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Quantum radiation from a sandwich black hole

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We discuss quantum radiation of a massless scalar field from a spherically symmetric nonsingular black hole with a finite lifetime. Namely, we discuss a sandwich black-hole model, where a black hole is originally created by a collapse of a null shell of mass M, and later, after some time ΔV, it is disrupted by the collapse of the other shell with negative mass $-M$. We assume that between the shells the metric is static and either coincides with the Hayward metric or with a special generalization of it. We show that in both cases for a sufficiently large parameter ΔV the radiation after the formation of the black hole practically coincides with the Hawking result. We also calculate the radiation, emitted from the black hole interior. This radiation contains a peak at the moment when the second shell intersects the inner horizon. In the standard sandwich metric (with the Hayward interior) this outburst of energy is exponentially large. In the modified metric, which includes an additional nontrivial redshift parameter, this exponent is suppressed. This is a result of a significant decrease of the surface gravity of the inner horizon in the latter case. We discuss possible consequences of this result in the context of the self-consistency requirement for nonsingular models with quantum radiation.

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I. INTRODUCTION

General relativity is a remarkable theory which allows one to understand spacetime and matter properties in a strong gravitational field. It predicts black holes and is important for a description of the early Universe. The general relativity predictions in the weak-field approximation are confirmed by observations. The recent discovery by LIGO of the coalescence of two black holes indicated that general relativity is valid in the strong-field regime, when its nonlinear effects are important. At the same time Einstein's general relativity theory is ultraviolet (UV) incomplete. A well known problem of the theory is the existence of singularities. Solutions of the Einstein equations, describing stationary black holes, have curvature singularities in their interior. It is generally believed that the theory should be modified in the domain where the curvature becomes high. Several different approaches to deal with this problem have been proposed. Theories of quantum gravity, such as string theory and loop gravity, are well-known examples. Recently a new, very promising UV-complete modification of general relativity was proposed [1–10]. It is called ghost-free gravity. Such a theory contains an infinite number of derivatives and, in fact, is nonlocal. A similar theory also appears naturally in the context of the noncommutative geometry deformation of Einstein gravity [11,12] (see the review [13] and references therein). The application of the ghost-free theory of gravity to the problem of singularities in cosmology and black holes can be found in Refs. [14–27]. In spite of a number of promising results, we still do not have a final conclusive solution of this problem in ghost-free gravity. This is due to the complexity of its equations, which are both nonlocal and nonlinear.

There exists another approach, which became quite popular recently. Roughly speaking it can be formulated as follows. Suppose there exists such a fundamental theory of gravity, in which black-hole solutions are regular and the singularities are absent. What might be the properties of black holes in such a theory? It is natural to expect, that the modified theory of gravity should include some fundamental length scale ℓ (or a related mass-scale parameter $\mu = \ell^{-1}$), and Einstein gravity gives an accurate description of the spacetime geometry in the domain where the curvature is less than ℓ^{-2}. A large number of nonsingular models of black holes have been proposed (see e.g. the discussion and references in Ref. [28]).

A natural requirement of nonsingular black-hole metrics is that at large radius they correctly reproduce Schwarzschild, Kerr or other black-hole solutions of general relativity. This means that the corresponding nonsingular black-hole metric possesses one or more arbitrary parameters (such as mass, angular momentum and charge). The curvature inside the black hole, being regular, nevertheless depends on the value of these parameters. In a general case it may infinitely grow for a special limit of the parameters. The requirement, that it does not happen and the curvature always remains finite and limited by some fundamental value ($\sim\ell^{-2}$), can be imposed as an additional principle, which restricts the variety of nonsingular black-hole models. The limiting curvature principle was first

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formulated by Markov [29,30] (see also Ref. [31]). For a spherically symmetric black hole this principle, in particular, implies that the apparent horizon cannot cross the center $r = 0$. In other words, besides the outer part of the apparent horizon there should also be an inner part, separated from the center. When a black hole evaporates the event horizon does not exist and the apparent horizon is closed. Such a model was first proposed in Ref. [32], and later was intensively discussed in the literature [16,33–43].

A general feature of nonsingular models of a black hole with a closed apparent horizon is the following. All the quanta which either fell into the black hole or were created inside it are finally emitted to an external observer after the complete evaporation of the black hole. Outgoing radial null rays in the black-hole interior are accumulated in the vicinity of the inner horizon. This is a consequence of the negative value of the surface gravity at the inner horizon. As a result one can expect that particles emitted from the inner horizon at the final stage of the black-hole evaporation would have large blueshift. In a self-consistent model, when the backreaction of the created particles is properly taken into account, this final burst of radiation should be somehow suppressed. In the absence of the theory of gravity which properly describes the black-hole interior and the final state of the evaporating black hole, one can estimate the quantum radiation in the adopted nonsingular black-hole model. In such a case a consistency requirement can be used as an additional test of the plausibility of the model [44].

The present paper discusses this problem. Namely, we study quantum radiation of massless particles from nonsingular black holes. To attack this problem we assume a number of simplifications. To describe a spherically symmetric black hole which has a finite lifetime we consider the following model. We assume that a black hole is formed as a result of the collapse of the null shell of positive mass M and ends its existence as a result of the collapse of another null shell with negative mass $-M$. We call it a *sandwich black hole*. Such a model was also considered earlier in the interesting paper [45], where the problem of the black hole entropy was discussed. Certainly such a model is quite different from a "real" evaporating black hole. However, it happens that because of its simplicity it might be useful for a study of quantum effects in black holes with a closed apparent horizon. We shall argue that some of its predictions might be quite robust and remain valid for more realistic "smooth models." To estimate the quantum radiation of the massless particles we use the result of Christensen and Fulling [46], who derived the two-dimensional quantum average of the stress-energy tensor from the conformal anomaly. It is well known that this approximation gives a rather good result for the Hawking radiation (see e.g. Ref. [47]).

The paper is organized as follows. In Sec. II we discuss generic properties of nonsingular black holes and collect some useful formulas for the quantum energy flux at infinity in an asymptotically flat spacetime. We also discuss a classical scattering problem for a massless particle propagating along radial geodesics from the past to future null infinity. We derive a useful expression for a gain function, which is the ratio of its final and initial energies. In Sec. III we describe a sandwich model of a nonsingular black hole. This section also contains general expressions for the gain function and quantum energy flux for a sandwich nonsingular black hole. In Sec. IV we discuss a special case, when the metric between the null shells coincides with the Hayward metric [35]. Such a "standard" model is characterized by two parameters: its mass and the time duration of its existence. We present both analytic and numerical results and discuss the behavior of the quantum radiation during the time of the black hole's formation, its existence, and its disruption. In Sec. V we discuss a sandwich model where the regular metric between the shells contains a nontrivial redshift factor. We calculate the quantum energy flux from such a modified sandwich black hole and demonstrate that the exponentially large peak of the radiation from the inner horizon, which is present in the "standard" model, can be suppressed by a proper choice of the redshift function. This suppression effect is a consequence of the reduction of the (negative) surface gravity of the inner horizon. A discussion of the obtained results and additional remarks are presented in Sec. VI.

II. NONSINGULAR MODELS OF EVAPORATING BLACK HOLES

A. Spherically symmetric regular black holes

The most general spherically symmetric metric in the four-dimensional spacetime can be written in the form

$$dS^2 = -A^2 F dV^2 + 2A dV dR + R^2 d\omega^2. \tag{2.1}$$

It contains two arbitrary functions of the advanced time V and radius R. These coordinates have the dimensionality of [length], and the dimensionality of dS^2 is [length^2]. In what follows it is convenient to deal with dimensionless objects. For this purpose one can use any length parameter as a standard scale. For example, in the metrics that we shall consider later it might be some fundamental scale ℓ, or the gravitational radius of the black hole. Sometimes, it is convenient to use their combination, or other scales. We denote this (unspecified at the moment) standard length scale by σ and denote

$$dS^2 = \sigma^2 ds^2, \qquad V = \sigma v, \qquad R = \sigma r,$$
$$F(V,R) = f(v,r), \qquad A(V,R) = \alpha(v,r). \tag{2.2}$$

Thus one has

$$ds^2 = -\alpha^2 f dv^2 + 2\alpha dv dr + r^2 d\omega^2. \tag{2.3}$$

It is easy to check that

$$f = g^{rr} = g^{\mu\nu}\nabla_\mu r \nabla_\nu r. \tag{2.4}$$

Points where $f = 0$ form an apparent horizon. We assume that the spacetime is asymptotically flat. In this case the function f at spatial infinity must take the value 1 in order to escape a solid angle deficit

$$f(v, r)|_{r\to\infty} = 1. \tag{2.5}$$

Using an ambiguity in the choice of v, we impose the following (gauge-fixing) condition:

$$\alpha(v, r)|_{r\to\infty} = 1. \tag{2.6}$$

In what follows we consider, so-called, nonsingular black holes (see, e.g., Ref. [28] and references therein). One of the conditions, which is valid for such metrics, is the requirement of the regularity of the metric at the center $r = 0$. Let R be the Ricci scalar, $S_{\mu\nu} = R_{\mu\nu} - \frac{1}{4} g_{\mu\nu} R$, and $C_{\mu\nu\alpha\beta}$ be the Weyl tensor. Let us define the following invariants that are quadratic in the curvature:

$$\mathcal{S}^2 = S_{\mu\nu}S^{\mu\nu}, \qquad \mathcal{C}^2 = C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta}. \tag{2.7}$$

We call a metric (2.1) finite at the center, if the metric functions f and α have the following expansions:

$$f(v, r) = f_0(v) + f_1(v)r + f_2(v)r^2 + \ldots, \tag{2.8}$$

$$\alpha(v, r) = \alpha_0(v) + \alpha_1(v)r + \alpha_2(v)r^2 + \ldots. \tag{2.9}$$

We call a finite-at-the-center metric (2.1) *regular* if the invariants R, $\mathcal{S}^2$ and $\mathcal{C}^2$ are finite at $r = 0$. This regularity condition implies that

$$f_0(v) = 1, \qquad f_1(v) = \alpha_1(v) = 0. \tag{2.10}$$

An important consequence is that for a nonsingular black hole the apparent horizon cannot cross the center $r = 0$. In a general case, when $\alpha(v) \neq 1$, the rate of the proper time τ at the center differs from the rate of time v

$$d\tau = \alpha_0(v) dv. \tag{2.11}$$

The conditions (2.10) imply that the geometry near $r = 0$ is locally flat, and, in particular, there is no solid-angle deficit.

B. Static spacetime

Let us make a few remarks, concerning a special case of the metric (2.1), when both metric functions, f and α, are time independent. Denote by $\xi = \xi^\alpha \partial_\alpha = \partial_v$ the corresponding Killing vector. Then one has

$$\xi^2 = -\alpha^2 f. \tag{2.12}$$

One can check that for such a metric the Killing horizon coincides with the apparent horizon, and α is regular at the horizon function. The surface gravity κ is defined as follows:

$$\xi^\beta \xi_{\alpha;\beta} \stackrel{\mathrm{H}}{=} \kappa \xi_\alpha. \tag{2.13}$$

Simple calculations give

$$\kappa = \frac{1}{2}(\alpha \partial_r f)|_H. \tag{2.14}$$

C. Regular evaporating black hole models

We assume now that a regular metric (2.3) describes a black hole, which was created as a result of spherical collapse, and it disappears after some finite time, for example, as a result of its quantum evaporation. For such a system there exist parameters v^- and v^+, such that

$$f = \alpha = 1, \quad \text{for } v < v^- \quad \text{and} \quad v > v^+. \tag{2.15}$$

For definiteness, we can choose v^- and v^+ so, that in the domain $v^- < v < v^+$ the spacetime curvature does not vanish. The condition (2.6) fixes the coordinate v up to a constant. We use this freedom to put $v^- = 0$. We also denote $q = v^+$.

Consider an incoming radial null ray described by the equation $v = \text{const}$. It propagates from the past null infinity, $\mathcal{I}^-$, and reaches the center $r = 0$. After passing the center, it becomes an outgoing radial null ray. We shall use diagrams where the angle variables are suppressed, so that the radial null ray will be presented by a line which is reflected at the origin $r = 0$. We choose the retarded null time parameter u_- so that at $r = 0$ one has $u_- = v$. In the initial flat domain, where $v < 0$, one has $u_- = v - 2r$. However, in a general case, for $v > 0$ this relation between u_- and v is not valid. In particular, in the final flat domain, where $v > q$, the null coordinate $u_+ = v - 2r$ differs from u_-, and one has the relations

$$u_+ = u_+(u_-), \qquad u_- = u_-(u_+). \tag{2.16}$$

One can rewrite the first relation in the form $u_+ = u_+(v)$. This relation can be interpreted as establishing a map between $\mathcal{I}^-$, parametrized by v, and $\mathcal{I}^+$, parametrized by u_+.

Let us describe a simple algorithm which allows one to find the required map.

Case I. Consider first outgoing rays with $u_- < 0$. They cross both of the null surfaces $v = 0$ and $v = q$. The radius of the first intersection is

$$r_- = -\frac{1}{2}u_-. \tag{2.17}$$

Denote by $r = r(v)$ a solution of the differential equation

$$\frac{dr}{dv} = \frac{1}{2}\mathcal{F}(v,r), \qquad \mathcal{F} = \alpha f, \tag{2.18}$$

with the initial condition

$$r(0) = -u_-/2. \tag{2.19}$$

This solution describes an outgoing null ray passing through $(0, r_-)$. Denote by r_+ the radius r where this ray crosses the second null shell, $r_+ = r(q)$. Since in the final domain, where $v > q$, the spacetime is flat, one has

$$u_+ = q - 2r(q). \tag{2.20}$$

Equations (2.17)–(2.20) determine the required map.

Case II. Let $0 < u_- < q$ and let $r(v)$ be a solution of Eq. (2.18) with the initial condition $r(u_-) = 0$. Then the relation (2.20) determines the map.

Case III. Let $u_- > q$. Then one has $u_+ = u_-$.

D. Quantum fluxes at $\mathcal{I}^+$

In what follows, we study quantum radiation of a massless scalar field from regular "black holes." For the calculation of the Hawking radiation one often uses a decomposition of the quantum modes in spherical harmonics. In such an analysis it was shown that the main contribution to the radiation is given by S-modes [47]. If one reduces to considering only S-modes, the theory is effectively reduced to the quantum theory of the two-dimensional (2D) massless scalar field in the (t, r) sector of the black hole geometry. We shall use a similar 2D approximation for the estimation of the quantum energy fluxes from a regular black hole [48].

An effective action for a two-dimensional conformal scalar field is

$$S = -\frac{1}{2}\int d^2x\sqrt{-g}(\nabla\hat{\varphi})^2, \tag{2.21}$$

where the two-dimensional metric is given by

$$ds^2 = -\alpha^2 f dv^2 + 2\alpha dv dr. \tag{2.22}$$

Let us notice that the rate of the energy emission, $\dot{E}$, is a dimensionless quantity. For the quantum radiation it is proportional to the Planck length squared, $\ell_{\rm Pl}^2$. Thus one has

$$\dot{E} = (\ell_{\rm Pl}/\sigma)^2\mathcal{E}, \tag{2.23}$$

where $\mathcal{E}$ is the dimensionless rate of the energy emission. The energy rate flux of massless particles, created from the initial vacuum state, is given by the following expression:

$$\mathcal{E} = \frac{1}{48\pi}\left[-2\frac{d^2P}{du^2} + \left(\frac{dP}{du}\right)^2\right], \tag{2.24}$$

where

$$P = \ln\left|\frac{du_-}{du_+}\right|. \tag{2.25}$$

This relation directly follows from a general result obtained by Fulling and Christensen [46] for the quantum average of the stress-energy tensor of a massless scalar field in two dimensions, reconstructed from the conformal anomaly. The same expression can also be obtained by the variation of the Polyakov effective action with respect to the metric [49,50].

The expression (2.24) for the energy flux contains two terms. The second one is a square of the first derivative of P and hence it is always positive. The first one, proportional to the second derivative of P, can be either positive or negative. Hence, for some periods of the retarded time u_+ the flux of the energy from the black hole can be negative. However, the total emitted energy is always positive. This can be easily checked. Really

$$E_{\rm tot} = \frac{1}{48\pi}\left[\int_{-\infty}^{\infty} du_+\left(\frac{dP}{du_+}\right)^2 - 2\frac{dP}{du_+}\Big|_{-\infty}^{\infty}\right]. \tag{2.26}$$

For a nonsingular black hole, which exits during a finite interval of time, the boundary terms vanish.

Quantum effects in nonsingular black holes were discussed earlier [44,51]. The main goal of this paper is to study the quantum energy flux from a sandwich black hole. In order to calculate this flux in the adopted 2D approximation it is sufficient to study the propagation of the radial null geodesics from $\mathcal{I}^-$ to $\mathcal{I}^+$. The function $u_-(u_+)$, which establishes a map $\mathcal{I}^+ \to \mathcal{I}^-$, allows one to find the function P, which enters the expression (2.24) for the energy flux on $\mathcal{I}^+$. In the next section we demonstrate that the function P is a logarithm of the ratio of the energy of a classical massless particle at $\mathcal{I}^+$ to its initial energy at $\mathcal{I}^-$. It is interesting that P is also related to the so-called *radiation entropy* $\Delta S_{\rm rad}(u_+)$ at retarded time u_+, defined in Ref. [45]. Namely, one has

$$\Delta S_{\rm rad}(u_+) = -\frac{1}{12}P. \tag{2.27}$$

E. Gain function

To better understand the main features of quantum particle production by a black hole it is instructive to consider the propagation of classical massless particles (photons) in its geometry. The motion of radial incoming null rays in the metric (2.3) is rather simple. For these rays one has $v = \text{const}$, and the corresponding four-momentum is $l^\mu = (0, \dot{r}, 0, 0)$. The dot means a derivative with respect to the affine parameter λ [52]. The geodesic equation for the null ray is

$$\frac{D^2 x^\mu}{d\lambda^2} \equiv \frac{d^2 x^\mu}{d\lambda^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{d\lambda} \frac{dx^\beta}{d\lambda} = 0. \tag{2.28}$$

For the radial rays this equation is identically satisfied in the (θ, ϕ) sector, while the only nonvanishing component of the Christoffel symbol $\Gamma^r_{\mu\nu}$ in the (v, r) sector is

$$\Gamma^r_{rr} = \frac{\partial_r \alpha}{\alpha}. \tag{2.29}$$

The equation (2.28) is identically satisfied for $\mu = v$, and for $\mu = r$ it gives

$$(\alpha \dot{r})^{\dot{}} = 0. \tag{2.30}$$

This means that the quantity $\alpha \dot{r}$ is constant along the radial incoming null ray. Consider a photon, which starts its motion at $\mathcal{I}^-$, where $\alpha = 1$ and its energy is $E_- = -\dot{r}$. When such a ray arrives to the center $r = 0$, where $f = 1$, one has

$$\dot{r}_0 = -E_- / \alpha_0. \tag{2.31}$$

Here α_0 is the value of the redshift function α at the center. The geometry near the center is regular. Let us introduce Cartesian coordinates (X, Y, Z) in its vicinity, and choose their orientation so that the infalling photon passes the center $X = Y = Z = 0$ moving in the negative direction along the X axis. Before it crosses the center one has $r = X$. After this, the photon continues its motion along the X direction; however, for the outgoing photon one now has $r = -X$. This means that

$$\dot{r}_0 = E_- / \alpha_0. \tag{2.32}$$

For the outgoing null ray one has

$$\dot{r} = \frac{1}{2} \alpha f \dot{v} \tag{2.33}$$

and $f = 1$ at the center. Thus

$$(\alpha \dot{v})_0 = 2 \frac{E_-}{\alpha_0}. \tag{2.34}$$

The outgoing null rays obey the equation

$$\frac{dr}{dv} = \frac{1}{2} \alpha f. \tag{2.35}$$

In a general case the metric functions α and f depend on both r and v. We denote the four-momentum of the radial outgoing null ray by k^μ, and denote by λ its affine parameter. Thus one has $k^\mu = (\dot{v}, \dot{r}, 0, 0)$, where a dot denotes a derivative with respect to λ. The only nonvanishing components of $\Gamma^v_{\mu\nu}$ in the (v, r) sector are

$$\Gamma^v_{vv} = \frac{1}{2\alpha} [\alpha^2 \partial_r f + 2\alpha f \partial_r \alpha + 2 \partial_v \alpha]. \tag{2.36}$$

Using Eq. (2.35) and the relation

$$\dot{\alpha} = \dot{v} \partial_v \alpha + \dot{r} \partial_r \alpha, \tag{2.37}$$

one can write the v component of the equation (2.28) in the form

$$(\alpha \dot{v})^{\dot{}} = -\frac{1}{2} \alpha (\alpha f)' \dot{v}^2. \tag{2.38}$$

The first integral of this equation is

$$\alpha \dot{v} = (\alpha \dot{v})_{v_0} \exp\left[-\frac{1}{2} \int_{v_0}^{v} \partial_r (\alpha f) dv\right]. \tag{2.39}$$

Suppose there exists a spacetime domain where the metric is static, so that $\partial_v \alpha = \partial_v f = 0$. We denote by $\xi = \xi^\mu \partial_\mu = \partial_v$ the corresponding Killing vector. Then the energy of a photon, which moves in this domain

$$E = -g_{\mu\nu} \xi^\mu k^\nu = \alpha^2 f \dot{v} - \alpha \dot{r} = \frac{1}{2} \alpha^2 f \dot{v}, \tag{2.40}$$

is conserved.

Consider the case of an evaporating black hole. If the spacetime after black-hole evaporation is flat one can use the relation (2.39) as follows. For large v the spacetime is flat, so that $f = \alpha = 1$ in this domain, and the energy E_+ of the outgoing ray is

$$E_+ = \frac{1}{2} \dot{v}_{\mathcal{I}^+}. \tag{2.41}$$

Denote by v_0 the advanced time v when the incoming null ray crosses the center; then using Eqs. (2.39) and (2.34) one gets

$$\beta = \frac{E_+}{E_-} = \frac{1}{\alpha_0} \exp\left[-\frac{1}{2} \int_{v_0}^{\infty} \partial_r (\alpha f) dv\right]. \tag{2.42}$$

We call the ratio β of the final energy of a photon to its initial energy a *gain function*.

Let us show now that the gain function is related to the function P defined by Eq. (2.25). Namely one has

$$P = \ln \beta. \tag{2.43}$$

Consider two nearby incoming radial null rays with parameters v_0 and $v_0 + \delta v$. They pass the center $r = 0$ with the proper time interval difference $\delta\tau = \alpha(v)\delta v$. When the second incoming ray reaches the center, the first one has already passed it. It crosses the world line of the second ray at a point $(v + \delta v, \ \delta r_0 = \delta\tau)$, where $\delta r|_0 = \delta\tau/2$. Using Eq. (2.35) one can find how this separation δr between two nearby outgoing rays, calculated for $v = \mathrm{const}$, changes along the outgoing ray. Namely, one has

$$\frac{d\delta r}{dv} = \frac{1}{2}\partial_r(\alpha f)\delta r. \tag{2.44}$$

By integrating this equation one gets

$$\delta r|_{\mathcal{I}^+} = \delta r_0 \exp\left[\frac{1}{2}\int_{v_0}^{\infty}\partial_r(\alpha f)dv\right]. \tag{2.45}$$

Since for a fixed value v $\delta u_+ = 2\delta r|_{\mathcal{I}^+}$ and $\delta u_- = \delta v_0$, using the above results one obtains

$$\frac{du_-}{du_+} = \frac{\delta u_-}{\delta u_+} = \beta. \tag{2.46}$$

Thus the relation (2.43) is proved.

The obtained result has a quite simple explanation. Consider a high-frequency wave packet $\sim\exp[i\Phi(u_-)]$ emitted from $\mathcal{I}^-$. Its frequency, as measured by an observer near $\mathcal{I}^-$ is $\omega_- = d\Phi/du_-$. In the adiabatic 2D approximation such a packet, when it arrives at $\mathcal{I}^+$ has the form $\sim\exp(i\Phi(u_-(u_+))$, and its frequency is

$$\omega_+ = d\Phi/du_+ = \frac{d\Phi}{du_-}\frac{du_-}{du_+} = \beta\omega_-. \tag{2.47}$$

This relation is in accordance with the definition of the gain function β.

III. QUANTUM RADIATION FROM A SANDWICH BLACK HOLE

A. Double-shell model of a nonsingular black hole

A black hole after its formation becomes a source of quantum radiation. An external observer registers an outgoing flux of Hawking radiation. As a result, the black hole mass decreases and the black hole shrinks. One of the options is that the black hole completely disappears in this process. For a nonsingular spherical black hole this means that the apparent horizon is closed, and the event horizon does not exist. Strictly speaking, this object is not a black hole (according to the standard definition), but its long-time "imitation." For simplicity, we shall use the term "black hole" for these objects as well.

An important new feature of such nonsingular black holes is that besides the standard Hawking flux there exists

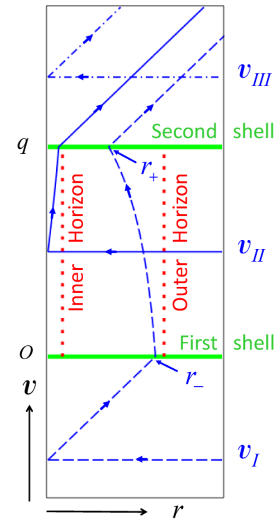

FIG. 1. Spacetime of a sandwich black hole in (v, r) coordinates. The first null shell with the positive mass is shown by a horizontal line $v = 0$, while the second one, with the negative mass, is shown by the horizontal line $v = q$. The spacetime before the first shell and after the second one is flat. We denote these domains as I and III, respectively. The domain II is located between the shells. Inner and outer horizons in the domain II are the inner and outer branches of the apparent horizon, respectively. Incoming and outgoing radial null rays are schematically shown in this diagram.

an additional quantum radiation, coming from the black hole interior. One should expect that, if the black hole completely disappears as a result of evaporation, then the total energy loss by the evaporating black hole must be equal to the initial mass of this object. As we shall demonstrate this condition imposes a severe self-consistency restriction on the nonsingular models of black holes. To illustrate this we consider a simple model. Namely, we assume that a black hole is formed as a result of the spherical collapse of a null shell of mass M, it exists for a time ΔV, and then disappears, as a result of the collapse of another null shell of mass $-M$ (see Fig. 1). During the time interval ΔV the metric is a static nonsingular one. The corresponding metric (in the dimensionless form) is

$$ds^2 = \alpha(-\mathcal{F}dv^2 + 2dvdr) + r^2 d\omega^2, \tag{3.1}$$

where $\mathcal{F} = \alpha f$. α and $\mathcal{F}$ are functions of r for $0 < v < q$, while outside this interval $\alpha = \mathcal{F} = 1$. We call such a model a *sandwich black hole*. Certainly, this model is quite different from the expected behavior of the evaporating black hole. However, they have the following common feature: a finite time of the "black hole's" existence [53].

B. Gain function for a sandwich black hole

For the sandwich black hole with a static interior the formula for the energy gain is simplified. Consider an incoming radial photon with the initial energy E_i. After crossing the first shell at the radius r_- it passes through the spacetime between the shells and leaves the sandwich black hole with the energy E_f, crossing the second shell at r_+. One has

$$\beta = \frac{\mathcal{F}_-}{\mathcal{F}_+} = \frac{dr_-}{dr_+}. \tag{3.2}$$

Here $\mathcal{F}_\pm$ is the quantity $\mathcal{F}$ calculated at the points of the entrance, r_-, and of the exit, r_+, of the photon in the domain between the shells, respectively. In the last equality we used the relation (2.42). For a photon, propagating along a horizon $r_+ = r_-$, one has $\mathcal{F}_- = \mathcal{F}_+ = 0$. For this case one uses the original formula (2.39). Since $\frac{1}{2}(\partial_r(\alpha f)_{\rm H} = \kappa_{\rm H}$, one has

$$\beta_{\rm H} = \exp(-\kappa_{\rm H} q). \tag{3.3}$$

Consider a beam of radial type II incoming photons with the energy E_i crossing the first shell between r_- and $r_- + \Delta r_-$. They will cross the second shell in the interval $(r_+, r_+ + \Delta r_+)$ with the energy E_f. Then

$$E_- \Delta r_- = E_+ \frac{dr_-}{dr_+} \Delta r_+ = E_- \frac{\mathcal{F}_-}{\mathcal{F}_+} \Delta r_+ = E_+ \Delta r_+. \tag{3.4}$$

We define the rate of the energy flux $\mathcal{E}_\pm = E_\pm / \Delta r_\pm$. Then one has

$$\mathcal{E}_+ = \beta^2 \mathcal{E}_-. \tag{3.5}$$

Let us now calculate the gain function for the type II photons [see Eq. (2.39)]. Such a photon starts its motion from $\mathcal{I}^-$ with $v_0 \in (0, q)$. For such an incoming photon $l_\mu = (-E_-, 0)$, where E_- is its energy at $\mathcal{I}^-$. After passing $r = 0$ this photon becomes outgoing with the same energy, and one has

$$\ln \beta = -\frac{1}{2} \int_{v_0}^{q} \partial_r(\alpha f) dv - \ln \alpha_0. \tag{3.6}$$

For a double-shell model with a constant metric in the interior one gets

$$\beta = \frac{1}{\mathcal{F}_+}. \tag{3.7}$$

For the type III null ray the gain function is 1.

C. Quantum radiation from a sandwich black hole

We use the dimensionless form of the metric (2.3), keeping the scale parameter σ arbitrary. In the adopted model of a sandwich black hole

$$f = \alpha = 1, \quad \text{for } v < 0 \quad \text{and} \quad v > q, \tag{3.8}$$

while for $v \in (0, q)$ these functions depend only on r. The equation (2.18) between the shells can be easily solved with the following result. Denote

$$Q = \int_0^r \frac{dr}{\mathcal{F}}, \qquad \mathcal{F} = \alpha f. \tag{3.9}$$

Then the solution is

$$v - 2Q(r) = C = \text{const}. \tag{3.10}$$

1. Type I rays

This is the case when $u_- < 0$, so that the corresponding outgoing null ray intersects both of the null shells. In this case one has

$$u_+ = q - 2r_+, \qquad u_- = -2r_-, \tag{3.11}$$

$$Q(r_-) = Q(r_+) - q/2. \tag{3.12}$$

Equation (3.11) establishes relations between the retarded times u_+ and u_- and the radii r_- and r_+ of the points where this null ray crosses the null shells, while the relation (3.12) determines a map between r_- and r_+. The general expression (2.24) for the energy flux $\mathcal{E}$ can be transformed into a form that is more convenient for the calculations. Using Eq. (3.11) one gets

$$P = \ln\left|\frac{dr_-}{dr_+}\right|, \tag{3.13}$$

$$\mathcal{E} = \frac{1}{192\pi}\left[-2\frac{d^2P}{dr_+^2} + \left(\frac{dP}{dr_+}\right)^2\right]. \tag{3.14}$$

Using Eq. (3.12) one gets

$$\frac{dr_-}{dr_+} = \frac{\mathcal{F}(r_-)}{\mathcal{F}(r_+)} \tag{3.15}$$

and, hence,

$$\frac{dP}{dr_+} = \frac{1}{\mathcal{F}(r_+)}\left[\frac{d\mathcal{F}(r_-)}{dr_-} - \frac{d\mathcal{F}(r_+)}{dr_+}\right], \tag{3.16}$$

$$\frac{d^2P}{dr_+^2} = \frac{1}{\mathcal{F}^2(r_+)}\left[-\frac{d\mathcal{F}(r_-)}{dr_-}\frac{d\mathcal{F}(r_+)}{dr_+} + \left(\frac{d\mathcal{F}(r_+)}{dr_+}\right)^2 + \mathcal{F}(r_-)\frac{d^2\mathcal{F}(r_-)}{dr_-^2} - \mathcal{F}(r_+)\frac{d^2\mathcal{F}(r_+)}{dr_+^2}\right]. \tag{3.17}$$

If we introduce the function

$$B(r) = -2\mathcal{F}(r)\frac{d^2\mathcal{F}(r)}{dr^2} + \left(\frac{d\mathcal{F}(r)}{dr}\right)^2, \tag{3.18}$$

then

$$\mathcal{E} = \frac{1}{192\pi}\frac{B(r_-) - B(r_+)}{\mathcal{F}^2(r_+)}, \qquad r_- = r_-(r_+). \tag{3.19}$$

2. Type II rays

In this case $0 < u_- < q$ and the corresponding outgoing null rays intersect only the second null shell with negative mass. One has

$$u_+ = q - 2r_+, \qquad u_- = q - 2Q(r_+). \tag{3.20}$$

For the calculation of the energy flux one can use Eq. (3.14) with

$$P = -\ln \mathcal{F}(r_+). \tag{3.21}$$

To obtain derivatives of P one can use Eqs. (3.16) and (3.17) by putting $\mathcal{F}(r_-) = 1$ in these relations. The final result is

$$\mathcal{E} = -\frac{1}{192\pi}\frac{B(r_+)}{\mathcal{F}^2(r_+)}. \tag{3.22}$$

For the type III rays one has $u_+ = u_-$ and the quantum radiation vanishes, $\mathcal{E} = 0$.

D. Sandwich model with $\alpha = 1$

Let us notice, that in the case when $\alpha = 1$ the above expressions for the energy flux can be further simplified. We define a function κ by the relations

$$\kappa^2 = |\xi^2 w^2|, \qquad w_\mu = \frac{1}{2}\nabla_\mu \ln|\xi^2|, \qquad |\xi^2| = f. \tag{3.23}$$

The quantity κ is nothing but the redshifted proper acceleration of the Killing observer at the point r

$$\kappa = \frac{1}{2}\frac{df(r)}{dr}. \tag{3.24}$$

Denote by $R(r)$ the scalar curvature at the point r

$$R = -\frac{d^2 f(r)}{dr^2}. \tag{3.25}$$

Then one can show that the function $B(r)$, which enters Eqs. (3.19) and (3.22), can be written in the form

$$B(r) = 4\kappa^2(r) + 2R(r)f(r). \tag{3.26}$$

Both κ and R are finite and smooth functions at all radii.

E. Quantum radiation from the near-horizon domains

Let us demonstrate now how the relation (3.19) can be used to calculate exactly the quantum energy flux in some special cases. Namely, we assume that there exists a point r_{H} where $\mathcal{F}(r_{\mathrm{H}}) = 0$. This point belongs to either the outer or inner branch of the apparent horizon. The outgoing null ray, which crosses the first shell at $r_- = r_{\mathrm{H}}$, propagates along the horizon, and crosses the second shell at the same radius $r_+ = r_{\mathrm{H}}$. Consider a narrow beam of outgoing null rays, propagating in the vicinity of this horizon. Denote $y = r - r_{\mathrm{H}}$; then in the vicinity of the horizon one has

$$\mathcal{F} = \mathcal{F}'_{\mathrm{H}} y + \frac{1}{2}\mathcal{F}''_{\mathrm{H}} y^2 + \frac{1}{6}\mathcal{F}'''_{\mathrm{H}} y^3 + \dots. \tag{3.27}$$

We also denote by $y_\pm$ the values of y for the intersection of the null ray with the first or second null shell, respectively. Using Eq. (3.19) one finds

$$\mathcal{E} = \frac{1}{192\pi}\frac{\mathcal{F}'''_{\mathrm{H}}}{\mathcal{F}'_{\mathrm{H}}}\frac{y_+^2 - y_-^2}{y_+^2}. \tag{3.28}$$

In order to establish a relation between y_- and y_+, it is sufficient to use the linearized version of the equation (2.18)

Its solution with the initial data $y(0) = y_-$ is

$$y(v) = y_- \exp(\kappa_{\mathrm{H}} v), \tag{3.30}$$

where $\kappa_{\mathrm{H}} = \frac{1}{2}\mathcal{F}'_{\mathrm{H}}$ is the surface gravity at the horizon. Thus one has

$$y_+ = y_- \exp(\kappa_{\mathrm{H}} q), \tag{3.31}$$

and

$$\mathcal{E}_{\mathrm{H}} = \frac{\mathcal{F}'''_{\mathrm{H}}}{384\pi\kappa_{\mathrm{H}}}[1 - \exp(-2\kappa_{\mathrm{H}} q)]. \tag{3.32}$$

IV. STANDARD SANDWICH MODEL

A. Metric

To specify a sandwich model, one needs to specify a static metric between the shells. We start with a simple example. Namely, we put $A = 1$ in Eq. (2.1). We assume that $F = 1$ outside the interval $(0, \Delta V)$, while inside it has the form

$$F = 1 - \frac{2MR^2}{R^3 + 2M\ell^2}. \tag{4.1}$$

The corresponding metric was considered by Hayward [35] (see also Ref. [39]). We call this spacetime a *standard sandwich model*.

The metric function (4.1) contains two parameters: the mass M and the "fundamental length" ℓ. In what follows we assume that the mass M is large enough, so that the function F has two positive zeros $R_{1,2}$. In such a case it is convenient to use $R_{1,2}$ as two new parameters, instead of M and ℓ. Moreover, we use the radius of the inner horizon R_2 as a scale parameter σ. In addition to Eq. (2.2) we use the following notation:

$$p = R_1/R_2. \tag{4.2}$$

It is possible to check that $f(r)$, corresponding to the metric function (4.1) has the form

$$f(r) = \frac{(r-r_1)(r-r_2)(r-r_0)}{r^3 - r_1 r_2 r_0}, \tag{4.3}$$

where

$$r_1 = p, \qquad r_2 = 1, \qquad r_0 = -\frac{p}{p+1}. \tag{4.4}$$

The parameter p is the position of the outer horizon in dimensionless units, while the inner horizon is located at $r = 1$. The quantity r_0 is negative. We call it an *imaginary horizon*. The metrics (2.3) and (4.3) are uniquely specified by two quantities: the mass parameter p and the duration of the black hole's existence, $q = \Delta V/R_2$. Because of its rather simple form, a part of the results can be obtained in an analytical form. The dimensionless surface gravities, calculated for each of these horizons, are

$$\kappa_1 = \frac{(p-1)(p+2)}{2p(p^2+p+1)},$$
$$\kappa_2 = -\frac{(p-1)(2p+1)}{2(p^2+p+1)},$$
$$\kappa_0 = \frac{(p+1)(p+2)(2p+1)}{2p(p^2+p+1)}. \tag{4.5}$$

The motion of the incoming radial null rays in this geometry is rather simple. They are described by the equation $v = \text{const}$. Such rays pass through the center $r = 0$ and become outgoing. Outgoing null rays in the standard sandwich black-hole geometry are shown in Fig. 2. As we described earlier, there exist three different kinds of these rays. Type I rays have their origin as incoming rays with $v < 0$. In their propagation they cross both of the shells. Rays that cross the first null shell with $r < 1$ propagate near the origin, being accumulated near the inner horizon from its inner side. Type I rays that cross the first null shell between $r = 1$ and $r = p$ are also attracted to the inner horizon and are accumulated near it from its outer side. Type I rays that cross the first shell at $r > p$ are propagating away from the outer horizon. All incoming rays of type II, with the origin at $0 < v < q$, after passing the inner horizon cross the center $r = 0$ and return to the inner horizon from inside. They are accumulated in the narrow domain in its vicinity. In other words, the inner horizon plays the role of an attractor for the outgoing rays, while the outer horizon repulses the rays. After the collapse of the second shell with a negative mass, all the outgoing null rays freely propagate to the future null infinity $\mathcal{I}^+$.

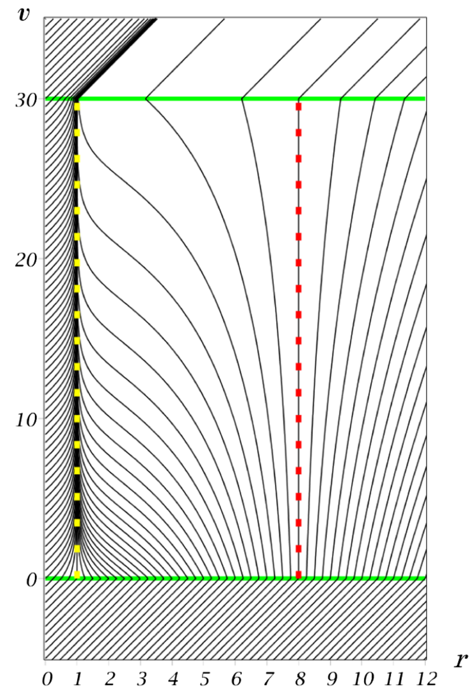

FIG. 2. This plot shows the outgoing radial null rays $u = \text{const}$ propagating in the standard sandwich black hole with $p = 8$ and $q = 30$.

To establish a relation between the parameters u_- and u_+, which is required for the calculation of the quantum energy flux, one needs to calculate the function Q, defined by Eq. (3.9), which in the case under consideration takes the form

$$Q = \int_0^r \frac{dr}{f}. \tag{4.6}$$

One has the following expression for f^{-1} in the metric (4.3):

$$f^{-1} = 1 + \frac{1}{2\kappa_1(r-r_1)} + \frac{1}{2\kappa_2(r-r_2)} + \frac{1}{2\kappa_0(r-r_0)}. \tag{4.7}$$

A simple calculation gives

$$Q(r) = r + \frac{1}{2\kappa_2}\ln|r-1| + \frac{1}{2\kappa_1}\ln\left(\frac{|r-p|}{p}\right) + \frac{1}{2\kappa_0}\ln\left(\frac{|r-r_0|}{|r_0|}\right). \tag{4.8}$$

Equations (3.12) and (3.20) allow one to find relations between u_+ and corresponding u_- for type I and II rays, respectively. For $u_- > q$, that is for rays of type III, one has $u_- = u_+$. Figure 3 shows this function $u_-(u_+)$ for a standard sandwich black hole with special values $p = 4$ and $q = 8$. For large negative u_+ the function $u_-(u_+)$ asymptotically tends to the straight line $u_+ = u_-$. This region corresponds to the "early" null rays with $v \ll 0$. After passing the center they cross both shells at large radii. In other words, they always propagate in the domain with small (but not vanishing) gravitational potential. This potential is smaller when the value of $-v$ is larger.

The conformal structure of the spacetime of the standard sandwich black hole is shown in the Carter-Penrose diagram of Fig. 4. To compactify the null coordinates (u, v) we used new coordinates $\bar{u} = \arctan(u/\gamma)$ and $\bar{v} = \arctan(v/\gamma)$ and corresponding Cartesian spacetime coordinates $\zeta = \bar{v} - \bar{u}$ and $\eta = \bar{v} + \bar{u}$. The parameter γ is an arbitrary positive constant. We chose $\gamma = 3$, for the better presentation of the diagram. In these coordinates null rays $u = \text{const}$ and $v = \text{const}$ are represented by straight lines with the slope ± 1. The future null infinity $\mathcal{I}^+$ and the past null infinity $\mathcal{I}^-$ are given by segments of the lines $\eta + \zeta = \pi$ and $\eta - \zeta = \pi$. At both null infinities the asymptotic Killing vector is normalized to unity. The solid (green) lines in Fig. 4 depict the shells of infalling null matter at $v = 0$ and $v = q$. The dashed (red) lines between them correspond to the inner and outer horizons of the standard sandwich black hole. The curve corresponding to the center of the black hole $r = 0$ can be calculated using the function $u_-(u_+)$ (see Sec. II B). The upper part of this curve (above the second shell) is a vertical straight line.

Figure 5 shows the logarithm of the gain function $P(r_+) = \ln \beta$. This function has a peak in the vicinity of the inner horizon, though the maximum of the gain function

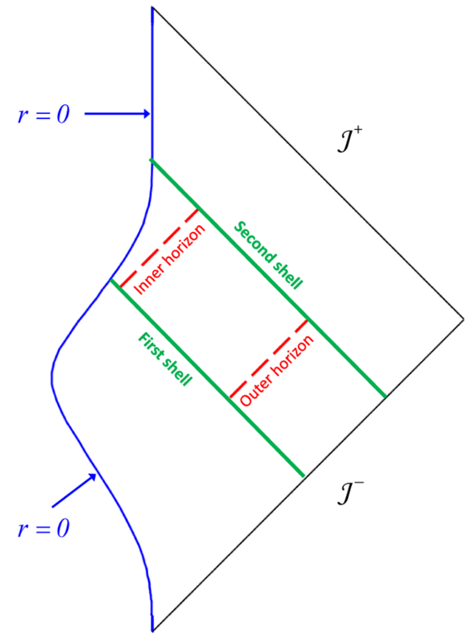

FIG. 4. This is a Carter-Penrose diagram for the standard sandwich black hole with the parameters $p = 3$, $q = 3$.

is not exactly on the inner horizon. For the given parameters of the standard sandwich black hole $p = 4$, $q = 8$ this maximum is located slightly below the horizon. Note that in a generic case the maximum can be either below or above the inner horizon. For the black holes with a long lifespan $q \gg |\kappa_2|^{-1}$ the peak is exponentially narrow $\sim \exp(-|\kappa_2|q)$ and looks very sharp, but it is, in fact, a smooth function on the top. Its amplitude is of the order of $|\kappa_2|q$.

B. Hawking radiation

We demonstrate now that the Hawking result for the quantum energy flux from a black hole is correctly

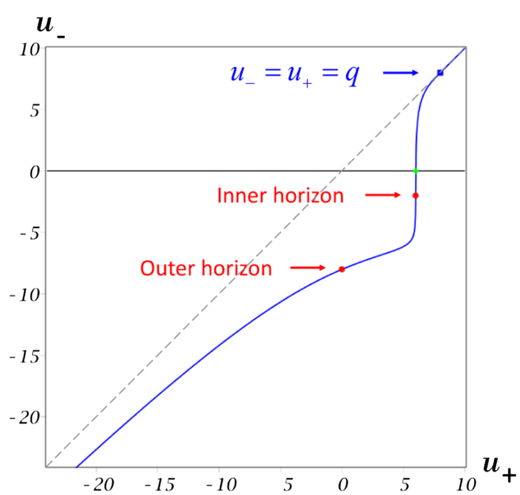

FIG. 3. This plot shows the function $u_-(u_+)$ for the standard sandwich black hole with $p = 4$, $q = 8$. The dashed line depicts an asymptotic $u_- \to u_+$, when $u_+ \to -\infty$ or $u_+ > q$.

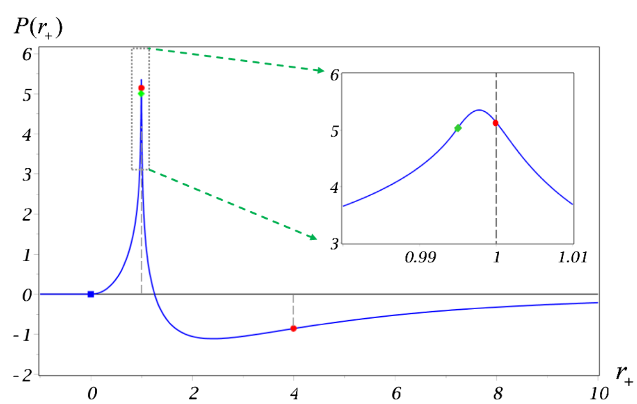

FIG. 5. This plot illustrates the function $P(r_+) = \ln \beta$ for the standard sandwich black hole with $p = 4$, $q = 8$. Two dashed vertical lines represent the position of the inner $r = 1$ and outer $r = p$ horizons. The values of P on the horizons are marked by (red) dots. The (blue) square marks P at the point $r_+ = 0$. The (green) diamond marks P at the point corresponding to $r_- = 0$.

reproduced when the mass parameter p and the lifespan q are large. In this case the metric function outside the external horizon f can be approximated as follows:

$$f = 1 - \frac{p}{r}, \tag{4.9}$$

and one has

$$\kappa = \frac{p}{2r^2}, \qquad R = \frac{2p}{r^3}, \qquad B = \frac{p(4r-3p)}{r^4}. \tag{4.10}$$

The null rays with $r_+ > p$ cross both shells. Let us denote $Y = y + \ln(y-1)$. Then, in the adopted approximation, the relation (3.12) takes the form

$$Y(y_+) = \frac{q}{2p} + Y(y_-), \qquad y_\pm = \frac{r_\pm}{p}. \tag{4.11}$$

The function Y vanishes at $y = y_*$, where

$$y_* = W(e^{-1}) \approx 1.27846 \tag{4.12}$$

where $W(z)$ is a Lambert W function. If $y_+ < y_*$ then $Y(y_+) < 0$ and Eq. (4.11) implies that $Y(y_-) < 0$. That is, both r_+ and r_- are close to the outer horizon, and the energy flux connected with this beam of null rays can be estimated by using Eq. (3.32). Let us consider the case when $y_+ \gg y_*$. In this domain $Y(y_+)$ can be approximated as $Y(y_+) \sim y_+$ and one has

$$B(y_+)p^2 \ll 1, \qquad f(y_+) \approx 1, \tag{4.13}$$

so that the relation (3.19) can be approximated as follows:

$$\mathcal{E} = \frac{B(r_-)}{192\pi}. \tag{4.14}$$

If $Y(y_+) < q/(2p)$ than Eq. (4.11) shows that $y_- < y_*$ and $B(r_-) \approx p^{-2}$. When $Y(y_+) \gg q/(2p)$ the quantity $B(r_)$ is small. To summarize, in the interval $r_+ \in (y_* p, q/2)$ the energy flux is approximately constant

$$\mathcal{E} = \mathcal{E}_{\text{Hawk}} = \frac{\kappa^2}{48\pi} = \frac{1}{192\pi p^2}, \tag{4.15}$$

and for $r_+ > q/2$ it quickly decreases and vanishes. The expression (4.15) correctly reproduces the result for the Hawking flux.

Figure 6 shows the result of the numerical calculation of the energy flux from the standard sandwich black hole in its external domain. A peak near the outer horizon describes the emission of photons, created near the outer horizon, which are "released" when the second shell crosses it. The properties of this part of radiation are model dependent. However, soon after this, the flux of the photons is stabilized and reaches the values $\mathcal{E}_{\text{Hawk}}$, given by Eq. (3.32). When $q \gg p$ the duration of the phase of the Hawking radiation is $\Delta u \approx q$ and the total energy emitted during this phase is $q/(192\pi p^2)$. It is instructive to present the obtained results in the dimensional form. We recall that we put $\sigma = r_- \approx \ell$. The duration of time of the Hawking radiation phase is $\Delta U = \Delta V$ and the rate of the energy emission, $\dot{E}$ and total emitted energy, ΔE, are

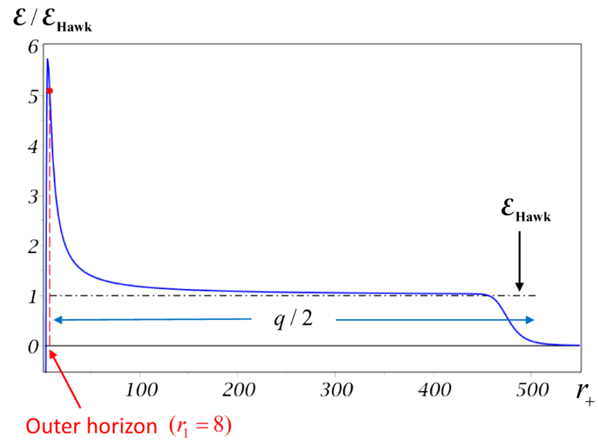

FIG. 6. This plot depicts the flux of particles emitted by the standard sandwich black hole with $p = 8$, $q = 1000$ at the moment $v = q$ in the range of radii from the outer horizon $r_1 = 8$ until infinity. The dashed line denotes the asymptotic of the Hawking radiation in the limit of $q \to \infty$. At radii greater than $\approx q/2$ the flux vanishes. It corresponds to the coordinate $u+- \approx 0$. Thus the tail of the Hawking radiation lasts $\approx q$.

$$\dot{E} = \frac{\ell_{\text{Pl}}^2}{192\pi r_g^2} = \frac{\pi}{12}(kT)^2, \qquad kT = \frac{1}{4\pi r_g}, \tag{4.16}$$

$$\Delta E = \frac{\pi}{12}(kT)^2 \Delta V. \tag{4.17}$$

Equation (4.16) correctly reproduces the result of Ref. [46] for the 2D energy flux, calculated from the trace anomaly.

Equation (4.17) implies that the total energy of the Hawking radiation emitted by a standard sandwich black hole is proportional to the duration of its existence, ΔV. This is a natural result, which clearly demonstrates that the quanta emitted in this process are created with a constant rate near the horizon [54].

C. Quantum radiation from the inner horizon

As a result of the focusing property of the inner horizon, the beam of the outgoing null rays crossing the second null shell for small $r_+ \ll p$ is sharply concentrated near $r_+ = 1$. For this reason for the calculation of the quantum radiation from the inner horizon one can use the following approximation:

$$Q(r_-) = \frac{1}{2\kappa_1}\ln\left(\frac{|r_- - p|}{p}\right) + r_- + \frac{1}{2\kappa_2}\ln|r_- - 1| + \frac{1}{2\kappa_0}\ln\left(\frac{|r_- - r_0|}{|r_0|}\right),$$

$$Q(r_+) = \frac{1}{2\kappa_2}\ln(|r_+ - 1|) + 1 + \frac{1}{2\kappa_1}\ln\left(\frac{|p-1|}{p}\right) + \frac{1}{2\kappa_0}\ln\left(\frac{|1-r_0|}{|r_0|}\right). \tag{4.18}$$

Consider type I rays. By solving Eq. (3.12) one finds r_+ as a function of r_-. Using the notations

$$y_+ = r_+ - 1, \qquad y = r_- - 1, \qquad \mathcal{B} = e^{-\kappa_2 q}, \qquad Y = \mathcal{B} y_+, \tag{4.19}$$

one obtains

$$Y = Z(y),$$

$$Z(y) = y e^{2\kappa_2 y}\left(\frac{p-1-y}{p-1}\right)^{\kappa_2/\kappa_1}\left(\frac{1+y-r_0}{1-r_0}\right)^{\kappa_2/\kappa_0},$$

$$\frac{dr_-}{dr_+} = \frac{\mathcal{B}}{Z'}, \qquad P = -\ln|Z'| + \ln\mathcal{B},$$

$$\frac{dP}{dr_+} = -\mathcal{B}\frac{Z''}{Z'^2}, \qquad \frac{d^2P}{dr_+^2} = -\mathcal{B}^2\left(\frac{Z'''}{Z'^3} - 2\frac{Z''^2}{Z'^4}\right),$$

$$\mathcal{E} = \frac{\mathcal{B}^2}{192\pi}\left(2\frac{Z'''}{Z'^3} - 3\frac{Z''^2}{Z'^4}\right).$$

Let us notice, that the parameter q enters the expression for $\mathcal{E}$ only in the combination $\mathcal{B} = e^{-\kappa_2 q}$, which is a common scaling factor of transformation between r_- and r_+ coordinates. Namely, after rescaling $r_+ - 1 = \mathcal{B}^{-1}Y$ the expression for $\mathcal{B}^{-2}\mathcal{E}$ as a function of Y has a universal behavior independent of q. Since the surface gravity of the inner horizon, κ_2, is negative the scaling factor $\mathcal{B}$ for large q is exponentially large. This fact reflects the following general property of the radiation from the inner horizon. Quanta, propagating near it, experience a huge blueshift and $\mathcal{B}$ is the corresponding blueshift factor. Using this fact one can estimate the total energy of the radiation, emitted from the inner horizon as follows:

$$\Delta E \sim \mathcal{E}\Delta u_+ \sim \mathcal{B}\Delta E_0, \tag{4.20}$$

where ΔE_0 is the scale-invariant value of the energy, which is of the order of 1.

D. Numerical results

We already mentioned that the numerical calculations for large p and q are in agreement with the Hawking result for standard sandwich black holes, provided the duration parameter q is large enough. Now let us discuss quantum radiation of the standard sandwich black hole in other domains. Figure 7 shows the energy flux from a standard sandwich black hole with parameters $p = 4$, $q = 2$. For such a small value of the duration parameter q there is not enough time to develop constant Hawking radiation. At the moment of time when the outer horizon crosses the second shell there exists a flash of radiation, which changes its sign from positive (outside the horizon) to negative (in the domain inside the outer horizon). The radiation emitted between the outer and inner horizons remains relatively small. The very intense outburst of energy occurs near the inner horizon. We choose this small value of q just to be able to show on the plot the radiation from all the domains. As we already mentioned in the previous section, for higher values of the duration parameter q the amplitude of the radiation burst grows as $\sim\exp(-2\kappa_2 q)$, while the width of the peak decreases as $\sim\exp(\kappa_2 q)$. Figure 8 shows the energy burst from the inner horizon for the standard sandwich black hole with parameters $p = 4$, $q = 8$. One can also see in this figure that for the radii smaller than 1 the energy flux becomes negative. This region in more detail is shown in Fig. 9. An analysis shows that this region is connected with type II rays.

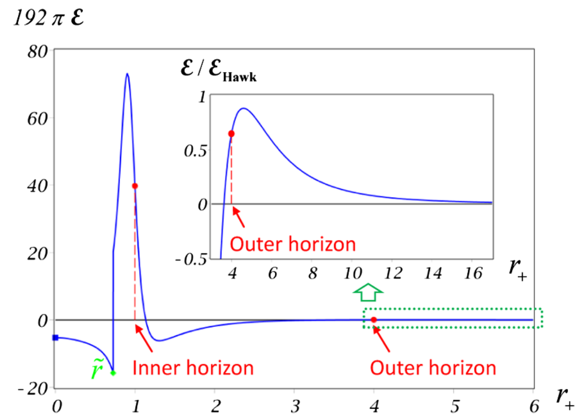

FIG. 7. This plot depicts the flux of particles emitted by the standard sandwich black hole with $p = 4$, $q = 2$ at the moment $v = q$ in the range of radii from $r = 0$ until $r = 16$.

Let us summarize. Until now we studied what was called a standard sandwich black-hole model. In such a model the metric between two null shells was chosen to coincide with the Hayward metric [35]. A characteristic property of this geometry is that $\alpha = 1$, so that a falling photon, when it reaches the center, $r = 0$, has the same energy, as at the infinity. In other words, there is no red- or blueshift for such photons. One of the consequences of this assumption is that the surface gravity at the inner horizon is high, and for large mass parameter p it is $\kappa_2 \approx -1$. As we demonstrated the quantum radiation from the inner horizon of such a black hole is high. For large duration parameter, q, the energy emitted from it is proportional to $\exp(2q)$ and easily exceeds the mass of the black hole $\sim p/2$. This property shows that such standard models are internally inconsistent.

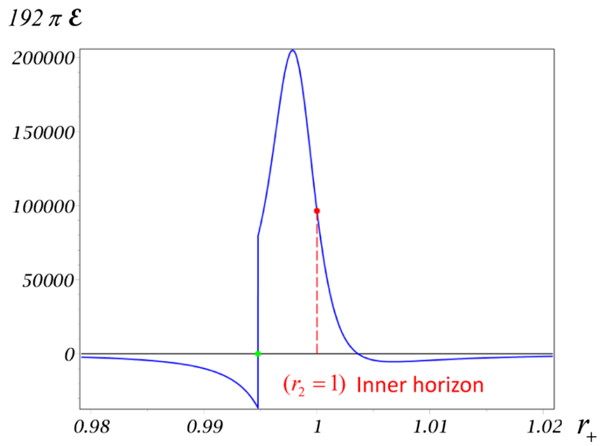

FIG. 8. This plot depicts the flux of particles emitted by the standard sandwich black hole with $p = 4$, $q = 8$ at the moment $v = q$ in the range of radii in the close vicinity of the inner horizon $r_2 = 1$. The maximum amplitude of the radiation grows exponentially with q as $\exp(-2\kappa_2 q)$, while the width of the peak shrinks as $\exp(\kappa_2 q) \ll 1$ (note that κ_2 is negative).

Certainly, the standard sandwich model is quite different from a "realistic" black hole, where the mass decrease is not abrupt, but rather is a smooth and continuous function of v. However one can expect that the main conclusion, concerning the quantum inconsistency of such a model, still remains valid. A reason for this is the following. The effect of the huge energy outburst from the inner horizon is deeply connected with the formula (2.42) for the energy gain parameter β. For the Hayward metric $\alpha_0 = 1$ and $\partial_r(f)|_{f=0}$ at the inner horizon is negative. For a static metric this quantity is negative and of the order of -1 (in the adopted units). This property remains valid almost all the time during

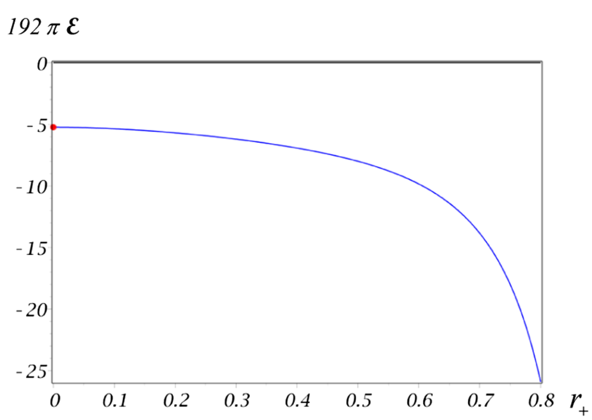

FIG. 9. This plot depicts the flux of particles emitted by the standard sandwich black hole with $p = 4$, $q = 8$ at the moment $v = q$ in the range of radii inside the inner horizon from the center $r = 0$ to $r = 0.8r_2$. For any given parameter p the shape of the curve does not depend on q until the radius $\tilde{r} = r_+(0)$. The position of this point depends on q and at large q we have $r_2 - \tilde{r} \approx \exp(\kappa_2 q) \ll 1$.

evaporation of the black hole, provided its initial mass is much larger than the Planck mass.

E. Possible role of quantum fluctuations

In the derivation of this result we assumed that the inner horizon is an infinitely sharp surface. One might assume that quantum fluctuations, that smear the horizon, can dramatically modify the expression for the flux, emitted from the inner horizon. Let us discuss this option and demonstrate that such a mechanism for the energy outburst suppression apparently does not work. In order to discuss the possible role of quantum fluctuations one can use the formalism developed in Refs. [55,56]. Namely, let us assume that the mass parameter M in the metric function (4.1) fluctuates and it is of the form

$$M = M_0 + \mu(v), \qquad \mu(v) = \mu_0 \cos(\omega v + \phi). \tag{4.21}$$

As a result, the map between $\mathcal{I}^-$ and $\mathcal{I}^+$, which is determined by the radial null rays, would depend on $\mu(u)$. To describe a fluctuating horizon one should consider the phase ϕ and the amplitude μ_0 as stochastic variables. As a result of averaging, the position of the horizon becomes uncertain, and the horizon is effectively smeared. The characteristic width of this broadening is determined by the average value of μ_0.

Let us consider a classical massless particle, propagating in the vicinity of the inner horizon, and find how its gain function is modified under the action of the fluctuations. To estimate the gain function we use the expression (2.42). In our case $\alpha = \alpha_0 = 1$. Let us expand the function f in the vicinity of the horizon

$$f \approx \kappa_2 (r - 1) + \partial_M f|_{r=1} \mu(v). \tag{4.22}$$

Thus

$$\beta \approx \exp(-\kappa_2 q) \exp b, \tag{4.23}$$

$$b = -\frac{1}{2\omega} \partial_r \partial_M f|_{r=1} \mu_0 [\sin(\omega q + \phi) - \sin(\phi)]. \tag{4.24}$$

For a small amplitude μ_0 one gets $\langle \exp b \rangle \approx 1 + O(\langle \mu_0 \rangle^2)$. Thus small fluctuations of the horizon only slightly modify the gain function and its leading term has the same form $\sim \exp(-\kappa_2 q)$ as in the absence of fluctuations. This implies that the same conclusion should be valid for the value of the outburst radiation from the inner horizon in this model.

V. QUANTUM RADIATION FROM A MODIFIED SANDWICH BLACK HOLE

A. Modified model

We consider now a modified version of the sandwich black hole, where the redshift factor α is present. Namely, we consider a double-shell model and choose a modification [28] of the Hayward metric between the null shells in the form

$$ds^2 = -\alpha^2 f dv^2 + 2\alpha dv dr + r^2 d\omega^2,$$
$$f = \frac{(r-p)(r-1)(r+\frac{p}{p+1})}{r^3 + \frac{p^2}{p+1}},$$
$$\alpha = \frac{r^n + 1}{r^n + 1 + p^k}. \tag{5.1}$$

A similar static modification of the Hayward metric with a time delay at the center was proposed in Ref. [57]. It was shown there that to be physically plausible, regular black hole metrics should incorporate a nontrivial time delay between an observer at infinity and an observer in the regular center. A generalization to the rotating case was also considered in Ref. [58].

To stress that this model differs from the standard one by the presence of the redshift function α we call it an *α-sandwich model*. As earlier, we assume that p is large. In order to preserve the correct Schwarzschild asymptotic form of the metric one must put $n \geq 2$. To preserve the value of the surface gravity of the outer horizon at the level of the order p^{-1}, one must have $n \geq k+1$. The surface gravity of the inner horizon is of the order of $\kappa_2 \sim -p^{-k}$. If we assume that q is of the order of the Hawking evaporation time, p^3, then the "dangerous" blueshift factor, which enters the expression for the rate of the energy emission by the inner horizon, $\exp(-2\kappa_2 q)$ becomes $\sim \exp(2p^{3-k})$. For large black holes $p \gg 1$ this factor does not grow with the black hole mass, if we assume that $k \geq 4$. To be more specific we present calculations for the special case $(k=4,\ n=6)$.

Figure 10 shows the outgoing null rays in the α-sandwich black hole geometry. Incoming radial null rays, as earlier, are described by the equation $v = \text{const}$. A comparison with Fig. 2 shows that the attraction of the rays to the inner horizon is weakened. This is a reflection of the fact, that the absolute value of the surface gravity κ_2 is smaller, than in the standard model. For the chosen value of q the outgoing rays are accumulated at the radius $r \approx 2.1969$ (in Fig. 10 it is marked on the second shell by the yellow box). In this aspect the quantum radiation of an α-sandwich black hole differs from that of the standard one, where the peak of the radiation is located in the close vicinity of the inner horizon. For larger q the accumulation point shifts closer to the inner horizon. Note that typically the width of this accumulation region is larger than in the standard case.

B. Accumulation effect

The difference between the α-sandwich black hole and the standard one comes from the effect of the α function on the pace of time between the horizons. The choice of the smooth function α in Eq. (5.1) guarantees that it has little effect on the properties of the α-sandwich black hole near and above the outer horizon, but it considerably slows down time below some radius between r_2 and r_1. As a consequence the surface gravity of the inner horizon is much smaller than that of the outer horizon.

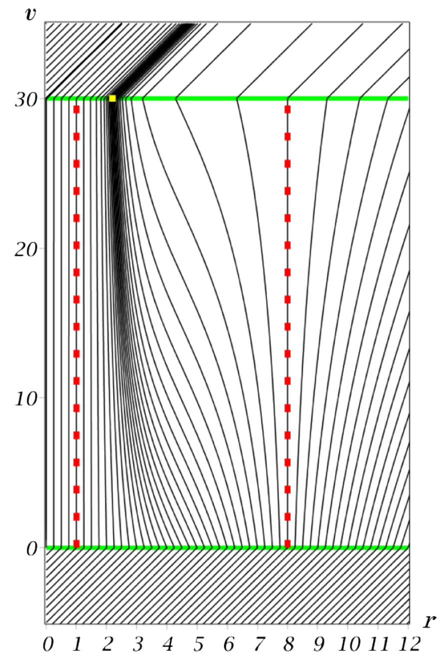

FIG. 10. This plot shows the null rays $u = \text{const}$ propagating in the α-sandwich black hole with $p = 8$, $q = 30$, and $k = 4$, $n = 6$.

The effect of the accumulation of the outgoing null rays has a simple explanation. Denote as usual by $r_\pm$ the radii of intersection of a null ray with null shells. The rays emitted in the interval Δr_- from the first shell cross the second shell in the interval Δr_+. The gain function β, which is the ratio $\Delta r_-/\Delta_+$, describes the compression of the beam of the null rays. The larger the value of β, the larger the compression. Hence, the radius r_+, where the outgoing rays are accumulated, corresponds to the ray with the maximum gain function $\beta(r_+)$. From the definition of the gain function (3.2) it follows that the maximum is achieved when

$$\frac{d}{dr_+}\beta = \frac{d}{dr_+}\left(\frac{\mathcal{F}(r_-)}{\mathcal{F}(r_+)}\right) = 0. \tag{5.2}$$

It is easy to check that this condition is satisfied when r_- and r_+ are connected by the relation

$$\mathcal{F}'(r_-) = \mathcal{F}'(r_+). \tag{5.3}$$

The behavior of the function $\mathcal{F}'(r)$ is qualitatively similar for all sandwich black holes in question. It is presented in Fig. 11. From the symmetry considerations it is evident that the function $\mathcal{F}'(r)$ vanishes at $r = 0$. Then it

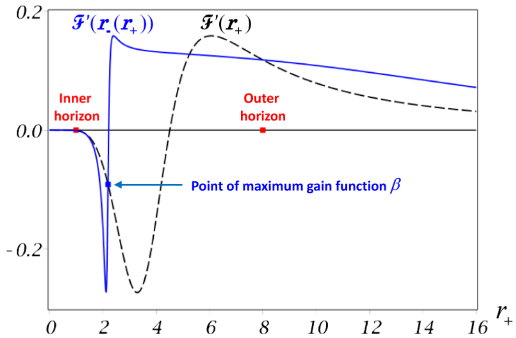

FIG. 11. This plot depicts the functions $\mathcal{F}'(r_+)$ (black dashed line) and $\mathcal{F}'(r_-(r_+))$ (blue solid line) for the α-sandwich black hole with $p = 8$, $q = 30$, and $k = 4$, $n = 6$. The maximum gain function β is achieved at the point, where they cross each other at negative values. For the given parameters this point is at $r_+ = \rho \approx 2.1969$.

becomes negative. It has the value $2\kappa_2$, which is also negative, on the inner horizon. It reaches a minimum and then it increases. $\mathcal{F}'(r)$ changes its sign at some radius between the horizons. On the outer horizon it is positive and is equal to $2\kappa_1$. Finally it asymptotically vanishes at large radii. Because of such a behavior this function has the following property. Choose a point $r = \rho$ in the vicinity of the inner horizon, where the slope $\mathcal{F}''(\rho)$ is negative. Then there exists a point $r = \zeta(\rho) < r_2$, where the slope $\mathcal{F}''(\zeta(\rho))$ is positive, such that the following relation is valid:

$$\mathcal{F}'(\rho) = \mathcal{F}'(\zeta(\rho)). \tag{5.4}$$

We call $\zeta(\rho)$ a point conjugated to ρ. In order for the top of the gain function at $v = q$ be located at ρ, the null ray passing through it must be emitted at $v = 0$ at $\zeta(\rho)$. That is the following condition must be satisfied:

$$q = 2\int_{\rho}^{\zeta(\rho)} \frac{dr}{|\mathcal{F}(r)|}. \tag{5.5}$$

The relations (5.4) and (5.5) can be used to determine $\rho = \rho(q)$ for the position of the accumulation point at time q.

C. Carter-Penrose diagram

Figure 12 shows the function $u_-(u_+)$ for the α-sandwich black-hole with parameters $p = 4$, $q = 8$, and $k = 4$, $n = 6$. The difference with Fig. 3 appears from the redshift factor α near the center of the black hole. For the chosen parameters of the α-sandwich black hole the outgoing null rays emitted from the center $r = 0$ in the interval between the shells $v \in [0, q]$ do not have enough time to propagate far from the center. They are concentrated near $r = 0$ and, when the second shell comes, are released altogether from this narrow region and create an almost rectangular pulse of high-frequency radiation propagating to infinity; the duration of this pulse is approximately $\alpha_0 q$. This radiation results from the domain, corresponding to $0 < u_- < q$ (see the almost vertical part of the curve in Fig. 12).

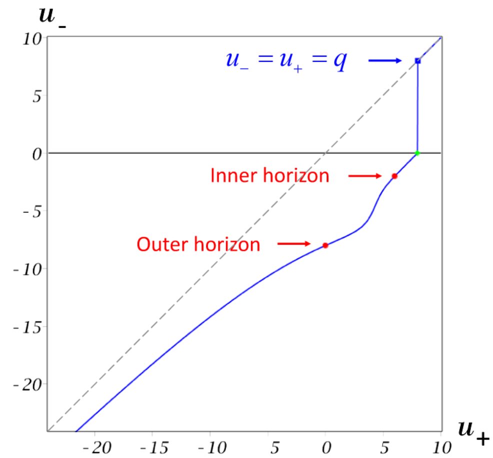

FIG. 12. This plot shows the function $u_-(u_+)$ for the α-sandwich black hole with $p = 4$, $q = 8$, and $k = 4$, $n = 6$. The dashed line depicts an asymptotic $u_- \to u_+$, when $u_+ \to -\infty$ or $u_+ > q$. The curve connecting the green and the blue dots, which mark the points $u_- = 0$ and $u_- = u_+ = q$ respectively, is very close to a vertical line.

The Carter-Penrose diagram for the α-sandwich black-hole with parameters $p = 3$, $q = 3$, and $k = 4$, $n = 6$ is shown in Fig. 13. We used the same coordinate transformations as in the Carter-Penrose diagram for the standard sandwich black hole (Fig. 4). Consider null rays in the vicinity of the inner horizon. In the case of the α-sandwich black hole, they cross the $r = 0$ curve at larger

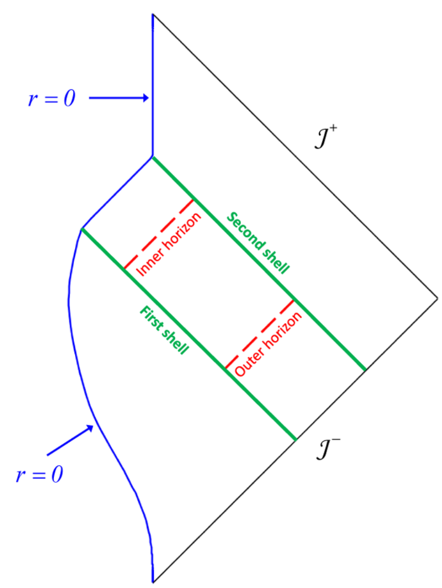

FIG. 13. This is a Carter-Penrose diagram for the modified α-sandwich black hole with the parameters $p = 3$, $q = 3$, and $k = 4$, $n = 6$.

angles than in the standard case (Fig. 4). This reflects the smaller gain function and the smaller ultraviolet shift of quanta in the modified case. Note that in Fig. 13 the $r=0$ curve between the shells looks very similar to the null line, but, in fact, it is a timelike curve and its slope is a little bit bigger than 1 by about $\alpha_0 = 1/(1+p^4)$.

D. Gain function

The logarithm of the gain function for the α-sandwich black hole is shown in Fig. 14. It has a peak at the point of the accumulation of rays. In comparison with the case of a standard black hole (see Fig. 5) the peak of the gain function has a smaller amplitude, looks smoother, and is concentrated not at the inner horizon but rather mostly at some finite radius above it. The position of the peak can be found by solving numerically Eqs. (5.4) and (5.5). For example, the numerical solution for the chosen values of the parameters $p=4$ and $q=8$ gives the value $\rho=1.842$, which identically coincides with the position of the maximum of the peak of the logarithm of the gain function in the plot presented at Fig. 14. We choose small values of the parameters p and q in order to be able to illustrate the behavior of the gain function in the total domain $r_+ =\in [0,\infty)$. When q becomes large, the height of the peak grows while its width becomes very narrow.

If we are interested in the case when both parameters p and q are large, one can obtain an approximate solution of the equations (5.4) and (5.5). Thus we assume that p is large. We also assume that the time of the existence of the sandwich black hole is larger than the Hawking evaporation time, $q \geq p^3$.

For the α-sandwich black holes of the type (5.1) the integral (5.5) can be taken exactly and be expressed in terms of elementary functions. Though the result is very cumbersome, one can find its leading asymptotic for $q \geq p^3$. The result is

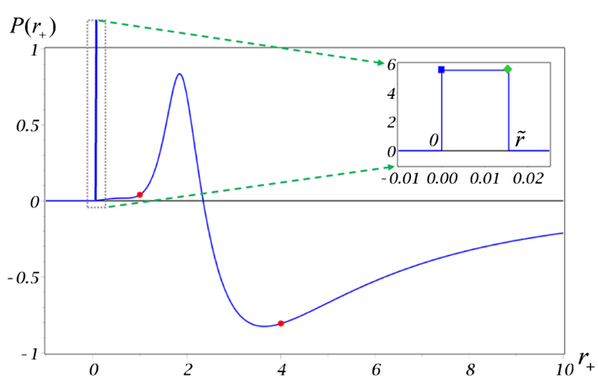

FIG. 14. This plot illustrates the logarithm of the gain function $P(r_+) = \ln\beta = \ln(du_-/du_+)$ for the α-sandwich black hole with $p=4$, $q=8$, and $k=4$, $n=6$. The two red circles represent fluxes at the inner $r=1$ and outer $r=p$ horizons. The blue square marks P at the point $r_+=0$. The green diamond marks P at the point corresponding to $r_-=0$.

$$\frac{q}{p^4} \approx \frac{1}{6}\left[6\,\mathrm{arctanh}\left(\frac{1}{r}\right) + 2\arctan r - 5\pi + 4\arctan(2r-\sqrt{3}) + 4\arctan(2r+\sqrt{3})\right]. \tag{5.6}$$

We checked numerically that Eq. (5.6) is extremely accurate for all values of $q \geq p^3$. One can see that the dependence of ρ on the lifespan of the black hole comes only via the combination q/p^4. In the leading approximation the dependence of the integral on $\zeta(\rho)$ drops out. Any inaccuracy in the upper limit of the integral $\Delta\zeta(\rho) \sim p^{2/3}$ leads to a negligible correction $\Delta\rho \sim p^{-10/3}$.

This approximation can also be used to derive another fairly good approximation for $\mathcal{F}(\rho(q))$, which is valid for all $p^3 < q < \infty$ with an accuracy of about 20%:

$$\mathcal{F}(\rho) = \begin{cases} -\frac{1}{p^4}\left(1-\frac{2q}{3p^4}\right)\left(\frac{7}{2}\frac{q}{p^4}\right)^{-8/7}, & p^3 < q < p^4, \\ -\frac{8}{p^4}\exp\left(-\frac{2q}{p^4}-\frac{5\pi}{6}\right), & q \geq p^4. \end{cases} \tag{5.7}$$

In the same approximation one gets

$$\zeta(\rho) \approx 5^{1/6} p^{2/3}, \quad \mathcal{F}(\zeta(\rho)) \approx -\frac{1}{6}(5^{5/6} p^{1/3} - 5). \tag{5.8}$$

The maximum of the peak of the gain function can be estimated as follows:

$$\beta_{\max} \sim \frac{|\mathcal{F}(\zeta(\rho))|}{|\mathcal{F}(\rho)|}. \tag{5.9}$$

One can see that for $p^3 < q < p^4$ the maximum of the gain function grows as a power of q, while for very large $q \geq p^4$ it grows exponentially with q. The width of the peak of the gain function is proportional to $\mathcal{F}(\rho)$.

A new feature of the α-sandwich model is the appearance of an almost rectangular pulse of the gain function at $r \in [0,\tilde{r}]$. Here $\tilde{r}$ is defined by the condition $r_-(\tilde{r}) = 0$. It means that the outgoing null ray emitted from the center $r=0$ at the moment of arrival of the first shell ($v=0$) reaches the second shell ($v=q$) at the radius $\tilde{r}$.

E. Quantum radiation

The Hawking radiation in the black hole exterior is practically the same as it is for the standard model with the same p and q parameters (see Fig. 15). In other words, the Hawking radiation practically does not depend on the internal structure of the black hole. The small difference in the flux is explained by the dependence of the surface gravity of the outer horizon on the fundamental constant ℓ. For large p this effect is negligible.

Figure 16 shows the energy flux from an α-sandwich black hole. Here we chose a small-mass black hole $p=4$ with the lifespan $q=8$. Though for these parameters the

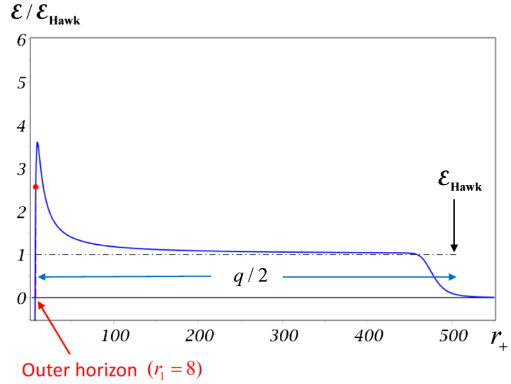

FIG. 15. This plot depicts the flux of particles emitted from in the α-sandwich black hole with $p = 8$, $q = 1000$, and $k = 4$, $n = 6$ at the moment $v = q$ in the range of radii from the outer horizon $r_1 = 8$ until infinity. The dashed line denotes the asymptotic of the Hawking radiation in the limit of $q \to \infty$. At radii greater than $\approx q/2$ the flux vanishes. It corresponds to the coordinate $u_- \approx 0$. The tail of the Hawking radiation lasts $\approx q$, i.e., the lifespan of the black hole.

plateau of the Hawking radiation in the exterior of the black hole is less pronounced, the structure of the quantum radiation in its interior can be depicted in more detail. The most important difference is the value of the amplitude of the peak-shaped radiation from the black hole interior. The position of the peak is shifted to a larger value of the radius. Numerical calculations show that it practically coincides with the position of the gain function for the same parameters p and q. More importantly, for α-sandwich black holes its height is greatly suppressed. If we increase the lifetime q of the black hole, the peak becomes higher, its width decreases, and its position shifts closer to the inner horizon. At the same time the quantum radiation in the

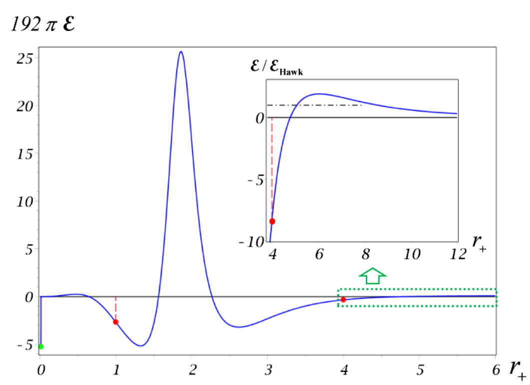

FIG. 16. This plot depicts the flux of particles emitted by the α-sandwich black hole with $p = 4$, $q = 8$, and $k = 4$, $n = 6$ at the moment $v = q$. The red dots in the plot correspond to the fluxes on the horizons at $r = 1$ and $r = 4$. We singled out the domain outside the outer horizon, rescaled it in units of the Hawking flux, and put it inside the figure. The green dot marks the point $r = \tilde{r}$. At radii $0 < r < \tilde{r}$ the flux is negative and almost constant.

black hole exterior becomes closer and closer to the constant Hawking flux of duration q.

The amplitude of the peak radiation can be estimated using Eq. (3.19). The maximum of quantum radiation at the moment $v = q$ comes out from the radius, which is very close to the radius ρ of the maximum of the gain function. For large p and q the value of the function $B(r_-)$ entering Eq. (3.19) can be roughly estimated at the point of the minimum of the function $\mathcal{F}$ [see Eq. (3.26)]. Its value at this point is $B(\zeta(\rho)) \sim 2.4p^{-2/3}$. Thus, the leading contribution to the flux density is

$$\mathcal{E}_{\max} \sim \frac{B(\zeta(\rho))}{192\pi\mathcal{F}^2(\rho)}. \tag{5.10}$$

In order to obtain the estimation for the rate of the energy flux at its maximum for large p and $q \geq p^3$ it is sufficient to substitute the expression (5.7) for $\mathcal{F}^2(\rho)$ in this expression. As a result we get

$$192\pi\mathcal{E}_{\max} \sim \begin{cases} 42p^{22/3}\frac{(\frac{q}{p^4})^{16/7}}{(1-\frac{2q}{3p^4})^2}, & p^3 < q < p^4, \\ 0.8p^{22/3}\exp(\frac{4q}{p^4}), & q \geq p^4. \end{cases}$$

In order to estimate the total energy emitted in this peak of radiation one has to know the width of the peak. The width of the peak of quantum radiation is basically the same as the width of the peak Δr of the gain function. The latter can be estimated from Eq. (5.9) and the condition $\Delta r \beta_{\max} \sim \int_{\tilde{r}}^{p} \beta(r_+) dr_+ = \int_0^p dr_- = p$:

$$[\text{width of the peak}] \sim \Delta r \sim p^{2/3}|\mathcal{F}(\rho)|. \tag{5.11}$$

Then the total energy flux for the black hole interior for $q \sim p^3$ can be estimated using Eq. (5.10) as

$$192\pi\Delta E_\alpha \sim 192\pi\mathcal{E}_{\max} \times \Delta r \sim |\mathcal{F}(\rho)|^{-1}. \tag{5.12}$$

Using Eq. (5.7) one can see that for a typical value of $q \sim p^3$ the asymptotic $|\mathcal{F}(\rho)| \sim p^{-20/7}$. Therefore, in this case $192\pi\Delta E_\alpha \sim p^{-20/7}$.

The corresponding energy flux for the standard model for the same value of $q \sim p^3$ is

$$\Delta E \sim \exp(p^3). \tag{5.13}$$

This result means that the inclusion of a properly chosen redshift function α allows one to suppress the exponential outburst of the energy flux from the sandwich black hole interior. We have to stress that even after suppression the estimation of the total emitted energy $\sim p^{2.9}$ is larger than the initial mass of the black hole $\sim p$. Is this inconsistency a consequence of the sandwich model adopted in this paper with its unphysical switching-on and switching-off procedure? One can hope, that for more realistic evaporating black hole models with a proper dependence of mass on time, this inconsistency can be cured.

Note that the density of the energy fluxes can be either positive or negative. Negative flux densities may appear for a short period of time because of the quantum nature of the emitted radiation. Nevertheless, as we already mentioned, taking into account Eq. (3.14) and asymptotic properties of the gain function that lead to the condition $P'|_{u_+=\pm\infty}=0$, one can show that the total flux of quantum radiation is *always positive*.

VI. DISCUSSION

In the present paper we discussed quantum radiation from spherically symmetric sandwich-type nonsingular black holes. We calculated the quantum energy flux for two types of black holes. In both cases the spacetime contains two null shells. The first shell, with a positive mass, creates the black hole. After some time this black hole is disrupted as a result of the collapse of the second shell with a negative mass. The two types of sandwich black holes in question differ by the choice of the metric between the shells. In the first case, which we called a standard sandwich black hole, this metric coincides with the Hayward solution. Such a model is characterized by two parameters: the mass and the duration parameters. Using the 2D approximation we calculated the energy flux of a quantum massless scalar field in the background of this geometry. We demonstrated that after the formation of the black hole the calculated energy flux correctly reproduces the Hawking result. For a black hole of large mass and with large duration parameter the flux is determined by the surface gravity of the outer horizon and practically does not depend on the inner structure of the black hole. This Hawking regime of radiation continues during the interval of time of the black hole existence. This result clearly indicates that the outgoing quantum flux is formed near the outer horizon during its existence. The attempts to explain it as a result of the creation of the flux at the moment of formation of the black hole, which sometimes can be found in the literature, are wrong. A steady Hawking flux terminates at the time, when a signal from the moment of intersection of the outer horizon by the second shell reaches an external observer.

Later the energy density of the radiation temporarily becomes negative. However, its amplitude remains rather small. This regime ends when a signal from the inner horizon reaches the observer. At this moment there exists a huge outburst of the radiation. The positive energy density in the peak is proportional to $\exp(2|\kappa_2|q\Delta V)$, where κ_2 is the (negative) surface density of the inner horizon. The width of the peak is $\sim\exp(-|\kappa_2|\Delta V)$, so that the total energy emitted during the outburst is exponentially large, $\Delta E\sim\exp(|\kappa_2|\Delta V)$. A similar result can be expected for any model of a black hole with the inner horizon with $|\kappa_1|\sim\ell^{-1}$. For a large duration $\Delta V\gg M$ the emitted energy from this domain is much larger than the black hole mass M. Thus such a model is not self-consistent.

Hence, the backreaction of the quanta propagating near the inner horizon on the geometry should greatly modify the geometry in this domain. One of the options is that after the backreaction is included, the surface gravity of the inner horizon becomes small. In order to demonstrate how the exponentially large radiation from the inner horizon can be suppressed, we considered a so-called "modified model," proposed in Refs. [28,57,58]. This model includes a nontrivial redshift function $\alpha(r)$. We normalized this factor so that at infinity it always has the value 1. In such a case the value α_0 of this parameter at the center characterizes the redshift of a photon, propagating from infinity to the center. An important new feature of an α-sandwich black hole is that the surface gravity of the inner horizon depends on the choice of α and can be made small.

We demonstrated that the gain function plays an important role in understanding features of the quantum radiation from the black hole interior. As we showed the peak of the gain function is a result of the effect of the accumulation of the outgoing null rays at some point in the black hole interior. The position of this point depends on the time q of the existence of the sandwich black hole. This effect is generic for nonsingular black holes. The reason is that in the vicinity of the center the gravity is not attractive, but repulsive. As a result the velocity of the infalling null rays inside the apparent horizon is slowed down and the radial density of the beam of photons increases. We obtained equations which determine the position of the peak of the gain function for a given parameter q. Numerical computations of the quantum energy flux from the interior of the black hole demonstrated that the amplitude and position of the peak of the energy flux are strongly correlated with similar characteristics as the peak of the gain function. We used this result to obtain an estimation of the energy flux from the black hole interior for large mass $\sim p$ and $q\geq p^3$. These results are applicable for both the standard and α-sandwich black holes. The main result is that the exponential peak of the energy outburst, which is characteristic of the standard sandwich black holes is strongly suppressed for the α-sandwich model. For a concrete α-sandwich model, which we considered in this paper, the total quantum energy, radiated from the interior of a black hole with $q\sim p^3$ is proportional to $\sim p^{2.9}$. It seams that the exponential dependence of the energy emitted in the standard sandwich model is a robust prediction of models with a trivial redshift function $\alpha=1$. The power in mass energy release in the α models is still larger than the initial mass of the black hole $\sim p$. However, it might be that this result is a manifestation of the adopted model roughness. In order to check this, one has to repeat the calculations, presented in this paper, with a more "realistic" evaporating nonsingular black hole model. Let us mention that an independent interesting approach to the self-consistency problem are attempts to construct a regular black hole model in the framework of a 2D dilaton gravity, that includes both quantum radiation and its backreaction (see e.g. Refs. [59–62]). Anyway, the search for (quantum) consistent nonsingular models of black holes is an interesting open problem which waits for its solution.

ACKNOWLEDGMENTS

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[54] Sometimes one can see in the literature a statement that all the particles of the Hawking radiation are created during the formation of the black hole. The relation (4.17) shows that such a statement is totally wrong. The decision of when to switch off the black hole can be made any time after its formation, so that ΔV is an arbitrary parameter.

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