$c\bar{b}$ spectrum and decay properties with coupled channel effects

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The mass spectrum of $c\bar{b}$ states has been obtained using the phenomenological relativistic quark model (RQM) with coupled channel effects. The Hamiltonian used in the investigation has confinement potential and confined one gluon exchange potential (COGEP). In the frame work of the RQM, a study of magnetic dipole and electric dipole transitions and radiative decays of $c\bar{b}$ states has been made. The weak decay widths in the spectator quark approximation have been estimated. An overall agreement is obtained with the experimental masses and decay widths.

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I. INTRODUCTION

The B_c meson is a double heavy quark-antiquark bound state, carries flavors explicitly, and provides a good platform for a systematic study of heavy quark dynamics. B_c mesons are predicted by the quark model to be members of the $J^P = 0^-$ pseudoscalar ground state multiplet [1]. The first successful observation of the B_c meson was made by the Collider Detector at Fermilab (CDF) Collaboration in 1998 from the first run at Tevatron through the semileptonic decay channel $B_c \to J/\Psi + l^+ + \bar{\nu}_l$ [2]. They measured the mass of B_c to be $m_{B_c} = 6400 \pm 390 \pm 130$ MeV and the lifetime $\tau_{B_c} = 0.46^{+0.18}_{-0.16} \pm 0.03$ ps. The more precise measurement of mass of the B_c, i.e., $m_{B_c} = 6275.6 \pm 2.9(\mathrm{stat}) \pm 5(\mathrm{syst})$ MeV was done by the CDF Collaboration through the exclusive non-leptonic decay $B_c \to J/\Psi\pi^+$ [3–5]. The results of the CDF Collaboration was confirmed by the observations made by the D0 Collaboration [6,7] at Tevatron. The LHCb has reported several new observations on B_c decays recently. More experimental data on the B_c meson are expected in the near future from the LHCb and Tevatron.

A suitable theoretical model is required to explain the properties such as mass spectrum, decays, reaction mechanisms, and the bound state behavior of mesons which involve heavy quarks. The properties of the light and heavy mesons were studied using the phenomenological models. The work of A. De Rujula *et al.* [8], proposed the first QCD based model for the study of hadron spectroscopy. The model had a reasonable success and predicted the masses of charmed mesons and baryons. Several nonrelativistic phenomenological potentials with radial dependencies for confinement along with one gluon exchange potential (OGEP) were examined by Bhaduri *et al.* [9]. The ground state heavy meson spectrum has been studied by Vijaya Kumar *et al.* [10]. Radiative decay properties of light vector mesons have been studied by Monteiro *et al.* [11]. Bottomonium spectrum and its decay properties have been studied in a nonrelativistic model using OGEP by Monteiro *et al.* [12]. The work of Bhagyesh *et al.* [13], in their nonrelativistic model, used Hulthén potential to study the orbitally excited quarkonium states. In these models, the relativistic effects were completely ignored.

There have been many calculations of baryon properties using relativistic models, like MIT bag models [14,15], cloudy bag models [16,17], chiral bag models [18,19], etc. Relativistic calculations, where constituent quarks are confined in a potential, have also been performed [20–22]. There are other bag models in literature too. In the Budapest bag model, the volume energy term is replaced by a surface energy term [23]. Another model, which effectively contains a surface tension term, is the "SLAC" bag model, developed by Bardeen *et al.* [24], which begins from a local field theory in which heavy quarks interact through a neutral scalar field. The work of Ferreria *et al.* [25,26] used relativistic quark model to study several properties of low lying hadrons. In this model, both the linear and quadratic confinement schemes were used. The work of Bander *et al.* [27] used a relativistic bound state formalism to make the simultaneous study of all meson systems. The work of Isgur *et al.* [28,29], in their relativized quark model, used a parametrized potential and incorporated relativistic kinematics to describe all of the mesons in the same frame work.

In nonrelativistic quark model (NRQM) formalism though, the mass spectra of the ground state $c\bar{b}$ meson has been produced successfully; the radiative decay rates, particularly hindered $M1$ decay rates are significantly influenced by relativistic effects. Therefore, it is necessary to include these effects for the correct description of the decays. Radiative decays are the most sensitive to relativistic effects. Hindered radiative decays are forbidden in the nonrelativistic limit due to the orthogonality of the initial

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and final meson wave functions. They have decay rates of the same order as the allowed ones. In the relativistic description of mesons, an important role is played by the confining quark-antiquark interaction, particularly its Lorentz structure. Thus, comparison of theoretical predictions with experimental data can provide valuable information on the nature of the confining potential. Hence, we use relativistic quark model formalism to study the properties of $c\bar{b}$ meson states.

The paper is organized in four sections. In Sec. II, we briefly review the theoretical background for the relativistic model, the framework of the coupled channel analysis, and the relativistic description of radiative decay widths. In Sec. III, we discuss the results and the conclusions are drawn in Sec. IV, with a comparison to other models.

II. THEORETICAL BACKGROUND

A. The relativistic harmonic model

We investigate the properties of $c\bar{b}$ states using confined one gluon exchange potential in the framework of the relativistic harmonic model (RHM) [20]. The Hamiltonian used has the confinement potential and a two body confined one gluon exchange potential (COGEP) [30–33].

The confinement potential has a Lorentz scalar and a vector harmonic oscillator potential part [34,35],

$$V_{\mathrm{CONF}}(r) = \frac{1}{2}(1+\gamma_0)A^2r^2 + M, \tag{1}$$

where γ_0 is the Dirac matrix, M is a constant mass, and A^2 is the confinement strength.

We use the following harmonic oscillator wave equation

$$\left(\frac{p^2}{E+M} + A^2r^2\right)\phi = (E-M)\phi, \tag{2}$$

the eigenvalue of which is given by

$$E_N^2 = M^2 + (2N+1)\Omega_N, \tag{3}$$

where Ω_N is the energy dependent oscillator size parameter given by

$$\Omega_N = A(E_N+M)^{1/2}, \tag{4}$$

where $\vec{p}$ is the momentum. For the detailed description of the RHM see [20,34,35].

B. Confined one gluon exchange potential

In the present existing models for low energy nuclear phenomena, the gluon degrees of freedom have been eliminated from the theoretical framework, and it is assumed that the gluon exchange can be incorporated into the theory through OGEP. But in deriving the OGEP [8], the gluon propagators used are similar to the free photon propagators used in obtaining the Fermi-Breit interaction in QED. Since the confinement of color means the confinement of quarks as well as gluons, the confined dynamics of gluons should play a decisive role in determining the hadron spectrum and in the hadron-hadron interaction. The confinement schemes for quarks and gluons have to be more closely connected to each other in QCD, and the confinement of gluons has to be taken into account. The COGEP is obtained from the scattering amplitude using confined gluon propagators [30–32,36]. Here $D^{ab}_{\mu\nu} = \partial^{ab}D_{\mu\nu}$ are the gluon propagators in the momentum representation in current confinement model (CCM) [37]. The CCM was developed for the confinement of gluons in the spirit of the RHM and aims at a unified confinement theory for the study of the quark-gluon bond system in the spirit of the RHM for the confinement of gluons. In the CCM, the coupled nonlinear terms in the Yang-Mill tensor is treated as a color gluon super current in analog with Ginzburg-Landu's theory of superconductivity. The coupled nonlinear terms in the equation of motion of a gluon are simulated by a self induced color current $j_\mu = \theta^{\vartheta}_{\mu}A_\nu(= m^2A_\mu)$, or equivalently, an effective mass term for all of the gluons with $m^2 = c^4r^2 - 2c^2\delta_{\mu 0}$. The equation of motion is solved using harmonic oscillator modes in the general Lorentz gauge, imposing a secondary gauge condition termed the oscillator gauge. The two confined gluon propagators are then obtained in this gauge using the properties of the harmonic oscillator wave functions. The RHM with the COGEP has been quite successful in obtaining the N-N phase shifts and in hadron spectroscopy [32].

The COGEP is obtained from the scattering amplitude [30–33]

$$\mathcal{M}_{fi} = \frac{g_s^2}{4\pi}\bar{\psi'}_1\gamma^\mu\psi_1 D^{ab}_{\mu\nu}(q)\bar{\psi'}_2\gamma^\nu\psi_2, \tag{5}$$

where $\bar{\psi} = \psi^\dagger\gamma_0$, $\psi_{1,2}$ are the wave functions of the quarks in the RHM. The $D_{00}(q)$ and $D_{\mathrm{ik}}(q)$ are the zero energy CCM gluon propagators in momentum representation, where $q = P_1' - P_1 = P_2 - P_2'$ is the four momentum transfer. $g_s^2/4\pi = \alpha_s$ is the quark gluon coupling constant. In CCM, propagators in the momentum representation are given by,

$$D_{00}(q) = 4\pi D_0(q) \tag{6}$$

The $D_{ik}(q)$ are given by,

$$D_{ik}(q) = -4\pi\left\{\delta_{ik} - \frac{a^\dagger_{q_i}a_{q_k}}{a_q\cdot a^\dagger_k}\right\}D_1(q), \tag{7}$$

where a_q and $a_q^\dagger$ are the creation and destruction operators in the momentum space.

The scattering amplitude (5) is written as

$$\mathcal{M}_{fi} = \frac{g_s^2}{4\pi}(\psi'^\dagger_1\psi_1\psi'^\dagger_2\psi_2)D_{00}(q) + (\psi'^\dagger_1\alpha_i\psi_1)(\psi'^\dagger_2\alpha_k\psi_2)D_{ik}(q). \tag{8}$$

We express the four component RHM wave function ψ in terms of a two component wave function ϕ by a similarity transformation, i.e.,

$$\psi'^\dagger_1\psi_1 = \psi'^\dagger_1 U'^\dagger_1(U'^\dagger_1)^{-1}U_1^{-1}U_1\psi_1 \tag{9}$$

$$= \phi'^\dagger_1(U'^\dagger_1)^{-1}U_1^{-1}\phi_1, \tag{10}$$

where

$$N = \sqrt{\frac{2(E+M)}{3E+M}} \tag{11}$$

and

$$U = \frac{1}{N[1+\frac{p^2}{(E+M)^2}]}\begin{pmatrix} 1 & \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E+M} \\ -\frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E+M} & 1 \end{pmatrix}. \tag{12}$$

The above expression can be simplified to

$$\psi'^\dagger_1\psi_1 = N^2\phi'^\dagger_1\left\{1+\left[\frac{P_1^2+q\cdot P_1+i\sigma_1\cdot(q\times P_1)}{(E+M)^2}\right]\right\}\phi_1. \tag{13}$$

We have

$$\psi'^\dagger_2\psi_2 = \phi'^\dagger_2(U'^\dagger_2)^{-1}U_2^{-1}U_2\phi_2, \tag{14}$$

i.e.,

$$\psi'^\dagger_2\psi_2 = N^2\phi'^\dagger_2\left\{1+\left[\frac{P_2^2+q\cdot P_2+i\sigma_2\cdot(q\times P_2)}{(E+M)^2}\right]\right\}\phi_2. \tag{15}$$

Similarly we can write,

$$\psi'^\dagger_1\alpha_i\psi_1 = \frac{N^2}{(E+M)}[\phi'^\dagger_1[2P_1+q+i(\sigma_1\times q)]\phi_1]_i \tag{16}$$

$$\psi'^\dagger_2\alpha_k\psi_2 = \frac{N^2}{(E+M)}[\phi'^\dagger_2[2P_2-q-(i\sigma_2\times q)]\phi_2]_k. \tag{17}$$

Substituting (13), (15), (16), and (17) in (8), the scattering amplitude now expressed in terms of the two component spinor ϕ and the momentum dependent operator U can be written as,

$$\mathcal{M}_{fi} = 4\pi\alpha_s N^4\phi_1^\dagger\phi_2^\dagger[U[P_1,P_2,q]]\phi_1\phi_2 \tag{18}$$

The function $U(P_1,P_2,q)$ is the particle interaction operator in the momentum representation, and by taking the Fourier transformation of each term in the scattering amplitude, we get the potential operator $U(\hat{P}_1,\hat{P}_2,r)$ in the coordinate space. We drop all of the higher order momentum dependent terms in $U(\hat{P}_1,\hat{P}_2,r)$ to obtain the scattering amplitude which is given by

$$\mathcal{M}_{fi} = 4\pi\alpha_s N^4\left[1+\frac{1}{(E+M)^2}[\sigma_1\cdot(\nabla\times\hat{P}_1)-\sigma_2\cdot(\nabla\times\hat{P}_2)]\right]D_0(\vec{r}) + 4\pi\alpha_s N^4 \times\left[\frac{1}{(E+M)^2}[2\sigma_2\cdot(\nabla\times\hat{P}_1)-2\sigma_1\cdot(\nabla\times\hat{P}_2)-\nabla^2[1-\sigma_1\cdot\sigma_2]-(\sigma_1\cdot\nabla)(\sigma_2\cdot\nabla)]D_1(\vec{r})\right]. \tag{19}$$

The terms which contribute to the central part of the COGEP are,

$$D_0(\vec{r}),\ \nabla^2[\sigma_1\cdot\sigma_2-1]D_1(\vec{r})\ \text{ and }\ (\sigma_1\cdot\nabla)(\sigma_2\cdot\nabla)D_1(\vec{r}).$$

In the CCM, the propagator $D_1(\vec{r})$ satisfies the differential equation

$$(-\nabla^2+c^4r^2)D_1(\vec{r}) = 4\pi\delta^3(\vec{r}). \tag{20}$$

The term $(\sigma_1\cdot\nabla)(\sigma_2\cdot\nabla)D_1(\vec{r})$ has angular dependence, but the tensor operator is constructed in such a way that the average value of the tensor operator over the angular variables vanishes. The averaging over the direction of r gives

$$(\sigma_1\cdot\nabla)(\sigma_2\cdot\nabla)D_1(\vec{r}) = (1/3)\sigma_1\cdot\sigma_2[\nabla^2D_1(\vec{r})]. \tag{21}$$

Substituting for $[\nabla^2D_1(\vec{r})]$, the central part of the COGEP becomes

$$V_{\text{COGEP}}^{\text{cent}}(\vec{r}) = \frac{\alpha_s N^4}{4}\vec{\lambda}_i\cdot\vec{\lambda}_j\left[D_0(\vec{r})+\frac{1}{(E+M)^2}[4\pi\delta^3(\vec{r})-c^4r^2D_1(\vec{r})]\left[1-\frac{2}{3}\vec{\sigma}_i\cdot\vec{\sigma}_j\right]\right], \tag{22}$$

where $D_0(\vec{r})$ and $D_1(\vec{r})$ are the propagators given by

$$D_0(\vec{r}) = \frac{\Gamma_{1/2}}{4\pi^{3/2}} c(cr)^{-3/2} W_{1/2;-1/4}(c^2r^2) \tag{23}$$

$$D_1(\vec{r}) = \frac{\Gamma_{1/2}}{4\pi^{3/2}} c(cr)^{-3/2} W_{0;-1/4}(c^2r^2), \tag{24}$$

where λ_i and λ_j are the color matrices, $\Gamma_{1/2} = \sqrt{\pi}$, W's are the Whittaker functions, and $c(\mathrm{fm}^{-1})$ is a constant parameter, which gives the range of the propagation of gluons and is fitted in the CCM to obtain the glueball spectra, and r is the distance from the confinement center.

The terms which contribute to the spin orbit part of the COGEP are

$$[\sigma_1 \cdot (\nabla \times \hat{P}_1) - \sigma_2 \cdot (\nabla \times \hat{P}_2)]D_0(\vec{r}) + [2\sigma_2 \cdot (\nabla \times \hat{P}_1) - 2\sigma_1 \cdot (\nabla \times \hat{P}_2)]D_1(\vec{r}), \tag{25}$$

operating ∇ on $D_0(\vec{r})$ and $D_1(\vec{r})$ and defining

$$\hat{P} = (\hat{P}_1 - \hat{P}_2)/2 \quad \text{and} \quad \hat{P}_{\mathrm{CM}} = \hat{P}_1 + \hat{P}_2.$$

The spin orbit part of COGEP is

$$V_{12}^{LS}(\vec{r}) = \frac{\alpha_s}{4} \frac{N^4}{(E+M)^2} \frac{\lambda_1 \cdot \lambda_2}{2r} \times [[r \times (\hat{P}_1 - \hat{P}_2) \cdot (\sigma_1 + \sigma_2)](D_0'(\vec{r}) + 2D_1'(\vec{r})) + [r \times (\hat{P}_1 + \hat{P}_2) \cdot (\sigma_1 - \sigma_2)](D_0'(\vec{r}) - D_1'(\vec{r}))]. \tag{26}$$

The spin orbit term has been split into the symmetric $(\sigma_1 + \sigma_2)$ and anti symmetric $(\sigma_1 - \sigma_2)$ terms.

The terms which contribute to the tensor part of the COGEP are,

$$\left[(\sigma_1 \cdot \nabla)(\sigma_2 \cdot \nabla)D_1(\vec{r}) - \left(\frac{1}{3}\sigma_1 \cdot \sigma_2[\nabla^2 D_1(\vec{r})]\right)\right]. \tag{27}$$

The tensor part of the COGEP is,

$$V_{12}^{\mathrm{TEN}}(\vec{r}) = -\frac{\alpha_s}{4} \frac{N^4}{(E+M)^2} \lambda_1 \cdot \lambda_2 \left[\frac{D_1''(\vec{r})}{3} - \frac{D_1'(\vec{r})}{3r}\right] S_{12}, \tag{28}$$

where

$$S_{12} = [3(\sigma_1 \cdot \hat{r})(\sigma_2 \cdot \hat{r}) - \sigma_1 \cdot \sigma_2]. \tag{29}$$

C. Coupled channel effects

In this section we briefly review coupled channel models. For detailed discussions on coupled channel effects see [38–52].

Current QCD inspired potential models generally neglect the hadron loop effects (continuum couplings). These couplings lead to two body strong decays of the meson; above threshold and below threshold they give rise to mass shifts of the bare meson states.

In the coupled channel model, the full hadronic state is given by [46,48,49]

$$|\psi\rangle = |A\rangle + \sum_{BC} |BC\rangle, \tag{30}$$

where we have considered open flavor strong decay $A \to BC$. Here A, B, C denote mesons.

The wave function $|\psi\rangle$ obeys the equation

$$H|\psi\rangle = M|\psi\rangle. \tag{31}$$

The Hamiltonian H for this combined system consists of a valence Hamiltonian H_0 and an interaction Hamiltonian H_I, which couples the valence and continuum sectors.

$$H = H_0 + H_I, \tag{32}$$

where

$$H_I = g \int d^3x \bar{\psi}\psi. \tag{33}$$

The matrix element of the valence-continuum coupling Hamiltonian is given by [48,49]

$$\langle BC|H_I|A\rangle = h_{fi}\delta(\vec{P}_A - \vec{P}_B - \vec{P}_C), \tag{34}$$

where h_{fi} is the decay amplitude.

The mass shift of hadron A, due to its continuum coupling to BC, can be expressed in terms of partial wave amplitude $\mathcal{M}_{LS}$ [46,49]

$$\Delta M_A^{(BC)} = \int_0^\infty dp \frac{p^2}{E_B + E_C - M_A - i\epsilon} \int d\Omega_p |h_{fi}(p)|^2 = \int_0^\infty dp \frac{p^2}{E_B + E_C - M_A - i\epsilon} \sum_{LS} |\mathcal{M}_{LS}|^2,$$
$$\Delta M_A^{(BC)} = \mathcal{P}\int_0^\infty dp \frac{p^2}{E_B + E_C - M_A} \sum_{LS} |\mathcal{M}_{LS}|^2 + i\pi \left(\frac{p * E_B * E_C}{M_A} \sum_{LS} |\mathcal{M}_{LS}|^2\right)\Bigg|_{E_B+E_C=M_A}. \tag{35}$$

The decay amplitude h_{fi} can be combined with a relativistic phase space to give the differential decay rate, which is

$$\frac{d\Gamma_{A\to BC}}{d\Omega} = 2\pi P \frac{E_B E_C}{M_A} |h_{fi}|^2, \tag{36}$$

where in the rest frame of A, we have $\vec{P}_A = 0$ and $P = |\vec{P}_B| = |\vec{P}_C|$;

$$P = \sqrt{[M_A^2 - (M_B + M_C)^2][M_A^2 - (M_B - M_C)^2]}/(2M_A) \tag{37}$$

The total decay rate is given by [46,49]

$$\Gamma_{A\to BC} = 2\pi P \frac{E_B E_C}{M_A} \sum_{LS} |\mathcal{M}_{LS}|^2. \tag{38}$$

D. Radiative decays

Radiative decays are a powerful tool for the study of the quark structure of mesons, and the calculation of corresponding amplitudes is a subject of increasing interest. We consider two types of radiative transitions of the B_c meson:

(a) Electric dipole (E1) transitions are those transitions in which the orbital quantum number is changed ($\Delta L = 1$, $\Delta S = 0$). E1 transitions do not change quark spin. Examples of such transitions are $n^3S_1 \to n'^3P_J\gamma (n > n')$ and $n^3P_J \to n'^3S_1\gamma (n \geq n')$. The partial widths for electric dipole (E1) transitions between states ${}^{2S+1}L_{iJ_i}$ and ${}^{2S+1}L_{fJ_f}$ are given by

$$\Gamma_{a\to b\gamma} = \frac{4\alpha}{9}\mu^2 \left(\frac{Q_c}{m_c} - \frac{Q_{\bar{b}}}{m_{\bar{b}}}\right)^2 \frac{E_b(k_0)}{m_a} k_0^3 |\langle b|r|a\rangle|^2 \begin{cases} (2J+1)/3, & {}^3S_1 \to {}^3P_J \\ 1/3, & {}^3P_J \to {}^3S_1 \\ 1/3, & {}^1P_1 \to {}^1S_0 \\ 1, & {}^1S_0 \to {}^1P_1, \end{cases} \tag{39}$$

where k_0 is the energy of the emitted photon,

$k_0 = \frac{m_a^2 - m_b^2}{2m_a}$ in relativistic model.

α is the fine structure constant. $Q_c = 2/3$ is the charge of the c quark, and $Q_{\bar{b}} = 1/3$ is the charge of the $\bar{b}$ quark in units of $|e|$, μ is reduced mass, $m_{\bar{b}}$ and m_c are the masses of b quark and c quark, respectively, and m_a and m_b are the masses of the initial and final mesons.

$$\mu = \frac{m_{\bar{b}} m_c}{m_{\bar{b}} + m_c}$$

and

$$\frac{E_b(k_0)}{m_a} = 1,$$
$$\langle b|r|a\rangle = \int_0^\infty r^3 R_b(r) R_a(r) dr, \tag{40}$$

is the radial overlap integral which has the dimension of length, with $R_{a,b}(r)$ being the normalized radial wave functions for the corresponding states.

(b) Magnetic dipole ($M1$) transitions are those transitions in which the spin of the quarks is changed ($\Delta S = 1$, $\Delta L = 0$), and thus the initial and final states belong to the same orbital excitation but have different spins. Examples of such transitions are vector to pseudoscalar ($n^3S_1 \to n'^1S_0 + \gamma$, $n \geq n'$) and pseudoscalar to vector ($n^1S_0 \to n'^3S_1 + \gamma$, $n > n'$) meson decays.

The magnetic dipole amplitudes between S-wave states are independent of the potential model.

The $M1$ partial decay width between S-wave states is [53–59]

$$\Gamma_{a\to b\gamma} = \delta_{L_a L_b} 4\alpha k_0^3 \frac{E_b(k_0)}{m_a} \left(\frac{Q_c}{m_c} + (-1)^{S_a+S_b} \frac{Q_{\bar{b}}}{m_{\bar{b}}}\right)^2 (2S_a+1) \times (2S_b+1)(2J_b+1)$$
$$\times \begin{Bmatrix} S_a & L_a & J_a \\ J_b & 1 & S_b \end{Bmatrix}^2 \begin{Bmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & S_a & S_b \end{Bmatrix}^2 \times \left[\int_0^\infty R_{n_b L_b}(r) r^2 j_0(kr/2) R_{n_a L_a}(r) dr\right]^2, \tag{41}$$

where $\int_0^\infty dr R_{n_b L_b}(r) r^2 j_0(kr/2) R_{n_a L_a}(r)$ is the overlap integral for the unit operator between the coordinate wave functions of the initial and the final meson states, $j_0(kr/2)$ is the spherical Bessel function. S_a, S_b, L_a, J_a, and J_b are the spin quantum number, orbital angular momentum, and total angular momentum of the initial and final meson states, respectively.

E. Weak decays

The weak decays of mesons provide information about the underlying quark dynamics within the system. The weak decays of the bound state of a quark and an antiquark, which carry heavy flavor c and b, enable us to probe the validity of the standard model of elementary particle interactions and determine several parameters of this model. A rough estimate of the B_c weak decay widths can be done by treating the $\bar{b}$-quark and c-quark decay independently, so that B_c decays can be divided into three classes [60,61]: (i) the $\bar{b}$-quark decay with spectator c quark, (ii) the c-quark decay with spectator $\bar{b}$ quark, and (iii) the annihilation $B_c^+ \to l^+ \nu_l$ $(c\bar{s}, u\bar{s})$, where $l = e, \mu, \tau$.

III. RESULTS AND DISCUSSION

A. Mass spectrum of $c\bar{b}$ states with coupled channel effects

The quark-antiquark wave functions in terms of oscillator wave functions corresponding to the relative and center of mass coordinates have been expressed here, which are of the form,

$$\Psi_{nlm}(r,\theta,\phi) = N\left(\frac{r}{b}\right)^l L_n^{l+\frac{1}{2}}\left(\frac{r^2}{b^2}\right)\exp\left(-\frac{r^2}{2b^2}\right) Y_{lm}(\theta,\phi), \quad (42)$$

where N is the normalizing constant given by

$$|N|^2 = \frac{2n!}{b^3 \pi^{1/2}} \frac{2^{[2(n+l)+1]}}{(2n+2l+1)!}(n+l)!. \quad (43)$$

$L_n^{l+\frac{1}{2}}$ are the associated Laguerre polynomials.

The six parameters are the mass of charm quark m_c, the mass of beauty quark $m_{\bar{b}}$, the harmonic oscillator size parameter b, the confinement strength A^2, the CCM parameter c, and the quark-gluon coupling constant α_s. The parameters $m_c, m_{\bar{b}}, A^2$ are obtained by a χ^2 square fit to the available experimental data of charmonium, bottomonium, and B_c meson mass spectra. The CCM parameter c is taken from ref ([30,37,62]), which was obtained by fitting the iota (1440 MeV)0^{-+} as a digluon glueball.There are several papers in literature where the size parameter b is defined [29,63]. We obtain the value "b" by minimizing the expectation value of the Hamiltonian, i.e, $\frac{\partial\langle\psi|H|\psi\rangle}{\partial b} = 0$. We

TABLE I. m_c and $m_{\bar{b}}$ for various theoretical models (in MeV).

Parameter	Ref. [64]	Ref. [65]	Ref. [66]	Ref. [67]	Ref. [68]
m_c	1800	1480	1480	1480	1550
$m_{\bar{b}}$	5174	5180	4880	4880	4880

then tune the parameter α_s to reproduce the experimental mass value. In the literature, we find different sets of values for m_c and $m_{\bar{b}}$, which are listed in Table I. The values of the strong coupling constant α_s in the literature are listed in Table II. The value of the strong coupling constant ($\alpha_s = 0.3$) used in our model is compatible with the perturbative treatment.

We use the following set of parameter values:

$$m_c = 1525.00 \pm 0.37 \text{ MeV}; \quad m_{\bar{b}} = 4825.00 \pm 0.29 \text{ MeV};$$
$$b = 0.3 \text{ fm}; \quad \alpha_s = 0.3; \quad A^2 = 550.00 \pm 0.78 \text{ MeV fm}^{-2};$$
$$c = 1.74 \text{ fm}^{-1}. \quad (44)$$

We evaluate the bare state masses and shifts due to BD, $B_s D_s$, $B^0 D^0$, $B^* D$, $B_s^* D_s$, $B^* D^*$, and $B_s^* D_s^*$ meson loops (with $M_B = 5279.26$ MeV, $M_{B_s} = 5366.77$ MeV, $M_{B^0} = 5279.58$ MeV, $M_{B^*} = 5324.6$ MeV, $M_{B_s^*} = 5415.4$ MeV, $M_D = 1869.61$ MeV, $M_{D_s} = 1968.30$ MeV, $M_{D_0} = 1864.84$ MeV, $M_{D^*} = 2006.96$ MeV and $M_{D_s^*} = 2112.1$ MeV). The mass shifts calculated in our model due to the hadron loop effects are listed in Table III.

For the case of a bound system of quark and antiquark of unequal mass, charge conjugation parity is no longer a good quantum number, so that states with different total spins but with the same total angular momentum, such as the ${}^3P_1 - {}^1P_1$ and ${}^3D_2 - {}^1D_2$ pairs, can mix via the spin orbit interaction or some other mechanism. The B_c meson states with $J = L$ are a linear combination of the spin triplet $|{}^3L_J\rangle$ and spin singlet $|{}^1L_J\rangle$ states, which we describe by the following by mixing:

$$|nL\rangle' = |n^1L_J\rangle \cos\theta_{nL} + |n^3L_J\rangle \sin\theta_{nL,} \quad (45)$$

$$|nL\rangle = -|n^1L_J\rangle \sin\theta_{nL} + |n^3L_J\rangle \cos\theta_{nL,}$$
$$J = L = 1, 2, 3, \ldots, \quad (46)$$

where θ_{nL} is a mixing angle, and the primed state has the heavier mass. For $L = J = 1$ we have the mixing of P states,

TABLE II. α_s for various theoretical models.

Parameter	Ref. [28]	Ref. [68]	Ref. [69]	Ref. [1]	Ref. [70]
α_s	0.21	0.265	0.357	0.361	0.391

TABLE III. Mass shifts (in MeV).

Bare $c\bar{b}$ State	BD	B_sD_s	B_0D_0	B^*D	$B_s^*D_s$	B^*D^*	$B_s^*D_s^*$	Total
1^1S_0	0	0	0	−5.661	−5.033	−10.434	−9.328	−30.456
1^3S_1	−2.046	−1.805	−2.052	−3.955	−3.496	−7.293	−6.488	−27.135
1^3P_0	−57.922	−57.406	−57.946	0	0	−19.088	−18.932	−211.294
1^1P_1	0	0	0	−18.49	−18.393	−37.603	−37.901	−112.387
1^3P_1	0	0	0	−38.390	−38.049	0	0	−76.439
1^3P_2	−40.618	−40.314	−40.632	0	0	0	0	−121.557
2^1S_0	0	0	0	−1.547	−1.361	−2.837	−2.523	−8.268
2^3S_1	−0.546	−0.476	−0.548	−1.929	−1.711	−1.049	−0.920	−7.179
1^3D_1	−30.675	−30.312	−30.682	−15.326	−15.146	−3.077	−3.044	−128.262
1^1D_2	0	0	0	−3.147	−3.111	−27.643	−27.49	−61.391
1^3D_2	0	0	0	−27.214	−27.552	−69.486	−68.957	−193.209
1^3D_3	−40.753	−40.359	−40.772	−54.308	−53.783	−20.835	−20.606	−230.663
2^3P_0	−148.72	−146.395	−148.828	0	0	−48.589	−47.903	−540.435
2^1P_1	0	0	0	−25.081	−24.744	−49.343	−48.741	−147.909
2^3P_1	0	0	0	−98.623	−97.088	0	0	−195.711
2^3P_2	−79.114	−77.890	−79.171	0	0	0	0	−236.175

$$|nP\rangle' = |n^1P_1\rangle \cos\theta_{nP} + |n^3P_1\rangle \sin\theta_{nP} \tag{47}$$

$$|nP\rangle = -|n^1P_1\rangle \sin\theta_{nP} + |n^3P_1\rangle \cos\theta_{nP.} \tag{48}$$

The values of the mixing angles for P states are $\theta_{1P} = 0.4°$ and $\theta_{2P} = 0.05°$.

Similarly, for $L = J = 2$, we have the mixing of D states,

$$|nD\rangle' = |n^1D_2\rangle \cos\theta_{nD} + |n^3D_2\rangle \sin\theta_{nD} \tag{49}$$

$$|nD\rangle = -|n^1D_2\rangle \sin\theta_{nD} + |n^3D_2\rangle \cos\theta_{nD.} \tag{50}$$

The value of the mixing angle for D states is $\theta_{1D} = 0.05°$. The calculated masses of the $c\bar{b}$ states are listed in Table IV. Our calculated mass value for $B_c(1S)$ is 6275.851 MeV, and for $B_c^*(1S)$ it is 6314.161 MeV. $B_c^*(1S)$ is heavier than $B_c(1S)$ by 38.193 MeV. This difference is justified by calculating the ${}^3S_1 - {}^1S_0$ splitting of the ground state which is given by

$$M({}^3S_1) - M({}^1S_0) = \frac{32\pi\alpha_s|\psi(0)|^2}{9m_cm_b}. \tag{51}$$

The mass of the first radial excitation $B_c(2S)$ is 6838.232 MeV, which is heavier than $B_c(1S)$ by

TABLE IV. B_c meson mass spectrum (in MeV).

State $n^{2S+1}L_J$	This work	Ref. [72]	Ref. [73]	Ref. [74]	Ref. [1]	Ref. [68]	Ref. [28]	Ref. [75]	Ref. [76]
1^1S_0	6275	6247	6253	6260	6264	6270	6271	$6280 \pm 30 \pm 190$	6286
1^3S_1	6314	6308	6317	6340	6337	6332	6338	6321 ± 20	6341
1^3P_0	6672	6689	6683	6680	6700	6699	6706	6727 ± 30	6701
$1P1$	6766	6738	6717	6730	6730	6734	6741	6743 ± 30	6737
$1P1'$	6828	6757	6729	6740	6736	6749	6750	6765 ± 30	6760
1^3P_2	6776	6773	6743	6760	6747	6762	6768	6783 ± 30	6772
2^1S_0	6838	6853	6867	6850	6856	6835	6855	$6960 \pm 80\pm$	6882
2^3S_1	6850	6886	6902	6900	6899	6881	6887	6990 ± 80	6914
1^3D_1	7078		7008	7010	7012	7072	7028		7019
$1D2$	7009		7001	7020	7012	7077	7041		7028
$1D2'$	7154		7016	7030	7009	7079	7036		7028
1^3D_3	6980		7007	7040	7005	7081	7045		7032
2^3P_0	6914		7088	7100	7108	7091	7122		
$2P1$	7259		7113	7140	7135	7126	7145		
$2P1'$	7322		7124	7150	7142	7145	7150		
2^3P_2	7232		7134	7160	7153	7156	7164		

562.381 MeV. This value agrees with the experimental value of $B_c(2S)$ $6842 \pm 4 \pm 5$ MeV [71]. The difference between the $B_c^*(2S)$ and $B_c^*(1S)$ masses turns out to be 536.412 MeV. Our prediction for masses of orbitally excited $c\bar{b}$ states are in good agreement with the other model calculations.

B. Radiative decays

The calculation of radiative (EM) transitions between the meson states can be performed from first principles in lattice QCD, but these calculation techniques are still in their developmental stage. At present, the potential model approaches provide the detailed predictions that can be compared to experimental results.

The possible $E1$ decay modes have been listed in Table V and the predictions for $E1$ decay widths are given. Also, our predictions have been compared with other theoretical models. Most of the predictions for $E1$ transitions are in qualitative agreement. However, there are some differences in the predictions, due to differences in the phase space arising from different mass predictions and also from the wave function effects. For the transitions involving $P1$ and $P1'$ states, which are mixtures of the spin singlet $^1P_1$ and spin triplet $^3P_1$ states, there exists a huge difference between the different theoretical predictions. These may be due to the different $^3P_1 - {}^1P_1$ mixing angles predicted by the different models. Wave function effects also appear in decays from radially excited states to ground state mesons such as $2^3P_0 \to 1^3S_1\gamma$. The overlap integral for these transitions in our model vanishes, and hence, we get zero decay width for these transitions.

The $M1$ transitions contribute little to the total widths of the $2S$ levels, because it cannot decay by annihilation. Allowed $M1$ transitions correspond to triplet-singlet transitions between S-wave states of the same n quantum number, while hindered $M1$ transitions are either triplet-singlet or singlet-triplet transitions between S-wave states of different n quantum numbers.

The possible radiative $M1$ transition modes are as follows, (i) $2^3S_1 \to 2^1S_0 + \gamma$, (ii) $2^3S_1 \to 1^1S_0 + \gamma$, (iii) $2^1S_0 \to 1^3S_1 + \gamma$, (iv) $1^3S_1 \to 1^1S_0 + \gamma$. In the above, (ii) and (iii) represent hindered transitions, and (i) and (iv) represent allowed transitions. In order to calculate decay rates of hindered transitions we need to include relativistic corrections. There are three main types of corrections: relativistic modification of the nonrelativistic wave functions, relativistic modification of the electromagnetic transition operator, and finite-size corrections. In addition to these, there are additional corrections arising from the quark anomalous magnetic moment. Corrections

TABLE V. E1 transition rates of B_c meson.

Transition	k_0 MeV	This work keV	Ref. [68] keV	Ref. [1] keV	Ref. [73] keV	Ref. [76] keV
$1^3P_0 \to 1^3S_1\gamma$	348.527	42.384	75.5	79.2	65.3	74.2
$1P1 \to 1^3S_1\gamma$	437.575	83.879	87.1	99.5	77.8	75.8
$1P1' \to 1^3S_1\gamma$	494.615	121.143	13.7	0.1	8.1	26.2
$1^3P_2 \to 1^3S_1\gamma$	446.371	89.04	122	112.6	102.9	126
$1P1 \to 1^1S_0\gamma$	473.213	106.088	18.4	0	11.6	32.5
$1P1' \to 1^1S_0\gamma$	529.934	148.992	147	56.4	131.1	128
$2^3S_1 \to 1^3P_0\gamma$	175.953	3.635	5.53	7.8	7.7	9.6
$2^3S_1 \to 1P1\gamma$	83.181	0.384	7.65	14.5	12.8	13.3
$2^3S_1 \to 1P1'\gamma$	22.416	0.00751	0.74	0	1.0	2.5
$2^3S_1 \to 1^3P_2\gamma$	73.879	0.269	7.59	17.7	14.8	14.5
$2^1S_0 \to 1P1\gamma$	70.979	0.238	1.05	0	1.9	6.4
$2^1S_0 \to 1P1'\gamma$	10.104	0.00068	4.40	5.2	15.9	13.1
$2^3P_0 \to 1^3S_1\gamma$	573.927	0		21.9	16.1	
$2P1 \to 1^3S_1\gamma$	88.815	0		22.1	15.3	
$2P1' \to 1^3S_1\gamma$	938.854	0		2.1	2.5	
$2^3P_2 \to 1^3S_1\gamma$	859.837	0		25.8	19.2	
$2P1 \to 1^1S_0\gamma$	917.035	0			3.1	
$2P1' \to 1^1S_0\gamma$	971.788	0			20.1	
$2^3P_0 \to 2^3S_1\gamma$	63.253	0.422	34.0	41.2	25.5	
$2P1 \to 2^3S_1\gamma$	397.439	104.751	45.3	54.3	32.1	
$2P1' \to 2^3S_1\gamma$	456.65	158.896	10.4	5.4	5.9	
$2^3P_2 \to 2^3S_1\gamma$	371.628	85.639	75.3	73.8	49.4	
$2P1 \to 2^1S_0\gamma$	409.075	114.223	13.8		8.1	
$2P1' \to 2^1S_0\gamma$	468.191	171.244	90.5		58.0	

TABLE VI. $M1$ transition rates for the B_c meson.

Transition	k_0 (MeV)	Γ (keV) This work	Γ (keV) Ref. [69]	Γ (keV) Ref. [68]	Γ (keV) Ref. [76]	Γ (keV) Ref. [70]	Γ (keV) Ref. [68]	Γ (keV) Ref. [1]
$1^3S_1 \to 1^1S_0\gamma$	38.193	0.0185	0.0189	0.033	0.059	0.060	0.073	0.135
$2^3S_1 \to 2^1S_0\gamma$	12.329	0.0018	0.0037	0.017	0.012	0.010	0.030	0.029
$2^3S_1 \to 1^1S_0\gamma$	550.614	0.193	0.1357	0.428	0.122	0.098	0.141	0.123
$2^3S_1 \to 1^3S_1\gamma$	515.410	0.123	0.1638	0.488	0.139	0.096	0.160	0.093

to the wave function that give contributions to the transition amplitude are of two categories:

(1) higher order potential corrections, which are distinguished as (a) the zero recoil effect and (b) recoil effects of the final state meson, and (2) color octet effects. The color octet effects are not included in potential model formulation and are not considered so far in radiative transitions. The spherical Bessel function $j_0(kr/2)$, introduced in equation (41), takes into account the so called finite-size effect (equivalently, resumming the multipole-expanded magnetic amplitude to all orders). For small k, $j_0(kr/2) \to 1$, so that transitions with $n' = n$ have favored matrix elements, though the corresponding partial decay widths are suppressed by smaller k^3 factors. For a large value of photon energy (k), transitions with $n \neq n'$ have favored the matrix element, since $j_0(kr/2)$ becomes very small. $M1$ transition rates are very sensitive to hyperfine splitting of the levels due to the k^3 factor in equation (41).

There have been many models which study the radiative decays of B_c meson using nonrelativistic and relativistic quark models. Eichten and Quigg [1] calculated the radiative $M1$ transition rates for the allowed and hindered transitions. They used the equation (41) in their potential model approach to determine the $M1$ transition rates of the B_c meson. Allowed transition rates for processes (i) and (iv) were found to be 0.0040 keV and 0.130 keV respectively. Hindered transition rates for the processes (ii) and (iii) were 0.253 keV and 0.223 keV respectively. Abd El-Hady *et al.* [69] have investigated the radiative decay properties of the B_c meson in a Bethe-Salpeter model. The allowed transition rates for processes (i) and (iii) were found to be 0.0037 keV and 0.0189 keV, respectively. The hindered transition rates for processes (ii) and (iv) were found to be 0.135 keV and 0.1638, respectively. Ebert *et al.* [68] have studied these $M1$ transitions, including full relativistic corrections in their relativistic model. They depend explicitly on the Lorentz structure of the nonrelativistic potential. Several sources of uncertainty make $M1$ transitions particularly difficult to calculate. The leading-order results depend explicitly on the constituent quark masses, and corrections depend on the Lorentz structure of the potential. They estimated the allowed transition rates to be 0.033 keV and 0.017 keV, respectively. For the hindered transition, decay rates were found to be 0.428 keV and 0.488 keV. Also, it is clear from their calculations that the predicted decay rates for hindered transitions are increased by relativistic effects, almost by a factor of 3, and are larger than the rates of allowed $M1$ transitions by an order of magnitude.

We have calculated the $M1$ transition rates for $c\bar{b}$ meson states using equation (41). The resulting radiative $M1$ transition rates of these states are presented in Table VI. In this table we give calculated values for decay rates of the $M1$ radiative transition in comparison with the other relativistic and nonrelativistic quark models. We see from these results that the relativistic effects play a very important role in determining the B_c meson $M1$ transition rates. The relativistic effects reduce the decay rates of allowed transitions and increase the rates of hindered transitions. The $M1$ transition rates calculated in our model agree well with the values predicted by other theoretical models.

C. Weak decays and lifetime of the B_c meson

In accordance with the classification given in Sec. II E, the total decay width can be written as the sum over partial widths

$$\Gamma(B_c \to X) = \Gamma(b \to X) + \Gamma(c \to X) + \Gamma(ann). \qquad (52)$$

In the spectator approximation:

$$\Gamma_1(\bar{b} \to X) = \frac{9G_F^2|V_{cb}|^2 m_b^5}{192\pi^3}. \qquad (53)$$

The calculated value of $\Gamma_1(\bar{b} \to X)$ is 1.041×10^{-3} eV and

$$\Gamma_2(c \to X) = \frac{5G_F^2|V_{cs}|^2 m_c^5}{192\pi^3}. \qquad (54)$$

The calculated value of $\Gamma_2(c \to X)$ is 8.958×10^{-4} eV.

In the above expressions, V_{cb} and V_{cs} are the elements of the Cabibbo-Kobayashi-Maskawa matrix, $G_F = 1.16637 \times 10^{-5}$ is the Fermi coupling constant, and m_c and m_b are the masses of c and b quarks, respectively. The decay widths are calculated using $|V_{bc}| = 0.044$ [77] and $|V_{cs}| = 0.975$ [77].

The leptonic partial widths probe the compactness of the quarkonium system and provide important information complementary to level spacings. The quark-antiquark assignments for the vector mesons, as well as the fractional

TABLE VII. Comparison of predictions for the pseudoscalar decay constant of the B_c meson (f_{B_c}).

This work	Ref. [66]	Ref. [64]	Ref. [67]	Ref. [75]
554.125	500	512	479	440 ± 20

TABLE VIII. Comparison of lifetime of B_c meson (in ps).

This work	Experiment [77]	Ref. [60]	Ref. [73]	Ref. [79]	Ref. [80]
0.339	0.452 ± 0.033	0.47	0.55 ± 0.15	0.50	0.75

values for the quark charges, are tested from the values of their leptonic decay widths. The decay of vector mesons into charged leptons proceeds through the virtual photon ($q\bar{q} \to l^+l^-$). The $^3S_1$ and $^3D_1$ states have quantum numbers of a virtual photon, $J^{PC} = 1^{--}$ and can annihilate into lepton pairs through one photon. Annihilation widths such as $c\bar{b} \to l\nu_l$ are given by the expression

$$\Gamma_3 = \frac{G_F^2}{8\pi}|V_{bc}|^2 f_{B_c}^2 M_{B_c} \sum_i m_i^2 \left(1 - \frac{m_i^2}{M_{B_c}^2}\right) C_i. \quad (55)$$

The calculated value of Γ_3 is 5.663×10^{-6} eV.

Here, m_i is the mass of the heavier fermion in the given decay channel. For lepton channels $C_i = 1$, while for quark channels $C_i = 3|V_{q\bar{q}}|^2$ and f_{B_c} is the pseudoscalar decay constant for the B_c meson.

Adding these results, we get the total decay width $\Gamma(\text{total}) = \Gamma_1 + \Gamma_2 + \Gamma_3 = 19.428 \times 10^{-4}$ eV, corresponding to a lifetime of $\tau = 0.339$ ps.

The pseudoscalar decay constant f_{B_c} is defined by:

$$\langle 0|\bar{b}(x)\gamma^\mu c(x)|B_c(k)\rangle = i f_{B_c} V_{cb} k^\mu, \quad (56)$$

where k^μ is the four momentum of the B_c meson. In the nonrelativistic limit, the pseudoscalar decay constant is proportional to the wave function at the origin and is given by van Royen-Weisskopf formula [78]

$$f_{B_c} = \sqrt{\frac{12}{M_{B_c}}}\psi(0). \quad (57)$$

Here $\psi(0)$ is wave function at the origin. The values of the decay constants in various theoretical models are listed in Table VII and in Table VIII, and we compare the lifetime of the B_c meson calculated in our model with other models.

D. Strong decays

The $c\bar{b}$ states, which lie below the BD threshold are stable against strong decays. However, the states which are above the BD threshold undergo two body strong decays. We have calculated strong decay widths of $c\bar{b}$ states which lie above the BD threshold using the equation (38). The decay widths are calculated within the $^3P_0$ pair creation model. The results are presented in Table IX.

TABLE IX. Strong decay widths of the B_c meson.

Transition	Γ (MeV)
$2^1P_1 \to B^* + D$	54.599
$2^3P_1 \to B^* + D$	2.145
$2^3P_2 \to B + D$	99.386
$2^3P_2 \to B^0 + D^0$	108.185
$2^3P_2 \to B^* + D$	31.247
$1^3D_2 \to B^* + D$	0.198
$1^3D_2 \to B_s^* + D_s$	5.837
$1^3D_2 \to B^* + D^*$	2.123
$1^3D_2 \to B_s^* + D_s^*$	20.885

IV. CONCLUSIONS

The complete spectrum of $c\bar{b}$ states has been calculated in a relativistic quark model with coupled channel effects. We have calculated the meson loop effects on the masses of $1S$, $2S$, $1P$, $2P$ and $1D$ $c\bar{b}$ states. The mass shifts calculated due to these loop effects are large. The ground state mass of $c\bar{b}$ state calculated in our model matches the experimental data. When the results for $c\bar{b}$ state mass spectrum are compared with the previous calculations, it is found that the predictions for the mass spectrum agree within a few MeV. The differences between the predictions in most cases do not exceed 30 MeV, and the higher orbitally excited states are 50–100 MeV heavier in our model. The hyperfine splitting of the ground state vector and pseudoscalar $c\bar{b}$ states in our model is in good agreement with the prediction made by other theoretical models. The ground state pseudoscalar B_c and vector B_c^* meson masses lie within the ranges quoted by Kwong and Rosner in their survey of techniques for estimating the masses of the $c\bar{b}$ ground state: i.e., $6194 \text{ MeV} < M_{B_c} < 6292$ MeV and $6284 \text{ MeV} < M_{B_c^*} < 6357$ MeV.

The difference of quark flavors forbids the B_c meson from annihilation into gluons. As a result, the excited B_c meson states lying below the BD production threshold (i.e., with $M < M_D + M_B = 7143.1 \pm 2.1$ MeV) undergo a radiative transition to a ground state, which then decays through weak decay process. Radiative decays are the dominant decay modes of the B_c excited states having widths of about a fraction of MeV. Therefore, it is very important to determine the masses and the radiative decay widths of the B_c meson accurately in order to understand the B_c spectrum and distinguishing exotic states. The radiative $E1$ and $M1$ decay rates of $c\bar{b}$ states have been calculated using spectroscopic parameters obtained from RQM. Most of our predictions for the $E1$ decay rates are in good agreement with the other theoretical calculations. The differences in the prediction for the decay rates in various theoretical models can be attributed to the differences in mass predictions, wave function effects and singlet-triplet mixing angles. The calculated $M1$ transition rates reasonably agree with the other theoretical model predictions as listed in Table IV. It is clearly seen in this calculation that the relativistic effects play an important role in determining the radiative transition rates, since the hindered transition rates are suppressed due to the wave function orthogonality in the NRQM formalism. The inclusion of these relativistic effects enhances the hindered transition rates and reduces the allowed transition rates. It is evident from the table that the hindered transition rates are larger than the allowed transition rates by an order of magnitude. Experimental results for the masses of excited states and radiative decays of the B_c meson are needed to clarify these predictions. Experimental results will give us more insight into the B_c spectrum and will help us to clarify the hyperfine splitting calculated in different models.

We have done an estimation of weak decay widths in the spectator quark approximation and calculated the lifetime of the $c\bar{b}$ state. We get about a 53% branching ratio for b-quark decays, about 46% for c-quark decays, and about a 1% branching ratio in the annihilation channel. The lifetime of $c\bar{b}$ state predicted in this calculation is listed in Table VIII and is found to be in good agreement with experimental value as well as with other theoretical predictions. The decay constant of the $c\bar{b}$ state (f_{B_c}) has been calculated and compared with other model predictions and it is found that the decay constant is consistent with these predictions. We have calculated two body strong decay widths of $c\bar{b}$ states in the framework of $^3P_0$ pair creation model.

A simple relativistic model employing COGEP and harmonic-oscillator confinement potential, along with the coupled channel effects used in this study, is successful to predict the various properties of the $c\bar{b}$ states, and this can shed further light on their nonleptonic transitions.

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