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Gravitational localization of scalar zero modes in $SU(5) \times Z_2$ branes

Nelson Pantoja and Rossana Rojas

Centro de Física Fundamental, Universidad de Los Andes, Mérida 5101, Venezuela

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The fluctuations of five-dimensional self-gravitating non-Abelian kinks which arise from the breaking of the $SU(5) \times Z_2$-symmetric theory are analyzed within the context of braneworlds. While tensor and vector sectors of these fluctuations behave like its counterparts in the standard Abelian Z_2 kinks, the mixing between the field excitations of the non-Abelian kink and the scalar components of the metric makes the pure scalar sector of the theory very interesting. The spectrum of these scalar fluctuations, which includes gravitationally trapped massless modes on the core of the wall associated with the broken symmetries, is discussed for the two classes of kinks that break $SU(5)$ into its maximal subgroups.

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I. INTRODUCTION

The central idea in braneworld scenarios is that matter and its interactions, as we know them, must be localized on the brane. Domain wall spacetimes, providing regularizations of the Randall-Sundrum braneworld [1,2], preserve the property of producing effective four-dimensional (4D) gravity on the brane [3–6]. In addition, while it has been shown that fermion fields may be localized on the brane via their interaction with the scalar field from which the wall is made [7–13], gauge field localization has been somewhat more elusive [14–20].

A lot of work has been done on the topic of localization in braneworlds in which the field theoretic domain walls considered arise in Abelian Z_2-symmetric theories, although it is also possible to consider (besides the Z_2 symmetry) continuous internal symmetry walls with interesting localization properties. There are well-known examples of domain walls in flat spacetime that arise in an $SU(5) \times Z_2$ symmetric theory [21–24] in which the full $SU(5) \times Z_2$ symmetry is not restored at the core of the wall [25]. This makes non-Abelian domain walls of this sort (rather than its extensions to the gravitating case) very interesting within the context of braneworlds [25–28].

It has been shown that non-Abelian branes in flat space that break E_6 to $SO(10) \times U(1)$ may produce, via the Dvali-Shifman (DS) mechanism [29], an $SU(5)$ effective theory on the brane [27]. On the other hand, self-gravitating $SU(5) \times Z_2$ domain walls, supporting 4D massless fermion excitations in its world volume, may also localize 4D massless non-Abelian excitations via the DS mechanism, depending on the symmetry breaking pattern [28].

While domain walls in Abelian Z_2-symmetric theories are topologically stable, there is no global stability criterium for the non-Abelian ones. This lead us to resort to perturbative analysis, after a solution is found, to establish at least their local stability [23,24]. Furthermore, the inclusion of gravity makes the perturbative analysis somewhat more intricate than the analogous one in flat spacetime due to the mixing between the field excitations of the non-Abelian kink Φ and the scalar components of the metric g_{ab}, in a theory invariant under diffeomorphisms acting both on the Φ and on g_{ab}.

In this work we make a perturbative analysis for the five-dimensional (5D) self-gravitating $SU(5) \times Z_2$ domain walls of [28] in terms of diffeomorphism-invariant quantities. This analysis not only shows that these walls are perturbatively stable, it also permits us to study the gravitationally trapped content from the point of view of 4D observers localized on the brane. We find, as expected, that the tensor and vector sectors of the fluctuations behave as its counterparts in the more familiar Z_2-symmetric domain walls. On the other hand, the scalar sector of the fluctuations in the $SU(5) \times Z_2$ case shows a rather different behavior from the Abelian one, linked to the particular symmetry breaking pattern considered and with a spectrum that includes normalizable massless modes, i.e., with massless scalar particles associated with the broken symmetries which are gravitationally trapped on the core of the wall.

II. SELF-GRAVITATING $SU(5) \times Z_2$ KINKS

In this section we briefly recall the results of [28] regarding the properties of 5D self-gravitating domain walls formed in the two possible symmetry breaking patterns of $SU(5) \times Z_2$. Let us consider the $(4+1)$-dimensional theory

$$S = \int d^4x d\xi \sqrt{-g} \left[\frac{1}{2} R - \mathrm{Tr}(\partial_a \Phi \partial^a \Phi) - V(\Phi) \right], \quad (1)$$

where R is the scalar curvature, g is the determinant of the metric, Φ is a scalar field that transforms in the adjoint representation of $SU(5)$, and $V(\Phi)$ is a potential such that (1) is invariant under the transformations

(i) $\Phi \to U \Phi U^\dagger$, $U = \exp\{i\omega_q \mathbf{T}^q\}$,

(ii) $Z_2\colon\ \Phi \to -\Phi$, $Z_2 \notin SU(5)$, where $\mathbf{T}^q$, $q = 1, \ldots, 24$, are traceless Hermitian generators of $SU(5)$.[1]

The Einstein-scalar field equations for this system are

$$R_{ab} - \frac{1}{2} g_{ab} R = T_{ab}, \tag{2}$$

where

$$T_{ab} = 2\mathrm{Tr}(\nabla_a \Phi \nabla_b \Phi) - g_{ab}(g^{cd}\mathrm{Tr}(\nabla_c \Phi \nabla_d \Phi) + V(\Phi)) \tag{3}$$

and

$$g^{ab}\nabla_a \nabla_b \phi_m = \frac{\partial V(\Phi)}{\partial \phi_m}, \qquad \Phi = \phi_m \mathbf{T}^m, \tag{4}$$

where $\nabla_c g_{ab} = 0$. Exact domain wall solutions of (2)–(4) are available for a sixth-order Higgs potential of the form

$$V(\Phi) = V_0 - \mu^2 \mathrm{Tr}[\Phi^2] + h(\mathrm{Tr}[\Phi^2])^2 + \lambda \mathrm{Tr}[\Phi^4] + \alpha(\mathrm{Tr}[\Phi^2])^3 + \beta(\mathrm{Tr}[\Phi^3])^2 + \gamma(\mathrm{Tr}\Phi^4)(\mathrm{Tr}\Phi^2) \tag{5}$$

for special values of the couplings which yield integrable models.

Assuming that the geometry preserves 4D-Poincaré invariance, the metric ansatz is

$$g_{ab} = e^{2A(\xi)} \eta_{\mu\nu} dx^\mu_a dx^\nu_b + d\xi_a d\xi_b, \tag{6}$$

with $\eta_{\mu\nu} = \mathrm{diag}(-1, +1, +1, +1)$ and a domain wall solution Φ of the form

$$\Phi(\xi) = \phi_M(\xi)\mathbf{M} + \phi_P(\xi)\mathbf{P} \tag{7}$$

is proposed, where ϕ_M, ϕ_P satisfy the boundary conditions

$$\phi_M(+\infty) = -\phi_M(-\infty), \qquad \phi_P(+\infty) = \phi_P(-\infty), \tag{8}$$

with $\mathbf{M}$ and $\mathbf{P}$ appropriately chosen orthogonal diagonal generators of $SU(5)$.

Imposing the integrability conditions

$$\mathrm{Tr}(\mathbf{MP}) = \mathrm{Tr}(\mathbf{M}^2\mathbf{P}) = \mathrm{Tr}(\mathbf{M}^3\mathbf{P}) = 0, \tag{9}$$

$$\mathrm{Tr}(\mathbf{P}^3\mathbf{M}) = \mathrm{Tr}(\mathbf{P}^3) = 0, \tag{10}$$

for the special values of the couplings given by

$$h = -12\mathrm{Tr}(\mathbf{P}^2\mathbf{M}^2)\lambda,$$

$$\alpha = \frac{4}{3}\left[\frac{2\mathrm{Tr}(\mathbf{M}^3)\mathrm{Tr}(\mathbf{P}^4) - 3\mathrm{Tr}(\mathbf{P}^2\mathbf{M})\mathrm{Tr}(\mathbf{M}^4)}{3\mathrm{Tr}(\mathbf{P}^2\mathbf{M}) - 2\mathrm{Tr}(\mathbf{M}^3)} - 6\mathrm{Tr}(\mathbf{P}^2\mathbf{M}^2)\right]\gamma,$$

$$\beta = \frac{1}{6}\left[\frac{\mathrm{Tr}(\mathbf{M}^4) - \mathrm{Tr}(\mathbf{P}^4)}{\mathrm{Tr}(\mathbf{P}^2\mathbf{M})[3\mathrm{Tr}(\mathbf{P}^2\mathbf{M}) - 2\mathrm{Tr}(\mathbf{M}^3)]}\right]\gamma, \tag{11}$$

an exact domain wall solution is given by [28]

$$\phi_M(\xi) = v \tanh b\xi, \qquad \phi_P(\xi) = v\kappa, \tag{12}$$

where μ^2, λ, and γ can be written explicitly in terms of v and b, and κ is a numerical constant which depends on the choice of $\mathbf{M}$ and $\mathbf{P}$. On the other hand,

$$A(\xi) = -\frac{v^2}{9}\left[2\ln(\cosh(b\xi)) + \frac{1}{2}\tanh^2(b\xi)\right], \tag{13}$$

and the spacetime is asymptotically 5D anti–de Sitter space with the cosmological constant $\Lambda = V(\Phi)_{\xi=-\infty} = V(\Phi)_{\xi=+\infty} = -\frac{8}{27}b^2 v^4$.

As discussed in [28] (see [23,24] for the flat space case), the choice of $\mathbf{M}$ and $\mathbf{P}$ relies on the desired asymptotic values for Φ at $\xi \to \pm\infty$. It is well known that in flat spacetime there are two symmetry breaking patterns of $SU(5)$ by a field Φ in the adjoint: involving the breaking in subgroups with the same rank as $SU(5)$ and occurring as minima of a fourth-order potential [30,31]. These are

$$SU(5) \times Z_2 \to SU(3) \times SU(2) \times U(1)/(Z_3 \times Z_2) \tag{14}$$

and

$$SU(5) \times Z_2 \to SU(4) \times U(1)/Z_4, \tag{15}$$

the second type of symmetry breaking pattern yielding the largest residual symmetry. In the remainder of this section, we quote the results of [28] concerning the symmetry breakings (14) and (15) for the gravitating case with the sixth-order potential and refer the reader to that work for a detailed discussion on these and other issues related to the localization of fermions and gauge fields within the context of braneworlds.

A. Symmetry breaking $SU(5) \times Z_2 \to SU(3) \times SU(2) \times U(1)/(Z_3 \times Z_2)$

For this symmetry breaking pattern, we have

$$\mathbf{M_A} = \frac{1}{\sqrt{40}}\mathrm{diag}(1, 1, 1, 1, -4), \tag{16}$$

$$\mathbf{P_A} = \frac{1}{2\sqrt{2}}\mathrm{diag}(1, 1, -1, -1, 0), \tag{17}$$

[1]We use units where $\hbar = G = c = 1$.

and $\kappa_A = \sqrt{5}$. With $H_\pm$ and $H_0 = H_+ \cap H_-$ being the unbroken symmetries at $\xi \to \pm\infty$ and $\xi = 0$, respectively, we have

$$H_\pm^{\mathbf{A}} = \frac{SU(3)_\pm \times SU(2)_\pm \times U(1)_\pm}{Z_2 \times Z_3}, \tag{18}$$

$$H_0^{\mathbf{A}} = \frac{SU(2)_+ \times SU(2)_- \times U(1)_M \times U(1)_P}{Z_2 \times Z_2}, \tag{19}$$

with the following embeddings:

$$SU(2)_\pm \subset SU(3)_\mp, \tag{20}$$

in the sense that the Cartan subalgebra of $SU(2)_+$ $[SU(2)_-]$ is a subspace of the Cartan-subalgebra space of $SU(3)_-$ $[SU(3)_+]$ corresponding to the particular basis chosen for the Lie algebra. In the model with gauge symmetry, H_0^A resembles the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, on which the Higgs sector of the minimal 4D left-right symmetric theory is based. This 5D non-Abelian wall may be thought of as the generalization to the gravitating case of the perturbatively stable flat space one for a quartic potential of Refs. [22–24].

B. Symmetry breaking $SU(5) \times Z_2 \to SU(4) \times U(1)/Z_4$

In this case,

$$\mathbf{M_B} = \frac{1}{\sqrt{60}} \mathrm{diag}(-2, -2, -2, 3, 3), \tag{21}$$

$$\mathbf{P_B} = \frac{1}{2} \mathrm{diag}(0, 0, 0, 1, -1), \tag{22}$$

and $\kappa_B = \sqrt{5/3}$. The unbroken symmetries are

$$H_\pm^{\mathbf{B}} = \frac{SU(4)_\pm \times U(1)_\pm}{Z_4}, \tag{23}$$

$$H_0^{\mathbf{B}} = \frac{SU(3) \times U(1)_M \times U(1)_P}{Z_3}, \tag{24}$$

where $SU(3)$ is embedded in different manners in $SU(4)_+$ and $SU(4)_-$. In this symmetry breaking pattern, if the symmetry were gauged, the $SU(3)$ gauge bosons on the wall would need to form massive glueballs of $SU(4)$ in order to escape into the bulk, so they can be localized on the wall via the DS mechanism [29].

III. DIFFEOMORPHISM-INVARIANT FLUCTUATIONS

Let g_{ab} and $\Phi = \phi_q \mathbf{T}^q$ be the metric and the scalar field solutions of the field equations of the theory (1). Let h_{ab} and $\boldsymbol{\varphi} = \varphi_q \mathbf{T}^q$ be the metric and scalar field fluctuations, respectively, around the above background. Following [6], we find that these fluctuations satisfy

$$\begin{aligned} &-\frac{1}{2} g^{cd} \nabla_c \nabla_d h_{ab} + R^c{}_{(ab)}{}^d h_{cd} + R^c_{(a} h_{b)c} \\ &\quad -\frac{1}{2} \nabla_a \nabla_b (g^{cd} h_{cd}) + \nabla_{(a} \nabla^c h_{b)c} \\ &\qquad = 4\mathrm{Tr}[\nabla_{(a} \Phi \nabla_{b)} \boldsymbol{\varphi}] + \frac{2}{3} h_{ab} V(\Phi) + \frac{2}{3} \left(\frac{\partial V(\Phi)}{\partial \phi_q} \varphi_q \right) g_{ab} \end{aligned} \tag{25}$$

and

$$\begin{aligned} &- h^{ab} \nabla_a \nabla_b \phi_q - \frac{1}{2} g^{ab} g^{cd} (\nabla_a h_{bd} + \nabla_b h_{ad} - \nabla_d h_{ab}) \nabla_c \phi_q \\ &\quad + g^{ab} \nabla_a \nabla_b \varphi_q - \frac{\partial^2 V(\Phi)}{\partial \phi_p \partial \phi_q} \varphi_p = 0, \end{aligned} \tag{26}$$

where $\nabla_c g_{ab} = 0$ and

$$\begin{aligned} V(\Phi + \boldsymbol{\varphi}) = V(\Phi) + \frac{\partial V(\Phi)}{\partial \phi_q} \varphi_q + \frac{1}{2} \frac{\partial^2 V(\Phi)}{\partial \phi_p \partial \phi_q} \varphi_p \varphi_q \\ + O(\boldsymbol{\varphi}^3). \end{aligned} \tag{27}$$

For the theory (1) with the sixth-order potential (5), we have

$$\begin{aligned} \frac{\partial V(\Phi)}{\partial \phi_q} = {} & (-2\mu^2 + 4h\mathrm{Tr}[\Phi^2] + 6\alpha (\mathrm{Tr}[\Phi^2])^2 \\ & + 2\gamma \mathrm{Tr}[\Phi^4]) \mathrm{Tr}[\Phi \mathbf{T}^q] + 6\beta \mathrm{Tr}[\Phi^3] \mathrm{Tr}[\Phi^2 \mathbf{T}^q] \\ & + 4(\lambda + \gamma \mathrm{Tr}[\Phi^2]) \mathrm{Tr}[\Phi^3 \mathbf{T}^q] \end{aligned} \tag{28}$$

and

$$\begin{aligned} \frac{\partial^2 V}{\partial \phi_q \partial \phi_p} = {} & (-2\mu^2 + 4h\mathrm{Tr}[\Phi^2] + 6\alpha (\mathrm{Tr}[\Phi^2])^2 + 2\gamma \mathrm{Tr}[\phi^4]) \mathrm{Tr}[\mathbf{T}^q \mathbf{T}^p] + 6\beta (\mathrm{Tr}[\Phi^3] (\mathrm{Tr}[\mathbf{T}^q \mathbf{T}^p \Phi] \\ & + \mathrm{Tr}[\mathbf{T}^p \mathbf{T}^q \Phi]) + 3\mathrm{Tr}[\mathbf{T}^q \Phi^2] \mathrm{Tr}[\mathbf{T}^p \Phi^2]) + 4(\lambda + \gamma \mathrm{Tr}[\Phi^2]) (\mathrm{Tr}[\mathbf{T}^q \mathbf{T}^p \Phi^2] + \mathrm{Tr}[\mathbf{T}^p \mathbf{T}^q \Phi^2] \\ & + \mathrm{Tr}[\mathbf{T}^q \Phi \mathbf{T}^p \Phi]) + 8(h + 3\alpha \mathrm{Tr}[\Phi^2]) \mathrm{Tr}[\mathbf{T}^q \Phi] \mathrm{Tr}[\mathbf{T}^p \Phi] + 8\gamma (\mathrm{Tr}[\mathbf{T}^q \Phi^3] \mathrm{Tr}[\mathbf{T}^p \Phi] + \mathrm{Tr}[\mathbf{T}^p \Phi^3] \mathrm{Tr}[\mathbf{T}^q \Phi]). \end{aligned} \tag{29}$$

In the background $\{g_{ab}, \Phi\}$ provided by the $SU(5)\times Z_2$ kinks discussed in the previous section, (28) reduces to

$$\frac{\partial V(\Phi)}{\partial \phi_q} = (\phi_M'' + 4A'\phi_M')\delta^{Mq}, \tag{30}$$

and the right-hand side of (29) turns out to be diagonal in q, p. However, although they get notably simplified, (25) and (26) are still extremely involved.

Indeed, as is well known, h_{ab} and $\boldsymbol{\varphi}$ are not diffeomorphism-invariant variables. Under an infinitesimal diffeomorphism of the form

$$x^a \to \bar{x}^a = x^a + \epsilon^a, \tag{31}$$

we have

$$h_{ab} \to \bar{h}_{ab} = h_{ab} - 2\nabla_{(a}\epsilon_{b)} \tag{32}$$

and

$$\boldsymbol{\varphi} \to \bar{\boldsymbol{\varphi}} = \boldsymbol{\varphi} - \epsilon^a \nabla_a \Phi, \tag{33}$$

where $\{h_{ab}, \boldsymbol{\varphi}\}$ and $\{\bar{h}_{ab}, \bar{\boldsymbol{\varphi}}\}$ describe the same physical perturbations. Hence, we must take care of the general coordinate invariance of the theory (1).

Since the background $\{g_{ab}, \Phi\}$ provided by the $SU(5)\times Z_2$ kinks preserves 4D-Poincaré invariance, we decompose h_{ab} (from the point of view of the four-dimensional observers confined to the wall) in scalar, vector, and tensor sectors. Following [32], we set

$$h_{\mu\nu} = 2e^{2A}(h_{\mu\nu}^{TT} + \partial_{(\mu}f_{\nu)} + \eta_{\mu\nu}\psi + \partial_\mu\partial_\nu E), \tag{34}$$

$$h_{\mu\xi} = h_{\xi\mu} = e^A(D_\mu + \partial_\mu C), \tag{35}$$

and

$$h_{\xi\xi} = 2\omega, \tag{36}$$

where

$$h_\mu^{TT\,\mu} = 0, \qquad \partial^\mu h_{\mu\nu}^{TT} = 0 \tag{37}$$

and

$$\partial^\mu f_\mu = 0, \qquad \partial^\mu D_\mu = 0. \tag{38}$$

From (32), with

$$\epsilon_a = (e^{2A}\epsilon_\mu, \epsilon_\xi), \tag{39}$$

where

$$\epsilon_\mu = \partial_\mu \epsilon + \zeta_\mu, \qquad \partial^\mu \zeta_\mu = 0, \tag{40}$$

we have that, under an infinitesimal diffeomorphism,

$$\psi \to \bar{\psi} = \psi - A'\epsilon_\xi, \qquad \omega \to \bar{\omega} = \omega + \epsilon_\xi', \tag{41}$$

$$E \to \bar{E} = E - \epsilon, \qquad C \to \bar{C} = C - e^A\epsilon' + e^{-A}\epsilon_\xi, \tag{42}$$

$$f_\mu \to \bar{f}_\mu = f_\mu - \zeta_\mu, \qquad D_\mu \to \bar{D}_\mu = D_\mu - e^A\zeta_\mu', \tag{43}$$

and

$$\bar{h}_{\mu\nu}^{TT} = h_{\mu\nu}^{TT}, \tag{44}$$

and from (33) we have

$$\bar{\boldsymbol{\varphi}} = \boldsymbol{\varphi} - \Phi'\epsilon_\xi \quad \text{since} \quad \Phi = \Phi(\xi). \tag{45}$$

The $h_{\mu\nu}^{TT}$ sector is automatically diffeomorphism invariant, i.e., $\bar{h}_{\mu\nu}^{TT} = h_{\mu\nu}^{TT}$, while it is also possible to define an invariant divergenceless vector (since we have one vector gauge function ζ_μ),

$$V_\mu = D_\mu - e^A f_\mu', \tag{46}$$

and two diffeomorphism-invariant scalar fluctuations (since we have two gauge functions, ϵ and ϵ_ξ), given by

$$\Gamma = \psi - A'(e^{2A}E' - e^A C) \tag{47}$$

and

$$\Theta = \omega + (e^{2A}E' - e^A C)', \tag{48}$$

such that $\bar{V}_\mu = V_\mu$, $\bar{\Gamma} = \Gamma$ and $\bar{\Theta} = \Theta$.

On the other hand, for the diffeomorphism-invariant non-Abelian scalar field fluctuation, we find

$$\boldsymbol{\chi} = \boldsymbol{\varphi} - \Phi'(e^{2A}E' - e^A C), \tag{49}$$

such that $\bar{\boldsymbol{\chi}} = \boldsymbol{\chi}$.

In the (generalized) longitudinal gauge [32], $E = C = 0$ and $f_\mu = 0$, the freedom of the coordinate transformations (31) is completely fixed, and we have $\Gamma = \psi$, $\Theta = \omega$, $V_\mu = D_\mu$, and $\boldsymbol{\chi} = \boldsymbol{\varphi}$. This leaves us with the tensor $\{h_{\mu\nu}^{TT}\}$, vector $\{V_\mu\}$, and scalar $\{\Gamma, \Theta, \boldsymbol{\chi}\}$ sectors, which decouple from each other at the linearized level.

The tensor and vector sectors of the fluctuations behave as their corresponding analogous ones in the standard Abelian Z_2 kinks, which are widely discussed in the literature. It is the scalar sector of the theory under consideration that is quite different from the Z_2 case. Nevertheless, in order to render this work self-consistent, we will also discuss briefly the tensor and vector fluctuations. In the following, the results will be expressed in the conformal coordinate

$$z = \int d\xi e^{-A(\xi)}, \tag{50}$$

such that

$$g_{ab} = e^{2A(z)}(\eta_{\mu\nu} dx_a^\mu dx_b^\nu + dz_a dz_b). \tag{51}$$

A. Tensor fluctuations

In the tensor sector $h_{\mu\nu}^{TT}$ with $\Psi_{\mu\nu} \equiv e^{3A/2} h_{\mu\nu}^{TT}$, the modes $\Psi_{\mu\nu}(x,z) \sim e^{ip\cdot x}\Psi_{\mu\nu}(z)$ satisfy the Schrödinger-like equation

$$(-\partial_z^2 + V_{Q_1})\Psi_{\mu\nu}(z) = m^2 \Psi_{\mu\nu}(z), \tag{52}$$

where

$$V_{Q_1} = \frac{9}{4}A'^2 + \frac{3}{2}A'' \tag{53}$$

and $p^\mu p_\mu = -m^2$. V_{Q_1} supports a massless bound state which can be identified with the graviton and a tower of non-normalizable massive states which propagate in the bulk. As is well known, the above equation can be factorized as

$$(-\partial_z^2 + V_{Q_1}(z)) = \left(\partial_z + \frac{3}{2}A'(z)\right)\left(-\partial_z + \frac{3}{2}A'(z)\right), \tag{54}$$

which implies the absence of modes with $m^2 < 0$, ensuring the stability of the system under tensor perturbations. As in the Abelian Z_2 kink [33], the normalizable massless tensor mode is given by $\Psi^0 \propto \exp\{3A(z)/2\}$, which reproduces 4D gravity on the core of the wall, while the continuum of massive modes gives small corrections to this behavior at short distances.

B. Vector fluctuations

The vector sector V_μ satisfies the equations

$$(\partial_z + 3A')V_\mu(x,z) = 0 \tag{55}$$

and

$$\partial^\beta \partial_\beta V_\mu(x,z) = 0, \tag{56}$$

whose solution is given by $V_\mu(x,z) = e^{-3A(z)}V_\mu(x)$, with $\partial^\beta\partial_\beta V_\mu(x) = 0$. On the other hand, from (46) we see that $V_\mu(x,z)$ must be an odd function of z and hence $V_\mu(x) = 0$. As in the Abelian Z_2 kink [32], there are no massless vector fluctuations localized on the wall.

C. Scalar fluctuations

In the scalar sector, we have the set of diffeomorphism-invariant fluctuations $\{\Gamma, \Theta, \boldsymbol{\chi}\}$, which are subject to two constraints. The first one only involves scalars without charge under $SU(5)$ and is given by

$$2\Gamma + \Theta = 0, \tag{57}$$

while the second one also involves the component χ_M of $\boldsymbol{\chi}$,

$$3A'\Theta - 3\Gamma' - \phi_M' \chi_M = 0. \tag{58}$$

From (57) and (58), it follows that the fluctuations Γ, Θ, and χ_M are not independent and correspond to a single physical scalar fluctuation.

With $\Omega \equiv e^{3A/2}\Gamma/\phi_M'$, the modes $\Omega(x,z) \sim e^{ip\cdot x}\Omega(z)$ satisfy

$$(-\partial_z^2 + V_{Q_2})\Omega(z) = m^2\Omega(z), \tag{59}$$

where V_{Q_2} is given by

$$V_{Q_2} = -\frac{5}{2}A'' + \frac{9}{4}A'^2 + A'\frac{\phi_M''}{\phi_M'} + 2\left(\frac{\phi_M''}{\phi_M'}\right)^2 - \frac{\phi_M'''}{\phi_M'}. \tag{60}$$

V_{Q_2} does not support normalizable massless states. Notice that

$$(-\partial_z^2 + V_{Q_2}) = \left(-\partial_z + \frac{Z'}{Z}\right)\left(\partial_z + \frac{Z'}{Z}\right), \tag{61}$$

where

$$Z = e^{3A/2}\frac{\phi_M'}{A'}, \tag{62}$$

implying the absence of modes with $m^2 < 0$. This scalar fluctuation has an exact analogous one in the Abelian Z_2 self-gravitating kink (see [32]).

Now, let us consider the scalar fluctuation $\boldsymbol{\chi} = \chi_q \mathbf{T}^q$. Let $\boldsymbol{\Xi} = \Xi_q \mathbf{T}^q$, with

$$\Xi_q \equiv e^{3A/2}\left(\chi_q - \frac{\Gamma}{A'}\phi_q'\right). \tag{63}$$

The modes $\Xi_q(x,z) \sim e^{ip\cdot x}\Xi_q(z)$ satisfy Schrödinger-like equations which depend on q.

First, let us consider perturbations along the $\mathbf{M}$ direction. For $q = M$ (it should be recalled that the fluctuations Γ, Θ, and χ_M are not independent), $\Xi_M(z)$ satisfies

$$(-\partial_z^2 + V_M)\Xi_M(z) = m^2\Xi_M(z), \tag{64}$$

where V_M is given by

$$V_M = -\frac{A'''}{A'} - \frac{3}{2}A'' + \frac{9}{4}A'^2 + 2\frac{A''^2}{A'^2} + \left(3A' - 2\frac{A''}{A'}\right)\frac{\phi''_M}{\phi'_M} + \frac{\phi'''_M}{\phi'_M}. \tag{65}$$

In terms of Z, as defined in (62), (64) can be rewritten as

$$(-\partial_z^2 + V_M) = \left(\partial_z + \frac{Z'}{Z}\right)\left(-\partial_z + \frac{Z'}{Z}\right), \tag{66}$$

and, from (61) and (66), it follows that the eigenvalues m^2 of the modes of Ω and Ξ_M always come in pairs, except possibly for the massless ones. Hence, there are no modes with $m^2 < 0$ for Ξ_M. On the other hand, the massless solution $\Xi_M^0(z)$ of (64) is given by

$$\Xi_M^0(z) \propto e^{3A/2}\frac{\phi'_M}{A'}, \tag{67}$$

which is, however, not normalizable since $\Xi_M^0(z)$ is not bounded for $z \to 0$. This is the mode which is expected to be related, when gravity is switched off, to the translational zero mode of the flat space $SU(5) \times Z_2$ kink [22–24]. As in the Abelian Z_2 kinks [5,34], once we include gravity, the massless mode is not normalizable and therefore not localized.

The behavior of the scalar fluctuation Ξ_M closely parallels that of the scalar fluctuation associated with the standard Abelian Z_2 kink [32]. Now, let us consider perturbations along the generators $\mathbf{T}^q$ of $SU(5)$ other than $\mathbf{M}$. For $q \neq M$, $\Xi_q(z)$ satisfies

$$(-\partial_z^2 + V_q)\Xi_q = m^2\Xi_q, \tag{68}$$

where V_q is given by

$$V_q = V_{Q_1} + e^{2A}\frac{\partial^2 V}{\partial\phi_q^2}, \tag{69}$$

with V_{Q_1} given by (53).

For $q = P$, we find that

$$\frac{\partial^2 V}{\partial\phi_P^2} = 4b^2\left(1 + \frac{4}{9}v^2\right) \tag{70}$$

for both symmetry breakings. V_P is always positive and hence does not support bound states with $m^2 \leq 0$. As expected, Φ is stable to perturbations along the Φ direction. Furthermore, these perturbations do not generate normalizable 4D massless modes independently of the symmetry breaking pattern.

For a $\mathbf{T}^q$ that is a generator of H_0 and where $\mathbf{T}^q \neq \mathbf{M}, \mathbf{P}$, we find for the symmetry breaking $\mathbf{A}$

$$\frac{\partial^2 V}{\partial\phi_q^2} = b^2\left(-8 + \frac{32}{9}v^2 + (3 \pm F)\left(6 - \frac{4}{9}v^2(F^2+1)\right)\right), \tag{71}$$

where $F = F(z)$ is the function defined by

$$F(z) = \tanh b\xi, \tag{72}$$

with $\xi = \xi(z)$ (50). These are perturbations along the six generators of $SU(2)_+ \times SU(2)_- \subset H_0^{\mathbf{A}}$; the $\pm$ signs correspond to the two different $SU(2)$'s. On the other hand, for the symmetry breaking $\mathbf{B}$, we find

$$\frac{\partial^2 V}{\partial\phi_q^2} = b^2\left(\frac{5}{2} + \frac{109}{36}v^2 + \left(\frac{3}{2} + \frac{1}{6}v^2\left(5 - \frac{11}{6}F^2\right)\right)F^2\right) \tag{73}$$

for the perturbations along the eight generators of $SU(3) \subset H_0^{\mathbf{B}}$. In both symmetry breaking patterns, V_q is always positive and hence does not support bound states with $m^2 \leq 0$. From the above results, it follows that perturbations along the generators of H_0 do not generate normalizable 4D massless modes.

Next, let us consider perturbations along those generators of $SU(5)$ that are not generators of H_0, which we will call broken generators. For a perturbation along $\mathbf{T}^q$, such that $\mathbf{T}^q$ is a generator of H_+ but not of H_- (the minus signs) or a generator of H_- but not of H_+ (the plus signs), we find for the symmetry breaking $\mathbf{A}$

$$\frac{\partial^2 V}{\partial\phi_q^2} = 2b^2F\left(1 + \frac{2}{3}v^2\left(1 - \frac{1}{3}F^2\right)\right)(F \pm 1), \tag{74}$$

and, for the symmetry breaking $\mathbf{B}$,

$$\frac{\partial^2 V}{\partial\phi_q^2} = 2b^2F\left(\pm 13\left(\frac{1}{2} + \frac{4}{9}v^2\right) + \left(1 + \frac{2}{3}v^2\left(1 - \frac{1}{3}F^2\right)\right)(F \pm 1)\right). \tag{75}$$

These are perturbations along the $n_\pm$ generators that are broken only at one side of the wall, with $n_\pm = 4$ for the symmetry breaking $\mathbf{A}$ and $n_\pm = 6$ for the symmetry breaking $\mathbf{B}$ ($n_\pm$ is the dimension of the coset $H_\pm/H_0$). The Schrödinger-like equation for these fluctuations cannot be written in terms of a fake superpotential, as we did for Ξ_M. However, on general grounds and from the shape of V_q for these perturbations (see Fig. 1), besides a mild resonance behavior, trapped massless modes are expected.

These gravitationally trapped massless modes correspond to rotations of Φ within the class described by $H_\pm/H_0$. In the nongravitating setting, the analogous zero

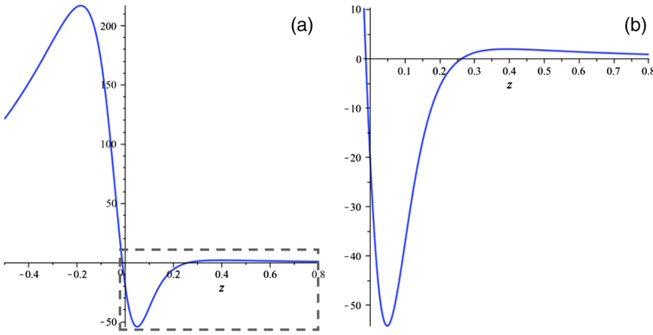

FIG. 1. (a) V_q for the $\Xi_q(z)$ modes of the fluctuations within the class described by H_+/H_0 (for the class described by H_-/H_0, V_q is the mirror image). (b) Enlargement of the area shown in (a).

modes behave as $F \pm 1$ and are not normalizable [23]. Hence, gravitationally trapped massless modes associated with these perturbations can be seen as the zero modes in flat space which turn out normalizable and get localized once we include gravity. For this type of perturbation, there are no bound states in the flat space case [23], nor do we find any modes with $m^2 < 0$ in the gravitating case.

Finally, for a q such that $\mathbf{T}^q$ is neither a generator of H_+ nor of H_-, we find that $\partial^2 V/\partial\phi_q^2 = 0$, and the modes $\Xi_q(z)$ satisfy

$$(-\partial_z^2 + V_{Q_1})\Xi_q = m^2\Xi_q. \tag{76}$$

As stated above, V_{Q_1} supports a massless bound state, a tower of non-normalizable massive states with $m^2 > 0$ and no modes with $m^2 < 0$. These are perturbations along the n_{br} generators that are broken everywhere, with $n_{br} = 8$ for the symmetry breaking **A** and $n_{br} = 2$ for the symmetry breaking **B**.

The above results show that the 5D self-gravitating $SU(5) \times Z_2$ domain walls of [28] are perturbatively stable. Additionally, on both symmetry breaking patterns, normalizable 4D massless modes of the kink fluctuations $\chi = \chi_q \mathbf{T}^q$ appear when $\mathbf{T}^q$ is not a generator of H_0. The existence of normalizable massless scalar modes is reminiscent of the situation envisaged in [35], although in the latter case these modes appears in Abelian Z_2-symmetric models with N scalar fields ($N \geq 2$) and a potential generated by a fake superpotential.

IV. SUMMARY AND OUTLOOK

Dynamical localization of gauge fields via the DS mechanism [29] in non-Abelian domain wall scenarios was put forward in [25,26] and discussed in [27] for a flat space model based on an E_6 symmetry group and in [28] for a gravitating $SU(5) \times Z_2$ model. Besides the fact that the stability of non-Abelian kinks is not guaranteed and should be proved at least perturbatively [23,24], it is clear that the 4D phenomenology on the brane may be influenced by the inclusion of gravity, which couples metric fluctuations with the wall fluctuations.

We have thus carried out a diffeomorphism-invariant analysis of the fluctuations of the 5D self-gravitating $SU(5) \times Z_2$ domain walls of [28], in order to determine their perturbative stability as well as their localization properties from the point of view of 4D observers.

We have found no modes with $m^2 < 0$ for the fluctuations of the self-gravitating $SU(5) \times Z_2$ kinks, implying that these are perturbatively stable as in the flat spacetime case [22–24].

Not surprisingly, the tensor and vector sectors of the fluctuations behave in the same way as the corresponding ones of the standard Z_2 kinks. There is a normalizable massless mode in the tensor sector which gives rise to 4D gravity on the brane and a tower of non-normalizable massive states which propagate in the bulk. There is no localized vector fluctuation.

On the other hand, the scalar sector of the fluctuations for the self-gravitating $SU(5) \times Z_2$ kinks greatly differs from its analogous one for the Abelian Z_2 case. We have found no normalizable 4D massless modes associated with the unbroken subgroup H_0 on the core of the wall, independently of the symmetry breaking pattern considered. However, there are as many normalizable 4D zero modes as there are broken generators, i.e., we find gravitationally localized massless 4D scalar particles without charge under H_0 which can be identified as (the 4D zero modes of) the Nambu-Goldstone fields associated with the symmetry breaking $SU(5) \times Z_2 \to H_0$. This gravitational trapping of Nambu-Goldstone bosons is presumably shared with other non-Abelian domain walls, a subject that deserves further investigation.

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