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Generalized multi-Galileons, covariantized new terms, and the no-go theorem for nonsingular cosmologies

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It has been pointed out that nonsingular cosmological solutions in second-order scalar-tensor theories generically suffer from gradient instabilities. We extend this no-go result to second-order gravitational theories with an arbitrary number of interacting scalar fields. Our proof follows directly from the action of generalized multi-Galileons, and thus is different from and complementary to that based on the effective field theory approach. Several new terms for generalized multi-Galileons on a flat background were proposed recently. We find a covariant completion of them and confirm that they do not participate in the no-go argument.

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I. INTRODUCTION

Inflation [1–3] is an attractive scenario because it gives a natural resolution of the horizon and flatness problems in standard Big Bang cosmology and accounts for the origin of density perturbations that are consistent with observations such as CMB. However, there are criticisms that even inflation cannot resolve the initial singularity [4] and the trans-Planckian problem for cosmological perturbations [5]. Alternative scenarios such as bounces and Galilean Genesis have therefore been explored by a number of authors (see, e.g., Ref. [6] for a review).

To avoid the initial singularity, there must be a period in which the Hubble parameter H is an increasing function of time. This indicates a violation of the null energy condition (NEC), possibly causing some kind of instability. It is easy to show that NEC-violating cosmological solutions are indeed unstable if the Universe is filled with a usual scalar field or a perfect fluid. However, this is not the case if the underlying Lagrangian depends on second derivatives of a scalar field [7], and one can construct explicitly a stable cosmological phase in which the NEC is violated in the Galileon-type scalar-field theory [8–10].

Nevertheless, this does not mean that such nonsingular cosmological solutions are stable at all times in the entire history; it has been known that gradient instabilities occur at some moment in many concrete examples (see, e.g., Refs. [11–17]), and in some cases the instabilities show up even in the far future after the NEC-violating stage [18–20]. Recently, it was shown that this is a generic nature of nonsingular cosmological solutions in the Horndeski/generalized Galileon theory [21–23], i.e., in the most general scalar-tensor theory having second-order field equations, provided that graviton geodesics are complete [24–26].

As the no-go result is obtained in the single-field Horndeski theory, one could evade this by considering theories with multiple scalar fields or higher derivative theories beyond Horndeski. The latter way is indeed successful within the Gleyzes-Langlois-Piazza-Vernizzi scalar-tensor theory [27–29], as pointed out in Refs. [26,30] based on the effective field theory (EFT) of cosmological perturbations [31]. Gradient instabilities can also be cured if higher spatial derivative terms arise in the action for curvature perturbations [16,17,32]. This occurs in a more general framework [33,34] than [27] including Hořava gravity [35]. In some cases it is possible, even without such general frameworks, that the strong coupling scale cuts off the instabilities [36].

The purpose of the present paper is to show that, in contrast to the case of the higher derivative extension, the no-go theorem for nonsingular cosmologies still holds in general multiscalar-tensor theories of gravity. In a subclass of the generalized multi-Galileon theory [37], the same conclusion as in the single-field case was obtained in [38]. It was found in [26] that the no-go theorem can also be extended to the EFT of multifield models in which a shift symmetry is assumed for the entropy mode [39]. (See Ref. [40] for the EFT of multifield inflation without the shift symmetry.) In this paper, we provide a new proof which follows directly from the full action of the generalized multi-Galileon theory.

This paper is organized as follows. In the next section, we give a brief review on the generalized multi-Galileon theory and extend the proof of the no-go theorem for nonsingular cosmologies to multifield models. Recently, several new terms were found that are not included in the generalized multi-Galileon theory but still yield second-order field equations [41]. To keep the proof as general as possible, we show in Sec. III that the main result is not changed by the addition of these new terms. In doing so, we find a covariant completion of the flat-space action of

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Ref. [41]. In Sec. IV we give a comment on the (in) completeness of graviton geodesics viewed from the original (non-Einstein) frame. We draw our conclusions in Sec. V.

II. NO-GO THEOREM IN GENERALIZED MULTI-GALILEON THEORY

A. Generalized multi-Galileon theory

The most general single-scalar-tensor theory whose field equations are of second order is given by the Horndeski action [21]. To begin with, let us review briefly how the same theory was rediscovered in a different way starting from the Galileon theory. The Galileon theory is a scalar-field theory on a fixed Minkowski background having the Galilean shift symmetry, $\partial_\mu\phi \to \partial_\mu\phi + b_\mu$, and second-order field equations [42]. To make the metric dynamical and consider an arbitrary spacetime, one can covariantize the Galileon theory by replacing ∂_μ with ∇_μ, but this procedure induces higher derivative terms in the field equations due to the noncommutativity of the covariant derivative. However, the resulting higher derivative terms can be removed by introducing nonminimal derivative coupling to the curvature. The covariant multi-Galileon theory is thus obtained [43]. Now the Galilean shift symmetry is lost and what is more important is the second-order nature of the field equations, as it guarantees the absence of Ostrogradski instabilities. One can further generalize the covariant Galileon theory by promoting $X := -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi/2$ in the action to arbitrary functions ϕ and X while retaining the second-order field equations [22]. This yields the Lagrangian

$$\mathcal{L} = G_2(X,\phi) - G_3(X,\phi)\Box\phi + G_4(X,\phi)R + \frac{\partial G_4}{\partial X}[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2] + G_5(X,\phi)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi - \frac{1}{6}\frac{\partial G_5}{\partial X}[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3], \tag{1}$$

where R is the Ricci scalar and $G_{\mu\nu}$ is the Einstein tensor. Interestingly, it can be shown that this Lagrangian is equivalent to the one obtained by Horndeski in an apparently different form [23], and therefore is the most general one having second-order field equations.

The multifield generalization can proceed in the following way. In Refs. [44–49], the Galileons on a fixed Minkowski background was generalized to multifield models, whose action is a functional of N scalar fields ϕ^I ($I = 1, 2, \ldots, N$) and their derivatives of order up to two. Covariantizing the multi-Galileons and introducing arbitrary functions of the scalar fields and their first derivatives so that no higher derivative terms appear in the field equations, one can arrive at the generalized multi-Galileon theory, the Lagrangian of which is given in an analogous form to Eq. (1) by [37]

$$\begin{aligned}\mathcal{L} = {} & G_2(X^{IJ},\phi^K) - G_{3L}(X^{IJ},\phi^K)\Box\phi^L + G_4(X^{IJ},\phi^K)R \\ & + G_{4,\langle IJ\rangle}(\Box\phi^I\Box\phi^J - \nabla_\mu\nabla_\nu\phi^I\nabla^\mu\nabla^\nu\phi^J) \\ & + G_{5L}(X^{IJ},\phi^K)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi^L - \frac{1}{6}G_{5I,\langle JK\rangle} \\ & \times (\Box\phi^I\Box\phi^J\Box\phi^K - 3\Box\phi^{(I}\nabla_\mu\nabla_\nu\phi^J\nabla^\mu\nabla^\nu\phi^{K)} \\ & + 2\nabla_\mu\nabla_\nu\phi^I\nabla^\nu\nabla^\lambda\phi^J\nabla_\lambda\nabla^\mu\phi^K),\end{aligned} \tag{2}$$

where

$$X^{IJ} := -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J, \tag{3}$$

$$G_{,\langle IJ\rangle} := \frac{1}{2}\left(\frac{\partial G}{\partial X^{IJ}} + \frac{\partial G}{\partial X^{JI}}\right). \tag{4}$$

In order for the field equations to be of second order, it is required that

$$G_{3IJK} := G_{3I,\langle JK\rangle}, \qquad G_{4IJKL} := G_{4,\langle IJ\rangle,\langle KL\rangle}, \tag{5}$$

$$G_{5IJK} := G_{5I,\langle JK\rangle}, \qquad G_{5IJKLM} := G_{5IJK,\langle LM\rangle}, \tag{6}$$

are symmetric in all of their indices $I, J, \ldots$. In what follows we will write $G_{4,\langle IJ\rangle}$ as G_{4IJ}. It is obvious that $G_{4IJ} = G_{4JI}$.

The multiscalar-tensor theory described by the Lagrangian (2) seems very general and includes the earlier works [50,51] and more recent ones [38,52–55] as specific cases. However, in contrast to the case of the single Galileon, it is *not* the most general multiscalar-tensor theory with second-order field equations. Indeed, as demonstrated in [56], the multi-Dirac-Born-Infeld (DBI) Galileon theory [57] is not included in the above one. To date, no complete multifield generalization of the Horndeski action has been known. Taking the same approach as Horndeski did rather than starting from the multi-Galileon theory, the authors of Ref. [58] obtained the most general second-order field equations of biscalar-tensor theories, but deducing the corresponding action and extending the biscalar result to the case of more than two scalars have not been successful so far. We will come back to this issue in the next section in light of the recent result reported in [41].

Although the generalized multi-Galileon theory is thus not the most general one, it is definitely quite general and so we choose to use the Lagrangian (2). This is one of the best things one can do at this stage to draw some general conclusions on the cosmology of multiple interacting scalar fields, and is considered as complementary to the approach based on the effective field theory of multifield inflation [26].

B. Stability of a nonsingular universe in generalized multi-Galileon theory

We now show that the no-go theorem in [25] can be extended to the case of the generalized multi-Galileon theory.

The quadratic actions for perturbations around a flat Friedmann background have been calculated in [56]. For tensor perturbations $h_{ij}(t,\vec{x})$ we have

$$S_h^{(2)} = \frac{1}{8}\int \mathrm{d}t\mathrm{d}^3x a^3 \left[\mathcal{G}_T \dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2}(\vec{\nabla}h_{ij})^2\right], \quad (7)$$

where

$$\mathcal{G}_T \coloneqq 2[G_4 - 2X^{IJ}G_{4IJ} - X^{IJ}(H\dot{\phi}^K G_{5IJK} - G_{5I,J})] \quad (8)$$

and

$$\mathcal{F}_T \coloneqq 2[G_4 - X^{IJ}(\ddot{\phi}^K G_{5IJK} + G_{5I,J})]. \quad (9)$$

Here we defined $G_{,I} \coloneqq \partial G/\partial\phi^I$. Stability requires

$$\mathcal{G}_T > 0, \qquad \mathcal{F}_T > 0, \quad (10)$$

at any moment in the whole cosmological history.

To study scalar perturbations in multifield models, it is convenient to use the spatially flat gauge. The quadratic action for scalar perturbations is of the form [56]

$$S_Q^{(2)} = \frac{1}{2}\int \mathrm{d}t\mathrm{d}^3x a^3 \left[\mathcal{K}_{IJ}\dot{Q}^I\dot{Q}^J - \frac{1}{a^2}\mathcal{D}_{IJ}\vec{\nabla}Q^I\cdot\vec{\nabla}Q^J - \mathcal{M}_{IJ}Q^IQ^J + 2\Omega_{IJ}Q^I\dot{Q}^J\right], \quad (11)$$

where Q^I's are the perturbations of the scalar fields defined by

$$\phi^I = \bar{\phi}^I(t) + Q^I(t,\vec{x}). \quad (12)$$

The explicit expressions for the matrices $\mathcal{K}_{IJ}$, $\mathcal{M}_{IJ}$, and Ω_{IJ} can be found in [56], but are not necessary for the following discussion. Since gradient instabilities manifest most significantly at high frequencies, only the structure of the matrix $\mathcal{D}_{IJ}$ is crucial to our no-go argument. We will use the fact that $\mathcal{D}_{IJ}$ is given by [56]

$$\mathcal{D}_{IJ} = \mathcal{C}_{IJ} - \frac{\mathcal{J}_{(I}\mathcal{B}_{J)}}{\Theta} + \frac{1}{a}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{a\mathcal{B}_I\mathcal{B}_J}{2\Theta}\right), \quad (13)$$

where $\mathcal{C}_{IJ}$ is the matrix satisfying the identity

$$\mathcal{C}_{IJ}X^{IJ} = 2H(\dot{\mathcal{G}}_T + H\mathcal{G}_T) - \dot{\Theta} - H\Theta - H^2\mathcal{F}_T, \quad (14)$$

with

$$\begin{aligned}\Theta \coloneqq &-\dot{\phi}^I X^{JK}G_{3IJK} + 2HG_4 \\ &- 8HX^{IJ}(G_{4IJ} + X^{KL}G_{4IJKL}) \\ &+ 2\dot{\phi}^I X^{JK}G_{4IJ,K} + \dot{\phi}^I G_{4,I} \\ &- H^2\dot{\phi}^I X^{JK}(5G_{5IJK} + 2X^{LM}G_{5IJKLM}) \\ &+ 2HX^{IJ}(3G_{5I,J} + 2X^{KL}G_{5IJK,L}). \end{aligned} \quad (15)$$

The explicit expressions for $\mathcal{J}_I$ and $\mathcal{B}_I$ in Eq. (13) are also unimportant, but we will use the equation [56]

$$\dot{\phi}^I\mathcal{J}_I + \ddot{\phi}^I\mathcal{B}_I + 2\dot{H}\mathcal{G}_T = 0. \quad (16)$$

This follows from the background equations, and corresponds in the minimally coupled single-field case to the familiar equation

$$\dot{\phi}^2 + 2M_{\mathrm{Pl}}^2\dot{H} = 0. \quad (17)$$

It is required for the stability of the scalar sector that the matrices $\boldsymbol{\mathcal{K}} = (\mathcal{K}_{IJ})$ and $\boldsymbol{\mathcal{D}} = (\mathcal{D}_{IJ})$ must be positive definite. Hence, a nonsingular cosmological solution is free from gradient instabilities if, for every nonzero column vector $\boldsymbol{v}$,

$$\boldsymbol{v}^{\mathrm{T}}\boldsymbol{\mathcal{D}}\boldsymbol{v} > 0, \quad (18)$$

where $\boldsymbol{v}^{\mathrm{T}}$ is the transpose of $\boldsymbol{v}$. Now, let $\boldsymbol{v}$ be

$$\boldsymbol{v} = \begin{pmatrix}\dot{\phi}^1 \\ \dot{\phi}^2 \\ \vdots \\ \dot{\phi}^N\end{pmatrix}. \quad (19)$$

Then, Eq. (18) reads

$$\boldsymbol{v}^{\mathrm{T}}\boldsymbol{\mathcal{D}}\boldsymbol{v} = 2X^{IJ}\mathcal{D}_{IJ} > 0. \quad (20)$$

Using Eqs. (13), (14), and (16) and doing some manipulation, one finds

$$X^{IJ}\mathcal{D}_{IJ} = H^2\left(\frac{1}{a}\frac{\mathrm{d}\xi}{\mathrm{d}t} - \mathcal{F}_T\right) > 0, \quad (21)$$

where

$$\xi \coloneqq \frac{a\mathcal{G}_T^2}{\Theta}. \quad (22)$$

The remaining part of the proof is parallel to that in the Horndeski case [25], because the structure of the inequality (21) is identical to the single-field counterpart. In a nonsingular universe, Θ never diverges because it is composed of H and ϕ^I as given in Eq. (15) and we require that the functions G_2, G_{3I}, … in the underlying Lagrangian remain finite in the entire cosmological history.[1] We also have $a\mathcal{G}_T^2 > 0$ which comes from the stability of the tensor perturbations.[2] Therefore, ξ cannot cross zero. From Eq. (21) we have

[1]Our postulate on this point is different from that adopted in Ref. [20], in which *singular* functions are introduced in the underlying Lagrangian to obtain nonsingular cosmological solutions.

[2]Our postulate on this point is different from that adopted in Ref. [59], in which all the coefficients in the quadratic action for cosmological perturbations vanish at the same moment.

$$\frac{\mathrm{d}\xi}{\mathrm{d}t} > a\mathcal{F}_T > 0, \tag{23}$$

indicating that ξ is a monotonically increasing function of t. Integrating Eq. (23) from some t_i to t_f, we obtain

$$\xi(t_\mathrm{f}) - \xi(t_\mathrm{i}) > \int_{t_\mathrm{i}}^{t_\mathrm{f}} a\mathcal{F}_T \mathrm{d}t'. \tag{24}$$

(We admit that ξ diverges at some t_* where $\Theta = 0$ occurs. In this case, t_i and t_f are taken to be such that $t_\mathrm{i} < t_\mathrm{f} < t_*$ or $t_* < t_\mathrm{i} < t_\mathrm{f}$.) If $\lim_{t\to-\infty}\xi = \mathrm{const}$, we take $t_\mathrm{i} \to -\infty$ in Eq. (24) and obtain

$$\int_{-\infty}^{t_\mathrm{f}} a\mathcal{F}_T \mathrm{d}t' < \xi(t_\mathrm{f}) - \xi(-\infty) < \infty. \tag{25}$$

Similarly, if $\lim_{t\to\infty}\xi = \mathrm{const}$ then we take $t_\mathrm{f} \to \infty$ to get

$$\int_{t_\mathrm{i}}^{\infty} a\mathcal{F}_T \mathrm{d}t' < \xi(\infty) - \xi(t_\mathrm{i}) < \infty. \tag{26}$$

Thus, we conclude that a nonsingular cosmological solution in the generalized multi-Galileon theory is stable in the entire history provided that either

$$\int_{-\infty}^{t} a\mathcal{F}_T \mathrm{d}t' \quad \text{or} \quad \int_{t}^{\infty} a\mathcal{F}_T \mathrm{d}t' \tag{27}$$

is convergent. (If $\Theta = 0$ occurs, both of the above integrals must be convergent.) As is argued in Refs. [26,30] and also in Sec. IV of the present paper, the convergence of the above integrals signals some kind of pathology in the tensor perturbations. If one prefers to avoid this pathology, all nonsingular cosmological solutions in the generalized multi-Galileon theory are inevitably plagued with gradient instabilities.

One might expect naively that, in the presence of multiple interacting scalar fields, a dominant field can transfer its energy to another field or matter before the instability of the former shows up, and thus the instability can be eliminated. We have shown that this is not the case in the generalized multi-Galileon theory.

The same conclusion was reached using the EFT of multifield cosmologies, in which a shift symmetry is assumed for the entropy mode [26]. Our proof is different from, and complementary to, that based on the EFT. The EFT approach amounts to writing all the terms allowed by symmetry, which leads to the theory of cosmological perturbations on a given background. Therefore, the adiabatic and entropy modes are decomposed by construction in the EFT. In contrast, our guiding principle is the second-order nature of the field equations, and so we start from the general action of second-order multiple scalar-tensor theories that governs the perturbation evolution as well as the background dynamics. It should be noticed that we have not performed the adiabatic/entropy decomposition, as it is unnecessary for our no-go argument. Although the relation between the second-order theory and the EFT of cosmological perturbations has been clarified in the single-field case [60], to date, it is not obvious how the EFT of multifield cosmology is related to the generalized multi-Galileon theory.

III. COVARIANTIZED NEW TERMS FOR MULTI-GALILEON THEORY

Very recently, the author of Ref. [41] proposed new terms for scalar multi-Galileon theory that are not included in the existing multi-Galileon Lagrangian but give rise to a second-order field equation. The Lagrangians for these "extended" multi-Galileons are given by [41,44]

$$\mathcal{L}_{\mathrm{ext1}} = A_{[IJ][KL]M}\delta^{\mu_1\mu_2\mu_3}_{\nu_1\nu_2\nu_3}\partial_{\mu_1}\phi^I\partial_{\mu_2}\phi^J\partial^{\nu_1}\phi^K\partial^{\nu_2}\phi^L \times \partial_{\mu_3}\partial^{\nu_3}\phi^M, \tag{28}$$

$$\mathcal{L}_{\mathrm{ext2}} = A_{[IJ][KL](MN)}\delta^{\mu_1\mu_2\mu_3\mu_4}_{\nu_1\nu_2\nu_3\nu_4}\partial_{\mu_1}\phi^I\partial_{\mu_2}\phi^J \times \partial^{\nu_1}\phi^K\partial^{\nu_2}\phi^L\partial_{\mu_3}\partial^{\nu_3}\phi^M\partial_{\mu_4}\partial^{\nu_4}\phi^N, \tag{29}$$

$$\mathcal{L}_{\mathrm{ext3}} = A_{[IJK][LMN]O}\delta^{\mu_1\mu_2\mu_3\mu_4}_{\nu_1\nu_2\nu_3\nu_4}\partial_{\mu_1}\phi^I\partial_{\mu_2}\phi^J\partial_{\mu_3}\phi^K \times \partial^{\nu_1}\phi^L\partial^{\nu_2}\phi^M\partial^{\nu_3}\phi^N\partial_{\mu_4}\partial^{\nu_4}\phi^O, \tag{30}$$

where the coefficients $A_{[IJ][KL]M}, \ldots$ are arbitrary functions of ϕ^I and X^{IJ}. These coefficients are antisymmetric in indices inside [ ] and symmetric in indices inside ( ). In order for the field equations to be of second order, we require that

$$A_{[IJ][KL]\underline{M,\langle NO\rangle}}, A_{[IJ][KL]\underline{(MN),\langle OP\rangle}}, A_{[IJK][LMN]\underline{O,\langle PQ\rangle}}, \tag{31}$$

are symmetric in underlined indices.

The Lagrangians (28)–(30) are those for scalar fields on fixed Minkowski spacetime. Let us explore a covariant completion of the above flat-space multiscalar theory. To make the metric dynamical, we first promote ∂_μ to ∇_μ. It is easy to see that this procedure is sufficient for $\mathcal{L}_{\mathrm{ext1}}$ and $\mathcal{L}_{\mathrm{ext3}}$:

$$\mathcal{L}'_{\mathrm{ext1}} = A_{[IJ][KL]M}\delta^{\mu_1\mu_2\mu_3}_{\nu_1\nu_2\nu_3}\nabla_{\mu_1}\phi^I\nabla_{\mu_2}\phi^J\nabla^{\nu_1}\phi^K\nabla^{\nu_2}\phi^L \times \nabla_{\mu_3}\nabla^{\nu_3}\phi^M, \tag{32}$$

$$\mathcal{L}'_{\mathrm{ext3}} = A_{[IJK][LMN]O}\delta^{\mu_1\mu_2\mu_3\mu_4}_{\nu_1\nu_2\nu_3\nu_4}\nabla_{\mu_1}\phi^I\nabla_{\mu_2}\phi^J\nabla_{\mu_3}\phi^K \times \nabla^{\nu_1}\phi^L\nabla^{\nu_2}\phi^M\nabla^{\nu_3}\phi^N\nabla_{\mu_4}\nabla^{\nu_4}\phi^O, \tag{33}$$

have second-order equations of motion for the metric and scalar fields. However, the simple covariantization of $\mathcal{L}_{\mathrm{ext2}}$,

$$\mathcal{L}_{\mathrm{cext2}} = A_{[IJ][KL](MN)}\delta^{\mu_1\mu_2\mu_3\mu_4}_{\nu_1\nu_2\nu_3\nu_4}\nabla_{\mu_1}\phi^I\nabla_{\mu_2}\phi^J \times \nabla^{\nu_1}\phi^K\nabla^{\nu_2}\phi^L\nabla_{\mu_3}\nabla^{\nu_3}\phi^M\nabla_{\mu_4}\nabla^{\nu_4}\phi^N, \tag{34}$$

yields higher derivative terms in the field equations. To cancel such terms, we add a counterterm, i.e., a coupling to the curvature tensor $\mathcal{L}_{\mathrm{curv2}}$. It turns out that the appropriate Lagrangian is the following:

$$\mathcal{L}_{\mathrm{curv2}} = B_{[IJ][KL]}\delta^{\mu_1\mu_2\mu_3\mu_4}_{\nu_1\nu_2\nu_3\nu_4} \times R^{\nu_3\nu_4}{}_{\mu_3\mu_4}\nabla_{\mu_1}\phi^I\nabla_{\mu_2}\phi^J\nabla^{\nu_1}\phi^K\nabla^{\nu_2}\phi^L, \qquad (35)$$

where

$$B_{[IJ][KL],\langle MN\rangle} = \frac{1}{2}A_{[IJ][KL](MN)} \qquad (36)$$

must be imposed. Thus, we find that the covariant completion of $\mathcal{L}_{\mathrm{ext2}}$ is given by

$$\mathcal{L}'_{\mathrm{ext2}} = \mathcal{L}_{\mathrm{curv2}} + \mathcal{L}_{\mathrm{cext2}} \qquad (37)$$

where $A_{[IJ][KL](MN)} = 2B_{[IJ][KL],\langle MN\rangle}$ and

$$B_{[IJ][KL]\underline{MNOP}} \coloneqq B_{[IJ][KL],\langle MN\rangle,\langle OP\rangle} \qquad (38)$$

is symmetric in underlined indices.

One can check that the multi-DBI Galileon theory at leading order in the X^{IJ} expansion [56] is obtained by taking

$$B_{[IJ][KL]} = \mathrm{const}\times(\delta_{IK}\delta_{JL} - \delta_{IL}\delta_{JK}), \qquad (39)$$

though it seems extremely difficult to see explicitly that the complete Lagrangian for the multi-DBI Galileons [57] can be reproduced by choosing appropriately the functions in the above Lagrangians.

Now the question is how the additional terms

$$\mathcal{L}_{\mathrm{ext}} \coloneqq \mathcal{L}'_{\mathrm{ext1}} + \mathcal{L}'_{\mathrm{ext2}} + \mathcal{L}'_{\mathrm{ext3}} \qquad (40)$$

change the stability of cosmological solutions. Obviously, $\mathcal{L}_{\mathrm{ext}}$ does not change the background equations due to antisymmetry. We see that, in the quadratic actions for scalar and tensor perturbations, only the $\mathcal{C}_{IJ}$ coefficients are modified as follows:

$$\mathcal{C}_{IJ} \to \mathcal{C}_{IJ} + \mathcal{C}^{\mathrm{ext}}_{IJ}, \qquad (41)$$

with

$$\mathcal{C}^{\mathrm{ext}}_{IJ} \coloneqq 32H(-A_{[IK][JL]M}X^{KL}\dot{\phi}^M + 2HB_{[IK][JL]}X^{KL} + 4HB_{[IK][JL],\langle MN\rangle}X^{KL}X^{MN}), \qquad (42)$$

and no other terms are affected by the addition of $\mathcal{L}_{\mathrm{ext}}$. Since $X^{IJ}\mathcal{C}^{\mathrm{ext}}_{IJ} = 0$ due to antisymmetry, $X^{IJ}\mathcal{D}_{IJ}$ remains the same even if one adds $\mathcal{L}_{\mathrm{ext}}$:

$$X^{IJ}\mathcal{D}_{IJ} \to X^{IJ}\mathcal{D}_{IJ}. \qquad (43)$$

Therefore, the new terms proposed in Ref. [41] do not change the no-go argument.

The new term $\mathcal{L}_{\mathrm{ext}}$ vanishes for the homogeneous background, which implies that $\mathcal{L}_{\mathrm{ext}}$ contributes only to the entropy modes at the level of perturbations. This is consistent with the result of [26], where it can be seen using the EFT that the instability occurs in the adiabatic direction.

IV. GRAVITON GEODESICS

We have thus seen that within the multifield extension of the generalized Galileons, nonsingular cosmological solutions are possible only if either integral in Eq. (27) is convergent, as in the single-field Horndeski case. In Ref. [25], this fact was noticed and a numerical example of a nonsingular cosmological solution with the convergent integral was obtained for the first time in the single-field context. Later, the authors of Ref. [61] followed Ref. [25] and presented another example.

One can move to the "Einstein frame" for tensor perturbations from the original frame (7) by performing a disformal transformation [62]. This is because one has two independent functions of t in performing a disformal transformation which can be fitted to make $\mathcal{F}_T$ and $\mathcal{G}_T$ into their standard forms: $\mathcal{F}_T \to M_{\mathrm{Pl}}^2$, $\mathcal{G}_T \to M_{\mathrm{Pl}}^2$. It is clearly explained in Ref. [26] that because gravitons propagate along null geodesics in the Einstein frame and the integral

$$\int a\mathcal{F}_T\mathrm{d}t \qquad (44)$$

is nothing but the affine parameter of the null geodesics in the Einstein frame, the convergent integral (27) implies past (future) incompleteness of graviton geodesics (see also Ref. [30]). This may signal some kind of pathology in the tensor perturbations, though it is not obvious whether the incompleteness of null geodesics in a disformally related frame causes actual problems.

Let us rephrase this potential pathology of gravitons without invoking the disformal transformation. The equation of motion for the tensor perturbation h_{ij} derived from the action (7) can be written in the form

$$Z^{\mu\nu}\mathcal{D}_\mu\mathcal{D}_\nu h_{ij} = 0, \qquad (45)$$

where

$$Z_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu = -\frac{\mathcal{F}_T^{3/2}}{\mathcal{G}_T^{1/2}}\mathrm{d}t^2 + a^2(\mathcal{F}_T\mathcal{G}_T)^{1/2}\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j, \qquad (46)$$

and $\mathcal{D}_\mu$ is the covariant derivative associated with the "metric" $Z_{\mu\nu}$. Equation (45) shows that graviton paths can be interpreted as null geodesics in the effective geometry defined by $Z_{\mu\nu}$. It turns out that the affine parameter λ of null geodesics in the metric $Z_{\mu\nu}$ is given by $\mathrm{d}\lambda = a\mathcal{F}_T\mathrm{d}t$. Therefore, the incompleteness of graviton geodesics can be made manifest even without working in the Einstein frame.

V. SUMMARY

In this paper, we have shown that all nonsingular cosmological solutions are plagued with gradient instabilities in the multifield generalization of scalar-tensor theories, if the graviton geodesic completeness is required. This extends the recent no-go arguments of Refs. [24,25,38]. We have given a direct proof using the generalized multi-Galileon action, so that our proof is different from and complementary to that obtained from the effective field theory of cosmological fluctuations [26]. Several new terms for multi-Galileons on a flat background were found recently [41]. We have covariantized these terms and shown that the inclusion of them does not change the no-go result.

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