Addendum to "Absorption of a massive scalar field by a charged black hole"

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In [1] we studied the absorption cross section of a scalar field of mass m impinging on a static black hole of mass M and charge Q. We presented numerical results using the partial-wave method, and analytical results in the high- and low-frequency limit. Our low-frequency approximation was only valid if the (dimensionless) field velocity v exceeds $v_c = 2\pi Mm$. In this addendum we give the complementary result for $v \lesssim v_c$, and we consider the possible physical relevance of this regime.

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In Ref. [1] we analyzed the scenario of a neutral scalar field of mass m and frequency ω absorbed by a Reissner-Nordström black hole of mass M and charge Q. Our stated aim was to provide a quantitative full-spectrum description of absorption, by bringing together numerical methods and analytical approximations. However, due to a tacit assumption, we gave an incomplete description of the low-frequency regime; with this addendum we fulfil our original objective.

The total absorption cross section σ may be written as a sum of partial absorption cross sections σ_l. In the low-frequency limit $M\omega \ll 1$, the dominant contribution arises from the monopole sector, $\sigma_{lf} \approx \sigma_{l=0}$. In Ref. [1] we obtained $\sigma_{lf} = \mathcal{A}/v$ [see Eq. (63)], where $v = \sqrt{1 - m^2/\omega^2}$ is the (dimensionless) velocity of the field, $\mathcal{A} = 4\pi r_+^2$ is the area of the black hole, and r_+ is the areal coordinate of the event horizon location.

However, our original result did not encompass the limit of small velocities. After taking this into account, the completed low-frequency approximation is

$$\sigma_{lf} = \begin{cases} \sigma_{lf}^{(1)} = \mathcal{A}/v, & v \gtrsim v_c, \\ \sigma_{lf}^{(2)} = \frac{4(\pi r_+)^2 (2Mm)}{v^2}, & v \lesssim v_c, \end{cases} \tag{1}$$

where $v_c = 2\pi Mm$ is the velocity of the transition.

One may obtain the approximation valid for $v \lesssim v_c$ by starting from Eq. (62) of Ref. [1], namely

$$\sigma = \frac{4\pi r_+^2 \rho^2}{v}, \tag{2}$$

where we are considering only the first term in the denominator of Eq. (62). For this purpose, we write[1]

$$\rho^2 = \frac{2\pi\eta}{e^{2\pi\eta} - 1} \tag{3}$$

as

$$\rho^2 = -\frac{2\pi Mm(1+v^2)}{v\sqrt{1-v^2}} \frac{1}{\exp\left(-\frac{2\pi Mm(1+v^2)}{v\sqrt{1-v^2}}\right) - 1}. \tag{4}$$

Substituting Eq. (4) in Eq. (2) and considering the limit for $v \to 0$ we obtain

$$\sigma_{lf}^{(2)} = \frac{4(\pi r_+)^2 (2Mm)}{v^2}. \tag{5}$$

In the uncharged case $Q = 0$ we recover Unruh's result for the case of a Schwarzschild black hole [cf. Eq. (97) of Ref. [2]].

In Fig. 1 we compare Eq. (1) with our numerical results. We can see that the numerical results present a transition between $\sigma_{lf}^{(1)}$ and $\sigma_{lf}^{(2)}$, which happens near $v = v_c$, with $v_c \approx 0.138$ in this case.

Recently a new dark matter candidate was proposed by Hui *et al.* [3], in the form of a scalar field with mass $m \approx 10^{-22}$ eV$/c^2$ and de Broglie wavelength $\lambda_B \approx 1$ kpc. Its corresponding velocity is $v \approx 4 \times 10^{-4}$, found from $v = (1 + \lambda_B^2/\lambda_C^2)^{-1/2}$, where $\lambda_C = h/mc \approx 0.4$ pc is the Compton wavelength. For a black hole mass $M_1 = 3.6 \times 10^6\ M_\odot$ (e.g. Sgr. A* [4]), we find $v_c \approx 1.7 \times 10^{-5}$ and thus $v > v_c$; whereas for a supermassive black hole of mass $M_2 = 2 \times 10^8\ M_\odot$ (e.g. Andromeda's supermassive black hole has mass $(1.1 - 2.3) \times 10^8\ M_\odot$ [5]) we find $v_c = 9.4 \times 10^{-4}$ and thus $v < v_c$. This suggests that both regimes of Eq. (1) are potentially relevant in the scenario of

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[1] There are 2π factors (in front of η) missing in Eq. (57) of Ref. [1].

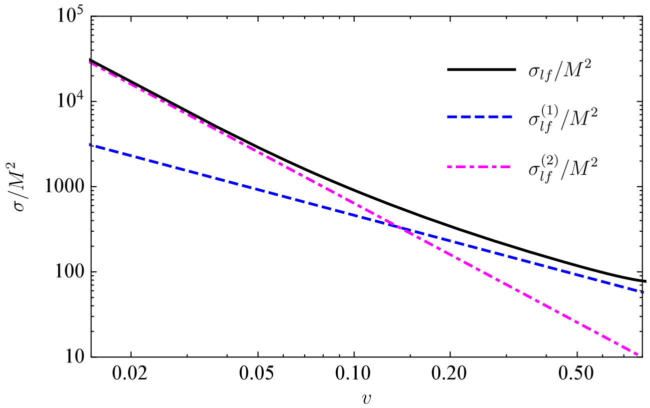

FIG. 1. Comparison between the partial absorption cross section $\sigma_{l=0}$ (solid line), and the approximate analytical results of Eq. (1) (broken lines), for the case $Q/M = 0.4$ and $Mm = 0.022$. A transition in behavior is visible near $v_c \approx 0.138$.

Hui *et al.*, and that disparate cross sections are possible. For example, $\sigma_{lf}^{(1)} \approx 3.6 \times 10^{24} \mathrm{m}^2$ for M_1 (Sgr. A*) and $\sigma_{lf}^{(2)} \approx 2.6 \times 10^{28} \mathrm{m}^2$ for M_2 (Andromeda).

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