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Cosmological models in modified gravity theories with extended nonminimal derivative couplings

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We construct gravitational modifications that go beyond Horndeski, namely theories with extended nonminimal derivative couplings, in which the coefficient functions depend not only on the scalar field but also on its kinetic energy. Such theories prove to be ghost-free in a cosmological background. We investigate the early-time cosmology and show that a de Sitter inflationary phase can be realized as a pure result of the novel gravitational couplings. Additionally, we study the late-time evolution, where we obtain an effective dark energy sector which arises from the scalar field and its extended couplings to gravity. We extract various cosmological observables and analyze their behavior at small redshifts for three choices of potentials, namely for the exponential, the power-law, and the Higgs potentials. We show that the Universe passes from deceleration to acceleration in the recent cosmological past, while the effective dark energy equation-of-state parameter tends to the cosmological-constant value at present. Finally, the effective dark energy can be phantomlike, although the scalar field is canonical, which is an advantage of the model.

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I. INTRODUCTION

Horndeski's theory [1] is the most general single-scalar tensor theory that has second-order field equations, for both the metric and the scalar fields in four dimensions. It was originally discovered in 1974, then rediscovered independently [2], and recently brought back to attention [3–5] (for a review see [6]). The generality of the theory is reminiscent of Lovelock's theorem [7], and it comes as no surprise that many of its terms, especially those that involve derivative couplings of the scalar with curvature terms, come from a dimensional reduction of higher dimensional Lovelock theories [8]. Note that having second-order field equations is crucial, in order to avoid Ostrogradski instabilities [9–11].

The advantage of Horndeski cosmological models is that they are able to screen the vacuum energy coming from any field theory, assuming that after this screening the space should be in a de Sitter vacuum [12,13]. These models allow us to understand the current accelerated expansion of the Universe as the result of a dynamical evolution toward a de Sitter attractor [14]. Thus, it was shown that Horndeski models with a de Sitter critical point for any kind of material content may provide a mechanism to alleviate the cosmological constant problem [15]. The cosmological scenario that results when considering the radiation and matter content was also studied, and it was concluded that their background dynamics is compatible with the latest observational data.

Despite the huge interest in these theories, extensions of Horndeski's theory have also been recently discussed. In [16] a new class of scalar-tensor theories was introduced, going beyond Horndeski's theory, where despite the fact that the equations of motion contain higher derivatives, they can be cast in a way that they contain only second-order ones [17]. Additionally, these generalized theories were shown to be free of ghost instabilities in the unitary gauge [18], and later on this was also verified using the Hamiltonian formalism [19–23], due to the existence of a primary constraint which prevents the propagation of extra degrees of freedom [23] (see also [24] and [25–27] for additional descriptions). We mention that these extended theories can also address the cosmological constant problem [28] via a self-tuning mechanism, similar to the analysis done in the original Horndeski theory for the so-called *Fab Four* theory [3,4] (the cosmological aspects of the Fab Four have been explored in [29]). A detailed analysis of the cosmological self-tuning and local solutions in the context of beyond Horndeski theories has also been explored in [30]. Recently, it was shown that the two additional Lagrangian pieces, appearing in theories beyond

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Horndeski, could be reexpressed in a very elegant and compact way, by allowing the potentials to also depend on the kinetic term of the scalar field [28].

One interesting subclass of Horndeski theory, which has been given much attention recently, includes the nonminimal (kinetic) coupling of matter to gravity by inserting derivative couplings between the geometry and the kinetic part of the scalar field [31], which leads to interesting new dynamical cosmological phenomena [32,33], including the existence of an effective cosmological constant [34,35]. The nonminimal derivative coupling leads to cosmological models with rich phenomenology, such as solutions containing a big bang, expanding universes with no beginning, cosmological bounces, eternally contracting universes, a big crunch, and a big rip avoidance [33,36–42]. In particular, it was shown that one is able to explain in a unique manner both a quasi–de Sitter phase and an exit from it without any fine-tuned potential [32]. Furthermore, one can successfully describe the sequence of cosmological epochs without any fine-tuned potential [43]. Using couplings of this type, it was found that in the absence of other matter sources or in the presence of only pressureless matter, the scalar field behaves as pressureless matter and its sound speed is vanishing [44]. These properties enable the scalar field to be a candidate of cold dark matter. It was also shown that if the kinetic term is coupled to more than one Einstein tensor, then the equation of state is always approximately equal to -1, independent from the potential flatness, and hence the scalar may also be considered a candidate for the inflaton. Tachyon models involving nonminimal derivative coupling have also been explored [45,46], while Chaplygin gas models in this framework were studied in [47]. Moreover, the dynamics of entropy perturbations in the two-field assisted dark energy model with mixed kinetic terms was also studied in [48]. Recently there has also been an investigation on how the derivative coupling can mimic cold dark matter at the cosmological level and also explain the flattening of galactic rotation curves [49].

The inflationary context within this theory has been extensively analyzed too. In the case of a power-law potential, and using the dynamical system method, all possible asymptotical regimes of the model were analyzed [50]. It was shown that for sloping potentials there exists a quasi–de Sitter asymptotic corresponding to an early inflationary universe. In contrast to a standard inflationary scenario, the kinetic-coupling inflation does not depend on a scalar field potential and is only determined by the coupling parameter. In addition to this, there is a unique nonminimal derivative coupling of the Standard Model Higgs boson to gravity which propagates no more degrees of freedom than general relativity sourced by a scalar field, reproduces a successful inflating background within the Standard Model Higgs parameters, and, finally, does not suffer from dangerous quantum corrections [51]. The slow-roll conditions have been found [52], and the reheating temperature was obtained [53,54] (see also recent analyses in [55] and [56]). Furthermore, the cosmological perturbations originated at the inflationary stage were studied and the consistency of the results with observational constraints coming from Planck 2013 data were investigated [57]. Moreover, these scenarios exhibit a gravitationally enhanced friction during inflation, where even steep potentials with theoretically natural model parameters can drive cosmic acceleration [58], while being compatible with the current observational data mainly due to the suppressed tensor-to-scalar ratio. Finally, the gravitational production of heavy X particles of mass of the order of the inflaton mass, produced after the end of inflation, was also studied [59], where it was found that this production is suppressed as the strength of the coupling is increased.

A combined perturbation and observational investigation of the scenario of nonminimal derivative coupling between a scalar field and curvature was performed in [60]. Using Type Ia supernovae (SNIa), baryon acoustic oscillations (BAO), and cosmic microwave background (CMB) observations, it was shown that, contrary to its significant effects on inflation, the nonminimal derivative coupling term has a negligible effect on the Universe acceleration, since it is driven solely by the usual scalar-field potential. Therefore, the scenario can provide a unified picture of early and late time cosmology, with the nonminimal derivative coupling term responsible for inflation, and the usual potential responsible for late-time acceleration.

Finally, nonminimal derivative couplings to gravity have also been explored in a variety of extended theories of gravity. For instance, one can incorporate an additional coupling to the Gauss Bonnet invariant, obtaining rich cosmological behavior, with both decelerated and accelerated phases [61,62]. Additionally, a large class of scalar-tensor models with interactions containing the second derivatives of the scalar field but not leading to additional degrees of freedom have also been extensively investigated [63]. These models exhibit peculiar features, such as an essential mixing of scalar and tensor kinetic terms, named kinetic braiding, and possess a rich cosmological phenomenology, including a late-time asymptotic de Sitter state, and a possible phantom-divide crossing, with neither ghosts nor gradient instabilities. Finally, the nonminimal derivative coupling to gravity has also been investigated in the context of the curvaton model [64], or in the framework of $N = 1$ four-dimensional new-minimal supergravity [65].

In this work we are interested in investigating a theory that goes beyond Horndeski, based on a generalization of nonminimal derivative coupling. In particular, we consider the latter coupling and introduce an additional arbitrary coefficient function of the field and its derivatives. We mention that this class is not included in Horndeski theory, since only specific combinations of it are allowed [2,66,67].

This paper is outlined in the following manner. In Sec. II, we present the action and deduce the gravitational field equations. In Sec. III, we apply the developed formalism to a spatially flat Friedmann-Robertson-Walker (FRW) background metric and present the modified Friedmann equations. The early-time cosmology is briefly analyzed in Sec. IV, and the late-time evolution is considered in Sec. V. In the latter, we study the full Friedmann equations and focus on important observables, by considering three well-known scalar potentials, such as the exponential, the power-law, and the Higgs potentials. Finally, in Sec. VI we discuss our results and conclude.

II. EXTENDED NONMINIMAL DERIVATIVE COUPLING

In this work, we consider a generalized nonminimal coupling of the scalar field derivative to gravity, by introducing an additional arbitrary coefficient function of the field and its derivatives. The action is given by

$$S = \int d^4x\sqrt{-g}\left\{\frac{R}{16\pi G} - \frac{1}{2}\{g_{\mu\nu} - [\beta + \zeta F(X,\phi)]G_{\mu\nu}\}\nabla^\mu\phi\nabla^\nu\phi - V(\phi)\right\} + S_m, \tag{2.1}$$

with $g_{\mu\nu}$ the metric, $g = \det(g_{\mu\nu})$, R the scalar curvature, β and ζ the derivative coupling parameters, $V(\phi)$ the scalar field potential, $X = -\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi$ the scalar kinetic energy, and $F(X,\phi)$ an arbitrary function. Note that in principle β can be absorbed inside $\zeta F(X,\phi)$; however, we prefer to keep it separately in order to be able to reproduce at any stage the results of the simple nonminimal derivative coupling by setting ζ to 0. Finally, we have included the usual matter action, corresponding to a matter fluid of energy density ρ_m and pressure p_m.

Variation of the action with respect to the metric leads to the field equations

$$\begin{aligned}
&\frac{1}{16\pi G}G_{\mu\nu} - \frac{1}{2}\nabla_\mu\phi\nabla_\nu\phi + \frac{1}{4}g_{\mu\nu}\nabla^\alpha\phi\nabla_\alpha\phi + \frac{\beta}{2}\Big\{-\frac{1}{2}g_{\mu\nu}G^{\alpha\beta}\nabla_\alpha\phi\nabla_\beta\phi + 2G_{(\mu}{}^{\lambda}\nabla_{\nu)}\phi\nabla_\lambda\phi + \frac{1}{2}R\nabla_\mu\phi\nabla_\nu\phi \\
&-\frac{1}{2}R_{\mu\nu}\nabla^\alpha\phi\nabla_\alpha\phi + \frac{1}{2}g_{\mu\nu}[(\Box\phi)^2 - \nabla_\alpha\nabla_\beta\phi\nabla^\alpha\nabla^\beta\phi - R_{\alpha\beta}\nabla^\alpha\phi\nabla^\beta\phi] + \nabla_\mu\nabla^\alpha\phi\nabla_\nu\nabla_\alpha\phi - \Box\phi\nabla_\mu\nabla_\nu\phi \\
&+R_\mu{}^\alpha{}_\nu{}^\beta\nabla_\alpha\phi\nabla_\beta\phi\Big\} + \frac{\zeta}{2}\left[\frac{1}{2}(XFG_{\mu\nu} + 2FP_\mu{}^\alpha{}_\nu{}^\beta\nabla_\alpha\phi\nabla_\beta\phi) + F_{,X}\nabla_\mu\phi\nabla_\nu\phi G^{\alpha\beta}\nabla_\alpha\phi\nabla_\beta\phi\right] \\
&-\frac{1}{2}\epsilon_\mu{}^{\alpha\sigma\gamma}\epsilon_\nu{}^{\beta\rho}{}_\gamma[F\nabla_\beta\nabla_\alpha\phi\nabla_\sigma\nabla_\rho\phi + 2\nabla_{(\alpha}F\nabla_{\beta)}\phi\nabla_\sigma\nabla_\rho\phi - \nabla_\alpha\phi\nabla_\beta\phi\nabla_\sigma\nabla_\rho F] + \frac{1}{2}V(\phi)g_{\mu\nu} = -\frac{1}{\sqrt{-g}}\frac{\delta S_m}{\delta g^{\mu\nu}},
\end{aligned} \tag{2.2}$$

where $P_{\mu\nu\alpha\beta}$ is the double dual of the Riemann tensor defined as [68]

$$P^{\mu\nu}{}_{\alpha\beta} \equiv \frac{1}{4}\delta^{\mu\nu\gamma\delta}_{\sigma\lambda\alpha\beta}R^{\sigma\lambda}{}_{\gamma\delta} = R^{\mu\nu}{}_{\alpha\beta} - 2R^\mu{}_{[\alpha}\delta^\nu{}_{\beta]} + 2R^\nu{}_{[\alpha}\delta^\mu_{\beta]} + R\delta^\mu_{[\alpha}\delta^\nu_{\beta]}, \tag{2.3}$$

and where $\nabla_{(\mu}\phi R^\alpha_{\nu)} = \frac{1}{2}(\nabla_\mu\phi R^\alpha_\nu + \nabla_\nu\phi R^\alpha_\mu)$. Additionally, variation of the action (2.1) with respect to ϕ provides the scalar field equation of motion, namely

$$\Box\phi - \beta G^{\mu\nu}\nabla_\mu\nabla_\nu\phi - \frac{\zeta}{2}[2\nabla_\mu(FG^{\mu\nu}\nabla_\nu\phi) + 2\nabla_\alpha(F_{,X}G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi\nabla^\alpha\phi) - F_{,\phi}G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi] - V_\phi = 0. \tag{2.4}$$

In this work we will consider the case where $F(\phi,X) = X$, which is adequate to capture the new features of the theories beyond Horndeski at hand. Hence the coupling constant ζ has dimensions of inverse mass to the power of six while β has dimensions of inverse mass to the power of two. Using the identity

$$\epsilon_{\mu\alpha\beta\gamma}\epsilon^{\nu\kappa\lambda\rho} = -\delta^{\nu\kappa\lambda\rho}_{\mu\alpha\beta\gamma} \tag{2.5}$$

and the fact that apart from the potential term the rest of the action is shift symmetric, we can now write our field equations in the following elegant way:

$$\frac{1}{16\pi G}G_\mu{}^\nu - \frac{1}{2}\nabla_\mu\phi\nabla^\nu\phi + \frac{1}{4}\delta_\mu{}^\nu\nabla^\alpha\phi\nabla_\alpha\phi + \frac{\beta}{2}\Big\{-\frac{1}{2}\delta_\mu{}^\nu G^{\alpha\beta}\nabla_\alpha\phi\nabla_\beta\phi + 2G_{(\mu}{}^\lambda\nabla^{\nu)}\phi\nabla_\lambda\phi + \frac{1}{2}R\nabla_\mu\phi\nabla^\nu\phi$$
$$-\frac{1}{2}R_\mu{}^\nu\nabla^\alpha\phi\nabla_\alpha\phi + \frac{1}{2}\delta_\mu{}^\nu[(\Box\phi)^2 - \nabla_\alpha\nabla_\beta\phi\nabla^\alpha\nabla^\beta\phi - R_{\alpha\beta}\nabla^\alpha\phi\nabla^\beta\phi] + \nabla_\mu\nabla^\alpha\phi\nabla^\nu\nabla_\alpha\phi$$
$$-\Box\phi\nabla_\mu\nabla^\nu\phi + R_\mu{}^{\alpha\nu\beta}\nabla_\alpha\phi\nabla_\beta\phi\Big\} + \frac{\zeta}{2}\left[\frac{1}{2}(X^2G_\mu{}^\nu + 2XP_{\mu\alpha}{}^{\nu\beta}\nabla^\alpha\phi\nabla_\beta\phi) + \nabla_\mu\phi\nabla^\nu\phi G^{\alpha\beta}\nabla_\alpha\phi\nabla_\beta\phi\right]$$
$$+\frac{1}{2}\delta^{\nu\beta\rho\gamma}_{\mu\alpha\sigma\gamma}[X\nabla^\beta\nabla_\alpha\phi\nabla^\sigma\nabla_\rho\phi + 2\nabla^{(\alpha}X\nabla_{\beta)}\phi\nabla^\sigma\nabla_\rho\phi - \nabla^\alpha\phi\nabla_\beta\phi\nabla^\sigma\nabla_\rho X] + \frac{1}{2}V(\phi)\delta_\mu{}^\nu = \frac{1}{2}T_\mu{}^\nu, \tag{2.6}$$

and

$$\nabla_\alpha J^\alpha = V_\phi, \tag{2.7}$$

where

$$J^\alpha = [g^{\alpha\beta} - \beta G^{\alpha\beta} - \zeta(XG^{\alpha\beta} + g^{\alpha\beta}G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi)]\nabla_\beta\phi. \tag{2.8}$$

III. COSMOLOGICAL EQUATIONS

In this section we are interested in investigating the cosmological implications of theories with extended non-minimal derivative couplings. Hence, we focus on a spatially flat FRW background metric of the form

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^idx^j, \tag{3.1}$$

where t is the cosmic time, x^i are the comoving spatial coordinates, $a(t)$ is the scale factor, and $H = \dot{a}/a$ is the Hubble parameter (a dot denotes differentiation with respect to t). Additionally, we consider the scalar field to be homogeneous, that is $\phi = \phi(t)$. Finally, as usual, we consider the matter sector to correspond to a perfect fluid.

In this case, the field equations (2.6) provide the two Friedmann equations,

$$H^2 = \frac{4\pi G}{3(1 - 12\pi G\beta\dot\phi^2 + 20\pi G\zeta\dot\phi^4)}[2V(\phi) + 2\rho_m + \dot\phi^2], \tag{3.2}$$

$$\begin{aligned}&2\dot H + 3H^2 + 4\pi G(2p_m - 2V + \dot\phi^2)\\&- 4\pi G\beta\dot\phi[(3H^2 + 2\dot H)\dot\phi + 4H\ddot\phi]\\&+ 4\pi G\zeta\dot\phi^3[(2\dot H + 3H^2)\dot\phi + 8H\ddot\phi] = 0,\end{aligned} \tag{3.3}$$

while Eq. (2.7) gives

$$\begin{aligned}&\ddot\phi + 3H\dot\phi + V_\phi + 3\beta H[(3H^2 + 2\dot H)\dot\phi + H\ddot\phi]\\&- 6\zeta H\dot\phi^2[(2\dot H + 3H^2)\dot\phi + 3H\ddot\phi] = 0.\end{aligned} \tag{3.4}$$

As we can easily see, despite the appearance of higher derivatives in Eq. (2.2), on the cosmological background they disappear [16], and we only have to deal with up to two derivatives. From the above expressions one can see that the Friedmann equations (3.2) and (3.3) can be written in the usual form, namely

$$H^2 = \frac{8\pi G}{3}(\rho_{\rm DE} + \rho_m), \tag{3.5}$$

$$2\dot H + 3H^2 = -8\pi G(p_{\rm DE} + p_m), \tag{3.6}$$

where we have defined an effective dark energy sector with energy density and pressure,

$$\rho_{\rm DE} \equiv \rho_\phi = \frac{\dot\phi^2}{2} + V(\phi) + H^2\dot\phi^2\left(\frac{9}{2}\beta - \frac{15}{2}\zeta\dot\phi^2\right), \tag{3.7}$$

$$\begin{aligned}p_{\rm DE} \equiv p_\phi = {}&\frac{\dot\phi^2}{2} - V(\phi) - \beta\frac{\dot\phi}{2}[(3H^2 + 2\dot H)\dot\phi + 4H\ddot\phi]\\&+ \zeta\frac{\dot\phi^3}{2}[(2\dot H + 3H^2)\dot\phi + 8H\ddot\phi],\end{aligned} \tag{3.8}$$

respectively. Therefore, in the scenario at hand, the dark energy equation-of-state parameter is given by

$$w_{\rm DE} \equiv \frac{p_{\rm DE}}{\rho_{\rm DE}}. \tag{3.9}$$

One can straightforwardly see that, in terms of the dark energy density and pressure, the scalar field evolution equation (3.4) can be written in the standard form

$$\dot\rho_{\rm DE} + 3H(\rho_{\rm DE} + p_{\rm DE}) = 0. \tag{3.10}$$

Furthermore, the matter energy density and pressure satisfy the standard evolution equation

$$\dot\rho_m + 3H(\rho_m + p_m) = 0. \tag{3.11}$$

Finally, we introduce the deceleration parameter q, which is an indicator of the accelerated expansion and is defined as

$$q = \frac{d}{dt}\frac{1}{H} - 1, \tag{3.12}$$

and thus negative values of q correspond to accelerating evolution. Additionally, in order to allow for an easy comparison between the observational and theoretical results, instead of the time variable t we can use the redshift z, defined as

$$1 + z = \frac{1}{a}, \tag{3.13}$$

where we have normalized the scale factor $a(z)$ so that its current value is $a(0) = 1$. Thus, time derivatives can be expressed as

$$\frac{d}{dt} = -H(z)(1 + z)\frac{d}{dz}. \tag{3.14}$$

IV. EARLY-TIME COSMOLOGY

In order to examine the early-time behavior of the scenario at hand we will neglect the matter content of the theory. We are interested in exponential cosmological solutions, which could describe the inflationary epoch. Note that since we desire to study the pure effects of the novel, extended nonminimal derivative couplings, we do not consider an explicit potential, since it is well known that a potential term can easily drive an exponential solution, with the best example being a simple cosmological constant.

Using the metric (3.1) we examine whether the cosmological equations (3.2)–(3.4) admit solutions in which the scale factor has an exponential dependence in time of the form

$$a(t) = e^{\frac{1}{2}H_0 t}, \tag{4.1}$$

with H_0 a constant. Inserting this in the first Friedmann equation (3.2), and setting as usual $8\pi G = 1$, we can easily see that the general solution for the scalar field has a linear time dependence; thus we depict our solution for the scalar field as

$$\phi(t) = \phi_1 t + \phi_0, \tag{4.2}$$

where ϕ_1, ϕ_0 are integration constants, which will be determined in the following, in order for the full system of equations to be satisfied. Inserting this expression for the scalar field in the Klein-Gordon equation (3.4) we deduce that it is satisfied if

$$\beta = \frac{2(3H_0^2\zeta\phi_1^2 - 2)}{3H_0^2}. \tag{4.3}$$

Substituting back to the Friedmann equations (3.2) and (3.3), we obtain

$$\zeta = \frac{2(3H_0^2 + 4\phi_1^2)}{3H_0^2\phi_1^4}. \tag{4.4}$$

Hence β and ζ are both positive, namely

$$\beta = \frac{4}{H_0^2} + \frac{1}{\phi_1^2}, \tag{4.5}$$

$$\zeta = \frac{2}{\phi_1^4} + \frac{8}{3H_0^2\phi_1^2}. \tag{4.6}$$

In summary, we can see that the scenario at hand easily admits de Sitter solutions that can describe the inflationary epoch of the Universe. In particular, for a given set of coupling parameters β and ζ, one obtains the de Sitter solution (4.1), with the scalar field evolving as in Eq. (4.2), where the solution parameters H_0 and ϕ_1 are determined by inverting Eqs. (4.5) and (4.6). The only requirement is the obtained H_0 and ϕ_1 to be real numbers, and this constrains the allowed (β, z) parameter space. For instance, note that if one of β or ζ is zero, the system does not admit an exponential solution unless there is a bare cosmological constant. We stress that the above de Sitter solution has been obtained without considering a potential term; i.e. it is an effect of the extended nonminimal derivative coupling terms considered in this work.

In general, at early times, where matter can be neglected, we can express the deceleration parameter (3.12) using the Friedmann equations (3.2) and (3.3) as

$$q = -\frac{\dot{H}}{H^2} - 1 = \frac{1}{2}(1 + 3w_{\rm tot}), \tag{4.7}$$

where we have defined the "total" equation-of-state parameter of the Universe as

$$w_{\rm tot} = \frac{\frac{\dot{\phi}^2}{2} - V(\phi) - \beta\frac{\dot{\phi}}{2}[(3H^2 + 2\dot{H})\dot{\phi} + 4H\ddot{\phi}] + \zeta\frac{\dot{\phi}^3}{2}[(2\dot{H} + 3H^2)\dot{\phi} + 8H\ddot{\phi}]}{\frac{\dot{\phi}^2}{2} + V(\phi) + H^2\dot{\phi}^2(\frac{9}{2}\beta - \frac{15}{2}\zeta\dot{\phi}^2)}. \tag{4.8}$$

Hence, the condition for accelerated expansion can then be formulated as $1 + 3w_{\rm tot} < 0$, or equivalently

$$\frac{\dot{\phi}\{\dot{\phi}[-3\beta\dot{H} + 3\zeta(\dot{H} - H^2)\dot{\phi}^2 + 2] - 6H\ddot{\phi}(\beta - 2\zeta\dot{\phi}^2)\} + 4V(\phi)}{\dot{\phi}^2[3H^2(3\beta - 5\zeta\dot{\phi}^2) + 1] + 2V(\phi)} < 0, \tag{4.9}$$

where for generality we have kept the potential term. Hence, one can use this requirement in order to find more general inflationary solutions, beyond the de Sitter one.

V. LATE-TIME COSMOLOGY

In the present section we investigate several cosmological models in the framework of gravitational theories with an extended nonminimal derivative coupling, focusing on the late-time evolution. In particular, we are interested in studying the full Friedmann equations (3.5) and (3.6), i.e., considering the matter sector as well, and we focus on important observables such as the dark energy equation-of-state parameter $w_{\rm DE}$ defined in (3.9), and the dark-matter and dark energy density parameters defined, respectively, as

$$\Omega_m = \frac{8\pi G}{3H^2}\rho_m, \qquad \Omega_{\rm DE} = \frac{8\pi G}{3H^2}\rho_{\rm DE}. \tag{5.1}$$

Additionally, concerning the scalar potential we will consider three well-known cases, namely the exponential potential [69–73]

$$V(\phi) = \Lambda_0 e^{-\mu\phi}, \tag{5.2}$$

with Λ_0 and μconstants, the power-law potential [74,75]

$$V(\phi) = V_0\phi^n, \tag{5.3}$$

with V_0 and n constants, and the Higgs potential [76]

$$V(\phi) = V_0 + \frac{1}{2}M^2\phi^2 + \frac{\lambda}{4}\phi^4, \tag{5.4}$$

where V_0 is a constant, while the constant $M^2 < 0$ may be related to the mass of the Higgs boson by the relation $m_H = V''(v)$, where $v^2 = -M^2/\lambda$ gives the minimum of the potential.

In general, in the above cases, and in the presence of matter, analytical solutions are impossible to be extracted, and thus we resort to numerical elaboration of the cosmological equations. Thus, we evolve the equations using as an independent variable the redshift z defined in (3.13).

In order to perform a numerical elaboration of the above cosmological equations, it proves convenient to rewrite them in a dimensionless way. In particular, we introduce the dimensionless variables $(\tau, h, \Phi, v(\Phi), \beta_0, \zeta_0, r, P)$, defined as

$$\tau = H_0 t, \quad H = H_0 h, \quad \phi = \sqrt{\frac{6}{8\pi G}}\Phi, \quad v(\Phi) = \frac{8\pi G}{3H_0^2}V(\Phi), \tag{5.5}$$

$$\beta = \frac{\beta_0}{9H_0^2}, \qquad \zeta = \frac{4\pi G}{45H_0^4}\zeta_0, \qquad \rho_m = \frac{3H_0^2}{8\pi G}r, \qquad p_m = \frac{3H_0^2}{8\pi G}P. \tag{5.6}$$

Using these new variables, the generalized Friedmann equations (3.2) and (3.3), the scalar field equation (3.4), as well as the matter conservation equation (3.11) take the form

$$\frac{da}{d\tau} = \sqrt{\frac{v(\Phi) + r + (d\Phi/d\tau)^2}{1 - \beta_0(d\Phi/d\tau)^2 + \zeta_0(d\Phi/d\tau)^4}}a, \tag{5.7}$$

$$\begin{aligned} &2\frac{dh}{d\tau} + 3h^2 + 3\left[P - v(\Phi) + \left(\frac{d\Phi}{d\tau}\right)^2\right] - \frac{\beta_0}{3}\frac{d\Phi}{d\tau}\left[\left(2\frac{dh}{d\tau} + 3h^2\right)\frac{d\Phi}{d\tau} + 4h\frac{d^2\Phi}{d\tau^2}\right] \\ &\quad + \frac{\zeta_0}{5}\left(\frac{d\Phi}{d\tau}\right)^3\left[\left(2\frac{dh}{d\tau} + 3h^2\right)\frac{d\Phi}{d\tau} + 8h\frac{d^2\Phi}{d\tau^2}\right] = 0, \end{aligned} \tag{5.8}$$

$$\frac{d^2\Phi}{d\tau^2} + 3h\frac{d\Phi}{d\tau} + \frac{1}{2}\frac{dv(\Phi)}{d\Phi} + \frac{\beta_0}{3}h\left[\left(2\frac{dh}{d\tau} + 3h^2\right)\frac{d\Phi}{d\tau} + h\frac{d^2\Phi}{d\tau^2}\right] - \frac{2}{5}\zeta_0 h\left(\frac{d\Phi}{d\tau}\right)^2\left[\left(2\frac{dh}{d\tau} + 3h^2\right)\frac{d\Phi}{d\tau} + 3h\frac{d^2\Phi}{d\tau^2}\right] = 0, \tag{5.9}$$

$$\frac{dr}{d\tau} + 3h(r + P) = 0, \tag{5.10}$$

respectively. Therefore, after solving Eqs. (5.8) and (5.9) for $dh/d\tau$ and $d^2\Phi/d\tau^2$, the cosmological field equations take the form

$$\frac{d\Phi}{d\tau} = \Pi, \tag{5.11}$$

$$\frac{da}{d\tau} = \sqrt{\frac{v(\Phi) + r + \Pi^2}{1 - \beta_0\Pi^2 + \zeta_0\Pi^4}}a, \tag{5.12}$$

$$\frac{dh}{d\tau}=\frac{1}{2\{h^2[5\beta_0+(5\beta_0^2-18\zeta_0)\Pi^2-9\beta_0\zeta_0\Pi^4+6\zeta_0^2\Pi^6]-5\beta_0\Pi^2+3\zeta_0\Pi^4+15\}}\\ \times\{-3\Pi^2\{2h^2[10\beta_0-9\zeta_0 P+9\zeta_0 v(\Phi)]+(5\beta_0^2-18\zeta_0)h^4+15\}-15(\beta_0 h^2+3)[h^2+P-v(\Phi)]\\ +9\zeta_0 h^2\Pi^4(3\beta_0 h^2+13)-18\zeta_0^2 h^4\Pi^6+2h\Pi(6\zeta_0\Pi^2-5\beta_0)v'(\Phi)\}, \tag{5.13}$$

$$\frac{d\Pi}{d\tau}=\frac{(5\beta_0\Pi^2-3\zeta_0\Pi^4-15)v'(\Phi)-6h\Pi[-5\beta_0 P-2\Pi^2(5\beta_0-3\zeta_0 P+3\zeta_0 v(\Phi))+9\zeta_0\Pi^4+5\beta_0 v(\Phi)+15]}{2\{h^2[5\beta_0+(5\beta_0^2-18\zeta_0)\Pi^2-9\beta_0\zeta_0\Pi^4+6\zeta_0^2\Pi^6]-5\beta_0\Pi^2+3\zeta_0\Pi^4+15\}}, \tag{5.14}$$

which must be solved together with Eq. (5.10) after the equation of state $P=P(r)$ of matter has been imposed. The initial conditions for the system (5.11)–(5.14) are $a(\tau_0)=a_0$, $h(\tau_0)=h_0$, $\Phi(\tau_0)=\Phi_0$, and $\Pi(\tau_0)=\Pi_0$, respectively. Furthermore, in terms of the dimensionless variables the deceleration parameter (3.12) becomes

$$q=\frac{d}{d\tau}\left(\frac{1}{h}\right)-1, \tag{5.15}$$

while the dark energy equation-of-state parameter reads as

$$w_{\rm DE}=\frac{\Pi^2-v(\Phi)-\frac{\beta_0}{9}\Pi[(2\frac{dh}{d\tau}+3h^2)\Pi+4h\frac{d\Pi}{d\tau}]+\frac{\zeta_0}{15}\Pi^3[(2\frac{dh}{d\tau}+3h^2)\Pi+8h\frac{d\Pi}{d\tau}]}{\Pi^2\{1+h^2[\beta_0-\zeta_0\Pi^2]\}+v(\Phi)}. \tag{5.16}$$

Finally, the dimensionless time-redshift relation (3.14) becomes

$$\frac{d}{d\tau}=-(1+z)h(z)\frac{d}{dz}. \tag{5.17}$$

A. Exponential potential

Let us start the analysis by considering the exponential potential (5.2), namely

$$V(\phi)=\Lambda_0\exp(-\mu\phi), \tag{5.18}$$

with Λ_0 and μ the potential parameters. In terms of the dimensionless variables introduced in (5.5), the above exponential potential takes the form

$$v(\Phi)=\lambda_0\exp(-\mu_0\Phi), \tag{5.19}$$

where

$$\lambda_0=\frac{8\pi G}{3H_0^2}\Lambda_0,\qquad \mu_0=\sqrt{\frac{6}{8\pi G}}\mu. \tag{5.20}$$

In the following we consider the time evolution of a dust matter fluid; namely we assume that $P=0$, and hence the redshift dependence of the auxiliary matter energy density is given by $r=(1+z)^3$. In this case, the dimensionless cosmological equations (5.11), (5.13), and (5.14) become

$$\frac{d\Phi}{dz}=-\frac{\Pi}{(1+z)h}, \tag{5.21}$$

$$\frac{dh}{dz}=-\left\{\frac{(1+z)^{-1}h^{-1}}{2\{h^2[5\beta_0+(5\beta_0^2-18\zeta_0)\Pi^2-9\beta_0\zeta_0\Pi^4+6\zeta_0^2\Pi^6]-5\beta_0\Pi^2+3\zeta_0\Pi^4+15\}}\right\}\\ \times\{-3\Pi^2[2h^2(10\beta_0+9\zeta_0\lambda_0 e^{-\mu_0\Phi}+(5\beta_0^2-18\zeta_0)h^4+15]-15(\beta_0 h^2+3)(h^2-\lambda_0 e^{-\mu_0\Phi})\\ +9\zeta_0 h^2\Pi^4(3\beta_0 h^2+13)-18\zeta_0^2 h^4\Pi^6-2h\Pi(6\zeta_0\Pi^2-5\beta_0)\mu_0\lambda_0 e^{-\mu_0\Phi}\}, \tag{5.22}$$

$$\frac{d\Pi}{dz}=\frac{(5\beta_0\Pi^2-3\zeta_0\Pi^4-15)\mu_0\lambda_0 e^{-\mu_0\Phi}+6h\Pi[-2\Pi^2(5\beta_0+3\zeta_0\lambda_0 e^{-\mu_0\Phi})+9\zeta_0\Pi^4+5\beta_0\mu_0\lambda_0 e^{-\mu_0\Phi}+15]}{2(1+z)h\{h^2[5\beta_0+(5\beta_0^2-18\zeta_0)\Pi^2-9\beta_0\zeta_0\Pi^4+6\zeta_0^2\Pi^6]-5\beta_0\Pi^2+3\zeta_0\Pi^4+15\}}, \tag{5.23}$$

respectively. Additionally, the parameter of the dark energy equation of state (3.9) reads as

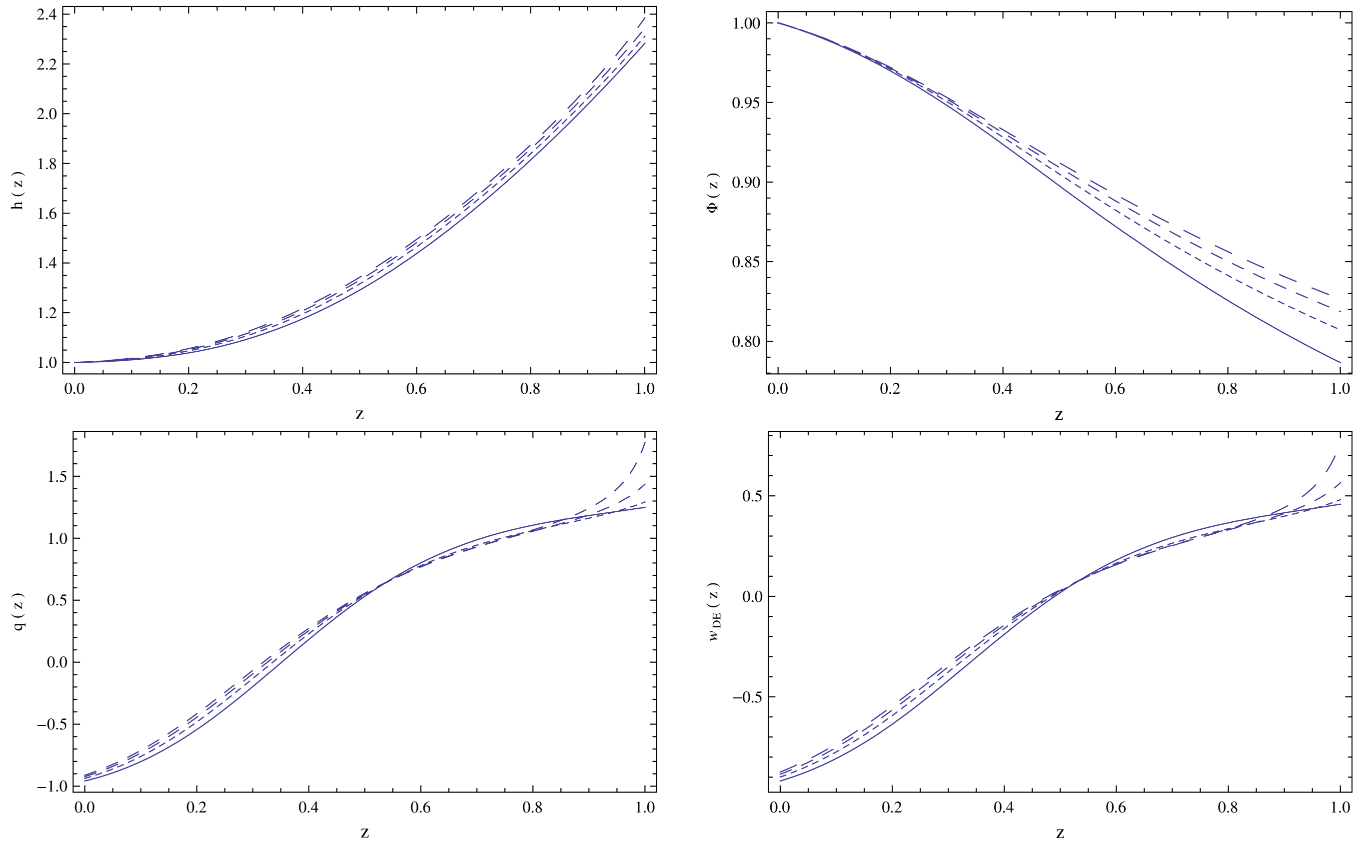

FIG. 1. Evolution of the dimensionless Hubble function (top left), of the dimensionless field (top right), of the deceleration parameter (bottom left), and of the dark energy equation-of-state parameter (bottom right), as a function of the redshift, for cosmology with extended nonminimal derivative coupling in the case of the exponential potential (5.18) and for dust matter. The initial conditions have been chosen as $h(0) = 1$, $\Phi(0) = 1$, and $\Pi(0) = 0.1$, while the parameters of the exponential potential have been fixed as $\mu_0 = -1.05$ and $\lambda_0 = 0.36$. Concerning the coupling parameters β_0 and ζ_0, we choose $\beta_0 = 1.90$ and $\zeta_0 = 1$ (solid curve), $\beta_0 = 2.68$ and $\zeta_0 = 2$ (dotted curve), $\beta_0 = 3.29$ and $\zeta_0 = 3$ (short dashed curve), and $\beta_0 = 3.79$ and $\zeta_0 = 4$ (dashed curve), respectively.

$$w_{\rm DE} = \{\Pi^2[1 + h^2(\beta_0 - \zeta_0\Pi^2)] + \lambda_0 e^{-\mu_0\Phi}\}^{-1}\left\{\Pi^2 - \lambda_0 e^{-\mu_0\Phi} - \frac{\beta_0}{9}\Pi\left\{\left[-2(1+z)h\frac{dh}{dz} + 3h^2\right]\Pi - 4(1+z)h^2\frac{d\Pi}{dz}\right\}\right.$$
$$\left. + \frac{\zeta_0}{15}\Pi^3\left\{\left[-2(1+z)h\frac{dh}{dz} + 3h^2\right]\Pi - 8(1+z)h^2\frac{d\Pi}{dz}\right\}\right\}. \qquad (5.24)$$

A crucial observation is that according to the above expression, $w_{\rm DE}$ could acquire values below -1 too, and thus the phantom regime can be exhibited. This is an advantage of the scenario at hand, since such a behavior is obtained although the scalar field is canonical, that it is a pure result of the extended, gravitational couplings.

In order to study the cosmological evolution of the dust universe in the presence of the exponential potential we integrate the system of Eqs. (5.21)–(5.23) numerically. We choose the potential parameters as $\lambda_0 = 0.36$, and $\mu_0 = -1.05$, while for the initial conditions we set $h(0) = 1$, $\Phi(0) = 1$, and $\Pi(0) = 0.1$. We are interested in studying the effect of the parameters β_0 and ζ_0 that determine the novel, extended nonminimal derivative coupling, on the cosmological evolution, restricting the analysis at late times, i.e., at the redshift range $0 \leq z \leq 1$.

In Fig. 1, we depict the evolution of the Hubble function, of the scalar field, of the deceleration parameter, and of the dark energy equation-of-state parameter, in terms of the redshift, for various values of β_0 and ζ_0. As we can see, the Hubble function, represented in the top left figure, is a monotonically increasing function of the redshift, indicating an expansionary cosmological evolution, while in the redshift range $0 \leq z \leq 0.1$ it remains almost constant. Its variation is not affected significantly by the change of the numerical values of the coupling parameters. Moreover, the dimensionless scalar field, depicted in the top right figure, is a monotonically decreasing function of the redshift, and therefore an increasing function of the cosmological time. Note that the scalar field behavior is strongly affected by variations of the numerical values of β_0 and ζ_0. The deceleration parameter, shown in the bottom left figure, starts with high positive values; however, acceleration arises quickly, with q crossing zero at a redshift $z \approx 0.35$. The variations of the coupling parameters have a small influence on the deceleration parameter behavior. Finally, the parameter $w_{\rm DE}$ of the dark energy equation of state, presented in the bottom right figure, starts with values of the order of $w_{\rm DE} \approx 0.4$–0.5 at $z = 1$, it reaches zero at $z \approx 0.5$, and it tends to -1 at $z = 0$, indicating that the dark energy sector behaves like a cosmological constant at present. Similar to the case of the dimensionless Hubble parameter, the changes in the numerical values of the parameters β_0 and ζ_0 have a small influence on $w_{\rm DE}$ evolution.

B. Power-law potential

Let us now investigate the cosmological evolution in the extended nonminimal derivative coupling gravitational theory in the presence of a simple power-law potential of the form (5.3), namely

$$V(\phi) = V_0 \phi^n, \tag{5.25}$$

where V_0 and n are constants. Hence, using the dimensionless variables introduced in (5.5), the dimensionless form of the power-law potential writes as

$$v(\Phi) = u_0 \Phi^n, \tag{5.26}$$

where

$$u_0 = \frac{6^{n/2}}{3H_0^2}(8\pi G)^{1-n/2} V_0. \tag{5.27}$$

In the case of the dust universe, namely imposing that $P = 0$, the dimensionless cosmological equations (5.11), (5.13), and (5.14) become

$$\frac{d\Phi}{dz} = -\frac{\Pi}{(1+z)h}, \tag{5.28}$$

$$\begin{aligned}\frac{dh}{dz} = &-\left\{\frac{(1+z)^{-1}h^{-1}}{2\{h^2[5\beta_0 + (5\beta_0^2 - 18\zeta_0)\Pi^2 - 9\beta_0\zeta_0\Pi^4 + 6\zeta_0^2\Pi^6] - 5\beta_0\Pi^2 + 3\zeta_0\Pi^4 + 15\}}\right\}\\ &\times\{-3\Pi^2[2h^2(10\beta_0 + 9\zeta_0 u_0\Phi^n) + (5\beta_0^2 - 18\zeta_0)h^4 + 15] - 15(\beta_0 h^2 + 3)(h^2 - u_0\Phi^n)\\ &+ 9\zeta_0 h^2\Pi^4(3\beta_0 h^2 + 13) - 18\zeta_0^2 h^4\Pi^6 + 2h\Pi(6\zeta_0\Pi^2 - 5\beta_0)nu_0\Phi^{n-1}\},\end{aligned} \tag{5.29}$$

$$\frac{d\Pi}{dz} = -\frac{(5\beta_0\Pi^2 - 3\zeta_0\Pi^4 - 15)v'(\Phi) - 6h\Pi\{-2\Pi^2[5\beta_0 + 3\zeta_0 v(\Phi)] + 9\zeta_0\Pi^4 + 5\beta_0 u_0\Phi^n + 15\}}{2(1+z)h\{h^2[5\beta_0 + (5\beta_0^2 - 18\zeta_0)\Pi^2 - 9\beta_0\zeta_0\Pi^4 + 6\zeta_0^2\Pi^6] - 5\beta_0\Pi^2 + 3\zeta_0\Pi^4 + 15\}}, \tag{5.30}$$

respectively. Additionally, the dark energy equation-of-state parameter (3.9) writes as

$$\begin{aligned}w_{\rm DE} = &\{\Pi^2[1 + h^2(\beta_0 - \zeta_0\Pi^2)] + u_0\Phi^n\}^{-1}\left\{\Pi^2 - u_0\Phi^n - \frac{\beta_0}{9}\Pi\left\{\left[-2(1+z)h\frac{dh}{dz} + 3h^2\right]\Pi - 4(1+z)h^2\frac{d\Pi}{dz}\right\}\right.\\ &\left.+\frac{\zeta_0}{15}\Pi^3\left\{\left[-2(1+z)h\frac{dh}{dz} + 3h^2\right]\Pi - 8(1+z)h^2\frac{d\Pi}{dz}\right\}\right\}.\end{aligned} \tag{5.31}$$

Similar to the exponential potential case, we can see that $w_{\rm DE}$ can acquire values of the phantom regime, which is an advantage of the scenario at hand.

In order to study the cosmological evolution of the dust universe in the presence of the power-law potential we integrate the system of Eqs. (5.28)–(5.30) numerically. We choose the potential parameters as $n = 1/4$ and $u_0 = 1.05$; for the initial conditions we set $h(0) = 1$, $\Phi(0) = 1$, and $\Pi(0) = 0.1$; and similar to the exponential potential of the previous subsection we restrict our analysis at late times, namely at the redshift range $0 \leq z \leq 1$.

As in the previous case, in Fig. 2 we depict the evolution of the Hubble function, of the scalar field, of the deceleration parameter, and of the dark energy equation-of-state parameter, in terms of the redshift, for various values of the coupling parameters β_0 and ζ_0.

The evolution of the Hubble function, presented in the top left graph, shows that the universe is expanding, with the Hubble function monotonically increasing with the redshift, while at $z \approx 0.2$ and below the Hubble function becomes almost a constant. The behavior of the Hubble function is relatively strongly affected by changes in the values of the coupling parameters β_0 and ζ_0, with the effect being stronger at higher redshifts. The dimensionless scalar field Φ, depicted in the top right graph, is a monotonically decreasing function of the redshift, and at high redshifts it also presents a strong dependence on the numerical values of β_0 and ζ_0. The deceleration parameter, presented in the

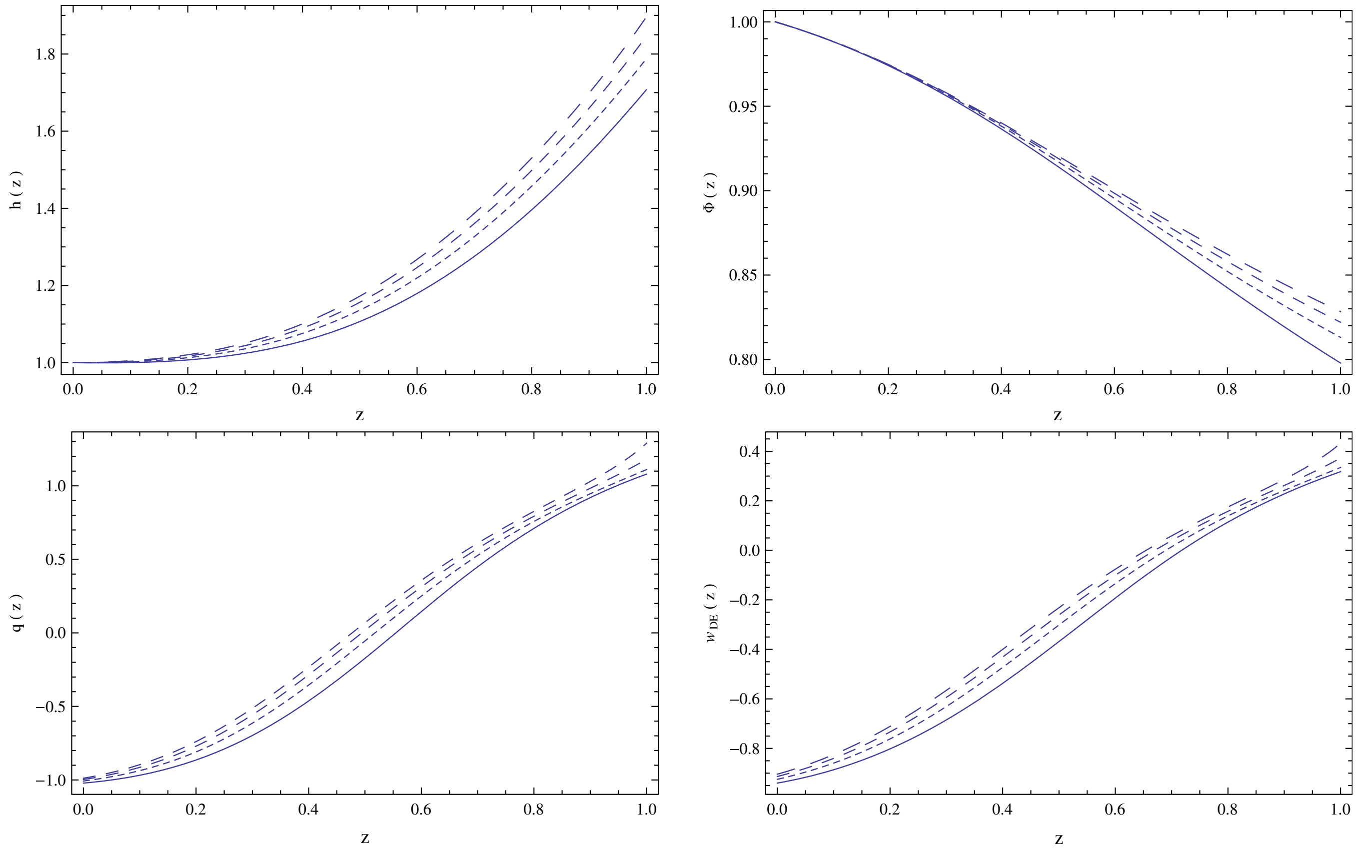

FIG. 2. Evolution of the dimensionless Hubble function (top left), of the dimensionless field (top right), of the deceleration parameter (bottom left), and of the dark energy equation-of-state parameter (bottom right), as a function of the redshift, for cosmology with extended nonminimal derivative coupling in the case of the power-law potential (5.25) and for dust matter. The initial conditions have been chosen as $h(0) = 1$, $\Phi(0) = 1$, and $\Pi(0) = 0.1$, while the parameters of the potential have been fixed as $n = 1/4$ and $u_0 = 1.05$. Concerning the coupling parameters β_0 and ζ_0, we choose $\beta_0 = 1.90$ and $\zeta_0 = 1$ (solid curve), $\beta_0 = 2.68$ and $\zeta_0 = 2$ (dotted curve), $\beta_0 = 3.29$ and $\zeta_0 = 3$ (short dashed curve), and $\beta_0 = 3.79$ and $\zeta_0 = 4$ (dashed curve), respectively.

bottom left graph, has positive values of the order of $q = 1$ at $z = 1$; however, acceleration is obtained at a redshift $z \approx 0.5$. Finally, the dark energy equation-of-state parameter, presented in the bottom right graph, has values of the order of $w_{DE} \approx 0.3$–0.4 at $z = 1$, it reaches zero at redshifts $z \approx 0.6$–0.7, and it tends to -1 at $z = 0$, implying that the dark energy sector behaves like a cosmological constant at present. We mention that both q and w_{DE} exhibit a relatively strong dependence on the numerical values of β_0 and ζ_0.

C. Higgs potential

As a final case we investigate the cosmological implications of theories with extended nonminimal derivative coupling, in the presence of the Higgs potential (5.4), namely

$$V(\phi) = V_0 + \frac{1}{2}M^2\phi^2 + \frac{\lambda}{4}\phi^4, \tag{5.32}$$

where V_0 is a constant, and where the constant $M^2 < 0$ can be related to the Higgs mass by the relation $m_H = V''(v)$, with $v^2 = -M^2/\lambda$ the minimum of the potential. Moreover, based on the determination of m_H from accelerator experiments one can infer for the Higgs self-coupling constant a value of the order of $\lambda \approx 1/8$ [77]. In terms of the dimensionless variables (5.5) the above Higgs-like potential becomes

$$v(\Phi) = v_0 - \frac{1}{2}m^2\Phi^2 + \frac{1}{4}\Lambda\Phi^4, \tag{5.33}$$

where

$$v_0 = \frac{8\pi G}{3H_0^2}V_0, \quad m^2 = 2\left(\frac{M}{H_0}\right)^2, \quad \Lambda = \frac{12\lambda}{8\pi G H_0^2}. \tag{5.34}$$

In the following paragraphs we study separately the cases of dust and radiation, respectively.

1. Cosmological evolution of a dust fluid

In the case of the dust matter sector, namely for $P = 0$, the dimensionless cosmological equations (5.11), (5.13), and (5.14) become

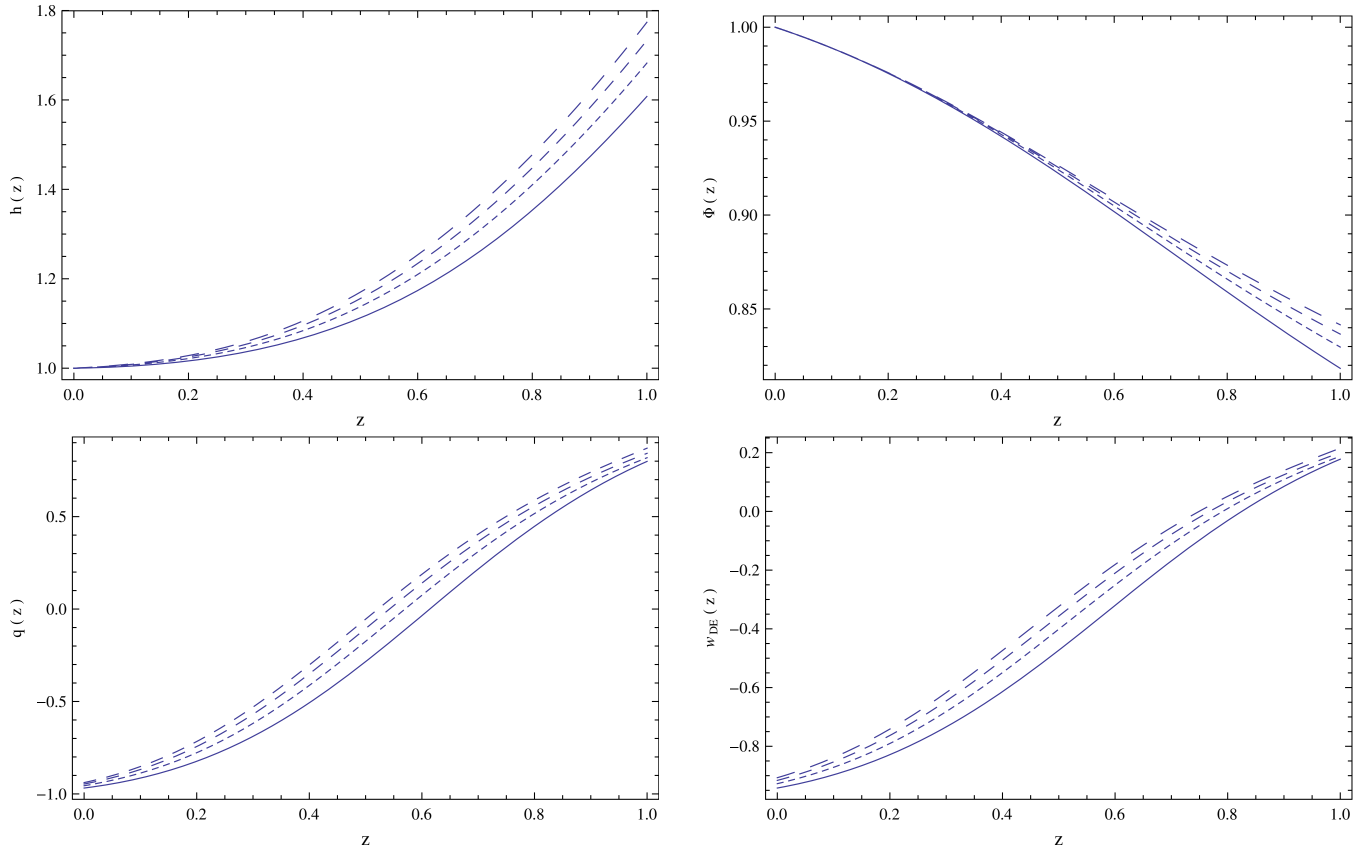

FIG. 3. Evolution of the dimensionless Hubble function (top left), of the dimensionless field (top right), of the deceleration parameter (bottom left), and of the dark energy equation-of-state parameter (bottom right), as a function of the redshift, for cosmology with extended nonminimal derivative coupling in the case of the Higgs-like potential (5.32) and for dust matter. The initial conditions have been chosen as $h(0) = 1$, $\Phi(0) = 1$, and $\Pi(0) = 0.1$, while the parameters of the exponential potential have been fixed as $v_0 = 0.99$, $m = 0.1$, $\lambda = 0.1$. Concerning the coupling parameters β_0 and ζ_0, we choose: $\beta_0 = 1.90$ and $\zeta_0 = 1$ (solid curve), $\beta_0 = 2.68$ and $\zeta_0 = 2$ (dotted curve), $\beta_0 = 3.29$ and $\zeta_0 = 3$ (short dashed curve), and $\beta_0 = 3.79$ and $\zeta_0 = 4$ (dashed curve), respectively.

$$\frac{d\Phi}{dz} = -\frac{\Pi}{(1+z)h}, \tag{5.35}$$

$$\begin{aligned}\frac{dh}{dz} = &-\left\{\frac{(1+z)^{-1}h^{-1}}{2\{h^2[5\beta_0 + (5\beta_0^2 - 18\zeta_0)\Pi^2 - 9\beta_0\zeta_0\Pi^4 + 6\zeta_0^2\Pi^6] - 5\beta_0\Pi^2 + 3\zeta_0\Pi^4 + 15\}}\right\}\\
&\times\left\{-3\Pi^2\left\{2h^2\left[10\beta_0 + 9\zeta_0\left(v_0 - \frac{1}{2}m^2\Phi^2 + \frac{1}{4}\Lambda\Phi^4\right)\right] + (5\beta_0^2 - 18\zeta_0)h^4 + 15\right\}\right.\\
&- 15(\beta_0 h^2 + 3)\left[h^2 - \left(v_0 - \frac{1}{2}m^2\Phi^2 + \frac{1}{4}\Lambda\Phi^4\right)\right] + 9\zeta_0 h^2\Pi^4(3\beta_0 h^2 + 13) - 18\zeta_0^2 h^4\Pi^6\\
&\left. + 2h\Pi(6\zeta_0\Pi^2 - 5\beta_0)(-m^2\Phi + \Lambda\Phi^3)\right\},\end{aligned} \tag{5.36}$$

$$\begin{aligned}\frac{d\Pi}{dz} = &-\{2(1+z)h\{h^2[5\beta_0 + (5\beta_0^2 - 18\zeta_0)\Pi^2 - 9\beta_0\zeta_0\Pi^4 + 6\zeta_0^2\Pi^6] - 5\beta_0\Pi^2 + 3\zeta_0\Pi^4 + 15\}\}^{-1}\\
&\times\left\{(5\beta_0\Pi^2 - 3\zeta_0\Pi^4 - 15)(-m^2\Phi + \Lambda\Phi^3) - 6h\Pi\left\{-2\Pi^2\left[5\beta_0 + 3\zeta_0\left(v_0 - \frac{1}{2}m^2\Phi^2 + \frac{1}{4}\Lambda\Phi^4\right)\right]\right.\right.\\
&\left.\left. + 9\zeta_0\Pi^4 + 5\beta_0\left(v_0 - \frac{1}{2}m^2\Phi^2 + \frac{1}{4}\Lambda\Phi^4\right) + 15\right\}\right\},\end{aligned} \tag{5.37}$$

respectively. Moreover, the dark energy equation-of-state parameter (3.9) reads as

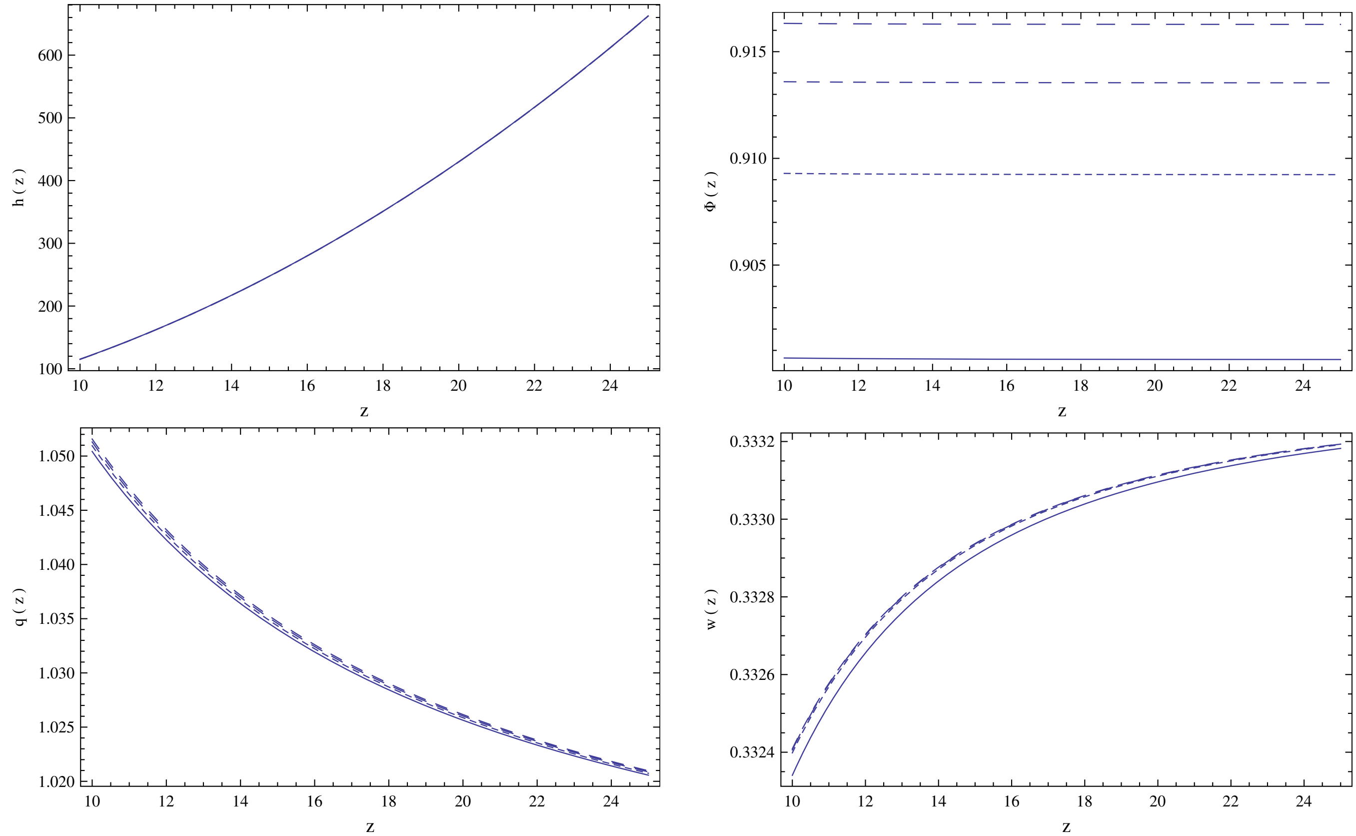

FIG. 4. Evolution of the dimensionless Hubble function (top left), of the dimensionless field (top right), of the deceleration parameter (bottom left), and of the total equation-of-state parameter (bottom right), as a function of the redshift, for cosmology with extended nonminimal derivative coupling in the case of the Higgs-like potential (5.32) and for radiation fluid. The initial conditions have been chosen as $h(0) = 1$, $\Phi(0) = 1$, and $\Pi(0) = 0.1$, while the parameters of the exponential potential have been fixed as $v_0 = 0.99$, $m = 0.1$, and $\lambda = 0.1$. Concerning the coupling parameters β_0 and ζ_0, we choose $\beta_0 = 1.897$ and $\zeta_0 = 1$ (solid curve), $\beta_0 = 2.683$ and $\zeta_0 = 2$ (dotted curve), $\beta_0 = 3.286$ and $\zeta_0 = 3$ (short dashed curve), and $\beta_0 = 3.794$ and $\zeta_0 = 4$ (dashed curve), respectively.

$$w_{\mathrm{DE}} = \{\Pi^2[1 + h^2(\beta_0 - \zeta_0\Pi^2)] + v(\Phi)\}^{-1}\left\{\Pi^2 - v(\Phi) - \frac{\beta_0}{9}\Pi\left\{\left[-2(1+z)h\frac{dh}{dz} + 3h^2\right]\Pi - 4(1+z)h^2\frac{d\Pi}{dz}\right\}\right.$$
$$\left. + \frac{\zeta_0}{15}\Pi^3\left\{\left[-2(1+z)h\frac{dh}{dz} + 3h^2\right]\Pi - 8(1+z)h^2\frac{d\Pi}{dz}\right\}\right\}. \quad (5.38)$$

Similar to the previous cases, we can see that w_{DE} can acquire values of the phantom regime, which is an advantage of the scenario at hand.

In order to study the cosmological evolution of the dust universe in the presence of the Higgs-like potential we integrate the system of Eqs. (5.35)–(5.37) numerically. We choose the potential parameters as $v_0 = 0.99$, $m = 0.1$, and $\lambda = 0.1$, and for the initial conditions we set $h(0) = 1$, $\Phi(0) = 1$, and $\Pi(0) = 0.1$; similar to the previous subsections we restrict our analysis at late times, namely at the redshift range $0 \leq z \leq 1$.

In Fig. 3 we depict the evolution of the Hubble function, of the scalar field, of the deceleration parameter, and of the dark energy equation-of-state parameter, in terms of the redshift, for various values of the coupling parameters β_0 and ζ_0.

The Hubble function, presented in the top left graph, is a monotonically increasing function of the redshift, and it becomes almost a constant in the redshift range $0 \leq z \leq 0.1$. The dimensionless scalar field Φ, presented in the top right graph, is a monotonically decreasing function of z. For the considered range of parameters, the deceleration parameter q, presented in the bottom left graph, acquires a value $q \approx 0.6$ at $z = 1$, and it decreases monotonically with respect to the redshift, crossing the $q = 0$ line at redshifts of the order of $z \approx 0.5$–0.6, while tending to the value -1 at present. Finally, the dark energy equation-of-state parameter, depicted in the bottom right

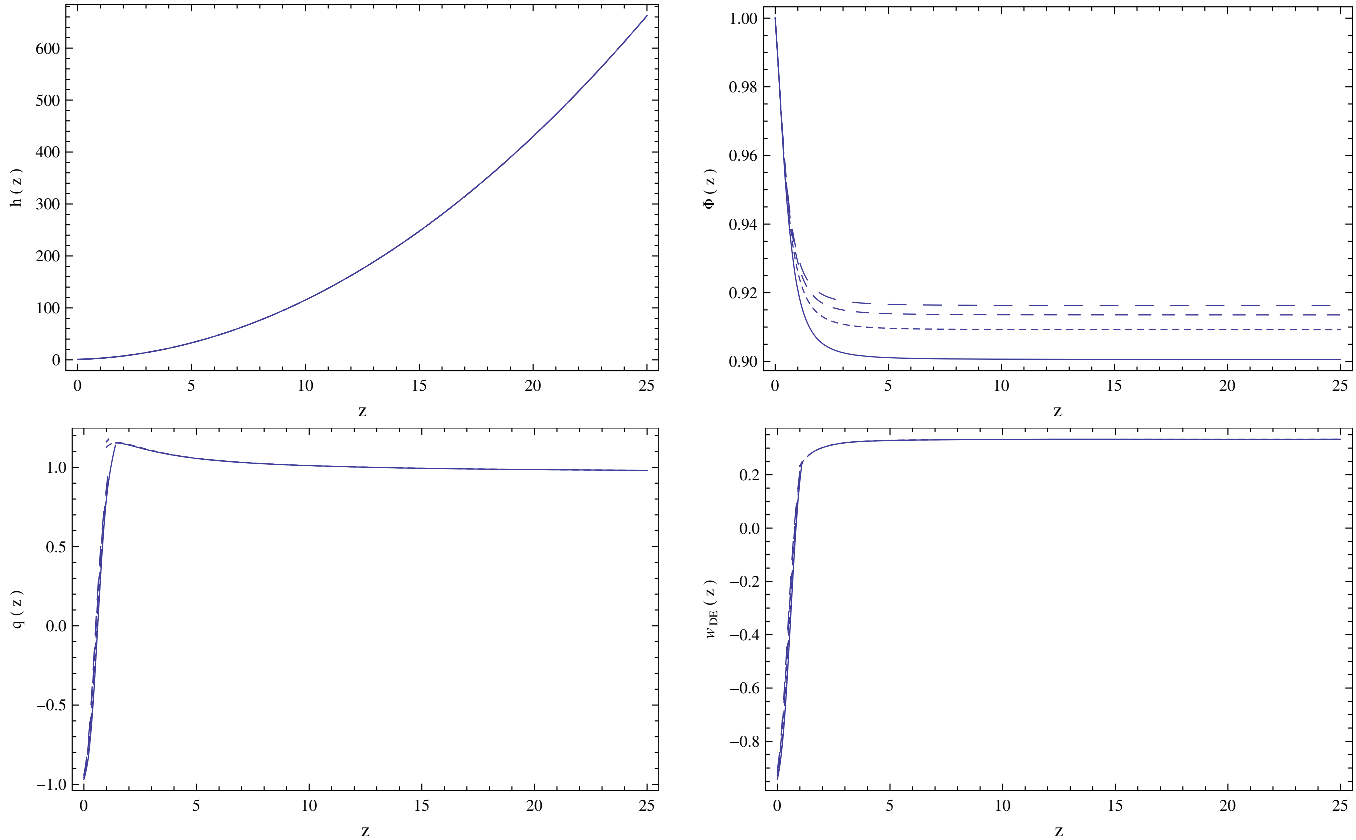

FIG. 5. Evolution of the dimensionless Hubble function (top left), of the dimensionless field (top right), of the deceleration parameter (bottom left), and of the total equation-of-state parameter (bottom right), as a function of the redshift, for cosmology with extended nonminimal derivative coupling in the case of the Higgs-like potential (5.32), in the redshift range $z \in [0, 25]$. The initial conditions have been chosen as $h(0) = 1$, $\Phi(0) = 1$, and $\Pi(0) = 0.1$, while the parameters of the exponential potential have been fixed as $v_0 = 0.99$, $m = 0.1$, and $\lambda = 0.1$. Concerning the coupling parameters β_0 and ζ_0, we choose $\beta_0 = 1.897$ and $\zeta_0 = 1$ (solid curve), $\beta_0 = 2.683$ and $\zeta_0 = 2$ (dotted curve), $\beta_0 = 3.286$ and $\zeta_0 = 3$ (short dashed curve), and $\beta_0 = 3.794$ and $\zeta_0 = 4$ (dashed curve), respectively.

graph, has a value $w_{\rm DE} \approx 0.2$ at $z = 1$, and it tends to -1 at $z = 0$, implying that the dark energy sector behaves like a cosmological constant at present.

2. Cosmological evolution of a radiation fluid

For completeness, in this subsection we investigate the case where the matter fluid corresponds to radiation, a case which is useful for early and intermediate stages of the universe evolution. In particular, we consider an equation of state of the form $p_m = \rho_m/3$, and thus the dimensionless matter density and pressure scale with respect to the redshift according to $r(z) = (1+z)^4$ and $P(z) = (1+z)^4/3$, respectively. Moreover, we restrict our analysis at intermediate times; that is, we focus on the redshift range $10 \leq z \leq 25$. In Fig. 4, we depict the evolution of the Hubble function, of the scalar field, of the deceleration parameter, and of the total equation-of-state parameter of the Universe, in terms of the redshift, for various values of the coupling parameters β_0 and ζ_0.

As we can see, the Hubble function, presented in the top left graph, is a monotonically increasing function of the redshift. In the considered redshift range the scalar field, shown in the top right graph, obtains constant values. The deceleration parameter, plotted in the bottom left graph, is positive, with a slight increase from $q = 1$ at $z = 25$ to $q = 1.05$ at $z = 10$, indicating a decelerating behavior, as expected. Finally, the total equation of the state of the Universe, depicted in the bottom right graph, is positive, with values of the order of $w \approx 0.33$, indicating a radiation-dominated expansion. We mention that all quantities, apart from the scalar field, do not have a strong dependence on the change of the numerical values of the coupling parameters β_0 and ζ_0, and hence the corresponding individual theories would not be easily distinguishable.

3. The unified picture of the evolution of the Universe in theories with extended nonminimal derivative coupling, in the presence of the Higgs potential

Finally, to conclude the investigation of the cosmological implications of theories with extended nonminimal derivative coupling, in the presence of the Higgs potential, we present a unified picture of the evolution of the Universe for the redshift range $z \in (0, 25)$. The variations of the Hubble function, of the dimensionless scalar field, of the deceleration parameter, and of the parameter of the dark energy equation of state are plotted in Fig. 5, respectively.

To obtain a unified picture of the evolution of the Universe in the presence of a gravitational theory with extended nonminimal derivative coupling, and in the presence of a Higgs-type potential, we take for the redshift z the range from 0 to 25. We further assume that in the range $z \in [5, 25]$ the matter content of the Universe can be described (at least approximately) by a radiation-type equation of state $p \approx \rho/3$. At the redshift $z \approx 5$ the Universe enters in the matter dominated era, with $p \approx 0$. In the present simplified model the transition from the radiation dominated era to the matter dominated stage is smooth, with all physical, geometrical, and thermodynamical quantities continued at the $z = 5$ transition redshift. The Hubble function and the scalar field Φ, represented in the upper panels of Fig. 5, are monotonically increasing and decreasing functions of the redshift for the entire period. The evolution of the Hubble function is not significantly influenced by the variation of the model parameters. The scalar field Φ is approximately a constant in the redshift range $z \in (3, 25]$, and its numerical values are strongly dependent on the model parameters. For redshifts $z < 3$ the scalar field starts to increase, reaching its maximum value at $z = 0$. In the range $z \in [0, 2)$ the variation of the field is basically independent on the model parameters. The deceleration parameter, depicted in the bottom left panel of Fig. 5, is approximately constant and positive in the redshift range $z \in (5, 25]$, with numerical values of the order of $q \approx 1$. After the beginning of the matter dominate phase at $z = 5$, the deceleration parameter increases, indicating a further deceleration of the Universe. But at $z \approx 2$, the Universe enters in an accelerating phase, and at $z = 0$ the Universe experiences an exponential, de Sitter–type expansion, with $q = -1$. The parameter w_{DE} of the dark energy equation of state (bottom right panel of Fig. 5) shows a similar dynamics. w_{DE} is approximately constant in the reshift range $z \in (3, 25]$, with $w_{\mathrm{DE}} \approx 0.30$. At $z \approx 3$, w_{DE} begins to decrease rapidly, and reaches the value $w_{\mathrm{DE}} = -1$ at $z = 0$, indicating that at this redshift the Universe is dominated by the effective dark energy generated by the extended nonminimal derivative coupling in the presence of a Higgs-type scalar field potential. The cosmological evolutions of both q and w_{DE} are basically independent on the variation of the numerical values of the model parameters.

VI. CONCLUSIONS

In this work we considered gravitational modifications that go beyond Horndeski; namely we presented theories with extended nonminimal derivative coupling, in which the coefficient functions depend not only on the scalar field but on its kinetic energy too. Such theories prove to be ghost-free in a cosmological background, and hence it is interesting to examine their cosmological implications. We first analyzed the cosmology of these novel gravitational modifications at early times, neglecting the matter sector, and we showed that a de Sitter inflation can be realized even in the absence of a potential term or of an explicit cosmological constant, and hence it is a pure result of the extended gravitational couplings.

Additionally, we studied the behavior of these cosmological scenarios at late times, where we obtained an effective dark energy sector arisen from the scalar field and its extended couplings to gravity. We extracted various cosmological observables such as the Hubble function, the deceleration parameter, and the dark energy equation-of-state parameter, and we numerically investigated their evolution at small redshifts, for three choices of potentials, namely for the exponential, the power-law, and the Higgs potentials. As we showed, in all cases the Universe passes from deceleration to acceleration in the recent cosmological past, while the effective dark energy equation-of-state parameter tends to the cosmological-constant value at present, in agreement with observations. Moreover, we showed that the phantom regime can be accessible too, which is an advantage of the scenarios since it is obtained despite the scalar field being canonical; i.e., it results purely from the novel, extended gravitational couplings.

The above features indicate that theories with an extended nonminimal derivative could be a good candidate for the description of early and late time universes. Hence one could proceed to more detailed analyses. In particular, one could use observational data from SNIa, BAO, and CMB in order to constrain the coefficient functions, as well as the new coupling parameters. Additionally, one could perform a complete dynamical analysis, in order to bypass the nonlinearities of the equations, and extract the global behavior at asymptotically late times. Moreover, one should analyze the perturbations in a thorough way, in order to extract the values for inflation-related observables such as the spectral index and the tensor-to-scalar ratio. Furthermore, the issue of the influence of the scalar degree of freedom in local gravity is an open problem; however, it lies beyond the scope of the present work. It would be interesting to examine whether there are any issues related to the fifth force, as it has been done previously on the original Horndeski theory [78,79]. Finally, it could be interesting to apply extended nonminimal derivative couplings to biscalar theories, such as those proposed recently in [80–83]. These investigations lie beyond the scope of the present work and are left for future projects.

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