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Gravitational perturbations of the Higgs field

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We study the possible effects of classical gravitational backgrounds on the Higgs field through the modifications induced in the one-loop effective potential and the vacuum expectation value of the energy-momentum tensor. We concentrate our study on the Higgs self-interaction contribution in a perturbed FRW metric. For weak and slowly varying gravitational fields, a complete set of mode solutions for the Klein-Gordon equation is obtained to leading order in the adiabatic approximation. Dimensional regularization has been used in the integral evaluation, and a detailed study of the integration of nonrational functions in this formalism has been presented. As expected, the regularized effective potential contains the same divergences as in flat spacetime, which can be renormalized without the need of additional counterterms. We find that, in contrast with other regularization methods, even though metric perturbations affect the mode solutions, they do not contribute to the leading adiabatic order of the potential. We also obtain explicit expressions of the complete energy-momentum tensor for general nonminimal coupling in terms of the perturbed modes. The corresponding leading adiabatic contributions are also obtained.

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I. INTRODUCTION

There are two equally fundamental aspects of the Higgs mechanism for electroweak symmetry breaking which have received remarkably different attention in the last years. On one hand, we have the prediction that a new scalar boson should be present in the spectrum of the theory. Such a new particle has been recently discovered by the ATLAS and CMS experiments at the LHC [1,2]. The most precise measurement to date of its mass comes from a combination of data from both experiments and is given by $m_H = 125.09 \pm 0.21(\text{stat}) \pm 0.11(\text{syst})$ GeV [3]. A large deal of experimental effort is being devoted to the study of the properties of the Higgs boson. Apart from improving the precision in the determination of its mass, measurements of its production and decay channels, self-coupling and couplings to other particles are being performed. So far, all of them are in excellent agreement with the predictions of the Standard Model (SM) [4–6].

On the other hand, the mechanism also predicts the existence of a Higgs field, i.e., a constant classical field $\hat{\phi} = v$ with v the Higgs vacuum expectation value (VEV).[1] given by $v = 246.221 \pm 0.002$ GeV [7]. It is precisely the interaction with the Higgs field what generates the masses of quarks, leptons and gauge bosons. The presence of this nonvanishing field which permeates all of space is a distinctive feature with respect to the rest of SM fields which have vanishing VEVs. Moreover, together with the homogeneous gravitational field created by the cosmological energy density, the Higgs field is the only SM field which is present today in the Universe on its largest scales. This fact opens the interesting possibility of probing the Higgs field not only by exciting its quanta in colliders, but by directly perturbing its VEV. Thus, for example, the fact that the Higgs field is a dynamical field sourced by massive particles suggests that the presence of a heavy particle could induce shifts in the masses of neighboring ones [8]. This effect does not need the production of on-shell Higgs particles, but because of the short range of the corresponding Yukawa interaction, it is negligible at distances beyond the Compton wavelength of the Higgs boson. Existing data does not seem to contain enough kinematic information in order to confirm or exclude it. A similar approach has been proposed in [9] in order to probe the Higgs couplings to electrons and light quarks. The idea of generating peculiar Higgs shifts was also considered in a different context in [10]. In that work a nonminimal coupling of the Higgs field to the spacetime curvature was considered. The nonminimal coupling modifies the effective potential inducing shifts of the VEV in high-curvature regions such as those near neutron stars or black holes [11].

In this work, we explore further the effects of classical gravitational fields on the Higgs VEV. We consider the

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[1]In the SM, the Higgs VEV is related to the Fermi coupling constant by $v = (\sqrt{2}G_F)^{-1/2}$. The value of this constant is known since the original works of Fermi in the early 30's.

SM Higgs minimally coupled to gravity. The Higgs VEV corresponds to the constant field configuration that minimizes the effective potential. This potential contains not only the classical (tree-level) contribution, but also loop corrections introduced by quantum effects of all the particles that couple to the Higgs, including the Higgs self-interactions [12]. More relevant from the point of view of the present paper is the fact that these quantum corrections are sensitive to the spacetime geometry. The aim of this work is precisely to start the study of the Higgs one-loop effective potential in weak and slowly varying gravitational backgrounds. For simplicity and as a first step, we limit ourselves to the contributions of the Higgs self-interactions. The fact that we assume weak gravitational backgrounds, i.e., whose curvature scale is much smaller than the Higgs mass, allows us to use an adiabatic approximation and avoid the problems generated by mode mixing and particle production typical of quantum field theory in curved spacetime. For the same reason, we can still define an effective quasi-potential [13,14] instead of using the full effective action since all the kinetic terms are suppressed with respect to the potential ones.

Our work deals with the calculation of vacuum expectation values of quadratic operators in curved spacetime [15,16]. These are divergent objects whose renormalization requires the introduction of additional counterterms depending on the curvature tensors. Different techniques have been used in the literature to work out these divergences which, because of the fact that they are determined by the short-distance physics, depend locally on the geometry of spacetime [17–26]. But, apart from the local divergent contributions, there are also finite nonlocal terms which are sensitive to the large-scale structure of the manifold and, in general, depend on the quantum state on which the expectation value is evaluated. In some particular simple geometries, such as conformally flat metrics, these finite contributions can be exactly computed in some cases from the knowledge of the trace anomaly, but in general only brute force methods, such as mode summation, are available to evaluate them [27–30]. This is precisely the approach we follow in this work. In particular, we extend the analysis performed in [31] to arbitrary dimension in order to calculate the integrals over the quantum modes using dimensional regularization. Several errors in [31] are also corrected in the present paper.

The work is organized as follows: in Sec. II, the effective action formalism is briefly reviewed. The field quantization in arbitrary $D+1$ dimensions in the adiabatic approximation is discussed in Sec. III. Section IV contains the full mode solutions to first order in metric perturbations. The general results for the Higgs effective potential and the method used to obtain them are described in Sec. V. The vacuum expectation value of the energy-momentum tensor is calculated in Sec. VI. The paper ends in Sec. VII with some discussions and conclusions.

II. ONE-LOOP EFFECTIVE ACTION

The classical action for a minimally coupled real scalar field with potential $V(\phi)$ in arbitrary $(D+1)$-dimensional curved spacetime reads

$$S[\phi, g_{\mu\nu}] = \int \mathrm{d}^{D+1}x\sqrt{g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right). \quad (1)$$

In the case of the real Higgs field, the classical potential is given by

$$V(\phi) = V_0 + \frac{1}{2}M^2\phi^2 + \frac{\lambda}{4}\phi^4 \quad (2)$$

with $M^2 < 0$. The minimum corresponds to $\phi = v$ with $v^2 = -M^2/\lambda$. The mass of the Higgs boson at tree-level is given by $m_H^2 = V''(v) = -2M^2$ and from the recently measured value of m_H at the LHC, the Higgs self-coupling is $\lambda \simeq 1/8$.

The action is minimized by the solutions $\phi = \hat{\phi}$ of the classical equation of motion:

$$\Box\hat{\phi} + V'(\hat{\phi}) = 0. \quad (3)$$

The quantum fluctuations around the classical solution $\delta\phi = \phi - \hat{\phi}$ satisfy the equation of motion

$$(\Box + m^2(\hat{\phi}))\delta\phi = 0 \quad (4)$$

with

$$m^2(\hat{\phi}) = V''(\hat{\phi}) = M^2 + 3\lambda\hat{\phi}^2. \quad (5)$$

The effective action which takes into account the effect of quantum fluctuations on the dynamics of the classical field can be written as

$$W[\hat{\phi}, g_{\mu\nu}] = \int \mathrm{d}^{D+1}x\sqrt{g}L_{\mathrm{eff}} \quad (6)$$

which can be expanded up to one-loop order as

$$W[\hat{\phi}, g_{\mu\nu}] = S[\hat{\phi}, g_{\mu\nu}] + W^{(1)}[\hat{\phi}, g_{\mu\nu}]. \quad (7)$$

The one-loop correction $W^{(1)}$ can be written as [21]

$$W^{(1)}[\hat{\phi}, g_{\mu\nu}] = \frac{i}{2}\ln\det(-K) = \frac{i}{2}\mathrm{Tr}\ln(-K) \quad (8)$$

where Tr denotes the functional trace and K is the quadratic operator associated to the quantum fluctuations

$$K(x,y) = (\Box_x + m^2(\hat{\phi}))\frac{\delta^{D+1}(x,y)}{\sqrt{g}}. \quad (9)$$

The corresponding Feynman's Green function

$$iG_F(x,y) = \langle 0|T(\delta\phi(x)\delta\phi(y))|0\rangle \quad (10)$$

satisfies

$$K(x,y)G_F(y,z) = -\frac{\delta^{D+1}(x,z)}{\sqrt{g}} \tag{11}$$

where the de Witt repeated indices rule has been assumed.

Following [32,33], let us consider the derivative of the one-loop effective action with respect to the mass parameter m^2, so that from (11) we can write

$$\frac{\mathrm{d}W^{(1)}}{\mathrm{d}m^2} = -\frac{i}{2}\mathrm{Tr}G_F \tag{12}$$

or writing the trace explicitly

$$\begin{aligned}\frac{\mathrm{d}W^{(1)}}{\mathrm{d}m^2} &= -\frac{1}{2}\int \mathrm{d}^{D+1}x\sqrt{g}iG_F(x,x)\\ &= -\frac{1}{2}\int \mathrm{d}^{D+1}x\sqrt{g}\langle 0|\delta\phi^2(x)|0\rangle.\end{aligned} \tag{13}$$

Thus, we can finally get a formal expression for the one-loop contribution to the effective Lagrangian as

$$L_{\mathrm{eff}}^{(1)}(x) = -\frac{1}{2}\int_0^{m^2(\hat{\phi})} \mathrm{d}m^2\langle 0|\delta\phi^2(x)|0\rangle. \tag{14}$$

In general, in a static homogeneous spacetime, $\hat{\phi}$ is a constant field and the effective Lagrangian defines the effective potential $V_1(\hat{\phi}) = -L_{\mathrm{eff}}^{(1)}(\hat{\phi})$. In time-dependent or inhomogeneous spacetimes, $\hat{\phi}$ changes in time or space and the effective potential is ill defined. In this case, the effective Lagrangian is a function of the classical fields; i.e., it will, in general, depend on $\hat{\phi}$ and $g_{\mu\nu}$ and arbitrary order derivatives,

$$L_{\mathrm{eff}} = L_{\mathrm{eff}}[\hat{\phi}, g_{\mu\nu}, \partial\hat{\phi}, \partial^2\hat{\phi}, \partial g_{\mu\nu}, \partial^2 g_{\mu\nu}, \ldots]. \tag{15}$$

However, in the case in which the background fields ($\hat{\phi}$, $g_{\mu\nu}$) evolve very slowly in space and time compared to the evolution of the fluctuations, the derivative terms in the effective Lagrangian are negligible, and the effective Lagrangian can be considered as an effective quasipotential [13,14]. As we will explicitly show in the next section, this is indeed the case for Higgs fluctuations in weak gravitational backgrounds so that we can still define the one-loop effective potential as

$$V_{\mathrm{eff}}(\hat{\phi}) = V(\hat{\phi}) + V_1(\hat{\phi}), \tag{16}$$

where

$$V_1(\hat{\phi}) = -L_{\mathrm{eff}}^{(1)}(\hat{\phi}) = \frac{1}{2}\int_0^{m^2(\hat{\phi})} \mathrm{d}m^2\langle 0|\delta\phi^2|0\rangle. \tag{17}$$

The equation of motion for the classical field, thus, reduces to

$$V'_{\mathrm{eff}}(\hat{\phi}) \simeq 0; \tag{18}$$

i.e., the effective (quasi)potential correctly determines the VEV for a slowly varying background metric.

The central object in this calculation is the vacuum expectation value of a quadratic operator (14). The standard Schwinger–de Witt representation [15,16] allows us to obtain a local expansion of G_F in curvatures over the mass parameter m^2. However, as mentioned before, this representation does not provide the full nonlocal finite contributions of the effective action in which we are interested in this work. Thus we will follow [15] and evaluate the expectation value from the explicit mode expansion of the quantum fields.

III. QUANTIZATION AND ADIABATIC APPROXIMATION

We will consider quantum fluctuations of the Higgs field in a $(D+1)$-dimensional spacetime metric which can be written as a scalar perturbation around a flat Robertson-Walker background

$$\mathrm{d}s^2 = a^2(\eta)\{[1+2\Phi(\eta,\mathbf{x})]\mathrm{d}\eta^2 - [1-2\Psi(\eta,\mathbf{x})]\mathrm{d}\mathbf{x}^2\}, \tag{19}$$

where η is the conformal time, $a(\eta)$ the scale factor, and Φ and Ψ are the scalar perturbations in the longitudinal gauge. This metric describes the spacetime geometry in cosmological contexts with density perturbations, but also, in the $a(\eta) = 1$ case, it provides a good description of weak gravitational fields generated by slowly rotating astrophysical objects like the Sun.

Up to first order in metric perturbations, Eq. (4) for the fluctuation field $\delta\phi$ reads

$$\begin{aligned}&\delta\phi'' + [(D-1)\mathcal{H} - \Phi' - D\Psi']\delta\phi' - [1+2(\Phi+\Psi)]\nabla^2\delta\phi\\ &\quad - \nabla\delta\phi\cdot\nabla[\Phi - (D-2)\Psi] + a^2(1+2\Phi)m^2(\hat{\phi})\delta\phi = 0,\end{aligned} \tag{20}$$

where $\mathcal{H} = a'/a$ is the comoving Hubble parameter.

In order to evaluate $V_1(\hat{\phi})$, we need to quantize the fluctuation field. Because of the inhomogeneities of the metric tensor, exact solutions for the perturbed Eq. (20) are not expected to be found. Nevertheless, a perturbative expansion of the solution in powers of metric perturbations can be obtained. Moreover, when the mode frequency ω is larger than the typical temporal or spatial frequency of the background metric, i.e., $\omega^2 \gg \mathcal{H}^2$ and $\omega^2 \gg \{\nabla^2\Phi, \nabla^2\Psi\}$, one can consider an adiabatic approximation in order to quantize the field fluctuations $\delta\phi$. Since $\omega \geq m_H$, the adiabatic approximation is extremely good during the whole matter and acceleration eras until present, and also during most of the radiation era, for all cosmological and astrophysical scales of interest.

Let us start with the canonical quantization procedure for the field perturbations $\delta\phi$. Thus, following [34,35], we build a complete set of mode solutions for (20), which are orthonormal with respect to the standard scalar product in curved spacetime [15],

$$(\delta\phi_k, \delta\phi_{k'}) = i\int_\Sigma [\delta\phi^*_{k'}(\partial_\mu\delta\phi_k) - (\partial_\mu\delta\phi^*_{k'})\delta\phi_k]\sqrt{g_\Sigma}\mathrm{d}\Sigma^\mu, \tag{21}$$

with $\mathrm{d}\Sigma^\mu = n^\mu \mathrm{d}\Sigma$. Here n^μ is a unit timelike vector directed to the future and orthogonal to the $\eta = \text{const}$ hypersurface Σ, i.e.,

$$\mathrm{d}\Sigma^\mu = \mathrm{d}^D\mathbf{x}\left(\frac{1-\Phi}{a}, \mathbf{0}\right), \tag{22}$$

whereas the determinant of the metric on the spatial hypersurface reads to first order in metric perturbations

$$\sqrt{g_\Sigma} = a^D(1 - D\Psi). \tag{23}$$

With this definition, the scalar product is independent on the choice of spatial hypersurface Σ.

In terms of orthonormal modes,

$$(\delta\phi_k, \delta\phi_{k'}) = \delta^D(\mathbf{k} - \mathbf{k}'), \tag{24}$$

the fluctuation field $\delta\phi$ can be expanded as

$$\delta\phi(\eta, \mathbf{x}) = \int \mathrm{d}^D\mathbf{k}[a_\mathbf{k}\delta\phi_k(\eta, \mathbf{x}) + a^\dagger_\mathbf{k}\delta\phi^*_k(\eta, \mathbf{x})]. \tag{25}$$

The corresponding creation and annihilation operators satisfy the standard commutation relations

$$[a_\mathbf{k}, a^\dagger_{\mathbf{k}'}] = \delta^D(\mathbf{k} - \mathbf{k}') \tag{26}$$

and the vacuum state associated to the quantum modes $\{\delta\phi_k\}$ is defined as usual by $a_\mathbf{k}|0\rangle = 0\ \forall\mathbf{k}$.

In order to construct the orthonormal set, we use a WKB ansatz,

$$\delta\phi_k(\eta, \mathbf{x}) = f_k(\eta, \mathbf{x})e^{i\theta_k(\eta, \mathbf{x})}, \tag{27}$$

and assume that $f_k(\eta, \mathbf{x})$ evolves slowly in space and time, whereas the evolution of $\theta_k(\eta, \mathbf{x})$ is rapid. In general, as mentioned above, such an adiabatic ansatz works whenever the Compton wavelength of the field perturbation is much smaller than the typical astrophysical or cosmological scales involved. In particular, in the adiabatic expansion we assume $\partial\theta \sim ma$ and $\partial f \sim \mathcal{H} f$.

Substituting (27) in (20), we obtain to the leading adiabatic order $\mathcal{O}((\partial\theta)^2)$

$$-\theta'^2_k + [1 + 2(\Phi + \Psi)](\nabla\theta_k)^2 + m^2a^2(1 + 2\Phi) = 0 \tag{28}$$

and to the next-to-leading order $\mathcal{O}(\partial\theta)$

$$\begin{aligned} &f_k\theta''_k + 2f'_k\theta'_k + [(D-1)\mathcal{H} - \Phi' - D\Psi']f_k\theta'_k \\ &\quad - f_k\nabla^2\theta_k - 2\nabla f_k\cdot\nabla\theta_k \\ &\quad - f_k\nabla\theta_k\cdot\nabla[\Phi - (D-2)\Psi] = 0. \end{aligned} \tag{29}$$

Notice that $\partial^2\theta \sim \mathcal{H}\partial\theta$ and that, in the adiabatic expansion, $\mathcal{H} \sim \partial\Phi$.

IV. PERTURBATIVE EXPANSION AND MODE SOLUTIONS

To solve these two equations, (28) and (29), we look for a perturbative expansion in the metric potentials. To obtain the lowest-order solution; i.e., in the absence of metric perturbations, we write (20) in the limit $\Phi = \Psi = 0$ and get

$$\delta\phi^{(0)\prime\prime} + (D-1)\mathcal{H}\delta\phi^{(0)\prime} - \nabla^2\delta\phi^{(0)} + a^2m^2(\hat{\phi})\delta\phi^{(0)} = 0, \tag{30}$$

where $a^2m^2(\hat{\phi})$ only depends on time. Fourier transforming the spatial coordinates, the following positive frequency solution with momentum $\mathbf{k}$ is obtained

$$\delta\phi^{(0)}_k(\eta, \mathbf{x}) = F_k(\eta)e^{i\mathbf{k}\cdot\mathbf{x} - i\int^\eta \omega_k(\eta')\mathrm{d}\eta'} \tag{31}$$

with

$$\omega^2_k = k^2 + m^2a^2 \tag{32}$$

and

$$F_k(\eta) = \frac{1}{(2\pi)^{D/2}}\frac{1}{a^{(D-1)/2}\sqrt{2\omega_k}}, \tag{33}$$

which is fixed by the normalization condition (24).

Once the unperturbed solution is known, we can look for the first-order corrections. Thus, the amplitude and phase of (27) are expanded in metric perturbations as follows

$$\begin{aligned} f_k(\eta, \mathbf{x}) &= F_k(\eta) + \delta f_k(\eta, \mathbf{x}) \\ \theta_k(\eta, \mathbf{x}) &= \mathbf{k}\cdot\mathbf{x} - \int^\eta \omega_k(\eta')\mathrm{d}\eta' + \delta\theta_k(\eta, \mathbf{x}) \end{aligned} \tag{34}$$

where δf_k and $\delta\theta_k$ are first order in perturbations. Substituting (35) in the leading equation (28), we obtain

(32) to the lowest order as expected, and to first order we get

$$\omega_k\delta\theta_k' + \mathbf{k}\cdot\boldsymbol{\nabla}\delta\theta_k + k^2(\Phi+\Psi) + m^2a^2\Phi = 0. \tag{35}$$

On the other hand, by substituting in the next-to-leading equation (30), we recover (33) to the lowest perturbative order, whereas to first order we get

$$\begin{aligned} &F_k\delta\theta_k'' + 2F_k'\delta\theta_k' + (D-1)\mathcal{H}F_k\delta\theta_k' - F_k\nabla^2\delta\theta_k \\ &\quad - 2\omega_k\delta f_k' - 2\mathbf{k}\cdot\boldsymbol{\nabla}\delta f_k - (D-1)\omega_k\mathcal{H}\delta f_k - \omega_k'\delta f_k \\ &\quad + \omega_kF_k\Phi' + D\omega_kF_k\Psi' - F_k\mathbf{k}\cdot\boldsymbol{\nabla}[\Phi-(D-2)\Psi] = 0. \end{aligned} \tag{36}$$

The two new equations (35) and (36) can also be solved by performing an additional Fourier transformation in the spatial coordinates since the equations coefficients only depend on time.

A. Phase solution $\delta\theta_k$

Equation (35) in Fourier space reads

$$\delta\theta_k'(\eta,\mathbf{p}) + i\frac{\mathbf{k}\cdot\mathbf{p}}{\omega_k}\delta\theta_k(\eta,\mathbf{p}) = -\omega_k\left[\Phi(\eta,\mathbf{p}) + \frac{k^2}{\omega_k^2}\Psi(\eta,\mathbf{p})\right], \tag{37}$$

where

$$\delta\theta_k(\eta,\mathbf{p}) = \frac{1}{(2\pi)^{3/2}}\int \mathrm{d}^3\mathbf{x}\,\delta\theta_k(\eta,\mathbf{x})e^{-i\mathbf{p}\cdot\mathbf{x}} \tag{38}$$

and analogous definitions apply for $\Phi(\eta,\mathbf{p})$, $\Psi(\eta,\mathbf{p})$ and $\delta f_k(\eta,\mathbf{p})$.[2] Defining

$$\begin{aligned} \beta_k(\eta_f,\eta_i) &= \int_{\eta_i}^{\eta_f}\frac{\mathrm{d}\eta'}{\omega_k(\eta')} \\ G_k(\eta,\mathbf{p}) &= -\omega_k\left[\Phi(\eta,\mathbf{p}) + \frac{k^2}{\omega_k^2}\Psi(\eta,\mathbf{p})\right], \end{aligned} \tag{39}$$

the solution of (37) is

$$\begin{aligned} &\delta\theta_k(\eta,\mathbf{p}) \\ &\quad = \int_0^\eta e^{-i\mathbf{k}\cdot\mathbf{p}\beta_k(\eta,\eta')}G_k(\eta',\mathbf{p})\mathrm{d}\eta' + e^{-i\mathbf{k}\cdot\mathbf{p}\beta_k(\eta,0)}\delta\theta_k(0,\mathbf{p}). \end{aligned} \tag{40}$$

The term $\delta\theta_k(0,\mathbf{p})$ stands for the initial boundary condition of the modes or, equivalently, the phase difference of the modes at the initial time. In principle, $\delta\theta_k(0,\mathbf{p})$ is not completely arbitrary since the orthonormalization condition of the modes (24) may constrain its functional dependence. We discuss this point at the end of this section.

B. Amplitude solution δf_k

Let us write

$$\delta f_k(\eta,\mathbf{p}) = F_k(\eta)P_k(\eta,\mathbf{p}) \tag{41}$$

and following a similar procedure with the next-to-leading-order equation (36), it can be rewritten in Fourier space as

$$P_k'(\eta,\mathbf{p}) + i\frac{\mathbf{k}\cdot\mathbf{p}}{\omega_k}P_k(\eta,\mathbf{p}) = \frac{H_k(\eta,\mathbf{p})}{2\omega_k}, \tag{42}$$

where

$$H_k(\eta,\mathbf{p}) = \omega_kQ_k'(\eta,\mathbf{p}) + T_k(\eta,\mathbf{p}) \tag{43}$$

with

$$Q_k(\eta,\mathbf{p}) = -i\frac{\mathbf{k}\cdot\mathbf{p}}{\omega_k^2}\delta\theta_k(\eta,\mathbf{p}) + \left[D - \frac{k^2}{\omega_k^2}\right]\Psi(\eta,\mathbf{p}) \tag{44}$$

and

$$\begin{aligned} T_k(\eta,\mathbf{p}) &= p^2\delta\theta_k(\eta,\mathbf{p}) \\ &\quad - i\mathbf{k}\cdot\mathbf{p}[\Phi(\eta,\mathbf{p}) - (D-2)\Psi(\eta,\mathbf{p})]. \end{aligned} \tag{45}$$

The corresponding solution is given by

$$\begin{aligned} &P_k(\eta,\mathbf{p}) \\ &\quad = \int_0^\eta e^{-i\mathbf{k}\cdot\mathbf{p}\beta_k(\eta,\eta')}\frac{H_k(\eta',\mathbf{p})}{2\omega_k(\eta')}\mathrm{d}\eta' + e^{-i\mathbf{k}\cdot\mathbf{p}\beta_k(\eta,0)}P_k(0,\mathbf{p}). \end{aligned} \tag{46}$$

The integration constant $P_k(0,\mathbf{p})$ is fixed by the normalization condition (24).

1. Time-independent gravitational potentials

For simplicity, in the rest of the work we focus on time-independent gravitational potentials. This case encompasses super-Hubble modes in both matter and radiation era, and also sub-Hubble modes in the matter era. This is also a good approximation to describe the gravitational potentials in the Solar System. In such a case, the constants $P_k(0,\mathbf{p})$ are given by

$$P_k(0,\mathbf{p}) = \frac{1}{2}\left(D - \frac{k^2}{\omega_k(0)^2}\right)\Psi(\mathbf{p}). \tag{47}$$

Integrating by parts in (46), the integration constant can be eliminated and the following expression is obtained:

[2] In the following, the wave vector of the quantum modes is denoted by $\mathbf{k}$, and $\mathbf{p}$ is used for that of metric perturbations.

$$P_k(\eta,\mathbf{p}) = \frac{1}{2}Q_k(\eta,\mathbf{p}) - i\int_0^\eta \left\{\frac{\mathbf{k}\cdot\mathbf{p}}{2\omega_k(\eta')}e^{-i\mathbf{k}\cdot\mathbf{p}\beta_k(\eta,\eta')} \times \left[Q_k(\eta',\mathbf{p}) + \frac{T_k(\eta',\mathbf{p})}{\mathbf{k}\cdot\mathbf{p}}\right]\right\}\mathrm{d}\eta'. \quad (48)$$

There are three types of contributions to P_k, depending on the number of time integrals involved. Thus, we can write

$$P_k(\eta,\mathbf{p}) = P_k^{(0)}(\eta,\mathbf{p}) + P_k^{(1)}(\eta,\mathbf{p}) + P_k^{(2)}(\eta,\mathbf{p}) \quad (49)$$

where

$$P_k^{(0)}(\eta,\mathbf{p}) = \frac{1}{2}\left(D - \frac{k^2}{\omega_k(\eta)^2}\right)\Psi(\mathbf{p}) \quad (50)$$

$$P_k^{(1)}(\eta,\mathbf{p}) = \int_0^\eta e^{-i\mathbf{k}\cdot\mathbf{p}\beta_k(\eta,\eta')}N_k^{(1)}(\eta,\eta',\mathbf{p})\mathrm{d}\eta' \quad (51)$$

$$P_k^{(2)}(\eta,\mathbf{p}) = \int_0^\eta\int_0^{\eta'} e^{-i\mathbf{k}\cdot\mathbf{p}\beta_k(\eta,\eta'')}N_k^{(2)}(\eta',\eta'',\mathbf{p})\mathrm{d}\eta''\mathrm{d}\eta' \quad (52)$$

with

$$N_k^{(1)}(\eta,\eta',\mathbf{p}) = \frac{i\mathbf{k}\cdot\mathbf{p}}{2\omega_k^2(\eta)\omega_k(\eta')}\left\{[\omega_k^2(\eta') - \omega_k^2(\eta)]\Phi(\mathbf{p}) + \left[k^2 + \omega_k^2(\eta)\left(\frac{k^2}{\omega_k^2(\eta')} - 2\right)\right]\Psi(\mathbf{p})\right\} \quad (53)$$

$$N_k^{(2)}(\eta',\eta'',\mathbf{p}) = \frac{(\mathbf{k}\cdot\mathbf{p})^2 - p^2\omega_k^2(\eta')}{2\omega_k^3(\eta')\omega_k(\eta'')} \times [\omega_k^2(\eta'')\Phi(\mathbf{p}) + k^2\Psi(\mathbf{p})] \quad (54)$$

where $p = |\mathbf{p}|$.

C. Orthonormalization condition

In order to quantize the field canonically, we must check that the modes $\delta\phi_k$ used to define the creation and annihilation operators are orthonormal (24). This may restrict the functional dependence of the initial conditions of our solution, i.e., $P_k(0,\mathbf{p})$ and $\delta\theta_k(0,\mathbf{p})$.[3] We already fixed $P_k(0,\mathbf{p})$ when imposing the correct normalization of the modes; hence, we can only play with $\delta\theta_k(0,\mathbf{p})$ to have orthogonal modes. The scalar product (21) can be computed using (27), (35), (40), and (46) to get

$$(\delta\phi_k, \delta\phi_{k'}) = \delta^D(\mathbf{k}-\mathbf{k}') + \tau_\Psi(\mathbf{k},\mathbf{k}') + \tau_{\delta\theta}(\mathbf{k},\mathbf{k}'), \quad (55)$$

where $\tau_{\Psi,\delta\theta}$ are first order in metric perturbation. The explicit expressions for $\tau_{\Psi,\delta\theta}$ are given in Appendix A. In this appendix it is shown that they are zero for $\forall\mathbf{k},\mathbf{k}'$ up to corrections beyond the leading adiabatic order for slowly varying gravitational fields. This result does not impose any restriction on the functional dependence of $\delta\theta_k(0,\mathbf{p})$.

Different initial conditions $\delta\theta_k(0,\mathbf{p})$ amount to different definitions of the vacuum. The discussion above guarantees that the modes given by (27), (35), (40), (46), are orthonormal for any choice of the vacuum. In the following we take $\delta\theta_k(0,\mathbf{p}) = 0$ as the initial condition for the modes.

V. HIGGS EFFECTIVE POTENTIAL

Once we have the expressions for the mode solutions of the perturbative equations, namely (40) and (41) [together with (46)]; we can proceed to calculate the one-loop contribution to the effective potential (17).

Let us first calculate $\langle 0|\delta\phi^2(\eta,\mathbf{x})|0\rangle$ to first order in metric perturbations. Because of the inhomogeneity of the background, this quantity depends on $(\eta,\mathbf{x})$ as follows

$$\langle 0|\delta\phi^2(\eta,\mathbf{x})|0\rangle = \langle\delta\phi^2\rangle_\mathrm{h}(\eta) + \langle\delta\phi^2\rangle_\mathrm{i}(\eta,\mathbf{x}) \quad (56)$$

where

$$\langle\delta\phi^2\rangle_\mathrm{h}(\eta) = \int \mathrm{d}^D\mathbf{k}F_k^2(\eta) \quad (57)$$

and

$$\langle\delta\phi^2\rangle_\mathrm{i}(\eta,\mathbf{x}) = 2\int \mathrm{d}^D\mathbf{k}F_k^2(\eta)[\mathrm{Re}P_k(\eta,\mathbf{x}) - \mathrm{Im}\delta\theta_k(\eta,\mathbf{x})]. \quad (58)$$

A. Homogeneous contribution $\langle\delta\phi^2\rangle_\mathrm{h}$

The homogeneous contribution $\langle\delta\phi^2\rangle_\mathrm{h}$ reads

$$\langle\delta\phi^2\rangle_\mathrm{h}(\eta) = \frac{1}{2(2\pi)^D a^{D-1}(\eta)}\int\frac{\mathrm{d}^D\mathbf{k}}{\sqrt{k^2+m^2a^2(\eta)}} = \frac{1}{2(2\pi)^D a^{D-1}(\eta)}\frac{2\pi^{D/2}}{\Gamma(D/2)}\int_0^\infty\frac{\mathrm{d}k k^{D-1}}{\sqrt{k^2+m^2a^2(\eta)}} \quad (59)$$

which is analogous to the Minkowskian result, except for the scale-factor dependence.

B. Nonhomogeneous contribution $\langle\delta\phi^2\rangle_\mathrm{i}$

The inhomogeneous component $\langle\delta\phi^2\rangle_\mathrm{i}$ can be dealt with more easily in momentum space. The only angular dependence of the quantum fluctuation wave vector $\mathbf{k}$

[3] $P_k(0,\mathbf{x})$ and $\delta\theta_k(0,\mathbf{x})$ are assumed to be real. If this were not the case, the phase of $P_k(0,\mathbf{x})$ could be absorbed into $\delta\theta_k(0,\mathbf{x})$ and the imaginary part of $\delta\theta_k(0,\mathbf{x})$ could also be absorbed into $P_k(0,\mathbf{x})$ in a trivial way.

enters as $\mathbf{k}\cdot\mathbf{p} = kp\hat{x}$ with $\hat{x} = \cos\theta$, where we have taken the k_z direction along $\mathbf{p}$. On the other hand, the contribution from $\delta\theta$ in (58) vanishes after integrating in $\hat{x}$. Then, we have

$$\langle\delta\phi^2\rangle_{\rm i}(\eta,\mathbf{p}) = \frac{1}{(2\pi)^D a^{D-1}(\eta)}\int \mathrm{d}^D\mathbf{k}\,\frac{P_k(\eta,\mathbf{p})}{\sqrt{k^2+m^2a^2(\eta)}}. \tag{60}$$

Since the integration on $\hat{x}$ can be performed in a straightforward way, let us define

$$\hat{P}_k(\eta,\mathbf{p}) = \int_{-1}^{1}\mathrm{d}\hat{x}(1-\hat{x}^2)^{(D-3)/2}P_k(\eta,\mathbf{p}), \tag{61}$$

where we have included the general integration measure in D dimensions. Hence, we can write (see Appendix B)

$$\langle\delta\phi^2\rangle_{\rm i}(\eta,\mathbf{p}) = \frac{1}{(2\pi)^D a^{D-1}(\eta)}\frac{2\pi^{(D-1)/2}}{\Gamma((D-1)/2)} \times\int_0^\infty \mathrm{d}k\,\frac{k^{D-1}\hat{P}_k(\eta,\mathbf{p})}{\sqrt{k^2+m^2a^2(\eta)}}. \tag{62}$$

Both integrals (60) and (62) are divergent in $D=3$ dimensions, and they should be regularized as discussed in the next section.

C. Regularization

Let us now discuss the regularization procedure based on standard dimensional regularization techniques.

1. Regularized homogeneous contribution $\langle\delta\phi^2\rangle_{\rm h}(\eta)$

The momentum integral in $\langle\delta\phi^2\rangle_{\rm h}$ (60) can be done using (B3) of Appendix B. After expanding for small ϵ with $D = 3-\epsilon$ dimensions, the final result is

$$\langle\delta\phi^2\rangle_{\rm h}(\eta) = \frac{m^2(\hat{\phi})}{16\pi^2}\left[\ln\left(\frac{m^2(\hat{\phi})}{\mu^2}\right) - N_\epsilon - \frac{3}{2}\right], \tag{63}$$

where μ is the renormalization scale and

$$N_\epsilon = \frac{2}{\epsilon} + \log 4\pi - \gamma \tag{64}$$

with γ the Euler-Mascheroni constant.

2. Regularized nonhomogeneous contribution $\langle\delta\phi^2\rangle_{\rm i}(\eta,\mathbf{x})$

Let us now consider the inhomogeneous contribution (62). We cannot apply directly standard dimensional regularization formulas because of the nontrivial k dependence of $\hat{P}_k(\eta,\mathbf{p})$. Thus, additional work is necessary.

First, it should be noticed that the dependence of $\hat{P}_k(\eta,\mathbf{p})$ on the direction of $\mathbf{p}$ only enters through the potentials, $\Phi(\mathbf{p})$ and $\Psi(\mathbf{p})$. Therefore, it can be expanded in the following way:

$$\hat{P}_k(\eta,\mathbf{p}) = \left[\sum_{l=0}^{\infty}P^{\Phi}_{k,l}(\eta)p^{2l}\right]\Phi(\mathbf{p}) + \left[\sum_{l=0}^{\infty}P^{\Psi}_{k,l}(\eta)p^{2l}\right]\Psi(\mathbf{p}). \tag{65}$$

The coefficients $P^{\{\Phi,\Psi\}}_{k,l}(\eta)$ are given in Appendix C. The $l=0$ terms only get contributions from the $P^{(0)}_k(\eta,\mathbf{p})$ term given in (50), and its integral vanishes in dimensional regularization. The $l>0$ terms involve time integrals of the form

$$\int_0^\eta \mathrm{d}\eta'\left(\prod_{i=1}^{2l-1}\int_{\eta'}^{\eta}\frac{\mathrm{d}\eta_i}{\omega_k(\eta_i)}\right)\frac{k^{2\alpha}}{\omega_k(\eta)^a\omega_k(\eta')^b} \tag{66}$$

for the contributions coming from $P^{(1)}_k(\eta,\mathbf{p})$ in (51), and

$$\int_0^\eta \mathrm{d}\eta'\int_0^{\eta'}\mathrm{d}\eta''\left(\prod_{i=1}^{2l-2}\int_{\eta''}^{\eta}\frac{\mathrm{d}\eta_i}{\omega_k(\eta_i)}\right)\frac{k^{2\alpha}}{\omega_k(\eta)^a\omega_k(\eta')^b\omega_k(\eta'')^c} \tag{67}$$

for those coming from $P^{(2)}_k(\eta,\mathbf{p})$ in (52), with α, a, b, $c\in\mathbb{Z}$. In order to simplify the functional dependence on k, we apply the generalized Feynman trick,

$$\frac{1}{A_1^{d_1}\cdots A_n^{d_n}} = \frac{\Gamma(d_1+\cdots+d_n)}{\Gamma(d_1)\cdots\Gamma(d_n)} \times\int_0^1\mathrm{d}x_1\cdots\int_0^1\mathrm{d}x_n\times\delta(x_1+\ldots+x_n-1) \times\frac{x_1^{d_1-1}\cdots x_n^{d_n-1}}{(x_1A_1+\cdots+x_nA_n)^{d_1+\cdots+d_n}}. \tag{68}$$

Then, let the parameters of the Feynman formula be defined by

$$n = 2l+1 \tag{69}$$

$$A_j = \begin{cases}\omega_k^2(\eta) & \text{if } j=1\\ \omega_k^2(\eta') & \text{if } j=2\\ \omega_k^2(\eta_{j-2}) & \text{if } 3\le j\le 2l+1\end{cases} \tag{70}$$

$$d_j = \begin{cases}a/2 & \text{if } j=1\\ b/2 & \text{if } j=2\\ 1/2 & \text{if } 3\le j\le 2l+1\end{cases} \tag{71}$$

for the case (66) [with a trivial modification for the expression (67)]. In this way, the k dependence only appears in $\sum_{i=1}^{2l+1} x_i \omega_{k,i}^2$ which can be simplified in the following way,

$$\sum_{i=1}^{2l+1} x_i \omega_{k,i}^2 = \sum_{i=1}^{2l+1} x_i (k^2 + m^2 a_i^2) = k^2 + m^2 \sum_{i=1}^{2l+1} x_i a_i^2, \qquad (72)$$

where we have used $\sum_{i=1}^{2l+1} x_i = 1$. Now, the k dependence is simple enough to use standard dimensional regularization formulas (Appendix B). The integration over the $\{x_i\}$ and the time integrals can be performed analytically (Appendix D).

As we did with $\hat{P}_k(\eta, \mathbf{p})$, we now decompose $\langle \delta\phi^2 \rangle_{\rm i}(\eta, \mathbf{p})$ into two terms proportional to $\Phi(\mathbf{p})$ and $\Psi(\mathbf{p})$, respectively,

$$\langle \delta\phi^2 \rangle_{\rm i}(\eta, \mathbf{p}) = \langle \delta\phi^2 \rangle_{\rm i}^{\Phi}(\eta, \mathbf{p}) \Phi(\mathbf{p}) + \langle \delta\phi^2 \rangle_{\rm i}^{\Psi}(\eta, \mathbf{p}) \Psi(\mathbf{p}). \qquad (73)$$

Then, integrating in dimensional regularization, we see that the $\mathcal{O}(1/\epsilon)$ terms cancel out, and the results are finite,

$$\langle \delta\phi^2 \rangle_{\rm i}^{\{\Phi,\Psi\}}(\eta, \mathbf{p}) = \frac{m^2}{4\pi^2 a^2(\eta)} \left[\sum_{l=1}^{\infty} R_l^{\{\Phi,\Psi\}}(\eta) p^{2l} \right], \qquad (74)$$

where $R_l^{\{\Phi,\Psi\}}$ are the already regularized integrals in k of $P_{k,l}^{\{\Phi,\Psi\}}$ divided by m^2 for convenience. The coefficients $R_l^{\{\Phi,\Psi\}}$ can be written as

$$R_l^{\{\Phi,\Psi\}}(\eta) = R_{l,\rm pol}^{\{\Phi,\Psi\}}(\eta) + R_{l,\rm log}^{\{\Phi,\Psi\}}(\eta), \qquad (75)$$

where, as shown in Appendix D, $R_{l,\rm pol}^{\{\Phi,\Psi\}}$ are polynomials in η, and $R_{l,\rm log}^{\{\Phi,\Psi\}}$ involve a logarithmic dependence on η.

The most important aspect of (74) is that all the divergent parts have canceled out. In particular, the divergent terms coming from $P_k^{(1)}(\eta, \mathbf{p})$ cancel exactly the ones from $P_k^{(2)}(\eta, \mathbf{p})$ order by order in p. This means that the UV behavior is the same as in an unperturbed FRW background and the inhomogeneous contributions are finite to the leading adiabatic order.

D. Nonhomogeneous contribution: Particular cases

1. Nonexpanding spacetimes

Let us consider weak gravitational fields generated by static sources. For the corresponding spacetime metric, we can take (19) with $a(\eta) = 1$ and static potentials $\Phi(\mathbf{x})$ and $\Psi(\mathbf{x})$ which allow us to use the previous results. This simplifies the calculations in several of the steps discussed above. For instance, all the time integrals can be done in a straightforward way, there is no need to apply the Feynamn trick since the ω's are all the same, and the coefficients $R_{l,\rm log}^{\{\Phi,\Psi\}}$ are zero (see Appendix D).

The results for a nonexpanding geometry read

$$R_l^{\Phi}(\eta) = R_{l,\rm pol}^{\Phi}(\eta) = 0 \qquad (76)$$

$$R_l^{\Psi}(\eta) = R_{l,\rm pol}^{\Psi}(\eta) = 0, \qquad (77)$$

which imply

$$\langle \delta\phi^2 \rangle_{\rm i}^{\Phi}(\eta, \mathbf{p}) = 0 \qquad (78)$$

$$\langle \delta\phi^2 \rangle_{\rm i}^{\Psi}(\eta, \mathbf{p}) = 0 \qquad (79)$$

and

$$\langle \delta\phi^2 \rangle_{\rm i}(\eta, \mathbf{p}) = 0. \qquad (80)$$

Thus, to the leading adiabatic order, the metric perturbations do not contribute to the Higgs effective potential in dimensional regularization. This is in contrast with previous results [31] using cutoff regularization, in which nonvanishing inhomogeneous contributions were obtained.

Although we have considered a particular coordinate choice in (19), corresponding to the longitudinal gauge, since in the absence of metric perturbations $V_{\rm eff}^{\rm h}(\hat{\phi})$ is a constant, the Stewart-Walker lemma [36] guarantees that the obtained effective potential is gauge invariant.

2. Expanding spacetimes: Cosmology

Now we consider the case of a perturbed expanding universe with scale factor $a(\eta)$ and constant metric perturbations $\Phi(\mathbf{x})$ and $\Psi(\mathbf{x})$. In particular, we will concentrate in the matter-dominated era, in which the metric perturbations are constant both for sub-Hubble and super-Hubble modes. In addition, we will also provide results for super-Hubble modes in the radiation era for which the metric perturbations are also constant.

For the Ψ contribution, we get for the matter and radiation eras with $a \propto \eta^2$ and $a \propto \eta$, respectively,

$$R_{l,\rm pol}^{\Psi}(\eta) = 0; \qquad R_{l,\rm log}^{\Psi}(\eta) = 0. \qquad (81)$$

The Φ terms are harder to compute since the $R_{l,\rm log}^{\Phi}$ contribution is not zero, and the integration over the Feynman parameters $\{x_i\}$ and the time integrals has to be performed by Taylor expanding the logarithm (see Appendix D). An exact analytical expression can be obtained for each order of the logarithm expansion given in terms of finite sums, which can be computed numerically for practical purposes. We have checked that the relative difference between $R_{l,\rm pol}^{\Phi}$ and $R_{l,\rm log}^{\Phi}$ terms is $\sim 10^{-4}$ for $l = 1, 2, 3$ and $\sim 10^{-2}$ for $l = 4, 5$. Then,

$$\frac{R^{\Phi}_{l,\mathrm{pol}}(\eta)+R^{\Phi}_{l,\log}(\eta)}{R^{\Phi}_{l,\mathrm{pol}}(\eta)}\leq 10^{-2}\quad \text{for } l=1,2,3,4,5.$$

This suggests that the exact Φ contribution also may be zero, as for the Ψ terms, so that for expanding geometries as well, static perturbations do not contribute to the Higgs effective potential to the leading adiabatic order.

E. Higgs effective potential

Taking into account (17), the one-loop contribution to the effective potential can be expressed as

$$V_1(\eta,\mathbf{x})=V_1^{\mathrm{h}}(\eta)+V_1^{\mathrm{i}}(\eta,\mathbf{x}). \tag{82}$$

Given the fact that, to the leading order, the nonhomogeneous contribution vanishes, the potential reads

$$V_1=V_1^{\mathrm{h}}(\eta)=\frac{1}{2}\int_0^{m^2(\hat{\phi})}\mathrm{d}m^2\langle\delta\phi^2\rangle_{\mathrm{h}}(\eta), \tag{83}$$

and substituting (63), we get

$$V_1(\hat{\phi})=\frac{m^4(\hat{\phi})}{64\pi^2}\left[\ln\left(\frac{m^2(\hat{\phi})}{\mu^2}\right)-N_\epsilon-\frac{3}{2}\right]. \tag{84}$$

As expected from previous works [17–26], the homogeneous contribution is constant even though the geometry is expanding. The N_ϵ term is proportional to $m^4(\hat{\phi})$, so that we have three kinds of divergences: constant, quadratic in $\hat{\phi}$ and quartic, which can be reabsorbed in the renormalization of the tree-level potential parameters V_0, M^2 and λ. This means that at the leading adiabatic order we obtain exactly the same divergences as in flat spacetime and we do not need additional counterterms to renormalize the effective potential.

Following the minimal subtraction scheme $\overline{\mathrm{MS}}$, we remove the terms proportional to N_ϵ. Thus, we are left with the complete renormalized homogeneous effective potential,

$$V_{\mathrm{eff}}(\hat{\phi})=V_0+\frac{1}{2}M^2\hat{\phi}^2+\frac{\lambda}{4}\hat{\phi}^4+\frac{m^4(\hat{\phi})}{64\pi^2}\left[\ln\left(\frac{m^2(\hat{\phi})}{\mu^2}\right)-\frac{3}{2}\right], \tag{85}$$

which agrees with the standard result in flat spacetime. Here, the physical mass M and coupling constant λ are defined at a given physical scale μ. Since the renormalized effective potential is independent of the renormalization scale μ, M^2 and the coupling constant should depend on μ according to the renormalization group equations

$$\beta(\lambda)\equiv\frac{\mathrm{d}\lambda}{\mathrm{d}(\log\mu)}=\frac{18\lambda^2}{(4\pi)^2}$$
$$\gamma_M(\lambda)\equiv\frac{\mathrm{d}(\log M^2)}{\mathrm{d}(\log\mu)}=\frac{6\lambda}{(4\pi)^2}. \tag{86}$$

VI. ENERGY-MOMENTUM TENSOR

In the previous sections, we have considered the one-loop correction to the effective potential. The complete set of perturbed modes obtained also allows us to evaluate the vacuum expectation value of the energy-momentum tensor. For the sake of completeness we will include also a possible nonminimal coupling to curvature, so that the equation for an arbitrary massive scalar field now reads

$$(\Box+m^2+\xi R)\varphi=0, \tag{87}$$

Notice that, to the leading adiabatic order, the curvature term is not going to modify the mode solutions found in Sec. IV; however, the energy-momentum tensor acquires new contributions. Thus,

$$\begin{aligned}T^\mu_\nu={}&-\delta^\mu_\nu\left(\frac{1}{2}-2\xi\right)(g^{\rho\sigma}\partial_\rho\varphi\partial_\sigma\varphi-m^2\varphi^2)\\&+(1-2\xi)g^{\mu\rho}\partial_\rho\varphi\partial_\nu\varphi-2\xi\varphi\nabla^\mu\nabla_\nu\varphi\\&+\frac{2}{D+1}\xi g^\mu_\nu(\varphi\Box\varphi+m^2\varphi^2)\\&-\xi\left(R^\mu_\nu-\frac{1}{2}Rg^\mu_\nu+\frac{2D}{D+1}\xi Rg^\mu_\nu\right)\varphi^2.\end{aligned} \tag{88}$$

Considering perturbations over a flat Robertson-Walker background (19), the vacuum expectation value of this tensor, $\langle T^\mu_\nu\rangle$, can be explicitly written to the leading adiabatic order in Fourier space as a mode sum in terms of the expansion (35) as

$$\langle T^0_0(\eta,\mathbf{p})\rangle=\rho(\eta,\mathbf{p})=\frac{1}{(2\pi)^D}\frac{1}{a^{D+1}}\int\mathrm{d}^D\mathbf{k}\,\frac{\omega_k}{2}\left[1+2\frac{k^2}{\omega_k^2}\Psi(\mathbf{p})+2P_k(\eta,\mathbf{p})+2i\frac{\mathbf{k}\cdot\mathbf{p}}{\omega_k^2}\delta\theta_k(\eta,\mathbf{p})-\frac{2\xi}{\omega_k^2}P''_k(\eta,\mathbf{p})\right] \tag{89}$$

$$\langle T^i_i(\eta,\mathbf{p})\rangle=-p_i(\eta,\mathbf{p})=-\frac{1}{(2\pi)^D}\frac{1}{a^{D+1}}\int\mathrm{d}^D\mathbf{k}\left[\frac{k_i^2}{2\omega_k}(1+2\Psi(\mathbf{p})+2P_k(\eta,\mathbf{p}))+2i\frac{k_ip_i}{2\omega_k}\delta\theta_k(\eta,\mathbf{p})+\xi\frac{p_i^2}{\omega_k}P_k(\eta,\mathbf{p})\right] \tag{90}$$

$$\langle T^i_0(\eta,\mathbf{p})\rangle=\frac{1}{(2\pi)^D}\frac{1}{a^{D+1}}\int\mathrm{d}^D\mathbf{k}\left[\frac{k_i}{2}\left(1+2P_k(\eta,\mathbf{p})+2i\frac{\mathbf{k}\cdot\mathbf{p}}{2\omega_k^2}\delta\theta_k(\eta,\mathbf{p})\right)+\frac{i}{2}p_i\delta\theta_k(\eta,\mathbf{p})+\xi\frac{ip_i}{\omega_k}P'_k(\eta,\mathbf{p})\right] \tag{91}$$

$$\langle T^i_j(\eta,\mathbf{p})\rangle = -\frac{1}{(2\pi)^D}\frac{1}{a^{D+1}}\int \mathrm{d}^D\mathbf{k}\left[\frac{k_i k_j}{2\omega_k}(1+2\Psi(\mathbf{p})+2P_k(\eta,\mathbf{p})) + i\frac{k_i p_j + k_j p_i}{2\omega_k}\delta\theta_k(\eta,\mathbf{p}) + \xi\frac{p_i p_j}{\omega_k}P_k(\eta,\mathbf{p})\right] \tag{92}$$

$$\langle T^\mu_\mu(\eta,\mathbf{p})\rangle = \frac{1}{(2\pi)^D}\frac{1}{a^{D+1}}\int \mathrm{d}^D\mathbf{k}\left[\frac{m^2}{2\omega_k}(1+2P_k(\eta,\mathbf{p})) - \frac{\xi}{\omega_k}(P''_k(\eta,\mathbf{p})+p^2P_k(\eta,\mathbf{p}))\right]. \tag{93}$$

The integration over the quantum modes can be performed using the same methods applied above and some tricks to reduce the integrals involving the components of $\mathbf{k}$, k_i or $k_i k_j$, to integrals of scalar character in $\mathbf{k}$ (Appendix B). After doing that, the homogeneous part is found to be diagonal, and the energy ρ and pressure p are given in the minimal substraction scheme $\overline{\mathrm{MS}}$ by

$$\rho = -p = \frac{m^4}{64\pi^2}\left[\log\left(\frac{m^2}{\mu^2}\right) - \frac{3}{2}\right], \tag{94}$$

where μ is the renormalization physical scale.

On the other hand, much as for the effective potential, the nonhomogeneous part of the energy-momentum tensor vanishes to this order.

While classical and weak gravitational fields are not able to change the UV behavior of quantum effects, it is expected that gravity should modify the IR parts of all quantum corrections. The result presented in this work shows that, within the dimensional regularization scheme, there are no gravitational corrections arising from a perturbed FRW metric up to first order in perturbations, and to the leading order in the adiabatic expansion, to the vacuum expectation value of the energy-momentum tensor of a scalar field. Then, gravitational corrections may appear beyond the leading adiabatic order, or through nonlinear terms.

In the considered regime, namely the one in which the Hubble scale is much smaller than the mass of the quantum field, corrections beyond the zero adiabatic order are negligible and they are unlikely to belong to the experimental realm in the near future.

On the other hand, although nonlinear contributions are expected to be smaller than the linear ones, they will be more important than the contribution from the first adiabatic order. Nevertheless, the computation of the second-order corrections to the energy-momentum tensor is a formidable task which is well beyond the scope of this work.

VII. DISCUSSION AND CONCLUSIONS

In this work, we have computed the one-loop corrections to the effective potential due to the self-interactions of the Higgs field and the vacuum expectation value of its energy-momentum tensor in a perturbed FRW background. Unlike previous results based on the Schwinger–de Witt approximation, we have calculated explicitly a complete orthonormal set of modes of the perturbed Klein-Gordon equation and the dimensional regularization procedure was used for the mode summation to the leading adiabatic order. The integrals containing metric perturbations involved nonrational functions of the momenta so that standard formulas in dimensional regularization were not suitable to evaluate them. New expressions have been developed for those cases which applied both to static and expanding backgrounds.

We have checked that the homogeneous contribution agrees with the Minkowski result as expected. On the other hand, we have found that to the leading adiabatic order, and to first order in metric perturbations, no additional contributions appear either in the regularized effective potential nor in the energy-momentum tensor. This is in contrast with previous results obtained with a cutoff regularization [31], in which quartic and quadratic inhomogeneous divergences appear in the calculation. Thus, we see that dimensional regularization ensures that the theory can be renormalized just absorbing the divergences in the tree-level parameters (at the leading adiabatic order).

We expect additional contributions from the metric perturbations at the next-to-leading adiabatic orders. Unlike the Schwinger–de Witt method which provides a local expansion of the effective action. The mode summation method used in this work could allow to determine the corresponding finite nonlocal contributions. In this sense, the explicit mode calculation obtained here together with the method developed to perform the integrals in dimensional regularization of nonrational functions of the momenta are a fundamental first step in this program. The results presented in this work would also allow to calculate the temperature effects on the Higgs effective potential using the explicit mode summation and, in general, the complete expressions of other expectations values in perturbed metric backgrounds. Work is in progress in these directions.

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APPENDIX A: ORTHONORMALIZATION CONDITION: $\tau_\Psi(\mathbf{k},\mathbf{k}')$ AND $\tau_{\delta\theta}(\mathbf{k},\mathbf{k}')$

In this appendix, we show that $\tau_\Psi(\mathbf{k},\mathbf{k}')$ and $\tau_{\delta\theta}(\mathbf{k},\mathbf{k}')$ appearing in (55) are zero to the leading adiabatic order. This implies that the modes given by (27), (35), (40), (41), (46) are orthonormal and, therefore, the scalar field $\delta\phi$ can be quantized within the canonical formalism.

The explicit expressions for $\tau_\Psi(\mathbf{k},\mathbf{k}')$ and $\tau_{\delta\theta}(\mathbf{k},\mathbf{k}')$ are

$$\tau_\Psi(\mathbf{k},\mathbf{k}') = \int \mathrm{d}^D\mathbf{x}\,\frac{(\omega_k-\omega_{k'})(k^2\omega_{k'}^2 - k'^2\omega_k^2)}{4(\omega_k\omega_{k'})^{5/2}}\Psi(\mathbf{x})\frac{e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{x}}}{(2\pi)^D} \tag{A1}$$

$$\tau_{\delta\theta}(\mathbf{k},\mathbf{k}') = \int \mathrm{d}^D\mathbf{x}\,\frac{(\omega_k-\omega_{k'})(\omega_{k'}^2\mathbf{k}\cdot\nabla\delta\theta_k(0,\mathbf{x}) - \omega_k^2\mathbf{k}'\cdot\nabla\delta\theta_{k'}(0,\mathbf{x}))}{4(\omega_k\omega_{k'})^{5/2}}\frac{e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{x}}}{(2\pi)^D}. \tag{A2}$$

First, let us focus on τ_Ψ in Fourier space:

$$\begin{aligned}\tau_\Psi(\mathbf{k},\mathbf{k}') &= \int \mathrm{d}^D\mathbf{x}\int\frac{\mathrm{d}^D\mathbf{p}}{(2\pi)^{D/2}}\frac{(\omega_k-\omega_{k'})(k^2\omega_{k'}^2 - k'^2\omega_k^2)}{4(\omega_k\omega_{k'})^{5/2}}\Psi(\mathbf{p})\frac{e^{i(\mathbf{k}-\mathbf{k}'+\mathbf{p})\cdot\mathbf{x}}}{(2\pi)^D}\\ &= \int\frac{\mathrm{d}^D\mathbf{p}}{(2\pi)^{D/2}}\frac{(\omega_k-\omega_{k'})(k^2\omega_{k'}^2 - k'^2\omega_k^2)}{4(\omega_k\omega_{k'})^{5/2}}\Psi(\mathbf{p})\delta^D(\mathbf{k}-\mathbf{k}'+\mathbf{p})\\ &= \frac{1}{(2\pi)^{D/2}}\frac{(\omega_k-\omega_{k'})(k^2\omega_{k'}^2 - k'^2\omega_k^2)}{4(\omega_k\omega_{k'})^{5/2}}\Psi(\mathbf{k}-\mathbf{k}').\end{aligned} \tag{A3}$$

Since Ψ varies over macroscopic scales, we can a assume an exponential damping for Ψ when $|\mathbf{k}-\mathbf{k}'|\gg|\nabla\Psi|\sim\mathcal{H}$; therefore, $\tau_\Psi(\mathbf{k},\mathbf{k}')\approx 0$ in this case. For $|\mathbf{k}-\mathbf{k}'|\sim\mathcal{H}$, we can Taylor expand the coefficient in front of $\Psi(\mathbf{k}-\mathbf{k}')$ in $\mathcal{H}/\omega_k$ to get

$$\tau_\Psi(\mathbf{k},\mathbf{k}')\approx\frac{1}{(2\pi)^{D/2}}\frac{m^2k^2}{2\omega_k^4}\left(\frac{\mathcal{H}}{\omega_k}\right)^2\Psi(\mathbf{k}-\mathbf{k}'), \tag{A4}$$

which is beyond the leading adiabatic order.

The same procedure works for $\tau_{\delta\theta}$, for instance,

$$\begin{aligned}\tau_{\delta\theta}(\mathbf{k},\mathbf{k}') &= \int \mathrm{d}^D\mathbf{x}\int\frac{\mathrm{d}^D\mathbf{p}}{(2\pi)^{D/2}}\frac{(\omega_k-\omega_{k'})i\mathbf{p}\cdot(\omega_{k'}^2\mathbf{k}\delta\theta_k(0,\mathbf{p}) - \omega_k^2\mathbf{k}'\delta\theta_{k'}(0,\mathbf{p}))}{4(\omega_k\omega_{k'})^{5/2}}\frac{e^{i(\mathbf{k}-\mathbf{k}'+\mathbf{p})\cdot\mathbf{x}}}{(2\pi)^D}\\ &= \int\frac{\mathrm{d}^D\mathbf{p}}{(2\pi)^{D/2}}\frac{(\omega_k-\omega_{k'})i\mathbf{p}\cdot(\omega_{k'}^2\mathbf{k}\delta\theta_k(0,\mathbf{p}) - \omega_k^2\mathbf{k}'\delta\theta_{k'}(0,\mathbf{p}))}{4(\omega_k\omega_{k'})^{5/2}}\delta^D(\mathbf{k}-\mathbf{k}'+\mathbf{p})\\ &= \frac{1}{(2\pi)^{D/2}}\frac{(\omega_k-\omega_{k'})i(\mathbf{k}-\mathbf{k}')\cdot(\omega_{k'}^2\mathbf{k}\delta\theta_k(0,\mathbf{k}-\mathbf{k}') - \omega_k^2\mathbf{k}'\delta\theta_{k'}(0,\mathbf{k}-\mathbf{k}'))}{4(\omega_k\omega_{k'})^{5/2}}.\end{aligned} \tag{A5}$$

The initial condition is supposed to not introduce power at small scales; therefore, $\delta\theta_k(0,\mathbf{k}-\mathbf{k}')$ is also exponentially damped for modes $|\mathbf{k}-\mathbf{k}'|\gg\mathcal{H}$. For $|\mathbf{k}-\mathbf{k}'|\sim\mathcal{H}$, we can Taylor expand in $\mathcal{H}/\omega_k$ to get

$$\begin{aligned}\tau_{\delta\theta}(\mathbf{k},\mathbf{k}')\approx{}&\frac{i}{(2\pi)^{D/2}}\frac{1}{4\omega_k^3}\mathbf{k}\cdot\frac{\mathbf{k}-\mathbf{k}'}{|\mathbf{k}-\mathbf{k}'|}\left(\frac{\mathcal{H}}{\omega_k}\right)^3\\ &\times((m^2-k^2)\delta\theta_k(0,\mathbf{k}-\mathbf{k}')\\ &+\omega_k^2\mathbf{k}\cdot\nabla\delta\theta_k(0,\mathbf{k}-\mathbf{k}')).\end{aligned} \tag{A6}$$

Thus, for $|\mathbf{k}-\mathbf{k}'|\sim\mathcal{H}$, $\tau_{\delta\theta}$ is also beyond the leading adiabatic order. Note that the nabla operator in (A6) is to be understood as acting over the index variable k, not over the argument $\mathbf{k}-\mathbf{k}'$.

APPENDIX B: DIMENSIONAL REGULARIZATION FORMULAS

The fundamental formula used in dimensional regularization in Euclidean space is [37,38]

$$\int \frac{\mathrm{d}^D\mathbf{k}}{(2\pi)^D}\frac{k^{2\alpha}}{(k^2+m^2)^\beta} = m^{2(\alpha-\beta)}\left(\frac{m^2}{4\pi}\right)^{D/2} \times \frac{\Gamma(D/2+\alpha)\Gamma(\beta-\alpha-D/2)}{\Gamma(\beta)\Gamma(D/2)}. \tag{B1}$$

This expression has been used to compute $\langle\delta\phi^2\rangle_\mathrm{h}$ in (60) in $D=3-\epsilon$. The left-hand side of the equation can be written as

$$\int \frac{\mathrm{d}^D\mathbf{k}}{(2\pi)^D}\frac{k^{2\alpha}}{(k^2+m^2)^\beta} = \frac{1}{(2\pi)^D}\frac{2\pi^{D/2}}{\Gamma(D/2)}\int_0^\infty \mathrm{d}k \frac{k^{D-1}k^{2\alpha}}{(k^2+m^2)^\beta}, \tag{B2}$$

then

$$\int_0^\infty \mathrm{d}k \frac{k^{D-1}k^{2\alpha}}{(k^2+m^2)^\beta} = \left[\frac{1}{(2\pi)^D}\frac{2\pi^{D/2}}{\Gamma(D/2)}\right]^{-1} m^{2(\alpha-\beta)}\left(\frac{m^2}{4\pi}\right)^{D/2} \times \frac{\Gamma(D/2+\alpha)\Gamma(\beta-\alpha-D/2)}{\Gamma(\beta)\Gamma(D/2)}. \tag{B3}$$

On the other hand, for the $\langle\delta\phi^2\rangle_\mathrm{i}$ term in (60), we have to deal with integrals of the following form,

$$\int \frac{\mathrm{d}^D\mathbf{k}}{(2\pi)^D}\frac{f(\mathbf{k}\cdot\mathbf{p})}{(k^2+m^2)^\beta}, \tag{B4}$$

where $f(\mathbf{k}\cdot\mathbf{p})$ is an analytical function. Taking the k_z direction along $\mathbf{p}$, we have $f(\mathbf{k}\cdot\mathbf{p}) = f(kp\hat{x})$ with $k=|\mathbf{k}|$, $p=|\mathbf{p}|$ and $\hat{x}=\cos(\theta_{D-2})$, θ_{D-2} being the angle between $\mathbf{k}$ and $\mathbf{p}$. When using spherical coordinates in D dimensions $\{\phi,\theta,\theta_2,\ldots,\theta_{D-2}\}$, the volume element can be expressed as

$$\mathrm{d}^D\mathbf{k} = k^{D-1}\sin^{D-2}(\theta_{D-2})\sin^{D-3}(\theta_{D-3})\ldots \times \sin(\theta)\mathrm{d}k\mathrm{d}\phi\mathrm{d}\theta\ldots\mathrm{d}\theta_{D-2}. \tag{B5}$$

The integrand of (B4) depends on $\cos(\theta_{D-2})$, so we can integrate in all the angular variables but θ_{D-2}. With that purpose, notice that the area of a sphere in a D-dimensional space is

$$\overbrace{\int_0^\pi\cdots\int_0^\pi}^{D-2}\int_0^{2\pi}\sin^{D-2}(\theta_{D-2})\sin^{D-3}(\theta_{D-3})\ldots \times \sin^2(\theta_2)\sin(\theta)\mathrm{d}\phi\mathrm{d}\theta\mathrm{d}\theta_2\ldots\mathrm{d}\theta_{D-2} = \frac{2\pi^{D/2}}{\Gamma(D/2)}. \tag{B6}$$

Since all the integrals involved can be factorized, the integration over all the angular variables but θ_{D-2} is simply given by the area of a sphere in $(D-1)$-dimensional space, i.e., $\frac{2\pi^{(D-1)/2}}{\Gamma((D-1)/2)}$. Therefore, Eq. (B4) can be expressed as

$$\int \frac{\mathrm{d}^D\mathbf{k}}{(2\pi)^D}\frac{f(\mathbf{k}\cdot\mathbf{p})}{(k^2+m^2)^\beta} = \frac{1}{(2\pi)^D}\frac{2\pi^{(D-1)/2}}{\Gamma((D-1)/2)}\int_0^\infty \mathrm{d}k \times \frac{k^{D-1}}{(k^2+m^2)^\beta}\hat{f}(kp), \tag{B7}$$

where $\hat{f}(kp) = \int_{-1}^1 \mathrm{d}\hat{x}(1-\hat{x}^2)^{(D-3)/2} f(kp\hat{x})$. Finally, Taylor expanding $\hat{f}(kp)$, the expression can be regularized order by order using Eq. (B3).

To regularize physical quantities like $\langle\delta\phi^2\rangle_\mathrm{h}$ and $\langle\delta\phi^2\rangle_\mathrm{i}$, two important aspects should be taken into consideration. First of all, the full physical expression should be computed in D dimensions, so that when taking $D=3-\epsilon$, all the terms are expanded in ϵ. Moreover, a physical scale μ^ϵ should be introduced to compensate the physical dimensions.

1. Integrals involving k_i or k_ik_j

Finally, we explain how to compute the integrals involving the components of $\mathbf{k}$, k_i, and k_ik_j, appearing in the expression of the energy-momentum tensor in Sec. VI. For these cases, the other vector quantity, namely the wave vector of the metric perturbations $\mathbf{p}$, can be used to produce scalar quantities that can be easily computed in terms of the expressions given above. For instance

$$\int \mathrm{d}^D\mathbf{k}\, g(k,\mathbf{k}\cdot\mathbf{p})k_i = Ap_i, \tag{B8}$$

taking the scalar product with $\mathbf{p}$ in each member we get that

$$A = \int \mathrm{d}^D\mathbf{k}\, g(k,\mathbf{k}\cdot\mathbf{p})\frac{\mathbf{k}\cdot\mathbf{p}}{p^2} \tag{B9}$$

which can be integrated using the expression (B7). For the remaining case, we have

$$\int \mathrm{d}^D\mathbf{k}\, g(k,\mathbf{k}\cdot\mathbf{p})k_ik_j = B\delta_{ij} + Cp_ip_j, \tag{B10}$$

where B and C can be computed solving the system obtained by taking the trace and contracting with p^ip^j. The results are

$$B = \frac{1}{(D-1)}\int \mathrm{d}^D\mathbf{k}\, g(k,\mathbf{k}\cdot\mathbf{p})\frac{(kp)^2-(\mathbf{k}\cdot\mathbf{p})^2}{p^2} \tag{B11}$$

$$C = \frac{1}{(D-1)}\int \mathrm{d}^D\mathbf{k}\, g(k,\mathbf{k}\cdot\mathbf{p})\frac{D(\mathbf{k}\cdot\mathbf{p})^2-(kp)^2}{p^4}. \tag{B12}$$

APPENDIX C: $P_{k,l}^{\{\Phi,\Psi\}}$

In this appendix, the exact expressions for the $P_{k,l}^{\{\Phi,\Psi\}}(\eta)$ coefficients of Eq. (65) are given. First, let us separate these coefficients as

$$P_{k,l}^{\Phi}(\eta) = P_{k,l}^{\Phi,(0)}(\eta) + P_{k,l}^{\Phi,(1)}(\eta) + P_{k,l}^{\Phi,(2)}(\eta), \quad \text{(C1)}$$

where the indices (0), (1), (2) stand for the contribution coming from $P_k^{(0)}$ in (50), $P_k^{(1)}$ in (51), and $P_k^{(2)}$ in (52), respectively. The same definition applies for the terms $P_{k,l}^{\Psi}$.

The $l = 0$ coefficients are given by

$$P_{k,0}^{\Phi}(\eta) = 0 \quad \text{(C2)}$$

$$P_{k,0}^{\Psi}(\eta) = P_{k,0}^{\Psi,(0)}(\eta) = \frac{1}{2}\frac{\sqrt{\pi}\Gamma((D-1)/2)}{\Gamma(D/2)}\left(D - \frac{k^2}{\omega_k^2(\eta)}\right). \quad \text{(C3)}$$

For $l > 0$, we have

$$P_{k,l}^{\Phi}(\eta) = P_{k,l}^{\Phi,(1)}(\eta) + P_{k,l}^{\Phi,(2)}(\eta) \quad \text{(C4)}$$

$$P_{k,l}^{\Psi}(\eta) = P_{k,l}^{\Psi,(1)}(\eta) + P_{k,l}^{\Psi,(2)}(\eta) \quad \text{(C5)}$$

with

$$P_{k,l}^{\Phi,(1)}(\eta) = \frac{(-1)^l}{2^{2l}}\frac{\sqrt{\pi}\Gamma((D-1)/2)}{(l-1)!\Gamma(D/2+l)}k^{2l}\int_0^{\eta} d\eta' \left(\prod_{i=1}^{2l-1}\int_{\eta'}^{\eta}\frac{d\eta_i}{\omega_k(\eta_i)}\right)\left[\frac{1}{\omega_k(\eta')} - \frac{\omega_k(\eta')}{\omega_k^2(\eta)}\right] \quad \text{(C6)}$$

$$P_{k,l}^{\Psi,(1)}(\eta) = \frac{(-1)^l}{2^{2l}}\frac{\sqrt{\pi}\Gamma((D-1)/2)}{(l-1)!\Gamma(D/2+l)}k^{2l}\int_0^{\eta} d\eta' \left(\prod_{i=1}^{2l-1}\int_{\eta'}^{\eta}\frac{d\eta_i}{\omega_k(\eta_i)}\right)\left[\frac{2}{\omega_k(\eta')} - \frac{k^2}{\omega_k^2(\eta)\omega_k(\eta')} - \frac{k^2}{\omega_k^3(\eta')}\right] \quad \text{(C7)}$$

$$P_{k,l}^{\Phi,(2)}(\eta) = \frac{(-1)^l}{2^{2l-1}}\frac{\sqrt{\pi}\Gamma((D-1)/2)}{(l-1)!\Gamma(D/2+l-1)}k^{2l}\int_0^{\eta} d\eta' \int_0^{\eta'} d\eta'' \left(\prod_{i=1}^{2l-2}\int_{\eta''}^{\eta}\frac{d\eta_i}{\omega_k(\eta_i)}\right)\left[\frac{\omega_k(\eta'')}{k^2\omega_k(\eta')} - \frac{(2l-1)\omega_k(\eta'')}{(D-2l-2)\omega_k^3(\eta')}\right] \quad \text{(C8)}$$

$$P_{k,l}^{\Psi,(2)}(\eta) = \frac{(-1)^l}{2^{2l-1}}\frac{\sqrt{\pi}\Gamma((D-1)/2)}{(l-1)!\Gamma(D/2+l-1)}k^{2l}\int_0^{\eta} d\eta' \int_0^{\eta'} d\eta'' \left(\prod_{i=1}^{2l-2}\int_{\eta''}^{\eta}\frac{d\eta_i}{\omega_k(\eta_i)}\right)\left[\frac{1}{\omega_k(\eta')\omega_k(\eta'')} - \frac{(2l-1)k^2}{(D-2l-2)\omega_k^3(\eta')\omega_k(\eta'')}\right]. \quad \text{(C9)}$$

The integral over k of all these terms can be regularized with the expressions given in Appendix B after applying the generalized Feynman trick discussed in Sec. V. After regularization, we are left with two terms: one polynomic in η, the other one logarithmic in η. The integration over the Feynman parameters $\{x_i\}$ and the time integrals can be done following the procedure discussed in Appendix D.

APPENDIX D: INTEGRATION OVER $\{x_i\}$ AND $\{\eta_i\}$

This appendix shows how to compute the integrals over $\{x_i\}$ and $\{\eta_i\}$ appearing in the $R_l^{\{\Phi,\Psi\}}$ coefficients in (74). These terms have the general form

$$\overbrace{\int d\eta_1 \cdots \int d\eta_{2N}}^{2N}\overbrace{\int_0^1 \frac{dx_1}{\sqrt{x_1}} \cdots \int_0^1 \frac{dx_{2N+1}}{\sqrt{x_{2N+1}}}}^{2N+1}\delta\left(\sum_{k=1}^{2N+1} x_k - 1\right)\left\{\mathrm{Pol}_1(\{x_i\},\{\eta_i\}) + \log\left[\sum_{k=1}^{2N+1} x_k a^2(\eta_k)\right]\mathrm{Pol}_2(\{x_i\},\{\eta_i\})\right\}, \quad \text{(D1)}$$

where the logarithmic contribution is included in the $R_{l,\log}^{\{\Phi,\Psi\}}$ part of (75), whereas the pure polynomic one coming from Pol_1 is included in $R_{l,\mathrm{pol}}^{\{\Phi,\Psi\}}$. Notice that we have redefined $2l$ appearing in expression (74), namely the power of p, to be $2N$ in (D1) in order to highlight its importance in the following discussion. Since the polynomials only introduce trivial modifications of the following formulas, let us focus on the expression

$$\overbrace{\int d\eta_1 \cdots \int d\eta_{2N}}^{2N}\overbrace{\int_0^1 \frac{dx_1}{\sqrt{x_1}} \cdots \int_0^1 \frac{dx_{2N+1}}{\sqrt{x_{2N+1}}}}^{2N+1}\delta\left(\sum_{k=1}^{2N+1} x_k - 1\right)\log\left[\sum_{k=1}^{2N+1} x_k a^2(\eta_k)\right]. \quad \text{(D2)}$$

There are $2N+1$ variables x_i from the Feynman trick and all of them are integrated from 0 to 1. There are also $2N+1$ time variables η_i, but only $2N$ of them are integrated. In particular, η_{2N+1} is not integrated. In order to recover the expressions given in the text, we have renamed η as η_{2N+1}, η' as η_{2N} and η'' as η_{2N-1}. From the general expression (D2), it is straightforward to prove that for $a(\eta)=1$, the logarithm vanishes since $\sum_{k=1}^{2N+1} x_k = 1$. Therefore, $R_{l,\log}^{\{\Phi,\Psi\}}=0$ in nonexpanding spacetimes.

First, we deal with the integration over the $\{x_i\}$. Defining new variables $y_i^2 = x_i$ for $i=1,\ldots,2N+1$, this integration can be written over the $2N$ sphere

$$\int_0^1 \frac{\mathrm{d}x_1}{\sqrt{x_1}} \cdots \int_0^1 \frac{\mathrm{d}x_{2N+1}}{\sqrt{x_{2N+1}}} \delta\left(\sum_{k=1}^{2N+1} x_k - 1\right) = 2^{2N} \int_{S^{2N}} \mathrm{d}^{2N}\Omega. \tag{D3}$$

Then, the logarithm can be expressed as

$$\log\left[\sum_{k=1}^{2N+1} y_k^2 a^2(\eta_k)\right] = \log\left[a^2(\eta_{2N+1})\right] + \log\left[1 + \sum_{k=1}^{2N} y_k^2 \left(\frac{a^2(\eta_k)}{a^2(\eta_{2N+1})} - 1\right)\right], \tag{D4}$$

where we have used that $y_{2N+1}^2 = 1 - \sum_{k=1}^{2N} y_k^2$. The first logarithm on the right-hand side is the usual logarithm of the scale factor which appears in dimensional regularization in a FRW metric and it cancels out at the end. On the other hand, since η_{2N+1} is an upper limit in all the time integrations (see next subsection), we have $\eta_k \leq \eta_{2N+1}$ for $k=1,\ldots,2N$. Thus, considering expanding universes, the argument of the logarithm is of the form $1+x$ with $-1 < x \leq 1$. Hence, it can be Taylor expanded as

$$\log\left[1 + \sum_{k=1}^{2N} y_k^2 \left(\frac{a^2(\eta_k)}{a^2(\eta_{2N+1})} - 1\right)\right] = \sum_{j=1}^{\infty} \frac{(-1)^{j+1}}{j} \left[\sum_{k=1}^{2N} y_k^2 \left(\frac{a^2(\eta_k)}{a^2(\eta_{2N+1})} - 1\right)\right]^j, \tag{D5}$$

where the last factor on the right-hand side can also be expanded using the multinomial theorem

$$\left[\sum_{k=1}^{2N} y_k^2 \left(\frac{a^2(\eta_k)}{a^2(\eta_{2N+1})} - 1\right)\right]^j = \sum_{\substack{l_1,l_2,\ldots,l_{2N}=0 \\ \sum_{i=1}^{2N} l_i = j}}^{j} \frac{j!}{l_1! l_2! \cdots l_{2N}!} \prod_{m=1}^{2N} \left[y_m^2 \left(\frac{a^2(\eta_m)}{a^2(\eta_{2N+1})} - 1\right)\right]^{l_m}. \tag{D6}$$

Therefore, the integration over the $2N$-sphere reduces to an integration of this kind:

$$\int_{S^{2N}} \mathrm{d}^{2N}\Omega\, y_1^{2l_1} y_2^{2l_2} \cdots y_{2N}^{2l_{2N}} = \frac{\sqrt{\pi} \prod_{i=1}^{2N} \Gamma(\frac{1}{2} + l_i)}{2^{2N} \Gamma(N + \frac{1}{2} + \sum_{i=1}^{2N} l_i)} \equiv \frac{1}{2^{2N}} \Gamma[\{l_i\}, 2N]. \tag{D7}$$

Then,

$$2^{2N} \int_{S^{2N}} \mathrm{d}^{2N}\Omega \log\left[1 + \sum_{k=1}^{2N} y_k^2 \left(\frac{a^2(\eta_k)}{a^2(\eta_{2N+1})} - 1\right)\right] = \sum_{j=1}^{\infty} \frac{(-1)^{j+1}}{j} \sum_{\substack{l_1,l_2,\ldots,l_{2N}=0 \\ \sum_{i=1}^{2N} l_i = j}}^{j} \frac{j!}{l_1! l_2! \cdots l_{2N}!} \Gamma[\{l_i\}, 2N] \prod_{m=1}^{2N} \left(\frac{a^2(\eta_m)}{a^2(\eta_{2N+1})} - 1\right)^{l_m}. \tag{D8}$$

Applying the binomial theorem to the last factors,

$$\left(\frac{a^2(\eta_m)}{a^2(\eta_{2N+1})} - 1\right)^{l_m} = \sum_{i_m=0}^{l_m} (-1)^{l_m - i_m} \binom{l_m}{i_m} \left[\frac{a^2(\eta_m)}{a^2(\eta_{2N+1})}\right]^{i_m}, \tag{D9}$$

and gathering all the results, we get

$$2^{2N} \int_{S^{2N}} \mathrm{d}^{2N}\Omega \log\left[1 + \sum_{k=1}^{2N} y_k^2 \left(\frac{a^2(\eta_k)}{a^2(\eta_{2N+1})} - 1\right)\right] = -\sum_{j=1}^{\infty} \sum_{\substack{l_1,l_2,\ldots,l_{2N}=0 \\ \sum_{i=1}^{2N} l_i = j}}^{j} \frac{(j-1)!}{l_1! l_2! \cdots l_{2N}!} \Gamma[\{l_i\}, 2N] \sum_{i_1,i_2,\ldots,i_{2N}=0}^{l_1,l_2,\ldots,l_{2N}} (-1)^{\sum_{m=1}^{2N} i_m} \prod_{m=1}^{2N} \binom{l_m}{i_m} \left[\frac{a^2(\eta_m)}{a^2(\eta_{2N+1})}\right]^{i_m}. \tag{D10}$$

Finally, the time integrations can be done in a straightforward way since the dependence on η_m of the scale factor is polynomial for the cosmologies considered in this work.

1. $R^{\Psi}_{l,\log} = 0$ for all cosmologies

In Sec. V D, it is mentioned that the $R^{\Psi}_{l,\log}$ coefficients are all zero for all the cases considered. In fact, these expressions vanish not because of the integration over $\{x_i\}$ but because the polynomial $\mathrm{Pol}_2(\{x_i\},\{\eta_i\})$ in (D1) is zero for the Ψ contribution. This can be shown by summing the already regularized expression for (C5). Although the limits of integration are apparently different in each of the terms (C7), (C9), the region of integration is the same. For instance, the first integral can be written as

$$\int_0^{\eta} \mathrm{d}\eta' \left(\prod_{i=1}^{2l-1} \int_{\eta'}^{\eta} \mathrm{d}\eta_i\right) = \int_0^{\eta} \mathrm{d}\eta' \left(\prod_{i=1}^{2l-1} \int_0^{\eta} \mathrm{d}\eta_i \theta(\eta_i - \eta')\right), \tag{D11}$$

where θ is the step function, while

$$\int_0^{\eta} \mathrm{d}\eta' \int_0^{\eta'} \mathrm{d}\eta'' \left(\prod_{i=1}^{2l-2} \int_{\eta''}^{\eta} \mathrm{d}\eta_i\right) = \int_0^{\eta} \mathrm{d}\eta' \int_0^{\eta} \mathrm{d}\eta'' \theta(\eta' - \eta'') \left(\prod_{i=1}^{2l-2} \int_0^{\eta} \mathrm{d}\eta_i \theta(\eta_i - \eta'')\right). \tag{D12}$$

Then, redefining in the last integral η' as η_{2l-1} and η'' as η', both integrals have the same form

$$\overbrace{\int_0^{\eta} \mathrm{d}\eta_1 \cdots \int_0^{\eta} \mathrm{d}\eta_{2N}}^{2N} \prod_{i=1}^{2N-1} \theta(\eta_i - \eta_{2N}). \tag{D13}$$

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