Electromagnetic properties of massive neutrinos in low-energy elastic neutrino-electron scattering

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A thorough account of electromagnetic interactions of massive neutrinos in the theoretical formulation of low-energy elastic neutrino-electron scattering is given. The formalism of neutrino charge, magnetic, electric, and anapole form factors defined as matrices in the mass basis is employed under the assumption of three-neutrino mixing. The flavor change of neutrinos traveling from the source to the detector is taken into account and the role of the source-detector distance is inspected. The effects of neutrino flavor-transition millicharges and charge radii in the scattering experiments are pointed out.

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I. INTRODUCTION

In the standard model neutrinos are massless left-handed fermions which very weakly interact with matter via exchange of the $W^{\pm}$ and Z^0 bosons. The development of our knowledge about neutrino masses and mixing [1–3] provides a basis for exploring neutrino properties and interactions beyond the standard model (BSM). In this respect, the study of nonvanishing electromagnetic characteristics of massive neutrinos is of particular interest [4–6]. It can help not only to shed light on whether neutrinos are Dirac or Majorana particles, but also to constrain the existing BSM theories and/or to hint at new physics.

The possible electromagnetic properties of massive neutrinos include the electric charge (millicharge), the charge radius, the dipole magnetic and electric moments, and the anapole moment. Their effects can be searched in astrophysical environments, where neutrinos propagate in strong magnetic fields and dense matter [7], and in laboratory measurements of neutrinos from various sources. In the latter case, a very sensitive and widely used method is provided by the direct measurement of low-energy elastic (anti)neutrino-electron scattering in reactor, accelerator, and solar experiments. A general strategy of such experiments consists in determining deviations of the scattering cross section differential with respect to the energy transfer from the value predicted by the standard model of the electroweak interaction.

So far, neither astrophysical observations nor laboratory measurements have evidenced nonvanishing electromagnetic properties of neutrinos, and only some constraints on their values have been obtained (the updated list of constraints is given in the review paper [5]). For example, the most stringent constraint on the neutrino millicharge obtained in the scattering experiments is

$$|e_{\nu_e}| \lesssim 1.5 \times 10^{-12} e, \tag{1}$$

which has been derived in Ref. [8] from the analysis of the reactor data [9] using the free-electron approximation for the differential cross section. If one goes beyond the free-electron approximation and takes into account the binding of electrons to atoms in the detector (the atomic-ionization effect), then one arrives at [10]

$$|e_{\nu_e}| < 1.1 \times 10^{-12} e. \tag{2}$$

This bound is orders of magnitude less stringent than those that follow from astrophysics [11],

$$|e_{\nu_e}| \lesssim 1.3 \times 10^{-19} e,$$

and the neutrality of matter [12]

$$|e_{\nu_e}| \lesssim 3 \times 10^{-21} e.$$

While neutrinos are generally believed to be electrically neutral particles, they are still expected to have nonzero charge radii. The current constraints from the scattering experiments ($|\langle r_\nu^2 \rangle| \lesssim 10^{-32} - 10^{-31}\,\mathrm{cm}^2$) differ only by 1 to 2 orders of magnitude from the values calculated within the minimally extended standard model with right-handed neutrinos ($|\langle r_{\nu_\ell}^2 \rangle| \sim 10^{-33}\ \mathrm{cm}^2,\ \ell = e,\ \mu,\ \tau$) [13]. This

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indicates that the standard model neutrino charge radii could be experimentally tested in the near future.

The experimental bounds for the neutrino millicharges and charge radii discussed above have been obtained under an implicit assumption that neutrinos do not change flavor when scattering on electrons in the detector. However, making this assumption for neutrino-electron scattering due to weak interaction is not necessarily justified in the case of electromagnetic interaction. It means that possible contributions from the neutrino flavor-transition electromagnetic properties should also be taken into account in the data analysis [14]. Therefore, the present work aims at filling the lacuna in the basic theoretical apparatus usually employed for interpretation and analysis of the data of experiments searching for electromagnetic interactions of massive neutrinos in the elastic neutrino-electron scattering.

The paper is organized as follows. Section II delivers a brief overview of neutrino electromagnetic form factors. In Sec. III general formulas for the scattering amplitude and differential cross section are presented. Then, in Sec. IV, the free-electron approximation and the stepping formula for the differential cross section are discussed. Section V is devoted to the role of the source-detector distance. The conclusions are drawn in Sec. VI.

II. ELECTROMAGNETIC INTERACTIONS OF MASSIVE NEUTRINOS

A detailed review of neutrino electromagnetic properties and interactions can be found in Refs. [4–6]. In this section we briefly outline the general form of the electromagnetic interactions of Dirac and Majorana neutrinos.

There are at least three massive neutrino fields ν_j with respective masses m_j ($j = 1, 2, 3$), which are mixed with the three active flavor neutrinos ν_e, ν_μ, ν_τ. Therefore, the effective electromagnetic interaction Hamiltonian can be presented as

$$\mathcal{H}_{\rm em}^{(\nu)} = j_\lambda^{(\nu)} A^\lambda = \sum_{j,k=1}^{3} \bar{\nu}_j \Lambda_\lambda^{jk} \nu_k A^\lambda, \tag{3}$$

where we take into account possible transitions between different massive neutrinos. The physical effect of $\mathcal{H}_{\rm em}^{(\nu)}$ is described by the effective electromagnetic vertex, which in the momentum-space representation depends only on the four-momentum $q = p_j - p_k$ transferred to the photon and can be expressed as follows:

$$\Lambda_\lambda(q) = \left(\gamma_\lambda - \frac{q_\lambda \not{q}}{q^2}\right)[f_Q(q^2) + f_A(q^2) q^2 \gamma^5] - i\sigma_{\lambda\rho} q^\rho [f_M(q^2) + i f_E(q^2)\gamma^5], \tag{4}$$

where $\sigma_{\lambda\rho} = i(\gamma_\lambda\gamma_\rho - \gamma_\rho\gamma_\lambda)/2$. Here $\Lambda_\lambda(q)$ is a 3×3 matrix in the space of massive neutrinos expressed in terms of the four Hermitian 3×3 matrices of form factors,

$$f_Q = f_Q^\dagger, \quad f_M = f_M^\dagger, \quad f_E = f_E^\dagger, \quad f_A = f_A^\dagger, \tag{5}$$

where Q, M, E, A refer, respectively, to the real charge, magnetic, electric, and anapole neutrino form factors. The Lorentz-invariant form of the vertex function (4) is also consistent with electromagnetic gauge invariance that implies four-current conservation.

For the coupling with a real photon in vacuum ($q^2 = 0$) one has

$$f_Q^{jk}(0) = e_{jk}, \quad f_M^{jk}(0) = \mu_{jk}, \quad f_E^{jk}(0) = \epsilon_{jk}, \quad f_A^{jk}(0) = a_{jk}, \tag{6}$$

where e_{jk}, μ_{jk}, ϵ_{jk}, and a_{jk} are, respectively, the neutrino charge, magnetic moment, electric moment, and anapole moment of diagonal ($j = k$) and transition ($j \neq k$) types.

Consider the diagonal case $j = k$. The Hermiticity of the electromagnetic current and the assumption of its invariance under discrete symmetries' transformations put certain constraints on the form factors, which are in general different for the Dirac and Majorana neutrinos. In the case of Dirac neutrinos, the assumption of CP invariance combined with the Hermiticity of the electromagnetic current J_μ implies that the electric dipole form factor vanishes, $f_E = 0$. At zero momentum transfer only $f_Q(0)$ and $f_M(0)$—which are called the electric charge and the magnetic moment, respectively—contribute to the Hamiltonian (3). The Hermiticity also implies that f_Q, f_A, and f_M are real. In contrast, in the case of Majorana neutrinos (regardless of whether CP invariance is violated or not) the charge, dipole magnetic, and electric moments vanish, $f_Q = f_M = f_E = 0$, so that only the anapole moment can be nonvanishing among the electromagnetic moments. Note that it is possible to prove [16–18] that the existence of a nonvanishing magnetic moment for a Majorana neutrino would bring about a clear evidence for CPT violation.

In the off-diagonal case $j \neq k$, the Hermiticity by itself does not imply restrictions on the form factors of Dirac neutrinos. It is possible to show [16] that, if the assumption of the CP invariance is added, the form factors f_Q, f_M, f_E, and f_A should have the same complex phase. For the Majorana neutrino, if CP invariance holds, there could be either a transition magnetic or a transition electric moment. Finally, as in the diagonal case, the anapole form factor of a Majorana neutrino can be nonzero.

It is usually believed that the neutrino electric charge $e_\nu = f_Q(0)$ is zero. In the standard model of $SU(2)_L \times U(1)_Y$ electroweak interactions it is possible to get [19] a general proof that neutrinos are electrically neutral, which is based on the requirement of electric charge quantization. The direct calculations of the neutrino charge in the standard model for massless (see, for instance, Refs. [20,21]) and massive neutrinos [22,23] also prove that, at least at the one-loop level, the neutrino electric charge is gauge independent and vanishes. However, if the neutrino has a mass, it still may become electrically millicharged. A brief discussion of different mechanisms for introducing millicharged particles including neutrinos can be found in Ref. [24]. In the case of millicharged massive neutrinos, electromagnetic gauge invariance implies that the diagonal electric charges e_{jj} ($j = 1, 2, 3$) are equal [6]. It should be mentioned that the most stringent experimental constraints on the electric charge of the neutrino can be obtained from the neutrality of matter.

Even if the electric charge of a neutrino is zero, the electric form factor $f_Q(q^2)$ can still contain nontrivial information about neutrino electrostatic properties [5]. A neutral particle can be characterized by a superposition of two charge distributions of opposite signs, so that the particle form factor $f_Q(q^2)$ can be nonzero for $q^2 \neq 0$. The mean charge radius (in fact, it is the squared charge radius) of an electrically neutral neutrino is given by

$$\langle r_\nu^2 \rangle = 6 \frac{d f_Q(q^2)}{dq^2}\bigg|_{q^2=0}, \tag{7}$$

which is determined by the second term in the power-series expansion of the neutrino charge form factor.

The most well studied and understood among the neutrino electromagnetic characteristics are the dipole magnetic and electric moments, which are given by the corresponding form factors at $q^2 = 0$:

$$\mu_\nu = f_M(0), \qquad \epsilon_\nu = f_E(0). \tag{8}$$

The diagonal magnetic and electric moments of a Dirac neutrino in the minimally extended standard model with right-handed neutrinos (derived for the first time in Ref. [25]) are, respectively,

$$\mu_{jj}^D = \frac{3 e_0 G_F m_j}{8\sqrt{2}\pi^2} \approx 3.2 \times 10^{-19} \mu_{\rm B} \left(\frac{m_j}{1\ {\rm eV}}\right),$$
$$\epsilon_{jj}^D = 0, \tag{9}$$

where $\mu_{\rm B}$ is the Bohr magneton. According to Eq. (9) the value of the neutrino magnetic moment is very small. However, in many other theoretical frameworks (beyond the minimally extended standard model) the neutrino magnetic moment can reach values that are of interest for the next generation of terrestrial experiments and also accessible for astrophysical observations.

The notion of an anapole moment for a Dirac particle was introduced by Zel'dovich [26] after the discovery of parity violation. In order to understand the physical characteristics of the anapole moment, it is useful to consider its effect in the interactions with external electromagnetic fields. The neutrino anapole moment contributes to the scattering of neutrinos with charged particles. In order to discuss its effects, it is convenient to consider strictly neutral neutrinos with $f_Q(0) = 0$ and define a reduced charge form factor $\tilde{f}_Q(q^2)$ such that

$$f_Q(q^2) = q^2 \tilde{f}_Q(q^2). \tag{10}$$

Then, from Eq. (7), apart from a factor 1/6, the reduced charge form factor at $q^2 = 0$ is just the squared neutrino charge radius:

$$\tilde{f}_Q(0) = \frac{1}{6} \langle r_\nu^2 \rangle. \tag{11}$$

Let us now consider the charge and anapole parts of the neutrino electromagnetic vertex function, as

$$\Lambda_\lambda^{Q,A}(q) = (\gamma_\lambda q^2 - q_\lambda \not{q})[\tilde{f}_Q(q^2) + f_A(q^2)\gamma^5]. \tag{12}$$

Since for ultrarelativistic neutrinos the effect of γ^5 is only a sign which depends on the helicity of the neutrino, the phenomenology of neutrino anapole moments is similar to that of neutrino charge radii.

III. BASIC FORMULAS FOR ELASTIC NEUTRINO-ELECTRON SCATTERING

We consider the process where an ultrarelativistic neutrino with energy E_ν originates from a source (reactor, accelerator, the Sun, etc.) and elastically scatters on an electron in a detector at energy-momentum transfer $q = (T, \mathbf{q})$. If the neutrino is born in the source in the flavor state $|\nu_\ell\rangle$, then its state in the detector is

$$|\nu_\ell(L)\rangle = \sum_{k=1}^{3} U_{\ell k}^* e^{-i\frac{m_k^2}{2E_\nu}L} |\nu_k\rangle, \tag{13}$$

where L is the source-detector distance. The matrix element of the transition $\nu_\ell(L) + e^- \to \nu_j + e^-$ due to weak interaction is given by

$$\mathcal{M}_j^{(w)} = \frac{G_F}{\sqrt{2}} \sum_{k=1}^{3} U_{\ell k}^* e^{-i\frac{m_k^2}{2E_\nu}L} [(g_V')_{jk} \bar{u}_j \gamma_\lambda (1-\gamma^5) u_k J_V^\lambda(q)$$
$$- (g_A')_{jk} \bar{u}_j \gamma_\lambda (1-\gamma^5) u_k J_A^\lambda(q)], \tag{14}$$

where

$$(g'_V)_{jk} = \delta_{jk} g_V + U^*_{ej} U_{ek},$$
$$(g'_A)_{jk} = \delta_{jk} g_A + U^*_{ej} U_{ek},$$

with $g_V = 2\sin^2\theta_W - 1/2$, $g_A = -1/2$, and $\bar{u}_j = u_j^\dagger \gamma^0$, where u_j (u_k) is the bispinor amplitude of the massive neutrino state $|\nu_j\rangle$ ($|\nu_k\rangle$) with four-momentum p_j (p_k). The electron transition vector and axial currents in the detector are

$$J_V^\lambda(q) = \langle f|\sum_d e^{i\mathbf{q}\cdot\mathbf{r}_d}\gamma_d^0\gamma_d^\lambda|i\rangle,$$
$$J_A^\lambda(q) = \langle f|\sum_d e^{i\mathbf{q}\cdot\mathbf{r}_d}\gamma_d^0\gamma_d^\lambda\gamma_d^5|i\rangle, \tag{15}$$

where the d sum runs over all electrons in the detector, and $|i\rangle$ and $|f\rangle$ are initial and final states of the detector, such that $\mathcal{E}_f - \mathcal{E}_i = T$, where $\mathcal{E}_i$ and $\mathcal{E}_f$ are the energies of these states.

The matrix element due to electromagnetic interaction is given by

$$\mathcal{M}_j^{(\gamma)} = \mathcal{M}_j^{(Q)} + \mathcal{M}_j^{(\mu)}, \tag{16}$$

with

$$\mathcal{M}_j^{(Q)} = \frac{4\pi\alpha}{q^2}\sum_{k=1}^{3} U^*_{\ell k} e^{-i\frac{m_k^2}{2E_\nu}L}\bar{u}_j\left(\gamma_\lambda - \frac{q_\lambda \not{q}}{q^2}\right) \times \left[(e_\nu)_{jk} + \frac{q^2}{6}\langle r_\nu^2\rangle_{jk}\right] u_k J_V^\lambda(q), \tag{17}$$

$$\mathcal{M}_j^{(\mu)} = -i\frac{2\pi\alpha}{m_e q^2}\sum_{k=1}^{3} U^*_{\ell k} e^{-i\frac{m_k^2}{2E_\nu}L}\bar{u}_j\sigma_{\lambda\rho}q^\rho(\mu_\nu)_{jk} u_k J_V^\lambda(q), \tag{18}$$

where the neutrino millicharge e_ν and magnetic moment μ_ν are measured in units of e and μ_B, respectively, and the following notation is employed:

$$(e_\nu)_{jk} = e_{jk}, \qquad \langle r_\nu^2\rangle_{jk} = \langle r^2\rangle_{jk} + 6\gamma^5 a_{jk},$$
$$(\mu_\nu)_{jk} = \mu_{jk} + i\gamma^5\varepsilon_{jk}.$$

Taking into account that $\gamma^5|\nu_\ell\rangle = -|\nu_\ell\rangle$, for ultrarelativistic neutrinos we have $\gamma^5 u_k \simeq -u_k$. Therefore, in such a case the effect of γ^5 in the above formulas is simply a multiplication by a factor of -1. Also, in such a case there is no interference between the helicity-conserving ($\mathcal{M}_j^{(w)}$ and $\mathcal{M}_j^{(Q)}$) and helicity-flipping ($\mathcal{M}_j^{(\mu)}$) amplitudes. Combining the helicity-conserving amplitudes, we find

$$\mathcal{M}_j^{(w,Q)} = \mathcal{M}_j^{(w)} + \mathcal{M}_j^{(Q)} = \frac{G_F}{\sqrt{2}}\sum_{k=1}^{3} U^*_{\ell k} e^{-i\frac{m_k^2}{2E_\nu}L} \times \{[(g'_V)_{jk} + \tilde{Q}_{jk}]\bar{u}_j\gamma_\lambda(1-\gamma^5)u_k J_V^\lambda(q) - (g'_A)_{jk}\bar{u}_j\gamma_\lambda(1-\gamma^5)u_k J_A^\lambda(q)\}, \tag{19}$$

where

$$\tilde{Q}_{jk} = \frac{2\sqrt{2}\pi\alpha}{G_F}\left[\frac{(e_\nu)_{jk}}{q^2} + \frac{1}{6}\langle r_\nu^2\rangle_{jk}\right].$$

In Eq. (19), it is taken into account that $q_\lambda J_V^\lambda(q) = 0$.

When evaluating the cross section, we neglect the neutrino masses and set $p_j = p'$ and $p_k = p$. Since the final massive state of the neutrino is not resolved in the detector, the differential cross section measured in the scattering experiment is given by

$$\frac{d\sigma}{dT} = \frac{1}{32\pi^2}\int_{T^2}^{(2E_\nu - T)^2}\frac{d\mathbf{q}^2}{E_\nu^2}\int_0^{2\pi} d\varphi_{\mathbf{q}}|\mathcal{M}_{fi}|^2\delta(T - \mathcal{E}_f + \mathcal{E}_i), \tag{20}$$

with the following absolute matrix element squared:

$$|\mathcal{M}_{fi}|^2 = \sum_{j=1}^{3}\{|\mathcal{M}_j^{(w,Q)}|^2 + |\mathcal{M}_j^{(\mu)}|^2\}, \tag{21}$$

where, as usual, averaging over initial and summing over final spin polarizations is assumed. The angle $\varphi_{\mathbf{q}}$ in Eq. (20) is the azimuthal angle of the momentum transfer $\mathbf{q}$ in the spherical coordinate system with the z axis directed along the incident neutrino momentum $\mathbf{p}$.

Using

$$\frac{1}{4}\mathrm{Sp}\{\not{p}'\gamma_\lambda(1-\gamma^5)\not{p}\gamma_{\lambda'}(1-\gamma^5)\} = 2[p_\lambda p'_{\lambda'} + p'_\lambda p_{\lambda'} - (p\cdot p')g_{\lambda\lambda'} - i\varepsilon_{\lambda\rho\lambda'\rho'}p'^\rho p^{\rho'}],$$

where $g_{\lambda\lambda'}$ is the metric tensor and $\varepsilon_{\lambda\rho\lambda'\rho'}$ is the Levi-Civita symbol, we obtain

$$
\begin{aligned}
|\mathcal{M}_{fi}^{(w,Q)}|^2 &= \sum_{j=1}^{3} |\tilde{\mathcal{M}}_j^{(w,Q)}|^2 \\
&= 4G_F^2\{C_1[2|p\cdot J_V(q)|^2 - (p\cdot p')J_V(q)\cdot J_V^*(q) - i\varepsilon_{\lambda\rho\lambda'\rho'}p'^{\rho}p^{\rho'}J_V^{\lambda}(q)J_V^{\lambda'*}(q)] \\
&\quad + C_2[(p\cdot J_A(q))(p'\cdot J_A^*(q)) + (p'\cdot J_A(q))(p\cdot J_A^*(q)) - (p\cdot p')J_A(q)\cdot J_A^*(q) \\
&\quad - i\varepsilon_{\lambda\rho\lambda'\rho'}p'^{\rho}p^{\rho'}J_A^{\lambda}(q)J_A^{\lambda'*}(q)] + 2\mathrm{Re}\{C_3[(p\cdot J_V(q))(p'\cdot J_A^*(q)) \\
&\quad + (p'\cdot J_A(q))(p\cdot J_V^*(q)) - (p\cdot p')J_V(q)\cdot J_A^*(q) - i\varepsilon_{\lambda\rho\lambda'\rho'}p'^{\rho}p^{\rho'}J_V^{\lambda}(q)J_A^{\lambda'*}(q)]\}\}. \qquad (22)
\end{aligned}
$$

Here

$$C_1 = \sum_{j,k,k'=1}^{3} U_{\ell k}^* U_{\ell k'} e^{-i\frac{\delta m_{kk'}^2}{2E_\nu}L}[(g'_V)_{jk} + \tilde{Q}_{jk}][(g'_V)_{jk'}^* + \tilde{Q}_{jk'}^*], \qquad (23)$$

$$C_2 = \sum_{j,k,k'=1}^{3} U_{\ell k}^* U_{\ell k'} e^{-i\frac{\delta m_{kk'}^2}{2E_\nu}L}(g'_A)_{jk}(g'_A)_{jk'}^*, \qquad (24)$$

$$C_3 = \sum_{j,k,k'=1}^{3} U_{\ell k}^* U_{\ell k'} e^{-i\frac{\delta m_{kk'}^2}{2E_\nu}L}[(g'_V)_{jk} + \tilde{Q}_{jk}](g'_A)_{jk'}^*, \qquad (25)$$

with $\delta m_{kk'}^2 = m_k^2 - m_{k'}^2$.

Using

$$\frac{1}{4}\mathrm{Sp}\{\not{p}'\sigma_{\lambda\rho}q^{\rho}\not{p}\sigma_{\lambda'\rho'}q^{\rho'}\} = -(p\cdot p')(p_\lambda + p'_\lambda)(p_{\lambda'} + p'_{\lambda'})$$

and the relations $p + p' = 2p - q$, $p\cdot p' = -q^2/2$, and $q_\lambda J_V^\lambda(q) = 0$, we receive

$$|\mathcal{M}_{fi}^{(\mu)}|^2 = \sum_{j=1}^{3} |\mathcal{M}_j^{(\mu)}|^2 = \frac{32\pi^2\alpha^2}{m_e^2|q^2|}|\mu_\nu(L,E_\nu)|^2|p\cdot J_V(q)|^2, \qquad (26)$$

where the absolute effective magnetic moment squared is given by [5]

$$|\mu_\nu(L,E_\nu)|^2 = \sum_{j=1}^{3}\left|\sum_{k=1}^{3} U_{\ell k}^* e^{-i\frac{m_k^2}{2E_\nu}L}(\mu_\nu)_{jk}\right|^2. \qquad (27)$$

In the case of Dirac antineutrinos, one must make the following substitutions in the above formulas: $U_{\ell k} \to U_{\ell k}^*$, $(g'_V)_{jk} \to -(g'_V)_{jk}^*$, $(g'_A)_{jk} \to -(g'_A)_{jk}^*$, $\varepsilon_{\lambda\rho\lambda'\rho'} \to -\varepsilon_{\lambda\rho\lambda'\rho'}$, $(e_\nu)_{jk} \to (e_{\bar\nu})_{jk} = -e_{kj}$, and

$$
\begin{aligned}
\langle r_\nu^2\rangle_{jk} &\to \langle r_{\bar\nu}^2\rangle_{jk} = -\langle r^2\rangle_{kj} + 6\gamma^5 a_{kj}, \\
(\mu_\nu)_{jk} &\to (\mu_{\bar\nu})_{jk} = -\mu_{kj} - i\gamma^5\varepsilon_{kj},
\end{aligned}
$$

where the effect of γ^5 is a multiplication by a factor of $+1$.

IV. FREE-ELECTRON APPROXIMATION

The simplest model of the electron system in the detector is a free-electron model, where it is assumed that electrons are free and at rest. This approximation is supposed to be applicable if the energy-transfer value T is much larger than the electron binding energy in the detector. The differential cross section (20) in the case of neutrino scattering on one free electron is

$$
\begin{aligned}
\frac{d\sigma}{dT} &= \frac{1}{32\pi^2}\int_{T^2}^{(2E_\nu - T)^2}\frac{d\mathbf{q}^2}{E_\nu^2} \\
&\quad\times\int_0^{2\pi} d\varphi_{\mathbf{q}}|\mathcal{M}_{fi}|^2\delta\Big(T - \sqrt{\mathbf{q}^2 + m_e^2} + m_e\Big), \qquad (28)
\end{aligned}
$$

The free-electron vector and axial currents (15) are

$$
\begin{aligned}
J_V^\lambda(q) &= \frac{1}{2\sqrt{E'_e m_e}}\bar{u}_e{}'\gamma^\lambda u_e, \\
J_A^\lambda(q) &= \frac{1}{2\sqrt{E'_e m_e}}\bar{u}_e{}'\gamma^\lambda\gamma^5 u_e,
\end{aligned}
$$

where $E'_e = m_e + T$ is the final electron energy, and u_e and u'_e are the initial and final electron bispinor amplitudes, which are normalized as $\bar{u}_e u_e = \bar{u}'_e u'_e = 2m_e$. For the absolute matrix elements squared (22) and (26), one thus has

$$
\begin{aligned}
|\mathcal{M}_{fi}^{(w,Q)}|^2 &= \frac{4G_F^2}{E'_e m_e}[(C_1 + C_2 - 2\mathrm{Re}\{C_3\})(p\cdot k)(p'\cdot k') \\
&\quad + (C_1 + C_2 + 2\mathrm{Re}\{C_3\})(p\cdot k')(p'\cdot k) \\
&\quad + (C_2 - C_1)(p\cdot p')m_e^2], \qquad (29)
\end{aligned}
$$

$$|\mathcal{M}_{fi}^{(\mu)}|^2 = \frac{32\pi^2\alpha^2}{m_e^3 E'_e|q^2|}|\mu_\nu(L,E_\nu)|^2(p\cdot k)(p\cdot k'), \qquad (30)$$

where $k = (m_e, 0)$ and $k' = k + q$ are the initial and final electron four-momenta.

From conservation of four-momentum, $p + k = p' + k'$, it follows that

$$p \cdot k = p' \cdot k' = E_\nu m_e, \qquad p \cdot k' = p' \cdot k = (E_\nu - T) m_e,$$
$$p \cdot p' = k \cdot k' - m_e^2 = T m_e,$$

and $q^2 = -2m_e T$. Using these relations in Eqs. (29) and (30), we obtain after performing integrations in Eq. (28) the differential cross section in the free-electron approximation as

$$\frac{d\sigma^{\mathrm{FE}}}{dT} = \frac{d\sigma^{\mathrm{FE}}_{(w,Q)}}{dT} + \frac{d\sigma^{\mathrm{FE}}_{(\mu)}}{dT}, \tag{31}$$

with

$$\frac{d\sigma^{\mathrm{FE}}_{(w,Q)}}{dT} = \frac{G_F^2 m_e}{2\pi} \left[C_1 + C_2 - 2\mathrm{Re}\{C_3\} + (C_1 + C_2 + 2\mathrm{Re}\{C_3\}) \left(1 - \frac{T}{E_\nu}\right)^2 + (C_2 - C_1) \frac{T m_e}{E_\nu^2} \right], \tag{32}$$

$$\frac{d\sigma^{\mathrm{FE}}_{(\mu)}}{dT} = \frac{\pi \alpha^2}{m_e^2} |\mu_\nu(L, E_\nu)|^2 \left(\frac{1}{T} - \frac{1}{E_\nu} \right). \tag{33}$$

When the energy-transfer value T is comparable to the electron binding energy, the free-electron approximation becomes not generally valid anymore. In particular, for atomic electrons it was found that as the value of T decreases the contribution to the cross section associated with the neutrino millicharge exhibits strong enhancement as compared to the free-electron case [10]. This is the so-called atomic-ionization effect, which is observed for ultrarelativistic charged projectiles and which can be estimated within the equivalent photon approximation. At the same time, if the neutrino millicharges are zero, i.e., $e_{jk} = 0$, the cross section for neutrino scattering on atomic electrons is well approximated by the stepping formula

$$\frac{d\sigma}{dT} = \frac{d\sigma^{\mathrm{FE}}}{dT} \sum_\beta n_\beta \theta(T - \varepsilon_\beta), \tag{34}$$

where n_β and ε_β are the number and binding energy of electrons in the (sub)shell β. The stepping approximation was first introduced in Ref. [27] on the basis of numerical calculations for the case of an iodine atomic target, and later it was supported by a general theoretical analysis [28,29]. Notable deviations of the weak and magnetic cross sections from the stepping formula (34) are found only close to the ionization threshold [30,31], where the cross-section values decrease relative to the free-electron approximation. The latter behavior is attributed to the effects of electron-electron correlations in atoms [29].

V. THE ROLE OF NEUTRINO FLAVOR OSCILLATIONS

It is clear that the manifestation of the neutrino electromagnetic properties in the discussed scattering process depends on the neutrino state $\nu_\ell(L)$ in the detector. Neutrino flavor oscillations are determined by the source-detector distance and the neutrino energy. Below we inspect their impact on the general formulas presented in Sec. III.

Introducing the flavor transition amplitude and probability,

$$\mathcal{A}_{\nu_\ell \to \nu_{\ell'}}(L, E_\nu) = \langle \nu_{\ell'} | \nu_\ell(L) \rangle = \sum_{k=1}^{3} U^*_{\ell k} U_{\ell' k} e^{-i \frac{m_k^2}{2E_\nu} L},$$
$$P_{\nu_\ell \to \nu_{\ell'}}(L, E_\nu) = |\mathcal{A}_{\nu_\ell \to \nu_{\ell'}}(L, E_\nu)|^2,$$

we arrive at

$$C_1 = g_V^2 + 2g_V P_{\nu_\ell \to \nu_e}(L, E_\nu) + P_{\nu_\ell \to \nu_e}(L, E_\nu) + 2g_V \sum_{\ell', \ell'' = e,\mu,\tau} \mathcal{A}_{\nu_\ell \to \nu_{\ell'}}(L, E_\nu) \mathcal{A}^*_{\nu_\ell \to \nu_{\ell''}}(L, E_\nu) \tilde{Q}_{\ell'' \ell'}$$
$$+ 2\mathrm{Re}\left\{ \mathcal{A}^*_{\nu_\ell \to \nu_e}(L, E_\nu) \sum_{\ell' = e,\mu,\tau} \mathcal{A}_{\nu_\ell \to \nu_{\ell'}}(L, E_\nu) \tilde{Q}_{e\ell'} \right\} + \sum_{\ell', \ell'', \ell''' = e,\mu,\tau} \mathcal{A}_{\nu_\ell \to \nu_{\ell'}}(L, E_\nu) \mathcal{A}^*_{\nu_\ell \to \nu_{\ell''}}(L, E_\nu) \tilde{Q}_{\ell'' \ell'''} \tilde{Q}_{\ell''' \ell'}, \tag{35}$$

$$C_2 = g_A^2 + 2g_A P_{\nu_\ell \to \nu_e}(L, E_\nu) + P_{\nu_\ell \to \nu_e}(L, E_\nu), \tag{36}$$

$$C_3 = g_V g_A + (g_V + g_A + 1) P_{\nu_\ell \to \nu_e}(L, E_\nu) + g_A \sum_{\ell', \ell'' = e,\mu,\tau} \mathcal{A}_{\nu_\ell \to \nu_{\ell'}}(L, E_\nu) \mathcal{A}^*_{\nu_\ell \to \nu_{\ell''}}(L, E_\nu) \tilde{Q}_{\ell'' \ell'}$$
$$+ \mathcal{A}^*_{\nu_\ell \to \nu_e}(L, E_\nu) \sum_{\ell' = e,\mu,\tau} \mathcal{A}_{\nu_\ell \to \nu_{\ell'}}(L, E_\nu) \tilde{Q}_{e\ell'}, \tag{37}$$

with

$$\tilde{Q}_{\ell'\ell} = \sum_{j,k=1}^{3} U_{\ell' j} U_{\ell k}^{*} \tilde{Q}_{jk} = \frac{2\sqrt{2}\pi\alpha}{G_F} \left[\frac{(e_\nu)_{\ell'\ell}}{q^2} + \frac{1}{6} \langle r_\nu^2 \rangle_{\ell'\ell} \right],$$

where

$$(e_\nu)_{\ell'\ell} = \sum_{j,k=1}^{3} U_{\ell' j} U_{\ell k}^{*} (e_\nu)_{jk} \quad \text{and}$$

$$\langle r_\nu^2 \rangle_{\ell'\ell} = \sum_{j,k=1}^{3} U_{\ell' j} U_{\ell k}^{*} \langle r_\nu^2 \rangle_{jk}$$

are the neutrino millicharge and charge radius in the flavor basis. In Eq. (35), it is taken into account that $\tilde{Q}_{\ell\ell'} = \tilde{Q}_{\ell'\ell}^{*}$ due to Hermiticity of the neutrino electromagnetic form factors f_Q and f_A.

Let us consider two typical cases of the scattering experiments: (i) short-baseline (reactor and accelerator neutrino experiments) and (ii) long-baseline (solar neutrino experiments). In the short-baseline experiments the effect of neutrino flavor change is insignificant, so that to a close approximation the neutrino flavor in the detector is the same as in the source. On the contrary, in the long-baseline experiments neutrinos can change their flavor many times when propagating from the source to the detector. Due to the finite energy resolution of the detector the interference effects in neutrino flavor oscillations over long distances appear to be washed out. In what follows, we formulate these behaviors mathematically.

In the short-baseline case we have $L \ll L_{kk'} = 2E_\nu/|\delta m_{kk'}^2|$ for any k and k'. This validates the approximation $e^{-i(\delta m_{kk'}^2/2E_\nu)L} = 1$. Using it, we find

$$\mathcal{A}_{\nu_\ell \to \nu_{\ell'}}(L, E_\nu) \mathcal{A}_{\nu_\ell \to \nu_{\ell''}}^{*}(L, E_\nu) = \delta_{\ell\ell'} \delta_{\ell\ell''},$$
$$P_{\nu_\ell \to \nu_e}(L, E_\nu) = \delta_{\ell e}.$$

Therefore, from Eqs. (35), (36), and (37) we derive, respectively,

$$C_1 = (g_V + \delta_{\ell e} + \tilde{Q}_{\ell\ell})^2 + \sum_{\ell'=e,\mu,\tau} (1 - \delta_{\ell'\ell}) |\tilde{Q}_{\ell'\ell}|^2, \quad (38)$$

$$C_2 = (g_A + \delta_{\ell e})^2, \quad (39)$$

$$C_3 = (g_V + \delta_{\ell e})(g_A + \delta_{\ell e}) + (g_A + \delta_{\ell e}) \tilde{Q}_{\ell\ell}. \quad (40)$$

This shows that the weak-electromagnetic interference term contains only flavor-diagonal neutrino millicharges and charge radii.

For the absolute effective magnetic moment squared (27) we get

$$|\mu_\nu(L, E_\nu)|^2 = \sum_{j=1}^{3} \sum_{k,k'=1}^{3} U_{\ell k}^{*} U_{\ell k'} (\mu_\nu)_{jk} (\mu_\nu)_{jk'}^{*}$$
$$= \sum_{\ell'=e,\mu,\tau} |(\mu_\nu)_{\ell'\ell}|^2, \quad (41)$$

where

$$(\mu_\nu)_{\ell'\ell} = \sum_{j,k=1}^{3} U_{\ell k}^{*} U_{\ell' j} (\mu_\nu)_{jk}$$

is the effective magnetic moment in the flavor basis.

In the long-baseline case we have $L \gg L_{kj} = 2E_\nu/|\delta m_{kk'}^2|$ for any k and k'. Taking into account the decoherence effects, we can set $\exp(-i\delta m_{kk'}^2/2E_\nu) = \delta_{kk'}$ in Eqs. (23), (24), and (25). Hence, we get

$$C_1 = g_V^2 + 2g_V P_{\nu_\ell \to \nu_e} + P_{\nu_\ell \to \nu_e} + \sum_{j,k=1}^{3} |U_{\ell k}|^2 |\tilde{Q}_{jk}|^2$$
$$+ 2g_V \sum_{j=1}^{3} |U_{\ell j}|^2 \tilde{Q}_{jj} + 2 \sum_{j,k=1}^{3} |U_{\ell k}|^2 \mathrm{Re}\{U_{ej} U_{ek}^{*} \tilde{Q}_{jk}\}, \quad (42)$$

$$C_2 = g_A^2 + 2g_A P_{\nu_\ell \to \nu_e} + P_{\nu_\ell \to \nu_e}, \quad (43)$$

$$C_3 = g_V g_A + (g_V + g_A + 1) P_{\nu_\ell \to \nu_e} + g_A \sum_{j=1}^{3} |U_{\ell j}|^2 \tilde{Q}_{jj}$$
$$+ 2 \sum_{j,k=1}^{3} |U_{\ell k}|^2 U_{ej} U_{ek}^{*} \tilde{Q}_{jk}, \quad (44)$$

where the flavor transition probability

$$P_{\nu_\ell \to \nu_e} = \sum_{k=1}^{3} |U_{\ell k}|^2 |U_{ek}|^2$$

does not depend both on the source-detector distance and on the neutrino energy.

For the absolute effective magnetic moment squared (27) we find

$$|\mu_\nu(L, E_\nu)|^2 = \sum_{j,k=1}^{3} |U_{\ell k}|^2 |(\mu_\nu)_{jk}|^2. \quad (45)$$

As in the case of Eq. (41), it is independent of the source-detector distance and neutrino energy.

VI. SUMMARY AND CONCLUDING REMARKS

We have considered theoretically the low-energy elastic neutrino-electron scattering, taking into account electromagnetic interactions of massive neutrinos. General formulas for the calculation of differential cross sections have been derived in the framework of three-neutrino mixing. The free-electron approximation and stepping formula for the differential cross sections have been discussed. The role of neutrino flavor oscillations has been outlined depending on the source-detector distance.

In contrast to the previous works on neutrino electromagnetic interactions in the processes of elastic neutrino-electron scattering, in the present study the cross section is formulated not in terms of some effective electromagnetic characteristics of the neutrino state $\nu_\ell(L)$ in a detector, but in terms of 3×3 matrices of neutrino electromagnetic form factors. It was shown that in the short-baseline experiments one studies these form factors in the flavor basis rather than in the fundamental, mass basis, which is more convenient for interpreting the results of the long-baseline experiments.

So far, in the analysis of the data of experiments on elastic neutrino-electron scattering the effect of the neutrino charge radius has been considered to be only a shift of the vector coupling constant, $g_V \to g_V + \frac{2}{3} M_W^2 \langle r^2_{\nu_\ell(L)} \rangle \sin^2\theta_W$ (see, for instance, Ref. [32]). However, one thus misses certain contributions to the cross section from the neutrino charge radius matrix, namely, those which do not interfere with the weak-interaction contribution. For example, the current most stringent constraints on the charge radius of the electron antineutrino obtained in this way are

$$-4.2 \times 10^{-32}\ \mathrm{cm}^2 < \langle r^2_{\bar{\nu}_e} \rangle < 6.6 \times 10^{-32}\ \mathrm{cm}^2, \qquad (46)$$

which are due to the TEXONO experiment with reactor antineutrinos [33]. The leading role in the derivation of the above bounds is played by the interference term $\propto g_V \langle r^2_{\bar{\nu}_e} \rangle$ in the cross section, while the term $\propto |\langle r^2_{\bar{\nu}_e} \rangle|^2$ is subsidiary. At the same time, according to Eq. (38), there is also the term $\propto |\langle r^2_{\bar{\nu}_e \to \bar{\nu}_\mu} \rangle|^2 + |\langle r^2_{\bar{\nu}_e \to \bar{\nu}_\tau} \rangle|^2$, where $\langle r^2_{\bar{\nu}_e \to \bar{\nu}_\mu} \rangle = \langle r^2_{\bar{\nu}} \rangle_{\mu e}$ and $\langle r^2_{\bar{\nu}_e \to \bar{\nu}_\tau} \rangle = \langle r^2_{\bar{\nu}} \rangle_{\tau e}$ are the transition charge radii in the flavor basis. The contributions from the flavor-transition charge radii do not interfere with the contribution from weak interaction. Hence, these charge radii can have values $\sim 10^{-32}$ cm^2, without notably affecting the constraints (46).

Finally, some comments should be made regarding contributions to the cross section from neutrino millicharges. The bound (2) has been derived in the region of small T values, where the weak-millicharge interference term is not important and where the atomic-ionization effect is to be taken into account. It follows from Eq. (38) that one must understand $|e_{\nu_e}|$ in Eq. (2) as

$$|e_{\nu_e}| = \sqrt{|(e_\nu)_{ee}|^2 + |(e_\nu)_{\mu e}|^2 + |(e_\nu)_{\tau e}|^2}.$$

In other words, the flavor-transition millicharges $(e_\nu)_{\mu e}$ and $(e_\nu)_{\tau e}$ also contribute to the cross section in addition to the usual, flavor-diagonal millicharge $(e_\nu)_{ee}$.

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