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Phenomenological profile of top squarks from natural supersymmetry at the LHC

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In supersymmetric models with radiatively driven naturalness and light Higgsinos, the top squarks may lie in the 0.5–3 TeV range, and thus, only a fraction of natural parameter space is accessible to LHC searches. We outline the range of top squark and lightest SUSY particle masses preferred by electroweak naturalness in the standard parameter space plane. We note that the branching fraction for $b \to s\gamma$ decay favors top squarks much heavier than 500 GeV. Such a range of top squark mass values is in contrast to previous expectations where $m(\mathrm{stop}) < 500$ GeV had been considered natural. In radiative natural SUSY, top squarks decay roughly equally via $\tilde{t}_1 \to b\tilde{W}_1$ and $t\tilde{Z}_{1,2}$ where $\tilde{W}_1$ and $\tilde{Z}_{1,2}$ are Higgsino-like electroweakinos. Thus, top squark pair production should yield all of $t\bar{t} + E_T^{\mathrm{miss}}$, $t\bar{b} + E_T^{\mathrm{miss}}$, $b\bar{t} + E_T^{\mathrm{miss}}$ and $b\bar{b} + E_T^{\mathrm{miss}}$ signatures at comparable rates. We propose that future LHC top squark searches take place within a semisimplified model which corresponds more closely to expectations from theory.

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I. INTRODUCTION

The supersymmetrized (SUSY) Standard Model (SM), e.g., the minimal supersymmetric Standard Model (MSSM), has for a long time intrigued particle theorists in that it is free of the scalar field quadratic divergences that plague nonsupersymmetric theories [1]. In addition, the MSSM has made three predictions which have since been verified by experiment: 1. The value of $\sin^2\theta_W \simeq 0.232$ which arises from unified gauge couplings at $m_{\mathrm{GUT}} \simeq 2 \times 10^{16}$ GeV that evolve via renormalization group (RG) evolution down to the weak scale within the context of the MSSM [2]. 2. The large top quark mass $m_t \simeq 173$ GeV [3] is exactly what is needed to initiate a radiative breakdown of electroweak symmetry in the MSSM [3]. 3. The measured value of the Higgs boson mass $m_h \simeq 125$ GeV [4–6] which falls squarely within the narrow window required by the MSSM [7].

In contrast, so far no evidence for direct production of superpartners has emerged at LHC, leading to mass limits $m_{\tilde{g}} \gtrsim 1900$ GeV [8,9] and $m_{\tilde{t}_1} \gtrsim 850$ GeV [10–12] in the context of various simplified models. The latter lower bound has been particularly disconcerting since it is in direct conflict with an oft-repeated mantra that one or more light third generation squarks ($m_{\tilde{t}_1} \lesssim 500$ GeV) are required for a *natural* SUSY solution to the Little Hierarchy (LH) problem. Here, the LH is characterized by the growing gap between the weak scale, as represented by $m_{W,Z,h} \sim 100$ GeV, and the superparticle mass scale m_{SUSY} which apparently lies within the multi-TeV range.

The light top squark narrative has lead to an "all hands on deck" call for exploring every conceivable gap of allowed masses and decay modes in the simplified model $m_{\tilde{t}_1}$ vs $m(\mathrm{LSP})$ (the LSP, lightest SUSY particle) plane. The impression has been made that by covering every possibility for existence of light top squarks, then one may be ruling out weak scale SUSY or else showing that whatever form SUSY takes, it is not as "we" understood it [13]. The top squark mass bound is also being invoked to justify costly decisions regarding future experimental facilities: if weak scale SUSY as we know it is ruled out, and the SM remains valid well into the multi-TeV range, then perhaps a 100 TeV hadron collider is the way to go as all bets from theory would be off. Alternatively, if SUSY remains just beyond the energy horizon, then perhaps ILC and an energy upgrade LHC (HE-LHC) operating with $\sqrt{s} \sim 28$–33 TeV are the right machines to build. Given the stakes involved, it is becoming critical to ensure the validity of our reasoning regarding the notions of electroweak naturalness and fine-tuning.

To address this issue, in Sec. II we briefly review several estimates of electroweak naturalness in the SM and in SUSY. We believe that several common measures are technically misapplied in the SUSY case. When corrected to allow for the fact that the soft parameters should be correlated, they reduce to the model independent measure Δ_{EW}, where EW denotes electroweak [14,15]. The latter measure also leads to bounds on top squarks and gluinos, but instead allows for $m_{\tilde{t}_1} \lesssim 3$ TeV and $m_{\tilde{g}} \lesssim 4$ TeV at little cost to naturalness since these masses enter into the value of

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m_Z as finite one- and two-loop corrections, respectively. In Sec. III, we present a top squark benchmark model from the two-extra-parameter nonuniversal Higgs model [16] (NUHM2) which allows for highly natural SUSY spectra with $m_h \simeq 125$ GeV. This leads to a grand overview plot of expectations for populating the $m_{\tilde{t}_1}$ vs $m_{\tilde{Z}_1}$ plane in Sec. IV. This plot presents a *guide* for top squark hunters at the LHC as to where in the plane their quarry of natural SUSY solutions lies for low values of $\Delta_{\rm EW}$. Here, we find $m_{\tilde{t}_1} \lesssim$ 1.2–1.8 TeV for $\Delta_{\rm EW} < 15$ while $m_{\tilde{t}_1} \lesssim 3$ TeV for $\Delta_{\rm EW} < 30$. Hardly any solutions lie in the highly scrutinized *compressed* region where $m_{\tilde{t}_1} \sim m_{\tilde{Z}_1}$. In Sec. V, we evaluate expectations for the flavor-changing decay $\mathrm{BF}(b \to s\gamma)$ vs $m_{\tilde{t}_1}$ and find for $m_{\tilde{t}_1} < 500$ GeV that one always expects large deviations from the measured value whereas for $m_{\tilde{t}_1} > 1.5$ TeV, then the SUSY loops decouple and one gains accord with experiment: in this sense, it comes as no great surprise that LHC top squark hunters have yet to sight their trophy. In Sec. VI, we outline top squark production and decay rates for natural SUSY, and in Sec. VII we outline a more realistic proposal for future top squark searches in a semisimplified model which corresponds more closely with predictions from theory. A summary and conclusions are given in Sec. VIII.[1]

II. BRIEF REVIEW OF NATURALNESS

A. Fine-tuning rule

For any observable $\mathcal{O}$, if the contributions to $\mathcal{O}$ are given by

$$\mathcal{O} = a + b + f(b) + c, \tag{1}$$

then we would claim the value of $\mathcal{O}$ is *natural* if each contribution on the right-hand side is comparable to or less than $\mathcal{O}$. If this were not the case, if say one contribution c were far larger than $\mathcal{O}$, then some other contribution would have to be fine-tuned to large opposite-sign values such as to maintain the measured value of $\mathcal{O}$. Thus, the naturalness measure

$$\Delta = |\text{largest contribution to RHS}|/|\mathcal{O}| \tag{2}$$

would be vindicated (here, RHS stands for right-hand-side). In the case of the quantity $f(b)$, if as a consequence of b getting large, then $f(b)$ becomes large negative, these two quantities are *dependent* and should be *combined* before evaluating naturalness. This is embodied by the fine-tuning rule articulated in Ref. [51]: in evaluating fine-tuning, it is not permissible to claim fine-tuning of dependent quantities one against another.

[1]Some early work on top squark phenomenology is given in Refs. [17–22]. Some recent examinations include Refs. [23–50].

B. Higgs mass fine-tuning in the SM

For illustration, in the case of the SM with a scalar potential given by

$$V = -\mu_{\rm SM}^2|\phi^\dagger\phi| + \lambda|\phi^\dagger\phi|^2, \tag{3}$$

the physical Higgs boson mass is given by

$$m_{H_{\rm SM}}^2 \simeq 2\mu_{\rm SM}^2 + \delta m_{H_{\rm SM}}^2, \tag{4}$$

where the largest contribution to $\delta m_{H_{\rm SM}}^2$ comes from the famous quadratic divergences:

$$\delta m_{H_{\rm SM}}^2 \simeq \frac{3}{4\pi^2}\left(-\lambda_t^2 + \frac{g^2}{4} + \frac{g^2}{8\cos^2\theta_W} + \lambda\right)\Lambda^2, \tag{5}$$

where λ_t is the SM top quark Yukawa coupling, g is the $SU(2)_L$ gauge coupling and Λ represents the energy scale cutoff on the quadratically divergent one-loop mass corrections. Since $2\mu_{\rm SM}^2$ is *independent* of $\delta m_{H_{\rm SM}}^2$, then $\mu_{\rm SM}^2$ can be freely dialed, or fine-tuned, to maintain the measured value of $m_{H_{\rm SM}} = 125.1$ GeV [6]. A valid measure of fine-tuning here would be $\Delta_{\rm SM} = |\delta m_{H_{\rm SM}}^2|/m_{H_{\rm SM}}^2$. Requiring $\Delta_{\rm SM} < 30$ implies an upper bound on the SM effective theory energy cutoff of $\Lambda \lesssim 5.8$ TeV.

C. Higgs mass fine-tuning in the MSSM

The situation in the MSSM is quite different [52]. In this case, the well-known quadratic divergences all cancel, but there remains a variety of intertwined logarithmic divergent contributions to m_h^2. In the MSSM, we have

$$\begin{aligned} m_h^2 &\simeq -2\{\mu^2(\text{weak}) + m_{H_u}^2(\text{weak})\} \\ &\sim -2\{\mu^2(\Lambda) + m_{H_u}^2(\Lambda) + \delta m_{H_u}^2(\Lambda)\}, \end{aligned} \tag{6}$$

where now μ is the superpotential Higgsino mass term and $m_{H_u}^2$ is the up-Higgs soft SUSY breaking squared mass. The quantity $\delta m_{H_u}^2$ is properly evaluated by integrating the renormalization group equation:

$$\frac{dm_{H_u}^2}{dt} = \frac{2}{16\pi^2}\left(-\frac{3}{5}g_1^2M_1^2 - 3g_2^2M_2^2 + \frac{3}{10}g_1^2S + 3f_t^2X_t\right), \tag{7}$$

where $t = \log(Q)$ with Q the renormalization scale, M_i ($i = 1$–3) are the various gaugino masses, g_i are the corresponding gauge coupling constants, f_t is the top Yukawa coupling,

$$S = m_{H_u}^2 - m_{H_d}^2 + \mathrm{Tr}[\mathbf{m}_Q^2 - \mathbf{m}_L^2 - 2\mathbf{m}_U^2 + \mathbf{m}_D^2 + \mathbf{m}_E^2], \tag{8}$$

and

$$X_t = m_{Q_3}^2 + m_{U_3}^2 + m_{H_u}^2 + A_t^2, \quad (9)$$

where $m_{H_d}^2$, $\mathbf{m}_Q^2$, $\mathbf{m}_L^2$, $\mathbf{m}_U^2$, $\mathbf{m}_D^2$, $\mathbf{m}_E^2$ are the soft masses for the down-type Higgs, left-handed squarks, left-handed sleptons, right-handed up-type squarks, right-handed down-type squarks, and right-handed charged sleptons, respectively, and A_t is the A-term for the top Yukawa coupling. To evaluate $\delta m_{H_u}^2$, it is common in the literature to set the gauge couplings, the S parameter and $m_{H_u}^2$ equal to zero so that a simple one step integration can be performed leading to

$$\delta m_{H_u}^2(\Lambda) \sim -\frac{3f_t^2}{8\pi^2}(m_{Q_3}^2 + m_{U_3}^2 + A_t^2)\log\left(\frac{\Lambda}{m_{\mathrm{SUSY}}}\right). \quad (10)$$

The fine-tuning measure $\Delta_{\mathrm{HS}} = |\delta m_{H_u}^2|/m_h^2 \lesssim 30$ requires at least one (and actually three) third generation squarks with mass less than 650 GeV [53].

The issue here is that, unlike the SM case, $\delta m_{H_u}^2(\Lambda)$ is *not independent* of the high scale value of $m_{H_u}^2(\Lambda)$. In fact, the larger $m_{H_u}^2(\Lambda)$ is, the larger is the cancelling correction $\delta m_{H_u}^2$. This violates the fine-tuning rule.

Instead, one ought to first *combine* dependent contributions, then evaluate the independent contributions to the observed value of m_h^2 to check whether they exceed its value. Upon regrouping $m_h^2 = -2\{\mu^2 + (m_{H_u}^2(\Lambda) + \delta m_{H_u}^2(\Lambda))\}$, where $m_{H_u}^2(\Lambda) + \delta m_{H_u}^2(\Lambda) = m_{H_u}^2(\mathrm{weak})$. Then, it is seen that the criteria for naturalness is that the *weak scale* values of μ^2 and $m_{H_u}^2$ are each comparable to m_h^2. This corrected measure allows for *radiatively driven naturalness*: large, unnatural values of $m_{H_u}^2$ at the high scale Λ may be driven to natural values at the weak scale via radiative corrections [14,15].

D. BG fine-tuning: Multiple or just one soft parameter?

The measure $\Delta_{\mathrm{BG}} \equiv \max_i |\frac{\partial \log m_Z^2}{\partial \log p_i}|$ was proposed by Ellis *et al.* [54] and investigated more thoroughly by Barbieri and Giudice [55]. Here, the p_i are fundamental parameters of the theory labeled by index i. To begin, one may express m_Z^2 in terms of weak scale SUSY parameters

$$m_Z^2 \simeq -2\mu^2 - 2m_{H_u}^2, \quad (11)$$

where the partial equality holds for moderate-to-large $\tan\beta$ values ($\tan\beta \equiv \langle H_u\rangle/\langle H_d\rangle$ is the ratio of the Higgs VEVs) and where we assume for now the radiative corrections are small. Next, one needs to know the explicit dependence of $m_{H_u}^2$ and μ^2 on the fundamental parameters. Semianalytic solutions to the one-loop renormalization group equations for $m_{H_u}^2$ and μ^2 can be found for instance in Ref. [56]. For the case of $\tan\beta = 10$, it is found that [57–59]

$$\begin{aligned} m_Z^2 \simeq {} & -2.18\mu^2 + 3.84M_3^2 + 0.32M_3M_2 + 0.047M_1M_3 - 0.42M_2^2 + 0.011M_2M_1 - 0.012M_1^2 - 0.65M_3A_t - 0.15M_2A_t \\ & - 0.025M_1A_t + 0.22A_t^2 + 0.004M_3A_b - 1.27m_{H_u}^2 - 0.053m_{H_d}^2 + 0.73m_{Q_3}^2 + 0.57m_{U_3}^2 + 0.049m_{D_3}^2 - 0.052m_{L_3}^2 \\ & + 0.053m_{E_3}^2 + 0.051m_{Q_2}^2 - 0.11m_{U_2}^2 + 0.051m_{D_2}^2 - 0.052m_{L_2}^2 + 0.053m_{E_2}^2 + 0.051m_{Q_1}^2 - 0.11m_{U_1}^2 \\ & + 0.051m_{D_1}^2 - 0.052m_{L_1}^2 + 0.053m_{E_1}^2, \end{aligned} \quad (12)$$

where all terms on the right-hand side are understood to be *GUT* scale parameters.

The conundrum is then: what constitutes fundamental parameters? If all GUT scale parameters on the RHS of Eq. (12) are fundamental, then for the doublet top squark soft term we would find $\Delta_{\mathrm{BG}} \sim 0.73m_{Q_3}^2/(m_Z^2/2)$, and so $\Delta_{\mathrm{BG}} < 30$ would imply $m_{Q_3} \lesssim 400$ GeV in accord with Eq. (10).

If instead we assume scalar mass universality as in the CMSSM, then the fourth and fifth lines of Eq. (12) combine to $0.027m_0^2$ and instead $\Delta_{\mathrm{BG}} = 0.027m_0^2/(m_Z^2/2) < 30$ would require $m_0 \lesssim 2$ TeV: multi-TeV scalars are natural as in focus-point SUSY [60].

In fact, in more fundamental supergravity theories with SUGRA breaking in a hidden sector, then *all* soft terms are computable as multiples of the more fundamental gravitino mass $m_{3/2}$ [61]. Then, all soft terms on the RHS of Eq. (12) are *dependent* and must be combined according to the fine-tuning rule. In this case, Eq. (12) collapses to a simpler form [51]:

$$m_Z^2 \simeq -2.18\mu^2 + a\cdot m_{3/2}^2, \quad (13)$$

and instead low fine-tuning requires $\mu \sim m_Z$ and also $\sqrt{|a\cdot m_{3/2}^2|} \sim m_Z$. Equating Eq. (11) with Eq. (13) shows that $a\cdot m_{3/2}^2 \sim -m_{H_u}^2(\mathrm{weak})$, and so we are led to consistency with the corrected implication of Δ_{HS}: the criteria for electroweak naturalness is that the weak scale values of $|m_{H_u}|$ and $|\mu|$ are $\sim m_{W,Z,h} \sim 100$ GeV.

E. The electroweak measure Δ_{EW}

The corrected versions of Δ_{HS} and Δ_{BG} are consistent with requiring low *electroweak* fine-tuning in m_Z^2.

Minimization of the scalar potential in the minimal supersymmetric Standard Model (MSSM) leads to the well-known relation [62]

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2 \tag{14}$$

$$\simeq -m_{H_u}^2 - \Sigma_u^u - \mu^2, \tag{15}$$

where Σ_u^u and Σ_d^d denote the one-loop corrections (expressions can be found in the Appendix of Ref. [15]) to the scalar potential and $m_{H_u}^2$ and $m_{H_d}^2$ are the Higgs soft masses at the weak scale. The second line is obtained from moderate to large values of $\tan\beta \gtrsim 5$ (as required by the Higgs mass calculation [7]). SUSY models requiring large cancellations between the various terms on the right-hand side of Eq. (15) to reproduce the measured value of m_Z^2 are regarded as unnatural, or fine-tuned. In contrast, SUSY models which generate terms on the RHS of Eq. (15) which are all less than or comparable to $m_{\rm weak}$ are regarded as natural. Thus, the *electroweak* naturalness measure $\Delta_{\rm EW}$ is defined as [14,15]

$$\Delta_{\rm EW} \equiv \max|\text{each additive term on RHS of Eq. (14)}|/(m_Z^2/2). \tag{16}$$

Including the various radiative corrections, over 40 terms contribute. The measure $\Delta_{\rm EW}$ is programmed in the Isajet spectrum generator Isasugra [63]. Neglecting radiative corrections, and taking moderate-to-large $\tan\beta \gtrsim 5$, then $m_Z^2/2 \sim -m_{H_u}^2 - \mu^2$, so the main criterion for naturalness is that *at the weak scale*

(i) $m_{H_u}^2 \sim -m_Z^2$ and
(ii) $\mu^2 \sim m_Z^2$ [64].

The value of $m_{H_d}^2$ (where $m_A \sim m_{H_d}$(weak) with m_A being the mass of the CP-odd Higgs boson) can lie in the TeV^2 range since its contribution to the RHS of Eq. (15) is suppressed by $1/\tan^2\beta$. The largest radiative corrections typically come from the top squark sector:

$$\Sigma_u^u(\tilde t_{1,2}) = \frac{3}{16\pi^2}F(m_{\tilde t_{1,2}}^2) \times \left[f_t^2 - g_Z^2 \mp \frac{f_t^2A_t^2 - 8g_Z^2(\frac{1}{4}-\frac{2}{3}\sin^2\theta_W)\Delta_t}{m_{\tilde t_2}^2 - m_{\tilde t_1}^2}\right], \tag{17}$$

where θ_W is the weak mixing angle, $\Delta_t = (m_{\tilde t_L}^2 - m_{\tilde t_R}^2)/2 + M_Z^2\cos 2\beta(\frac{1}{4}-\frac{2}{3}\sin^2\theta_W)$, $g_Z^2 = (g^2+g'^2)/8$, and $F(m^2) = m^2(\log(m^2/Q^2) - 1)$, with $Q^2 = m_{\tilde t_1}m_{\tilde t_2}$. Requiring highly mixed TeV-scale top squarks minimizes $\Sigma_u^u(\tilde t_{1,2})$ whilst lifting the Higgs mass m_h to ~125 GeV [15].

Using $\Delta_{\rm EW} < 30$ or better than 3% fine-tuning[2] then instead of earlier upper bounds, it is found that

(i) $m_{\tilde g} \lesssim 4$ TeV,
(ii) $m_{\tilde t_1} \lesssim 3$ TeV and
(iii) $m_{\tilde W_1,\tilde Z_{1,2}} \lesssim 300$ GeV.

Thus, gluinos and squarks may easily lie beyond the current reach of LHC at little cost to naturalness while only the Higgsino-like lighter charginos and neutralinos are required to lie near the weak scale. The lightest Higgsino $\tilde Z_1$ comprises a portion of the dark matter and would escape detection at LHC. The remaining dark matter abundance might be comprised of e.g. axions [66]. Owing to their compressed spectrum with mass gaps $m_{\tilde W_1} - m_{\tilde Z_1} \sim m_{\tilde Z_2} - m_{\tilde Z_1} \sim$ 10–20 GeV, the heavier Higgsinos are difficult to see at LHC owing to the rather small visible energy released from their 3-body decays $\tilde W_1 \to f\bar f'\tilde Z_1$ and $\tilde Z_2 \to f\bar f\tilde Z_1$ (where the f stands for SM fermions).

III. ILLUSTRATION FROM A SUSY BENCHMARK MODEL

In this section, we illustrate some aspects of top squark and Higgs boson masses and mixings for a sample SUSY benchmark model from the two-extra-parameter nonuniversal Higgs model (NUHM2[16]) with parameter space given by

$$m_0, \quad m_{1/2}, \quad A_0, \quad \tan\beta, \quad \mu, \quad m_A, \tag{18}$$

where the nonuniversal GUT scale parameters $m_{H_u}^2$ and $m_{H_d}^2$ have been exchanged for the more convenient weak scale values of μ and m_A. Here, we adopt parameter choices $m_0 = 5$ TeV, $m_{1/2} = 900$ GeV, $\tan\beta = 10$, $\mu = 125$ GeV and $m_A = 1$ TeV.

In Fig. 1 frame (a), we plot the values of the various third generation sfermion masses vs variation in the A_0 parameter. It is seen that for $A_0 \sim 0$, then the various sfermion masses range between 3 and 5 TeV. As A_0 becomes large positive or negative, the $A_{t,b,\tau}$ contributions to the MSSM RG equations tend to drive the soft masses $m_{Q_3}^2$ and $m_{U_3}^2$ to lower values due to the X_t [Eq. (9)] contribution to the RG running, which is amplified by the large top-quark Yukawa coupling f_t. The $\tilde\tau_{1,2}$ and $\tilde b_2$ mass values hardly change since their RG equations include X_τ and X_b which are only amplified by the much smaller τ- and b-Yukawa couplings. Also, the large A_t term causes large mixing in the top squark sector which enhances the splitting of the stop eigenstates. Only the value of $m_{\tilde t_1}$ is driven to sub-TeV values for $A_0 \lesssim -8.8$ TeV.

In Fig. 1(b), we show the value of m_h vs A_0. The Higgs mass at one loop is given by

[2]For higher values of $\Delta_{\rm EW}$, high fine-tuning sets in and is displayed visually in Fig. 2 of Ref. [65].

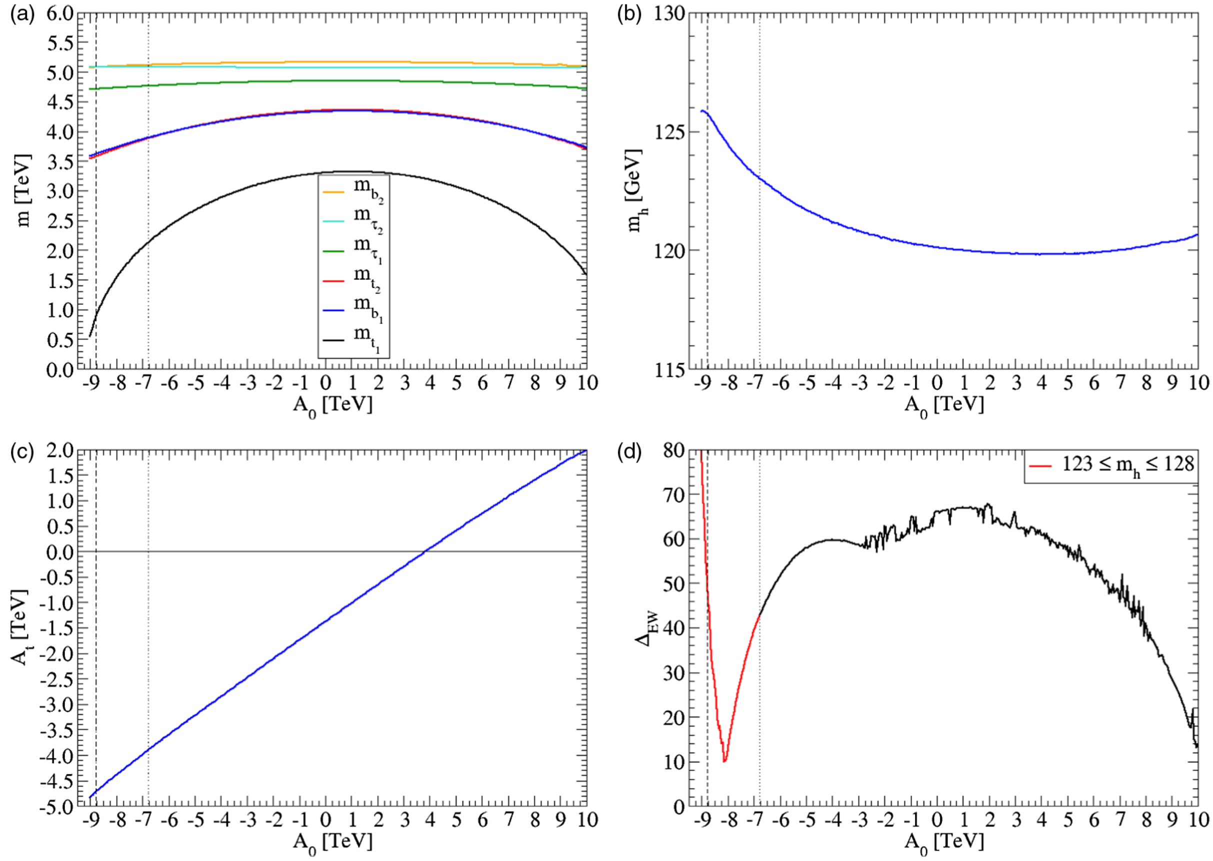

FIG. 1. In (a), we plot third generation sparticle masses vs A_0 for an RNS benchmark with $m_0 = 5$ TeV, $m_{1/2} = 900$ GeV, $\tan\beta = 10$ and with $\mu = 125$ GeV and $m_A = 1$ TeV. In (b), we plot the corresponding value of m_h and in (c) we plot A_t(weak) while in (d) we plot $\Delta_{\rm EW}$. The dashed vertical line denotes the current lower limit on $m_{\tilde{t}_1} \gtrsim 850$ GeV from ATLAS top squark searches [11] and left of the dotted vertical line denotes where $m_h > 123$ GeV. The red-shaded part corresponds to 123 GeV $\le m_h \le$ 128 GeV.

$$m_h^2 \simeq m_Z^2 \cos^2 2\beta + \frac{3g^2}{8\pi^2}\frac{m_t^4}{m_W^2}\left[\ln\frac{m_{\tilde{t}}^2}{m_t^2} + \frac{x_t^2}{m_{\tilde{t}}^2}\left(1 - \frac{x_t^2}{12 m_{\tilde{t}}^2}\right)\right], \tag{19}$$

where now $x_t = A_t - \mu\cot\beta$ and $m_{\tilde{t}}^2 \simeq m_{Q_3} m_{U_3}$. For a given value of $m_{\tilde{t}}^2$, this expression is maximal for large mixing in the stop sector with $x_t^{\rm max} = \sqrt{6} m_{\tilde{t}}$. We see from the plot that m_h is maximal for large negative A_0. This is because the *weak scale* value of A_t is large negative leading to large mixing in the stop sector. For large positive A_0, then the value of A_t largely cancels against gauge contributions in the A_t running so A_t runs to small values at the weak scale leading to small mixing and too small a value of m_h: see Fig. 1(c).

In Fig. 1(d), we show the calculated value of $\Delta_{\rm EW}$. Here, we see that $\Delta_{\rm EW} \sim 60$ for $A_0 \sim 0$, but for this value of A_0, the value of m_h is too small. For $A_0 \lesssim -7$ TeV, then we have large mixing leading to $m_h \sim 125$ GeV (shown by the red-shaded part of the curve), but also some suppression in the $\Sigma_u^u(\tilde{t}_{1,2})$ values leading to very natural solutions with $\Delta_{\rm EW} \sim 10$. For $A_0 \gtrsim +8$ TeV, then $\Delta_{\rm EW}$ drops below 30, but unfortunately m_h is too low at ∼120 GeV.

IV. NATURALNESS AND THE $m_{\tilde{t}_1}$ vs $m_{\tilde{Z}_1}$ PLANE

In this Section, we present a grand overview of the locus of natural SUSY models in the $m_{\tilde{t}_1}$ vs $m_{\tilde{Z}_1}$ mass plane. This plane was initially proposed as a template for top squark searches in Ref. [67] and has now served for several years to give a panoramic view of top squark search results in various simplified models from LHC data.

In Fig. 2, we present the results of the scan over NUHM2 parameter space from Ref. [65] where upper bounds on sparticle masses were derived from requiring not-to-large values of $\Delta_{\rm EW}$. The scan values were m_0: 0–20 TeV, $m_{1/2}$: 0.3–3 TeV, $-3 < A_0/m_0 < 3$, μ: 0.1–1.5 TeV, m_A: 0.15–20 TeV and $\tan\beta$: 3–60. The following were required: 1. Electroweak symmetry was radiatively broken. 2. The $\tilde{Z}_1$ was LSP. 3. The lightest chargino obeyed the LEP2 limit $m_{\tilde{W}_1} > 103.5$ GeV. 4. LHC8 bounds on $m_{\tilde{g}}$ and $m_{\tilde{q}}$ were respected. 5. $m_h = 125 \pm 2$ GeV. From the figure, we see that solutions with $\Delta_{\rm EW} < 15$ are clustered

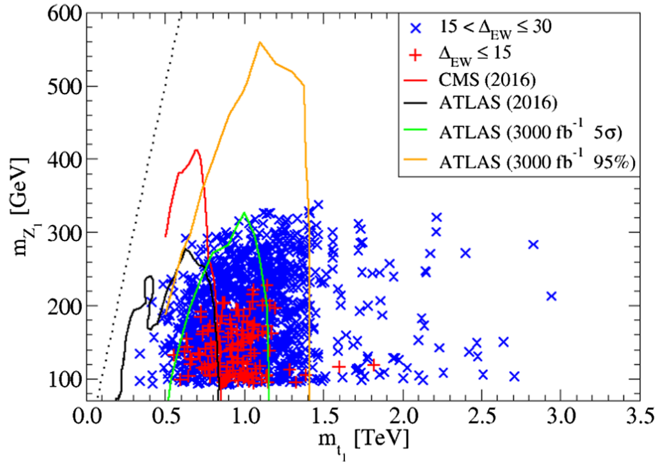

FIG. 2. The $m_{\tilde{t}_1}$ vs $m_{\tilde{Z}_1}$ mass plane for SUSY with radiatively driven naturalness and $\Delta_{\rm EW} < 15$ (red) and 30 (blue). The dotted line denotes the compressed region where $m_{\tilde{t}_1} = m_{\tilde{Z}_1}$.

with $m_{\tilde{t}_1} = 0.6$–1.3 TeV while if we allow for $\Delta_{\rm EW} < 30$ then $m_{\tilde{t}_1}$ can range up to 3 TeV.[3] The black-dotted line shows where $m_{\tilde{t}_1} \sim m_{\tilde{Z}_1}$ which is the compressed region, in which laborious searches for top squark production are taking place. Notice that essentially no highly natural solutions lie in this region. It is also important to note that the LSP is mainly Higgsino-like in this region in order to satisfy naturalness with low $\Delta_{\rm EW}$.

We also present for comparison several search contours from the ATLAS Collaboration. The region within the solid-black contour represents the area ruled out by current ATLAS searches at LHC13 for $pp \to \tilde{t}_1\tilde{t}_1^*$: for $\tilde{t}_1 \to t\tilde{Z}_1$ or $bW\tilde{Z}_1$ [11,68,69] and for $\tilde{t}_1 \to c\tilde{Z}_1$ [70]. These search results range up to $m_{\tilde{t}_1} \sim 850$ GeV which covers only a fraction of the expected range from natural SUSY. We note, however, that some of these limits might be significantly relaxed in the present case as they are obtained on the assumption of a specific decay channel in a simplified setup. For example, the limit from the ATLAS one lepton, jets plus missing energy search [68], in which all of the produced top squarks are assumed to decay into $t\tilde{Z}_1$, would be relaxed since the $\tilde{t}_1 \to t\tilde{Z}_{1,2}$ decay branch is about 50% in the natural SUSY parameter space, as we see below. Considering this, we also show in Fig. 2 as the red contour the limits presented in Ref. [43], which are obtained by recasting the CMS top squark mass limits [71] for models with light Higgsinos; the resultant upper bound on the top squark mass is again found to be about 850 GeV.

In Fig. 2, we show as well projected contours of what HL-LHC can achieve via the top squark search in the 0-lepton channel; the 5σ discovery and 95% CL exclusion contours with 3000 fb^{-1} integrated luminosity data are shown in the green and orange solid lines, respectively [72]. Here, we see that HL-LHC with 3000 fb^{-1} of integrated luminosity may be able to probe up to $m_{\tilde{t}_1} \sim 1.4$ TeV. This can be compared with a recent theory study [73] finding HL-LHC may probe top squark pair signatures to $m_{\tilde{t}_1} \sim 1.4$ TeV. A combination of the 0-lepton and 1-lepton search results may further push its reach by ∼50 GeV [72]. In either case, HL-LHC probes perhaps less than half the natural SUSY parameter space via top squark pair searches.

Before concluding this section, we comment on the excess events observed in the ATLAS top squark searches based on the one lepton, jets plus missing energy final states [68], where 2.2σ, 2.6σ, and 3.3σ excesses are observed in the signal categories, `SR1`, `bC2x_diag`, and `DM_low`, respectively. As discussed in Ref. [43], these excesses may be explained with a top squark with a mass of $\lesssim 750$ GeV and light Higgsinos with masses of $\lesssim 200$ GeV. However, such parameter region has already been excluded by other searches [11,71] as shown in Fig. 2, and thus these excesses are not accounted for in the present setup.[4]

V. THE BRANCHING FRACTION BF($b \to s\gamma$) VS $m_{\tilde{t}_1}$

Here, we examine expectations for the rare branching fraction BF($b \to s\gamma$) which takes place via Wt loops in the SM and via $\tilde{t}_i\tilde{W}_j$ and bH^+ loops in SUSY [74,75] (other SUSY loops also contribute but typically with much smaller amplitudes). The SM value for this decay is found to be [76] BF$(b \to s\gamma) = (3.36 \pm 0.23) \times 10^{-4}$ which is to be compared to the recent Belle measurement [77] that BF$(b \to s\gamma) = (3.01 \pm 0.22) \times 10^{-4}$. For the SUSY BF($b \to s\gamma$) calculation, we use the NLO results from [78] which is encoded in Isatools [63].

In Fig. 3, we show the predicted value of BF($b \to s\gamma$) from our scan over NUHM2 model parameters for points satisfying $\Delta_{\rm EW} < 15$ (red) and 30 (blue) vs $m_{\tilde{t}_1}$. The various constraints from above, including LHC search and compatibility with m_h, are included. We also indicate the Belle central value and $\pm 2\sigma$ bounds by the dashed and dot-dashed lines, respectively. From the plot, we see a large deviation between the predicted and measured values of BF($b \to s\gamma$) for light $m_{\tilde{t}_1}$ values. Especially noteworthy is that *no* values of BF($b \to s\gamma$) lie within the $\pm 2\sigma$ measured

[3]Let us compare this result with that obtained in Ref. [44]. The analysis presented in Ref. [44] shows that $\Delta_{\rm EW} < 30$ gives an upper bound on the mass of $\tilde{t}_1$ as $m_{\tilde{t}_1} \lesssim 1.6$ TeV, which is much lower than our result. This apparently severe bound results from the different strategy of the parameter scan, which turns out to be more restricted than ours. For example, they scan parameters in the ranges of 100 GeV $\le m_{\tilde{Q}_{3L},\tilde{U}_{3R}} \le 2.5$ TeV and 1 TeV $\le A_t \le 3$ TeV at the weak scale. As can be seen from Fig. 1(c), however, $A_t < -7$ TeV can give a very small value of $\Delta_{\rm EW}$, which is out of the range of the parameter scan in Ref. [44]. Top squark masses can also be as large as ∼3 TeV for $\Delta_{\rm EW} < 30$.

[4]We however note that by considering the bino LSP case with light Higgsinos, we may explain the excesses without conflicting with other limits, as discussed in Ref. [43].

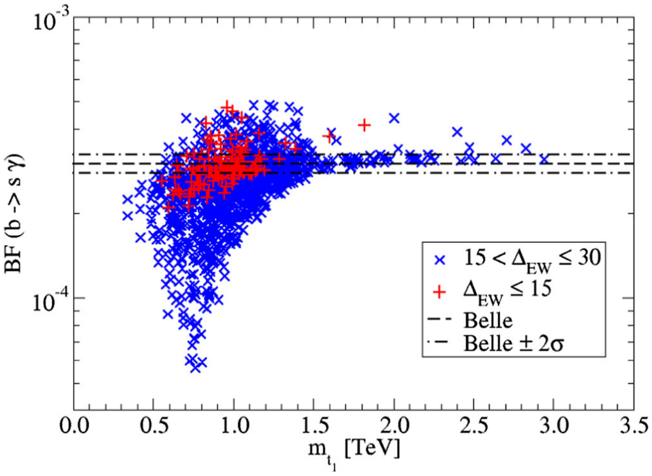

FIG. 3. Plot of $\mathrm{BF}(b \to s\gamma)$ vs $m_{\tilde{t}_1}$ for SUSY with radiatively driven naturalness and $\Delta_{EW} < 15$ (red) and 30 (blue).

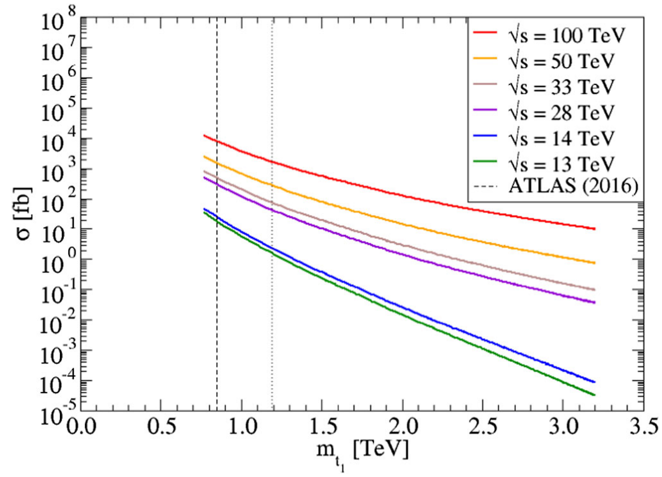

FIG. 4. NLO top squark pair production cross section vs $m_{\tilde{t}_1}$ for $\sqrt{s} = 13$, 14, 28, 33, 50 and 100 TeV. The dashed vertical line denotes the current lower limit on $m_{\tilde{t}_1} \gtrsim 850$ GeV from ATLAS top squark searches and the dotted vertical line denotes the projected reach of HL-LHC.

band for $m_{\tilde{t}_1} < 500$ GeV. Recall that this range of stop masses is often considered generally natural [53] before amending the calculations of Δ_{HS} and Δ_{BG}. As $m_{\tilde{t}_1}$ increases, the predicted range of $\mathrm{BF}(b \to s\gamma)$ rises asymptotically to be within the measured range: this occurs especially for $m_{\tilde{t}_1} > 1.5$ TeV. The intermediate region with $0.5\ \mathrm{TeV} < m_{\tilde{t}_1} < 1.5$ TeV contains points in agreement with the measured value, where the various $\tilde{t}_{1,2}\tilde{W}_{1,2}$ amplitudes, which can occur with either positive or negative values, cancel one with another. But even in this region of $m_{\tilde{t}_1}$ values, the bulk of points tend to deviate severely from the measured value. This is because the loop contributions can always be large since the Higgsino-like charginos and stops are both light. From examining the confrontation between predicted and measured values of $\mathrm{BF}(b \to s\gamma)$, it comes as no surprise that light stops have yet to be detected at LHC.

VI. TOP SQUARK PRODUCTION AND DECAY AT LHC

In this section, we consider top squark pair production and decay rates at the LHC. Top squark pair production proceeds dominantly through the QCD gg and $q\bar{q}$ annihilation channels. The NLO production rates for LHC with $\sqrt{s} = 13$ and 14 TeV are calculated using Prospino [79] and shown in Fig. 4 vs top squark mass $m_{\tilde{t}_1}$. We also show production rates for future proposed pp colliders operating with $\sqrt{s} = 28$, 33, 50 and 100 TeV. The vertical dashed line shows the approximate locus of the ATLAS/CMS bounds on $m_{\tilde{t}_1}$ from searches within the context of simplified models with a low value of $m_{\tilde{Z}_1}$. The dotted vertical line denotes the projected reach of HL-LHC for top squarks. We see that the total production cross section for $m_{\tilde{t}_1} \sim 850$ GeV at LHC14 are in the 10–20 fb range. By moving up to $m_{\tilde{t}_1} \sim 1200$ GeV, the cross section drops by about an order of magnitude to about 1 fb. At $m_{\tilde{t}_1} \sim 1.6$ TeV, $\sigma(pp \to \tilde{t}_1\tilde{t}_1^*)$ drops by another order of magnitude to about 0.1 fb. These total cross sections may be compared to the upper limit on $m_{\tilde{t}_1}$ from requiring $\Delta_{EW} < 30$ whereupon $m_{\tilde{t}_1} < 3$ TeV is required. For such large values of $m_{\tilde{t}_1}$, the total cross sections are in the 10^{-3} fb range. Probing such massive top squarks will likely require an LHC energy upgrade (HE-LHC with $\sqrt{s} \sim 28$–33 TeV) or else a future circular collider (FCC) with $\sqrt{s} \sim 50$–100 TeV [80–82].

In Fig. 5(a), we show the expected top squark branching fractions vs A_0 along the top squark model line. The branching fractions are from Isajet [63]. In the plot, the black curve denotes $\mathrm{BF}(\tilde{t}_1 \to b\tilde{W}_1)$ where for our model line $\tilde{W}_1$ is the lighter, mainly Higgsino-like, chargino. This mode occurs at the ∼50% rate and is rather model independent (within the context of natural SUSY with light Higgsinos). The $\tilde{W}_1$ further decays via 3-body mode into $\tilde{W}_1 \to f\bar{f}'\tilde{Z}_1$ where $\tilde{Z}_1$ is the Higgsino-like LSP. Since $m_{\tilde{W}_1} - m_{\tilde{Z}_1}$ (and $m_{\tilde{Z}_2} - m_{\tilde{Z}_1}$) are ∼10–20 GeV, most of the decay energy goes into making the $\tilde{Z}_1$ rest mass and is undetected. The $f\bar{f}'$ energy is rather soft leading to a few soft tracks. Thus, both the $\tilde{W}_1$ and $\tilde{Z}_2$ are only quasi-visible. Meanwhile, the b-jet from $\tilde{t}_1 \to b\tilde{W}_1$ decay may be quite hard, typically in the hundreds of GeV.

The red and blue curves denote the $\mathrm{BF}(\tilde{t}_1 \to t\tilde{Z}_2)$ and $\mathrm{BF}(\tilde{t}_1 \to t\tilde{Z}_1)$, respectively. Both these branching fractions come in at the 20–25% level thus covering the bulk of the remaining decays. While the $\tilde{Z}_1$ is invisible (it presumably comprises a portion of the dark matter), again the $\tilde{Z}_2$ and $\tilde{W}_1$ are quasivisible. Meanwhile, the top quarks are produced at large p_T and also their rest mass leads to energetic decay products. In addition, there is a non-negligible decay rate $\tilde{t}_1 \to t\tilde{Z}_3$ where $\tilde{Z}_3$ is bino-like and yields visible decays. These decays occur at the few percent level. Furthermore,

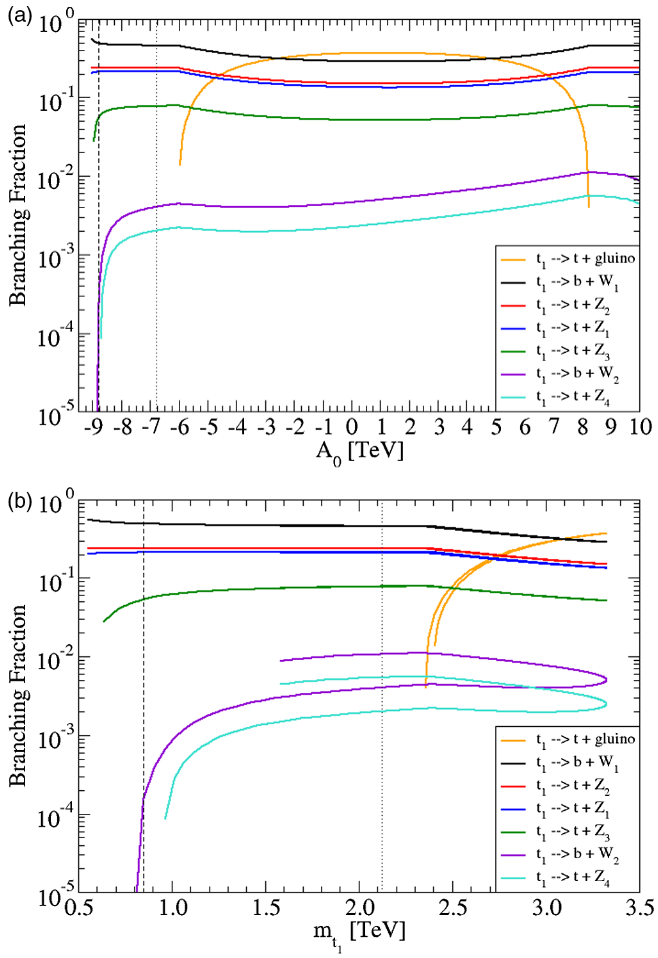

FIG. 5. Top squark branching fractions vs (a) A_0 and (b) $m_{\tilde{t}_1}$ along the RNS model line. Left of the dotted vertical line is where $m_h > 123$ GeV while left of the dashed vertical denotes where $m_{\tilde{t}_1} < 850$ GeV.

$\tilde{t}_1$ decays into wino-like $\tilde{W}_2$ and $\tilde{Z}_4$ can occur but at the subpercent level. The dip in branching fractions at the center of the plot is due to turn on of $\tilde{t}_1 \to t\tilde{g}$.

In Fig. 5(b), we show the same branching fractions vs $m_{\tilde{t}_1}$ along the model line. The branching fractions are again seen to be rather model independent except for $m_{\tilde{t}_1} \sim \mu$ (in the excluded range) where the decays into top quarks become kinematically forbidden. The branching fractions in this plot are double-valued since certain top squark mass values can occur for both large positive and large negative values of A_0. These mainly affect the tiny branching fractions into wino-like electroweakinos.

VII. PROSPECTS FOR TOP SQUARK DISCOVERY AT LHC AND BEYOND

The most direct implication of naturalness is the existence of light Higgsinos of mass $m_{\tilde{W}_1,\tilde{Z}_{1,2}} \sim 100$–$300$ GeV, the lighter the better. Given these expectations on $m(\mathrm{LSP})$, the LHC lower bound $m_{\tilde{t}_1} \gtrsim 850$ GeV applies, and we expect top squarks to lie in the mass range $m_{\tilde{t}_1} \sim 850$–3000 GeV at little cost to naturalness. This mass range is consistent with expectations from comparing the predicted $\mathrm{BF}(b \to s\gamma)$ to its measured value. Then, the highly scrutinized $\tilde{t}_1 - \tilde{Z}_1$ degeneracy rarely if ever applies, and we expect instead a rather large $m_{\tilde{t}_1} - m_{\tilde{Z}_1}$ mass difference. In this case, the top squark branching fraction predictions from Sec. VI are rather robust: they result over a huge range of NUHM2 parameter space and also under the natural general mirage mediation parameter space found in Ref. [83].[5] We would then expect, quite generally, the following collider signatures to obtain

(i) A. $\tilde{t}_1\tilde{t}_1^* \to b\bar{b} + E_T^{\mathrm{miss}} \sim 25\%$,
(ii) B. $\tilde{t}_1\tilde{t}_1^* \to b\bar{t},\ \bar{b}t + E_T^{\mathrm{miss}} \sim 50\%$,
(iii) C. $\tilde{t}_1\tilde{t}_1^* \to t\bar{t} + E_T^{\mathrm{miss}} \sim 25\%$.

These signatures should be accompanied by the usual initial state radiation plus perhaps additional semisoft tracks from associated light Higgsino $\tilde{W}_1$ and $\tilde{Z}_2$ decays.

The first signal channel, A, includes rather hard b-jets plus hard E_T^{miss} and should be plagued by backgrounds including $b\bar{b}Z$ production where $Z \to \nu\bar{\nu}$. One might create distributions using the m_{T2} variable applied to the $b\bar{b} + E_T^{\mathrm{miss}}$ final state to try to extract a kinematic upper edge which could yield an estimate of the top squark mass.

For signal channel B, we expect a hard t-jet along with a hard b-jet and E_T^{miss}. This channel would include $b\bar{b} + E_T^{\mathrm{miss}}$ along with an added $W \to f\bar{f}'$ where in the case of hadronic W decays, the W mass may be reconstructed. The dominant backgrounds would include $t\bar{t}$ production, $Wb\bar{b}$ production and WZ production where $Z \to \nu\bar{\nu}$ and $g \to b\bar{b}$, single top production and tbZ production. This "mixed top squark decay channel" has previously been emphasized by Graesser and Shelton [84].

Signal channel C contains a hard $t\bar{t}$ pair plus large E_T^{miss}. Major backgrounds would include $Zt\bar{t}$ production. The hard t-jets may benefit from a top-tagger [85].

A credible semisimplified model could be presented in the $m_{\tilde{t}_1}$ vs $m(\mathrm{Higgsino})$ mass plane where the several dominant decay branching fractions would be allowed to take place. Physically, this is what is expected to happen, and one would then include the dominant mixed decay mode where one $\tilde{t}_1$ decays to $b\tilde{W}_1$ while the other decays to $t\tilde{Z}_{1,2}$.

Finally, we comment on indirect searches for top squarks at the LHC. Since $m_{\tilde{t}_1}$:0.85–3 TeV is predicted in the radiatively driven natural SUSY, one may expect that its signature can be probed indirectly via the precise measurements of the Higgs decay branching ratios, as top squarks affect the $h \to \gamma\gamma$ and $h \to gg$ decay channels at one-loop level. As it turns out, however, the deviations of these decay branches from the SM prediction are too small to be detected even at the HL-LHC [86]. This observation

[5] The nonuniversal gaugino mass models [57] also predict similar top squark branching ratios [34,41].

again leads to the conclusion that future colliders such as ILC or an energy upgraded LHC are required for a thorough coverage of (just the top squark sector of) natural SUSY.

VIII. CONCLUSIONS

In this paper we have re-examined the phenomenology of top squarks expected from natural SUSY. We first noted that older expectations of very light top squarks based on requiring small $\delta m_{H_u}^2/m_h^2$ are technically flawed in that they neglect the contribution of $m_{H_u}^2$ to its own running. By properly including this contribution, then the $\Delta_{\rm HS}$ measure reduces to $\Delta_{\rm EW}$. The $\Delta_{\rm EW} < 30$ requires light Higgsinos $\sim$100–300 GeV while much heavier top squarks $m_{\tilde{t}_1} \sim$ 0.85–3 TeV are allowed at little cost to naturalness. In the latter case, the radiative corrections to $m_{H_u}^2$ aid in driving it from large unnatural high scale values to natural values at the weak scale—a situation known as radiatively driven natural SUSY or RNS. For the case of BG naturalness, if $\Delta_{\rm BG}$ is evaluated in multisoft-parameter effective theories, then one obtains an overestimate of fine-tuning as compared to the calculation for a more fundamental theory wherein the soft terms are all correlated. In the latter case, $\Delta_{\rm BG}$ reduces to $\Delta_{\rm EW}$.

Using $\Delta_{\rm EW}$, it is found that current LHC top squark search constraints have probed only a fraction of the allowed $m_{\tilde{t}_1}$ vs $m_{\tilde{Z}_1}$ parameter plane. The compressed region, which has been heavily searched, admits few or no solutions. Further, values of $m_{\tilde{t}_1} < 500$ GeV lead to typically large deviations in BF$(b \to s\gamma)$. Top squark production and decay rates are calculated in natural SUSY and lead to comparable mixtures of $E_T^{\rm miss}$ plus $b\bar{b}$, $t\bar{t}$ and tb signatures. It is emphasized that a semisimplified model containing the major admissible final states would be most helpful to truly constrain the natural SUSY parameter space or to discover top squarks.

What then is our guidance for top squark hunters? While we agree that—if possible—every corner of parameter space ought to be explored, it is also practical to focus search efforts, at least at first, on the theoretically most compelling scenarios. Much effort is being placed on top squark searches in the compressed region of parameter space with $m_{\tilde{t}_1} \sim m_{\tilde{Z}_1}$ with $m_{\tilde{t}_1} \lesssim 500$ GeV. However, if the stop was indeed that light, then likely the Higgs mass m_h would be closer to $m_h \sim 115-120$ GeV and also one would expect large measured deviations from the SM value of the $BF(b \to s\gamma)$ branching fraction. Instead, we advocate more detailed exploration of the high top squark mass region with 2-body decays to low mass Higgsino-like $\tilde{W}_1$ and $\tilde{Z}_{1,2}$ states. The likely most lucrative search would include all three top squark branching fractions as detailed in Sec. VI. We emphasize that focused searches in these directions are the most likely to bear the fruit of discovery.

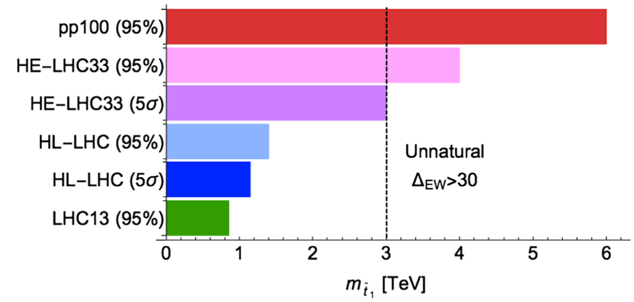

FIG. 6. Current limits on top squarks along with projected discovery and exclusion reaches of future possible colliders.

Even with more focused high mass top squark searches, it should be clear that plenty of perfectly natural SUSY solutions still exist with $m_{\tilde{t}_1}$ values well beyond the reach of even HL-LHC. To probe the entire expected natural SUSY top squark parameter space will likely require an energy upgrade of LHC to the $\sqrt{s} \sim 28$–33 TeV regime. To this end, in Fig. 6, we show the current exclusion limit on top squark masses from ATLAS/CMS for a light $\tilde{Z}_1 \sim 100$–200 GeV. We also show the HL-LHC projected reach and exclusion limits for 3000 fb^{-1} of integrated luminosity [72] along with the projected reach of future pp colliders with $\sqrt{s} = 33$ and 100 TeV [87]. In contrast to common notions, the display shows that HL-LHC has a very limited reach for natural SUSY in the top-squark pair production channel. Even if no top-squark signal is seen at HL-LHC, then there will be little impact on excluding natural SUSY (other channels such as same-sign diboson or soft dilepton plus jets appear more lucrative to HL-LHC) [88]. However, an energy upgrade to HE-LHC with $\sqrt{s} =$ 33 TeV will have a 5σ discovery reach to $m_{\tilde{t}_1} \sim 3$ TeV and a 95% CL exclusion reach to 4 TeV. Such a reach will either discover or exclude natural SUSY in the top squark sector. We also show the Snowmass projected reach [87] for top squark pairs for a 100 TeV collider. Such a machine is projected to probe up to $m_{\tilde{t}_1} \sim 6$ TeV. This reach probes beyond a 33 TeV machine only further into unnatural regions of parameter space.

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