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Quasi-two-body decays $B_{(s)} \to P\rho \to P\pi\pi$ in the perturbative QCD approach

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In this work, we calculate the CP-averaged branching ratios and the direct CP-violating asymmetries of the quasi-two-body decays $B_{(s)} \to P(\rho \to)\pi\pi$ by employing the perturbative QCD (PQCD) approach (here P stands for a light pseudoscalar meson π, K, η or η'). The vector current timelike form factor F_π, which contains the final-state interactions between the pion pair in the resonant region associated with the P-wave states $\rho(770)$ along with the two-pion distribution amplitudes, is employed to describe the interactions between the ρ and the pion pair under the hypothesis of the conserved vector current. We found that (a) the PQCD predictions for the branching ratios and the direct CP-violating asymmetries for most considered $B_{(s)} \to P(\rho \to)\pi\pi$ decays agree with currently available data within errors, (b) for $\mathcal{B}(B \to \pi^0\rho^0 \to \pi^0(\pi^+\pi^-)$, the PQCD prediction is much smaller than the measured one, and (c) for the $B^+ \to \pi^+(\rho^0 \to)\pi^+\pi^-$ decay mode, there is a negative CP asymmetry $(-27.5^{+3.0}_{-3.7})\%$, which agrees with other theoretical predictions but is different in sign from those reported by the *BABAR* and LHCb Collaborations.

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I. INTRODUCTION

Experimental data from different collaborations, like *BABAR* [1–5], Belle [6–9], and LHCb [10–12], provide valuable information for the three-body hadronic B-meson decays. For these decay modes, both the resonant and nonresonant contributions may appear, as well as the possible significant final-state interactions (FSIs) [13–15]. Different frameworks have been developed for the study of the three-body hadronic B-meson decays, based on the symmetry principles [16–24] or factorization theorems [25–34]. The QCD-improved factorization (QCDF) [31–34] has been widely used in the study of the three-body charmless hadronic B-meson decays [35–41]. In Refs. [40,41], the authors studied the nonresonant contributions using heavy meson chiral perturbation theory (HMChPT) [42–44] with some modifications and analyzed the resonant contributions with the isobar model in terms of the usual Breit-Wigner formalism [45]. The perturbative QCD (PQCD) approach based on the k_T factorization theorem [46,47] has also been adopted in Refs. [48–52].

As discussed in Refs. [46–49], the hard b-quark decay kernels containing two virtual gluons at leading order is not important due to the power-suppression. The contributions from the region, where there is at least one pair of light mesons having an invariant mass below $O(\bar{\Lambda} m_B)$ [46,47], $\bar{\Lambda} = m_B - m_b$ being the B-meson and b-quark mass difference, is dominant. It's reasonable that the dynamics associated with the pair of mesons can be factorized into a two-meson distribution amplitude $\Phi_{h_1h_2}$ [53]. As a result, one can describe the typical PQCD factorization formula for a $B \to h_1h_2h_3$ decay amplitude as the form of [46,47]

$$\mathcal{A} = \Phi_B \otimes H \otimes \Phi_{h_1h_2} \otimes \Phi_{h_3}. \tag{1}$$

With the hard kernel H describing the dynamics of the strong and electroweak interactions in three-body hadronic decays in a similar way as the one for the two-body $B \to h_1h_2$ decays, the Φ_B and Φ_{h_3} are the wave functions for the B meson and the final-state h_3, which absorb the nonperturbative dynamics in the process. The $\Phi_{h_1h_2}$ is the two-hadron (h_1 and h_2) distribution amplitude proposed in Refs. [53–59], which describes the structure of the final-state h_1-h_2 pair.

With the help of the two-pion distribution amplitudes, quasi-two-body decays $B \to K\rho \to K\pi\pi$, the subprocesses of the three-body decays $B \to K\pi\pi$, have been studied in Ref. [50] in the PQCD approach utilizing the framework discussed in [46–49]. The consistency between the PQCD predictions and the data supports the usability of the quasi-two-body framework in Ref. [50] for the study of the three-body hadronic B decays. In this work, we extend the previous studies in Ref. [50] to the quasi-two-body decays $B \to P\rho \to P\pi\pi$, with the P standing for the light

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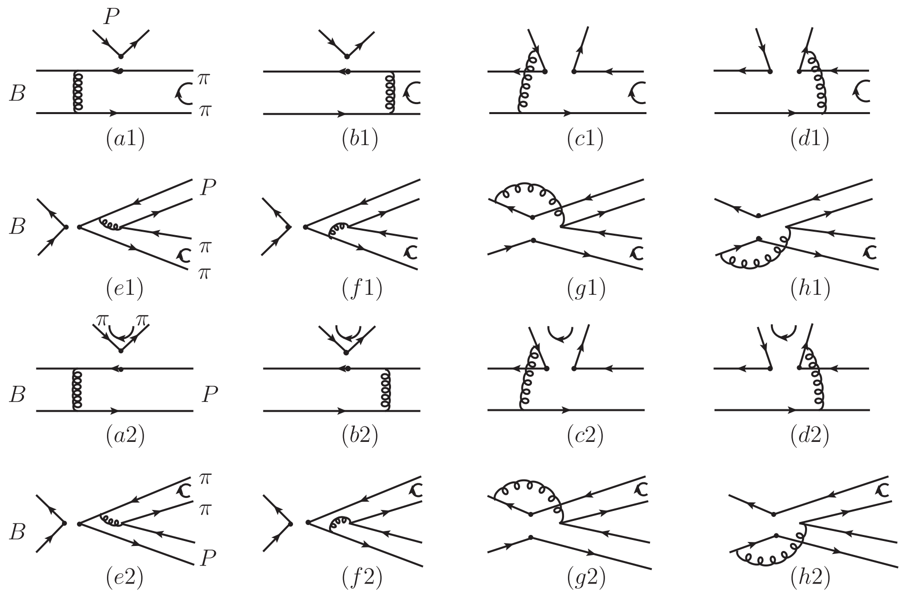

FIG. 1. Typical Feynman diagrams for the quasi-two-body decays $B \to P(\rho \to)\pi\pi$, where B stands for the $B^{\pm}$, B^0 or B_s meson and P denotes π, K, η or η'. With $\alpha = a$-d and $\beta = e$-h, the diagrams (α1) for the $B \to \rho \to \pi\pi$ transition and (α2) the $B \to P$ transition, as well as the diagrams (β1) and (β2) for annihilation contributions.

pseudoscalar mesons, $P = (\pi, K, \eta \,\text{or}\, \eta')$, as shown in Fig. 1. In the literature, much work has been done on the decays of $B \to P\rho$ in the two-body framework [30,34,60–63], and some of the experimental data can be found in [64–67]. From [50], we know that the width of the resonant state ρ and the interactions between the final-state pion pair will show their effects on the branching ratios, especially on the direct CP violations of the quasi-two-body decays. We should not neglect these effects in $B \to P\rho$ decays. In order to describe the strong interactions between the P-wave resonant state ρ and the final-state pion pair, the vector current timelike form factor F_π containing final-state interactions between the pion pair has been employed in Ref. [50]. Guaranteed by the Watson theorem [68], the results from the π-π scattering and τ decays for the timelike form factor F_π could be borrowed for the study of quasi-two-body B-meson decays. The detailed discussion of F_π can be found in [50] and its references.

This paper is organized as follows. In Sec. II, we give a brief introduction for the theoretical framework. The numerical values, some discussions, and the conclusions will be given in last two sections.

II. FRAMEWORK

For the quasi-two-body $B \to P(\rho \to)\pi\pi$ decays, the weak effective Hamiltonian can be specified as [69]

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}^* V_{uq} [C_1(\mu) O_1^u(\mu) + C_2(\mu) O_2^u(\mu)] - V_{tb}^* V_{tq} \left[\sum_{i=3}^{10} C_i(\mu) O_i(\mu) \right] \right\} + \text{H.c.}, \tag{2}$$

with $q = d, s$, $C_i(\mu)(i = 1, \ldots, 10)$ as the Wilson coefficients, and O_i the local four-quark operators.

We let the pion pair and the final-state P move along the direction of $n = (1, 0, 0_{\rm T})$ and $v = (0, 1, 0_{\rm T})$ in the light-cone coordinates, respectively. The B-meson momentum p_B, the total momentum of the pion pair, $p = p_1 + p_2$, and the final-state P momentum p_3 are chosen as

$$p_B = \frac{m_B}{\sqrt{2}}(1, 1, 0_{\rm T}), \quad p = \frac{m_B}{\sqrt{2}}(1, \eta, 0_{\rm T}), \quad p_3 = \frac{m_B}{\sqrt{2}}(0, 1-\eta, 0_{\rm T}), \tag{3}$$

where m_B is the mass of the B meson, the variable η is defined as $\eta = \omega^2/m_B^2$, the invariant mass squared $\omega^2 = p^2$. We define $\zeta = p_1^+/p^+$ as one of the pion pair's momentum fractions, in terms of which the other kinematic variables of the two pions are expressed as

$$p_1^- = (1-\zeta)\eta \frac{m_B}{\sqrt{2}}, \quad p_2^+ = (1-\zeta)\frac{m_B}{\sqrt{2}}, \quad p_2^- = \zeta\eta \frac{m_B}{\sqrt{2}}. \tag{4}$$

We employ x_B, z, x_3 to denote the momentum fraction of the positive quark in each meson, k_{BT}, $k_{\rm T}$, $k_{3\rm T}$ stands for the transverse momentum of the positive quark, respectively. The momentum k_B of the spectator quark in the B meson, the momentum k for the resonant state ρ, and k_3 for the final-state P are of the form of

$$k_B = \left(0, x_B \frac{m_B}{\sqrt{2}}, k_{BT}\right),$$
$$k = \left(\frac{m_B}{\sqrt{2}} z, 0, k_{\rm T}\right),$$
$$k_3 = \left(0, (1-\eta)x_3 \frac{m_B}{\sqrt{2}}, k_{3\rm T}\right). \quad (5)$$

The momentum fractions x_B, z, and x_3 run from zero to unity.

In this work, we use the wave function [70–74]

$$\Phi_B = \frac{i}{\sqrt{2N_c}}(\not{p}_B + m_B)\gamma_5 \phi_B(\mathbf{k_1}), \quad (6)$$

for the B^+, B^0, and B_s^0 mesons, and we adopt the widely used distribution amplitude [70–74]

$$\phi_B(x, b) = N_B x^2 (1-x)^2 \exp\left[-\frac{M_B^2 x^2}{2\omega_B^2} - \frac{1}{2}(\omega_B b)^2\right] \quad (7)$$

for them, with the normalization factor N_B depending on the value of ω_B and f_B, which is defined through the normalization relation $\int_0^1 dx \phi_B(x, b=0) = f_B/(2\sqrt{6})$. $\omega_B = 0.40 \pm 0.04$ GeV and $\omega_{B_s} = 0.50 \pm 0.05$ GeV [70,75,76] will be employed in the following numerical calculations.

For the final-state P (π, K, η or η'), we have the wave functions [71,72]

$$\Phi_P(P_3, x_3) \equiv \frac{i}{\sqrt{2N_C}} \gamma_5 [\not{P}_3 \phi_P^A(x_3) + m_{03}\phi_P^P(x_3) + m_{03}(\not{n}\not{v} - 1)\phi_P^T(x_3)], \quad (8)$$

where m_{03} is the corresponding meson chiral mass, and P_3 and x_3 are the momentum and the momentum fraction of P, respectively. The expressions of the relevant distribution amplitudes of the pion and kaon mesons are the following [77–82]:

$$\phi_\pi^A(x) = \frac{3f_\pi}{\sqrt{6}} x(1-x)[1 + 0.44 C_2^{3/2}(t)], \quad (9)$$

$$\phi_\pi^P(x) = \frac{f_\pi}{2\sqrt{6}}[1 + 0.43 C_2^{1/2}(t)], \quad (10)$$

$$\phi_\pi^T(x) = \frac{f_\pi}{2\sqrt{6}}(1-2x)[1 + 0.55(10x^2 - 10x + 1)], \quad (11)$$

$$\phi_K^A(x) = \frac{3f_K}{\sqrt{6}} x(1-x)[1 + 0.17 C_1^{3/2}(t) + 0.2 C_2^{3/2}(t)], \quad (12)$$

$$\phi_K^P(x) = \frac{f_K}{2\sqrt{6}}[1 + 0.24 C_2^{1/2}(t)], \quad (13)$$

$$\phi_K^T(x) = -\frac{f_K}{2\sqrt{6}}[C_1^{1/2}(t) + 0.35 C_3^{1/2}(t)]. \quad (14)$$

The distribution amplitudes $\phi_{\eta_{q(s)}}^{A,P,T}$ $(q = u, d)$ for $\eta_{q(s)}$ are given as [77–79,83]

$$\phi_{\eta_{q(s)}}^A(x) = \frac{f_{q(s)}}{2\sqrt{2N_c}} 6x(1-x)[1 + a_1^\eta C_1^{3/2}(2x-1) + a_2^\eta C_2^{3/2}(2x-1) + a_4^\eta C_4^{3/2}(2x-1)], \quad (15)$$

$$\phi_{\eta_{q(s)}}^P(x) = \frac{f_{q(s)}}{2\sqrt{2N_c}}\left[1 + \left(30\eta_3 - \frac{5}{2}\rho_{\eta_{q(s)}}^2\right) C_2^{1/2}(2x-1) - 3\left[\eta_3\omega_3 + \frac{9}{20}\rho_{\eta_{q(s)}}^2 (1 + 6a_2^\eta)\right] C_4^{1/2}(2x-1)\right], \quad (16)$$

$$\phi_{\eta_{q(s)}}^T(x) = \frac{f_{q(s)}}{2\sqrt{2N_c}}(1-2x)\left[1 + 6\left(5\eta_3 - \frac{1}{2}\eta_3\omega_3 - \frac{7}{20}\rho_{\eta_{q(s)}}^2 - \frac{3}{5}\rho_{\eta_{q(s)}}^2 a_2^\eta\right)(1 - 10x + 10x^2)\right], \quad (17)$$

with the Gegenbauer moments,

$$a_1^\eta = 0, \qquad a_2^\eta = 0.44, \qquad a_4^\eta = 0.25. \quad (18)$$

The parameters $\rho_{\eta_q} = 2m_q/m_0^q$ with $m_0^q = 1.07$ GeV for η_q and $\rho_{\eta_s} = 2m_s/m_0^s$ with $m_0^s = 1.92$ GeV for η_s [84]. The Gegenbauer polynomials $C_n^\nu(t)$ ($n = 1$, 2, 3, 4 and $\nu = 1/2, 3/2$) above could be found in Ref. [82].

In this paper, we consider the meson η, η' as a mixture from η_q and η_s,

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}, \quad (19)$$

with

$$\eta_q = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}), \qquad \eta_s = s\bar{s}. \quad (20)$$

The mixtures among the η_q, η_s and a possible glueball [85–88] will be neglected in this work. For the decay constant and the mixing angle ϕ, we have the forms as [89,90],

$$
\begin{aligned}
f_q &= (1.07 \pm 0.02) f_\pi, \\
f_s &= (1.34 \pm 0.06) f_\pi, \\
\phi &= 39.3^\circ \pm 1.0^\circ, \\
f_\pi &= 0.131\ \text{GeV}.
\end{aligned}
\tag{21}
$$

The two-pion distribution amplitudes are the same ones as those being used in Ref. [50],

$$
\Phi_{\pi\pi}^{\text{P}} = \frac{1}{\sqrt{2N_c}} \left[\not{p} \Phi_{v\nu=-}^{I=1}(z,\zeta,\omega^2) + \omega \Phi_s^{I=1}(z,\zeta,\omega^2) + \frac{\not{p}_1 \not{p}_2 - \not{p}_2 \not{p}_1}{w(2\zeta-1)} \Phi_{t\nu=+}^{I=1}(z,\zeta,\omega^2) \right],
\tag{22}
$$

with

$$
\Phi_{v\nu=-}^{I=1} = \phi_0 = \frac{3F_\pi(s)}{\sqrt{2N_c}} z(1-z) \times \left[1 + a_{2\rho}^0 \frac{3}{2}(5(1-2z)^2 - 1) \right] P_1(2\zeta - 1),
\tag{23}
$$

$$
\Phi_s^{I=1} = \phi_s = \frac{3F_s(s)}{2\sqrt{2N_c}} (1-2z) \times [1 + a_{2\rho}^s (10z^2 - 10z + 1)] P_1(2\zeta - 1),
\tag{24}
$$

$$
\Phi_{t\nu=+}^{I=1} = \phi_t = \frac{3F_t(s)}{2\sqrt{2N_c}} (1-2z)^2 \times \left[1 + a_{2\rho}^t \frac{3}{2}(5(1-2z)^2 - 1) \right] P_1(2\zeta - 1),
\tag{25}
$$

where the Legendre polynomial $P_1(2\zeta - 1) = 2\zeta - 1$. We make tiny corrections of the Gegenbauer moments for the two-pion distribution amplitudes compared with those in Ref. [50]. By referring to all the existing data of $B \to P(\rho \to)\pi\pi$ in Ref. [91], we adjust $a_{2\rho}^0$, $a_{2\rho}^s$, $a_{2\rho}^t$ to cater to the data, and we have the new Gegenbauer coefficients $a_{2\rho}^0 = 0.30$, $a_{2\rho}^s = 0.70$, $a_{2\rho}^t = -0.40$.

We adopt the same $F_\pi(s)$ in this work as that in Ref. [50], and the approximate relations $F_{s,t}(s) \approx (f_\rho^T / f_\rho) F_\pi(s)$ [50] will also be used in the following section. By taking the $\rho - \omega$ interference and the excited states into account, the form factor $F_\pi(s)$ can be written in the form

$$
F_\pi(s) = \left[\text{GS}_\rho(s, m_\rho, \Gamma_\rho) \frac{1 + c_\omega \text{BW}_\omega(s, m_\omega, \Gamma_\omega)}{1 + c_\omega} + \Sigma c_i \text{GS}_i(s, m_i, \Gamma_i) \right] [1 + \Sigma c_i]^{-1},
\tag{26}
$$

where $s = m^2(\pi\pi)$ is the two-pion invariant mass squared, $i = (\rho'(1450), \rho''(1700), \rho'''(2254))$, Γ is the decay width for the relevant resonance, and $m_{\rho,\omega,i}$ are the masses of the corresponding mesons, respectively. The function $\text{GS}_\rho(s, m_\rho, \Gamma_\rho)$ has been parametrized as the Gounaris-Sakurai (GS) model based on the Breit-Wigner (BW) model [45,92],

$$
\text{GS}_\rho(s, m_\rho, \Gamma_\rho) = \frac{m_\rho^2 [1 + d(m_\rho) \Gamma_\rho / m_\rho]}{m_\rho^2 - s + f(s, m_\rho, \Gamma_\rho) - i m_\rho \Gamma(s, m_\rho, \Gamma_\rho)},
\tag{27}
$$

with the functions

$$
\begin{aligned}
\Gamma(s, m_\rho, \Gamma_\rho) &= \Gamma_\rho \frac{s}{m_\rho^2} \left(\frac{\beta_\pi(s)}{\beta_\pi(m_\rho^2)} \right)^3, \\
d(m) &= \frac{3}{\pi} \frac{m_\pi^2}{k^2(m^2)} \ln \left(\frac{m + 2k(m^2)}{2m_\pi} \right) + \frac{m}{2\pi k(m^2)} - \frac{m_\pi^2 m}{\pi k^3(m^2)}, \\
f(s, m, \Gamma) &= \frac{\Gamma m^2}{k^3(m^2)} [k^2(s)[h(s) - h(m^2)] + (m^2 - s) k^2(m^2) h'(m^2)], \\
k(s) &= \frac{1}{2} \sqrt{s} \beta_\pi(s), \\
h(s) &= \frac{2}{\pi} \frac{k(s)}{\sqrt{s}} \ln \left(\frac{\sqrt{s} + 2k(s)}{2m_\pi} \right),
\end{aligned}
\tag{28}
$$

where $\beta_\pi(s) = \sqrt{1 - 4m_\pi^2/s}$. For the $\rho(770)$ resonant state, for example, the measured value of its resonance width is $\Gamma_\rho = 0.149$ GeV, to be used as input in the numerical calculations.

III. NUMERICAL RESULTS AND DISCUSSIONS

The following input parameters (the masses, decay constants and QCD scale are in units of GeV) will be used [91] in numerical calculations,

$$
\begin{aligned}
&\Lambda_{\overline{\text{MS}}}^{(f=4)} = 0.25, \quad m_{B^0} = 5.280, \quad m_{B_s} = 5.367, \\
&m_{B^\pm} = 5.279, \quad m_{\pi^\pm} = 0.140, \quad m_{\pi^0} = 0.135, \\
&m_{K^\pm} = 0.494, \quad m_{K^0} = 0.498, \quad m_\eta = 0.548, \\
&m_{\eta'} = 0.958, \quad m_{\rho^0} = 0.775, \quad m_{\rho^\pm} = 0.775, \\
&m_b = 4.8, \quad m_c = 1.275, \quad m_s = 0.095, \\
&f_B = 0.19 \pm 0.02, \quad f_{B_s} = 0.236 \pm 0.02, \\
&\tau_{B^0} = 1.519\ ps, \quad \tau_{B_s} = 1.512\ ps, \\
&\tau_{B^\pm} = 1.638\ ps, \quad f_\rho = 0.216 \pm 0.003, \\
&f_\rho^T = 0.184.
\end{aligned}
\tag{29}
$$

TABLE I. CP-averaged branching ratios and direct CP-violating asymmetries of $B_{(s)} \to K(\rho \to)\pi\pi$ decays calculated in PQCD approach together with experimental data [91].

Modes		Quasi-two-body results	Experiment
$B^+ \to K^+(\rho^0 \to)\pi^+\pi^-$	$\mathcal{B}(10^{-6})$	$4.04^{+0.75}_{-0.58}(\omega_B)^{+0.24}_{-0.20}(a^t_{2\rho})^{+0.27}_{-0.25}(a^s_{2\rho})^{+0.22}_{-0.21}(a^0_{2\rho})$	3.70 ± 0.50
	$\mathcal{A}_{CP}(\%)$	$50.7^{+3.8}_{-2.6}(\omega_B)^{+3.3}_{-4.7}(a^t_{2\rho})^{+0.0}_{-0.7}(a^s_{2\rho})^{+0.9}_{-1.5}(a^0_{2\rho})$	37.0 ± 10.0
$B^0 \to K^+(\rho^- \to)\pi^-\pi^0$	$\mathcal{B}(10^{-6})$	$8.17^{+1.93}_{-1.39}(\omega_B)^{+0.36}_{-0.31}(a^t_{2\rho})^{+0.46}_{-0.51}(a^s_{2\rho}) \pm 0.43(a^0_{2\rho})$	7.00 ± 0.90
	$\mathcal{A}_{CP}(\%)$	$39.7^{+2.6}_{-0.6}(\omega_B)^{+5.1}_{-5.4}(a^t_{2\rho})^{+0.5}_{-0.0}(a^s_{2\rho})^{+1.0}_{-0.9}(a^0_{2\rho})$	20.0 ± 11.0
$B^0_s \to K^-(\rho^+ \to)\pi^+\pi^0$	$\mathcal{B}(10^{-6})$	$19.68^{+7.63}_{-5.18}(\omega_{B_s}) \pm 0.01(a^t_{2\rho}) \pm 0.01(a^s_{2\rho})^{+0.05}_{-0.06}(a^0_{2\rho})$	…
	$\mathcal{A}_{CP}(\%)$	$21.8^{+3.7}_{-3.4}(\omega_{B_s}) \pm 0.3(a^t_{2\rho}) \pm 0.2(a^s_{2\rho}) \pm 1.2(a^0_{2\rho})$	…
$B^+ \to K^0(\rho^+ \to)\pi^+\pi^0$	$\mathcal{B}(10^{-6})$	$8.13^{+1.82}_{-1.23}(\omega_B) \pm 0.87(a^t_{2\rho})^{+0.44}_{-0.43}(a^s_{2\rho})^{+0.36}_{-0.39}(a^0_{2\rho})$	8.00 ± 1.50
	$\mathcal{A}_{CP}(\%)$	$13.8^{+3.1}_{-2.9}(\omega_B)^{+2.2}_{-1.9}(a^t_{2\rho})^{+0.2}_{-0.0}(a^s_{2\rho})^{+0.2}_{-0.3}(a^0_{2\rho})$	-12.0 ± 17.0
$B^0 \to K^0(\rho^0 \to)\pi^+\pi^-$	$\mathcal{B}(10^{-6})$	$4.39^{+1.12}_{-0.81}(\omega_B) \pm 0.38(a^t_{2\rho})^{+0.21}_{-0.22}(a^s_{2\rho})^{+0.19}_{-0.16}(a^0_{2\rho})$	4.70 ± 0.60
	$\mathcal{A}_{CP}(\%)$	$8.1^{+0.1}_{-0.2}(\omega_B)^{+0.8}_{-0.3}(a^t_{2\rho})^{+0.8}_{-0.6}(a^s_{2\rho}) \pm 0.0(a^0_{2\rho})$	…
$B^0_s \to \bar{K}^0(\rho^0 \to)\pi^+\pi^-$	$\mathcal{B}(10^{-6})$	$0.21^{+0.05}_{-0.01}(\omega_{B_s})^{+0.01}_{-0.00}(a^t_{2\rho})^{+0.01}_{-0.00}(a^s_{2\rho})^{+0.03}_{-0.01}(a^0_{2\rho})$	…
	$\mathcal{A}_{CP}(\%)$	$63.7^{+13.1}_{-15.2}(\omega_{B_s})^{+5.7}_{-7.0}(a^t_{2\rho})^{+3.1}_{-4.0}(a^s_{2\rho})^{+1.5}_{-2.0}(a^0_{2\rho})$	…

The values of the Wolfenstein parameters are the same as given in Ref. [91]: $A = 0.814^{+0.023}_{-0.024}$, $\lambda = 0.22537 \pm 0.00061$, $\bar{\rho} = 0.117 \pm 0.021$, $\bar{\eta} = 0.353 \pm 0.013$.

For the decay $B \to P(\rho \to \pi\pi)$, the differential branching ratio is written as [91]

$$\frac{d\mathcal{B}}{ds} = \tau_B \frac{|\vec{p}_\pi||\vec{p}_P|}{32\pi^3 m_B^3}|\mathcal{A}|^2, \tag{30}$$

where τ_B is the mean lifetime of the B meson, and s is the invariant mass squared $s = \omega^2 = p^2$. The kinematic variables $|\vec{p}_\pi|$ and $|\vec{p}_P|$ denote the magnitudes of one π meson in the pion pair and the P's momenta in the center-of-mass frame of the pion pair,

$$|\vec{p}_\pi| = \frac{1}{2}\sqrt{s - 4m_\pi^2},$$
$$|\vec{p}_P| = \frac{1}{2}\sqrt{[(m_B^2 - M_3^2)^2 - 2(m_B^2 + M_3^2)s + s^2]/s}. \tag{31}$$

By using the differential branching fraction in Eq. (30) and the decay amplitudes in the Appendix, we calculate and list the CP-averaged branching rations ($\mathcal{B}$) and direct CP-violating asymmetries ($\mathcal{A}_{CP}$) for $B_{(s)} \to K(\rho \to \pi\pi)$ in the third column of Table I, $B_{(s)} \to \pi(\rho \to \pi\pi)$ in Table II, and $B_{(s)} \to \eta^{(\prime)}(\rho \to \pi\pi)$ in Table III. The first error of these PQCD predictions comes from $\omega_B = (0.40 \pm 0.04)$ GeV for the B^+, B^0 mesons and $\omega_{B_s} = (0.50 \pm 0.05)$ GeV for the B_s meson, the second error is from $a^t_{2\rho} = -0.40 \pm 0.10$, and the other two errors result from $a^s_{2\rho} = 0.70 \pm 0.20$ and $a^0_{2\rho} = 0.30 \pm 0.05$, respectively.

From the numerical results as shown in the above three tables, one can address some issues as follows:

(i) Although we have made small changes for the three Gegenbauer moments $a_2^{0,s,t}$, the PQCD predictions for the branching ratios and direct CP asymmetries of the quasi-two-body decays $B^+ \to K^+(\rho^0 \to)\pi^+\pi^-$, $B^+ \to K^0(\rho^+ \to)\pi^+\pi^0$, $B^0 \to K^+(\rho^- \to)\pi^-\pi^0$ and $B^0 \to K^0(\rho^0 \to)\pi^+\pi^-$ agree well with those as given previously in Ref. [50]. The PQCD predictions for the decay rates of these four decay modes are consistent with currently available data [91]. For the decay $B^+ \to K^+(\rho^0 \to)\pi^+\pi^-$, the predicted direct CP asymmetry $\mathcal{A}_{CP} = (50.7^{+5.1}_{-5.6})\%$ matches the measured value $(37.0 \pm 10.0)\%$.

(ii) For the $B^+ \to \pi^+(\rho^0 \to)\pi^+\pi^-$ decay, the PQCD prediction for its branching ratio is well consistent with the world average $(8.3^{+1.2}_{-1.3}) \times 10^{-6}$ within errors, but its CP asymmetry is found to be negative: $\mathcal{A}_{CP} = (-27.5^{+3.0}_{-3.7})\%$ numerically. The *BABAR* and LHCb measurements for this quantity, however, prefer a positive CP asymmetry in the $m(\pi^+\pi^-)$ region peaked at m_ρ. The theoretical predictions based on the QCDF, PQCD and SCET all give a negative CP asymmetry of order -0.20 for $B^+ \to \rho^0\pi^+$ (see Table XIII of [93]). This puzzle concerning the sign of $\mathcal{A}_{CP}(\rho^0\pi^+)$ needs to be resolved in the near future.

(iii) The agreements of PQCD predictions with the data could be achieved for $B \to \pi(\rho \to)\pi\pi$ decays comparing with the results in Ref. [60]. The sum of the branching ratios of the $B^0 \to \pi^+(\rho^- \to)\pi^-\pi^0$ and $B^0 \to \pi^-(\rho^+ \to)\pi^+\pi^0$ decays are in consistent with the world average data. The calculated $\mathcal{A}_{CP}(B^0 \to \pi^-(\rho^+ \to)\pi^+\pi^0) = (8.2^{+2.0}_{-1.6})\%$ agree with the data

TABLE II. CP-averaged branching ratios and direct CP-violating asymmetries of $B_{(s)} \to \pi(\rho \to)\pi\pi$ decays calculated in PQCD approach together with experimental data [91].

Modes		Quasi-two-body results	Experiment
$B^+ \to \pi^+(\rho^0 \to)\pi^+\pi^-$	$\mathcal{B}(10^{-6})$	$8.84^{+1.48}_{-1.24}(\omega_B)^{+0.12}_{-0.13}(a^t_{2\rho})^{+1.17}_{-1.11}(a^s_{2\rho})^{+0.25}_{-0.26}(a^0_{2\rho})$	8.30 ± 1.20
	$\mathcal{A}_{CP}(\%)$	$-27.5^{+2.3}_{-3.1}(\omega_B)^{+0.9}_{-1.0}(a^t_{2\rho}) \pm 1.4(a^s_{2\rho}) \pm 0.9(a^0_{2\rho})$	$18.0^{+9.0}_{-17.0}$
$B^0 \to \pi^+(\rho^- \to)\pi^-\pi^0$	$\mathcal{B}(10^{-6})$	$7.85^{+2.60}_{-1.82}(\omega_B)^{+1.77}_{-1.58}(a^t_{2\rho})^{+0.94}_{-0.91}(a^s_{2\rho})^{+0.26}_{-0.25}(a^0_{2\rho})$	23.00 ± 2.30[a]
	$\mathcal{A}_{CP}(\%)$	$-31.4^{+3.4}_{-3.3}(\omega_B)^{+3.2}_{-4.0}(a^t_{2\rho})^{+1.1}_{-1.6}(a^s_{2\rho})^{+0.9}_{-0.7}(a^0_{2\rho})$	-8.0 ± 8.0
$B^0 \to \pi^-(\rho^+ \to)\pi^+\pi^0$	$\mathcal{B}(10^{-6})$	$18.78^{+6.92}_{-4.80}(\omega_B)^{+0.56}_{-0.55}(a^t_{2\rho})^{+0.20}_{-0.21}(a^s_{2\rho}) \pm 0.01(a^0_{2\rho})$	23.00 ± 2.30[a]
	$\mathcal{A}_{CP}(\%)$	$8.2^{+1.9}_{-1.5}(\omega_B) \pm 0.3(a^t_{2\rho})^{+0.2}_{-0.1}(a^s_{2\rho})^{+0.6}_{-0.5}(a^0_{2\rho})$	13.0 ± 6.0
$B^0_s \to \pi^+(\rho^- \to)\pi^-\pi^0$	$\mathcal{B}(10^{-6})$	$0.38 \pm 0.05(\omega_{B_s}) \pm 0.01(a^t_{2\rho})^{+0.00}_{-0.01}(a^s_{2\rho})^{+0.02}_{-0.03}(a^0_{2\rho})$	$\cdots$
	$\mathcal{A}_{CP}(\%)$	$-4.9^{+0.0}_{-1.7}(\omega_{B_s})^{+1.3}_{-4.4}(a^t_{2\rho})^{+0.0}_{-2.5}(a^s_{2\rho})^{+0.6}_{-1.5}(a^0_{2\rho})$	$\cdots$
$B^0_s \to \pi^-(\rho^+ \to)\pi^+\pi^0$	$\mathcal{B}(10^{-6})$	$0.41 \pm 0.05(\omega_{B_s})^{+0.00}_{-0.02}(a^t_{2\rho}) \pm 0.01(a^s_{2\rho})^{+0.02}_{-0.03}(a^0_{2\rho})$	$\cdots$
	$\mathcal{A}_{CP}(\%)$	$-36.7^{+0.0}_{-2.5}(\omega_{B_s})^{+2.8}_{-5.4}(a^t_{2\rho})^{+0.1}_{-0.3}(a^s_{2\rho})^{+0.0}_{-0.3}(a^0_{2\rho})$	$\cdots$
$B^+ \to \pi^0(\rho^+ \to)\pi^+\pi^0$	$\mathcal{B}(10^{-6})$	$5.53^{+2.65}_{-1.79}(\omega_B)^{+0.76}_{-0.71}(a^t_{2\rho})^{+0.49}_{-0.47}(a^s_{2\rho})^{+0.00}_{-0.02}(a^0_{2\rho})$	10.90 ± 1.40
	$\mathcal{A}_{CP}(\%)$	$34.9^{+7.3}_{-6.9}(\omega_B)^{+1.6}_{-2.1}(a^t_{2\rho})^{+1.6}_{-1.7}(a^s_{2\rho})^{+1.9}_{-1.8}(a^0_{2\rho})$	2.0 ± 11.0
$B^0 \to \pi^0(\rho^0 \to)\pi^+\pi^-$	$\mathcal{B}(10^{-6})$	$0.11^{+0.06}_{-0.03}(\omega_B)^{+0.02}_{-0.00}(a^t_{2\rho})^{+0.01}_{-0.00}(a^s_{2\rho})^{+0.01}_{-0.00}(a^0_{2\rho})$	2.00 ± 0.50
	$\mathcal{A}_{CP}(\%)$	$-14.2^{+17.1}_{-4.3}(\omega_B)^{+3.6}_{-0.7}(a^t_{2\rho})^{+11.3}_{-9.2}(a^s_{2\rho})^{+2.8}_{-0.0}(a^0_{2\rho})$	$\cdots$
$B^0_s \to \pi^0(\rho^0 \to)\pi^+\pi^-$	$\mathcal{B}(10^{-6})$	$0.35^{+0.06}_{-0.05}(\omega_{B_s}) \pm 0.01(a^t_{2\rho}) \pm 0.00(a^s_{2\rho}) \pm 0.03(a^0_{2\rho})$	$\cdots$
	$\mathcal{A}_{CP}(\%)$	$-24.6^{+2.8}_{-0.0}(\omega_{B_s})^{+1.9}_{-0.0}(a^t_{2\rho})^{+0.0}_{-1.6}(a^s_{2\rho})^{+0.0}_{-2.6}(a^0_{2\rho})$	$\cdots$

[a]Branching fraction for the decay $B^0 \to \rho^\pm\pi^\mp$ in [91].

$(13.0 \pm 6.0)\%$. We also obtain $\mathcal{A}_{CP}(B^0 \to \pi^+(\rho^- \to)\pi^-\pi^0) = (-31.4^{+4.9}_{-5.5})\%$ which needs to be tested precisely in the future experiments.

(iv) We calculated the branching ratios and CP violations of the quasi-two-body $B \to \eta^{(\prime)}(\rho \to)\pi\pi$ and find that $\mathcal{A}_{CP}(B^+ \to \eta(\rho^+ \to)\pi^+\pi^0) = (-0.3^{+0.4}_{-0.2})\%$ and $\mathcal{A}_{CP}(B^+ \to \eta'(\rho^+ \to)\pi^+\pi^0) = (21.0^{+2.4}_{-2.5})\%$ agree with the data. The contributions of the tree diagrams are larger than the penguin ones by roughly a factor of 200 for the decay $B^+ \to \eta(\rho^+ \to)\pi^+\pi^0$ and a factor of 40 for the $B^+ \to \eta'(\rho^+ \to)\pi^+\pi^0$. The tree contribution is therefore dominant for the decay

TABLE III. CP-averaged branching ratios and direct CP-violating asymmetries of $B_{(s)} \to \eta^{(\prime)}(\rho \to)\pi\pi$ decays calculated in PQCD approach together with experimental data [91].

Modes		Quasi-two-body results	Experiment
$B^+ \to \eta(\rho^+ \to)\pi^+\pi^0$	$\mathcal{B}(10^{-6})$	$6.74^{+2.04}_{-1.50}(\omega_B)^{+0.29}_{-0.27}(a^t_{2\rho})^{+0.10}_{-0.09}(a^s_{2\rho})^{+0.02}_{-0.01}(a^0_{2\rho})$	7.00 ± 2.90
	$\mathcal{A}_{CP}(\%)$	$-0.3^{+0.2}_{-0.0}(\omega_B)^{+0.3}_{-0.2}(a^t_{2\rho})^{+0.0}_{-0.1}(a^s_{2\rho}) \pm 0.0(a^0_{2\rho})$	11.0 ± 11.0
$B^0 \to \eta(\rho^0 \to)\pi^+\pi^-$	$\mathcal{B}(10^{-6})$	$0.17^{+0.03}_{-0.02}(\omega_B)^{+0.03}_{-0.02}(a^t_{2\rho})^{+0.01}_{-0.00}(a^s_{2\rho})^{+0.02}_{-0.00}(a^0_{2\rho})$	<1.5
	$\mathcal{A}_{CP}(\%)$	$16.3^{+3.3}_{-1.6}(\omega_B)^{+9.1}_{-7.2}(a^t_{2\rho})^{+0.0}_{-1.9}(a^s_{2\rho})^{+0.0}_{-1.8}(a^0_{2\rho})$	$\cdots$
$B^0_s \to \eta(\rho^0 \to)\pi^+\pi^-$	$\mathcal{B}(10^{-6})$	$0.10^{+0.04}_{-0.02}(\omega_{B_s}) \pm 0.00(a^t_{2\rho}) \pm 0.00(a^s_{2\rho}) \pm 0.00(a^0_{2\rho})$	$\cdots$
	$\mathcal{A}_{CP}(\%)$	$19.2^{+0.1}_{-0.2}(\omega_{B_s})^{+0.0}_{-0.4}(a^t_{2\rho})^{+0.2}_{-0.4}(a^s_{2\rho})^{+1.5}_{-1.7}(a^0_{2\rho})$	$\cdots$
$B^+ \to \eta'(\rho^+ \to)\pi^+\pi^0$	$\mathcal{B}(10^{-6})$	$4.56^{+1.44}_{-1.02}(\omega_B)^{+0.16}_{-0.13}(a^t_{2\rho})^{+0.04}_{-0.03}(a^s_{2\rho})^{+0.02}_{-0.01}(a^0_{2\rho})$	9.70 ± 2.20
	$\mathcal{A}_{CP}(\%)$	$21.0^{+1.7}_{-1.9}(\omega_B) \pm 1.6(a^t_{2\rho}) \pm 0.2(a^s_{2\rho})^{+0.3}_{-0.2}(a^0_{2\rho})$	26.0 ± 17.0
$B^0 \to \eta'(\rho^0 \to)\pi^+\pi^-$	$\mathcal{B}(10^{-6})$	$0.17^{+0.05}_{-0.04}(\omega_B)^{+0.01}_{-0.00}(a^t_{2\rho}) \pm 0.01(a^s_{2\rho}) \pm 0.01(a^0_{2\rho})$	<1.3
	$\mathcal{A}_{CP}(\%)$	$12.8^{+0.0}_{-1.2}(\omega_B)^{+21.1}_{-23.6}(a^t_{2\rho})^{+8.3}_{-7.3}(a^s_{2\rho})^{+0.1}_{-0.8}(a^0_{2\rho})$	$\cdots$
$B^0_s \to \eta'(\rho^0 \to)\pi^+\pi^-$	$\mathcal{B}(10^{-6})$	$0.23^{+0.08}_{-0.06}(\omega_{B_s})^{+0.00}_{-0.01}(a^t_{2\rho}) \pm 0.00(a^s_{2\rho})^{+0.00}_{-0.01}(a^0_{2\rho})$	$\cdots$
	$\mathcal{A}_{CP}(\%)$	$37.9^{+0.3}_{-0.5}(\omega_{B_s}) \pm 0.2(a^t_{2\rho}) \pm 0.3(a^s_{2\rho}) \pm 0.2(a^0_{2\rho})$	$\cdots$

$B^+ \to \eta(\rho^+ \to)\pi^+\pi^0$. Its direct CP asymmetry is really small in size. We also give predictions for $B^0 \to \eta(\rho^0 \to)\pi^+\pi^-$ and $B^0 \to \eta'(\rho^0 \to)\pi^+\pi^-$ decays.

(v) For all the $B_s \to K(\pi,\eta^{(\prime)})\rho \to K(\pi,\eta^{(\prime)})\pi\pi$ decay channels considered in this paper, we can compare our PQCD predictions with those as given in the Table VII and Table VIII of Refs. [82,94]. From the CP-averaged branching ratios, for example, our results for decays $B_s \to K(\pi,\eta^{(\prime)})\rho \to K(\pi,\eta^{(\prime)})\pi\pi$ are a little larger than the corresponding ones in Table VII of Ref. [82]. As verified in Ref. [50], it may be more appropriate to treat $B \to K(\pi,\eta^{(\prime)})\rho$ as the quasi-two-body decays. For $B_s^0 \to \pi^-(\rho^+ \to)\pi^+\pi^0$ and $B_s^0 \to \eta(\rho^0 \to)\pi^+\pi^-$ decays, we obtain sizeable negative CP asymmetries which could be examined in the forthcoming experiments. Our PQCD predictions for the direct CP asymmetries of $B_s^0 \to K^-(\rho^+ \to)\pi^+\pi^0$, $B_s^0 \to \bar{K}^0(\rho^0 \to)\pi^+\pi^-$ $B_s^0 \to \eta(\rho^0 \to)\pi^+\pi^-$ and $B_s^0 \to \eta'(\rho^0 \to)\pi^+\pi^-$ decays are positive and sizable.

(vi) For the $B^0 \to \pi^0\rho^0 \to \pi^0\pi^+\pi^-$ decay process, PQCD prediction is $\mathcal{B} = (0.11^{+0.07}_{-0.03}) \times 10^{-6}$ at leading order in the quasi-two-body framework in this work, such a branching ratio is much smaller than the value $(2.0 \pm 0.5) \times 10^{-6}$ in [91]. Similar with the $\pi\pi$, πK or $\rho\rho$ puzzles discussed in Refs. [84, 95–102], the $B \to \pi\rho$ puzzle has been noticed by some groups [103–109]. For example, in Ref. [105], the authors examined the role of $\sigma\pi$ channel in the Dalitz plot analysis of $\rho\pi$ decays and concluded that the effect of σ to $B^0 \to \rho^0\pi^0$ is not important. While, in [106], the authors found that $B^0 \to \rho^0\pi^0$ process could receive large contributions from the heavy-meson B^* and B_0 backgrounds. Since the isospin-violating effect is visible in the $e^+e^- \to \pi^+\pi^-$ data at $s = m_\omega^2$ [110], the ρ^0-ω mixing need to be taken into studies [105,111–115]. We leave the gap between the data in [91] and the PQCD prediction $\mathcal{B} = (0.11^{+0.07}_{-0.03}) \times 10^{-6}$ to the future studies.

For the considered $B/B_s \to P(\rho \to)\pi\pi$ decays, we know that the introduction of the resonance width Γ_ρ is one of the crucial differences between the two-body formalism and the quasi-two-body one and may play an important role in our theoretical predictions for the CP-averaged branching ratios and the CP-violating asymmetries. In order to check the Γ_ρ dependence of these physical observables, we vary Γ_ρ in Eqs. (26)–(27) in the range of $0 \le \Gamma_\rho \le 0.149$ GeV and list our PQCD predictions in Table IV. For the sake of simplicity, we take the experimentally measured decay mode $B^+ \to K^+(\rho^0 \to)\pi^+\pi^-$ as an example, and make numerical calculations for the seven fixed values of Γ_ρ. From the numerical results in Table IV, we find easily that

(i) Our PQCD predictions for the branching ratios are very sensitive on the variations of the given value of the resonance width Γ_ρ. For $\Gamma_\rho = \Gamma_\rho^{\rm exp} = 0.149$ GeV, the PQCD prediction $\mathcal{B}(B^+ \to K^+(\rho^0 \to)\pi^+\pi^-) \approx 4.0 \times 10^{-6}$ agrees well with the measured value $(3.7 \pm 0.5) \times 10^{-6}$ [91].

(ii) For CP asymmetries $\mathcal{A}_{CP}$, the Γ_ρ dependence is indeed negligible.

TABLE IV. For the measured decay mode $B^+ \to K^+(\rho^0 \to)\pi^+\pi^-$, the Γ_ρ dependence of the PQCD predictions for the branching ratios and the direct CP-violating asymmetries, assuming $0 \le \Gamma\rho \le 0.149$ GeV.

Γ_ρ(GeV)	0	0.005	0.015	0.060	0.090	0.120	0.149
$\mathcal{B}(10^{-6})$	5370.2	105.5	35.4	9.2	6.3	4.9	4.0
$\mathcal{A}_{CP}(\%)$	50.9	53.3	52.8	51.8	51.2	50.8	50.7

IV. CONCLUSION

In this paper, we calculated the CP-averaged branching ratios and direct CP-violating asymmetries of the quasi-two-body decays $B_{(s)} \to (\pi,K,\eta,\eta')\rho \to (\pi,K,\eta,\eta')\pi\pi$ by using the PQCD factorization approach. The two-pion distribution amplitude $\Phi_{\pi\pi}^{\rm P}$ with the P-wave timelike form factor F_π was employed to describe the resonant state ρ and its interactions with the pion pair. General agreements between the PQCD predictions and the data achieved by making a little adjustments of the Gegenbauer moments of the P-wave two-pion distribution amplitudes. We listed the PQCD predictions for those considered decay channels, which will be tested at the LHCb and Belle-II experiment.

From the numerical results, we found the following points:

(i) Except for the $B \to \pi^0\rho^0 \to \pi^0(\pi^+\pi^-)$ decay mode, the PQCD predictions for the branching ratios of other $B_{(s)} \to (\pi,K,\eta,\eta')\rho \to (\pi,K,\eta,\eta')\pi\pi$ decays agree with currently available data within errors.

(ii) For $\mathcal{B}(B \to \pi^0\rho^0 \to \pi^0(\pi^+\pi^-))$ decay, the PQCD prediction is about $(0.11^{+0.07}_{-0.03}) \times 10^{-6}$ and is much smaller than the measured one: $(2.0 \pm 0.5) \times 10^{-6}$.

(iii) For $B^+ \to \pi^+(\rho^0 \to)\pi^+\pi^-$ decay mode, we found a negative CP asymmetry $(-27.5^{+3.0}_{-3.7})\%$, which agrees with theoretical predictions based on QCDF or other factorization approaches, but different in sign from the measured ones in the $m(\pi^+\pi^-)$ region peaked at m_ρ, as reported by *BABAR* and LHCb Collaboration. Such difference should be tested in the forthcoming experimental measurements.

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APPENDIX: DECAY AMPLITUDES

The total decay amplitude for each considered decay mode in this work are given as follows:

$$\begin{aligned}\mathcal{A}(B^+ \to K^+(\rho^0 \to)\pi^+\pi^-) = \frac{G_F}{2}\Big\{ & V_{ub}^* V_{us}\Big[\Big(\frac{C_1}{3}+C_2\Big)(F_{e\rho}^{LL}+F_{a\rho}^{LL}) + \Big(C_1+\frac{C_2}{3}\Big)F_{eP}^{LL} + C_2 M_{eP}^{LL} \\ & + C_1(M_{e\rho}^{LL}+M_{a\rho}^{LL})\Big] - V_{tb}^* V_{ts}\Big[\Big(\frac{C_3}{3}+C_4+\frac{C_9}{3}+C_{10}\Big)(F_{e\rho}^{LL}+F_{a\rho}^{LL}) \\ & + \Big(\frac{C_5}{3}+C_6+\frac{C_7}{3}+C_8\Big)(F_{e\rho}^{SP}+F_{a\rho}^{SP}) + (C_3+C_9)(M_{e\rho}^{LL}+M_{a\rho}^{LL}) \\ & + (C_5+C_7)(M_{e\rho}^{LR}+M_{a\rho}^{LR}) + \frac{3C_8}{2}M_{eP}^{SP} + \frac{3C_{10}}{2}M_{eP}^{LL} \\ & + \frac{3}{2}\Big(C_7+\frac{C_8}{3}+C_9+\frac{C_{10}}{3}\Big)F_{eP}^{LL}\Big]\Big\}, \end{aligned} \tag{A1}$$

$$\begin{aligned}\mathcal{A}(B^0 \to K^+(\rho^- \to)\pi^-\pi^0) = \frac{G_F}{\sqrt{2}}\Big\{ & V_{ub}^* V_{us}\Big[\Big(\frac{C_1}{3}+C_2\Big)F_{e\rho}^{LL} + C_1 M_{e\rho}^{LL}\Big] - V_{tb}^* V_{ts}\Big[(C_3+C_9)M_{e\rho}^{LL} \\ & + \Big(\frac{C_3}{3}+C_4+\frac{C_9}{3}+C_{10}\Big)F_{e\rho}^{LL} + \Big(\frac{C_5}{3}+C_6+\frac{C_7}{3}+C_8\Big)F_{e\rho}^{SP} \\ & + (C_5+C_7)M_{e\rho}^{LR} + \Big(\frac{C_3}{3}+C_4-\frac{1}{2}\Big(\frac{C_9}{3}+C_{10}\Big)\Big)F_{a\rho}^{LL} + \Big(C_3-\frac{C_9}{2}\Big)M_{a\rho}^{LL} \\ & + \Big(\frac{C_5}{3}+C_6-\frac{1}{2}\Big(\frac{C_7}{3}+C_8\Big)\Big)F_{a\rho}^{SP} + \Big(C_5-\frac{C_7}{2}\Big)M_{a\rho}^{LR}\Big]\Big\}, \end{aligned} \tag{A2}$$

$$\begin{aligned}\mathcal{A}(B_s^0 \to K^-(\rho^+ \to)\pi^+\pi^0) = \frac{G_F}{\sqrt{2}}\Big\{ & V_{ub}^* V_{ud}\Big[\Big(\frac{C_1}{3}+C_2\Big)F_{eP}^{LL} + C_1 M_{eP}^{LL}\Big] - V_{tb}^* V_{td}\Big[(C_3+C_9)M_{eP}^{LL} \\ & + \Big(\frac{C_3}{3}+C_4+\frac{C_9}{3}+C_{10}\Big)F_{eP}^{LL} + (C_5+C_7)M_{eP}^{LR} \\ & + \Big(\frac{C_3}{3}+C_4-\frac{1}{2}\Big(\frac{C_9}{3}+C_{10}\Big)\Big)F_{aP}^{LL} + \Big(\frac{C_5}{3}+C_6-\frac{1}{2}\Big(\frac{C_7}{3}+C_8\Big)\Big)F_{aP}^{SP} \\ & + \Big(C_3-\frac{C_9}{2}\Big)M_{aP}^{LL} + \Big(C_5-\frac{C_7}{2}\Big)M_{aP}^{LR}\Big]\Big\}, \end{aligned} \tag{A3}$$

$$\begin{aligned}\mathcal{A}(B^+ \to K^0(\rho^+ \to)\pi^+\pi^0) = \frac{G_F}{\sqrt{2}}\Big\{ & V_{ub}^* V_{us}\Big[\Big(\frac{C_1}{3}+C_2\Big)F_{a\rho}^{LL} + C_1 M_{a\rho}^{LL}\Big] - V_{tb}^* V_{ts}\Big[\Big(C_3-\frac{C_9}{2}\Big)M_{e\rho}^{LL} \\ & + \Big(\frac{C_3}{3}+C_4-\frac{1}{2}\Big(\frac{C_9}{3}+C_{10}\Big)\Big)F_{e\rho}^{LL} + \Big(\frac{C_5}{3}+C_6-\frac{1}{2}\Big(\frac{C_7}{3}+C_8\Big)\Big)F_{e\rho}^{SP} \\ & + \Big(C_5-\frac{C_7}{2}\Big)M_{e\rho}^{LR} + \Big(\frac{C_3}{3}+C_4+\frac{C_9}{3}+C_{10}\Big)F_{a\rho}^{LL} + (C_3+C_9)M_{a\rho}^{LL} \\ & + \Big(\frac{C_5}{3}+C_6+\frac{C_7}{3}+C_8\Big)F_{a\rho}^{SP} + (C_5+C_7)M_{a\rho}^{LR}\Big]\Big\}, \end{aligned} \tag{A4}$$

$$\begin{aligned}\mathcal{A}(B^0\to K^0(\rho^0\to)\pi^+\pi^-) = \frac{G_F}{2}\Bigg\{ V_{ub}^*V_{us}\Bigg[\left(C_1+\frac{C_2}{3}\right)F_{eP}^{LL}+C_2M_{eP}^{LL}\Bigg] - V_{tb}^*V_{ts}\Bigg[\frac{3C_8}{2}M_{eP}^{SP} \\ -\left(\frac{C_3}{3}+C_4-\frac{1}{2}\left(\frac{C_9}{3}+C_{10}\right)\right)(F_{e\rho}^{LL}+F_{a\rho}^{LL}) - \left(C_3-\frac{C_9}{2}\right)(M_{e\rho}^{LL}+M_{a\rho}^{LL}) \\ -\left(\frac{C_5}{3}+C_6-\frac{1}{2}\left(\frac{C_7}{3}+C_8\right)\right)(F_{e\rho}^{SP}+F_{a\rho}^{SP}) - \left(C_5-\frac{C_7}{2}\right)(M_{e\rho}^{LR}+M_{a\rho}^{LR}) \\ +\frac{3}{2}\left(C_7+\frac{C_8}{3}+C_9+\frac{C_{10}}{3}\right)F_{eP}^{LL}+\frac{3C_{10}}{2}M_{eP}^{LL}\Bigg]\Bigg\},\end{aligned} \tag{A5}$$

$$\begin{aligned}\mathcal{A}(B_s^0\to K^0(\rho^0\to)\pi^+\pi^-) = \frac{G_F}{2}\Bigg\{ V_{ub}^*V_{ud}\Bigg[\left(C_1+\frac{C_2}{3}\right)F_{eP}^{LL}+C_2M_{eP}^{LL}\Bigg] - V_{tb}^*V_{td}\Bigg[\frac{3C_8}{2}M_{eP}^{SP} \\ +\left(-\frac{C_3}{3}-C_4+\frac{5C_9}{3}+C_{10}+\frac{3}{2}\left(C_7+\frac{C_8}{3}\right)\right)F_{eP}^{LL} + \left(-C_3+\frac{C_9}{2}+\frac{3C_{10}}{2}\right)M_{eP}^{LL} \\ -\left(C_5-\frac{C_7}{2}\right)(M_{eP}^{LR}+M_{aP}^{LR}) - \left(\frac{C_3}{3}+C_4-\frac{1}{2}\left(\frac{C_9}{3}+C_{10}\right)\right)F_{aP}^{LL}\Bigg) - \left(C_3-\frac{C_9}{2}\right)M_{aP}^{LL} \\ -\left(\frac{C_5}{3}+C_6-\frac{1}{2}\left(\frac{C_7}{3}+C_8\right)\right)F_{aP}^{SP}\Bigg]\Bigg\},\end{aligned} \tag{A6}$$

$$\begin{aligned}\mathcal{A}(B^+\to \pi^+(\rho^0\to)\pi^+\pi^-) = \frac{G_F}{2}\Bigg\{ V_{ub}^*V_{ud}\Bigg[\left(\frac{C_1}{3}+C_2\right)(F_{e\rho}^{LL}+F_{a\rho}^{LL}-F_{aP}^{LL}) + \left(C_1+\frac{C_2}{3}\right)F_{eP}^{LL} \\ + C_1(M_{e\rho}^{LL}+M_{a\rho}^{LL}-M_{aP}^{LL})+C_2M_{eP}^{LL}\Bigg] - V_{tb}^*V_{td}\Bigg[\frac{3C_8}{2}M_{eP}^{SP} \\ +\left(\frac{C_3}{3}+C_4+\frac{C_9}{3}+C_{10}\right)(F_{e\rho}^{LL}+F_{a\rho}^{LL}-F_{aP}^{LL}) + (C_3+C_9)(M_{e\rho}^{LL}+M_{a\rho}^{LL}-M_{aP}^{LL}) \\ +\left(-C_5+\frac{C_7}{2}\right)M_{eP}^{LR} + \left(\frac{C_5}{3}+C_6+\frac{C_7}{3}+C_8\right)(F_{e\rho}^{SP}+F_{a\rho}^{SP}-F_{aP}^{SP}) \\ +(C_5+C_7)(M_{e\rho}^{LR}+M_{a\rho}^{LR}-M_{aP}^{LR}) + \Bigg(-\frac{C_3}{3}-C_4+\frac{5}{3}C_9 \\ +C_{10}+\frac{3}{2}\left(C_7+\frac{C_8}{3}\right)\Bigg)F_{eP}^{LL} + \left(-C_3+\frac{C_9}{2}+\frac{3C_{10}}{2}\right)M_{eP}^{LL}\Bigg]\Bigg\},\end{aligned} \tag{A7}$$

$$\begin{aligned}\mathcal{A}(B^0\to \pi^-(\rho^+\to)\pi^+\pi^0) = \frac{G_F}{\sqrt{2}}\Bigg\{ V_{ub}^*V_{ud}\Bigg[\left(C_1+\frac{C_2}{3}\right)F_{a\rho}^{LL} + \left(\frac{C_1}{3}+C_2\right)F_{eP}^{LL}+C_2M_{a\rho}^{LL}+C_1M_{eP}^{LL}\Bigg] \\ - V_{tb}^*V_{td}\Bigg[\left(\frac{C_3}{3}+C_4+\frac{C_9}{3}+C_{10}\right)F_{eP}^{LL}+(C_4+C_{10})M_{a\rho}^{LL} \\ +\left(C_3+\frac{C_4}{3}-C_5-\frac{C_6}{3}-C_7-\frac{C_8}{3}+C_9+\frac{C_{10}}{3}\right)F_{a\rho}^{LL} \\ +(C_3+C_9)M_{eP}^{LL}+(C_5+C_7)M_{eP}^{LR}+\left(C_5-\frac{C_7}{2}\right)M_{aP}^{LR} \\ +\left(\frac{4}{3}\left(C_3+C_4-\frac{C_9}{2}-\frac{C_{10}}{2}\right)-C_5-\frac{C_6}{3}+\frac{1}{2}\left(C_7+\frac{C_8}{3}\right)\right)F_{aP}^{LL} \\ +\left(\frac{C_5}{3}+C_6-\frac{1}{2}\left(\frac{C_7}{3}+C_8\right)\right)F_{aP}^{SP}+\left(C_6-\frac{C_8}{2}\right)M_{aP}^{SP} \\ +\left(C_3+C_4-\frac{C_9}{2}-\frac{C_{10}}{2}\right)M_{aP}^{LL}+(C_6+C_8)M_{a\rho}^{SP}\Bigg]\Bigg\},\end{aligned} \tag{A8}$$

$$\mathcal{A}(B^0\to\pi^+(\rho^-\to)\pi^-\pi^0)=\frac{G_F}{\sqrt{2}}\Big\{V_{ub}^*V_{ud}\Big[\Big(\frac{C_1}{3}+C_2\Big)F_{e\rho}^{LL}+\Big(C_1+\frac{C_2}{3}\Big)F_{aP}^{LL}+C_1M_{e\rho}^{LL}+C_2M_{aP}^{LL}\Big]$$
$$-V_{tb}^*V_{td}\Big[\Big(\frac{C_3}{3}+C_4+\frac{C_9}{3}+C_{10}\Big)F_{e\rho}^{LL}+(C_3+C_9)M_{e\rho}^{LL}+\Big(\frac{C_5}{3}+C_6+\frac{C_7}{3}+C_8\Big)F_{e\rho}^{SP}$$
$$+(C_5+C_7)M_{e\rho}^{LR}+(C_6+C_8)M_{aP}^{SP}+\Big(\frac{4}{3}\Big(C_3+C_4-\frac{C_9}{2}-\frac{C_{10}}{2}\Big)$$
$$-C_5-\frac{C_6}{3}+\frac{1}{2}\Big(C_7+\frac{C_8}{3}\Big)\Big)F_{a\rho}^{LL}+\Big(\frac{C_5}{3}+C_6-\frac{1}{2}\Big(\frac{C_7}{3}+C_8\Big)\Big)F_{a\rho}^{SP}+\Big(C_3+C_4-\frac{C_9}{2}-\frac{C_{10}}{2}\Big)M_{a\rho}^{LL}$$
$$+\Big(C_5-\frac{C_7}{2}\Big)M_{a\rho}^{LR}+\Big(C_6-\frac{C_8}{2}\Big)M_{a\rho}^{SP}+(C_4+C_{10})M_{aP}^{LL}$$
$$+\Big(C_3+\frac{C_4}{3}-C_5-\frac{C_6}{3}-C_7-\frac{C_8}{3}+C_9+\frac{C_{10}}{3}\Big)F_{aP}^{LL}\Big]\Big\},\tag{A9}$$

$$\mathcal{A}(B_s^0\to\pi^-(\rho^+\to)\pi^+\pi^0)=\frac{G_F}{\sqrt{2}}\Big\{V_{ub}^*V_{us}\Big[\Big(C_1+\frac{C_2}{3}\Big)F_{a\rho}^{LL}+C_2M_{a\rho}^{LL}\Big]-V_{tb}^*V_{ts}\Big[(C_6+C_8)M_{a\rho}^{SP}$$
$$+\Big(C_3+\frac{C_4}{3}-C_5-\frac{C_6}{3}-C_7-\frac{C_8}{3}+C_9+\frac{C_{10}}{3}\Big)F_{a\rho}^{LL}+\Big(C_3+\frac{C_4}{3}-\frac{1}{2}\Big(C_9+\frac{C_{10}}{3}\Big)$$
$$-C_5-\frac{C_6}{3}+\frac{1}{2}\Big(C_7+\frac{C_8}{3}\Big)\Big)F_{aP}^{LL}+\Big(C_4-\frac{C_{10}}{2}\Big)M_{aP}^{LL}+\Big(C_6-\frac{C_8}{2}\Big)M_{aP}^{SP}+(C_4+C_{10})M_{a\rho}^{LL}\Big]\Big\},\tag{A10}$$

$$\mathcal{A}(B_s^0\to\pi^+(\rho^-\to)\pi^-\pi^0)=\frac{G_F}{\sqrt{2}}\Big\{V_{ub}^*V_{us}\Big[\Big(C_1+\frac{C_2}{3}\Big)F_{aP}^{LL}+C_2M_{aP}^{LL}\Big]-V_{tb}^*V_{ts}\Big[\Big(C_4-\frac{C_{10}}{2}\Big)M_{a\rho}^{LL}+\Big(C_3+\frac{C_4}{3}-\frac{1}{2}\Big(C_9+\frac{C_{10}}{3}\Big)$$
$$-C_5-\frac{C_6}{3}+\frac{1}{2}\Big(C_7+\frac{C_8}{3}\Big)\Big)F_{a\rho}^{LL}+\Big(C_6-\frac{C_8}{2}\Big)M_{a\rho}^{SP}+(C_4+C_{10})M_{aP}^{LL}+(C_6+C_8)M_{aP}^{SP}$$
$$+\Big(C_3+\frac{C_4}{3}-C_5-\frac{C_6}{3}-C_7-\frac{C_8}{3}+C_9+\frac{C_{10}}{3}\Big)F_{aP}^{LL}\Big]\Big\},\tag{A11}$$

$$\mathcal{A}(B^+\to\pi^0(\rho^+\to)\pi^+\pi^0)=\frac{G_F}{2}\Big\{V_{ub}^*V_{ud}\Big[\Big(C_1+\frac{C_2}{3}\Big)F_{e\rho}^{LL}+\Big(\frac{C_1}{3}+C_2\Big)(-F_{a\rho}^{LL}+F_{eP}^{LL}+F_{aP}^{LL})$$
$$+C_2M_{e\rho}^{LL}+C_1(-M_{a\rho}^{LL}+M_{eP}^{LL}+M_{aP}^{LL})\Big]-V_{tb}^*V_{td}\Big[\frac{3C_8}{2}M_{e\rho}^{SP}$$
$$+\Big(-\frac{C_3}{3}-C_4-\frac{3}{2}\Big(C_7+\frac{C_8}{3}\Big)+\frac{5C_9}{3}+C_{10}\Big)F_{e\rho}^{LL}$$
$$+\Big(-\frac{C_5}{3}-C_6+\frac{1}{2}\Big(\frac{C_7}{3}+C_8\Big)\Big)F_{e\rho}^{SP}+\Big(-C_3+\frac{C_9}{2}+\frac{3C_{10}}{2}\Big)M_{e\rho}^{LL}$$
$$+\Big(\frac{C_3}{3}+C_4+\frac{C_9}{3}+C_{10}\Big)(-F_{a\rho}^{LL}+F_{eP}^{LL}+F_{aP}^{LL})$$
$$+\Big(\frac{C_5}{3}+C_6+\frac{C_7}{3}+C_8\Big)(-F_{a\rho}^{SP}+F_{aP}^{SP})+\Big(-C_5+\frac{C_7}{2}\Big)M_{e\rho}^{LR}$$
$$+(C_3+C_9)(-M_{a\rho}^{LL}+M_{eP}^{LL}+M_{aP}^{LL})$$
$$+(C_5+C_7)(-M_{a\rho}^{LR}+M_{eP}^{LR}+M_{aP}^{LR})\Big]\Big\},\tag{A12}$$

$$\begin{aligned}\mathcal{A}(B^0\to\pi^0(\rho^0\to)\pi^+\pi^-) = & -\frac{G_F}{2\sqrt{2}}\Big\{V_{ub}^*V_{ud}\Big[\Big(C_1+\frac{C_2}{3}\Big)(F_{e\rho}^{LL}-F_{a\rho}^{LL}+F_{eP}^{LL}-F_{aP}^{LL})\\ & +C_2(M_{e\rho}^{LL}-M_{a\rho}^{LL}+M_{eP}^{LL}-M_{aP}^{LL})\Big]-V_{tb}^*V_{td}\Big[\frac{3C_8}{2}(M_{e\rho}^{SP}+M_{eP}^{SP})\\ & +\Big(-\frac{C_3}{3}-C_4-\frac{3}{2}\Big(C_7+\frac{C_8}{3}\Big)+\frac{5C_9}{3}+C_{10}\Big)(F_{e\rho}^{LL}+F_{eP}^{LL})\\ & +\Big(-\frac{C_5}{3}-C_6+\frac{1}{2}\Big(\frac{C_7}{3}+C_8\Big)\Big)F_{e\rho}^{SP}+\Big(-C_3+\frac{C_9}{2}+\frac{3C_{10}}{2}\Big)(M_{e\rho}^{LL}+M_{eP}^{LL})\\ & +\Big(-C_5+\frac{C_7}{2}\Big)(M_{e\rho}^{LR}+M_{eP}^{LL})-\Big(2C_6+\frac{C_8}{2}\Big)(M_{a\rho}^{SP}+M_{sP}^{SP})\\ & -\Big(\frac{7C_3}{3}+\frac{5C_4}{3}-2\Big(C_5+\frac{C_6}{3}\Big)-\frac{1}{2}\Big(C_7+\frac{C_8}{3}-\frac{2}{3}(C_9-C_{10})\Big)\Big)(F_{a\rho}^{LL}+F_{aP}^{LL})\\ & -\Big(\frac{C_5}{3}+C_6-\frac{1}{2}\Big(\frac{C_7}{3}+C_8\Big)\Big)(F_{a\rho}^{SP}+F_{aP}^{SP})-\Big(C_5-\frac{C_7}{2}\Big)(M_{a\rho}^{LR}+M_{aP}^{LR})\\ & -\Big(C_3+2C_4-\frac{C_9}{2}+\frac{C_{10}}{2}\Big)(M_{a\rho}^{LL}+M_{aP}^{LL})\Big]\Big\},\end{aligned} \tag{A13}$$

$$\begin{aligned}\mathcal{A}(B_s^0\to\pi^0(\rho^0\to)\pi^+\pi^-) = & \frac{G_F}{2\sqrt{2}}\Big\{V_{ub}^*V_{us}\Big[\Big(C_1+\frac{C_2}{3}\Big)(F_{a\rho}^{LL}+F_{aP}^{LL})+C_2(M_{a\rho}^{LL}+M_{aP}^{LL})\Big]\\ & -V_{tb}^*V_{ts}\Big[\Big(2\Big(C_3+\frac{C_4}{3}-C_5-\frac{C_6}{3}\Big)-\frac{1}{2}\Big(C_7+\frac{C_8}{3}-C_9-\frac{C_{10}}{3}\Big)\Big)(F_{a\rho}^{LL}+F_{aP}^{LL})\\ & +\Big(2C_4+\frac{C_{10}}{2}\Big)(M_{a\rho}^{LL}+M_{aP}^{LL})+\Big(2C_6+\frac{C_8}{2}\Big)(M_{a\rho}^{SP}+M_{aP}^{SP})\Big]\Big\},\end{aligned} \tag{A14}$$

$$\begin{aligned}\mathcal{A}(B^+\to\eta_q(\rho^+\to)\pi^+\pi^0) = & \frac{G_F}{2}\Big\{V_{ub}^*V_{ud}\Big[\Big(C_1+\frac{C_2}{3}\Big)F_{e\rho}^{LL}+\Big(\frac{C_1}{3}+C_2\Big)(F_{a\rho}^{LL}+F_{eP}^{LL}+F_{aP}^{LL})\\ & +C_2M_{e\rho}^{LL}+C_1(M_{a\rho}^{LL}+M_{eP}^{LL}+M_{aP}^{LL})\Big]-V_{tb}^*V_{td}\Big[\Big(C_5-\frac{C_7}{2}\Big)M_{e\rho}^{LR}+\Big(\frac{7C_3}{3}+\frac{5C_4}{3}-2\Big(C_5+\frac{C_6}{3}\Big)\\ & -\frac{1}{2}\Big(C_7+\frac{C_8}{3}-\frac{2}{3}(C_9-C_{10})\Big)\Big)F_{e\rho}^{LL}+\Big(\frac{C_5}{3}+C_6-\frac{1}{2}\Big(\frac{C_7}{3}+C_8\Big)\Big)F_{e\rho}^{SP}\\ & +\Big(C_3+2C_4-\frac{C_9}{2}+\frac{C_{10}}{2}\Big)M_{e\rho}^{LL}+\Big(\frac{C_3}{3}+C_4+\frac{C_9}{3}+C_{10}\Big)(F_{a\rho}^{LL}+F_{eP}^{LL}+F_{aP}^{LL})\\ & +\Big(\frac{C_5}{3}+C_6+\frac{C_7}{3}+C_8\Big)(F_{a\rho}^{SP}+F_{aP}^{SP})+(C_3+C_9)(M_{a\rho}^{LL}+M_{eP}^{LL}+M_{aP}^{LL})\\ & +(C_5+C_7)(M_{a\rho}^{LR}+M_{eP}^{LR}+M_{aP}^{LR})+\Big(2C_6+\frac{C_8}{2}\Big)M_{e\rho}^{SP}\Big]\Big\},\end{aligned} \tag{A15}$$

$$\begin{aligned}\mathcal{A}(B^+\to\eta_s(\rho^+\to)\pi^+\pi^0) = & \frac{G_F}{\sqrt{2}}\Big\{-V_{tb}^*V_{td}\Big[\Big(C_3+\frac{C_4}{3}-C_5-\frac{C_6}{3}+\frac{1}{2}\Big(C_7+\frac{C_8}{3}-C_9-\frac{C_{10}}{3}\Big)\Big)F_{e\rho}^{LL}\\ & +\Big(C_4-\frac{C_{10}}{2}\Big)M_{e\rho}^{LL}+\Big(C_6-\frac{C_8}{2}\Big)M_{e\rho}^{SP}\Big]\Big\},\end{aligned} \tag{A16}$$

$$\mathcal{A}(B^+\to\eta(\rho^+\to)\pi^+\pi^0)=\mathcal{A}(B^+\to\rho^+\eta_q)\cos\phi-\mathcal{A}(B^+\to\rho^+\eta_s)\sin\phi, \tag{A17}$$

$$\mathcal{A}(B^+\to\eta'(\rho^+\to)\pi^+\pi^0)=\mathcal{A}(B^+\to\rho^+\eta_q)\sin\phi+\mathcal{A}(B^+\to\rho^+\eta_s)\cos\phi, \tag{A18}$$

$$\mathcal{A}(B^0 \to \eta_q(\rho^0 \to)\pi^+\pi^-) = -\frac{G_F}{2\sqrt{2}}\left\{V_{ub}^* V_{ud}\left[\left(C_1 + \frac{C_2}{3}\right)(F_{e\rho}^{LL} - F_{a\rho}^{LL} - F_{eP}^{LL} - F_{aP}^{LL})\right.\right.$$
$$+ C_2(M_{e\rho}^{LL} - M_{a\rho}^{LL} - M_{eP}^{LL} - M_{aP}^{LL})\Big] - V_{tb}^* V_{td}\left[\left(C_5 - \frac{C_7}{2}\right)M_{e\rho}^{LR}\right.$$
$$+ \left(\frac{7C_3}{3} + \frac{5C_4}{3} - 2\left(C_5 + \frac{C_6}{3}\right) - \frac{1}{2}\left(C_7 + \frac{C_8}{3} - \frac{2}{3}(C_9 - C_{10})\right)\right)F_{e\rho}^{LL}$$
$$+ \left(\frac{C_5}{3} + C_6 - \frac{1}{2}\left(\frac{C_7}{3} + C_8\right)\right)F_{e\rho}^{SP} + \left(C_3 + 2C_4 - \frac{C_9}{2} + \frac{C_{10}}{2}\right)M_{e\rho}^{LL}$$
$$- \left(-\frac{C_3}{3} - C_4 - \frac{3}{2}\left(C_7 + \frac{C_8}{3}\right) + \frac{5C_9}{3} + C_{10}\right)(F_{a\rho}^{LL} + F_{aP}^{LL})$$
$$- \left(-\frac{C_5}{3} - C_6 + \frac{1}{2}\left(\frac{C_7}{3} + C_8\right)\right)(F_{a\rho}^{SP} + F_{aP}^{SP}) + \left(2C_6 + \frac{C_8}{2}\right)M_{e\rho}^{SP}$$
$$- \left(-C_3 + \frac{C_9}{2} + \frac{3C_{10}}{2}\right)(M_{a\rho}^{LL} + M_{eP}^{LL} + M_{aP}^{LL})$$
$$- \left(-C_5 + \frac{C_7}{2}\right)(M_{a\rho}^{LR} + M_{eP}^{LR} + M_{aP}^{LR}) - \frac{3C_8}{2}(M_{a\rho}^{SP} + M_{eP}^{SP} + M_{aP}^{SP})$$
$$\left.\left.- \left(-\frac{C_3}{3} - C_4 + \frac{3}{2}\left(C_7 + \frac{C_8}{3}\right) + \frac{5C_9}{3} + C_{10}\right)F_{eP}^{LL}\right]\right\}, \tag{A19}$$

$$\mathcal{A}(B^0 \to \eta_s(\rho^0 \to)\pi^+\pi^-) = -\frac{G_F}{2}\left\{-V_{tb}^* V_{td}\left[\left(C_3 + \frac{C_4}{3} - C_5 - \frac{C_6}{3} + \frac{1}{2}\left(C_7 + \frac{C_8}{3} - C_9 - \frac{C_{10}}{3}\right)\right)F_{e\rho}^{LL}\right.\right.$$
$$\left.\left.+ \left(C_4 - \frac{C_{10}}{2}\right)M_{e\rho}^{LL} + \left(C_6 - \frac{C_8}{2}\right)M_{e\rho}^{SP}\right]\right\}, \tag{A20}$$

$$\mathcal{A}(B^0 \to \eta(\rho^0 \to)\pi^+\pi^-) = \mathcal{A}(B^0 \to \rho^0\eta_q)\cos\phi - \mathcal{A}(B^0 \to \rho^0\eta_s)\sin\phi, \tag{A21}$$

$$\mathcal{A}(B^0 \to \eta'(\rho^0 \to)\pi^+\pi^-) = \mathcal{A}(B^0 \to \rho^0\eta_q)\sin\phi + \mathcal{A}(B^0 \to \rho^0\eta_s)\cos\phi, \tag{A22}$$

$$\mathcal{A}(B_s^0 \to \eta_q(\rho^0 \to)\pi^+\pi^-) = \frac{G_F}{2\sqrt{2}}\left\{V_{ub}^* V_{us}\left[\left(C_1 + \frac{C_2}{3}\right)(F_{a\rho}^{LL} + F_{aP}^{LL}) + C_2(M_{a\rho}^{LL} + M_{aP}^{LL})\right]\right.$$
$$- V_{tb}^* V_{ts}\left[-\frac{3}{2}\left(C_7 + \frac{C_8}{3} - C_9 - \frac{C_{10}}{3}\right)(F_{a\rho}^{LL} + F_{aP}^{LL})\right.$$
$$\left.\left.+ \frac{3C_8}{2}(M_{a\rho}^{SP} + M_{aP}^{SP}) + \frac{3C_{10}}{2}(M_{a\rho}^{LL} + M_{aP}^{LL})\right]\right\} \tag{A23}$$

$$\mathcal{A}(B_s^0 \to \eta_s(\rho^0 \to)\pi^+\pi^-) = \frac{G_F}{2}\left\{V_{ub}^* V_{us}\left[\left(C_1 + \frac{C_2}{3}\right)F_{eP}^{LL} + C_2 M_{eP}^{LL}\right] - V_{tb}^* V_{ts}\left[\frac{3C_8}{2}M_{eP}^{SP}\right.\right.$$
$$\left.\left.+ \frac{3}{2}\left(C_7 + \frac{C_8}{3} + C_9 + \frac{C_{10}}{3}\right)F_{eP}^{LL} + \frac{3C_{10}}{2}M_{eP}^{LL}\right]\right\}, \tag{A24}$$

$$\mathcal{A}(B_s^0 \to \eta(\rho^0 \to)\pi^+\pi^-) = \mathcal{A}(B_s^0 \to \rho^0\eta_q)\cos\phi - \mathcal{A}(B_s^0 \to \rho^0\eta_s)\sin\phi, \tag{A25}$$

$$\mathcal{A}(B_s^0 \to \eta'(\rho^0 \to)\pi^+\pi^-) = \mathcal{A}(B_s^0 \to \rho^0\eta_q)\sin\phi + \mathcal{A}(B_s^0 \to \rho^0\eta_s)\cos\phi, \tag{A26}$$

where G_F is the Fermi coupling constant and V_{ij}'s are the Cabibbo-Kobayashi-Maskawa matrix elements. The functions $(F_{e\rho}^{LL}, F_{a\rho}^{LL}, M_{e\rho}^{LL}, M_{a\rho}^{LL}, \cdots)$ appearing in the above equations are the individual decay amplitudes corresponding to different currents, and their explicit expressions can be found in the Appendix of Ref. [50].

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