PHYSICAL REVIEW D **95,** 044043 (2017)

Pair production in near extremal Kerr-Newman black holes

Chiang-Mei Chen*

Department of Physics, National Central University, Chungli 32001, Taiwan and Center for Mathematics and Theoretical Physics, National Central University, Chungli 32054, Taiwan

Sang Pyo Kim†

Department of Physics, Kunsan National University, Kunsan 54150, Korea

Jia-Rui Sun‡

School of Physics and Astronomy, Sun Yat-Sen University, Guangzhou 510275, China

Fu-Yi Tang§

Department of Physics, National Central University, Chungli 32001, Taiwan

(Received 12 July 2016; published 28 February 2017)

The spontaneous pair production of charged scalars in a near-extremal Kerr-Newman (KN) black hole is analytically studied. It is shown that the existence condition for pair production is equivalent to the violation of the Breitenlohner-Freedman bound in an AdS_2 space. The mean number of produced pairs in the extremal black hole has a thermal interpretation, in which the effective temperature for the Schwinger effect in the AdS_2 space persistently holds, while the mean number in the near-extremal black hole has an additional factor of the Schwinger effect in the Rindler space. In addition, the holographic dual conformal field theory (CFT) descriptions of charged scalar pair production are realized in both the J and Q pictures in terms of the KN/CFTs correspondence.

DOI: 10.1103/PhysRevD.95.044043

I. INTRODUCTION

Spontaneous pair production from a black hole provides important information about quantum aspects of the vacuum in curved spacetime. Pair production, especially from a charged black hole, intertwines two remarkable quantum processes—namely, the Schwinger mechanism [1] caused by electromagnetic force, and the Hawking radiation caused by tunneling through the horizon [2]. The leading term of the pair-production rate is expected to come from the associated processes occurring at the near-horizon region: the separation of virtual pairs by the causal boundary for the Hawking radiation and by the dominating electric field for the Schwinger mechanism. Therefore, one can mainly focus on quantum field theory in the near-horizon geometry of charged black holes and obtain the leading term of the pair-production rate outside the black hole. In particular, in the near-extremal limit, the near-horizon geometry turns out to have an enhanced symmetry. A well-known example is the near-horizon geometry of a near-extremal Reissner-Nordström (RN) black hole, which is the Bertotti-Robinson solution with $\mathrm{AdS}_2 \times S^2$ isometry. The existence of such AdS structures in the background spacetime allows one to obtain analytical results for the field equation.

In the previous study [3–5], the spontaneous pair production of charged scalars and spinors has been systematically investigated in the near-extremal RN black hole without backreactions. Interestingly, it has been shown that the existence condition for pair production is equivalent to the violation of the Breitenlohner-Freedman (BF) bound in the AdS_2 (or warped AdS_3) spacetime, which, in turn, just guarantees the cosmic censorship conjecture during the pair-production process. From the dual conformal field theory (CFT) side, the violation of the BF bound for the probe field makes the conformal dimensions of its dual operator complex numbers, which indicates instability for the dual CFT. Following the techniques in the RN/CFT correspondence [6–9], the holographic description of pair production in the near-extremal RN black hole has been found.

The pair production from a near-extremal RN black hole exhibits interesting features [3,4]—in particular, a thermal interpretation. The $\mathrm{AdS}_2 \times S^2$ geometry in the near-horizon region of the extremal RN black hole gives, except for different quantum numbers for S^2, the same Schwinger effect in a constant electric field in the AdS_2 space, which has a thermal interpretation in terms of the effective temperature consisting of the Unruh temperature for accelerating charges due to the electric field and the AdS curvature [10]. The effective temperature is an

*cmchen@phy.ncu.edu.tw
†sangkim@kunsan.ac.kr
‡sunjiarui@sysu.edu.cn
§foue.tang@gmail.com

extension of the Unruh temperature in AdS_2 [11] to the Schwinger effect in that space. The pair-production rate in the near-extremal RN black hole, however, has an additional factor due to a small deviation from the extremality, which also has another interpretation of the Schwinger effect in a Rindler space with the surface gravity of the black hole for the acceleration [12]. The Schwinger effect and the thermal interpretation in Ref. [10] seem to persistently hold as far as pairs are dominantly produced in the region of $AdS_2 \times \mathcal{C}$ for a compact space $\mathcal{C}$.

Another interesting example is the near-horizon geometry of a near-extremal Kerr-Newman (KN) black hole which contains a warped $AdS_3 = AdS_2 \times S^1$ structure. It is thus expected that we will obtain analytical results for the pair production of probe fields in that geometry. In addition, as has been shown, the RN/CFT duality can be incorporated into the Q (charge) picture of the more general KN/CFTs dualities [13–18], while there exists another J (angular momentum) dual CFT picture in terms of the Kerr/CFT duality [19–21]. Hence, it is interesting to check the holographic description of the pair production in the near-extremal KN black hole in both the Q and the J pictures.

In this paper, we will study the spontaneous pair-production process of charged scalars in the near-extremal KN black hole without backreactions. The Klein-Gordon equation of the probe charged scalar field can be separated into radial and angular equations. Though the angular coordinates are not spherical symmetric and the separation constant can only be numerically determined, the angular part is expected to contribute the same factor to the flux at the near-horizon region and at the asymptotic AdS boundary. Therefore, the angular part will not affect the vacuum persistence amplitude, the mean number of produced pairs, or the absorption cross-section ratio. The radial equation, our main consideration, can be solved exactly in terms of the hypergeometric functions. Following previous works [3,4], we adopt the "particle viewpoint" and impose the boundary condition of no incoming flux at the asymptotic boundary. Using the fluxes, we then derive the Bogoliubov coefficients (vacuum persistence and mean number of pairs) and the absorption cross-section ratio.

We also propose a thermal interpretation for the pair production in the extremal KN black hole, in which the effective temperature of the Unruh temperature for accelerating charges and the AdS curvature gives exactly the same form as that in the extremal RN black hole. The thermal interpretation holds for the near-extremal KN black hole, as it does for the near-extremal RN black hole. To explain a physical origin of the leading term for the pair production, we employ the Hamilton-Jacobi action approach and then compute the instanton action from the phase-integral formula in the complex plane of the near-horizon region. The Boltzmann factor for the Schwinger effect is a consequence of two simple poles at both the inner and outer horizons, in which there is no reason to exclude the quantum tunneling process from the inner horizon just adjacent to the outer horizon of the near-extremal black hole. Finally, based on the near-extremal KN/CFTs correspondence [13–18], we investigate the holographic dual CFTs descriptions of the scalar pair production in both the J and Q pictures by comparing the absorption cross-section ratio computed both from the near-extremal KN black hole and from their dual CFTs, and show that they agree with each other—namely, the pair production in a near-extremal KN black hole indeed can be characterized by two individual CFTs by taking appropriate limits.

The outline of the paper is as follows: In Sec. II, we review basic properties of the near-horizon geometry of the near-extremal KN black hole. In Secs. III and IV, we analytically solve the field equation for the probe charged scalar field in the near-extremal KN black hole and obtain the vacuum persistence amplitude, the mean number of produced pairs and the absorption cross-section ratio. In Sec. V, the thermal interpretation of the pair production is presented. Then, in Sec. VI we give the holographic descriptions of the pair production based on the KN/CFTs dualities. The conclusion is drawn in Sec. VII, and in the Appendix we list some useful properties of the hypergeometric functions.

II. NEAR-HORIZON GEOMETRY OF KERR-NEWMAN

The four-dimensional Einstein-Maxwell theory

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}(R - F_{[2]}^2) \tag{1}$$

admits the Kerr-Newman (KN) black hole solution, in natural units $c = G = \hbar = 1$, as follows:

$$\begin{aligned} ds^2 &= -\frac{\Delta - a^2\sin^2\theta}{\Sigma}\left[d\hat{t} + \frac{(2M\hat{r} - Q^2)a\sin^2\theta}{\Delta - a^2\sin^2\theta}d\hat{\varphi}\right]^2 \\ &\quad + \Sigma\left(\frac{d\hat{r}^2}{\Delta} + d\theta^2 + \frac{\Delta\sin^2\theta}{\Delta - a^2\sin^2\theta}d\hat{\varphi}^2\right), \\ A_{[1]} &= \frac{Q\hat{r}}{\Sigma}(d\hat{t} - a\sin^2\theta d\hat{\varphi}), \end{aligned} \tag{2}$$

where

$$\Sigma = \hat{r}^2 + a^2\cos^2\theta, \qquad \Delta = \hat{r}^2 - 2M\hat{r} + a^2 + Q^2. \tag{3}$$

This is the most general black hole, carrying three physical quantities: mass M, electric charge Q, and angular momentum $J = Ma$. The inner horizon $\hat{r}_-$ and the outer horizon $\hat{r}_+$ are located at $\hat{r}_\pm = M \pm \sqrt{M^2 - a^2 - Q^2}$. The thermodynamical properties of KN black holes are described by two essential quantities, the Hawking temperature and the Bekenstein-Hawking entropy:

$$T_{\mathrm{H}} = \frac{\hat{\kappa}}{2\pi} = \frac{\hat{r}_+ - \hat{r}_-}{4\pi(\hat{r}_+^2 + a^2)},$$
$$S_{\mathrm{BH}} = \frac{\hat{A}_+}{4} = \pi(\hat{r}_+^2 + a^2), \tag{4}$$

where $\hat{\kappa}$ and $\hat{A}_+$ are the surface gravity and the area of the outer horizon. Besides this, the first law of thermodynamics of the KN black hole reads

$$\delta M = T_{\mathrm{H}}\delta S_{\mathrm{BH}} + \Phi_{\mathrm{H}}\delta Q + \Omega_{\mathrm{H}}\delta J, \tag{5}$$

in which $\Phi_{\mathrm{H}} = \frac{Q\hat{r}_+}{\hat{r}_+^2 + a^2}$ is the electric potential at the polar points of the horizon and $\Omega_{\mathrm{H}} = \frac{a}{\hat{r}_+^2 + a^2}$ is the angular velocity of the KN black hole.

From the "orthodox" limit for rotating back holes [22], it is convenient to rewrite the KN metric in the ADM form as

$$\begin{aligned} ds^2 = &-\frac{\Sigma\Delta}{(\hat{r}^2 + a^2)^2 - \Delta a^2\sin^2\theta}d\hat{t}^2 + \frac{\Sigma}{\Delta}d\hat{r}^2 + \Sigma d\theta^2 \\ &+ \frac{(\hat{r}^2 + a^2)^2 - \Delta a^2\sin^2\theta}{\Sigma} \\ &\times \sin^2\theta\left[d\hat{\varphi} - \frac{a(2M\hat{r} - Q^2)}{(\hat{r}^2 + a^2)^2 - \Delta a^2\sin^2\theta}d\hat{t}\right]^2. \end{aligned} \tag{6}$$

In the extremal limit, i.e. $M^2 = r_0^2$ with $r_0^2 \equiv Q^2 + a^2$, the horizons degenerate to $\hat{r} = \hat{r}_+ = \hat{r}_- = r_0$. To derive the near-horizon geometry of a near-extremal KN black hole, the angular velocity Ω_{H} at the horizon should be firstly "removed" by the coordinate transformation

$$\hat{\varphi} \to \varphi - \frac{a}{r_0^2 + a^2}\hat{t}, \tag{7}$$

and then, taking the following near-horizon and near-extremal limit with $\varepsilon \to 0$,

$$\hat{r} \to r_0 + \varepsilon r, \quad \hat{t} \to \frac{r_0^2 + a^2}{\varepsilon}t, \quad M \to r_0 + \varepsilon^2\frac{B^2}{2r_0}, \tag{8}$$

one obtains the near-horizon solution of the near-extremal KN black hole as

$$\begin{aligned} ds^2 = &\Gamma(\theta)\left[-(r^2 - B^2)dt^2 + \frac{dr^2}{r^2 - B^2} + d\theta^2\right] \\ &+ \gamma(\theta)(d\varphi + brdt)^2, \end{aligned} \tag{9}$$

$$A_{[1]} = -Q\left(\frac{r_0^2 - a^2\cos^2\theta}{r_0^2 + a^2\cos^2\theta}rdt + \frac{r_0 a\sin^2\theta}{r_0^2 + a^2\cos^2\theta}d\varphi\right), \tag{10}$$

where

$$\Gamma(\theta) = r_0^2 + a^2\cos^2\theta,$$
$$\gamma(\theta) = \frac{(r_0^2 + a^2)^2\sin^2\theta}{r_0^2 + a^2\cos^2\theta},$$
$$b = \frac{2ar_0}{r_0^2 + a^2}. \tag{11}$$

The spacetime (9) contains a warped AdS_3 geometry, which allows the dual CFTs description for the KN black hole.

III. SCALAR FIELD IN KN BLACK HOLE

The action for a probe charged scalar field Φ with mass m and charge q is given by

$$S = \int d^4x\sqrt{-g}\left(-\frac{1}{2}D_\alpha\Phi^* D^\alpha\Phi - \frac{1}{2}m^2\Phi^*\Phi\right), \tag{12}$$

where $D_\alpha \equiv \nabla_\alpha - iqA_\alpha$, with ∇_α being the covariant derivative in a curved spacetime. The corresponding Klein-Gordon equation is

$$(\nabla_\alpha - iqA_\alpha)(\nabla^\alpha - iqA^\alpha)\Phi - m^2\Phi = 0. \tag{13}$$

The scalar field in the metric (9) is assumed to have the following form:

$$\Phi(t, r, \theta, \varphi) = \mathrm{e}^{-i\omega t + in\varphi}R(r)\Theta(\theta), \tag{14}$$

which separates the Klein-Gordon equation as

$$\begin{aligned} \partial_r[(r^2 - B^2)\partial_r R] + \Bigg(&\frac{[\omega(r_0^2 + a^2) - qQ^3 r + 2nar_0 r]^2}{(r_0^2 + a^2)^2(r^2 - B^2)} \\ &- m^2(r_0^2 + a^2) - \lambda\Bigg)R = 0, \end{aligned} \tag{15}$$

$$\begin{aligned} \frac{1}{\sin\theta}\partial_\theta(\sin\theta\partial_\theta\Theta) - \Bigg(&\frac{[n(r_0^2 + a^2\cos^2\theta) + qQar_0\sin^2\theta]^2}{(r_0^2 + a^2)^2\sin^2\theta} \\ &- m^2a^2\sin^2\theta - \lambda\Bigg)\Theta = 0, \end{aligned} \tag{16}$$

where λ is a separation constant. In addition, the radial equation (15) may be viewed as the equation of motion for a probe scalar field with the effective mass $m_{\mathrm{eff}}^2 = m^2 - \frac{(2nar_0 - qQ^3)^2}{(r_0^2 + a^2)^3} + \frac{\lambda}{r_0^2 + a^2}$ propagating in an AdS_2 spacetime with the AdS radius $L = \sqrt{r_0^2 + a^2}$. It has been shown from the pair production of scalars and spinors in the near-extremal RN black hole that the existence condition for pair production is the appearance of an instability—i.e., the violation of the Breitenlohner-Freedman (BF) bound of the field in the AdS_2 spacetime—which guarantees the existence of the propagating modes of the field [3,4]. For the near-extremal KN black hole, the corresponding relation is

$$m_{\mathrm{eff}}^2 < -\frac{1}{4(r_0^2+a^2)}$$
$$\Rightarrow m^2(r_0^2+a^2) - \frac{(2nar_0 - qQ^3)^2}{(r_0^2+a^2)^2} + \lambda + \frac{1}{4} < 0. \quad (17)$$

The radial flux of the scalar field (14) in the spacetime (9) can be expressed as

$$D = \int d\theta d\varphi i\sqrt{-g}g^{rr}(\Phi D_r\Phi^* - \Phi^* D_r\Phi)$$
$$= i(r_0^2+a^2)(r^2-B^2)(R\partial_r R^* - R^*\partial_r R)\mathfrak{S}, \quad (18)$$

where the contribution from the angular part is symbolically denoted by

$$\mathfrak{S} = 2\pi\int d\theta \sin\theta\Theta\Theta^*, \quad (19)$$

which will contribute the "same" factor for the flux at the near-horizon region and at the asymptotic region.[1] Therefore, it does not show up in the flux ratios, and consequently, it will not affect the physical quantities such as the mean number of produced pairs and the absorption cross-section ratio, etc.

IV. PAIR PRODUCTION

The general solution of the radial equation (15) can be found in terms of the hypergeometric functions,

$$R(r) = c_1(r-B)^{\frac{i}{2}(\tilde{\kappa}-\kappa)}(r+B)^{\frac{i}{2}(\tilde{\kappa}+\kappa)}$$
$$\times F\left(\frac{1}{2}+i\tilde{\kappa}+i\mu, \frac{1}{2}+i\tilde{\kappa}-i\mu; 1+i\tilde{\kappa}-i\kappa; \frac{1}{2}-\frac{r}{2B}\right)$$
$$+ c_2(r-B)^{-\frac{i}{2}(\tilde{\kappa}-\kappa)}(r+B)^{\frac{i}{2}(\tilde{\kappa}+\kappa)}$$
$$\times F\left(\frac{1}{2}+i\kappa+i\mu, \frac{1}{2}+i\kappa-i\mu; 1-i\tilde{\kappa}+i\kappa; \frac{1}{2}-\frac{r}{2B}\right), \quad (20)$$

with parameters

$$\tilde{\kappa} = \frac{\omega}{B}, \qquad \kappa = \frac{qQ^3 - 2nar_0}{r_0^2+a^2},$$
$$\mu = \sqrt{\kappa^2 - m^2(r_0^2+a^2) - \lambda - \frac{1}{4}}, \quad (21)$$

in which μ^2 is positive due to the BF-bound violation in Eq. (17). It turns out that this solution is a generalization of the corresponding result for the RN black holes [3] with different parameters.

In the near-horizon region, the solution (20) can be expanded around $r = B$ (we assume $\tilde{\kappa} > \kappa$ to cover the extremal limit $B \to 0$):

$$R_{\mathrm{H}}(r) \approx c_{\mathrm{H}}^{(\mathrm{in})}(2B)^{\frac{i}{2}(\tilde{\kappa}+\kappa)}(r-B)^{-\frac{i}{2}(\tilde{\kappa}-\kappa)}$$
$$+ c_{\mathrm{H}}^{(\mathrm{out})}(2B)^{\frac{i}{2}(\tilde{\kappa}+\kappa)}(r-B)^{\frac{i}{2}(\tilde{\kappa}-\kappa)}, \quad (22)$$

where

$$c_{\mathrm{H}}^{(\mathrm{in})} = c_2, \qquad c_{\mathrm{H}}^{(\mathrm{out})} = c_1. \quad (23)$$

In the asymptotic boundary ($r \to \infty$) of the metric (9), the hypergeometric function can be transformed into another form (see the Appendix), so that we have

$$R_{\mathrm{B}}(r) \approx c_{\mathrm{B}}^{(\mathrm{in})}(r-B)^{-\frac{1}{2}-\frac{i}{2}(\tilde{\kappa}+\kappa)-i\mu}(r+B)^{\frac{i}{2}(\tilde{\kappa}+\kappa)}$$
$$+ c_{\mathrm{B}}^{(\mathrm{out})}(r-B)^{-\frac{1}{2}-\frac{i}{2}(\tilde{\kappa}+\kappa)+i\mu}(r+B)^{\frac{i}{2}(\tilde{\kappa}+\kappa)}$$
$$\approx c_{\mathrm{B}}^{(\mathrm{in})} r^{-\frac{1}{2}-i\mu} + c_{\mathrm{B}}^{(\mathrm{out})} r^{-\frac{1}{2}+i\mu}, \quad (24)$$

where

$$c_{\mathrm{B}}^{(\mathrm{in})} = c_1(2B)^{\frac{1}{2}+i\tilde{\kappa}+i\mu}\frac{\Gamma(1+i\tilde{\kappa}-i\kappa)\Gamma(-2i\mu)}{\Gamma(\frac{1}{2}-i\kappa-i\mu)\Gamma(\frac{1}{2}+i\tilde{\kappa}-i\mu)}$$
$$+ c_2(2B)^{\frac{1}{2}+i\kappa+i\mu}\frac{\Gamma(1-i\tilde{\kappa}+i\kappa)\Gamma(-2i\mu)}{\Gamma(\frac{1}{2}+i\kappa-i\mu)\Gamma(\frac{1}{2}-i\tilde{\kappa}-i\mu)}, \quad (25)$$

$$c_{\mathrm{B}}^{(\mathrm{out})} = c_1(2B)^{\frac{1}{2}+i\tilde{\kappa}-i\mu}\frac{\Gamma(1+i\tilde{\kappa}-i\kappa)\Gamma(2i\mu)}{\Gamma(\frac{1}{2}-i\kappa+i\mu)\Gamma(\frac{1}{2}+i\tilde{\kappa}+i\mu)}$$
$$+ c_2(2B)^{\frac{1}{2}+i\kappa-i\mu}\frac{\Gamma(1-i\tilde{\kappa}+i\kappa)\Gamma(2i\mu)}{\Gamma(\frac{1}{2}+i\kappa+i\mu)\Gamma(\frac{1}{2}-i\tilde{\kappa}+i\mu)}. \quad (26)$$

Using the approximate solutions near both the inner and outer boundaries, the corresponding fluxes of each mode can be obtained via (18)

[1] Note that this asymptotic region means the $r \to \infty$ limit of Eq. (9), but it is still in the near-horizon region of the near-extremal KN black hole in Eq. (6) due to the coordinate transformations in Eq. (8). Although Eq. (9) only describes the near-horizon property of the near-extremal KN black hole in the original $\hat{t}$, $\hat{r}$ and $\hat{\varphi}$ coordinates, it contains a warped AdS_3 geometry in the new t, r and φ coordinates which has the new near-horizon limit $r \to B$ and asymptotic region $r \to \infty$, and the Schwinger effect occurs in the whole region from the horizon to the asymptotic boundary of the geometry in Eq. (9). In addition, the warped AdS_3 geometry allows a holographic dual CFT description at the asymptotic boundary $r \to \infty$ [18,21].

$$\begin{aligned}
D_{\mathrm{B}}^{(\mathrm{in})} &= -2(r_0^2+a^2)\mu|c_{\mathrm{B}}^{(\mathrm{in})}|^2\mathfrak{S},\\
D_{\mathrm{H}}^{(\mathrm{in})} &= -2B(r_0^2+a^2)(\tilde{\kappa}-\kappa)|c_{\mathrm{H}}^{(\mathrm{in})}|^2\mathfrak{S},\\
D_{\mathrm{B}}^{(\mathrm{out})} &= 2(r_0^2+a^2)\mu|c_{\mathrm{B}}^{(\mathrm{out})}|^2\mathfrak{S},\\
D_{\mathrm{H}}^{(\mathrm{out})} &= 2B(r_0^2+a^2)(\tilde{\kappa}-\kappa)|c_{\mathrm{H}}^{(\mathrm{out})}|^2\mathfrak{S},
\end{aligned} \tag{27}$$

where $\mathfrak{S}$ is defined in Eq. (19).

For scalar particles, the flux conservation is $|D_{\mathrm{incident}}| = |D_{\mathrm{reflected}}| + |D_{\mathrm{transmitted}}|$, and the Bogoliubov relation is $|\alpha|^2 - |\beta|^2 = 1$, in which the vacuum persistence amplitude $|\alpha|^2$ and the mean number of produced pairs $|\beta|^2$ are given by the ratios of the fluxes in the Coulomb gauge

$$|\alpha|^2 \equiv \frac{D_{\mathrm{incident}}}{D_{\mathrm{reflected}}}, \qquad |\beta|^2 \equiv \frac{D_{\mathrm{transmitted}}}{D_{\mathrm{reflected}}}. \tag{28}$$

Moreover, from the viewpoint of the scattering process of an incident flux from the asymptotic boundary, we can define the absorption cross-section ratio as

$$\sigma_{\mathrm{abs}} \equiv \frac{D_{\mathrm{transmitted}}}{D_{\mathrm{incident}}} = \frac{|\beta|^2}{|\alpha|^2}. \tag{29}$$

The spontaneous pair production can be revealed by two different boundary conditions—namely, the outer boundary condition in which there is no incoming flux at the asymptotic outer boundary, and the inner boundary condition in which there is no outgoing flux at the black hole horizon. It has been shown that these two boundary conditions are actually equivalent to each other due to the unitarity of the scattering matrix and give the same result [3]. In the following, we will adopt the particle point of view—i.e., impose the outer boundary condition (see Fig. 1) by setting $c_{\mathrm{B}}^{(\mathrm{in})} = 0$. By substituting

$$c_1 = -c_2(2B)^{i(\kappa-\tilde{\kappa})}\frac{\Gamma(1-i\tilde{\kappa}+i\kappa)\Gamma(\frac{1}{2}+i\tilde{\kappa}-i\mu)\Gamma(\frac{1}{2}-i\kappa-i\mu)}{\Gamma(\frac{1}{2}+i\kappa-i\mu)\Gamma(\frac{1}{2}-i\tilde{\kappa}-i\mu)\Gamma(1-i\kappa+i\tilde{\kappa})} \tag{30}$$

into Eq. (26), we have

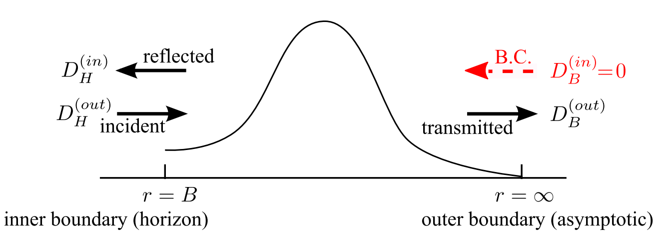

FIG. 1. Outer boundary condition: No incoming flux at asymptotic.

$$\begin{aligned}
c_{\mathrm{B}}^{(\mathrm{out})} &= -c_2(2B)^{\frac{1}{2}+i\kappa-i\mu}\frac{\sinh(2\pi\mu)\sinh(\pi\tilde{\kappa}-\pi\kappa)}{\cosh(\pi\tilde{\kappa}-\pi\mu)\cosh(\pi\kappa+\pi\mu)}\\
&\quad\times\frac{\Gamma(1-i\tilde{\kappa}+i\kappa)\Gamma(2i\mu)}{\Gamma(\frac{1}{2}-i\tilde{\kappa}+i\mu)\Gamma(\frac{1}{2}+i\kappa+i\mu)},
\end{aligned} \tag{31}$$

and we obtain the vacuum persistence amplitude and the mean number, respectively,

$$|\alpha|^2 = \frac{D_{\mathrm{incident}}}{D_{\mathrm{reflected}}} = \frac{|D_{\mathrm{H}}^{(\mathrm{out})}|}{|D_{\mathrm{H}}^{(\mathrm{in})}|} = \frac{\cosh(\pi\kappa-\pi\mu)\cosh(\pi\tilde{\kappa}+\pi\mu)}{\cosh(\pi\kappa+\pi\mu)\cosh(\pi\tilde{\kappa}-\pi\mu)}, \tag{32}$$

$$|\beta|^2 = \frac{D_{\mathrm{transmitted}}}{D_{\mathrm{reflected}}} = \frac{|D_{\mathrm{B}}^{(\mathrm{out})}|}{|D_{\mathrm{H}}^{(\mathrm{in})}|} = \frac{\sinh(2\pi\mu)\sinh(\pi\tilde{\kappa}-\pi\kappa)}{\cosh(\pi\kappa+\pi\mu)\cosh(\pi\tilde{\kappa}-\pi\mu)}, \tag{33}$$

and the absorption cross-section ratio,

$$\sigma_{\mathrm{abs}} = \frac{D_{\mathrm{transmitted}}}{D_{\mathrm{incident}}} = \frac{|D_{\mathrm{B}}^{(\mathrm{out})}|}{|D_{\mathrm{H}}^{(\mathrm{out})}|} = \frac{\sinh(2\pi\mu)\sinh(\pi\tilde{\kappa}-\pi\kappa)}{\cosh(\pi\kappa-\pi\mu)\cosh(\pi\tilde{\kappa}+\pi\mu)}. \tag{34}$$

Similarly to the RN black hole case, $|\beta|^2$ is related with σ_{abs} through $|\beta|^2(\mu\to-\mu)\to-\sigma_{\mathrm{abs}}$. If we take the extremal limit $B\to 0$; i.e. $\tilde{\kappa}\to\infty$, we have

$$\begin{aligned}
|\alpha|^2 &= \frac{\cosh(\pi\kappa-\pi\mu)}{\cosh(\pi\kappa+\pi\mu)}e^{2\pi\mu},\\
|\beta|^2 &= \frac{\sinh(2\pi\mu)}{\cosh(\pi\kappa+\pi\mu)}e^{\pi\mu-\pi\kappa},\\
\sigma_{\mathrm{abs}} &= \frac{\sinh(2\pi\mu)}{\cosh(\pi\kappa-\pi\mu)}e^{-\pi\mu-\pi\kappa}.
\end{aligned} \tag{35}$$

For the sake of self-completeness, the angular equation for $\Theta(\theta)$ can be expressed as

$$\frac{1}{\sin\theta}\partial_\theta(\sin\theta\partial_\theta\Theta) - \left(\frac{n^2}{\sin^2\theta} - \lambda_1\sin^2\theta - \lambda_2\right)\Theta = 0, \tag{36}$$

where

$$\begin{aligned}
\lambda_1 &= m^2a^2 - \frac{(qQr_0-na)^2}{(r_0^2+a^2)^2}a^2,\\
\lambda_2 &= \lambda - \frac{2na(qQr_0-na)}{r_0^2+a^2}.
\end{aligned} \tag{37}$$

This equation can be transformed into the confluent Heun equation. But the angular part contributes the "same" factor for the flux both at the near-horizon region ($r = B$) and at the asymptotic region ($r \to \infty$). Therefore, its explicit solution is not necessary for our consideration of pair production. However, the regularity for the angular solution should give a constraint on the separation parameter λ, which can be found numerically. Another interesting remark is that the existence condition for the pair production (violation of the BF bound) for RN black holes just turns out to guarantee the cosmic censorship conjecture during the pair-production process. Such a connection, however, is not so obvious for KN black holes.

V. THERMAL INTERPRETATION

In terms of instanton actions $S_a = 2\pi\kappa$, $\tilde{S}_a = 2\pi\tilde{\kappa}$ and $S_b = 2\pi\mu$ from the relativistic field equation, we write the mean number of produced pairs (33) as

$$\mathcal{N} = |\beta|^2 = \left(\frac{\mathrm{e}^{-S_a+S_b} - \mathrm{e}^{-S_a-S_b}}{1+\mathrm{e}^{-S_a-S_b}}\right)\left(\frac{1-\mathrm{e}^{-\tilde{S}_a+S_a}}{1+\mathrm{e}^{-\tilde{S}_a+S_b}}\right). \tag{38}$$

Note that the mean number (38) has a similar form to that of charged scalars in a near-extremal RN black hole in Ref. [3], except for different quantum numbers, since the near-horizon geometry is the warped $\mathrm{AdS}_3 \times S^1$ for the near-extremal KN black hole, while it is $\mathrm{AdS}_2 \times S^2$ for the near-extremal RN black hole. Following Refs. [12,23], by introducing an effective temperature and its associated one

$$\begin{aligned} T_{\mathrm{KN}} &= \frac{\bar{m}}{S_a - S_b} = T_U + \sqrt{T_U^2 + \frac{\mathcal{R}}{8\pi^2}}, \\ \bar{T}_{\mathrm{KN}} &= \frac{\bar{m}}{S_a + S_b} = T_U - \sqrt{T_U^2 + \frac{\mathcal{R}}{8\pi^2}}, \end{aligned} \tag{39}$$

where

$$\bar{m} = \sqrt{m^2 - \frac{\lambda + 1/4}{2}\mathcal{R}}, \tag{40}$$

and

$$\begin{aligned} T_U &= \frac{\kappa}{2\pi\bar{m}(r_0^2+a^2)} = \frac{qQ^3 - 2nar_0}{2\pi\bar{m}(r_0^2+a^2)^2}, \\ \mathcal{R} &= -\frac{2}{r_0^2+a^2}, \end{aligned} \tag{41}$$

the mean number (38) can be expressed as

$$\mathcal{N} = \mathrm{e}^{\frac{\bar{m}}{T_{\mathrm{KN}}}} \times \left(\frac{\mathrm{e}^{-\frac{\bar{m}}{T_{\mathrm{KN}}}} - \mathrm{e}^{-\frac{\bar{m}}{\bar{T}_{\mathrm{KN}}}}}{1+\mathrm{e}^{-\frac{\bar{m}}{\bar{T}_{\mathrm{KN}}}}}\right) \times \left\{\frac{\mathrm{e}^{-\frac{\bar{m}}{T_{\mathrm{KN}}}}(1-\mathrm{e}^{-\frac{\hat{\omega}-q\Phi_{\mathrm{H}}-n\Omega_{\mathrm{H}}}{T_{\mathrm{H}}}})}{1+\mathrm{e}^{-\frac{\hat{\omega}-q\Phi_{\mathrm{H}}-n\Omega_{\mathrm{H}}}{T_{\mathrm{H}}}}\mathrm{e}^{-\frac{\bar{m}}{T_{\mathrm{KN}}}}}\right\}. \tag{42}$$

Here, T_{H} is the Hawking temperature, and $\Phi_{\mathrm{H}}, \Omega_{\mathrm{H}}$ are the chemical potential and angular velocity at $r = B$:

$$T_{\mathrm{H}} = \frac{\hat{B}}{2\pi}, \quad \Phi_{\mathrm{H}} = -\frac{Q^3\hat{B}}{r_0^2+a^2}, \quad \Omega_{\mathrm{H}} = \frac{2ar_0\hat{B}}{r_0^2+a^2}, \tag{43}$$

in which $\hat{\omega} = \varepsilon\omega$ and $\hat{B} = \varepsilon B$ are quantities measured in the "original" coordinates of KN black holes (2). However, the result (42) is independent on the parameter ε; thus, here and after we do not elaborately distinguish $(\hat{\omega}, \hat{B})$ from (ω, B) in such a situation.

The physical interpretation of each term of Eq. (42) is that the first parenthesis is the Schwinger effect with the effective temperature T_{KN} in AdS_2 [10], and the second parenthesis is the Schwinger effect in the Rindler space [24], in which the Unruh temperature is given by the Hawking temperature and the charge has the chemical potentials of Φ_{H} and Ω_{H}, while the effective temperature for the Schwinger effect due to the electric field on the horizon is determined by T_{KN}.

The mean number of produced pairs above and the absorption cross-section ratio in the previous section have been obtained using the exact solution in the near-horizon geometry of an extremal or near-extremal KN black hole. Below, by applying the phase-integral formula, we derive the instanton actions from the Hamilton-Jacobi action for the field equation, which lead to the mean number. This method allows one to understand the physical origin of each term as a consequence of simple poles in the complex plane of space [25], and further also connects the interpretation to other physical systems involving the Schwinger effect in curved spacetimes [26].

The Hamilton-Jacobi action together with the phase-integral formula explains the origin of the subleading terms as well as the leading Boltzmann factor in Eq. (38). By substituting $R(r) = \mathrm{e}^{iS(r)}$ into the radial equation (15), we obtain the Hamilton-Jacobi action in the complex plane of $z = r$:

$$S(z) = \int \frac{dz}{z^2 - B^2}\sqrt{(\omega - \kappa z)^2 - \bar{m}^2(r_0^2+a^2)(z^2 - B^2)}. \tag{44}$$

The phase-integral formula for the action along a path Γ in the z plane gives a complex amplitude

$$N_S = \mathrm{e}^{iS_\Gamma}, \tag{45}$$

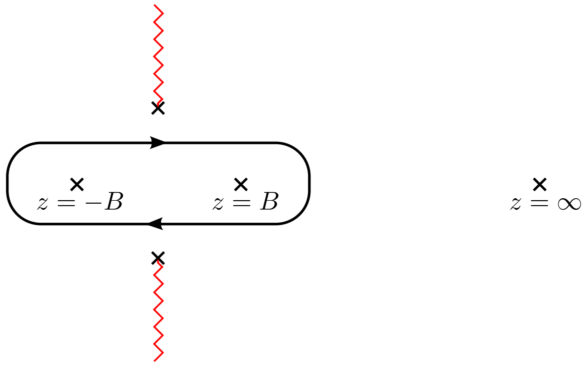

FIG. 2. The contour integral for the leading Boltzmann factor in the extremal KN black hole.

whose imaginary part due to simple poles determines the pair-production rate or the decay rate of the vacuum [3,25].

First, in the case of the near-extremal KN black hole, the action (44) has two simple poles at $z=\pm B$ and another simple pole at the infinity $z=\infty$ from the large z expansion, and thereby the residues $S_\pm$ and S_∞, respectively. Further, the square root has a pair of complex roots in the complex plane, so a branch cut may be introduced as in Fig. 2 to exclude the roots and make the integrand an analytic function. Then, the Cauchy residue theorem along the contour of Fig. 2 enclosing the simple poles at $z=\pm B$ gives the leading term

$$e^{i(-2\pi i)(\frac{\omega-\kappa B}{2B}-\frac{\omega+\kappa B}{2B}+\mu)} = e^{-(S_a-S_\infty)} = e^{-\frac{\bar{m}}{T_{\rm KN}}}, \qquad (46)$$

where the third term in the parenthesis is the residue from the infinity. On the other hand, the contour integral of Fig. 3 provides another instanton action for the exact pair-production rate (38)

$$\tilde{S}_a = (S_- - S_\infty) - (S_+ - S_\infty) = 2\pi\frac{\omega}{B} = 2\pi\tilde{\kappa}. \qquad (47)$$

A few comments are in order. The directions of the contours are chosen by the causality reason. The Schwinger effect in a

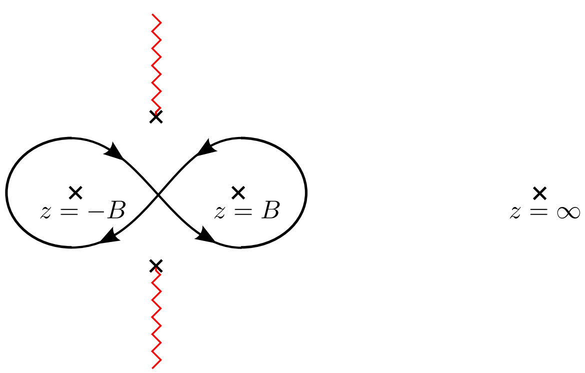

FIG. 3. The contour integral for the subleading term in the near-extremal KN black hole.

near-extremal black hole is a consequence of both the inner and outer horizons, which differs from the Hawking radiation of charges in a nonextremal black hole originated from the outer horizon only.

Second, in the case of the extremal KN black hole, we use the small z expansion, which has a simple pole at $z=0$ and the residue $-\kappa$. Thus, the clockwise contour integral gives the leading term

$$e^{i(-2\pi i)(-\kappa+\mu)} = e^{-(S_a-S_\infty)} = e^{-\frac{\bar{m}}{T_{\rm KN}}}, \qquad (48)$$

where the second term comes from the simple pole at infinity. Note that Eq. (48) is the same as Eq. (46).

VI. DUAL CFT DESCRIPTION

The KN black hole was shown to have twofold dual CFT descriptions [16,18], which are called the J (angular momentum) picture in terms of the Kerr/CFT correspondence [19,21] and the Q (charge) picture based on the RN/CFT correspondence [6,9]. Following previous studies on the holographic description of the pair production in the near-extremal RN black hole [5], it is interesting to analyze the holographic dual of the pair production in the KN black hole in both of the two pictures.

The absorption cross-section ratio of the charged scalar field in Eq. (34) can be rewritten as

$$\sigma_{\rm abs} = \frac{\sinh(2\pi\mu)}{\pi^2}\sinh(\pi\tilde{\kappa}-\pi\kappa)\left|\Gamma\left(\frac{1}{2}+i(\mu-\kappa)\right)\right|^2 \times\left|\Gamma\left(\frac{1}{2}+i(\mu+\tilde{\kappa})\right)\right|^2, \qquad (49)$$

with the parameters $\tilde{\kappa}, \kappa$, and μ defined in (21). From the field/operator duality, the charged scalar field in the near-extremal KN black hole is dual to a scalar operator in the two-dimensional CFT. In the present case, the conformal dimensions $(h_{\rm L}, h_{\rm R})$ of the operator can be read from the asymptotic expansion of the bulk charged scalar field at the AdS boundary in Eq. (24), which are given by [20]

$$h_{\rm L} = h_{\rm R} = \frac{1}{2}+i\mu. \qquad (50)$$

On the other hand, the absorption cross-section ratio of the dual scalar operator in the two-dimensional CFT has the universal form, namely

$$\sigma_{\rm abs} \sim T_{\rm L}^{2h_{\rm L}-1}T_{\rm R}^{2h_{\rm R}-1}\sinh\left(\frac{\tilde{\omega}_{\rm L}}{2T_{\rm L}}+\frac{\tilde{\omega}_{\rm R}}{2T_{\rm R}}\right)\left|\Gamma\left(h_{\rm L}+i\frac{\tilde{\omega}_{\rm L}}{2\pi T_{\rm L}}\right)\right|^2 \times\left|\Gamma\left(h_{\rm R}+i\frac{\tilde{\omega}_{\rm R}}{2\pi T_{\rm R}}\right)\right|^2, \qquad (51)$$

where $T_{\rm L}$ and $T_{\rm R}$ are the left- and right-hand temperatures of the dual CFT; and $\tilde{\omega}_{\rm L} = \omega_{\rm L} - q_{\rm L}\Phi_{\rm L}$ and $\tilde{\omega}_{\rm R} = \omega_{\rm R} - q_{\rm R}\Phi_{\rm R}$ are the total excited energy of the left- and right-hand sectors, in which $(q_{\rm L}, q_{\rm R})$ and $(\Phi_{\rm L}, \Phi_{\rm R})$ are, respectively, the charges and chemical potential of the dual left- and right-hand operators.

Before comparing Eq. (49) with Eq. (51), let us recall that for the near-extremal KN black hole, its entropy and temperature are

$$S_{\rm BH} = \pi(\hat{r}_+^2 + a^2) \Rightarrow S_{\rm BH} \sim \pi(r_0^2 + a^2 + 2r_0 B), \quad (52)$$

$$T_{\rm H} = \frac{\hat{r}_+ - \hat{r}_-}{4\pi(\hat{r}_+^2 + a^2)} \Rightarrow T_{\rm H} = \frac{B}{2\pi}. \quad (53)$$

Then we will discuss the holographic description of the pair production in the near-extremal KN black hole in the J and Q pictures, respectively.

A. J picture

The left- and right-hand central charges for the J picture are [16,18]

$$c_{\rm L}^J = c_{\rm R}^J = 12J, \quad (54)$$

and the associated temperatures are

$$T_{\rm L}^J = \frac{\hat{r}_+^2 + \hat{r}_-^2 + 2a^2}{4\pi a(\hat{r}_+ + \hat{r}_-)}, \qquad T_{\rm R}^J = \frac{\hat{r}_+ - \hat{r}_-}{4\pi a}, \quad (55)$$

which leads to the near-extreme results

$$T_{\rm L}^J = \frac{r_0^2 + a^2}{4\pi a r_0}, \qquad T_{\rm R}^J = \frac{B}{2\pi a}. \quad (56)$$

The CFT microscopic entropy is calculated from the Cardy formula via

$$S_{\rm CFT} = \frac{\pi^2}{3}(c_{\rm L}T_{\rm L} + c_{\rm R}T_{\rm R}) = \pi(r_0^2 + a^2 + 2r_0 B), \quad (57)$$

which agrees with area entropy of the near-extremal KN black hole.

Furthermore, we need to identify the first law of thermodynamics of the black hole with that of the dual CFT; i.e., $\delta S_{\rm BH} = \delta S_{\rm CFT}$

$$\frac{\delta M - \Omega_{\rm H}\delta J - \Phi_{\rm H}\delta Q}{T_{\rm H}} = \frac{\tilde{\omega}_{\rm L}}{T_{\rm L}} + \frac{\tilde{\omega}_{\rm R}}{T_{\rm R}}, \quad (58)$$

in which the angular velocity and chemical potential (at $r = B$) are

$$\Omega_{\rm H} = \frac{2ar_0}{r_0^2 + a^2}B, \qquad \Phi_{\rm H} = -\frac{Q^3 B}{r_0^2 + a^2}. \quad (59)$$

Note that in the J picture, $T_{\rm L} = T_{\rm L}^J$ and $T_{\rm R} = T_{\rm R}^J$. In addition, the charge of the probe scalar field is turned off in order to only "feel" the rotation of the KN black hole; thus, the variation of the conserved charges of the KN black hole are $\delta M = \omega$, $\delta J = -n$, $\delta Q = 0$. Subsequently, the excitation of the total energy for the dual J-picture CFT are determined to be

$$\tilde{\omega}_{\rm L}^J = n \quad \text{and} \quad \tilde{\omega}_{\rm R}^J = \frac{\omega}{a}, \quad (60)$$

and then we have $\frac{\tilde{\omega}_{\rm L}}{2T_{\rm L}} = -\pi\kappa$ and $\frac{\tilde{\omega}_{\rm R}}{2T_{\rm R}} = \pi\tilde{\kappa}$, in which q is set to zero. Consequently, in the J picture of the KN/CFT duality, the absorption cross-section ratio of the scalar field (with $q = 0$) in Eq. (49) matches well with that of its dual scalar operator in Eq. (51).

B. Q picture

The left- and right-hand central charges for the Q picture are [16,18]

$$c_{\rm L}^Q = c_{\rm R}^Q = \frac{6Q^3}{\ell}, \quad (61)$$

where the parameter ℓ is the measure of the $U(1)$ bundle formed by the background Maxwell field, which has a geometrical interpretation of the radius of the embedded extra circle in the fifth dimension, and the associated left- and right-hand temperatures are

$$T_{\rm L}^Q = \frac{(\hat{r}_+^2 + \hat{r}_-^2 + 2a^2)\ell}{4\pi Q(\hat{r}_+\hat{r}_- - a^2)}, \qquad T_{\rm R}^Q = \frac{(\hat{r}_+^2 - \hat{r}_-^2)\ell}{4\pi Q(\hat{r}_+\hat{r}_- - a^2)}, \quad (62)$$

which, in the near-extreme case, can be expressed as

$$T_{\rm L}^Q = \frac{(r_0^2 + a^2)\ell}{2\pi Q^3}, \qquad T_{\rm R}^Q = \frac{r_0 B\ell}{\pi Q^3}. \quad (63)$$

Besides, the CFT microscopic entropy computed from the Cardy formula

$$S_{\rm CFT} = \frac{\pi^2}{3}(c_{\rm L}T_{\rm L} + c_{\rm R}T_{\rm R}) = \pi(r_0^2 + a^2 + 2r_0 B) \quad (64)$$

reproduces the macroscopic entropy of the near-extremal KN black hole, too.

In the Q picture, the modes characterizing the rotation of the charged probe scalar field—namely n—should be turned off in order that the charged probe will only interact with the charge of the black hole; i.e., $\delta M = \omega$, $\delta J = 0$, $\delta Q = -q$. Again, we use the identification between the thermodynamics of the black hole and its dual CFT—namely, Eq. (58), with $T_{\rm L} = T_{\rm L}^Q$ and $T_{\rm R} = T_{\rm R}^Q$. Then the excitation of the total energy of the dual Q picture CFT are obtained as

$$\tilde{\omega}_{\mathrm{L}}^{Q} = -q\ell \quad \text{and} \quad \tilde{\omega}_{\mathrm{R}}^{Q} = \frac{2r_0\ell\omega}{Q^3}; \tag{65}$$

namely, $\frac{\tilde{\omega}_{\mathrm{L}}}{2T_{\mathrm{L}}} = -\pi\kappa$ and $\frac{\tilde{\omega}_{\mathrm{R}}}{2T_{\mathrm{R}}} = \pi\tilde{\kappa}$. Therefore, in the Q picture of the KN/CFT duality, the absorption cross-section ratio of the charged scalar field (with $n = 0$) in Eq. (49) also matches well with that of its dual scalar operator in Eq. (51). Moreover, since the mean number of produced pairs $|\beta|^2$ of the charged probe scalar field is associated with its absorption cross-section ratio via $|\beta|^2(\mu \to -\mu) \to -\sigma_{\mathrm{abs}}$, the holographic description of $|\beta|^2$ can be understood in both the J and Q pictures.

VII. CONCLUSIONS

In this paper, we have extended the previous study on the spontaneous production of charged pairs in the near-extremal RN black hole to the case of the near-extremal KN black hole. The near-horizon warped AdS_3 geometry of the near-extremal KN black hole allows us to obtain the analytic form of the vacuum persistence amplitude, the mean number of produced pairs, and the absorption cross-section ratio of charged scalars. We have found a universal feature of the Schwinger mechanism in extremal charged black holes, nonrotating or rotating. The violation of the BF bound of the charged scalar in the AdS_2 space is a necessary condition for pair production. But its connection to the cosmic censorship conjecture is not so obvious as in the RN black holes.

The pair-production rate has a thermal interpretation, with the effective temperature consisting of the Unruh temperature for accelerating charge and the AdS curvature. We have employed the Hamilton-Jacobi action and the phase-integral formula to compute the decay rate of the field due to spontaneous pair production. It is shown that the Boltzmann factor for the Schwinger effect in the near-extremal KN black hole comes from two simple poles located at the inner and outer horizons. There is no *a priori* reason to prevent the quantum tunneling process from the inner horizon, since it is located just adjacent to the outer horizon, contrary to a nonextremal KN black hole.

Since the near-extremal KN black hole has the twofold dual CFT descriptions associated with the J picture and the Q picture, we have thus analyzed the holographic dual CFT descriptions of the pair production by comparing the absorption cross-section ratio of the bulk charged scalar field and that of its dual scalar operator in the dual CFT in both of these two pictures and found they agree with each other. Furthermore, in addition to presenting a clear description of the spontaneous pair production of scalars, our results also give more information about the CFT dual of the near-extremal KN black hole. Note that although we only focused on the near-horizon region of the near-extremal KN black hole, similar analysis can be used to study the Schwinger effect in nonextremal black holes (away from the near-horizon region) in the low-frequency limit. It also would be interesting to take into account the backreaction to the background geometry to give a more precise picture of the pair-production process.

ACKNOWLEDGMENTS

The authors would like to thank Pauchy Hwang for the warm hospitality at National Taiwan University, where part of this work was done, and also Rong-Gen Cai for useful discussions at the 2016 Annual Meeting of the Chinese Society of Gravitation and Relativistic Astrophysics at Changsha. S. P. K. also would like to thank Robert Mann and Don N. Page for useful discussions during the BIRS-CMO Workshop on Black Holes' New Horizon and Fiorenzo Bastianelli for the warm hospitality and useful discussions at Bologna University. The work of C. M. C. was supported by the Ministry of Science and Technology of Taiwan under Grant No. MOST 102-2112-M-008-015-MY3. The work of S. P. K. was supported by the Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (Grants No. NRF-2015R1D1A1A01060626). J. R. S. was supported by the National Natural Science Foundation of China (Grant No. 11205058), the Fundamental Research Funds for the Central Universities, and in part by the Open Project Program of the State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, China (Grant No. Y5KF161CJ1).

APPENDIX: HYPERGEOMETRIC FUNCTIONS

There are a number of mathematical properties for the hypergeometric function $F(\alpha,\beta;\gamma;z)$. In particular, the following ones are useful for our analysis:

(i) Transformation formula:

$$\begin{aligned} F(\alpha,\beta;\gamma;z) = {} & \frac{\Gamma(\gamma)\Gamma(\beta-\alpha)}{\Gamma(\beta)\Gamma(\gamma-\alpha)}(1-z)^{-\alpha}F\left(\alpha,\gamma-\beta;\alpha-\beta+1;\frac{1}{1-z}\right) \\ & + \frac{\Gamma(\gamma)\Gamma(\alpha-\beta)}{\Gamma(\alpha)\Gamma(\gamma-\beta)}(1-z)^{-\beta}F\left(\beta,\gamma-\alpha;\beta-\alpha+1;\frac{1}{1-z}\right). \end{aligned} \tag{A1}$$

(ii) Special values:

$$F(\alpha,\beta;\gamma;0)=1,\qquad F(\alpha,\beta;\gamma;1)=\frac{\Gamma(\gamma)\Gamma(\gamma-\alpha-\beta)}{\Gamma(\gamma-\alpha)\Gamma(\gamma-\beta)}. \tag{A2}$$

In addition, the following properties of the gamma function are also needed in our computation:

$$\Gamma(\alpha+1)=\alpha\Gamma(\alpha),\qquad \Gamma(\alpha)\Gamma(1-\alpha)=\frac{\pi}{\sin(\alpha\pi)}, \tag{A3}$$

$$\left|\Gamma\left(\frac{1}{2}+iy\right)\right|^2=\frac{\pi}{\cosh(\pi y)},\qquad |\Gamma(1+iy)|^2=\frac{\pi y}{\sinh(\pi y)},\qquad |\Gamma(iy)|^2=\frac{\pi}{y\sinh(\pi y)}. \tag{A4}$$

[1] J. S. Schwinger, On gauge invariance and vacuum polarization, Phys. Rev. **82**, 664 (1951).
[2] M. K. Parikh and F. Wilczek, Hawking Radiation as Tunneling, Phys. Rev. Lett. **85**, 5042 (2000).
[3] C.-M. Chen, S. P. Kim, I.-C. Lin, J.-R. Sun, and M.-F. Wu, Spontaneous pair production in Reissner-Nordstrom black holes, Phys. Rev. D **85**, 124041 (2012).
[4] C.-M. Chen, J.-R. Sun, F.-Y. Tang, and P.-Y. Tsai, Spinor particle creation in near extremal Reissner-Nordstrom black holes, Classical Quantum Gravity **32**, 195003 (2015).
[5] C.-M. Chen, J.-R. Sun, and F.-Y. Tang, Pair production in near extremal charged black holes, arXiv:1510.00141.
[6] C.-M. Chen, J.-R. Sun, and S.-J. Zou, The RN/CFT correspondence revisited, J. High Energy Phys. 01 (2010) 057.
[7] C.-M. Chen, Y.-M. Huang, and S.-J. Zou, Holographic duals of near-extremal Reissner-Nordstrom black holes, J. High Energy Phys. 03 (2010) 123.
[8] C.-M. Chen and J.-R. Sun, Hidden conformal symmetry of the Reissner-Nordstrøm black holes, J. High Energy Phys. 08 (2010) 034.
[9] C.-M. Chen and J.-R. Sun, Holographic dual of the Reissner-Nordström black hole, J. Phys. Conf. Ser. **330**, 012009 (2011).
[10] R. G. Cai and S. P. Kim, One-loop effective action and Schwinger effect in (anti-)de Sitter space, J. High Energy Phys. 09 (2014) 072.
[11] S. Deser and O. Levin, Accelerated detectors and temperature in (anti-)de Sitter spaces, Classical Quantum Gravity **14**, L163 (1997).
[12] S. P. Kim, H. K. Lee, and Y. Yoon, Thermal interpretation of Schwinger effect in near-extremal RN black hole, arXiv:1503.00218.
[13] T. Hartman, K. Murata, T. Nishioka, and A. Strominger, CFT duals for extreme black holes, J. High Energy Phys. 04 (2009) 019.
[14] T. Hartman, W. Song, and A. Strominger, Holographic derivation of Kerr-Newman scattering amplitudes for general charge and spin, J. High Energy Phys. 03 (2010) 118.
[15] J. Rasmussen, On the CFT duals for near-extremal black holes, Mod. Phys. Lett. A **26**, 1601 (2011).
[16] C.-M. Chen, Y.-M. Huang, J.-R. Sun, M.-F. Wu, and S.-J. Zou, Twofold hidden conformal symmetries of the Kerr-Newman black hole, Phys. Rev. D **82**, 066004 (2010).
[17] B. Chen, C.-M. Chen, and B. Ning, Holographic Q-picture of Kerr-Newman-AdS-dS black hole, Nucl. Phys. **B853**, 196 (2011).
[18] C.-M. Chen and J.-R. Sun, The Kerr-Newman/CFTs correspondence, Int. J. Mod. Phys. Conf. Ser. **07**, 227 (2012).
[19] M. Guica, T. Hartman, W. Song, and A. Strominger, The Kerr/CFT correspondence, Phys. Rev. D **80**, 124008 (2009).
[20] I. Bredberg, T. Hartman, W. Song, and A. Strominger, Black hole superradiance from Kerr/CFT, J. High Energy Phys. 04 (2010) 019.
[21] G. Compere, The Kerr/CFT correspondence and its extensions: A comprehensive review, Living Rev. Relativ. **15**, 11 (2012).
[22] J. M. Bardeen and G. T. Horowitz, Extreme Kerr throat geometry: A vacuum analog of $AdS_2\times S^2$, Phys. Rev. D **60**, 104030 (1999).
[23] S. P. Kim, Schwinger effect, Hawking radiation and gauge-gravity relation, Int. J. Mod. Phys. A **30**, 1545017 (2015).
[24] C. Gabriel and P. Spindel, Quantum charged fields in $(1+1)$ Rindler space, Ann. Phys. (N.Y.) **284**, 263 (2000).
[25] S. P. Kim and D. N. Page, Improved approximations for fermion pair production in inhomogeneous electric fields, Phys. Rev. D **75**, 045013 (2007).
[26] S. P. Kim, Geometric origin of Stokes phenomenon for de Sitter radiation, Phys. Rev. D **88**, 044027 (2013).