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Analysis of the right-handed Majorana neutrino mass in an $SU(4) \times SU(2)_L \times SU(2)_R$ Pati-Salam model with democratic texture

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In this paper, we attempt to build a unified model with the democratic texture, that has some unification between up-type Yukawa interactions Y_ν and Y_u. Since the $S_{3L} \times S_{3R}$ flavor symmetry is chiral, the unified gauge group is assumed to be Pati-Salam type $SU(4)_c \times SU(2)_L \times SU(2)_R$. The breaking scheme of the flavor symmetry is considered to be $S_{3L} \times S_{3R} \to S_{2L} \times S_{2R} \to 0$. In this picture, the four-zero texture is desirable for realistic masses and mixings. This texture is realized by a specific representation for the second breaking of the $S_{3L} \times S_{3R}$ flavor symmetry. Assuming only renormalizable Yukawa interactions, type-I seesaw mechanism, and neglecting CP phases for simplicity, the right-handed neutrino mass matrix M_R can be reconstructed from low energy input values. Numerical analysis shows that the texture of M_R basically behaves like the "waterfall texture." Since M_R tends to be the "cascade texture" in the democratic texture approach, a model with type-I seesaw and up-type Yukawa unification $Y_\nu \simeq Y_u$ basically requires fine-tunings between parameters. Therefore, it seems to be more realistic to consider universal waterfall textures for both Y_f and M_R, e.g., by the radiative mass generation or the Froggatt-Nielsen mechanism. Moreover, analysis of eigenvalues shows that the lightest mass eigenvalue M_{R1} is too light to achieve successful thermal leptogenesis. Although the resonant leptogenesis might be possible, it also requires fine-tunings of parameters.

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I. INTRODUCTION

The flavor puzzle is one of the most stringent problems in the current particle physics. In particular, the fermion mixing matrices $U_{\rm CKM}$ [1,2] and $U_{\rm PMNS}$ [3,4] are curiously different. Various models and ideas have been considered to explain the underlying flavor dynamics of the standard model (SM). Typical approaches treat the flavor symmetries [5] and/or specific flavor textures [6,7]. In the latter approach, many researchers have studied the democratic texture [8–25]. In this approach, Yukawa interactions are assumed to have the "democratic matrix" (1), which is realized by $S_{3L} \times S_{3R}$ symmetry.

In order to explore a more fundamental understanding of flavor, building some unified model is a standard method. The grand unified theory (GUT) with the democratic texture is only discussed in [26,27], as far as the author knows. However, since these papers assumed a degenerated neutrino Yukawa matrix Y_ν, unification between Y_ν and other Y_f is difficult. In this paper, we attempt to build another unified model with the democratic texture, which has some unification between up-type Yukawa interactions Y_ν and Y_u. Since the $S_{3L} \times S_{3R}$ flavor symmetry is chiral, the unified gauge group is assumed to be Pati-Salam (PS) type $SU(4)_c \times SU(2)_L \times SU(2)_R$ (G_{422}) [28]. The breaking scheme of the flavor symmetry is considered to be $S_{3L} \times S_{3R} \to S_{2L} \times S_{2R} \to 0$. In this picture, the four-zero texture [29–32] is desirable for realistic masses and mixings. This texture is realized by a specific representation for the second breaking of the $S_{3L} \times S_{3R}$ flavor symmetry [33–35].

Assuming only renormalizable Yukawa interactions, type-I seesaw mechanism [36], and neglecting CP phases for simplicity, the right-handed neutrino mass matrix M_R can be reconstructed from low energy input values. Numerical analysis shows that the texture of M_R basically behaves like the "waterfall texture" in Table I. Since M_R tends to be the "cascade texture" in the democratic texture approach, a model with type-I seesaw and up-type Yukawa unification $Y_\nu \simeq Y_u$ basically requires fine-tunings between parameters (including its CP phases, errors of the input parameters, and schemes of gauge symmetry breaking). If we realize the breaking scheme $S_{3L} \times S_{3R} \to S_{2L} \times S_{2R} \to 0$ by some mechanism, the sector of ν_R might be too complicated to obtain cascade Y_f and waterfall M_R in a unified picture. Therefore, it seems to be more realistic to consider universal waterfall textures for both Y_f and M_R, e.g., by the radiative mass generation [37] or the Froggatt-Nielsen mechanism [38].

TABLE I. The cascade and waterfall texture, with $1 \gg \delta \gg \epsilon$ [39].

Cascade	Waterfall
$\begin{pmatrix} \epsilon & \epsilon & \epsilon \\ \epsilon & \delta & \delta \\ \epsilon & \delta & 1 \end{pmatrix}$	$\begin{pmatrix} \epsilon^2 & \epsilon\delta & \epsilon \\ \epsilon\delta & \delta^2 & \delta \\ \epsilon & \delta & 1 \end{pmatrix}$

Moreover, analysis of eigenvalues shows that the lightest mass eigenvalue M_{R1} is too light to achieve successful thermal leptogenesis [40]. Although the resonant leptogenesis [41,42] might be possible, it also requires fine-tunings of parameters.

In this study, we assume only renormalizable Yukawa interactions. However, this strong tendency to the waterfall texture originates from the seesaw relation $M_R \sim Y_u^T Y_u$. Therefore, it would be rather robust for nonrenormalizable Yukawa interactions, as far as the type-I seesaw mechanism is assumed.

This paper is organized as follows. The next section is a review of the Yukawa matrices with the democratic texture. In Sec. III, we construct a unified model with the $S_{3L} \times S_{3R}$ flavor symmetry. Section IV is a numerical analysis of mass matrix M_R in this model. Section V is devoted to conclusions.

II. THE FOUR-ZERO TEXTURE FROM THE DEMOCRATIC MATRIX APPROACH

The democratic matrix is defined as

$$Y_f^0 = \frac{K_f}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \equiv \frac{K_f}{3} D, \tag{1}$$

which is invariant under $S_{3L} \times S_{3R}$, the permutation symmetry between rows and columns. It is diagonalized by the unitary matrix $U_{\rm DC}$,

$$U_{\rm DC} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & -\frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}, \tag{2}$$

and eigenvalues are given by $Y_{fi}^0 = {\rm diag}(0, 0, K_f)$. Then, the democratic matrix produces mass only for the third generation. In order to provide masses for the first and second generations, the breaking scheme of the flavor symmetry is chosen as $S_{3L} \times S_{3R} \to S_{2L} \times S_{2R} \to 0$. Then, Yukawa matrices are represented as

$$Y_f = \frac{K_f}{3} D + \delta_f Y_f^\delta + \epsilon_f Y_f^\epsilon, \tag{3}$$

where Y_f^δ, Y_f^ϵ breaks $S_{3L} \times S_{3R}$ and $S_{2L} \times S_{2R}$, respectively. This breaking scheme is discussed in several papers [35,43–47]. The origin and specific realization of this breaking scheme have not been discussed by the authors who proposed it. For example, the radiatively generated light fermion masses by broken S_3 symmetry [37] could explain this breaking scheme. In Ref. [37], S_3 breaking effects induce departures from the democratic texture only radiatively, and light fermion masses are suppressed by typical loop factors $[1/(16\pi^2)]^{1-2}$. It naturally predicts the hierarchical relation

$$K_f \gg \delta_f \gg \epsilon_f, \tag{4}$$

which is required from realistic masses and mixings. A pedagogical explanation is also found in the review [48]. The following discussion is equivalent to Ref. [35].

The term $\delta_f Y_f^\delta$ is invariant under $S_{2L} \times S_{2R}$ between first and second indices, in order to provide mass only for the second generation. The most general form of the $S_{2L} \times S_{2R}$ invariant symmetric Y_f^δ is

$$Y_f^\delta = \begin{pmatrix} a & a & b \\ a & a & b \\ b & b & c \end{pmatrix}. \tag{5}$$

For later convenience, we parametrize $\delta_f Y_f^\delta$ as follows:

$$\delta_f Y_f^\delta = \delta_f \begin{pmatrix} \frac{\sqrt{2}r}{3} + \frac{1}{6} & \frac{\sqrt{2}r}{3} + \frac{1}{6} & -\frac{r}{3\sqrt{2}} - \frac{1}{3} \\ \frac{\sqrt{2}r}{3} + \frac{1}{6} & \frac{\sqrt{2}r}{3} + \frac{1}{6} & -\frac{r}{3\sqrt{2}} - \frac{1}{3} \\ -\frac{r}{3\sqrt{2}} - \frac{1}{3} & -\frac{r}{3\sqrt{2}} - \frac{1}{3} & \frac{2}{3} - \frac{2\sqrt{2}r}{3} \end{pmatrix}. \tag{6}$$

In Eq. (6), there are only two free parameters r, δ_f. However, it does not lose generality, because one of the parameters in Eq. (5) can be absorbed by the redefinition of K_f. Similarly, $\epsilon_f Y_f^\epsilon$ provide mass for the first generations. References [34,35] proposed that $\epsilon_f Y_f^\epsilon$ may be the doublet complex tensorial representation of the $S_{3(L+R)}$ diagonal subgroup:

$$\epsilon_f Y_f^\epsilon = \begin{pmatrix} \epsilon_1 & i\epsilon_2 & -\epsilon_1 - i\epsilon_2 \\ -i\epsilon_2 & -\epsilon_1 & \epsilon_1 + i\epsilon_2 \\ -\epsilon_1 + i\epsilon_2 & \epsilon_1 - i\epsilon_2 & 0 \end{pmatrix}. \tag{7}$$

In this case, the Yukawa matrices are approximately diagonalized as

$$U_{\rm DC}^\dagger Y_f U_{\rm DC} = U_{\rm DC}^\dagger \left[\frac{K_f}{3} D + \delta_f Y_f^\delta + \epsilon_f Y_f^\epsilon\right] U_{\rm DC} = \begin{pmatrix} 0 & \epsilon_f e^{i\phi_f} & 0 \\ \epsilon_f e^{-i\phi_f} & \delta_f & r\delta_f \\ 0 & r\delta_f & K_f \end{pmatrix}, \tag{8}$$

where $\epsilon_f e^{i\phi_f} = \sqrt{3}(\epsilon_1 + i\epsilon_2)$. Then, these Yukawa matrices lead to the "four-zero texture" or the "modified Fritzsch texture" [29–32]. This relationship between the democratic texture and the four-zero texture is studied by several authors [33–35]. In Eq. (8), $r \sim O(1)$ is required to obtain the successful Cabibbo-Kobayashi-Maskawa (CKM) matrix. This is a natural condition because $S_{3L} \times S_{3R}$ breaking would produce a relation $Y_{22} \sim Y_{23}$.

For simplicity, we neglect all CP phases of the Yukawa matrices [cf. $\phi_f = 0$ in Eq. (8)]. The effect of CP phases is discussed later. However, the qualitative result is considered to be rather robust with finite CP phases.

For the real Yukawa matrices, Eq. (8) is perturbatively diagonalized as

$$B_f^\dagger U_{\rm DC}^\dagger Y_f U_{\rm DC} B_f = {\rm diag}(y_{1f}, y_{2f}, y_{3f}), \tag{9}$$

where

$$y_{1f} \simeq -\frac{\epsilon_f^2}{\delta_f} - \frac{r^2\epsilon_f^2}{K_f}, \qquad y_{2f} \simeq \delta_f + \frac{\epsilon_f^2}{\delta_f} - \frac{r^2\delta_f^2}{K_f},$$
$$y_{3f} \simeq K_f + \frac{r^2\delta_f^2}{K_f}. \tag{10}$$

The unitary matrix B_f at leading order is found to be

$$B_f \simeq \begin{pmatrix} 1 & -\frac{\epsilon_f}{\delta_f} & 0 \\ \frac{\epsilon_f}{\delta_f} & 1 & r\frac{\delta_f}{K_f} \\ -r\frac{\epsilon_f}{K_f} & -r\frac{\delta_f}{K_f} & 1 \end{pmatrix} \simeq \begin{pmatrix} 1 & -\sqrt{-\frac{y_{f1}}{y_{f2}}} & 0 \\ \sqrt{-\frac{y_{f1}}{y_{f2}}} & 1 & r\frac{y_{f2}}{y_{f3}} \\ -r\sqrt{-\frac{y_{f1}}{y_{f2}}}\frac{y_{f2}}{y_{f3}} & -r\frac{y_{f2}}{y_{f3}} & 1 \end{pmatrix}. \tag{11}$$

Note that $y_{f1}/y_{f2} \simeq -\epsilon_f^2/\delta_f^2$ is always negative.

Therefore, the CKM matrix $V_{\rm CKM} = B_u^\dagger B_d$ (without complex phase) is calculated as

$$V_{\rm CKM} \simeq \begin{pmatrix} 1 & \sqrt{\frac{m_u}{m_c}} & -r\sqrt{\frac{m_u}{m_c}}\frac{m_c}{m_t} \\ -\sqrt{\frac{m_u}{m_c}} & 1 & -r\frac{m_c}{m_t} \\ 0 & r\frac{m_c}{m_t} & 1 \end{pmatrix} \begin{pmatrix} 1 & -\sqrt{\frac{m_d}{m_s}} & 0 \\ \sqrt{\frac{m_d}{m_s}} & 1 & r\frac{m_s}{m_b} \\ -r\sqrt{\frac{m_d}{m_s}}\frac{m_s}{m_b} & -r\frac{m_s}{m_b} & 1 \end{pmatrix} \tag{12}$$

$$\simeq \begin{pmatrix} 1 & -\left[\sqrt{\frac{m_d}{m_s}} - \sqrt{\frac{m_u}{m_c}}\right] & r\left[\sqrt{\frac{m_u}{m_c}}\frac{m_s}{m_b} - \sqrt{\frac{m_u}{m_c}}\frac{m_c}{m_t}\right] \\ \left[\sqrt{\frac{m_d}{m_s}} - \sqrt{\frac{m_u}{m_c}}\right] & 1 & r\left[\frac{m_s}{m_b} - \frac{m_c}{m_t}\right] \\ r\left[\sqrt{\frac{m_d}{m_s}}\frac{m_c}{m_t} - \sqrt{\frac{m_d}{m_s}}\frac{m_s}{m_b}\right] & -r\left[\frac{m_s}{m_b} - \frac{m_c}{m_t}\right] & 1 \end{pmatrix}. \tag{13}$$

Here, we omit the minus sign in the square root ($\sqrt{-m_u/m_c} \to \sqrt{m_u/m_c}$). It predicts V_{cb} and V_{ts} at leading order as follows:

$$V_{cb} \simeq -V_{ts} \simeq r\left[\frac{m_s}{m_b} - \frac{m_c}{m_t}\right]. \tag{14}$$

If the parameters K_f, δ_f, ϵ_f have CP phases, each CKM matrix element obtains overall phases and relative phases, such as $\sqrt{\frac{m_d}{m_s}} - \sqrt{\frac{m_u}{m_c}} \to e^{i\phi}\left[\sqrt{\frac{m_d}{m_s}} - e^{i\eta}\sqrt{\frac{m_u}{m_c}}\right]$. In particular, the best value of χ^2 fit $r = \sqrt{81/32} \simeq 1.59$ [35] gives excellent agreement between the prediction and the observation of absolute values of the CKM matrix elements.

III. $SU(4)_c \times SU(2)_L \times SU(2)_R$ MODEL WITH DEMOCRATIC TEXTURE

In order to explore a more fundamental understanding of flavor, building some unified model is a standard method. The grand unified theory (GUT) with the democratic texture is only discussed in [26,27], as far as the author knows. However, since these papers assumed degenerated Y_ν, unification between Y_ν and other Y_f is difficult. In this paper, we attempt to build another unified model with the democratic texture, which has some unification between Y_ν and Y_u. Since the $S_{3L} \times S_{3R}$ flavor symmetry is chiral,[1] the

[1] In the $SO(10)$ GUT, the flavor symmetry should be single S_3, and the condition $c_f = 0$ similar to Eq. (31) should be assumed.

TABLE II. The charge assignments of the SM fermions and Higgs fields under the gauge and the flavor symmetries.

	$SU(4)_c$	$SU(2)_L$	$SU(2)_R$	S_{3L}	S_{3R}
$\Psi_{Li} = (q^\alpha_{Li}, l_{Li})$	4	2	1	$1_L + 2_L$	1_R
$\Psi_{Ri} = (q^\alpha_{Ri}, l_{Ri})$	4	1	2	1_L	$1_R + 2_R$
Φ	1	2	2	1_L	1_R
Σ	15	2	2	1_L	1_R
Δ_R	10	1	3	1_L	1_R

unified gauge group is assumed to be Pati-Salam (PS) type $SU(4)_c \times SU(2)_L \times SU(2)_R$ (G_{422}) [28].

To produce realistic fermion masses, we consider the minimal contents of Higgs fields with the following representations under the G_{422} group:

$$\Phi:(1,2,2), \qquad \Sigma:(15,2,2), \qquad \Delta_R:(10,1,3). \tag{15}$$

Although other representations are also possible, such as (4,1,2) in [49,50], we consider only renormalizable interactions to control Yukawa interactions.

The field contents of the unified model are in Table II. These Higgs contents are sufficient to break the PS gauge group G_{224} to the SM gauge group $G_{\rm SM}$. For example, a breaking scheme of the gauge symmetry with these Higgs contents is discussed in the context of the noncommutative geometry [51,52]. We do not discuss the energy scales and order of the symmetry breakings. However, the final result is considered to be rather independent from breaking schemes.

The renormalizable Yukawa interactions invariant under G_{422} are found to be

$$\mathcal{L}_{\rm Yukawa} = \bar\Psi_{Ri}(Y^1_{ij}\Phi + Y^{15}_{ij}\Sigma)\Psi_{Lj} + {\rm H.c.} \tag{16}$$

Note that Yukawa matrices $Y^{1,15}$ become symmetric matrices if we impose the left-right symmetry between $\Psi_L \leftrightarrow \Psi_R$. These $Y^{1,15}$ are divided into $S_{3L} \times S_{3R}$ preserving and breaking parts respectively:

$$Y^1 = K_1 D + \delta Y_1, \qquad Y^{15} = K_{15} D + \delta Y_{15}. \tag{17}$$

In order to obtain the desirable masses and mixings, we assume $K_{15} = 0$ and δY_1 does not have $S_{3L} \times S_{3R}$ breaking elements δ_f. Then Y_{15} is treated as a perturbation, as in the previous study [26]. Vacuum expectation values of these Higgs fields are taken to be

$$\langle\Phi\rangle = {\rm Diag}(1,1,1,1) \times \begin{pmatrix} v_u^1 & 0 \\ 0 & v_d^1 \end{pmatrix},$$
$$\langle\Sigma\rangle = {\rm Diag}(1,1,1,-3) \times \begin{pmatrix} v_u^{15} & 0 \\ 0 & v_d^{15} \end{pmatrix}, \tag{18}$$

in the representation space of $\Psi_{L,R} = (q^1_{L,R}, q^2_{L,R}, q^3_{L,R}, l_{L,R})$.

This setup leads to the following mass matrices [53–55]:

$$M_u = v_u^1(K_1 D + \delta Y_1) + v_u^{15}\delta Y_{15} = v_u^1 K_1 D + v_u^1 \delta Y_1 + v_u^{15}\delta Y_{15}, \tag{19}$$

$$M_\nu^D = v_u^1(K_1 D + \delta Y_1) - 3v_u^{15}\delta Y_{15} = v_u^1 K_1 D + v_u^1 \delta Y_1 - 3v_u^{15}\delta Y_{15}, \tag{20}$$

$$M_d = v_d^1(K_1 D + \delta Y_1) + v_d^{15}\delta Y_{15} = v_d^1 K_1 D + v_d^1 \delta Y_1 + v_d^{15}\delta Y_{15}, \tag{21}$$

$$M_e = v_d^1(K_1 D + \delta Y_1) - 3v_d^{15}\delta Y_{15} = v_d^1 K_1 D + v_d^1 \delta Y_1 - 3v_d^{15}\delta Y_{15}. \tag{22}$$

In particular, effective Yukawa matrices are explicitly written as

$$Y_u = \begin{pmatrix} 0 & \epsilon_u & 0 \\ \epsilon_u & \delta_u & r\delta_u \\ 0 & r\delta_u & K_u \end{pmatrix}, \qquad Y_d = \begin{pmatrix} 0 & \epsilon_d & 0 \\ \epsilon_d & \delta_d & r\delta_d \\ 0 & r\delta_d & K_d \end{pmatrix}, \tag{23}$$

$$Y_\nu = \begin{pmatrix} 0 & \epsilon_\nu & 0 \\ \epsilon_\nu & \delta_\nu & r\delta_\nu \\ 0 & r\delta_\nu & K_\nu \end{pmatrix}, \qquad Y_e = \begin{pmatrix} 0 & \epsilon_e & 0 \\ \epsilon_e & \delta_e & r\delta_e \\ 0 & r\delta_e & K_e \end{pmatrix}, \tag{24}$$

with

$$K_{u,d} = K_{\nu,e}, \qquad \delta_{u,d} = -\frac{1}{3}\delta_{\nu,e}, \qquad \epsilon_{u,d} = \epsilon_{\nu,e}. \tag{25}$$

These conditions lead to the famous Georgi-Jarlskog relation [56]

$$m_d = 3m_e, \qquad m_s = \frac{1}{3}m_\mu, \qquad m_b = m_\tau, \tag{26}$$

and similar formulas hold for up-type fermions.

IV. ANALYSIS OF THE RIGHT-HANDED MAJORANA NEUTRINO MASS MATRIX

In this section, we analyze the right-handed neutrino mass matrix M_R in the PS model with the four-zero Yukawa textures. Many papers have studied this kind of model, such as SO(10) GUT with the four-zero texture [30,31,57,58]. However, the purpose of this paper is to analyze texture of M_R quantitatively in a united model with the democratic texture.

M_R emerges from the following interaction:

$$\mathcal{L}_{\rm Majorana} = \bar{\Psi}^c_{Ri} Y^{10}_{ij} \Delta_R \Psi_{Rj} + \text{H.c.}, \tag{27}$$

when Δ_R obtain a vacuum expectation value

$$\langle \Delta_R \rangle = \text{Diag}(0,0,0,1) \times \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}. \tag{28}$$

Because Y^{10} is transformed as $(\mathbf{1_R} + \mathbf{2_R}) \times (\mathbf{1_R} + \mathbf{2_R})$, it has two S_{3R} invariant terms [14]

$$Y^{10} = K_{10} D + c_{10} 1_3 + \delta Y_{10}, \tag{29}$$

where 1_3 is the 3×3 identity matrix.

To obtain the observed light neutrino masses, we assume the type-I seesaw mechanism [36]

$$m_\nu = \frac{v^2}{2} Y_\nu^T M_R^{-1} Y_\nu. \tag{30}$$

In this case,

$$\delta Y_{10} \gg c_{10} \simeq 0 \tag{31}$$

is required by phenomenological reason. The numerical analysis shown later reveals that Y_ν with a large $c_{10} \gg \delta Y_{10}$ are incompatible to obtain the observed large neutrino mixings.

If the flavor symmetry breaking $S_{3L} \times S_{3R} \to S_{2L} \times S_{2R} \to 0$ also controls the structure of M_R, and if there is no fine-tuning between the parameters, the form of M_R should be the following cascade texture in Table I:

$$M_R \sim v_R \begin{pmatrix} \epsilon & \epsilon & \epsilon \\ \epsilon & \delta & \delta \\ \epsilon & \delta & 1 \end{pmatrix}. \tag{32}$$

The light neutrino mass, Eq. (30), is diagonalized by

$$m_\nu \equiv V_\nu^* m_\nu^{\rm diag} V_\nu^\dagger, \tag{33}$$

where $m_\nu^{\rm diag} = \text{diag}(m_1, m_2, m_3)$. This mass matrix is rewritten as

$$m_\nu = B_e^* U_{\rm PMNS}^* m_\nu^{\rm diag} U_{\rm PMNS}^\dagger B_e^\dagger, \tag{34}$$

with the neutrino mixing matrix $U_{\rm PMNS} = B_e^\dagger V_\nu$ and B_e (11) for the charged leptons.

Ignoring all of the complex phases for simplicity, we can reconstruct M_R by the seesaw formula:

$$M_R = \frac{v^2}{2} Y_\nu^T m_\nu^{-1} Y_\nu \tag{35}$$

$$= \frac{v^2}{2} Y_\nu^T B_e U_{\rm PMNS} (m_\nu^{\rm diag})^{-1} U_{\rm PMNS}^T B_e^T Y_\nu. \tag{36}$$

As a benchmark, $M_R(\Lambda_{\rm GUT}) = Y_\nu(\Lambda_{\rm GUT})^T m_\nu(\Lambda_{\rm GUT}) Y_\nu \times (\Lambda_{\rm GUT})$ at the GUT scale $\Lambda_{\rm GUT} = 2 \times 10^{16}$ GeV can be evaluated as

$$\frac{M_R(\Lambda_{\rm GUT})}{[\rm GeV]} \simeq \frac{[\rm meV]}{m_1} \begin{pmatrix} 1.876 \times 10^7 & -3.623 \times 10^8 & -1.009 \times 10^{11} \\ -3.623 \times 10^8 & 6.996 \times 10^9 & 1.948 \times 10^{12} \\ -1.009 \times 10^{11} & 1.948 \times 10^{12} & 5.424 \times 10^{14} \end{pmatrix} \tag{37}$$

$$+ \frac{[\rm meV]}{m_2} \begin{pmatrix} 3.302 \times 10^7 & -2.173 \times 10^9 & -2.849 \times 10^{11} \\ -2.173 \times 10^9 & 1.429 \times 10^{11} & 1.874 \times 10^{13} \\ -2.849 \times 10^{11} & 1.874 \times 10^{13} & 2.457 \times 10^{15} \end{pmatrix} \tag{38}$$

$$+ \frac{[\rm meV]}{m_3} \begin{pmatrix} 6.255 \times 10^7 & 1.012 \times 10^{10} & 3.975 \times 10^{11} \\ 1.012 \times 10^{10} & 1.637 \times 10^{12} & 6.431 \times 10^{13} \\ 3.975 \times 10^{11} & 6.431 \times 10^{13} & 2.526 \times 10^{15} \end{pmatrix}. \tag{39}$$

The parameters used here are summarized in Table III. The fermion masses at the GUT scale $m_f(\Lambda_{\rm GUT})$ are taken from [59]. In most cases of this model, the order of light neutrino masses m_i becomes the normal hierarchy. The inverted hierarchy $m_1 \simeq m_2 \gg m_3$ is unnatural because the hierarchy of M_R should overcome the ratio m_t^2/m_c^2. The renormalization of the neutrino mass can be neglected for the normal hierarchy case [60,61]. Then, neutrino mixing angles and mass square differences are taken from the latest global fit [62], without renormalization running. A similar parameter set is used in [63].

TABLE III. Input values (for the SM) at the scale $M_{\rm GUT} = 2\times 10^{16}$ GeV. A similar parameter set is used in [63].

m_u (MeV)	0.48	θ^l_{12}	33.48°
m_c (GeV)	0.235	θ^l_{23}	42.3°
m_t (GeV)	74.0	θ^l_{13}	8.5°
m_e (MeV)	0.470	Δm^2_{31} (eV^2)	2.457×10^{-3}
m_μ (MeV)	99.14	Δm^2_{21} (eV^2)	7.50×10^{-5}
m_τ (MeV)	1685		

Equations (37)–(39) shows that the right-handed neutrino mass matrix $M_R \sim Y_u^T Y_u$ rather tends to be the waterfall texture in Table I,

$$M_R \sim v_R \begin{pmatrix} \epsilon^2 & \epsilon\delta & \epsilon \\ \epsilon\delta & \delta^2 & \delta \\ \epsilon & \delta & 1 \end{pmatrix}, \tag{40}$$

for each small mass eigenvalue m_i. Then, it seems to be difficult to explain this texture by the breaking scheme $S_{3L}\times S_{3R}\to S_{2L}\times S_{2R}\to 0$. Hereafter we precisely check the form of the M_R by numerical analysis.

A. Numerical results

Using the mass difference values Δm^2_{3i} in Table III,

$$m_3 = \pm\sqrt{m_1^2 + 2457}\ [\mathrm{meV}],$$
$$m_2 = \pm\sqrt{m_1^2 + 75}\ [\mathrm{meV}], \tag{41}$$

the mass matrices M_R (37)–(39) is expressed as a function of m_1, $M_R(\Lambda_{\rm GUT}) = M_R(m_1)$.

Figure 1 shows lighter matrix elements $(M_R)_{11}, (M_R)_{12}$, $(M_R)_{13}$, and $(M_R)_{22}$ of the $M_R(m_1)$ at the GUT scale $\Lambda_{\rm GUT} = 2\times 10^{16}$[GeV], as a function of m_1. The signatures of m_2 and m_3 are taken as the top of the figures. From Fig. 1, we can see the hierarchical structure of the M_R. These matrix elements basically behave like the waterfall texture $(M_R)_{22}\sim (M_R)_{13} \gg (M_R)_{22} \gg (M_R)_{11}$. Several changes of sign are due to cancellations among Eqs. (37)–(39).

This behavior shows that the cascade texture $(M_R)_{22}\gg (M_R)_{13}\sim (M_R)_{22}\sim (M_R)_{11}$ cannot be realized without fine-tunings of parameters in this model. In particular, the four-zero texture for M_R [equivalent to $(M_R)_{11} = (M_R)_{13} = 0$] is also difficult to realize without fine-tuning. However, in this analysis, approximate four-zero texture

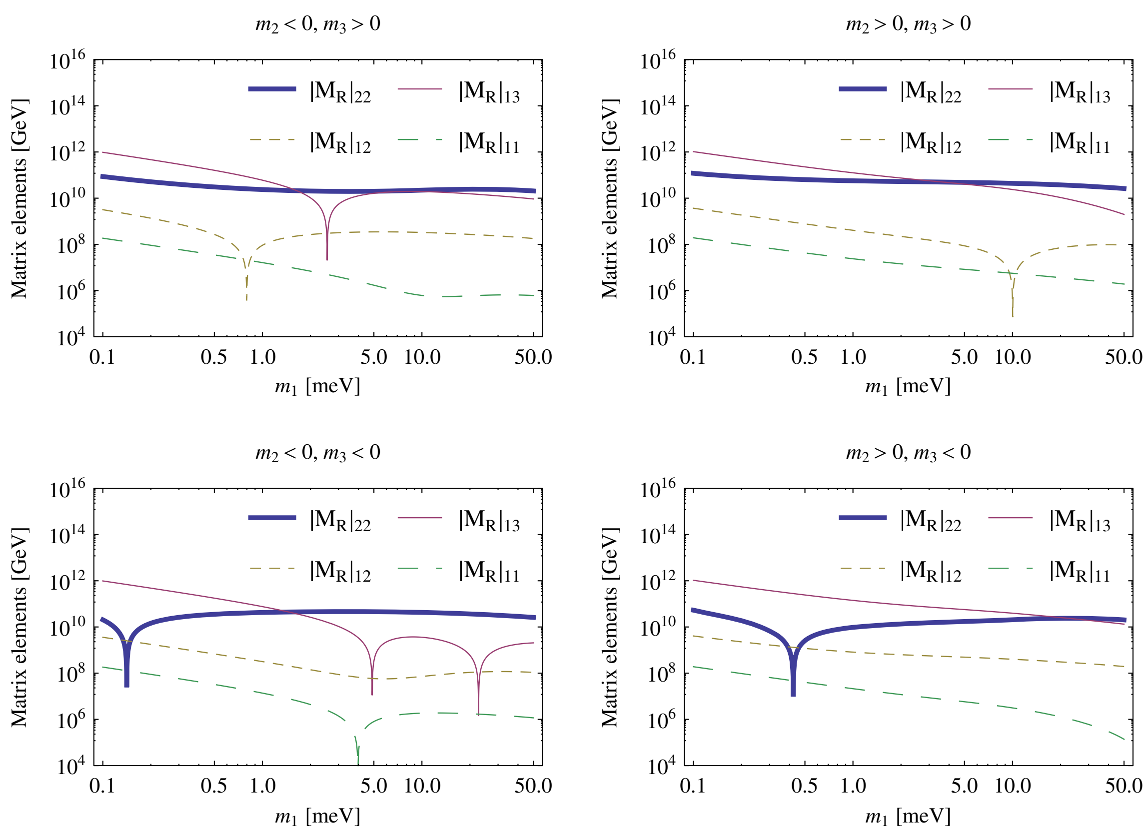

FIG. 1. Lighter matrix elements $(M_R)_{11}$, $(M_R)_{12}$, $(M_R)_{13}$, and $(M_R)_{22}$ of the $M_R(m_1)$ at the GUT scale $\Lambda_{\rm GUT} = 2\times 10^{16}$[GeV], as a function of m_1. The signatures of m_2 and m_3 are taken as the top of the figures.

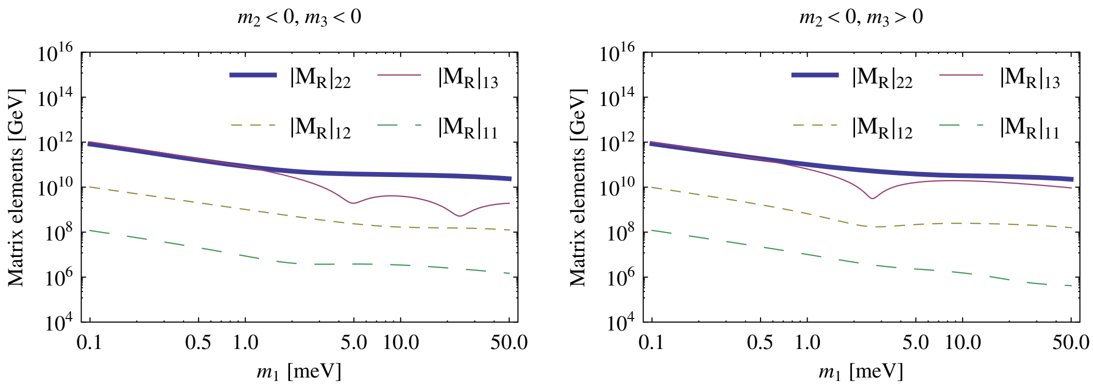

FIG. 2. Lighter matrix elements $(M_R)_{11}$, $(M_R)_{12}$, $(M_R)_{13}$, and $(M_R)_{22}$ of the $M_R(m_1)$, with finite dirac CP phase $\delta_{CP} = \pi/2$ of the PMNS matrix. Other parameters are taken to be the same as Fig. 1 (for m_2, only negative sign $m_2 < 0$ is presented).

$(M_R)_{12} \gg (M_R)_{13} \sim (M_R)_{11}$ is realized around $m_1 \sim 4$ meV with $m_{2,3} < 0$.

So far, the parameters of the model have been assumed to be real. Here we will discuss the effect of CP phases shortly. Figure 2 shows lighter matrix elements $(M_R)_{11}$, $(M_R)_{12}, (M_R)_{13}$, and $(M_R)_{22}$ of the $M_R(m_1)$, with finite dirac CP phase $\delta_{CP} = \pi/2$ of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix. Other parameters are taken to be the same as Fig. 1 (for m_2, only negative sign $m_2 < 0$ is presented). In Fig 2, the cancellations of $(M_R)_{ij}$ found in Fig. 1 vanish by the finite CP phases, and the cascade texture is evidently impossible with this parameter set. By assuming finite CP phases for other parameters, we found that the cancellations are basically smoothed or vanished. It is plausible that M_R strongly tend to be the waterfall texture (40). Therefore, in this democratic matrix approach, a model with type-I seesaw and up-type Yukawa unification $Y_\nu \simeq Y_u$ basically requires fine-tunings between parameters (including its CP phases, errors of the input parameters, and gauge symmetry breaking schemes). If we realize the breaking scheme $S_{3L} \times S_{3R} \to S_{2L} \times S_{2R} \to 0$ by some mechanism, the sector of ν_R might be too complicated to obtain cascade Y_f and waterfall M_R in a unified picture. Therefore, it seems to be more realistic to consider universal waterfall textures for both Y_f and M_R, e.g., by the radiative mass generation [37] or the Froggatt-Nielsen mechanism [38].

B. Mass eigenvalues and thermal leptogenesis

Figure 3 shows three mass eigenvalues M_{Ri} of the $M_R(m_1)$ at the GUT scale $\Lambda_{\rm GUT} = 2 \times 10^{16}$[GeV], as a function of m_1. The parameters are taken to be the same as Fig. 1 (for m_2, only negative sign $m_2 < 0$ is presented). Basically the eigenvalues M_{Ri} are strongly hierarchical, because M_R has large hierarchy such as $M_R \sim Y_u^T Y_u$. The largest eigenvalue M_{R3} changes its sign around $m_1 \sim 2$ meV. This is due to cancellation for the 33 element of M_R, between Eq. (37) and Eq. (38) around the region $m_2 \sim 5 m_1$. Similarly, the cancellation for $(M_R)_{11}$ induces the change of sign for two smaller eigenvalues, M_{R1} and M_{R2}.

These figures exhibit that the lightest mass eigenvalue tends to be rather small, $M_{R1} \lesssim 10^5$ GeV, except the cancellation regions. The successful thermal leptogenesis [40] requires $M_{R1} > 4.9 \times 10^8$ GeV for the hierarchical M_{Ri} [64,65]. Then, it is nearly impossible to explain the observed baryon asymmetry by the thermal leptogenesis in this model. The resonant leptogenesis [41,42] would be

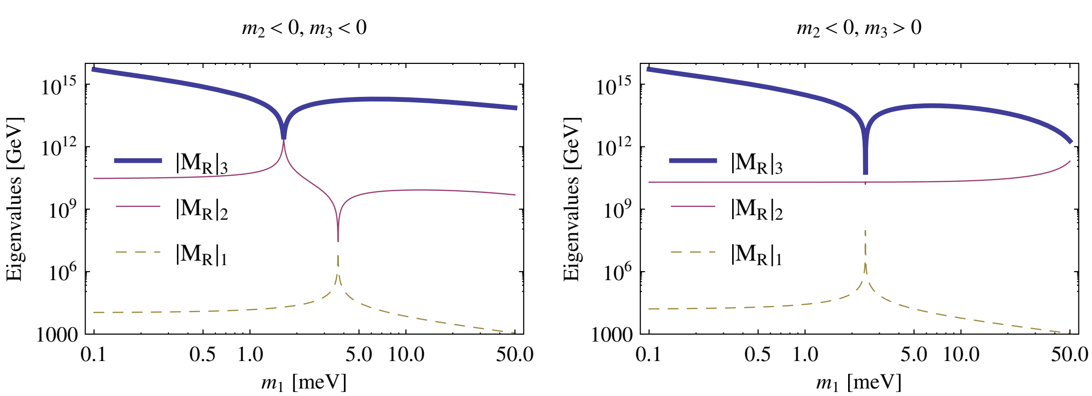

FIG. 3. Three mass eigenvalues M_{Ri} of the $M_R(m_1)$ at the GUT scale $\Lambda_{\rm GUT} = 2 \times 10^{16}$[GeV], as a function of m_1. The parameters are taken to be the same as Fig. 1 (for m_2, only negative sign $m_2 < 0$ is presented).

possible in the cancellation region with $M_{R1} \simeq M_{R2}$ ($m_3 < 0, m_1 \simeq 3$ meV). Similar results for SO(10) are found in Ref. [58]. However, this cancellation region can be easily vanished by finite CP phases. Therefore, successful leptogenesis also requires fine-tunings of the parameters in this model.

In this study, we assume only renormalizable Yukawa interactions. However, this strong tendency to the waterfall texture originates from the seesaw relation $M_R \sim Y_u^T Y_u$. Therefore, it would be rather robust for nonrenormalizable Yukawa interactions, as far as the type-I seesaw mechanism is assumed.

V. CONCLUSIONS

In this paper, we attempt to build a unified model with the democratic texture, which has some unification between up-type Yukawa interactions Y_ν and Y_u. Since the $S_{3L} \times S_{3R}$ flavor symmetry is chiral, the unified gauge group is assumed to be Pati-Salam (PS) type $SU(4)_c \times SU(2)_L \times SU(2)_R$ (G_{422}). The breaking scheme of the flavor symmetry is considered to be $S_{3L} \times S_{3R} \to S_{2L} \times S_{2R} \to 0$. In this picture, the four-zero texture is desirable for realistic mass and mixings. This texture is realized by a specific representation for the second breaking of the $S_{3L} \times S_{3R}$ flavor symmetry.

Assuming only renormalizable Yukawa interactions, type-I seesaw mechanism, and neglecting CP phases for simplicity, the right-handed neutrino mass matrix M_R can be reconstructed from low energy input values. Numerical analysis shows that the texture of M_R basically behaves like the waterfall texture in Table I. Since M_R tends to be the cascade texture in the democratic texture approach, a model with type-I seesaw and up-type Yukawa unification $Y_\nu \simeq Y_u$ basically requires fine-tunings between parameters (including its CP phases, errors of the input parameters, and schemes of gauge symmetry breaking). If we realize the breaking scheme $S_{3L} \times S_{3R} \to S_{2L} \times S_{2R} \to 0$ by some mechanism, the sector of ν_R might be too complicated to obtain cascade Y_f and waterfall M_R in a unified picture. Therefore, it seems to be more realistic to consider universal waterfall textures for both Y_f and M_R, e.g., by the radiative mass generation or the Froggatt-Nielsen mechanism.

Moreover, analysis of eigenvalues shows that the lightest mass eigenvalue M_{R1} is too light to account the baryon asymmetry of the universe by the thermal leptogenesis. Although the resonant leptogenesis might be possible, it also requires fine-tunings of parameters.

In this study, we assume only renormalizable Yukawa interactions. However, this strong tendency to the waterfall texture originates from the seesaw relation $M_R \sim Y_u^T Y_u$. Therefore, it would be rather robust for nonrenormalizable Yukawa interactions, as far as the type-I seesaw mechanism is assumed.

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