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Faddeev fixed center approximation to $\pi\bar{K}K^*$ system and the $\pi_1(1600)$

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We investigate the three-body system of $\pi\bar{K}K^*$ by using the fixed-center approximation to the Faddeev equation, taking the interaction between π and $\bar{K}$, π and K^*, and $\bar{K}$ and K^* from the chiral unitary approach. The study is made assuming scattering of a π on a $\bar{K}K^*$ cluster, which is known to generate the $f_1(1285)$ state. The resonant structure around 1650 MeV shows up in the modulus squared of the π-$(\bar{K}K^*)_{f_1(1285)}$ scattering amplitude and suggests that a π-$(\bar{K}K^*)_{f_1(1285)}$ state, with "exotic" quantum numbers $J^{PC}=1^{-+}$, can be formed. This state can be identified as the observed $\pi_1(1600)$ resonance. We suggest that this is the origin of the present $\pi_1(1600)$ resonance and propose to look at the $\pi f_1(1285)$ mode in future experiments to clarify the issue.

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I. INTRODUCTION

The mesons are described as bound states of quarks and antiquarks in the classical quark model. Until now, most of the known mesons can be described very well within the quark model [1]. However, there is a growing set of experimental observations of resonance-like structures with quantum numbers which are forbidden for the quark-antiquark ($q\bar{q}$) system or situated at masses which cannot be explained by the classical quark model [2,3]. From the experimental side, new observations in the heavy quark sector have reported several mesons with nonconventional features [4–10].

A state with quantum numbers $J^{PC}=1^{-+}$ cannot be described as simple quark antiquark pairs [11]. For $J^{PC}=1^{-+}$ the angular momentum l between the quark and the antiquark must be even, since $P=-(-1)^l$. The positive C-parity then requires the total quark spin s to be zero, since $C=(-1)^{l+s}$. This then implies $J=l$ and therefore excludes $J=1$. But, the quantum numbers of these exotic states could be obtained within the hybrid configurations by adding a gluonic excitation to the $q\bar{q}$ pair and such exotic hybrid configurations should be observed as additional states in the meson spectrum. In the light quark sector there are three quite well-established exotic candidates with $J^{PC}=1^{-+}$: $\pi_1(1400)$, $\pi_1(1600)$, and $\pi_1(2015)$. Over the past two decades, both experimental and theoretical sides have put forth many efforts to investigate these exotic mesons [12]. The $\pi_1(1600)$ state was observed by the E852 Collaboration in the $\rho\pi$ channel with the reaction $\pi^- p\to\pi^-\pi^+\pi^- p$ [13,14], in the $\eta'\pi$ channel with the reaction $\pi^- p\to\eta'\pi^- p$ [15], in the $f_1(1285)\pi$ channel with the reaction $\pi^- p\to\eta\pi^+\pi^-\pi^- p$ [16], and in the $b_1\pi$ channel with the reaction $\pi^- p\to\pi^+\pi^-\pi^-\pi^0\pi^0 p$ [17]. Later, COMPASS Collaboration at CERN showed further evidence for $\pi_1(1600)$ in the $\rho\pi$ channel [18] with mass $M_{\pi_1(1600)}=1660\pm10^{+0}_{-64}$ MeV and a width of $\Gamma_{\pi_1(1600)}=269\pm21^{+42}_{-64}$ MeV. However, the CLAS Collaboration at JLab did not find the evidence of $\pi_1(1600)$ state through the photo-production process $\gamma p\to\pi^+\pi^+\pi^-(n)_{\rm missing}$ [19,20].

Within different theoretical approaches, there are many investigations of the light 1^{-+} hybrid meson properties in Refs. [21–28]. However, the calculations of the mass of the lightest 1^{-+} meson in those works are different. For example, in Ref. [27], it is found that the $\pi_1(1600)$ could be the lightest exotic quantum number hybrid meson, while the results in Ref. [28] favor $\pi_1(1400)$ as the lightest hybrid state. Furthermore, the decay properties of the 1^{-+} hybrid state are studied within the framework of the QCD sum rules in Ref. [29] and the chiral corrections to the $\pi_1(1600)$ state are calculated up to one-loop order in Ref. [30]. There are also other interpretations that $\pi_1(1600)$ might be a four-quark state [31] or a molecule/four-quark mixing state [32].

On the basis of the experimental and theoretical studies of the 1^{-+} hybrid mesons, the identification of the $\pi_1(1600)$ state is a debated issue, thus it is still worth studying the $\pi_1(1600)$ state in different ways.

In this article, we investigate the $\pi_1(1600)$ state in three-body system of $\pi\bar{K}K^*$ but keep the strong correlations of the $\bar{K}K^*$ system[1] which generate $f_1(1285)$ resonance in the

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[1]Note that the $|\bar{K}K^*\rangle$ state has no well-defined C- and G-parity, but it is known that the combination $\frac{1}{\sqrt{2}}(|\bar{K}K^*\rangle+|K\bar{K}^*\rangle)$ is C- and G-parity eigenstate with $C=+1$ and $G=+1$ (see more details in Ref. [33]), and $f_1(1285)$ is a bound state of $\frac{1}{\sqrt{2}}(|\bar{K}K^*\rangle+|K\bar{K}^*\rangle)$. However, as we shall see later, the output of our calculation with $|\bar{K}K^*\rangle$ is the same as $\frac{1}{\sqrt{2}}(|\bar{K}K^*\rangle+|K\bar{K}^*\rangle)$ for $f_1(1285)$. Thus, in this work, we take only $|\bar{K}K^*\rangle$ for $f_1(1285)$.

isospin $I = 0$ sector [33,34]. In such a situation the use of the fixed center approximation (FCA) to the Faddeev equation is justified [35–37]. The FCA to the Faddeev equations has been used with success recently in Ref. [38] for the case of $N\bar{K}K$ system, with results very similar to those obtained in full Faddeev calculations in Refs. [39,40] and in the variational estimate in Ref. [41]. With FCA to the Faddeev equations, the $\Delta_{5/2^+}(2000)$ puzzle is solved in the study of the $\pi - (\Delta\rho)_{N_{5/2^-}(1675)}$ system [42]. In Refs. [43–46], by taking the FCA to Faddeev equations the three-body systems of $\rho K\bar{K}$, $\eta K\bar{K}$, $\eta' K\bar{K}$, $\rho D\bar{D}$, and $\rho D^*\bar{D}^*$ were investigated. Besides, the $\pi(1300)$ resonance was obtained in the study of three-pseudoscalar $\pi K\bar{K}$ and $\pi\pi\eta$ coupled system by solving the Faddeev equations within an approach based on unitary chiral dynamics [47]. For 2^{-+} pseudotensor mesons, it was shown that, in Ref. [48], the $\pi_2(1670)$, $\eta_2(1645)$, and $K_2^*(1770)$ can be regarded as molecules made of a pseudoscalar and a tensor meson, where the latter is itself made of two vector mesons.

In the present work we will use the FCA to Faddeev equations to investigate the $\pi\bar{K}K^*$ system. When studied in s-wave, provided the strength of the interactions allows for it, the π-$(\bar{K}K^*)_{f_1(1285)}$ system could give rise to the exotic π_1 states with quantum numbers $I^G(J^{PC}) = 1^-(1^{-+})$. In terms of two-body $\pi\bar{K}$ and πK^* scattering amplitudes obtained from the chiral unitary approach [33,49,50], we perform an analysis of the π-$(\bar{K}K^*)_{f_1(1285)}$ scattering amplitude, which will allow us to identify dynamically generated resonances with the exotic states discussed above.

In the next section, we present the FCA formalism and ingredients to analyze the π-$(\bar{K}K^*)_{f_1(1285)}$ system. In Sec. III, our results and discussions are presented. Finally, a short summary is given in Sec. IV.

II. FORMALISM AND INGREDIENTS

The FCA approximation to Faddeev equations assumes a pair of particles (1 and 2) forming a cluster. Then particle 3 interacts with the components of the cluster, undergoing all possible multiple scattering with those components. This is depicted in Fig. 1. In terms of the two partition functions T_1 and T_2, which sum all diagrams of the series of Fig. 1 that begin with the interaction of particle 3 with the particle 1 of the cluster (T_1), or with the particle 2 (T_2), the FCA equations are

$$T_1 = t_1 + t_1 G_0 T_2, \tag{1}$$

$$T_2 = t_2 + t_2 G_0 T_1, \tag{2}$$

$$T = T_1 + T_2, \tag{3}$$

where T is the total scattering amplitude. The amplitudes t_1 and t_2 represent the unitary scattering amplitudes with coupled channels for the interactions of particle 3 with particle 1 and 2, respectively. In the present work, we consider $\bar{K}K^*$ as a bound state of the $f_1(1285)$, thus $\bar{K}$ and K^* are particles 1 and 2, respectively. The π meson

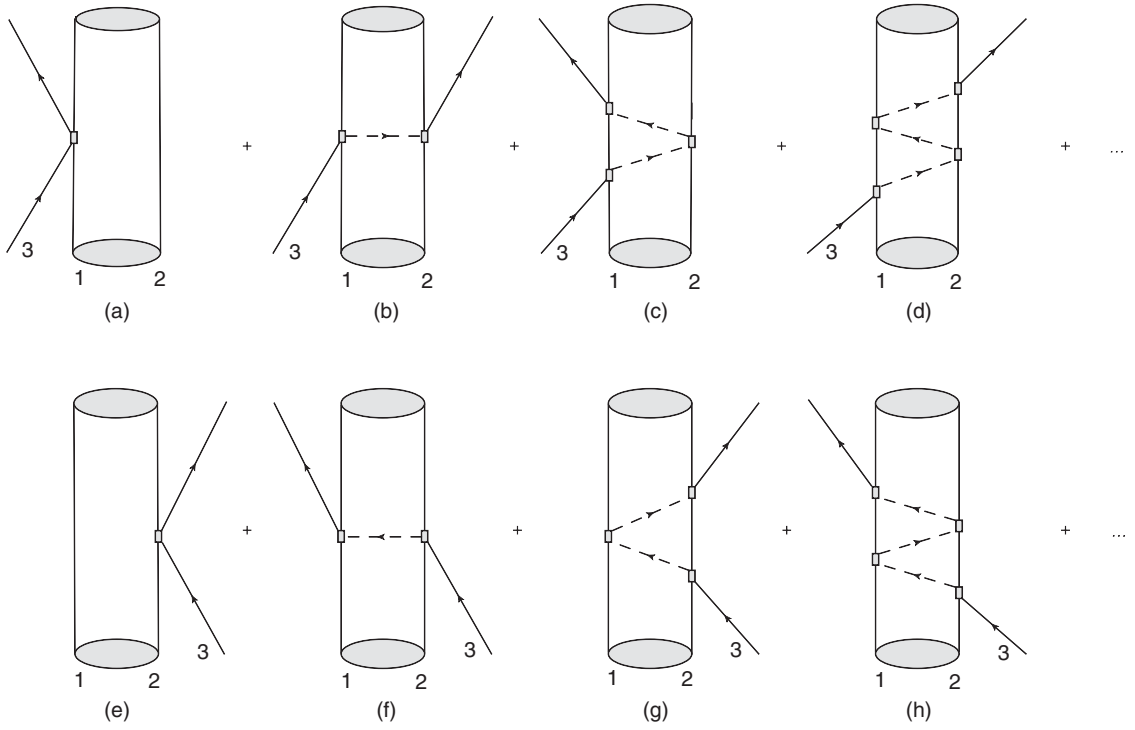

FIG. 1. Diagrammatic representation of the FCA to Faddeev equations.

is particle 3. Then, t_1 is the combination of the $I=1/2$ and $3/2$ unitarized two-body πK scattering amplitude, while t_2 is the $I=1/2$ and $3/2$ unitarized two-body πK^* scattering amplitude. In the above equations, G_0 is the loop function for the π meson propagating inside the $(\bar{K}K^*)_{f_1(1285)}$ cluster which is discussed below. The analysis of the π-$(\bar{K}K^*)_{f_1(1285)}$ scattering amplitude will allow us to study dynamically generated resonances.

For the evaluation of the two body amplitudes t_1 and t_2 in terms of the unitary amplitudes in the isospin basis, we need first to consider the interaction of a π and a $\bar{K}K^*$ cluster. The $\bar{K}K^*$ in isospin zero is written as,

$$|\bar{K}K^*\rangle_{I=0} = \frac{1}{\sqrt{2}}\left|\left(\frac{1}{2},-\frac{1}{2}\right)\right\rangle - \frac{1}{\sqrt{2}}\left|\left(-\frac{1}{2},\frac{1}{2}\right)\right\rangle, \quad (4)$$

where the kets in the right-hand side indicate the I_z components of the particles $\bar{K}$ and K^*, $|(I_z^{\bar{K}}, I_z^{K^*})\rangle$.

Following the procedures of Refs. [38,42], t_1 and t_2 can be easily obtained in terms of two-body amplitudes t_{31} and t_{32}. Here we write explicitly the case of $I_{\pi\bar{K}K^*} = I^z_{\pi\bar{K}K^*} = 1$,

$$\begin{aligned}\langle \pi\bar{K}K^*|t|\pi\bar{K}K^*\rangle &= \left(\langle 11| \otimes \frac{1}{\sqrt{2}}\left(\left\langle\left(\frac{1}{2},-\frac{1}{2}\right)\right| - \left\langle\left(-\frac{1}{2},\frac{1}{2}\right)\right|\right)\right)(t_{31}+t_{32})\left(|11\rangle \otimes \frac{1}{\sqrt{2}}\left(\left|\left(\frac{1}{2},-\frac{1}{2}\right)\right\rangle - \left|\left(-\frac{1}{2},\frac{1}{2}\right)\right\rangle\right)\right)\\ &= \left(\frac{1}{\sqrt{2}}\left\langle\left(\frac{3}{2}\frac{3}{2},-\frac{1}{2}\right)\right| - \frac{1}{\sqrt{6}}\left\langle\left(\frac{3}{2}\frac{1}{2},\frac{1}{2}\right)\right| - \frac{1}{\sqrt{3}}\left\langle\left(\frac{1}{2}\frac{1}{2},\frac{1}{2}\right)\right|\right)t_{31}\left(\frac{1}{\sqrt{2}}\left|\left(\frac{3}{2}\frac{3}{2},-\frac{1}{2}\right)\right\rangle - \frac{1}{\sqrt{6}}\left|\left(\frac{3}{2}\frac{1}{2},\frac{1}{2}\right)\right\rangle\right.\\ &\quad \left. - \frac{1}{\sqrt{3}}\left|\left(\frac{1}{2}\frac{1}{2},\frac{1}{2}\right)\right\rangle\right) + \left(\frac{1}{\sqrt{6}}\left\langle\left(\frac{3}{2}\frac{1}{2},\frac{1}{2}\right)\right| + \frac{1}{\sqrt{3}}\left\langle\left(\frac{1}{2}\frac{1}{2},\frac{1}{2}\right)\right| - \frac{1}{\sqrt{2}}\left\langle\left(\frac{3}{2}\frac{3}{2},-\frac{1}{2}\right)\right|\right)t_{32}\left(\frac{1}{\sqrt{6}}\left|\left(\frac{3}{2}\frac{1}{2},\frac{1}{2}\right)\right\rangle\right.\\ &\quad \left. + \frac{1}{\sqrt{3}}\left|\left(\frac{1}{2}\frac{1}{2},\frac{1}{2}\right)\right\rangle - \frac{1}{\sqrt{2}}\left|\left(\frac{3}{2}\frac{3}{2},-\frac{1}{2}\right)\right\rangle\right), \end{aligned} \quad (5)$$

where the notation followed in the last term for the states is $|(I_{\bar{K}\pi}I^z_{\bar{K}\pi}, I^z_{K^*})\rangle$ for t_{31}, while $|(I_{K^*\pi}I^z_{K^*\pi}, I^z_{\bar{K}})\rangle$ for t_{32}. This leads to the following amplitudes[2] for the single-scattering contribution [Figs. 1(a) and 1(e)],

$$t_1 = \frac{2}{3}t^{I=3/2}_{\pi K} + \frac{1}{3}t^{I=1/2}_{\pi K}, \quad (6)$$

$$t_2 = \frac{2}{3}t^{I=3/2}_{\pi K^*} + \frac{1}{3}t^{I=1/2}_{\pi K^*}. \quad (7)$$

On the other hand, it is worth noting that the argument of the total scattering amplitude T is the total invariant mass s of the three-body system, while the arguments of t_1 and t_2 are s_1 and s_2, where s_i $(i=1,2)$ is the invariant mass of the interaction particle π and the particle $\bar{K}$ $(i=1)$ or K^* $(i=2)$. The value of s_i is given by

$$s_1 = m_\pi^2 + m_{\bar{K}}^2 + \frac{M_R^2 + m_{\bar{K}}^2 - m_{K^*}^2}{2M_R^2}(s - m_\pi^2 - M_R^2), \quad (8)$$

$$s_2 = m_\pi^2 + m_{K^*}^2 + \frac{M_R^2 + m_{K^*}^2 - m_{\bar{K}}^2}{2M_R^2}(s - m_\pi^2 - M_R^2), \quad (9)$$

where M_R is the mass of the $f_1(1285)$ state, and we take $M_R = 1281.3$ MeV.

[2]Because of charge conjugation symmetry, the amplitude for $\pi\bar{K}$ scattering is the same as that for πK scattering.

Then, following the approach developed in Refs. [51,52], we can easily obtain the S-matrix for the single-scattering term [Fig. 1(a) and 1(e)] as

$$\begin{aligned} S^{(1)} &= S_1^{(1)} + S_2^{(1)}\\ &= \frac{(2\pi)^4}{V^2}\delta^4(k + k_R - k' - k'_R)\frac{1}{\sqrt{2\omega_\pi}}\frac{1}{\sqrt{2\omega'_\pi}}\\ &\quad \times \left(-it_1 F_R\left[\frac{m_{K^*}(\vec{k}-\vec{k}')}{m_{\bar{K}}+m_{K^*}}\right]\frac{1}{\sqrt{2\omega_{\bar{K}}}}\frac{1}{\sqrt{2\omega'_{\bar{K}}}}\right.\\ &\quad \left. -it_2 F_R\left[\frac{m_{\bar{K}}(\vec{k}-\vec{k}')}{m_{\bar{K}}+m_{K^*}}\right]\frac{1}{\sqrt{2\omega_{K^*}}}\frac{1}{\sqrt{2\omega'_{K^*}}}\right), \end{aligned} \quad (10)$$

where V stands for the volume of a box in which the states are normalized to unity, while k, k' (k_R, k'_R) refer to the momentum of the initial, final scattering particle (R for the cluster), ω_π $(\omega_{\bar{K}}, \omega_{K^*})$ and ω'_π $(\omega'_{\bar{K}}, \omega'_{K^*})$ are the energies of the initial and final scattering particles.

In Eq. (10), F_R is the form factor of $f_1(1285)$ as a bound state of $\bar{K}K^*$. This form factor was taken to be unity neglecting the $\vec{k}$, $\vec{k}'$ momentum in Refs. [51,52] where only states below threshold were considered. To consider states above threshold, we project the form factor into the s-wave, the only one that we consider. Hence

$$F_R\left[\frac{m_{K^*}(\vec{k}-\vec{k}')}{m_{\bar{K}}+m_{K^*}}\right] \Rightarrow FFS_1(s) = \frac{1}{2}\int_{-1}^{1} F_R(k_1)d(\cos\theta), \quad (11)$$

$$F_R\left[\frac{m_{\bar{K}}(\vec{k}-\vec{k}')}{m_{\bar{K}}+m_{K^*}}\right] \Rightarrow FFS_2(s) = \frac{1}{2}\int_{-1}^{1} F_R(k_2)d(\cos\theta), \tag{12}$$

with

$$k_1 = \frac{m_{K^*}}{m_{\bar{K}}+m_{K^*}} k\sqrt{2(1-\cos\theta)}, \tag{13}$$

$$k_2 = \frac{m_{\bar{K}}}{m_{\bar{K}}+m_{K^*}} k\sqrt{2(1-\cos\theta)}, \tag{14}$$

and

$$k = \frac{\sqrt{(s-(m_{\bar{K}}+m_{K^*}+m_\pi)^2)(s-(m_{\bar{K}}+m_{K^*}-m_\pi)^2)}}{2\sqrt{s}}, \tag{15}$$

is the module of the momentum of the π meson in $\pi\bar{K}K^*$ center-of-mass frame when $\sqrt{s}$ is above the threshold of the $\pi\bar{K}K^*$ system; otherwise, k equals zero. The expression of F_R is given below.

The double scattering contributions are from Figs. 1(b) and 1(f). The expression for the S-matrix for the double scattering $[S_2^{(2)} = S_1^{(2)}]$ is given by

$$\begin{aligned} S_1^{(2)} = &-it_1t_2\frac{(2\pi)^4}{V^2}\delta^4(k+k_R-k'-k'_R) \\ &\times \frac{1}{\sqrt{2\omega_\pi}}\frac{1}{\sqrt{2\omega'_\pi}}\frac{1}{\sqrt{2\omega_{\bar{K}}}}\frac{1}{\sqrt{2\omega'_{\bar{K}}}}\frac{1}{\sqrt{2\omega_{K^*}}}\frac{1}{\sqrt{2\omega'_{K^*}}} \\ &\times \int \frac{d^3q}{(2\pi)^3}F_R(q)\frac{1}{q^{02}-\vec{q}^2-m_\pi^2+i\epsilon}, \end{aligned} \tag{16}$$

with

$$q^0 = \frac{s+m_\pi^2-M_R^2}{2\sqrt{s}}. \tag{17}$$

One of the ingredients in the calculation is the form factor $F_R(q)$ for the bound state $f_1(1285)$ of a pair of $\bar{K}K^*$. Following the approach of Refs. [51,52], we can easily get the following expression for the form factor $F_R(q)$,

$$\begin{aligned} F_R(q) = &\frac{1}{N}\int_{|\vec{p}|<\Lambda,|\vec{p}-\vec{q}|<\Lambda} \\ &\times d^3\vec{p}\frac{1}{2\omega_{\bar{K}}(\vec{p})}\frac{1}{2\omega_{K^*}(\vec{p})} \\ &\times \frac{1}{M_R-\omega_{\bar{K}}(\vec{p})-\omega_{K^*}(\vec{p})}\frac{1}{2\omega_{\bar{K}}(\vec{p}-\vec{q})}\frac{1}{2\omega_{K^*}(\vec{p}-\vec{q})} \\ &\times \frac{1}{M_R-\omega_{\bar{K}}(\vec{p}-\vec{q})-\omega_{K^*}(\vec{p}-\vec{q})}, \end{aligned} \tag{18}$$

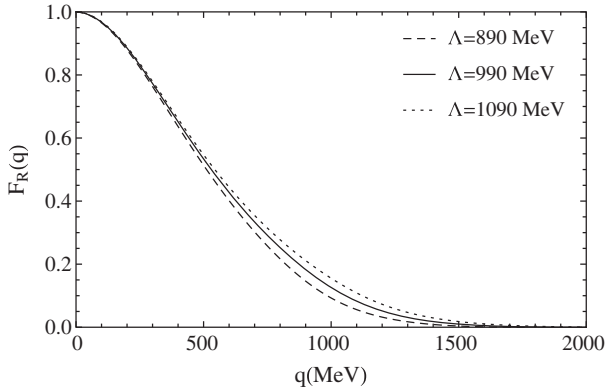

FIG. 2. Form factor of the $f_1(1285)$ as a $\bar{K}K^*$ bound state.

where the normalization factor N is

$$N = \int_{|\vec{p}|<\Lambda} d^3\vec{p}\left(\frac{1}{2\omega_{\bar{K}}(\vec{p})}\frac{1}{2\omega_{K^*}(\vec{p})}\frac{1}{M_R-\omega_{\bar{K}}(\vec{p})-\omega_{K^*}(\vec{p})}\right)^2. \tag{19}$$

The parameter Λ is used to regularize the loop functions in the chiral unitary approach [33].

In this work we take Λ around 990 MeV such that the $f_1(1285)$ is obtained [33]. The condition $|\vec{p}-\vec{q}| < \Lambda$ implies that the form factor is exactly zero for $q > 2\Lambda$. Therefore the integration in Eq. (18) has upper limit of 2Λ.

We show the form factor $F_R(q)$ in Fig. 2 with $\Lambda = 890$, 990, and 1090 MeV. From Fig. 2 we see that the form factor $F_R(q)$ is not sensitive to the value of Λ, especially for $q < 600$ MeV, and we find that the results of the total scattering amplitude T are very similar with $\Lambda = 990 \pm 100$ MeV, hence we take $\Lambda = 990$ MeV in the following such that the $f_1(1285)$ is obtained [33].

With the results of $F_R(q)$, we can easily calculate the form factors $FFS_i(s)$ for single scattering. In Fig. 3, we show the projection over the s-wave of the form factor for the single scattering contribution as a function of the total invariant mass of the $\pi\bar{K}K^*$ system. The solid and dashed curves are the results of FFS_1 and FFS_2, respectively.

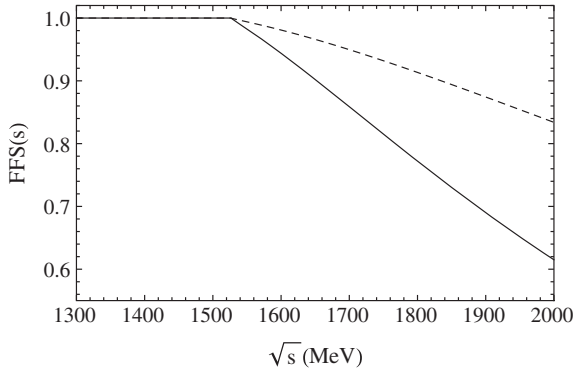

FIG. 3. Form factor for the single-scattering contribution.

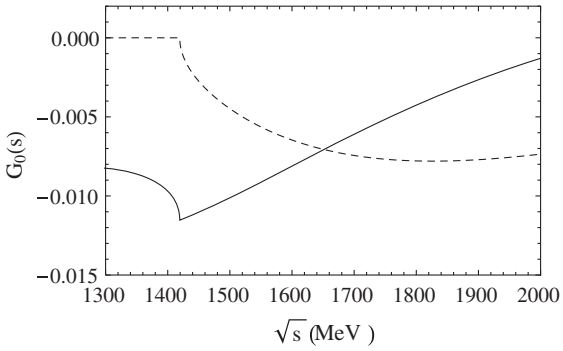

FIG. 4. Real (solid line) and imaginary (dashed line) parts of the G_0 function.

We see that the FFS_1 and FFS_2 are very close to one below $\sqrt{s} = 1800$ MeV, which indicates that the corrections from these two form factors are very small and only affect moderately the results of T beyond 1800 MeV.

Before proceeding further, we examine the normalization for the S matrix, which is given by

$$S = -iT\frac{(2\pi)^4}{V^2}\delta^4(k + k_R - k' - k'_R) \times \frac{1}{\sqrt{2\omega_\pi}}\frac{1}{\sqrt{2\omega'_\pi}}\frac{1}{\sqrt{2\omega_{f_1(1285)}}}\frac{1}{\sqrt{2\omega'_{f_1(1285)}}}. \quad (20)$$

By comparing Eq. (20) with Eq. (10) for the single scattering and Eq. (16) for the double scattering, we see that we have to give a weight to t_1 and t_2 such that Eqs. (10) and (16) get the weight factors that appear in the general formula of Eq. (20). This is achieved by replacing

$$t_1 \to \tilde{t}_1 = t_1\sqrt{\frac{2\omega_{f_1(1285)}}{2\omega_{\bar{K}}}}\sqrt{\frac{2\omega'_{f_1(1285)}}{2\omega'_{\bar{K}}}}, \quad (21)$$

$$t_2 \to \tilde{t}_2 = t_2\sqrt{\frac{2\omega_{f_1(1285)}}{2\omega_{K^*}}}\sqrt{\frac{2\omega'_{f_1(1285)}}{2\omega'_{K^*}}}. \quad (22)$$

By solving Eqs. (1) and (2) and summing the two partitions T_1 and T_2, we get

$$T = \frac{\tilde{t}_1 + \tilde{t}_2 + 2\tilde{t}_1\tilde{t}_2 G_0}{1 - \tilde{t}_1\tilde{t}_2 G_0^2} + \tilde{t}_1[FFS_1 - 1] + \tilde{t}_2[FFS_2 - 1], \quad (23)$$

where G_0 depends on the invariant mass square s and is given by

$$G_0(s) = \frac{1}{2\omega_{f_1(1285)}}\int\frac{d^3\vec{q}}{(2\pi)^3}F_R(q)\frac{1}{q^{02} - \vec{q}^2 - m_\pi^2 + i\epsilon}. \quad (24)$$

In Fig. 4, we show the real and imaginary parts of the G_0 as a function of the invariant mass of the $\pi\bar{K}K^*$ system.

III. RESULTS AND DISCUSSION

To perform the evaluation of Faddeev equations under the FCA, we need the calculation of the two-body interaction amplitudes (t_1 and t_2) of $\pi\bar{K}$ and πK^*, which are investigated in Refs. [33,49,50] as mentioned before. These two-body scattering amplitudes depend on the subtraction constants $a_{\pi\bar{K}}$ and $a_{\pi K^*}$, which are assumed as effective parameters in our calculation. We take them as used in Refs. [49,50]: $a_{\pi K^*} = -1.85$ and $\mu = 1000$ MeV for $I_{\pi K^*} = 1/2$; $a_{\pi K} = -1.38$ and $\mu = m_K$ for $I_{\pi K} = 1/2$; $a_{\pi K} = -4.64$ and $\mu = m_K$ for $I_{\pi K} = 3/2$. Then we calculate the total scattering amplitude T and associate the peaks/bumps in the modulus squared $|T|^2$ to resonances.

In Ref. [49], only the πK^* interaction in $I_{\pi K^*} = 1/2$ sector was studied where two $K_1(1270)$ states were obtained. In this work we need also the parameter $a_{\pi K^*}$ for the case of $I_{\pi K^*} = 3/2$, which is taken the same as for $I_{\pi K^*} = 1/2$ as used in Ref. [49].

In the FCA, we keep the wave function of the cluster unchanged by the presence of the third particle. In order to estimate uncertainties of the FCA due to this frozen condition we admit that the wave function of the cluster could be modified by the presence of the third particle, which is the normal situation in a full Faddeev calculation. Indeed, $\pi f_1(1285)$ may couple to other s-wave meson-meson channels, such as π meson and other excited f_1 states or $\bar{K}$ meson and K_1 states. However, other excited f_1 states may not have large $\bar{K}K^*$ component[3] or the thresholds of these channels are far from the energy region we considered. Furthermore, including such contributions, the $\pi - (\bar{K}K^*)_{f_1(1285)}$ scattering amplitude would become more complex due to additional parameters from the nondiagonal transitions, and we cannot determine or constrain these parameters. Hence, we will leave these contributions to future studies when more experimental data become available. For the sake of simplicity we do not include other channels in our calculation.

As pointed before, the form factor, $F_R(q)$, is not sensitive to the value of Λ. Then, in order to quantify uncertainties of the FCA, we perform calculations with

[3]One might think that the inclusion of $h_1(1380)$ and $b_1(1235)$ states might improve the situation, since those resonances couple also dominantly to the $\bar{K}K^*$ channel [34]. However, the quantum numbers of $h_1(1380)$ and $b_1(1235)$ are different with $f_1(1285)$. The transition between $\pi - (\bar{K}K^*)_{h_1(1380)}$, $\pi - (\bar{K}K^*)_{b_1(1235)}$ and $\pi - (\bar{K}K^*)_{f_1(1285)}$ should be zero.

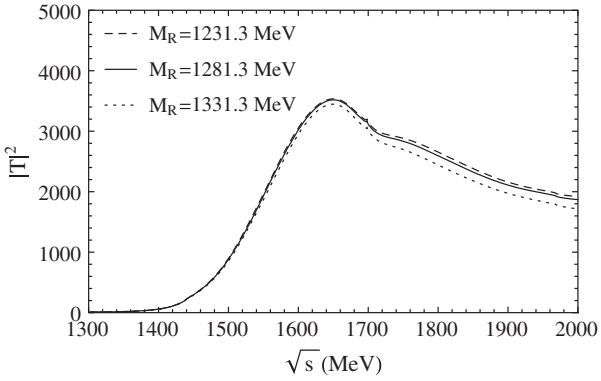

FIG. 5. Modulus squared of the $\pi\bar{K}K^*$ three-body scattering amplitude.

different values of M_R. In Fig. 5, we show the modulus squared of the total $\pi-(\bar{K}K^*)_{f_1(1285)}$ scattering amplitude with $M_R = 1231.3$, 1281.3, and 1331.3 MeV, where we see a clear bump structure around $\sqrt{s} \sim 1650$ MeV for the three cases. From the PDG [1], this structure can be assigned to $\pi_1(1600)$, with mass 1660 MeV. Furthermore, taking $\sqrt{s} = 1660$ MeV we get $\sqrt{s_1} = 792$ MeV and $\sqrt{s_2} = 1244$ MeV from Eqs. (8) and (9). At these energy points, the interactions of $\pi\bar{K}$ and πK^* are strong enough to produce the $\pi_1(1600)$ state.

Note that the location of the peak is quite stable against variation of the parameters of $a_{\pi\bar{K}}$ and $a_{\pi K^*}$ in the ranges of values to reproduce the results of Refs. [49,50] within uncertainties. This may indicate that the $\pi_1(1600)$ state can be generated from $\pi f_1(1285)$ where $f_1(1285)$ is present in the $\bar{K}K^*$ interaction. This may be the origin of the $\pi_1(1600)$ state and the future measurements about the $\pi f_1(1285)$ mode can be used to test our finding here.

On the other hand, from Fig. 5 we see that there is no any bump structure around $\sqrt{s} \sim 1400$ MeV, which can be assigned as the $\pi_1(1400)$ state. This may indicate that the $\pi_1(1400)$ can not be dynamically generated from the $\pi f_1(1285)$ interaction.

IV. SUMMARY

In this work, we have performed a Faddeev calculation for the π-$f_1(1285)$ system treating $f_1(1285)$ state as a $\bar{K}K^*$ bound state as found in previous studies of the $\bar{K}$-K^* system [33,34]. We have used the FCA to describe the π-$(\bar{K}K^*)_{f_1(1285)}$ system in terms of the two-body interactions, $\pi\bar{K}$ and πK^*, provided by the chiral unitary approach as investigated in Refs. [49,50]. There is a clear and stable bump structure around 1650 MeV in the module squared of the total scattering amplitude indicating the formation of a resonant $\pi\bar{K}K^*$ state around this energy. This state has "exotic" quantum numbers $J^{PC} = 1^{-+}$. From PDG, we can associated this resonance to the exotic $\pi_1(1600)$ state with mass 1660 MeV and large uncertainties for the width [1]. This may be the origin of the $\pi_1(1600)$ resonance that is treated as a hybrid state in Refs. [29,30], a four-quark state in Ref. [31] or a molecule/four-quark mixing state in Ref. [32]. Future measurements about the $\pi f_1(1285)$ mode can be used to test our calculations and clarify the issue.

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[1] K. A. Olive *et al.* (Particle Data Group Collaboration), Chin. Phys. C **38**, 090001 (2014).
[2] E. Klempt and A. Zaitsev, Phys. Rep. **454**, 1 (2007).
[3] N. Brambilla *et al.*, Eur. Phys. J. C **74**, 2981 (2014).
[4] S. K. Choi *et al.* (Belle Collaboration), Phys. Rev. Lett. **91**, 262001 (2003).
[5] D. Acosta *et al.* (CDF Collaboration), Phys. Rev. Lett. **93**, 072001 (2004).
[6] V. M. Abazov *et al.* (D0 Collaboration), Phys. Rev. Lett. **93**, 162002 (2004).
[7] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett. **110**, 252001 (2013).
[8] Z. Q. Liu *et al.* (Belle Collaboration), Phys. Rev. Lett. **110**, 252002 (2013).
[9] C. Adolph *et al.* (COMPASS Collaboration), Phys. Rev. Lett. **115**, 082001 (2015).
[10] V. M. Abazov *et al.* (D0 Collaboration), Phys. Rev. Lett. **117**, 022003 (2016).
[11] C. Amsler and N. A. Tornqvist, Phys. Rep. **389**, 61 (2004).
[12] C. A. Meyer and E. S. Swanson, Prog. Part. Nucl. Phys. **82**, 21 (2015).
[13] G. S. Adams *et al.* (E852 Collaboration), Phys. Rev. Lett. **81**, 5760 (1998).
[14] S. U. Chung *et al.*, Phys. Rev. D **65**, 072001 (2002).
[15] E. I. Ivanov *et al.* (E852 Collaboration), Phys. Rev. Lett. **86**, 3977 (2001).
[16] J. Kuhn *et al.* (E852 Collaboration), Phys. Lett. B **595**, 109 (2004).
[17] M. Lu *et al.* (E852 Collaboration), Phys. Rev. Lett. **94**, 032002 (2005).
[18] M. Alekseev *et al.* (COMPASS Collaboration), Phys. Rev. Lett. **104**, 241803 (2010).

[19] B. A. Mecking *et al.* (CLAS Collaboration), Nucl. Instrum. Methods Phys. Res., Sect. A **503**, 513 (2003).
[20] M. Nozar *et al.* (CLAS Collaboration), Phys. Rev. Lett. **102**, 102002 (2009).
[21] N. Isgur and J. E. Paton, Phys. Rev. D **31**, 2910 (1985).
[22] F. E. Close and P. R. Page, Nucl. Phys. **B443**, 233 (1995).
[23] P. R. Page, E. S. Swanson, and A. P. Szczepaniak, Phys. Rev. D **59**, 034016 (1999).
[24] D. Ebert, R. N. Faustov, and V. O. Galkin, Phys. Rev. D **79**, 114029 (2009).
[25] H. C. Kim and Y. Kim, J. High Energy Phys. 01 (2009) 034.
[26] J. J. Dudek, R. G. Edwards, M. J. Peardon, D. G. Richards, and C. E. Thomas, Phys. Rev. D **82**, 034508 (2010).
[27] C. A. Meyer and Y. Van Haarlem, Phys. Rev. C **82**, 025208 (2010).
[28] L. Bellantuono, P. Colangelo, and F. Giannuzzi, Eur. Phys. J. C **74**, 2830 (2014).
[29] H. X. Chen, Z. X. Cai, P. Z. Huang, and S. L. Zhu, Phys. Rev. D **83**, 014006 (2011).
[30] B. Zhou, Z. F. Sun, X. Liu, and S. L. Zhu, arXiv: 1603.06367.
[31] H. X. Chen, A. Hosaka, and S. L. Zhu, Phys. Rev. D **78**, 054017 (2008).
[32] S. Narison, Phys. Lett. B **675**, 319 (2009).
[33] L. Roca, E. Oset, and J. Singh, Phys. Rev. D **72**, 014002 (2005).
[34] M. F. M. Lutz and E. E. Kolomeitsev, Nucl. Phys. **A730**, 392 (2004).
[35] A. Gal, Int. J. Mod. Phys. A **22**, 226 (2007).
[36] R. C. Barrett and A. Deloff, Phys. Rev. C **60**, 025201 (1999).
[37] S. S. Kamalov, E. Oset, and A. Ramos, Nucl. Phys. **A690**, 494 (2001).
[38] J. J. Xie, A. Martinez Torres, and E. Oset, Phys. Rev. C **83**, 065207 (2011).
[39] A. Martinez Torres, K. P. Khemchandani, and E. Oset, Phys. Rev. C **79**, 065207 (2009).
[40] A. Martinez Torres and D. Jido, Phys. Rev. C **82**, 038202 (2010).
[41] D. Jido and Y. Kanada-En'yo, Phys. Rev. C **78**, 035203 (2008).
[42] J. J. Xie, A. Martinez Torres, E. Oset, and P. Gonzalez, Phys. Rev. C **83**, 055204 (2011).
[43] M. Bayar, W. H. Liang, T. Uchino, and C. W. Xiao, Eur. Phys. J. A **50**, 67 (2014).
[44] W. Liang, C. W. Xiao, and E. Oset, Phys. Rev. D **88**, 114024 (2013).
[45] B. Durkaya and M. Bayar, Phys. Rev. D **92**, 036006 (2015).
[46] M. Bayar, X. L. Ren, and E. Oset, Eur. Phys. J. A **51**, 61 (2015).
[47] A. Martinez Torres, K. P. Khemchandani, D. Jido, and A. Hosaka, Phys. Rev. D **84**, 074027 (2011).
[48] L. Roca, Phys. Rev. D **84**, 094006 (2011).
[49] L. S. Geng, E. Oset, L. Roca, and J. A. Oller, Phys. Rev. D **75**, 014017 (2007).
[50] F. K. Guo, R. G. Ping, P. N. Shen, H. C. Chiang, and B. S. Zou, Nucl. Phys. **A773**, 78 (2006).
[51] L. Roca and E. Oset, Phys. Rev. D **82**, 054013 (2010).
[52] J. Yamagata-Sekihara, L. Roca, and E. Oset, Phys. Rev. D **82**, 094017 (2010); **85**, 119905(E) (2012).