PHYSICAL REVIEW D **95,** 054001 (2017)

Electromagnetic form factors of Λ_b in the Bethe-Salpeter equation approach

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(Received 19 December 2016; published 3 March 2017)

The heavy baryon Λ_b is regarded as composed of a heavy quark and a scalar diquark which has good spin and isospin quantum numbers. In this picture, we calculate the electromagnetic form factors of Λ_b in the Bethe-Salpeter equation approach in the spacelike region. We find that the shapes of the electromagnetic form factors of Λ_b are similar to those of Λ, with a peak at $\omega = 1$ (for the magnetic form factor) and $\omega \simeq 1.1$ (for the electric form factor)($\omega = v' \cdot v$ is the velocity transfer between the initial state (with velocity v) and the final state (with velocity v') of Λ_b), but the amplitudes are much smaller than those of Λ.

DOI: 10.1103/PhysRevD.95.054001

I. INTRODUCTION

The spacelike (SL) nucleon electromagnetic (EM) form factors describe the spatial distributions of electric charge and current inside the nucleon, and they are intimately related to the nucleon's internal structure. They are not only important observable parameters but also a vital key to understanding the strong interaction [1,2]. They are measured through the elastic electron-proton scattering ($e + p \to e + p$) with the exchange of a virtual photon of squared momentum $q^2 \leq 0$, which is SL. The annihilation reactions ($e^+ + e^- \to \bar{p} + p$), $q^2 > 0$ are timelike (TL). There have been some theoretical investigations of EM form factors in both the SL and TL regions [3–8] and many experimental results on the EM form factors of baryons [9–21] and mesons [22–25] during the past two decades.

The EM form factors of Λ and Σ were calculated in the framework of the light-cone sum rule (LCSR) up to twist six [26,27] in the SL region. The authors provided a fit approach to predict the magnetic moment of a hadron. The Q^2-dependent EM form factors of the Λ baryon were obtained and were fitted by the dipole formula to estimate the magnetic moment of the Λ baryon. It was found that the magnetic form factor approaches zero faster than the dipole formula with the increase of Q^2.

In the present paper, we will study the EM form factors of Λ_b in the quark-diquark picture in the SL region. In this picture, Λ_b is regarded as a bound state of two particles: one is a heavy quark and the other is a quasiparticle made of two quarks or a diquark. This model has been successful in describing some baryons [28–31]. Since the parity of the b quark is positive, the parity of the diquark involved in the ground state baryon should also be positive. Since the isospin of Λ_b and the b quark are zero, the isospin of the diquark (ud) should be zero. Hence, the spin of the diquark is also zero. In this picture, the Bethe-Salpeter (BS) equation for Λ_b has been studied extensively [32–36]. Then Λ_b can be described as $b(ud)_{00}$ (the first and second subscripts correspond to the spin and the isospin of the (ud) diquark, respectively). Then, using the covariant instantaneous approximation and applying the kernel which includes the scalar confinement and the one-gluon-exchange terms, we will calculate the EM form factors in the BS equation approach and compare the results with the EM form factors of Λ.

The paper is organized as follows. In Sec. II, we will establish the BS equation for Λ_b as a bound state of $b(ud)_{00}$. In Sec. III we will derive EM form factors for Λ_b in the BS equation approach. In Sec. IV, the numerical results for the EM form factors of Λ_b will be given. Finally, the summary and discussion will be given in Sec. V.

II. BS EQUATION FOR Λ_b

In the previous work [32–35], the BS wave function of the $b(ud)_{00}$ system is defined as

$$\chi(x_1, x_2, P) = \langle 0|T\psi(x_1)\varphi(x_2)|P\rangle, \tag{1}$$

where $\psi(x_1)$ is the field operator of the b quark at the position x_1, $\varphi(x_2)$ is the field operator of the scalar diquark

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at the position x_2, and $P = Mv$ is the momentum of the baryon. We use M, m_q, and m_D to represent the masses of the baryon, the b quark, and the diquark, respectively, and v to represent the baryon's velocity. We define the BS wave function in momentum space:

$$\chi(x_1, x_2, P) = e^{iPX} \int \frac{d^4p}{(2\pi)^4} e^{ipx} \chi_P(p), \tag{2}$$

where $X = \lambda_1 x_1 + \lambda_2 x_2$ is the coordinate of center mass, $\lambda_1 = \frac{m_q}{m_q + m_D}$, $\lambda_2 = \frac{m_D}{m_q + m_D}$, and $x = x_1 - x_2$. As in Refs. [32–35], we can prove that the BS equation for the $b(ud)_{00}$ system has the following form in momentum space,

$$\chi_P(p) = S_F(p_1) \int \frac{d^4p}{(2\pi)^4} K(P, p, q) \chi_P(q) S_D(p_2), \tag{3}$$

where $p_1 = \lambda_1 P + p$ and $p_2 = \lambda_2 P - p$, $K(P, p, q)$ are the kernel that is the sum of all two-particle-irreducible diagrams and $S_F(p_1)$ and $S_D(p_2)$ are propagators of the quark and the scalar diquark, respectively. According to the potential model [32,37], the kernel is assumed to have the following form:

$$-iK(P, p, q) = I \otimes I V_1(p, q) + \gamma_\mu \otimes \Gamma^\mu V_2(p, q), \tag{4}$$

where $\Gamma^\mu = (p_2 + q_2)^\mu \frac{\alpha_{\mathrm{seff}} Q_0^2}{Q^2 + Q_0^2}$ is introduced to describe the structure of the scalar diquark [32,38], and Q_0^2 is a parameter that freezes Γ^μ when Q^2 is very small. In the high-energy region, the diquark form factor is proportional to $1/Q^2$, which is consistent with perturbative QCD calculations [39]. V_1 and V_2 are the scalar confinement and one-gluon-exchange terms that have the following forms in the covariant instantaneous approximation [32,33,36,40]:

$$\tilde{V}_1(p_t - q_t) = \frac{8\pi\kappa}{[(p_t - q_t)^2 + \varepsilon^2]^2} - (2\pi)^2 \delta^3(p_t - q_t) \times \int \frac{d^3k}{(2\pi)^3} \frac{8\pi\kappa}{(k^2 + \varepsilon^2)^2}, \tag{5}$$

$$\tilde{V}_2(p_t - q_t) = -\frac{16\pi}{3} \frac{\alpha_{\mathrm{seff}}^2 Q_0^2}{[(p_t - q_t)^2 + \varepsilon^2][(p_t - q_t)^2 + Q_0^2]}, \tag{6}$$

where p_t and q_t are the transverse projections of the relative momenta along the momentum P and are defined as $p_t^\mu = p^\mu - p_l v^\mu$ and $q_t^\mu = q^\mu - q_l v^\mu$, where $p_l = v \cdot p$ and $q_l = v \cdot q$, the second term of $\tilde{V}_1$ is introduced to avoid infrared divergence at the point $p_t = q_t$, and ε is a small parameter to avoid the divergence in numerical calculations. The range of the parameter κ is $0.02 \sim 0.08\ \mathrm{GeV}^3$ [34,35]. By analyzing the EM form factors of the proton, one can get the value of Q_0^2. Generally, the range of Q_0^2 is $1 \sim 10\ \mathrm{GeV}^2$ [33,38,41–43]. In the Donnacheie-Landshoff model, Q_0^2 was taken to be of order 1 GeV^2, and in the scattering model, the value of Q_0^2 is $3 \sim 4\ \mathrm{GeV}^2$ [38]. In Ref. [41], the author gave the value $Q_0^2 = 3.22\ \mathrm{GeV}^2$ by fitting the experimental data. In Ref. [44], it was found that the value of $Q_0^2 = 10$ or $3\ \mathrm{GeV}^2$ is in agreement with deep inelastic scattering data including perturbative QCD corrections. So, in our paper, we will take the value of Q_0^2 to be 1.0, 3.2, 10 GeV^2, respectively, to see how our results depend on Q_0^2.

The quark and diquark propagators can be written as the following,

$$S_F(p_1) = \frac{i}{2\omega_q} \left[\frac{\not{v}\omega_q + (\not{p}_t + m_q)}{\lambda_1 M + p_l - \omega_q + i\epsilon} + \frac{\not{v}\omega_q - (\not{p}_t + m_q)}{\lambda_1 M + p_l + \omega_q - i\epsilon} \right], \tag{7}$$

$$S_D(p_2) = \frac{i}{2\omega_D} \left[\frac{1}{\lambda_2 M - p_l - \omega_D + i\epsilon} - \frac{1}{\lambda_2 M - p_l + \omega_D - i\epsilon} \right], \tag{8}$$

where $\omega_q = \sqrt{m_q^2 - p_t^2}$ and $\omega_D = \sqrt{m_D^2 - p_t^2}$. Considering $\not{v} u(v, s) = u(v, s)$ ($u(v, s)$ is the spinor of Λ_b with helicity s), $\chi_P(p)$ can be written as [34]

$$\chi_P(p) = (f_1 + f_2\gamma_5 + f_3\gamma_5\not{p}_t + f_4\not{p}_t + f_5\sigma_{\mu\nu}\varepsilon^{\mu\nu\alpha\beta} p_{t\alpha} p_{t\beta}) u(v, s), \tag{9}$$

where f_i ($i = 1, \ldots, 5$) are the Lorentz-scalar functions of p_t^2 and p_l. Considering the properties of $\chi_P(p)$ under parity and Lorentz transformations, Eq. (9) can be simplified as the following:

$$\chi_P(p) = (f_1 + \not{p}_t f_2) u(v, s). \tag{10}$$

Defining $\tilde{f}_{1(2)} = \int \frac{dp_l}{2\pi} f_{1(2)}$, and using the covariant instantaneous approximation, $p_l = q_l$, we find that the scalar BS wave functions satisfy the coupled integral equation as follows:

$$\tilde{f}_1(p_t) = \frac{1}{4\omega_D\omega_q(-M+\omega_D+\omega_q)}\int\frac{d^3q_t}{(2\pi)^3}\{[(\omega_q+m_q)(\tilde{V}_1+2\omega_D\tilde{V}_2) - p_t\cdot(p_t+q_t)\tilde{V}_2]\tilde{f}_1(q_t) + [-(\omega_q+m_q)(q_t+p_t)\cdot q_t\tilde{V}_2 + p_t\cdot q_t(\tilde{V}_1-2\omega_D\tilde{V}_2)]\tilde{f}_2(q_t)\} - \frac{1}{4\omega_D\omega_q(M+\omega_D+\omega_l)}\int\frac{d^3q_t}{(2\pi)^3}\{[(\omega_q-m_q)(\tilde{V}_1-2\omega_D\tilde{V}_2)+4p_t\cdot(p_t+q_t)\tilde{V}_2]\tilde{f}_1(q_t) + [(m_q-\omega_q)(q_t+p_t)\cdot q_t\tilde{V}_2 - p_t\cdot q_t(\tilde{V}_1+2\omega_D\tilde{V}_2)]\tilde{f}_2(q_t)\}, \tag{11}$$

$$\tilde{f}_2(p_t) = \frac{1}{4\omega_D\omega_q(-M+\omega_D+\omega_q)}\int\frac{d^3q_t}{(2\pi)^3}\left\{\left[(\tilde{V}_1+2\omega_D\tilde{V}_2) - (-\omega_q+m_q)\frac{(p_t+q_t)\cdot p_t}{p_t^2}\tilde{V}_2\right]\tilde{f}_1(q_t) + \left[((m_q-\omega_q)(\tilde{V}_1+2\omega_D\tilde{V}_2)\frac{p_t\cdot q_t}{p_t^2} - (q_t^2+p_t\cdot q_t)\tilde{V}_2\right]\tilde{f}_2(q_t)\right\} - \frac{1}{4\omega_D\omega_q(M+\omega_D+\omega_q)}\int\frac{d^3q_t}{(2\pi)^3}\left\{\left[-(\tilde{V}_1-2\omega_D\tilde{V}_2)+(\omega_q+m_q)\frac{(p_t+q_t)\cdot p_t}{p_t^2}\tilde{V}_2)\right]\tilde{f}_1(q_t) + \left[\frac{(m_q+\omega_q)(-\tilde{V}_1-2\omega_D\tilde{V}_2)}{p_t^2}p_t\cdot q_t + (q_t^2+p_t\cdot q_t)\tilde{V}_2)\right]\tilde{f}_2(q_t)\right\}. \tag{12}$$

It is noted that the second part of $\tilde{f}_i$ $(i = 1, 2)$ in Eqs. (11) and (12) are of order $1/M_{\Lambda_b}$, which is very important for obtaining the magnetic form factor.

In general, in the SL region, the BS wave function can be normalized in the condition of the covariant instantaneous approximation [34,40],

$$i\delta^{i_1i_2}_{j_1j_2}\int\frac{d^4qd^4p}{(2\pi)^8}\bar{\chi}_P(p,s)\left[\frac{\partial}{\partial P_0}I_p(p,q)^{i_1i_2j_2j_1}\right]\chi_P(q,s') = \delta_{ss'}, \tag{13}$$

where $i_{1(2)}$ and $j_{1(2)}$ represent the color indices of the quark and the diquark, respectively, $s^{(\prime)}$ is the spin index of the baryon Λ_b, $I_p(p,q)^{i_1i_2j_2j_1}$ is the inverse of the four-point propagator written as follows:

$$I_p(p,q)^{i_1i_2j_2j_1} = \delta^{i_1j_1}\delta^{i_2j_2}(2\pi)^4\delta^4(p-q)S_q^{(-1)}(p_1)S_D^{(-1)}(p_2). \tag{14}$$

III. SL EM FORM FACTORS OF Λ_b

Generally, the expressions of SL EM form factors of the spin-1/2 baryon B are defined by the matrix element of the EM current between the baryon states [24,26,27],

$$\langle B(P',s')|j_\mu(x=0)|B(P,s)\rangle = \bar{u}(P',s')\left[\gamma_\mu F_1(Q^2) - i\frac{\sigma_{\mu\nu}q^\nu}{2M}F_2(Q^2)\right]u(P,s), \tag{15}$$

where $F_1(Q^2)$ and $F_2(Q^2)$ are Dirac and Pauli form factors, respectively, $u(P,s)$ denotes the baryon spinor with momentum P and spin s, M is the baryon mass, $Q^2 = -q^2 = -(P-P')^2$ is the squared momentum transfer, and j_μ is the EM current relevant to the baryon. It is noted that Eq. (15) represents the microscopic description of the SL form factors of the baryon B which include two contributions coming from the quark and the diquark, respectively. In particular, for the proton and the neutron, the form factors F_1 and F_2 have the following values at the point $Q^2 \to 0$, which corresponds to the exchange of the low-virtuality photon,

$$F_{1p(n)}(0) = 1(0), \tag{16}$$

$$F_{2p(n)}(0) = \kappa_{p(n)}, \tag{17}$$

where the indices p and n represent the proton and the neutron, respectively, and $\kappa_p = \mu_p - 1$ (μ_p is the magnetic momentum of the proton), $\kappa_n = \mu_n$ are the anomalous magnetic momenta of the proton and the neutron, respectively. In the perturbative QCD theory for the helicity-conserving form factor $F_1(Q^2)$, a dominant scaling behavior at large momentum transfer is predicted [45]:

$$F_1 \sim \left(\frac{1}{Q^2}\right)^{n-1}, \tag{18}$$

where n is the number of valence quarks in the hadron. The power counting can be justified by QCD factorization theorems which separate short-distance quark-gluon interactions from soft-hadron wave functions [46–51]. Hence, for a baryon, we have

$$F_1 \sim \frac{1}{Q^4}. \tag{19}$$

The Pauli form factor F_2 requires a helicity flip between the final and initial baryons, which in turn requires, thinking of the quarks as collinear, a helicity flip at the quark level, which is suppressed at high Q^2. F_2 should have the following behavior at high Q^2 [52,53]:

$$F_2 \sim \frac{1}{Q^6}. \tag{20}$$

The Dirac and Pauli form factors are related to the magnetic and electric form factors $G_M(Q^2)$ and $G_E(Q^2)$,

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2), \tag{21}$$

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2), \tag{22}$$

where M is the mass of a baryon. At small Q^2, G_E and G_M can be thought of as Fourier transforms of the charge and magnetic current densities of the baryon. However, at large momentum transfer this view does not apply. Considering Eqs. (19)–(22), at the large momentum transfer, $|G_E|/|G_M|$ should be a stable value.

In our present work, we will calculate the EM form factors of Λ_b. When we consider the quark current contribution we have

$$\begin{aligned}\langle \Lambda_b(v', s')| j_\mu^{\text{quark}} |\Lambda_b(v, s)\rangle \\ = \bar{u}(v', s')[g_{1q}(Q^2)\gamma_\mu + g_{2q}(Q^2)(v' + v)_\mu] u(v, s),\end{aligned} \tag{23}$$

where $j_\mu^{\text{quark}} = \bar{b}\gamma_\mu b$, $v^{(\prime)} = P^{(\prime)}/M_{\Lambda_b}$ is the velocity of Λ_b.

Define $\omega = v' \cdot v = \frac{Q^2}{2M_{\Lambda_b}^2} + 1$ as the velocity transfer, g_{1q}, and g_{2q} become functions of ω [32,34,54]. When $\omega = 1$, to order $\frac{1}{M_{\Lambda_b}}$, we have the following relation [32]:

$$g_{1q}(1) + 2g_{2q}(1) = 1 + \mathcal{O}(1/M_{\Lambda_b}^2). \tag{24}$$

In our work, we will use Eq. (24) to normalize BS wave functions and neglect $1/M_b^2$ corrections [54]. This relation has been proven to be a good approximation [54] for a heavy baryon and proposed in [55–58] for mesons.

In the quark-diquark model, the electromagnetic current j_μ coupling to Λ_b is simply the sum of the quark and diquark currents; see Fig. 1. So, we have the relation [24]

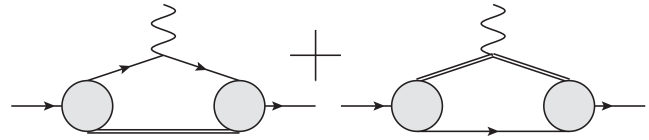

FIG. 1. The EM current is the sum of the quark current and the diquark current [59].

$$j_\mu = j_\mu^{\text{quark}} + j_\mu^{\text{diquark}}, \tag{25}$$

where $j_\mu^{\text{diquark}} = \bar{D}\Gamma_\mu D$, Γ_μ is the vertex among the photon and the diquark which includes the scalar diquark form factor. Hence, we have

$$\begin{aligned}\langle \Lambda_b(v', s')| j_\mu |\Lambda_b(v, s)\rangle \\ = \bar{u}(v', s')[g_1(Q^2)\gamma_\mu + g_2(Q^2)(v' + v)_\mu] u(v, s).\end{aligned} \tag{26}$$

Comparing Eqs. (26) and (15), we have

$$g_1 = F_1 - \frac{F_2}{2}, \tag{27}$$

$$g_2 = \frac{F_2}{4}. \tag{28}$$

It can be shown that the matrix elements of the quark current and the diquark current can be written as the following:

$$\begin{aligned}\langle \Lambda_b(v', s')| j_\mu^{\text{quark}}(x=0) |\Lambda_b(v, s)\rangle \\ = \int \frac{d^4q}{(2\pi)^4} \bar{\chi}(p')\gamma_\mu \chi(p) S_D^{-1}(p_2),\end{aligned} \tag{29}$$

$$\begin{aligned}\langle \Lambda_b(v', s')| j_\mu^{\text{diquark}}(x=0) |\Lambda_b(v, s)\rangle \\ = \int \frac{d^4q}{(2\pi)^4} \bar{\chi}(p')\Gamma_\mu \chi(p) S_q^{-1}(p_1).\end{aligned} \tag{30}$$

Hence, we can calculate g_1 and g_2 as follows:

$$g_1(\omega) = g_{1q}(\omega) - g_{1D}(\omega), \tag{31}$$

$$g_2(\omega) = g_{2q}(\omega) - g_{2D}(\omega), \tag{32}$$

where $g_{iq}(\omega)$ and $g_{iD}(\omega)$ ($i = 1, 2$) are from quark and diquark current contributions, respectively. The minus signs in Eqs. (31) and (32) are due to the relative charge between the quark and the diquark. So, we have

$$\begin{aligned}\bar{u}(v', s')[g_{1q}(\omega)\gamma_\mu + g_{2q}(\omega)(v' + v)_\mu] u(v, s) \\ = \int \frac{d^4q}{(2\pi)^4} \bar{\chi}(p')\gamma_\mu \chi(p) S_D^{-1}(p_2),\end{aligned} \tag{33}$$

$$\bar{u}(v', s')[g_{1D}(\omega)\gamma_\mu + g_{2D}(\omega)(v'+v)_\mu]u(v,s) = \int \frac{d^4q}{(2\pi)^4}\bar{\chi}(p')\Gamma_\mu\chi(p)S_q^{-1}(p_1). \quad (34)$$

IV. NUMERICAL ANALYSIS

A. Solution of the BS wave functions

In order to solve Eqs. (11) and (12), we define $M_{\Lambda_b} = m_b + m_D + E$, where E is the binding energy. Taking $m_b = 5.02$ and $M_{\Lambda_b} = 5.62$ GeV, we have $m_D + E = 0.6$ GeV for Λ_b [33]. We choose the diquark mass m_D to be from 0.70 to 0.80 GeV for Λ_b. So, the binding energy E is from -0.2 to -0.1 GeV. The parameter κ is taken to change from 0.02 to 0.08 GeV^3 [35]. Hence, for each m_D, we can get a best value of α_{seff}, corresponding to a value of κ. Generally, $\tilde{f}_i$ $(i = 1, 2)$ should decrease to zero when $p_t \to +\infty$. We change variables as the following:

$$p_t = \varepsilon + 3\log\left[1 + 0.3\frac{1+t}{1-t}\right], \quad (35)$$

where ε is a small parameter in order to avoid divergence in numerical calculations, the range of t is from -1 to 1. Now we can use the Gaussian quadrature method to solve Eq. (11) and (12). Dividing the integration region into n small pieces (n is sufficiently large), the integral equations in Eqs. (11) and (12) become the following matrix equations:

$$f_{1i} = A_{1ij}f_{1j} + B_{1ij}f_{2j} + A_{2ij}f_{1j} + B_{2ij}f_{2j}, \quad (36)$$

$$f_{2i} = A'_{1ij}f_{1j} + B'_{1ij}f_{2j} + A'_{2ij}f_{1j} + B'_{2ij}f_{2j}. \quad (37)$$

Comparing Eqs. (11) and (12) and (36) and (37), it is very easy to get the matrices $A^{(\prime)}_{(1,2)}$ and $B^{(\prime)}_{(1,2)}$ (where A and B contain Jacobian determinants). After fixing parameters m_D, Q_0^2 and assigning the mass of Λ_b, for each value of κ, we can obtain a value of α_{seff} when we solve the eigenvalue equation (36) and (37) with the eigenvalue 1. Solving matrix equations (36) and (37), we can get numerical solutions of the BS wave functions. In Table I, we give the values of α_{seff} for $m_D = 0.70, 0.75, 0.80$ GeV for different κ when $Q_0^2 = 3.2$ GeV^2. In Table II, we give the values of α_{seff} for $Q_0^2 = 1.0$, 3.2, 10.0 GeV^2 for different κ when $m_D = 0.75$ GeV.

TABLE I. When $Q_0^2 = 3.2$ GeV^2, these are the values of α_{seff} for Λ_b with different m_D (GeV) and κ(GeV^3).

	α_{seff} ($\kappa = 0.02$)	α_{seff} ($\kappa = 0.04$)	α_{seff} ($\kappa = 0.06$)	α_{seff} ($\kappa = 0.08$)
$m_D = 0.70$	0.72	0.76	0.78	0.80
$m_D = 0.75$	0.76	0.78	0.80	0.82
$m_D = 0.80$	0.80	0.82	0.84	0.86

TABLE II. When $m_D = 0.75$ GeV, the values of α_{seff} for Λ_b with different Q_0^2 (GeV^2) and κ(GeV^3).

	α_{seff} ($\kappa = 0.02$)	α_{seff} ($\kappa = 0.04$)	α_{seff} ($\kappa = 0.06$)	α_{seff} ($\kappa = 0.08$)
$Q_0^2 = 1.0$	0.82	0.86	0.88	0.90
$Q_0^2 = 3.2$	0.76	0.78	0.80	0.82
$Q_0^2 = 10.0$	0.72	0.74	0.76	0.78

In Figs. 2, 3, 4, we plot $\tilde{f}_i$ $(i = 1, 2)$ depending on $|p_t|$. We can see from these figures that, for different α_{seff} and κ, the shapes of BS wave functions are quite similar. All the wave functions decrease to zero when $|p_t|$ is larger than about 2.5 GeV due to the confinement interaction. We find that the uncertainty of Q_0^2 has a smaller impact on BS wave functions than that of κ for the same value of m_D.

B. Calculation of EM form factors of Λ_b

In order to solve Eq. (33), we use the following definitions:

$$\int \frac{d^4p}{(2\pi)^4} f'_1(p')f_1(p)S_D^{-1}(p_2) = k_0, \quad (38)$$

$$\int \frac{d^4p}{(2\pi)^4} f'_1(p')p_t^\mu f_2(p)S_D^{-1}(p_2) = k_1 v^\mu + k_2 v'^\mu, \quad (39)$$

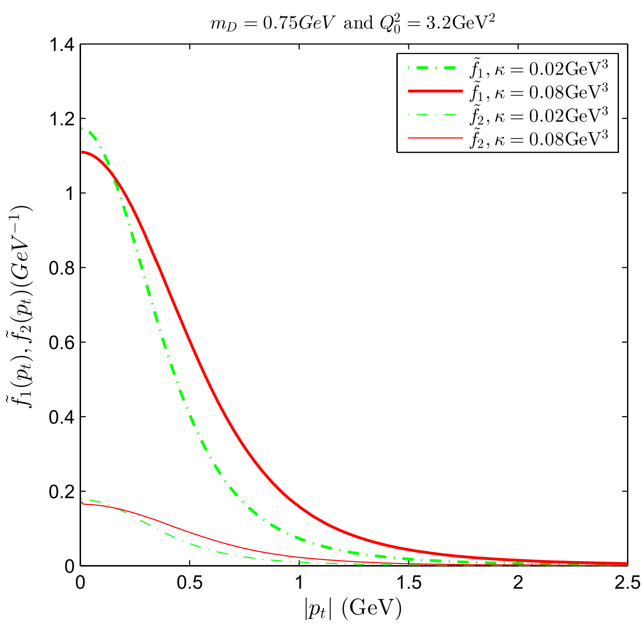

FIG. 2. The BS wave functions for Λ_b when $m_D = 0.75$ GeV and $Q_0^2 = 3.2$ GeV^2.

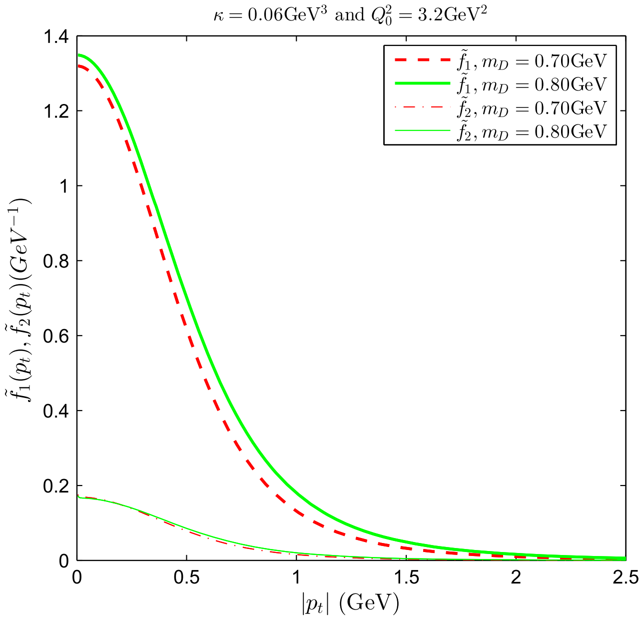

FIG. 3. The BS wave functions for Λ_b when $\kappa = 0.06$ GeV^3 and $Q_0^2 = 3.2$ GeV^2.

$$\int \frac{d^4p}{(2\pi)^4} f_2'(p') p_t^{\mu\prime} f_1(p) S_D^{-1}(p_2) = k_3 v^\mu + k_4 v'^\mu, \quad (40)$$

$$\int \frac{d^4p}{(2\pi)^4} f_2'(p') p_t'^\mu p_t^\nu f_2(p) S_D^{-1}(p_2) = k_5 g^{\mu\nu} + k_6 v'^\mu v^\nu + k_7 v^\mu v'^\nu, \quad (41)$$

where k_i $(1 = 1, 2, 3 \ldots 7)$ are functions of ω. It is easy to prove

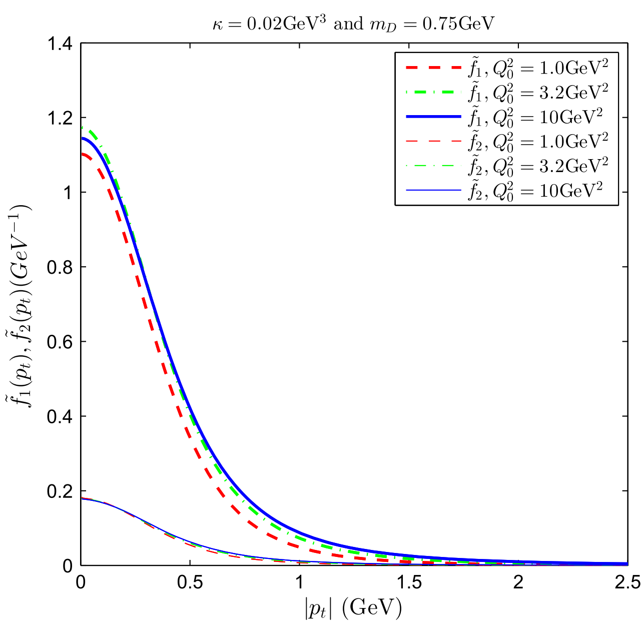

FIG. 4. The BS wave functions for Λ_b when $\kappa = 0.02$ GeV^3 and $m_D = 0.75$ GeV.

$$k_1 = -\omega k_2, \quad (42)$$

$$k_4 = -\omega k_3, \quad (43)$$

$$k_6 = 0, \quad (44)$$

$$k_5 = -\omega k_7. \quad (45)$$

Then, we have

$$k_0 = \int \frac{d^4p}{(2\pi)^4} f_1'(p') f_1(p) S_D^{-1}(p_2), \quad (46)$$

$$k_2 = \frac{1}{1-\omega^2} \int \frac{d^4p}{(2\pi)^4} f_1'(p') p_t \cdot v' f_2(p) S_D^{-1}(p_2), \quad (47)$$

$$k_3 = \frac{1}{1-\omega^2} \int \frac{d^4p}{(2\pi)^4} f_2'(p') p_t' \cdot v f_1(p) S_D^{-1}(p_2), \quad (48)$$

$$k_5 = \frac{1}{3} \int \frac{d^4p}{(2\pi)^4} f_2'(p') p_t' \cdot p_t f_2(p) S_D^{-1}(p_2). \quad (49)$$

Define θ to be the angle between p_t and v_t' where $v_t' = v' - (v \cdot v')v$, then we have

$$|v_t'| = \sqrt{\omega^2 - 1}, \quad (50)$$

$$p_t \cdot v_t' = -|p_t||v_t'| \cos\theta. \quad (51)$$

Then, we obtain the following relations:

$$p_t \cdot v_t' = -|p_t| \sqrt{\omega^2 - 1} \cos\theta, \quad (52)$$

$$p_t' \cdot v = p_l(1 - \omega^2) + |p_t| \omega \sqrt{\omega^2 - 1} \cos\theta + m_D(\omega - 1)^2. \quad (53)$$

$$p_t \cdot p_t' = \left(p_l\omega - |p_t|\sqrt{\omega^2 - 1}\cos\theta - m_D\omega\right) \times |p_t|\sqrt{\omega^2 - 1}\cos\theta - |p_t|^2. \quad (54)$$

Substituting Eqs. (7), (8), and (50)–(54) into Eqs. (46)–(49), integrating p_l, and using the relation $\tilde{f}'_{1(2)} = \int \frac{dp_l'}{2\pi} f'_{1(2)}$, k_i $(i = 0,\ 2,\ 3,\ 5)$ can be expressed in terms of $\tilde{f}^{(\prime)}_{(1,2)}$. Similarly, for solving Eq. (34), we repeat the above process with S_F^{-1} being replaced by $S_D^{-1}(p_2)$, and k_i $(i = 0, 1, 2 \ldots 7)$ being replaced by k_i'. Furthermore. in Eqs. (53) and (54), we replace m_D with $-m_b$. Finally, we obtain the following expressions for g_{1q}, g_{2q}, g_{1D}, and g_{2D}:

$$g_{1q} = k_0 - (\omega + 1)(k_2 + k_3) + \frac{k_5}{\omega}, \quad (55)$$

$$g_{2q} = 2\left(k_2 - \frac{k_5}{\omega}\right), \tag{56}$$

$$g_{1D} = 0, \tag{57}$$

$$g_{2D} = k'_0 + 2(1-\omega)k'_2 + \left(2 + \frac{1}{\omega}\right)k'_5. \tag{58}$$

Substituting Eqs. (27), (28), (31), and (32) into Eqs. (21) and (22) and considering the diquark contribution, the EM form factors G_E and G_M can be written as

$$G_E = g_{1q} - 2\omega(g_{2q} - g_{2D}), \tag{59}$$

$$G_M = g_{1q} + 6(g_{2q} - g_{2D}). \tag{60}$$

In Ref. [26], the electric form factor of Λ depends on Q^2 from 1–7 GeV, corresponding to ω from 1.5 to 4. With the normalization condition Eq. (24), solving Eqs. (33) and (34), we give the EM form factors $G_E(\omega)$ and $G_M(\omega)$ in Figs. 5–8.

From Figs. 5–10, we find that for different Q_0^2, m_D, and κ, the shapes of G_E and G_M are similar. In the range of ω from 1.0 to 4.5, the trends of G_E and G_M for Λ_b are similar to those for Λ, respectively, but changing more slowly than Λ [26,60]. From these figures, we also find that G_M decreases more rapidly than G_E as ω increases and Q_0^2 leads to smaller impact on electromagnetic form factors than κ for the same value of m_D. We find that G_E has a peak at $\omega \simeq 1.1$ and G_M has a peak at $\omega = 1$. Comparing with Ref. [60], we find that the amplitudes of the peaks for Λ_b are much smaller than those for Λ.

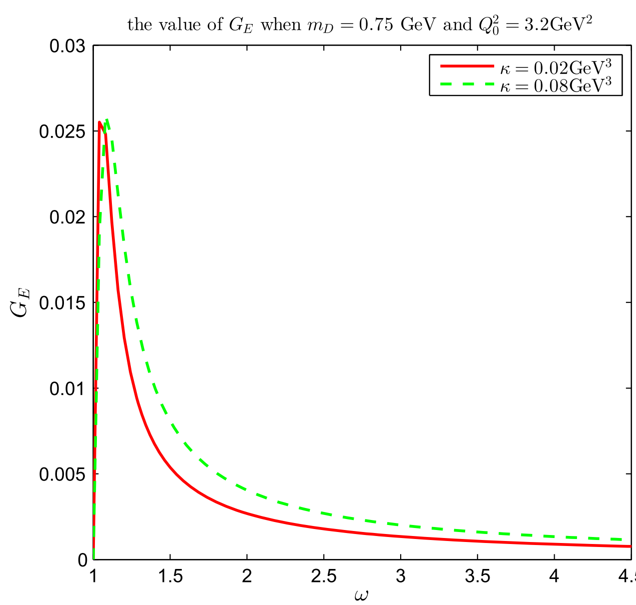

FIG. 5. ω-dependence of the electric form factor of Λ_b for $m_D = 0.75$ GeV, $Q_0^2 = 3.2$ GeV^2 and different values of κ.

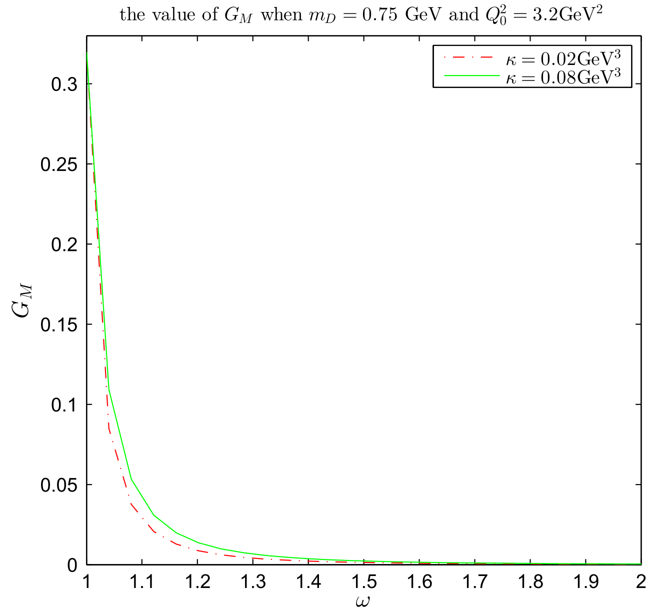

FIG. 6. ω dependence of the magnetic form factor of Λ_b for $m_D = 0.75$ GeV, $Q_0^2 = 3.2$ GeV^2, and different values of κ.

In the dipole model, $G_M(Q^2) = \frac{\mu}{(1+Q^2/m_0^2)^2}$, $\mu \propto 1/M$ (For $\Lambda_{(b)}$, M is the mass of $s(b)$ quark) corresponds to the baryon magnetic moment and $m_0 = \sqrt{0.89}$ GeV is a parameter [27]. There is no data for EM form factors of Λ_b at present. However, for Λ and Λ_b baryons, the ratio of $|G_E|$ and $|G_M|$, RM, should be of order M_s/M_b,

$$RM = \left|\frac{G_{M_s}}{G_{M_b}}\right| \sim \frac{M_s}{M_b}. \tag{61}$$

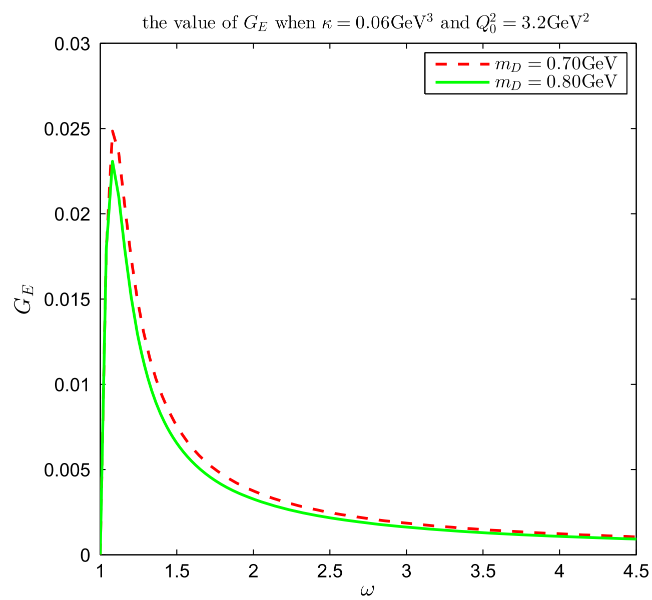

FIG. 7. ω dependence of the electric form factor of Λ_b for $\kappa = 0.06$ GeV^3, $Q_0^2 = 3.2$ GeV^2, and different values of m_D.

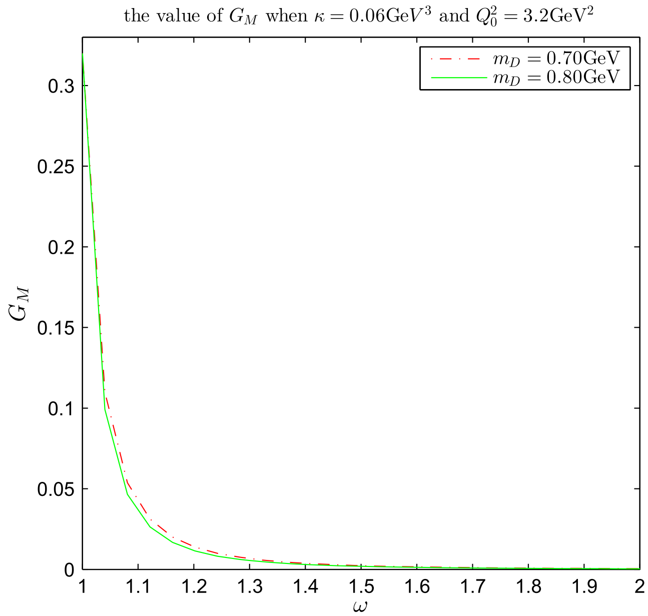

FIG. 8. ω dependence of the magnetic form factor of Λ_b for $\kappa = 0.06$ GeV^3, $Q_0^2 = 3.2$ GeV^2, and different values of m_D.

For Λ and Λ_b, RM is about 0.11 in the dipole model. From Ref. [26], we know that the magnetic form factor of Λ decreases faster than that in the dipole model. So, we expect the real value of RM could be about $10^{-2} \sim 10^{-1}$. In the range of ω from 1.5 to 4.5, our result for $|G_{M\Lambda_b}|$ varies from about 0.007 to 0, and in Ref. [26], $|G_{M\Lambda}|$ varies from about 0.38 to 0. In the range of ω from 1.0 to 3.0, our result for $|G_{M\Lambda_b}|$ varies from about 0.33 to 0, and in Ref. [60], $|G_{M\Lambda}|$ varies from about 1.2 to 0.06. The ratio agrees roughly with our expectation.

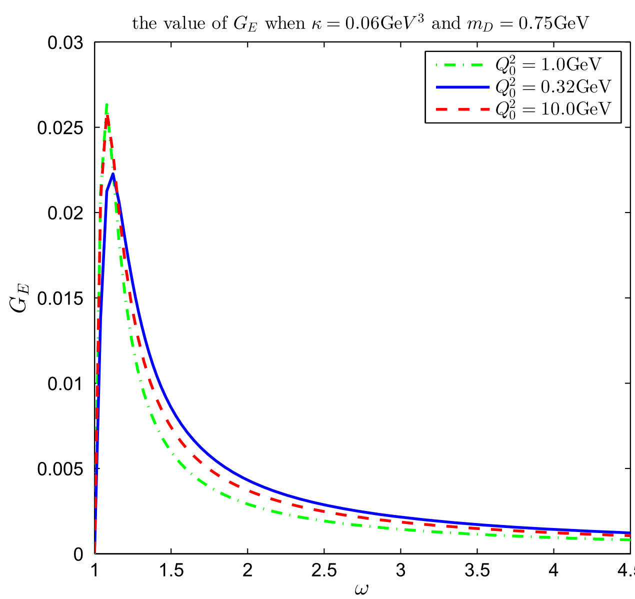

FIG. 9. ω dependence of the electric form factor of Λ_b for $\kappa = 0.06$ GeV^3, $m_D = 0.75$ GeV, and different values of Q_0^2.

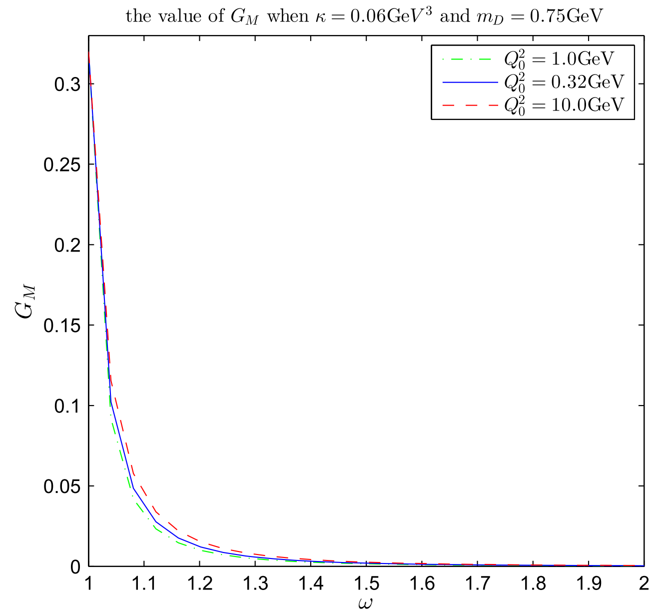

FIG. 10. ω dependence of the magnetic form factor of Λ_b for $\kappa = 0.06$ GeV^3, $m_D = 0.75$ GeV, and different values of Q_0^2.

V. SUMMARY AND DISCUSSION

Many data about Λ_b have been collected in experiments. In the quark-diquark picture, Λ_b is regarded as a bound state of a heavy b quark and a light scalar diquark based on the fact that the light degrees of freedom in Λ_b have good spin and isospin quantum numbers. In this picture, we established the BS equation for Λ_b. Then we solved the BS equation numerically by applying the kernel which includes the scalar confinement and the one-gluon-exchange terms. Then, we calculated the EM form factors of Λ_b and compared the results with those of Λ. We find that the shapes of the electromagnetic form factors of Λ_b are similar to those of Λ [26,60], with a peak at $\omega = 1$ for G_M and $\omega \simeq 1.1$ for G_E, but the amplitudes are much smaller than those of Λ. Since the b-quark mass is larger than the s-quark mass, the radius of Λ_b is smaller than that of Λ. Hence, the average transverse momentum of Λ_b is larger than that of Λ. Then the BS wave function of Λ_b changes more slowly with respect to p_t than that of Λ. Therefore, as ω increases, the electromagnetic form factors of Λ_b, as the overlap integrals of the initial and finial states, change more slowly than those of Λ, and the amplitudes of the peaks become smaller correspondingly. For different values of m_D and κ, the electric form factors of Λ_b change in the range 0.33–0 as ω changes form 1.0 to 4.5 and the magnetic form factors of Λ_b change in the range $0.025 \sim 0$ as ω changes form about 1.1 to 4.5.

Depending on the parameters m_D, κ, and Q_0^2 in our model, our results vary in some ranges. We studied the uncertainties for G_E and G_M that can be caused by κ, m_D, and Q_0^2 and found that these uncertainties are at most about 27% due to κ, 12% due to m_D, and 21% due to Q_0^2. Our results need to be tested in future experimental measurements. In the future, our model can be used to study other baryons such as the proton, the neutron, Λ, and Λ_c.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China under Contracts No. 11175020 and No. 11575023.

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