Through calculating, we obtain the equation

$$\left(\frac{dr}{d\tau}\right)^2=-\frac{1}{g_{rr}}+\frac{g_{\phi\phi}E^2+2g_{t\phi}EL+g_{tt}L^2}{(g_{t\phi}^2-g_{tt}g_{\phi\phi})g_{rr}}=E^2-V^2 \tag{35}$$

The stable circular orbit satisfies two conditions

$$\frac{dr}{d\tau}=0, \qquad \frac{dV^2}{dr}=0, \tag{36}$$

Solving the Eqs. (36) and (35), we obtain [23,30]

$$E=\pm\frac{g_{tt}+g_{t\phi}\Omega_\phi}{\sqrt{-g_{tt}-2g_{t\phi}\Omega_\phi-g_{\phi\phi}\Omega_\phi^2}},$$
$$L=\pm\frac{g_{t\phi}+g_{\phi\phi}\Omega_\phi}{\sqrt{-g_{tt}-2g_{t\phi}\Omega_\phi-g_{\phi\phi}\Omega_\phi^2}}, \tag{37}$$

where the angular velocity is defined by

$$\Omega_\phi = \frac{-g_{t\phi,r} + \sqrt{(g_{t\phi,r})^2 - g_{tt,r}g_{\phi\phi,r}}}{g_{\phi\phi,r}}. \tag{38}$$

The rotation velocity for any ω is given by the following equation

$$v = \frac{L}{\sqrt{g_{\phi\phi}}} = \frac{1}{\sqrt{g_{\phi\phi}}} \frac{g_{t\phi} + g_{\phi\phi}\Omega_\phi}{\sqrt{-g_{tt} - 2g_{t\phi}\Omega_\phi - g_{\phi\phi}\Omega_\phi^2}}, \tag{39}$$

where the parameter ω dominates the circular orbits. The rotation velocities on the equatorial plane are shown in Fig. 3 and Fig. 4 for two examples $\omega = -2/3$ and $\omega = -1/2$, they are different from those in Kerr black hole. At the same time, ω have large influence on circular orbits. When ω is close to $-1/3$ and α is close to 0, the rotation velocity on the equatorial plane is more asymptotically flat. We take different charge Q to draw the rotation velocities, we find that Q has weak influence on the rotation velocities in the equatorial plane. Because the cosmological constant is small, its influence on rotation velocity can be ignored.

Comparing Fig. 3, Fig. 4, and Fig. 5, we find that when the parameter α is very small, the rotation velocities on the equatorial plane will be asymptotically flat in large distance r. Kiselev suggests that when quintessential dark energy work, the rotation curves in spiral galaxies will be asymptotically flat with distance r [31]. In their paper, they study the rotation velocities in spherically symmetric black hole in quintessential dark energy. Here we generalize their results to the Kerr-Newman-AdS black hole around by quintessential dark energy.

V. SUMMARY

Using the Newman-Janis algorithm, we obtain Kerr-Newman solutions in quintessential dark energy. Because the Newman-Janis algorithm does not include the cosmological constant, we cannot use this method to derive the Kerr-Newman-AdS solution around by quintessential dark energy. Through direct complex computation, we extend the Kerr-Newman solution to Kerr-Newman-AdS in quintessential dark energy. By analyzing the horizon equation, we obtain the value of α for $\omega = -2/3, -1/2$. When $\Lambda = 0$, we find that $\alpha \leq \sqrt{2}/5$ for $\omega = -1/2$ and $\alpha < 1/6$ for $\omega = -2/3$ which is the same with one given by [21] in quintessential dark energy, showing that the black hole charge cannot change the value of α. When $\Lambda \neq 0$ and four horizons especially r_q exist, we obtain the constraint equation on α, implying that the black hole spin and cosmological constant make the maximum value of α to become more small. With the state parameter ω ranging from -1 to $-1/3$, the maximum value of α change Λ to 1. If $\omega \to -1$, r_q arrives at r_c and α is close to the cosmological constant. For all Kerr-Newman-AdS solutions in quintessential dark energy, the naked singularity appears when $\Sigma^2 = 0$. Finally, we calculate the geodetic motion on equatorial plane for three situations of $\omega = -2/3, -1/2$ and $-1/3$. We find that the parameters Q, a, Λ have small influence on rotation velocity, while the parameters α and ω have large influence on rotation velocity. For small value of α, the rotation velocity on the equatorial plane is asymptotically flat and it can explain the rotation curves in spiral galaxies.

The Kerr-Newman-AdS solution around by quintessential dark energy maybe useful in astrophysics. In the future we want to study the effects of rotation and charge in a more thorough manner, and the influence of quintessential dark energy on Blandford-Znajek mechanism and black hole accretion disk.

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