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Effects of the $\mathrm{U_Y(1)}$ Chern-Simons term and its baryonic contribution on matter asymmetries and hypermagnetic fields

S. Rostam Zadeh* and S. S. Gousheh†

Department of Physics, Shahid Beheshti University, G.C., Evin, Tehran 19839, Iran

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In this paper, we study the significance of the $\mathrm{U_Y(1)}$ Chern-Simons term in general, and its baryonic contribution in particular, for the evolution of the matter asymmetries and the hypermagnetic field in the temperature range 100 GeV $\leq T \leq$ 10 TeV. We show that an initial helical hypermagnetic field, denoted by $B_Y^{(0)}$, can grow matter asymmetries from zero initial value. However, the growth which is initially quadratic with respect to $B_Y^{(0)}$ saturates for values larger than a critical value. The inclusion of the baryonic contribution reduces this critical value, leading to smaller final matter asymmetries. Meanwhile, $B_Y(T_{\mathrm{EW}})$ becomes slightly larger than $B_Y^{(0)}$. In the absence of the $\mathrm{U_Y(1)}$ Chern-Simons term, the final values of matter asymmetries grow without saturation. Conversely, we show that an initial matter asymmetry can grow an initial seed of a hypermagnetic field, provided the Chern-Simons term is taken into account. The growth process saturates when the matter asymmetry drops abruptly. When the baryonic contribution is included, the saturation occurs at an earlier time, and $B_Y(T_{\mathrm{EW}})$ becomes larger. We also show that the baryonic asymmetry and the magnetic field strength can be within the acceptable range of present day data, provided the inverse cascade process is also taken into account; however, the magnetic field scale obtained from this simple model is much lower than the ones usually assumed for gamma-ray propagation.

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I. INTRODUCTION

The origin of matter is still one of the great mysteries of nature. There is observational evidence that the matter in the present-day Universe is the remnant of a small matter-antimatter asymmetry $\eta_B \sim 10^{-10}$ in the early Universe, i.e., just before the primordial plasma entered the hadronization phase. The value of this asymmetry has been determined independently in two different ways: first from the abundances of light elements in the intergalactic medium [1] and second from the power spectrum of temperature fluctuations in the cosmic microwave background (CMB) [2]. The observational discovery of the cosmic expansion [3] and CMB [4] strengthens the big bang theory, which asserts that the Universe was hot during its early stages [5] and antimatter was present when pair creation and annihilation processes were in thermal equilibrium. As the temperature decreased in the plasma of the early Universe, almost all of the particles and antiparticles were annihilated, and a small amount of matter remained. The discovery of C, P, [6] and CP [7] violation raised the possibility that the matter-antimatter asymmetry may have been created dynamically by baryogenesis, as well as leptogenesis, from an initial state which is matter-antimatter symmetric. In a seminal paper, Sakharov stated three necessary conditions for successful baryogenesis, which are the existence of baryon number violation processes, C and CP violation, and deviation from thermal equilibrium [8]. The idea of baryogenesis was elevated by the paradigm of cosmic inflation [9], which states that the Universe had an accelerated expansion in its very early history explaining its spatial flatness and the isotropy of the CMB temperature. Therefore, any preexisting baryon asymmetry was diluted and negligible at the end of inflation [10].

A seemingly unrelated but important discovery, which can be rightfully called another great mystery of nature, was the detection of a long-range magnetic field coherent over scales of the order of 30 Kpc with a strength of order μG over the plain of the disk of the Milky Way galaxy [11]. Interestingly, similar magnetic fields have been observed in other spiral and barred galaxies [12–14] as well as galaxy clusters [15–17] and high redshift protogalactic structures [18]. It is generally believed that these magnetic fields are produced from the amplification of some seed fields [19] of which the strength and origin are largely unknown [11,20]. The fact that the magnetic fields are present ubiquitously at high redshifts strengthens the idea that their origin is cosmological, and magnetic fields may have pervaded the Universe in its hot early stages [18]. The presence of coherent magnetic fields in the low-density intergalactic medium, which has been reported recently [21–26], supports the idea of primordial magnetism as well.

Assuming that the seed fields are primordial, they should have been generated out of thermal equilibrium [27]. Therefore, most of the scenarios presented for the generation of the seed fields in the early Universe operate either at a phase transition [28–34] or during the inflation [35–41]. The

*S_Rostamzadeh@sbu.ac.ir
†ss-gousheh@sbu.ac.ir

inflationary mechanisms have received a lot of attention, since they have the advantage of achieving superhorizon correlations and therefore generate much more coherent magnetic fields in the early Universe. However, the conformal invariance of the electromagnetism leads to the conservation of the magnetic flux [35], and hence the strength of generated magnetic fields decreases exponentially due to rapid expansion of the inflationary Universe. Various mechanisms have been proposed to solve this problem by breaking the conformal invariance [42]. In most of these scenarios, the generated magnetic fields are helical as well (e.g., from axion dynamics during inflation). The helical magnetic fields further evolve experiencing the inverse cascade process, which increases their scale in the radiation-dominated era after inflation. Another interesting mechanism suggested in the literature for the generation of a magnetic field is through the chiral vortical effect [43]. This effect operates in the presence of cosmic chiral asymmetry and vorticity [44–46]. According to this effect, first, the spin-orbit coupling tends to align the spins of the fermions due to the vorticity ($\omega = \nabla \times v$). Then, particle helicity determines the direction of the momentum and the electric current corresponding to each particle. If there is a disbalance between left-handed and right-handed particles, a net nonzero electric current emerges which creates a magnetic field [45].

It is well known that at high temperatures non-Abelian long-range magnetic fields cannot exist because their corresponding gauge bosons obtain a magnetic mass gap $\sim g^2 T$ [47]. Thus, the only long-range magnetic field surviving in the plasma is associated with the Abelian U(1) group of which the vector particle remains massless [48]. Moreover, electric fields decay quickly due to the large conductivity of the plasma. In the symmetric phase, the hypercharge fields couple to the fermions chirally. This leads to the fermion number violation through the Abelian anomaly, $\partial_\mu j^\mu \sim \frac{g'^2}{4\pi^2}\mathbf{E_Y}.\mathbf{B_Y}$. Here, g' is the $\mathrm{U_Y}(1)$ gauge coupling [49]. The anomalous coupling of the hypercharge fields to fermion number densities appearing in the above equation also shows up as the $\mathrm{U_Y}(1)$ Chern-Simons term.

At high temperatures and finite fermion densities, the Chern-Simons terms emerge in the effective Lagrangian densities of $\mathrm{SU}(2)_\mathrm{L}$ and $\mathrm{U_Y}(1)$ gauge fields due to their chiral couplings to fermions [49–51]. The $\mathrm{U_Y}(1)$ Chern-Simons term leads to the appearance of a new anomalous term in the magnetohydrodynamic equations which are subsequently called the anomalous magnetohydrodynamic (AMHD) equations [49,52]. As mentioned earlier, the evolution equations of the anomalous charge densities acquire a hypermagnetic source term as well (the Abelian anomaly). The mutual effects of the fermions and hypermagnetic fields on each other might have major effects in cosmology [49,51–53]. As a matter of fact, some authors believe that the evolutions of matter-antimatter asymmetries and the hypermagnetic field are intertwined [46,49,51,52,54–62].

There exist n_G global charges, i.e., $N_i = B/n_G - L_i$, which are exactly conserved in the Standard Model. Here, n_G is the number of generations, B is the baryon number, and L_i is the lepton number of the ith generation. Assigning n_G chemical potentials μ_i, $i = 1, \ldots, n_G$, to these charges and also introducing μ_Y corresponding to the weak hypercharge which will be fixed due to the hypercharge neutrality of the plasma, $\langle Y \rangle = 0$, one can describe the electroweak plasma in complete thermal equilibrium [63].

It was discussed years ago that right-handed electrons that have a very small Yukawa coupling with Higgs bosons $h_e = 2.94 \times 10^{-6}$ and do not take part in any weak interaction are decoupled from the thermal ensemble at temperatures above $T_{RL} \sim 10$ TeV [64]. This is due to the fact that, in this range of temperatures, the rates $\Gamma_{RL} \sim h_e^2 T$ of the relevant reactions[1] (direct and inverse Higgs decays in processes $e_L \bar{e}_R \leftrightarrow \phi^{(0)}$ and $\nu_e^L \bar{e}_R \leftrightarrow \phi^{(+)}$ and their conjugate processes) are much lower than the Hubble expansion rate $H \sim T^2$. Thus, neglecting the Abelian anomaly, the right-handed electron number is partially conserved, and its associated chemical potential can be added to the aforementioned $n_G + 1 = 4$ (for three generations) chemical potentials of the electroweak theory [49].

Considering the above fact, the authors of Ref. [64] suggested the following scenario in which a right-handed electron asymmetry might preserve a primordial baryon asymmetry from the weak sphalerons: at temperatures above T_{RL},[2] the weak sphalerons could not wash out the asymmetry of right-handed electrons, and therefore that of baryons. However, at temperatures below T_{RL}, the chirality flip processes turn the right-handed electrons into left-handed leptons, while roughly at these temperatures, the weak sphalerons gradually start to fall out of equilibrium.[3] Thus, it was conjectured that they might not be able to transform the left-handed leptons into antiquarks to wipe out the remaining baryon and lepton asymmetry [64].

Afterward, in related works, the authors of Refs. [49,54–57] assumed the presence of the large-scale hypermagnetic fields in the plasma and considered the Abelian anomalous effects for right-handed electrons, which led to the generation of baryon and lepton asymmetries. The reverse effect has been studied by assuming an asymmetry for right-handed electrons while considering the Abelian anomalous effects. This situation gives rise to the generation of long-range hypermagnetic fields, when the full

[1]It is discussed in the third paper of Ref. [64] that some gauge and fermion scattering processes (such as $e_R H \leftrightarrow L_e A$, where $A = Y$ or W and $e_R L_f \leftrightarrow L_e f_R$) also contribute to the chirality flip rate of electrons.

[2]In the first paper of Ref. [64], the value of T_{RL} was computed as $T_{RL} \simeq 1$ TeV.

[3]More accurate computations for the temperature at which the weak sphalerons fall out of equilibrium have been done recently [65].

range of the frequency spectrum for the hypermagnetic field is taken into account [52].

In our previous work [66], we studied the simultaneous evolution of baryon asymmetries, the first-generation lepton asymmetries and long-range hypermagnetic fields, considering the Abelian anomalous effects. For that purpose, we presented the general form of the $\mathrm{U_Y}(1)$ Chern-Simons term, which showed how chemical potentials of various fermion species contribute to it with different coefficients (see Eq. (2.7) of Ref. [66]). Most importantly, we emphasized that the chemical potentials of right-handed and left-handed particles contribute with opposite signs to the coefficient of the $\mathrm{U_Y}(1)$ Chern-Simons term, in contrast to what has been used in some of the previous works. To explore the consequences of this one correction, we used a simple model presented in one of these works as a testing ground and implemented our correction, while keeping all other main assumptions of the model unaltered so that the results would be comparable. We then compared our results with theirs. The simplifying assumptions implemented in the model were the following: only the contribution of the first-generation leptonic chemical potentials to the $\mathrm{U_Y}(1)$ Chern-Simons term were considered, and that of the baryonic ones was ignored. Only the electron chirality flip processes via inverse Higgs decays were considered.[4] These processes violate chiral electron numbers and tend to reduce the electron chiral asymmetry,[5] especially when they enter into thermal equilibrium below T_{RL}. Moreover, the Higgs asymmetry was assumed to be zero,[6] and also the weak sphaleron processes were neglected.[7]

As mentioned earlier, the evolution of matter asymmetries and hypermagnetic fields are strongly coupled, since they have mutual effects on one another through the Abelian anomaly and the $\mathrm{U_Y}(1)$ Chern-Simons term. However, in some of the previous works, the Chern-Simons term is neglected, and it is assumed to be a negligible backreaction process with unimportant effects on baryogenesis and magnetogenesis. Moreover, some other former studies which have considered this Chern-Simons term have neglected the baryonic contribution to it. The main purpose of this paper is to explore the detailed consequences of taking the $\mathrm{U_Y}(1)$ Chern-Simons term into account. To be more precise, we compare the simultaneous evolutions of matter asymmetries and hypermagnetic fields with and without taking the Chern-Simons term into account. Moreover, we explore the consequences of including the contributions of baryonic chemical potentials to this term, along with the usual leptonic contributions. To accomplish this task, we choose the simple model presented in Ref. [54] and used in our previous work [66] with the aforementioned simplifying assumptions and use it again as a testing ground which permits us to focus on our main goal. Indeed, including other processes such as the weak sphalerons affects the results and therefore prevents us from identifying and focusing on the effects of our desired terms. We solve the set of coupled differential equations for the baryon and the first-generation lepton asymmetries and the hypermagnetic field for various ranges of initial conditions in the temperature range 100 GeV $\leq T \leq$ 10 TeV and wherever possible compare the results with those of our previous study.

The outline of our paper is as follows. In Sec. II, we obtain a simplified form for the coefficient of the $\mathrm{U_Y}(1)$ Chern-Simons term containing the baryon and the first-generation lepton chemical potentials. In Secs. III and IV, we derive the dynamical equations for the hypermagnetic field as well as the baryon and the first-generation lepton asymmetries by considering the Abelian anomaly equations and the inverse Higgs decay processes and using the simplified coefficient of the $\mathrm{U_Y}(1)$ Chern-Simons term obtained in Sec. II. In Sec. V, we solve the set of coupled differential equations for fermion asymmetries and the hypermagnetic field numerically and display the results. We also use the conventions discussed in Appendix A of Ref. [46] and the anomaly equations summarized in Appendix B of that reference. In Sec. VI, we summarize the results and state our conclusions.

II. STATIC CHERN-SIMONS TERMS

In the static limit, one can use the method of dimensional reduction to obtain the effective action for the soft $\mathrm{SU}(2)_\mathrm{L}$ and $\mathrm{U_Y}(1)$ gauge fields in which the Chern-Simons terms $c_E n_{CS}$ and $c'_E n'_{CS}$ emerge, respectively [68,69]. Here, the Chern-Simons densities n_{CS} and n'_{CS} are given by [51]

$$n_{CS} = \frac{g^2}{32\pi^2}\epsilon_{ijk}\left(A_i^a G_{jk}^a - \frac{g}{3} f^{abc} A_i^a A_j^b A_k^c\right),$$
$$n'_{CS} = \frac{g'^2}{32\pi^2}\epsilon_{ijk} Y_i Y_{jk}, \tag{2.1}$$

where A_μ^a and Y_μ are the $\mathrm{SU}(2)_\mathrm{L}$ and $\mathrm{U_Y}(1)$ gauge fields and $G_{\mu\nu}^a$, $Y_{\mu\nu}$, g, and g' are their relevant field strength tensors and gauge couplings.

Let us define the notations needed in the expressions for c_E and c'_E. Since the non-Abelian gauge interactions are in thermal equilibrium at all temperatures of interest [46], they produce a strong driving force to equalize the asymmetries carried by different components of a given multiplet. Therefore, we can let μ_{Q_i} denote the common

[4] None of the chirality flip reactions mentioned in footnote 1 were considered. Indeed, the inverse Higgs decays were fast enough for our investigations.

[5] The evolution of electron chiral asymmetry $\Delta\mu = \mu_{e_R} - \mu_{e_L}$ and Maxwellian magnetic fields are strongly coupled in the broken phase [67]; therefore, the value of this asymmetry before the electroweak phase transition (EWPT) is important.

[6] This assumption leads to the absence of any net contribution from direct Higgs decays to chirality flip processes.

[7] For some of the issues concerning the weak sphalerons and their consequences, see Sec. II.

chemical potential of up and down left-handed quarks with different colors, $\mu_{u_{Ri}}$ ($\mu_{d_{Ri}}$) denote the common chemical potential of right-handed up (down) quarks with different colors, and $\mu_{L_i}(\mu_{R_i})$ denote the common chemical potential of left-handed (right-handed) leptons, where i is the generation index. Then, the general forms of c_E and c'_E as given by Eqs. (2.4) and (2.7) of our previous study are [66]

$$c_E = \sum_{i=1}^{n_G}(3\mu_{Q_i} + \mu_{L_i}),$$
$$c'_E = \sum_{i=1}^{n_G}\left[-2\mu_{R_i} + \mu_{L_i} - \frac{2}{3}\mu_{d_{Ri}} - \frac{8}{3}\mu_{u_{Ri}} + \frac{1}{3}\mu_{Q_i}\right], \quad (2.2)$$

where n_G is the number of generations.

As mentioned in Sec. I, the simultaneous evolution of matter asymmetries and hypermagnetic fields has been studied in some of the previous works. However, the $\mathrm{U_Y}(1)$ Chern-Simons term has been either completely neglected or only the contribution of the first-generation leptonic chemical potentials to its coefficient [c'_E as given by Eq. (2.2)] been taken into account and that of the baryonic ones been neglected. These are precisely the issues that we want to explore in this paper, namely, the consequences of considering the $\mathrm{U_Y}(1)$ Chern-Simons term and also its baryonic contribution. For this purpose, we choose the simple model used in our previous work [66] as a testing ground, along with all of its simplifying assumptions, including the neglect of the weak sphaleron processes. These processes, the properties of which are well studied in the absence of the hypermagnetic fields, have very high rates in the symmetric phase [65], which keeps them in thermal equilibrium and leads to the vanishing of c_E as given by Eq. (2.2) (see Table 1 of Ref. [46]). This puts a constraint on the chemical potentials and strongly affects the scope of the aforementioned effects that we want to study. Therefore, the inclusion of weak sphalerons in the model[8] adds an unnecessary complication which would obscure our results. Hence, the chosen simple model is a proper testing ground, to which we now return.

[8]To properly include the effects of weak sphalerons in the presence of hypermagnetic fields, one can include the term corresponding to the weak sphalerons in the evolution equations of left-handed fermion asymmetries and let c_E evolve freely in accordance with the evolution of its constituents as given by Eq. (2.2). When we do this for the model under study, we find that c_E stays very close to zero in the whole interval; albeit, near the EWPT, the effect of the hypermagnetic fields via the Abelian anomalous effects becomes strong enough to force the system slightly out of equilibrium. Although the departure of c_E from zero is small in this case, its consequences are non-negligible. However, the extent of this effect is model dependent. We plan to complete the study mentioned in this footnote by including weak sphalerons and other comparable effects, and report on it elsewhere.

The expression for c'_E as given by Eq. (2.2) can be simplified by considering the fast processes operating on the quarks. Assuming that the rates of all Yukawa interactions for quarks (up-type Yukawa in processes $d^i_L + \phi^{(+)} \leftrightarrow u^i_R$ and $u^i_L + \phi^{(0)} \leftrightarrow u^i_R$, down-type Yukawa in processes $u^i_L \leftrightarrow \phi^{(+)} + d^i_R$ and $d^i_L \leftrightarrow \phi^{(0)} + d^i_R$, and their conjugate reactions [46]) are much higher than the Hubble expansion rate, we obtain

$$\mu_{u_{Ri}} - \mu_{Q_i} = \mu_0, \qquad \mu_{d_{Ri}} - \mu_{Q_i} = -\mu_0, \quad (2.3)$$

where μ_0 is the chemical potential of the Higgs field. Let us assume that all up or down quarks belonging to different generations with distinct handedness have the same chemical potential (i.e., $\mu_{u_{Ri}} = \mu_{u_R}, \mu_{d_{Ri}} = \mu_{d_R}, \mu_{Q_i} = \mu_Q$: $i = 1, 2, 3$) due to the flavor mixing in the quark sector (see Sec. 2.1 of the third paper of Ref. [64]). Then, we obtain

$$\mu_{u_R} - \mu_Q = \mu_0, \qquad \mu_{d_R} - \mu_Q = -\mu_0. \quad (2.4)$$

Since we have the simplifying assumption of zero Higgs asymmetry, we get

$$\mu_{u_R} = \mu_{d_R} = \mu_Q. \quad (2.5)$$

In other words, assuming zero Higgs asymmetry, the fast processes tend to equalize all quark chemical potentials. Using Eq. (2.5), we can simplify Eq. (2.2) in the form

$$c'_E = \sum_{i=1}^{n_G}[-2\mu_{R_i} + \mu_{L_i} - 3\mu_Q]. \quad (2.6)$$

Recalling that $N_c = 3$ and $N_w = 2$ are the ranks of non-Abelian gauge groups and $n_G = 3$ is the number of generations, the whole baryonic chemical potential can be calculated as

$$\mu_B = \frac{1}{N_c}\sum_{i=1}^{n_G}[N_c N_w \mu_{Q_i} + N_c \mu_{u_{R_i}} + N_c \mu_{d_{R_i}}] = 12\mu_Q. \quad (2.7)$$

Therefore, the simplified form of c'_E in terms of the baryonic and the first-generation leptonic chemical potentials takes the form

$$c'_E = -2\mu_{e_R} + \mu_{e_L} - \frac{3}{4}\mu_B. \quad (2.8)$$

III. EVOLUTION EQUATION FOR THE HYPERMAGNETIC FIELD

Let us recall the generalized diffusion equation for the hypermagnetic field derived from the AMHD equations in our previous work (Eq. (3.6) of Ref. [66]),

$$\frac{\partial \mathbf{B_Y}}{\partial t} = \frac{1}{\sigma}\nabla^2 \mathbf{B_Y} + \alpha_Y \nabla \times \mathbf{B_Y},$$
$$\text{where } \alpha_Y(T) = -c'_E \frac{g'^2}{8\pi^2\sigma}. \tag{3.1}$$

In the above equation, $\sigma \sim 100T$ is the hyperconductivity of the plasma [70], and c'_E is given by Eq. (2.8). Choosing the simplest nontrivial configuration of the hypermagnetic field, which is

$$Y_x = Y(t)\sin k_0 z, \qquad Y_y = Y(t)\cos k_0 z, \qquad Y_z = Y_0 = 0, \tag{3.2}$$

and using it in Eq. (3.1), one obtains the evolution equation for the hypermagnetic field amplitude $B_Y(t) = k_0 Y(t)$ in the form

$$\frac{dB_Y}{dt} = B_Y\left[-\frac{k_0^2}{\sigma} + \frac{k_0 g'^2}{4\pi^2\sigma}\left(\mu_{e_R} - \frac{\mu_{e_L}}{2} + \frac{3}{8}\mu_B\right)\right]. \tag{3.3}$$

In the above equation, the coupling of the evolution of the hypermagnetic field to those of the chemical potentials is apparent. In the next section, we discuss the latter; however, let us first obtain the relevant expression for the Abelian anomaly ($\sim\mathbf{E_Y}.\mathbf{B_Y}$) appearing in the dynamical equations of the fermionic asymmetries.

Let us recall the generalized Ohm law derived from the AMHD equations in our previous work (Eq. (3.4) of Ref. [66]),

$$\mathbf{E_Y} = -\mathbf{V}\times\mathbf{B_Y} + \frac{\nabla\times\mathbf{B_Y}}{\sigma} - \alpha_Y \mathbf{B_Y}. \tag{3.4}$$

Using the above equation with $\sigma = 100T$ and α_Y and c'_E as given by Eqs. (3.1) and (2.8), for the simple configuration of the hypermagnetic field given by Eq. (3.2), the form of the Abelian anomaly simplifies to

$$\mathbf{E_Y}.\mathbf{B_Y} = \frac{B_Y^2}{100}\left[\frac{k_0}{T} - \frac{g'^2}{4\pi^2 T}\left(\mu_{e_R} - \frac{\mu_{e_L}}{2} + \frac{3}{8}\mu_B\right)\right]. \tag{3.5}$$

IV. DYNAMICAL EQUATIONS FOR THE LEPTON AND BARYON ASYMMETRIES

In the Standard Model, the $\mathrm{U_Y}(1)$ Abelian anomaly violates the first-generation lepton numbers in the form

$$\partial_\mu j^\mu_{e_R} = -\frac{1}{4}(Y_R^2)\frac{g'^2}{16\pi^2}Y_{\mu\nu}\tilde{Y}^{\mu\nu} = \frac{g'^2}{4\pi^2}(\mathbf{E_Y}.\mathbf{B_Y}),$$
$$\partial_\mu j^\mu_{\nu_e^L} = \partial_\mu j^\mu_{e_L} = +\frac{1}{4}(Y_L^2)\frac{g'^2}{16\pi^2}Y_{\mu\nu}\tilde{Y}^{\mu\nu} = -\frac{g'^2}{16\pi^2}(\mathbf{E_Y}.\mathbf{B_Y}), \tag{4.1}$$

where $\tilde{Y}^{\mu\nu}$ is the dual field strength tensor and the relevant hypercharges are $Y_R = -2$ and $Y_L = -1$. Therefore, the system of dynamical equations for the corresponding asymmetries, taking into account the Abelian anomaly Eqs. (4.1) and inverse Higgs decay processes, takes the form[9]

$$\frac{d\eta_{e_R}}{dt} = +\frac{g'^2}{4\pi^2 s}(\mathbf{E_Y}.\mathbf{B_Y}) + 2\Gamma_{RL}(\eta_{e_L} - \eta_{e_R}),$$
$$\frac{d\eta_{\nu_e^L}}{dt} = \frac{d\eta_{e_L}}{dt} = -\frac{g'^2}{16\pi^2 s}(\mathbf{E_Y}.\mathbf{B_Y}) + \Gamma_{RL}(\eta_{e_R} - \eta_{e_L}). \tag{4.2}$$

In Eqs. (4.2), $\eta_f = (n_f - n_{\bar{f}})/s$ with $f = \{e_R, e_L, \nu_e^L\}$ is the matter asymmetry, $s = 2\pi^2 g^* T^3/45$ is the entropy density of the Universe, and $g^* = 106.75$ is the number of relativistic degrees of freedom. Γ_{RL} is the rate of inverse Higgs decay reactions, and the factor 2 multiplying it in the first line is because of the equivalent rates of reaction branches ($e_L\bar{e}_R \to \phi^{(0)}$ and $\nu_e^L\bar{e}_R \to \phi^{(+)}$ and their conjugate processes). Since the SU(2) gauge interactions are very fast, $\eta_{e_L} \approx \eta_{\nu_e^L}$, and the evolution equation of the neutrino asymmetry is unnecessary.

Let us define the variable $x = t/t_{\mathrm{EW}} = (T_{\mathrm{EW}}/T)^2$, in accordance with the Friedmann law, where $t_{\mathrm{EW}} = M^*_{\mathrm{Pl}}/2T^2_{\mathrm{EW}}$ and $M^*_{\mathrm{Pl}} = M_{\mathrm{Pl}}/1.66\sqrt{g^*}$ is the reduced Planck mass. Then, $\Gamma_{RL} = \Gamma_0(1-x)/2t_{\mathrm{EW}}\sqrt{x}$ with $\Gamma_0 = 121$ [54,55,64]. Recalling the equation $n_f - n_{\bar{f}} = \mu_f T^2/6$ for fermions and defining $y_f = 10^4\mu_f/T$, the fermion asymmetry will be $\eta_f = 10^{-4}y_f T^3/6s$. Using Eq. (3.5), Eqs. (4.2) can be rewritten in terms of the variables y_f in the form

$$\frac{dy_R}{dx} = [B_0 x^{1/2} - A_0 y_T]\left(\frac{B_Y(x)}{10^{20}\,\mathrm{G}}\right)^2 x^{3/2} - \Gamma_0\frac{1-x}{\sqrt{x}}(y_R - y_L),$$
$$\frac{dy_L}{dx} = \frac{-1}{4}[B_0 x^{1/2} - A_0 y_T]\left(\frac{B_Y(x)}{10^{20}\,\mathrm{G}}\right)^2 x^{3/2} - \Gamma_0\frac{1-x}{2\sqrt{x}}(y_L - y_R), \tag{4.3}$$

where

$$B_0 = 25.6\left(\frac{k_0}{10^{-7}T_{\mathrm{EW}}}\right), \qquad A_0 = 77.6, \quad \text{and}$$
$$y_T = y_R - \frac{y_L}{2} + \frac{3}{8}y_B. \tag{4.4}$$

We have chosen the overall scale of B_0 and A_0 to normalize the hypermagnetic field at 10^{20} G.

In the Standard Model, the anomalous processes change the baryon asymmetry $\eta_B = (n_B - n_{\bar{B}})/s$ as well as the lepton asymmetry of each generation $\eta_{L_i} = (n_{L_i} - n_{\bar{L}_i})/s$,

[9] We have used Appendix B of Ref. [55], but with the assumption of zero Higgs asymmetry. See also Eq. (2.6) in Sec. 2.1 of Ref. [59] for the general form of the equations.

respecting the conservation law $\eta_B/3 - \eta_{L_i} = \text{constant}$. Using this fact for the first-generation asymmetries, one can obtain the evolution equation of the baryon asymmetry in the form

$$\frac{1}{3}\frac{d\eta_B}{dt} = \frac{d\eta_{e_R}}{dt} + \frac{d\eta_{e_L}}{dt} + \frac{d\eta_{\nu_e^L}}{dt}, \quad \text{or}$$
$$\frac{1}{3}\frac{dy_B}{dx} = \frac{dy_R}{dx} + 2\frac{dy_L}{dx}, \tag{4.5}$$

where $y_B = 4\times 10^4 \pi^2 g^* \eta_B/15$ is the scaled baryon asymmetry. Finally, we use Eqs. (4.5) and (4.3) to obtain

$$\frac{dy_B}{dx} = \frac{3}{2}\left[B_0 x^{1/2} - A_0\left(y_R - \frac{y_L}{2} + \frac{3}{8}y_B\right)\right]\left(\frac{B_Y(x)}{10^{20}G}\right)^2 x^{3/2}. \tag{4.6}$$

We also rewrite Eq. (3.3) in terms of x and the variables y_f to obtain

$$\frac{dB_Y}{dx} = 3.5\left(\frac{k_0}{10^{-7}T_{\mathrm{EW}}}\right)\left[\frac{y_T}{\pi} - 0.1\left(\frac{k_0}{10^{-7}T_{\mathrm{EW}}}\right)\sqrt{x}\right]B_Y(x), \tag{4.7}$$

where y_T is given by Eq. (4.4).

V. RESULTS

The simplified form of the $\mathrm{U_Y}(1)$ Chern-Simons coefficient c'_E is given in Eq. (2.8) and affects the evolution equations (4.3), (4.6), and (4.7) through α_Y as given by Eq. (3.1). In this section, we study the effect of the Chern-Simons term on the evolution of matter asymmetries and hypermagnetic fields for a variety of initial conditions. To accomplish this task, we compare the results for three different choices of α_Y, namely, $\alpha_Y^{(0)} = \frac{g'^2}{8\pi^2\sigma}(2\mu_{e_R} - \mu_{e_L})$ (in the absence of the baryonic contribution), $\alpha_Y = \frac{g'^2}{8\pi^2\sigma}(2\mu_{e_R} - \mu_{e_L} + \frac{3}{4}\mu_B)$ [as given by Eqs. (3.1) and (2.8)], and $c\alpha_Y$, where $c = \{0, 0.1, 0.2\}$, i.e., the attenuated Chern-Simons term, with a given set of initial conditions. Moreover, k_0 is set to $k_{\max} = 10^{-7}T_{\mathrm{EW}}$, which is the maximum wave number surviving Ohmic dissipation.

A. Matter asymmetry generation by hypermagnetic fields

First, the evolution equations are solved numerically by assuming zero initial matter asymmetries but an initial amplitude of the hypermagnetic field $B_Y^{(0)} = 10^{21}$ G for two different cases, namely, $\alpha_Y^{(0)} = \frac{g'^2}{8\pi^2\sigma}(2\mu_{e_R} - \mu_{e_L})$ and $\alpha_Y = \frac{g'^2}{8\pi^2\sigma}(2\mu_{e_R} - \mu_{e_L} + \frac{3}{4}\mu_B)$. The results are presented as time plots in Fig. 1. As can be seen, in both cases, matter asymmetry generation occurs in the presence of

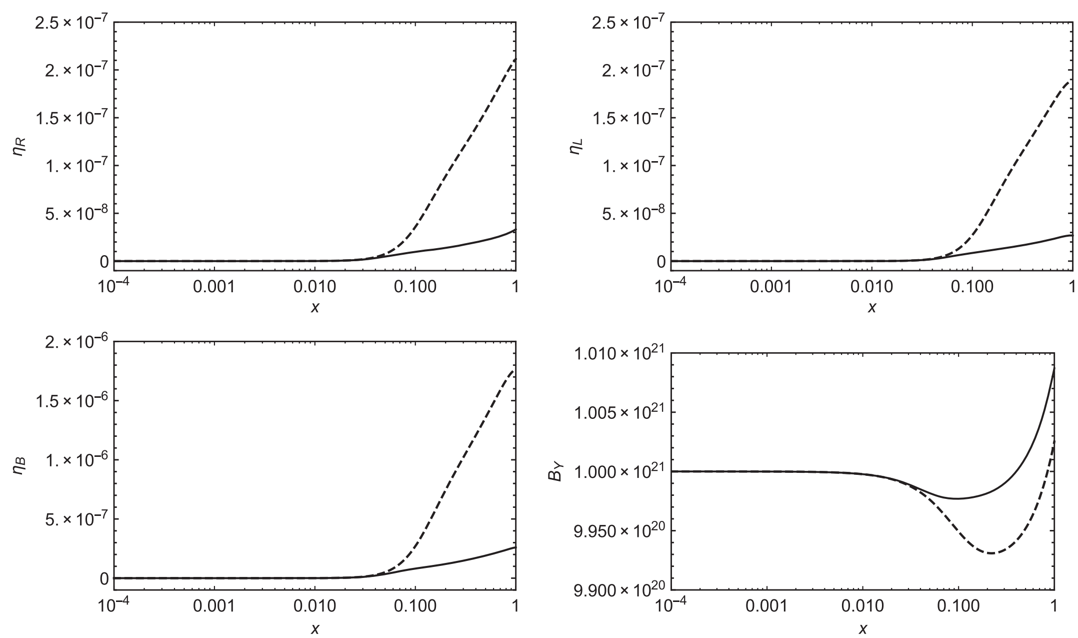

FIG. 1. The time plots of the first-generation leptonic asymmetries $\eta_R = \eta_{e_R}$ and $\eta_L = \eta_{e_L} = \eta_{\nu_e^L}$, baryonic asymmetry η_B, and the hypermagnetic field amplitude B_Y for $k_0 = 10^{-7}T_{\mathrm{EW}}$ with initial conditions $B_Y^{(0)} = 10^{21}G$ and zero initial matter asymmetries for two different cases. Case 1 (dashed lines): $\alpha_Y^{(0)} = \frac{g'^2}{8\pi^2\sigma}(2\mu_{e_R} - \mu_{e_L})$; case 2 (solid lines): $\alpha_Y = \frac{g'^2}{8\pi^2\sigma}(2\mu_{e_R} - \mu_{e_L} + \frac{3}{4}\mu_B)$. The starting point is at $T_0 = 10$ TeV, $x_0 = \frac{t_0}{t_{\mathrm{EW}}} = (\frac{T_{\mathrm{EW}}}{T_0})^2 = 10^{-4}$ and the final point is at $T_f = T_{\mathrm{EW}}$, $x_f = \frac{t_f}{t_{\mathrm{EW}}} = (\frac{T_{\mathrm{EW}}}{T_f})^2 = 1$. Case 1 is obtained from our previous work [66] and is reproduced here for comparison. The maximum relative error for these plots is of the order of 10^{-20}.

hypermagnetic fields; however, the final values of the asymmetries at the onset of the electroweak phase transition (EWPT) for the second case are almost seven times smaller than those of the first case. Moreover, the hypermagnetic field amplitude behaves nearly the same with a little more increase in its final value for the second case. For the rest of this subsection, we use α_Y as given by Eqs. (3.1) and (2.8), that is, including the baryonic contribution.

Let us examine the importance of the $\mathrm{U_Y}(1)$ Chern-Simons term via attenuating its effect by multiplying it with an adjustable parameter $c \le 1$. We numerically solve the evolution equations with the aforementioned initial conditions for three different values of c: $\{0.2, 0.1, 0\}$ and present the results as time plots, along with the case $c = 1$ obtained earlier, in Fig. 2. Figure 2 shows that the smaller the value of c, the larger the matter asymmetries and the weaker the hypermagnetic field at T_{EW}. The case $c = 0$ also shows that the hypermagnetic field is able to generate substantial matter asymmetries through the Abelian anomaly even in the absence of the $\mathrm{U_Y}(1)$ Chern-Simons term. Therefore, taking into account the $\mathrm{U_Y}(1)$ Chern-Simons term leads to a severe decrease in the generated matter asymmetries but a very small increase in the strength of the hypermagnetic field, all at $T = T_{\mathrm{EW}}$.

Let us return to our first investigation but change $B_Y^{(0)}$ in the range 10^{17} G $< B_Y^{(0)} < 10^{22}$ G. We have solved the equations and obtained the final values of the matter asymmetries and the hypermagnetic field amplitude at $T = T_{\mathrm{EW}}$. We do not display the results for space limitation, and suffice it to point out the salient features of this investigation. This investigation is analogous to the one done in our previous work (Fig. 2 of Ref. [66]), and the results are qualitatively similar. That is, the final asymmetries increase approximately quadratically for $B_Y^{(0)} \lesssim 10^{19.5}$ G and saturate for $B_Y^{(0)} \gtrsim 10^{20.5}$ G. However, the saturated values are about seven times smaller than those of our previous work where we used $\alpha_Y^{(0)}$. The amplitude B_Y stays relatively unchanged except for $B_Y^{(0)} \gtrsim 10^{20}$ G, where it increases slightly above its initial value, indicating a mild resonance effect.

Next, we repeat the above investigation in the absence of the $\mathrm{U_Y}(1)$ Chern-Simons term by setting $c = 0$. Interestingly, we observe that the final asymmetries again increase quadratically with increasing $B_Y^{(0)}$ due to the Abelian anomaly without any saturation. Moreover, the final value of B_Y decreases slightly compared to its initial value $B_Y^{(0)}$.

B. Hypermagnetic fields growth by matter asymmetries

In continuation, we examine the possibility of producing a hypermagnetic field from initial matter asymmetries, when no initial seed of the hypermagnetic field is present in the plasma. We observe that no hypermagnetic field with

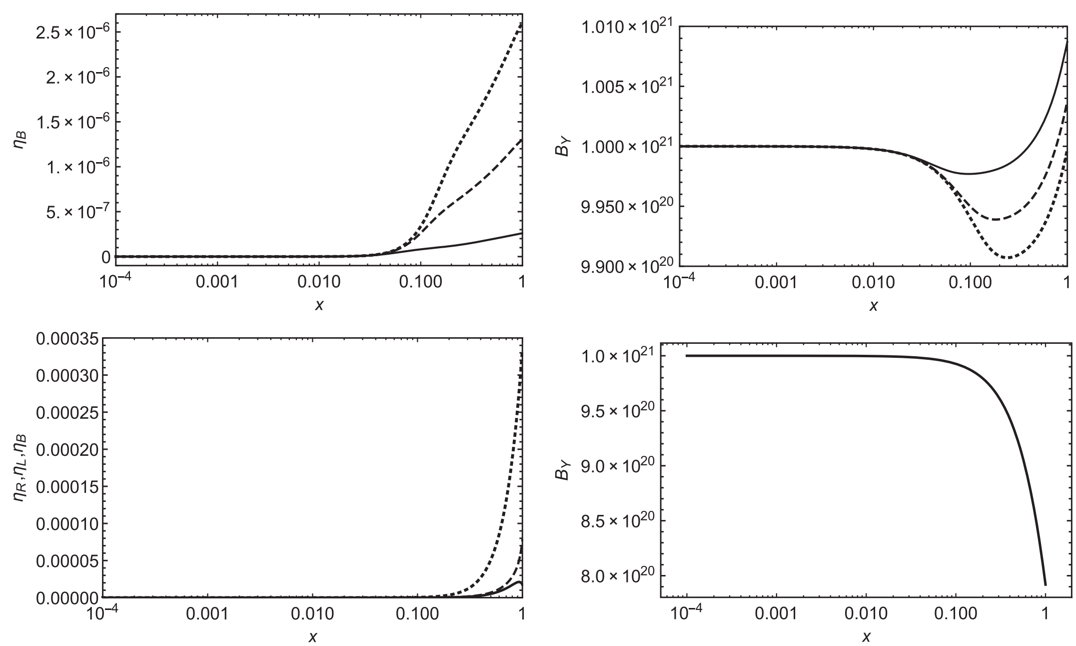

FIG. 2. Top figures: The time plots of baryonic asymmetry η_B and the hypermagnetic field amplitude B_Y for $k_0 = 10^{-7} T_{\mathrm{EW}}$ with initial conditions $B_Y^{(0)} = 10^{21} G$, zero initial matter asymmetries, and the attenuated hypermagnetic helicity coefficient $c\alpha_Y$ for three different values of c. That is, $c = 1$ (solid lines), $c = 0.2$ (dashed lines), and $c = 0.1$ (dotted lines). Bottom figures: The time plots of the first-generation leptonic asymmetries $\eta_R = \eta_{e_R}$ (dashed line) and $\eta_L = \eta_{e_L} = \eta_{\nu_e^L}$ (solid line), baryonic asymmetry η_B (dotted line), and the hypermagnetic field amplitude B_Y in the absence of the $\mathrm{U_Y}(1)$ Chern-Simons term ($c = 0$). The maximum relative error for these plots is of the order of 10^{-16}.

the simple wave configuration as given by Eq. (3.2) can be generated. The following integral form for the evolution equation of the hypermagnetic field amplitude (4.7) clarifies that the amplitude stands at zero if its initial value is zero:

$$B_Y(x)=B_Y^{(0)}\exp\left[\frac{3.5k_0}{10^{-7}T_{\rm EW}}\int_{x_0}^{x}\left(\frac{y_T(x')}{\pi}-\frac{0.1k_0}{10^{-7}T_{\rm EW}}\sqrt{x'}\right)dx'\right],$$
$$\text{where}\;\; y_T(x')=y_R(x')-\frac{y_L(x')}{2}+\frac{3}{8}y_B(x'). \tag{5.1}$$

In the next step, we examine the possibility of growing a very weak seed of the hypermagnetic field, e.g., $B_Y^{(0)}=10^{-2}$ G, by initial baryon and right-handed electron asymmetries which respect the constraint $\eta_B^{(0)}/3-\eta_{L_e}^{(0)}=0$. We solve the evolution equations with $y_R^{(0)}=10^3$ for two different cases, i.e., $\alpha_Y^{(0)}=\frac{g'^2}{8\pi^2\sigma}(2\mu_{e_R}-\mu_{e_L})$ and $\alpha_Y=\frac{g'^2}{8\pi^2\sigma}(2\mu_{e_R}-\mu_{e_L}+\frac{3}{4}\mu_B)$, and display the time plots in Fig. 3. As can be seen, in both cases, the hypermagnetic field becomes strong in the presence of the initial matter asymmetries. Although the final amplitude of the hypermagnetic field for the second case is about five times larger than that of the first case, the final baryon asymmetry is about 40 times smaller as compared to the first case. Moreover, the anomalous processes which reduce the asymmetries and amplify the hypermagnetic field start up much sooner in the second case, i.e., near the point $x\sim 0.04$.

Let us again investigate the significance of the $\mathrm{U_Y}(1)$ Chern-Simons term, the coefficient of which is given by Eq. (2.8), via reducing its effect by multiplying it with the adjustable parameter $c\leq 1$. We have solved the dynamical equations with the aforementioned initial conditions for three different values of $c:\{0.2,0.1,0\}$, and the resulting time plots, along with the case $c=1$ already obtained, are presented in Fig. 4. Again, it can be seen that, as the value of c becomes smaller, the final matter asymmetries increase, but the final hypermagnetic field amplitude decreases. More importantly, the case $c=0$ shows that even large matter asymmetries are not able to strengthen the hypermagnetic field in the absence of the $\mathrm{U_Y}(1)$ Chern-Simons term.

Finally, we solve the dynamical equations with $B_Y^{(0)}=10^{-2}$ G, $y_R^{(0)}$ in the range $10^{-2}<y_R^{(0)}<10^3$ and initial baryon asymmetry fulfilling the condition $\eta_B^{(0)}/3-\eta_{L_e}^{(0)}=0$ and obtain the final values at $T=T_{\rm EW}$. Again, we do not display the results for space limitation, and suffice it to point out the salient features of this investigation. We find that for $10^{-2}<y_R^{(0)}<10^{1.52}$ the final value of the hypermagnetic field amplitude B_Y grows until it becomes as large as 10^{20} G, then increases with a much smaller slope for $10^{1.52}<y_R^{(0)}<10^3$. Moreover, the matter asymmetries stay very close to zero except for $10^1<y_R^{(0)}<10^2$, where η_B

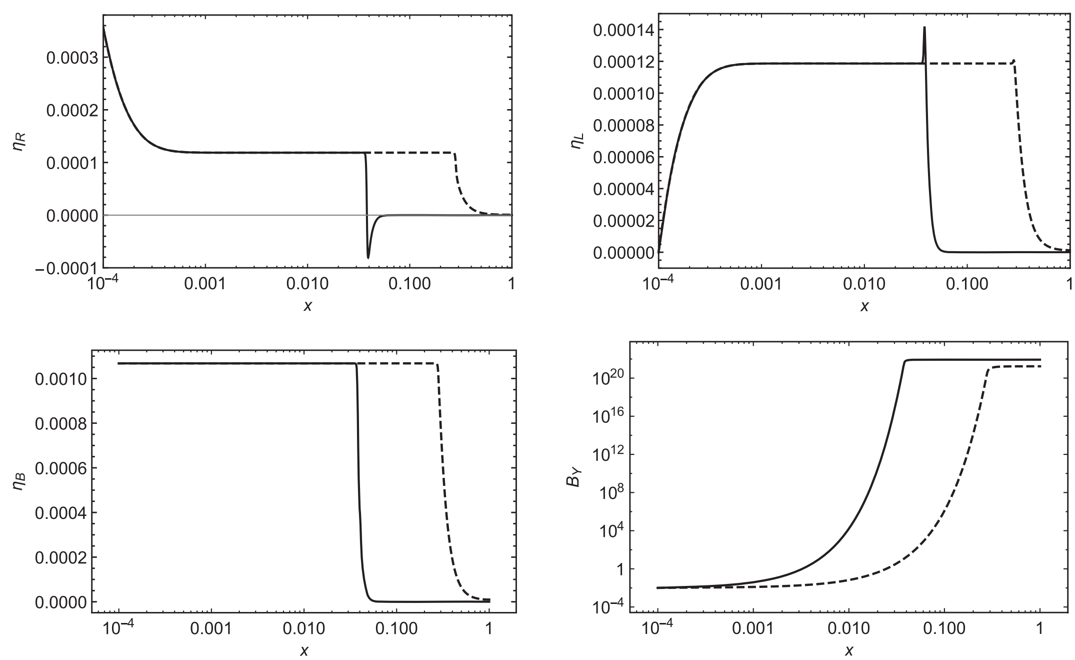

FIG. 3. The time plots of the first-generation leptonic asymmetries $\eta_R=\eta_{e_R}$ and $\eta_L=\eta_{e_L}=\eta_{\nu_e^L}$, baryonic asymmetry η_B, and the hypermagnetic field amplitude B_Y, for $k_0=10^{-7}T_{\rm EW}$ with initial conditions $B_Y^{(0)}=10^{-2}G$, and $y_R^{(0)}=10^3$ and $\eta_B^{(0)}$ respecting the condition $\eta_B^{(0)}/3-\eta_{L_e}^{(0)}=0$ for two different cases. Case 1 (dashed lines): $\alpha_Y^{(0)}=\frac{g'^2}{8\pi^2\sigma}(2\mu_{e_R}-\mu_{e_L})$; case 2 (solid lines): $\alpha_Y=\frac{g'^2}{8\pi^2\sigma}(2\mu_{e_R}-\mu_{e_L}+\frac{3}{4}\mu_B)$. The maximum relative error for these plots is of the order of 10^{-15}.

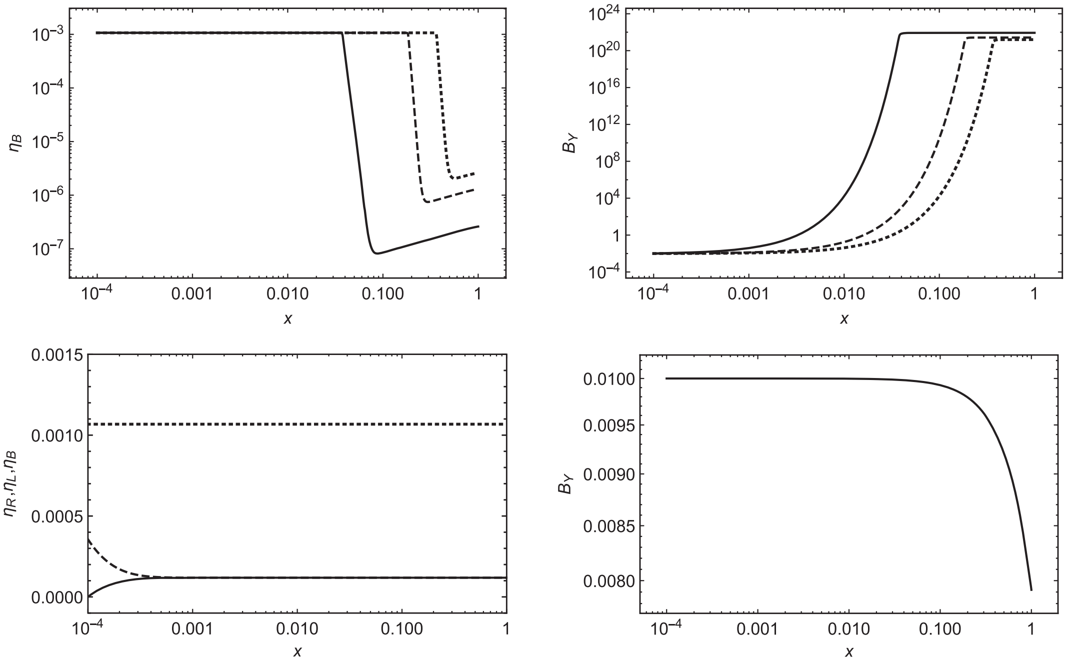

FIG. 4. Top two plots: The time plots of baryonic asymmetry η_B and the hypermagnetic field amplitude B_Y for $k_0 = 10^{-7} T_{\mathrm{EW}}$ with initial conditions $B_Y^{(0)} = 10^{-2} G$, and $y_R^{(0)} = 10^3$ and $\eta_B^{(0)}$ respecting the conservation law $\eta_B^{(0)}/3 - \eta_{L_e}^{(0)} = 0$ for three different values of c, namely, $c = 1$ (solid lines), $c = 0.2$ (dashed lines), and $c = 0.1$ (dotted lines). The lower two plots are for the case $c = 0$ and show the time plots of the first-generation leptonic asymmetries, $\eta_R = \eta_{e_R}$ (dashed line) and $\eta_L = \eta_{e_L} = \eta_{\nu_e^L}$ (solid line), baryonic asymmetry η_B (dotted line), and the hypermagnetic field amplitude B_Y. The maximum relative error for these plots is of the order of 10^{-15}.

and η_L attain a maximum and η_R attains a negative minimum value close to $y_R^{(0)} \approx 10^{1.55}$. The behavior described above is somehow similar to the behavior observed in the fifth investigation of our previous work except that there was no negative value for the final value of η_R and the matter asymmetries reach their extremum values around $y_R^{(0)} \approx 10^{2.55}$. Two interesting points can be emphasized about the results: the first one is that at $y_R^{(0)} = 10^{1.52}$ a strong hypermagnetic field and large amounts of matter asymmetries are obtained at $T = T_{\mathrm{EW}}$; another one is that at $y_R^{(0)} = 10^{1.56}$ the final amount of $\eta_R - \eta_L$ becomes maximum. This chiral asymmetry is important for the evolution of Maxwellian magnetic fields in the broken phase [67].

We also repeat the above investigation in the absence of the Abelian Chern-Simons term by choosing $c = 0$. We find that the behavior is totally different and none of the interesting features of the previous case can be seen. Indeed, there is no amplification of the hypermagnetic fields. Moreover, the final baryonic asymmetry is the same as its initial value, and the final right-handed and left-handed lepton asymmetries are equal, with their sum being equal to $\eta_R^{(0)}$.

VI. SUMMARY AND DISCUSSION

In this paper, we have studied the effect of the $\mathrm{U_Y}(1)$ Chern-Simons term, and its baryonic contribution, on the evolution of the matter asymmetries and the hypermagnetic field, within the context of a simple model and in the temperature range $100\ \mathrm{GeV} < T < 10\ \mathrm{TeV}$. To do the latter, we have compared the results when the coefficient of the $\mathrm{U_Y}(1)$ Chern-Simons term, i.e., c'_E, includes only the usual first-generation leptonic contribution, with the results when the baryonic contribution is also included. To study the first part, i.e., the importance of the $\mathrm{U_Y}(1)$ Chern-Simons term in general, we have studied the effect of multiplying c'_E, which now includes the baryonic contribution, by an attenuating parameter $0 \leq c < 1$. The baryonic contribution added is subject to the condition $\eta_B^{(0)}/3 - \eta_{L_e}^{(0)} = 0$. One of the effects of this condition is to increase the initial magnitude of c'_E. Comparison of the results for the matter asymmetries and hypermagnetic fields with and without the inclusion of the baryonic contribution shows that the results are qualitatively similar. The differences, along with the effect of attenuating the amplitude of the $\mathrm{U_Y}(1)$ Chern-Simons term to the point of eliminating it altogether, are described below.

We first discuss the generation of matter asymmetries by an initial hypermagnetic field. Our study has shown that an initial nonzero hypermagnetic field can grow matter asymmetries from zero initial value. However, the growth which is initially quadratic with respect to $B_Y^{(0)}$ saturates for values larger than a critical value denoted by $B_{Y,C}^{(0)}$. Therefore, the larger the $B_{Y,C}^{(0)}$, the larger the final saturated values of the matter asymmetries. The values of $B_{Y,C}^{(0)}$ for the cases with and without the baryonic contribution are approximately $10^{20.5}$ and 10^{21} G, respectively, leading to about seven times smaller final matter asymmetries in the first case. This comparison also indicates that $B_{Y,C}^{(0)}$ increases with attenuating c_E', a conclusion which is confirmed with the use of attenuating parameter. In this regard, the interesting point is that when the Chern-Simons term is eliminated altogether by setting $c = 0$, the growth of the matter asymmetries continues to be quadratic with respect to $B_Y^{(0)}$ without any saturation, as though $B_{Y,C}^{(0)}$ has moved to infinity. On the other hand, the change in the final value of the hypermagnetic field, denoted by $B_Y(T_{\rm EW})$, is very small in either case. For the case shown in Fig. 1, when the baryonic contribution is added, it increases by 1%, as compared to 0.2% when it is not. Both of these cases are indications of a mild resonance. Moreover, as the attenuating parameter c decreases, $B_Y(T_{\rm EW})$ decreases as well, becoming equal to its initial value for $c \approx 0.1$ and decreasing by 20% when $c = 0$.

Next, we discuss the generation of hypermagnetic field by an initial matter asymmetry. As mentioned before, the generation of a nonzero $B_Y(T_{\rm EW})$ is possible only if its initial value is nonzero. The time plots show that in general one can identify a particular time, denoted by $t_{\rm Tr}$, where the important transitions start. Figure 3 shows that the inclusion of the baryonic contribution leads to a decrease in $t_{\rm Tr}$, i.e., the transitions start at a higher temperature. Moreover, at $t_{\rm Tr}$, the matter asymmetries drop rather sharply, and the growth of the hypermagnetic field, which had been steady heretofore, saturates. For the case displayed, $B_Y(T_{\rm EW})$ becomes about five times larger when the baryonic contribution is included, while the final matter asymmetries become about 40 times smaller. Figure 4, which displays the effects of the attenuation parameter, shows that the features just described are generic consequences of changing the value of c_E'. Figure 4 also shows a very interesting case of $c = 0$. In this case, the matter asymmetries do not change, except for balancing out due to chirality flip processes. More importantly, the minute seed of the hypermagnetic field not only does not grow but also drops by 20%. Another interesting outcome of the investigation which includes the range $10^{-2} \le y_R^{(0)} \le 10^3$ is that when no attenuation parameter is taken into account almost all of the matter asymmetries are expended to grow $B_Y(T_{\rm EW})$. Close to the point $y_R^{(0)} \approx 10^{1.5}$, the rate of growth of $B_Y(T_{\rm EW})$ suddenly slows down considerably, and the final matter asymmetries attain their extremum values. Surprisingly, the extremum of η_R is a negative minimum. Hence, a relatively large chiral asymmetry is generated at this point, which is important for the subsequent evolution of the Maxwellian magnetic field in the EWPT and the broken phase. The corresponding point in the absence of the baryonic contribution is $y_R^{(0)} \approx 10^{2.4}$.

We mentioned in Sec. I that the baryon asymmetry of the Universe (BAU) is $\eta_B \sim 10^{-10}$ as extracted from the observational data. Let us also briefly state some features of the observational data about the magnetic fields and then check the compatibility of our results with these data.

The observations of the CMB temperature anisotropy put an upper bound on the strength B_0 of the present magnetic fields, $B_0 \lesssim 10^{-9}$ G on the CMB scales $\lambda_0 \gtrsim 1$ Mpc [71]. Furthermore, the observations of the gamma rays from blazars not only provide both lower and upper bounds on the strength B_0 but also indicate the existence of the large-scale magnetic fields with the scales as large as $\lambda_0 \simeq 1$ Mpc [25,26,72]. The strength B_0 of the present intergalactic magnetic fields (IGMFs) reported in Ref. [25] is $B_0 \simeq 10^{-15}$ G. Two different cases are also investigated in Ref. [26]. In the first case, in which blazars are assumed to produce both gamma rays and cosmic rays, the authors find 1×10^{-17} G $< B_0 < 3 \times 10^{-14}$ G. However, in the second case, in which the cosmic-ray component is excluded, they report that the 10^{-17} G lower limit remains valid but the upper limit depends on the spectral properties of the source. Reference [72] estimates the strength of the IGMFs to be in the range $B_0 \simeq 10^{-17} - 10^{-15}$ G, which is consistent with the above mentioned results of Refs. [25,26]. Moreover, a nonvanishing helicity of the present large-scale magnetic fields is also inferred with the strength $B_0 \simeq 5.5 \times 10^{-14}$ G in Ref. [73].

Aside from the cosmic expansion which leads to the trivial adiabatic evolution of the cosmic magnetic fields, several other effects such as the viscous diffusion, the inverse cascade, the Abelian anomalous effects, etc., affect their evolution as well. In the trivial case, the strength $B(t)$ and the scale $\lambda(t)$ are proportional to $a^{-2}(t)$ and $a(t)$, respectively, where $a(t)$ is the Friedmann-Robertson-Walker (FRW) scale factor. However, in the inverse cascade mechanism, $\lambda(t)$ grows faster than $a(t)$ due to the turbulence in the plasma [58]. In this case, the magnetic helicity is approximately conserved, but the energy is transferred from small scales to large scales [74], and the spectrum develops with a characteristic scaling law [75]. After recombination, the plasma becomes neutral, and the magnetic fields evolve trivially. One can use the scaling relation to express the spectrum of the primordial magnetic fields in terms of λ_0 and B_0 as (see Ref. [58] and Appendix C of Ref. [59])

$$\begin{aligned} B(T) &\simeq (1 \times 10^{20}\ \mathrm{G}) \left(\frac{T}{100\ \mathrm{GeV}}\right)^{7/3} \left(\frac{B_0}{10^{-14}\ \mathrm{G}}\right) g_B(T), \\ \lambda(T) &\simeq (2 \times 10^{-29}\ \mathrm{Mpc}) \left(\frac{T}{100\ \mathrm{GeV}}\right)^{-5/3} \left(\frac{\lambda_0}{1\ \mathrm{pc}}\right) g_\lambda(T), \end{aligned} \tag{6.1}$$

where $g_B(T)$ and $g_\lambda(T)$ are O(1) factors. The following linear relation can also be obtained for the magnetic fields that have experienced the inverse cascade process [76,77]

$$\frac{\lambda_0}{1\ \mathrm{pc}} \simeq a\frac{B_0}{10^{-14}\ \mathrm{G}}, \tag{6.2}$$

where the range of a is inferred to be from O(0.1) to O(1) [59]. Let us now use these inverse cascade results to see whether our results are compatible with the observations.

The inverse cascade mechanism that we want to invoke in the broken phase needs magnetic helicity in order to operate efficiently. So, let us first investigate whether our helical hypermagnetic field leads to a helical Maxwellian magnetic field after the electroweak phase transition, via calculating the magnetic helicity before and after the symmetry breaking. In the symmetric phase, the hypermagnetic helicity is defined as $\overline{\mathbf{Y}.\mathbf{B_Y}}$, where the overline represents the volume average. We calculate this quantity for our simple wave configuration of the hypermagnetic field and obtain $\overline{\mathbf{Y}.\mathbf{B_Y}} = \mathbf{Y}.\mathbf{B_Y} = k_0 y^2(t) = B_Y^2(t)/k_0$. During Standard Model electroweak symmetry breaking, three out of four gauge fields of $\mathrm{SU}(2)_L \times U(1)$ acquire mass, i.e., $W^{\pm}$ and Z, while one combination, i.e., the photon, remains massless. A thorough study of this evolution in the plasma of the early Universe is beyond the scope of this work. Therefore, we choose the following simple model presented in Sec. 2 of Ref. [59], which assumes that the system passes abruptly from the symmetric phase to the broken phase (in a way similar to that of Ref. [78]). Then, we can estimate the strength B and the magnetic helicity $\overline{\mathbf{A}.\mathbf{B}}$ of the magnetic field after the symmetry breaking. Let us recall the relations

$$\begin{aligned} Z_\mu &= c_W W_\mu^3 - s_W Y_\mu, \\ A_\mu &= s_W W_\mu^3 + c_W Y_\mu, \end{aligned} \tag{6.3}$$

where s_W and c_W are the sine and cosine of the weak mixing angle θ_W and $s_W^2 = 0.23$. It can be seen that the hypermagnetic field $\mathbf{B_Y}$ has components in both $\mathbf{B_Z}$ and $\mathbf{B_A}$. As the Higgs condensate grows at the EWPT, the W and Z fields get mass and decay. Following the simple model presented in Ref. [59], we assume that the Z component of $\mathbf{B_Y}$ decays rapidly at the EWPT. Therefore, the $\mathbf{B_Z}$ component of $\mathbf{B_Y}$ vanishes, and the electromagnetic component $\mathbf{B_A}$ remains. Moreover, the thermal expectation value $\langle W_\mu^a \rangle = 0$, since in the symmetric phase the non-Abelian gauge fields $W_\mu^a(x)$ acquire mass from their self-interactions in the plasma [47] and are screened. Then, we obtain the electromagnetic component in the form $\mathbf{E} = c_W \mathbf{E_Y}$ and $\mathbf{B} = c_W \mathbf{B_Y}$. This means that the strength decreases by about 10% ($B \simeq 0.88 B_Y$) and the magnetic helicity decreases around 20% ($\overline{\mathbf{A}.\mathbf{B}} \simeq 0.77\, \overline{\mathbf{Y}.\mathbf{B_Y}}$). Although the helicity is decreased, the Maxwellian magnetic fields of the broken phase are still helical. Hereafter, we consider the simplifying assumption of neglecting the decrease in the magnitudes of these quantities, since it does not significantly affect our order-of-magnitude estimates of the strength B_0 and the scale λ_0 of present magnetic fields.

Using the relation $\lambda = k_0^{-1}$, the scale of the hypermagnetic field used in our investigations is estimated as $\lambda(T_{\mathrm{EW}} \simeq 100\ \mathrm{GeV}) = (10^{-7} T_{\mathrm{EW}})^{-1} = 6.45 \times 10^{-28}$ pc. Let us first assume that the magnetic fields evolve trivially from EWPT until the present ($T_0 \simeq 2\mathrm{K} \simeq 17.2 \times 10^{-14}$ GeV). Then, using the mentioned relation $\lambda(t) \propto a(t) \propto T^{-1}$, the present scale of the magnetic fields is obtained as

$$\lambda(T_0) = \lambda(T_{\mathrm{EW}}) \left(\frac{100\,\mathrm{GeV}}{17.2 \times 10^{-14}\,\mathrm{GeV}} \right) \simeq 3.75 \times 10^{-13}\ \mathrm{pc}, \tag{6.4}$$

which is much lower than the acceptable scales of the present magnetic fields. When we decrease the wave number k_0 to $10^{-3} k_{\mathrm{max}}$, the saturated value of the baryonic asymmetry mentioned in Sec. V A becomes $\eta_B \simeq 10^{-10}$. Indeed, no wave number lower than this one can give the BAU in our model. The scale λ corresponding to this k_0 is $\lambda(T_{\mathrm{EW}}) \simeq 6.45 \times 10^{-25}$ pc, leading to $\lambda(T_0) \simeq 3.75 \times 10^{-10}$ pc, which is still far from the current scales of magnetic fields. These calculations show that for obtaining the present large-scale magnetic fields it is necessary to rely on an inverse cascade process which starts after the EWPT.

Let us assume that the inverse cascade process is the only nontrivial process which starts immediately after the EWPT. Then, using Eqs. (6.1), and Eq. (6.2) with $a \simeq 0.1$, we can roughly estimate λ_0 and B_0 for $\lambda(T_{\mathrm{EW}}) \simeq 6.45 \times 10^{-25}$ pc ($k_0 = 10^{-3} k_{\mathrm{max}}$) and $B(T_{\mathrm{EW}}) \simeq 3.225 \times 10^{19}$ G to obtain

$$B_0 \simeq 3.225 \times 10^{-15}\ \mathrm{G}, \quad \text{and} \quad \lambda_0 \simeq 3.225 \times 10^{-2}\ \mathrm{pc}. \tag{6.5}$$

It can be seen that the above value of B_0, along with $\eta_B \sim 10^{-10}$ already used, are within the acceptable range of present-day data. However, the value of λ_0 is much smaller than the scale usually assumed for gamma-ray propagation, which is about $\sim$1 Mpc [25,26,72].

The above results are obtained using a single-mode wave configuration of the hypermagnetic field, which is maximally helical, since its helicity density $h_Y = \mathbf{Y}.\mathbf{B_Y} = k_0 y^2(t)$ is related to its energy density $\rho_Y = \mathbf{B_Y}.\mathbf{B_Y}/2 = k_0^2 y^2(t)/2$ via the relation $k_0 h_Y = 2\rho_Y$ (or equivalently, $\rho_k = \frac{k}{2} h_k$ in Fourier space). The use of this field configuration seems to be an oversimplification; however, as we shall argue below, it is adequate for our purposes. As mentioned in Sec. I, a helical magnetic field may have been generated during the inflation (see also Ref. [79]). Even if the generated field is partially helical, it would become maximally helical through an inverse cascade mechanism after the inflation [80]. Nevertheless, let us predict the consequences of choosing a more complicated initial field configuration, namely, a superposition of the fields with different values of k_0.

To accomplish this task, we first study an analogous case in the broken phase, which investigates the evolution of the magnetic fields, taking into account the chiral anomaly [67]. It has been shown that for a continuous spectrum magnetic field a very important effect emerges; that is, the initial spectrum reddens with time, while the total helicity remains (nearly) conserved, similar to the well-studied turbulence-driven inverse cascade phenomenon for the helical magnetic fields. However, in this case, the magnetic energy and helicity transfer from shorter to longer scales occur not because of the turbulence but due to the chiral anomaly. In continuation, the authors of Ref. [67] have analyzed a special helical single-mode solution of the system of chiral magnetohydrodynamic equations (exactly like our simple wave configuration) and have shown that their qualitative conclusions reached in Ref. [67] remain valid [81]. In particular, they have shown an important property of the helical single-mode solutions in the presence of a homogeneous axial chemical potential, which is the inverse cascade phenomenon, i.e., the transfer of energy and magnetic helicity from short to large scales.

Similar works have also been done in the symmetric phase which show the same effect [45,56]. Indeed, the evolution equations of the hypermagnetic fields and the fermionic chemical potentials, taking into account the Abelian anomalous effects in the symmetric phase, are similar to those of the magnetic fields and the axial chemical potential ($\Delta\mu = \mu_L - \mu_R$) considering the chiral anomalous effects in the broken phase. Therefore, it seems that for the superposition of the fields with different values of k_0 as an initial configuration, a fast decay of one helicity mode and an exponential growth of its adjacent lower helicity mode occurs, while the total helicity remains constant. This also leads to the total magnetic energy dissipation, since $\rho_k = \frac{k}{2} h_k$ for helical fields. Finally, the helicity concentrates around the longest mode, which can be chosen to be the k_0 studied in this paper. Therefore, the study of the single mode can reveal the important features of the system and imply the behavior of the system in the presence of more complicated configurations of the hypermagnetic field.

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