String theory and the dark glueball problem

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We study cosmological constraints on dark pure Yang-Mills sectors. Dark glueballs are overproduced for large regions of ultraviolet parameter space. The problem may be alleviated in two ways: via a large preferential reheating into the visible sector, motivating certain inflation or modulus decay models, or via decays into axions or moduli, which are strongly constrained by nucleosynthesis and ΔN_{eff} bounds. String models frequently have multiple hidden Yang-Mills sectors, which are subject to even stronger constraints due to the existence of multiple dark glueballs.

I. INTRODUCTION

Hidden gauge sectors are ubiquitous in string theory. Initially they arose in the perturbative heterotic string, beginning with the ten-dimensional $E_8 \times E_8$ heterotic string [1] itself, its orbifold compactifications [2–7], free fermionic realizations (e.g. [8,9]), and later its smooth Calabi-Yau [10–12] compactifications. They also arise in other contexts, for example on D-branes in type II models (e.g. [13–15]); in rational conformal field theory orientifolds [16,17]; on singularities in G_2 compactifications of M-theory (e.g. [18–20]); and on seven-branes in F-theory. In fact, in F-theory there is growing evidence [21–25] that dark gauge sectors are generic in a sharp sense: the set of seven-brane configurations at a generic point in seven-brane moduli space has multiple disconnected gauge sectors.

In this paper we study the cosmological implications of gauged hidden sectors from an ultraviolet (UV) perspective, focusing on the simplest case of pure Yang-Mills theory. This is well motivated: if N of the hidden gauge factors in a much larger set of hidden sectors are either pure super Yang-Mills theories or have no symmetry protection for matter masses (other than supersymmetry), then N pure Yang-Mills sectors arise in the infrared. The associated N UV gauge couplings may each take a variety of values, determined by moduli stabilization, giving N hierarchical confinement scales due to the exponential dependence of each on its UV gauge coupling.

These sectors are cosmologically relevant if reheating via inflaton or modulus decay reheats not only the visible sector but some of the hidden sectors as well. Such a scenario was studied in the case of a single hidden sector in [26]. Though not focused specifically on dark glueballs, the relic abundance inferred from [26] depends critically on the confinement scale and the ratio of visible to hidden sector entropy densities determined by reheating; this is the result from which many of ours follow. This scenario also has an effective $3 \rightarrow 2$ self-interaction that causes the dark sector to "cannibalize" itself, a phenomenon of recent interest [27–33]. Dark matter that is comprised of dark glueballs has also been the subject of a number of studies [34–38].

In this paper we will show the converse: in ultraviolet theories (such as string theory) with many Yang-Mills sectors and a variety of dark confinement scales, the associated dark glueballs are poor dark matter candidates, but place valuable cosmological constraints on the ultraviolet theory. The problem exists already in the case of a single dark glueball, as its relic abundance oversaturates the observed dark matter relic abundance for much of the natural UV parameter space. This oversaturation is simple to understand, as the dark confinement scale may take a variety of values and there is no reason to have a dark glueball "miracle" analogous to the weakly interacting massive particle miracle. The problem is exacerbated in theories with many dark glueballs since each may have a different confinement scale, and if any fall into the dangerous regions of parameter space that glueball will oversaturate.

We study two possible ways in which the problem may be ameliorated, via dark glueball decay into axions or moduli, or via preferential reheating into the visible sector. Each mechanism faces constraints of its own, the former from nucleosynthesis bounds on glueball lifetimes and on the effective number of neutrinos ΔN_{eff} present at late times, and the latter on inflationary or modulus decay model building. In the case of symmetric reheating most of the parameter space is ruled out, even after taking into account possible decays.

II. THE RELIC ABUNDANCE OF DARK GLUEBALLS

We consider a scenario in which a dark Yang-Mills sector with gauge group G and confinement scale Λ is

reheated to a temperature $T'_{\text{rh}} > \Lambda$. The dark sector is a thermal bath of dark gluons, and as the dark sector cools through a transition temperature $T'_\Lambda \sim \Lambda$ the energy density in gluons is converted into glueballs. For dark sector temperature $T' < T'_\Lambda$, number depleting $3 \to 2$ interactions change the dependence of T' on the scale factor $a(t)$ relative to that of noninteracting nonrelativistic particles, giving a dark to visible temperature ratio

$$\frac{T'}{T} \propto \frac{a}{\ln(a)}. \tag{1}$$

Physically, this unusual temperature dependence arises because the interactions increase the average kinetic energy per glueball, so the dark sector cannibalizes itself to stay warm. Freeze-out occurs when these interactions cease to be effective, leaving a dark glueball relic.

Through this process, comoving entropy density is conserved in each sector due to thermal equilibrium and minimal interactions between the sectors, so that the ratio

$$\xi := \frac{s}{s'} \tag{2}$$

is a constant. For sufficiently high T_{rh} both sectors are relativistic, since the dark sector is by the assumption $T'_{\text{rh}} > \Lambda$, giving the additional relation $\xi = g_S T^3 / g'_S T'^3$. In this case the initial entropy ratio could instead be thought of as an initial temperature ratio, $\xi_T := T/T' = (g'_S \xi / g_S)^{1/3}$. We call $\xi = 1$ the democratic scenario.

This cosmological scenario was studied by Carlson *et al.* in [26], who treated the lightest glueball as a scalar field ϕ. Let us review their results. The annihilation rate for an average particle via $3 \to 2$ interactions is determined by an effective operator

$$\mathcal{O}_{3 \to 2} = \frac{1}{\Lambda} \frac{f}{5!} \phi^5, \tag{3}$$

with the rate given by

$$\Gamma(3 \to 2) = \frac{\sqrt{5} f^2 n'^2}{2304 \pi \Lambda^5} = \lambda \Lambda \left(\frac{n'}{\Lambda^3}\right)^2, \tag{4}$$

where $\lambda = \sqrt{5} f^2 / (2304\pi)$. Note that Λ and f are unrelated. As the Universe expands this rate is eventually not high enough to further deplete the glueball number and therefore the glueballs decouple at a temperature $T'_d \leq \Lambda$. Using the fact that ξ is constant and comparing it to the visible sector entropy density today, the relic abundance is

$$\Omega h^2 = \frac{T'_d}{3.6 \text{ eV} \xi}. \tag{5}$$

At decoupling, Einstein's equations may be radiation or matter dominated. In the case of radiation domination at decoupling, T'_d may be determined by solving the transcendental equation (the small deviation from [26] in the numerical constants comes from a slightly improved value of N_{eff})

$$\frac{\Lambda}{T'_d} + 2\ln\left(\frac{\Lambda}{T'_d}\right) = \frac{3}{4}\ln\left(\frac{\lambda g'^{7/4}}{\Omega h^2}\right) - \frac{5}{4}\ln(g'^{1/4}\xi) + 43.4. \tag{6}$$

In the case where the Universe is matter dominated at decoupling, T'_d is determined by

$$\frac{\Lambda}{T'_d} + \frac{3}{2}\ln\left(\frac{\Lambda}{T'_d}\right) = \frac{2}{3}\ln\left(\frac{\lambda g'^{7/4}}{\Omega h^2}\right) - \frac{2}{3}\ln(g'^{1/4}\xi) + 38.07. \tag{7}$$

The appearance of Λ is a substitute for the dark matter mass m' of [26], and g' is the number of effective relativistic degrees of freedom in the dark sector. This is motivated by the fact that glueballs are expected to have mass $m' = c\Lambda$ with $c \gtrsim 1$ an $O(1)$ coefficient. We take $c = 1$ for simplicity, since it does not significantly affect our conclusions.

Using these results, [26] studied the implications of the observed dark matter relic abundance from decoupling.

We instead take an ultraviolet perspective, where high scale physics such as moduli stabilization in string theory could set a wide range of values for the ultraviolet gauge coupling α_{UV}; Λ depends exponentially on α_{UV}. We will see that the glueball relic abundance is linear in Λ to a good approximation, and therefore the glueball is a poor dark matter candidate since α_{UV} must be exponentially fine-tuned to obtain a relic abundance close to the observed value. However, we will see that dark glueballs can place strong constraints on the ultraviolet theory.

Let us compute the relic abundance in terms of the confinement scale rather than the decoupling temperature. To do so, we use (5) to trade T'_d for the relic abundance in (6) and (7). In the case of radiation domination at decoupling this leads to

$$\Omega h^2 = \frac{\Lambda}{3.6 \text{ eV}} \frac{4}{5\xi W(7.45 \times 10^{12} f^{6/5} g'^{4/5} \xi^{-2/5} (\frac{3.6 \text{ eV}}{\Lambda})^{3/5})}, \tag{8}$$

where $W(x)$ is the Lambert W-function or product logarithm, which is the inverse of $f(x) = xe^x$ much as log is the inverse of $f(x) = e^x$. In the case of matter domination at decoupling the relic abundance is

$$\Omega h^2 = \frac{\Lambda}{3.6 \text{ eV}} \frac{6}{5\xi W(1.28 \times 10^{17} f^{8/5} g'^{6/5} (\frac{3.6 \text{ eV}}{\Lambda})^{4/5})}. \tag{9}$$

These calculations are valid for $\Lambda/T'_d > 1$.

What is the relic abundance outside of this regime? Naively considering $\Lambda/T'_d < 1$ is not physically sensible, since for temperatures $T' > \Lambda$ the dark sector is comprised of relativistic gluons and the effective field theory in which

$3 \rightarrow 2$ interactions were computed is not valid. Instead, as the Universe cools in this other regime glueballs form and immediately decouple, i.e. $T'_d \simeq \Lambda$. This together with (5) gives a relic abundance

$$\Omega h^2 \simeq \frac{\Lambda}{3.6 \text{ eV}\xi}, \qquad (10)$$

which closely matches the results of [36], which set the $3 \rightarrow 2$ interactions to zero.

How strongly do $3 \rightarrow 2$ interactions affect the relic abundances (8) and (9)? Specifically, how much do they deplete the relic abundance (10) that would be obtained in the absence of these interactions? This can be approximated by noting the mild dependence of $W(x)$ on $x > 0$, which is similar to the mild dependence of $\log(x)$ on similar x. For example, though $W(10)$ is $\mathcal{O}(1)$, $W(10^{1000})$ is only $\mathcal{O}(10^3)$. Since both (8) and (9) have $\Omega h^2 \simeq (\Lambda/\text{eV}) \times (1/\xi W(x))$, the order of magnitude of the dark glueball relic abundance is primarily set by Λ and ξ.

In particular, if (10) oversaturates the observed relic abundance by many orders of magnitude, $3 \rightarrow 2$ interactions cannot ameliorate the situation.
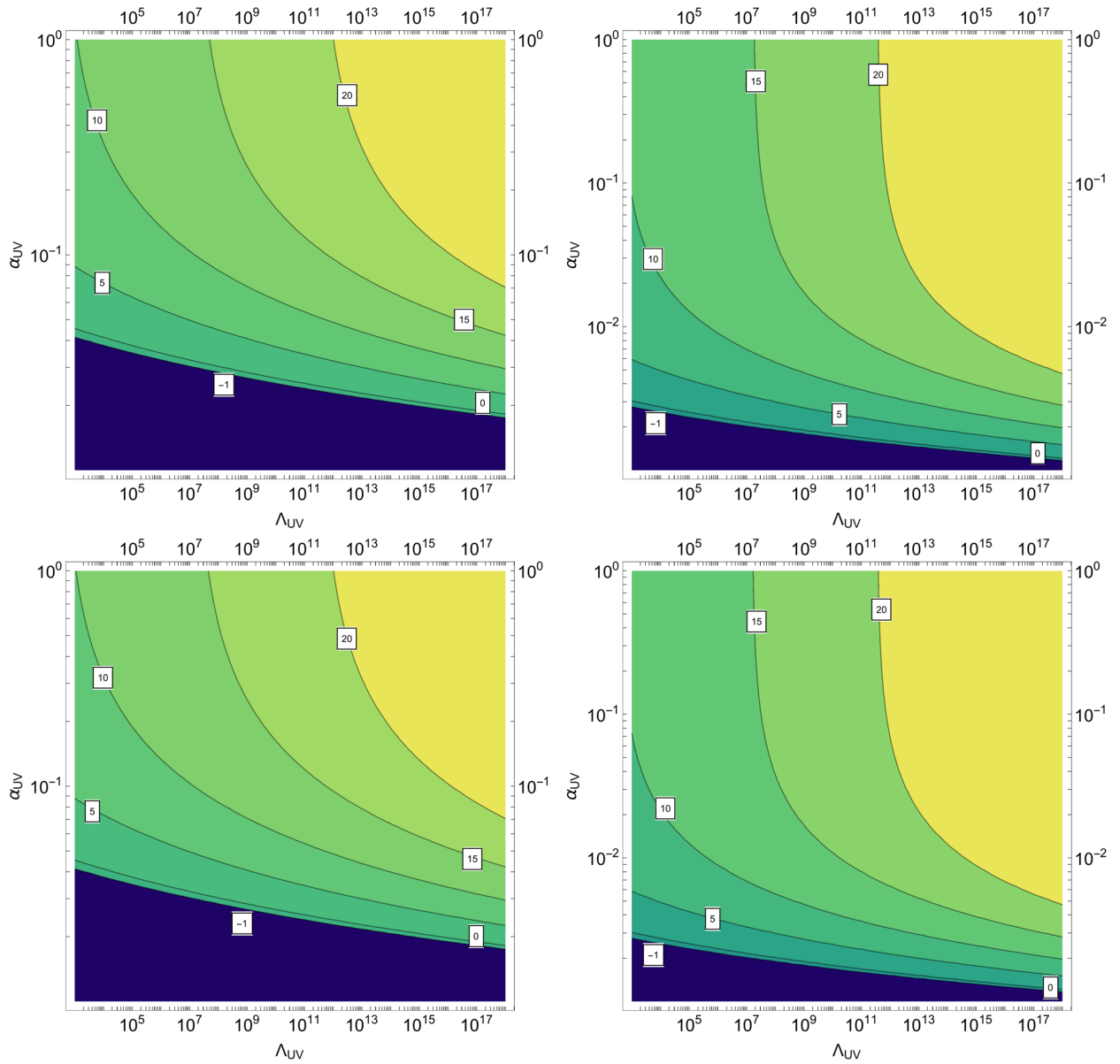

FIG. 1. Glueball relic abundance as a function of α_{UV} and Λ_{UV} for $\xi = 1$, $f = .1$, $g' = 1$. In each figure the relic abundance is oversaturated outside of the blue region. (Upper left panel) $G = SU(2)$ and radiation domination at decoupling. (Upper right panel) $G = E_8$ and radiation domination at decoupling. (Lower left panel) $G = SU(2)$ and matter domination at decoupling. (Lower right panel) $G = E_8$ and matter domination at decoupling.
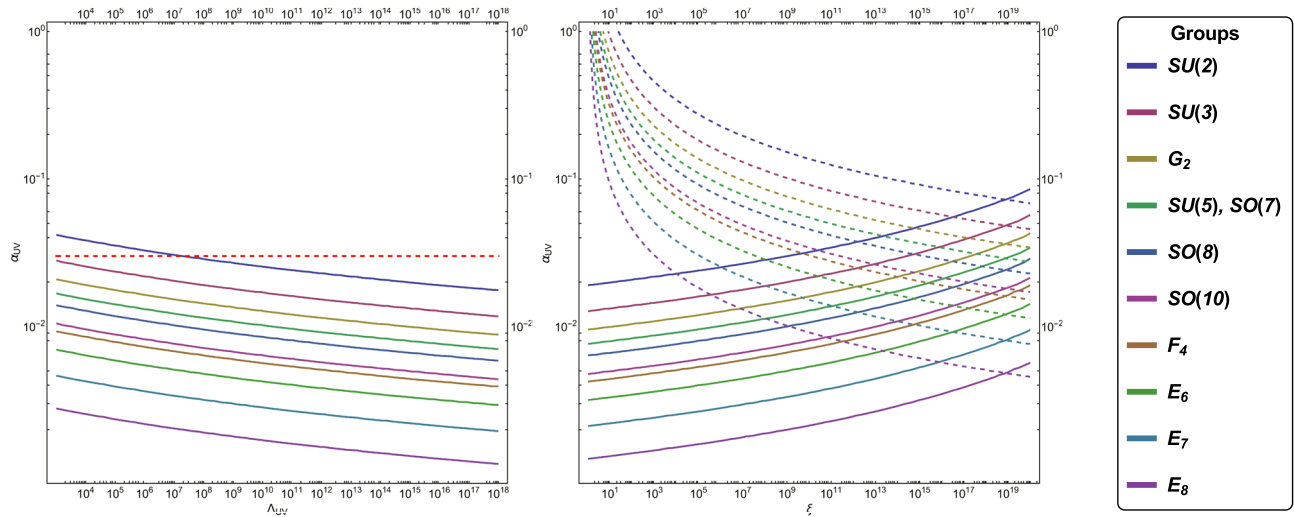
FIG. 2. (Left panel) Contours where the glueball relic abundance saturates the observed relic abundance for various groups as a function of α_{UV} and Λ_{UV} (in GeV), with oversaturation occurring in the region above each contour. The horizontal red line marks $\alpha_{\mathrm{UV}} = \alpha_{\mathrm{GUT}}$. (Right panel) The glueball relic abundance matches the observed relic abundance along dark contours, with oversaturation above. To the right of the dashed contours the dark sector is not reheated above its confinement scale.

A. Overproduction of democratic dark glueballs

Let us study the glueball relic abundance in the democratic case $\xi = 1$, focusing on its dependence on Λ and ξ for natural values of ultraviolet parameters.

We compute Λ via the beta function of super Yang-Mills theory, which gives

$$\Lambda \equiv \Lambda_{\mathrm{IR}} = \Lambda_{\mathrm{UV}} e^{-\frac{2\pi}{3C_2(G)\alpha_{\mathrm{UV}}}}, \qquad (11)$$

where $C_2(G)$ is the dual Coxeter number of the gauge group G, α_{UV} is the ultraviolet gauge coupling evaluated at scale Λ_{UV}, and Λ_{IR} is the scale at which α diverges. We use the supersymmetric beta functions all the way down to low scale for both simplicity and generosity. The former applies since this choice avoids the introduction of the scale Λ_{SUSY}, and the latter applies since the supersymmetric beta functions give rise to lower confinement scales; the oversaturation problem that we will encounter is only exacerbated by using nonsupersymmetric beta functions below Λ_{SUSY}.

The groups that we study are $SU(2)$, $SU(3)$, G_2, $SO(7)$, $SU(5)$, $SO(8)$, $SO(10)$, F_4, E_6, E_7, and E_8, which are two of the most commonly studied grand unified groups [$SU(5)$ and $SO(10)$] together with the group factors that may appear geometrically for general values of complex structure moduli in $d = 4$ F-theory [21]. These groups have $C_2(G)$ given by 2, 3, 4, 5, 5, 6, 8, 9, 12, 18, and 30, respectively. These values imply that, for a fixed Λ_{UV}, a change from one group to another can give rise to the same Λ by an $O(1)$–$O(10)$ change in α_{UV}. The same relic abundance for glueballs of different groups can therefore be obtained by a relatively small α_{UV} change.

In Fig. 1 we take $G = SU(2)$ and E_8 glueballs as prototypes, since they have the lowest and highest confinement scales, respectively, for a fixed Λ_{UV} and α_{UV}. The relic abundances are computed in both the case of radiation domination and matter domination at decoupling, taking $\xi = 1$ and studying the natural parameter space $10^{-3} \leq \alpha_{\mathrm{UV}} \leq 1$, 10^3 GeV $\leq \Lambda_{\mathrm{UV}} \leq 10^{18}$ GeV. On a log-log scale we see that there is little difference between the two cases. The blue band represents undersaturation of the observed relic abundance [39] $\Omega_{\mathrm{obs}} h^2 = 0.1199 \pm 0.0027$, with saturation occurring at the edge. The $\Omega h^2 = 1$ contour sits very close to the observed relic abundance contour, and the $\Omega h^2 = 10^5, 10^{10}, 10^{15}, 10^{20}$ contours make up the remaining parameter space. This figure demonstrates that for $\xi = 1$, the smallest [$SU(2)$] and largest (E_8) glueball relic abundances for these groups oversaturate the observed value by many orders of magnitude for much of this parameter space.

Having demonstrated how rapidly Ωh^2 increases in the Λ_{UV}-α_{UV} plane, in Fig. 2 we present the contours on which each of the groups we study saturates the observed relic abundance. Over half of the parameter space is ruled out for all of the groups, and for some groups it is much more. For $\Lambda_{\mathrm{UV}} > 10^9$ GeV the observed relic abundance is oversaturated for $\alpha = \alpha_{\mathrm{GUT}} \simeq .03$ for all groups, though moduli stabilization may fix the ultraviolet gauge coupling at significantly different values.

In conclusion, stable dark glueballs in the democratic scenario $\xi = 1$ oversaturate the relic abundance for much of the ultraviolet parameter space, putting strong constraints on ultraviolet theories that realize dark Yang-Mills sectors. Henceforth we will call this the dark glueball problem, for

brevity, and in the next two sections we will study mechanisms that could potentially solve it.

III. CONSTRAINTS FROM PREFERENTIAL REHEATING

One potential solution to the dark glueball problem is to reheat preferentially into the visible sector, constraining models of reheating via inflaton or modulus decay. Preferential reheating into the visible sector, i.e. $\xi > 1$, may solve the problem by either depleting the dark glueball relic abundance or by leaving the regime of validity $T'_{\rm rh} \gtrsim \Lambda$ for the production mechanism of [26].

Let us study the former by deriving bounds on ξ that are sufficient to not oversaturate the observed dark matter relic abundance, beginning with models that do not exhibit $3 \to 2$ interactions. This will give an approximate lower bound on ξ that is rough, but good enough for some purposes since $3 \to 2$ interactions cannot significantly suppress a very large relic abundance. The bound that avoids oversaturation is

$$\xi \gtrsim \frac{\Lambda}{3.6 \text{ eV}\Omega_{\rm obs}h^2} = 2.3 \frac{\Lambda}{\text{eV}}. \qquad (12)$$

Thus, in the absence of $3 \to 2$ interactions, confinement scales $\Lambda \gtrsim 1$ eV require there to be more entropy in the visible sector, i.e. $\xi > 1$. As a reference point, hidden sectors with confinement scales $\Lambda = \Lambda_{\rm QCD} \simeq 10^6$ eV require $\xi \gtrsim 10^6$. The bounds are weakest for lower rank groups, since they have lower confinement scales, but the constraint can be significant even for low rank groups. For example, with $\alpha = \alpha_{\rm GUT} \simeq .03$ and $G = SU(2)$, the approximate bound is $\xi \gtrsim 1.5 \times 10^{10}$. We will study the accuracy of this approximate bound momentarily by taking into account $3 \to 2$ interactions.

Alternatively, the dark glueball problem may be solved if the production mechanism of [26] is not in effect. This arises as follows. The visible sector reheat temperature after inflation (or modulus decay) must satisfy $T_{\rm rh} \lesssim M_{\rm GUT}$. The relationship $\xi = s/s' = g_S T^3/(g'_S T'^3)$ implies $T'_{rh} = (g_{S,{\rm rh}}/(g'_{S,{\rm rh}}\xi))^{1/3}T_{\rm rh} \simeq \xi^{-1/3}T_{\rm rh}$, where the latter approximation gives a gauge group independent relationship that will suffice for our purposes since the g_S dependence will make little qualitative difference on a log-log scale. Then the bounds $T'_{\rm rh} \gtrsim \Lambda$ and $T_{\rm rh} \lesssim M_{\rm GUT}$ together imply $\xi \lesssim (M_{\rm GUT}/\Lambda)^3$, so that the bound associated with leaving the regime of validity for glueball production is

$$\xi \gtrsim \left(\frac{M_{\rm GUT}}{\Lambda}\right)^3. \qquad (13)$$

For confinement scales that we study $\Lambda < M_{\rm GUT}$ and this bound implies that glueball production is valid for $\xi \lesssim 1$, which includes the democratic scenario. If ξ is increased from $\xi = 1$ with fixed Λ, however, eventually the bound will be satisfied, in which case the dark sector reheats to a temperature below the confinement scale and the glueball production mechanism we study is not in effect. Other production mechanisms may potentially arise, most plausibly when $T'_{\rm rh} \simeq \Lambda$, but we will leave such studies to future work and will clearly delineate regions of parameter space where (13) is violated.

Summarizing, if either of the bounds (12) or (13) are satisfied then the glueball relic abundance is not oversaturated.

Let us see when these bounds are satisfied for various groups and values of $\alpha_{\rm UV}$, fixing $\Lambda_{\rm UV} = 10^{16}$ GeV. The bound (12) can be slightly weakened by the incorporation of $3 \to 2$ interactions, which are taken into account in the right panel of Fig. 2 with $f = 0.1$. The solid and dashed lines are those in which the bounds for the relic abundance (with $3 \to 2$ interactions) and the regime of validity are saturated, respectively, for a particular group. Oversaturation occurs for a glueball with a fixed G for points in the parameter space above the associated solid contour, but below the associated dashed contour.

Both bounds must be taken into account: for example, for a fixed $\alpha \gtrsim .1$ satisfying the relic abundance bound would require $\xi \gtrsim 10^{20}$, but the regime of validity bound may be satisfied for smaller values of ξ, avoiding oversaturation. Conversely, for fixed $\alpha_{\rm UV} \lesssim 7 \times 10^{-3}$ there are values of ξ that violate the relic abundance bound but not the regime of validity bound. For any fixed $\alpha_{\rm UV}$ and G the minimum value of ξ sufficient to avoid the dark glueball problem can be read from the associated solid and dashed contours. For a fixed G, the value of $\alpha_{\rm UV}$ that requires the largest ξ to satisfy the bounds occurs when both bounds are saturated, which occurs for the $\alpha_{\rm UV}$ at which the associated solid and dashed contours intersect. Interestingly, this always occurs for $.005 < \alpha_{\rm UV} < .1$, which is a range that contains $\alpha_{\rm GUT}$.

IV. CONSTRAINTS FROM DECAYS TO MODULI AND AXIONS

An additional mechanism for evading the consequences of the above analysis is to allow the glueballs to decay to lighter degrees of freedom. In the present context we are assuming a hidden sector devoid of matter charged under the confining group, and we do not assume a renormalizable coupling (or "portal") to the fields of the Standard Model [40]. Indeed, the hidden sectors in (type II/F-theory) string theory are often truly hidden; either they are pure Yang-Mills theories or there are no fields charged under both the hidden and the visible sector. It is thus natural to consider these sectors to be coupled via nonrenormalizable couplings only, on which we will focus subsequently. This well-motivated assumption leaves only potentially light moduli and/or axionic fields as decay channels.

We have in mind the geometrical moduli generic to all string compactifications. While their masses are *a priori* undetermined, general arguments in supergravity [41–43] suggest that the lightest such modulus ought to have a mass comparable to that of the gravitino mass—roughly the size of the soft scalar masses in the observable sector—and thus (presumably) on the order of 10 TeV. Argument from the successful predictions of big bang nucleosynthesis (BBN) imply that the masses of these moduli must be no less than approximately 50 TeV, and we will take this number as a benchmark throughout the remainder of the paper. Note that the argument from BBN persists even in the absence of low-energy supersymmetry.

Let us again denote the glueball in the low-energy effective field theory by ϕ and designate its mass by m_ϕ, where $m_\phi \simeq \Lambda$. Let us denote a generic modulus field as χ. Then assuming the decay into such moduli ($\phi \to \chi\chi$) is kinematically accessible, we can estimate the lifetime by utilizing a dimension-6 operator such as

$$\mathcal{O}_6 = \frac{\mathrm{Tr}(G^a_{\mu\nu}G^{\mu\nu}_a)\chi\chi}{M_s^2} \to \frac{\Lambda^3}{M_s^2}\phi\chi\chi, \quad (14)$$

where the trace is over the gauge degrees of freedom, and we replace the field strengths with $\phi\Lambda^3$ in the effective theory below the confinement scale. The scale M_s is the scale at which the supergravity effective theory is valid. We will take $M_s = M_{\mathrm{GUT}}$ in explicit computations.

The width associated with (14) is given by

$$\Gamma_6 = \frac{1}{4\pi}\frac{1}{m_\phi}\left(\frac{\Lambda^3}{M_s^2}\right)^2\sqrt{1 - \frac{4m_\chi^2}{m_\phi^2}}, \quad (15)$$

where m_χ is the modulus mass. Successful BBN requires that the Universe be radiation dominated at the time that the relative abundances of protons and neutrons are set, roughly 0.1 s. As the glueballs quickly come to dominate the energy density of the Universe upon confinement, we must therefore demand that the lifetime associated with (15) be no longer than this value. Taking $m_\chi = 50$ TeV, $M_s = M_{\mathrm{GUT}} = 10^{16}$ GeV, and $m_\phi = \Lambda$, this implies a constraint

$$\Lambda \geq 2.4 \times 10^8 \text{ GeV}. \quad (16)$$

Glueballs with masses below the bound in (16) but above $2m_\chi \simeq 100$ TeV decay after the onset of BBN and spoil its successful predictions.

Alternatively one might hope that decays into even lighter objects could remedy the situation. A well-motivated candidate would be a light axionic field. Such states are common in string theory; indeed, the moduli fields themselves will have an imaginary component that behaves like an axion in the low-energy effective theory.

For our purposes it is sufficient to consider a single such axionic state, with an associated decay constant f_a. We will assume that all interactions between the glueball and the axion are suppressed by this scale. In practice, one commonly finds $f_a \simeq M_s$ in typical string models, but we will be agnostic as to the precise value of this constant. Prior to taking into account nonperturbative effects, the axion enjoys a shift symmetry and therefore only appears in the Lagrangian through derivative interactions. Thus an operator such as (14) is forbidden, and one must instead turn to a dimension-8 interaction governed by

$$\mathcal{O}_8 = \frac{\mathrm{Tr}(G^a_{\mu\nu}G^{\mu\nu}_a)\partial_\rho a\partial^\rho a}{f_a^4} \to \frac{\Lambda^3}{f_a^4}\phi\partial_\rho a\partial^\rho a, \quad (17)$$

resulting in a decay width given by

$$\Gamma_8 = \frac{1}{64\pi}m_\phi^3\left(\frac{\Lambda^3}{f_a^4}\right)^2 \simeq \frac{\Lambda}{64\pi}\left(\frac{\Lambda}{f_a}\right)^8, \quad (18)$$

where we are making the assumption $m_a \ll m_\phi$, for simplicity. The eight powers of Λ/f_a suppression make the resulting glueball lifetime fantastically long. For example, a glueball whose confinement scale saturated the bound in (16) would require

$$f_a \leq 1.1 \times 10^{12} \text{ GeV} \quad (19)$$

to allow for a decay before the onset of BBN. A glueball with confinement scale $\Lambda = 1$ TeV would require $f_a \leq 9.7 \times 10^5$ GeV to decay before BBN. Fixing $f_a = M_s = 10^{16}$ GeV, we find $\tau(\phi \to aa) > 0.1$ s for all confinement scales below 8×10^{11} GeV, showing that those cases which cannot decay to geometrical moduli promptly enough will not be rescued by decays to axions if $f_a \simeq 10^{16}$ GeV.

Under these assumptions we have established that hidden Yang-Mills sectors with $\Lambda \gtrsim 3$ eV are cosmologically ruled out unless the upper bound on f_a is satisfied for 50 TeV $\lesssim \Lambda \lesssim 10^9$ GeV, the confinement scale is quite high ($\Lambda \gtrsim 10^9$ GeV), or an extreme measure of preferential reheating is engineered.

One might argue the latter constraint could be relaxed if faster decay rates could be motivated. We have chosen the operators (14) and (17) to reflect the underlying structure of the UV theory above the scale of confinement. However, treating the glueball as a gauge-singlet scalar does naively allow the operator

$$\mathcal{O}_5 = \frac{\phi}{M_s}\partial_\mu\chi\partial^\mu\chi \to \Gamma_5 = \frac{1}{64\pi}\frac{m_\phi^3}{M_s^2}\sqrt{1 - \frac{4m_\chi^2}{m_\phi^2}}, \quad (20)$$

which could mediate glueball decay to moduli fields. The operator in (20) allows for prompt ($\tau < 0.1$ s) decays of
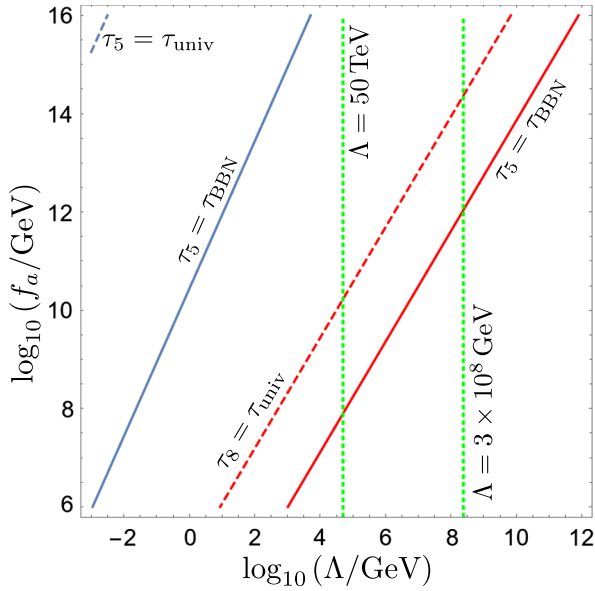
FIG. 3. Glueball lifetimes as a function of f_a and Λ, with contours for $\tau_n = \tau_{\text{univ}}$ and $\tau_n = \tau_{\text{BBN}} = 0.1 s$ for $n = 5, 8$ corresponding to decay via dimension-5 and -8 operators.

glueballs to moduli for all cases in which the decay is kinematically accessible, assuming a modulus mass $m_\chi = 50$ TeV and $M_s = 10^{16}$ GeV. This is to be expected: the operator in (20) is precisely the operator that is postulated to allow sufficiently heavy moduli to decay into light degrees of freedom prior to BBN, thereby solving the cosmological moduli problem.[1] For much lighter glueball states (lower confinement scales), the moduli decay channel closes and (20) with $\chi \to a$ and $M_s \to f_a$ now governs decay to axions (where we might assume $m_a \ll m_\phi$). In order to keep the lifetime sufficiently short, we must now lower the value of f_a away from 10^{16} to compensate for the much lower glueball mass. For example, taking an intermediate value $f_a = 10^{12}$ GeV, we find that the glueball decays sufficiently quickly into massless axions provided $\Lambda > 11$ GeV. A glueball whose mass is comparable to the QCD scale ($\Lambda = 100$ MeV) would require $f_a < 9 \times 10^8$ GeV to decay into axions during the brief window between confinement and the beginning of BBN.

However, we do not believe that (20) is present due to the structure of dark gluon interactions with moduli in the UV theory and the emergent nature of the glueball ϕ below the confinement scale.

Results on axion decays are summarized in Fig. 3. In the case of decay via the dimension-8 (-5) operator the glueball is stable and oversaturates the observed relic abundance to

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[1]One might choose to suppress the operator in (20) by the confinement scale Λ, as opposed to the much larger scale M_s. Or one might argue that a superrenormalizable coupling like $\Lambda\phi\chi\chi$ be utilized. Both modifications would only serve to shorten the lifetime, so we do not consider them here.

the left of the red (blue) dashed line. It is unstable but spoils nucleosynthesis between the red (blue) dashed and solid lines. To the right of the red (blue) solid line the glueball is unstable and decays prior to nucleosynthesis.

V. CONSTRAINTS FROM DARK RADIATION

In the better motivated case in which decay operators arise from the underlying structure of the hidden gauge fields, we find that decays to moduli require a confinement scale above 10^8 GeV and prompt decays to axions are possible for lower confinement scales if the axion decay constant is sufficiently small. If the decay operators descend from "naive" effective field theory, the glueball will decay into (cosmologically safe) moduli, prior to BBN, provided that such decays are kinematically accessible (i.e. $m_\phi \simeq 2m_\chi \simeq 100$ TeV). Furthermore, decays into light axions are possible for confinement scales between the modulus mass and the onset of BBN, provided the axion decay constant takes intermediate values 10^{10} GeV $\lesssim f_a \lesssim 10^{16}$ GeV. However, these axionic decay products are not necessarily cosmologically benign.

The prediction from BBN for the abundance of ^{4}He is sensitive to the Hubble parameter at the time BBN begins, and is thus sensitive to the number of relativistic degrees of freedom present in the cosmos at that time, $g_*^{\text{hid}}|_{\text{BBN}}$ [44]. The value of $g_*^{\text{hid}}|_{\text{CMB}}$ is constrained again at the time of cosmic microwave background (CMB) formation [39]. The upper bounds, at 95% confidence, on these quantities are given by [45]

$$g_*^{\text{hid}}\big|_{\text{BBN}} \leq 2.52\xi_T^4\big|_{\text{BBN}},$$
$$g_*^{\text{hid}}\big|_{\text{CMB}} \leq 0.18\xi_T^4\big|_{\text{CMB}}, \tag{21}$$

where we recall that $\xi_T = T/T'$.

Any non-Abelian group which avoids overproduction of glueball dark matter by achieving a very low confinement scale, governed by (10), would therefore have a potential problem from dark radiation at the time of BBN ($T_{\text{BBN}} \simeq 1$ MeV) and/or the time at CMB formation ($T_{\text{CMB}} \simeq 0.25$ eV). The situation is ameliorated somewhat by the fact that the visible sector has fewer degrees of freedom at those late times than it did at some earlier, primordial time [46]. Thus the visible sector should be "warmer" than the hidden sector at the time of BBN and CMB formation (assuming the hidden sector has remained a plasma of relativistic gluons throughout this history). Indeed one expects

$$\xi_T^3\big|_{\text{BBN}} = \xi_T^3\big|_{\text{rh}} \left(\frac{g_{*s}^{\text{vis}}(T_{\text{rh}})}{g_{*s}^{\text{vis}}(T_{\text{BBN}})} \right)$$
$$= \xi_T^3\big|_{\text{rh}} \left(\frac{g_{*s}^{\text{vis}}(T_{\text{rh}})}{10.75} \right), \tag{22}$$

$$\xi_T^3|_{\text{CMB}} = \xi_T^3|_{\text{rh}} \left(\frac{g_{*s}^{\text{vis}}(T_{\text{rh}})}{g_{*s}^{\text{vis}}(T_{\text{CMB}})} \right)$$

$$= \xi_T^3|_{\text{rh}} \left(\frac{g_{*s}^{\text{vis}}(T_{\text{rh}})}{3.36} \right), \tag{23}$$

where $\xi_T|_{\text{rh}}$ is the ratio of temperatures at some primordial scale at which both the visible and hidden sectors are populated through some sort of reheating. Assuming the reheat temperature is such as to populate the entire minimal supersymmetric Standard Model (MSSM) field content, one finds from (21)

$$g_*^{\text{hid}}|_{\text{BBN}} \leq 148.6 \xi_T^4|_{\text{rh}},$$

$$g_*^{\text{hid}}|_{\text{CMB}} \leq 50 \xi_T^4|_{\text{rh}}. \tag{24}$$

Taking the case of $SU(N)$ for concreteness, the unconfined gluons will contribute $g_*^{\text{hid}}|_{\text{rh}} = 2(N^2 - 1)$ at the time they are first produced in the early Universe. This number will persist for as long as the group remains unconfined. Taking a democratic ansatz $\xi_T|_{\text{rh}} = 1$, which differs from the previous $\xi = 1$ ansatz by a mild g_S, g_S' dependence, this limits the hidden sector gauge group to $SU(8)$ or smaller for $1 \text{ MeV} \gtrsim \Lambda \gtrsim 0.1 \text{ eV}$, and $SU(5)$ or smaller for $\Lambda \lesssim 0.1 \text{ eV}$. Larger unconfined groups can be accommodated with (mild) preferential reheating favoring the visible sector.

Axions produced at the time of glueball decay will be relativistic, and thus contribute to the bounds in (21) as well. For simplicity, let us consider the case in which the lifetime of the intermediate glueball state is brief relative to the Hubble parameter at that epoch. As argued in the previous section, this will be true of most cases in which the glueball decays before the onset of BBN. Under these circumstances we may approximate the thermodynamics by assuming all of the entropy in relativistic gluons is transmitted into a *single* species of relativistic axions.

As a result, the resulting fluid of relativistic axions will experience a "reheating" proportional to $(g_{*s}^{\text{hid}})^{1/3}$, partially offsetting the visible sector reheating associated with the factors in (22) and (23). Again assuming the reheat temperature is such as to populate the entire MSSM field content, the bounds in (24) are modified to

$$g_*^{\text{hid}}|_{\text{BBN}} \leq \frac{148.6}{(g_{*s}^{\text{hid}}(T_{\text{rh}}))^{4/3}} \xi_T^4|_{\text{rh}},$$

$$g_*^{\text{hid}}|_{\text{CMB}} \leq \frac{50}{(g_{*s}^{\text{hid}}(T_{\text{rh}}))^{4/3}} \xi_T^4|_{\text{rh}}. \tag{25}$$

In the case of democratic reheating ($\xi_T|_{\text{rh}} = 1$) the bound in (25) arising from CMB observations falls below unity for $g_{*s}^{\text{hid}}(T_{\text{rh}}) > 18$, suggesting that a glueball which decays into axions prior to the time of CMB formation will produce too much dark radiation for *any* hidden sector

gauge group larger than $SU(3)$. Of course, larger progenitor gauge groups can be entertained if some preferential reheating into the visible sector is engineered. It is worth pointing out that in a world where the visible sector consists solely of the Standard Model, the visible sector reheats less (by a factor of 2) and consequently *any* decay into a single axion species will violate ΔN_{eff} bounds in the democratic reheating limit.

VI. COMPATIBILITY WITH STRING MODEL BUILDING

Let us briefly turn to the question of whether these bounds are compatible with string-inspired supergravity models. The answer depends on the string theory in question as well as on the uplifting and moduli stabilization scheme. In the heterotic theories, the string scale is fixed one order of magnitude above the grand unified theory (GUT) scale. Compatibility with low-energy observations fixes the gauge coupling at the GUT scale to roughly $\alpha_{\text{GUT}} = .03$, and the gauge couplings of the other hidden sector gauge groups are then expected to be in a similar range, $\mathcal{O}(10^{-1})$–$\mathcal{O}(10^{-2})$. In the type II theories, the string coupling is usually taken to be $0.1 < g_s < 1$, where the lower bound comes from compatibility with the truncation of the string α' expansion. The string scale and the actual gauge coupling on the stack of D_{3+q}-branes depends on the volume $\mathcal{V}$ of the overall compactification manifold and the volume $\mathcal{V}_q$ of the q-cycle wrapped by D_{3+q}-branes and is given by

$$\alpha_q = \frac{g_s}{2\mathcal{V}_q}, \qquad \frac{M_s}{M_P} = \frac{g_s}{\sqrt{4\pi\mathcal{V}}}. \tag{26}$$

Hence for a high string scale (around the GUT scale) we get $\mathcal{V} = \mathcal{O}(10)$–$\mathcal{O}(100)$ and consequently $\mathcal{V}_q = \mathcal{O}(1)$–$\mathcal{O}(10)$, which means $\alpha_q = \mathcal{O}(10^{-1})$–$\mathcal{O}(10^{-2})$. For intermediate $[\mathcal{V} \sim \mathcal{O}(10^{13})]$ or TeV-scale $[\mathcal{V} \sim \mathcal{O}(10^{27})]$ string scales, much larger values for $\mathcal{V}_q$ and thus much lower values for g_q are possible, and it then becomes a question of the moduli stabilization and the uplifting scheme to answer whether the Kähler moduli can be consistently stabilized in this regime.

VII. DISCUSSION

We have shown that theories with multiple disconnected gauge sectors—as is typical in string theory—will often suffer a dark glueball problem.

In the simplest scenario of symmetric reheating via inflaton or modulus decay, any confining Yang-Mills hidden sector with the confinement scale $\Lambda \gtrsim 3 \text{ eV}$ will generate too much cold dark matter to be compatible with current measurements. A wide range of values of the ultraviolet gauge coupling naturally give rise to such a Λ via renormalization group evolution, leading to the

overproduction of dark matter. Thus, these considerations can rule out any model with even one such Yang-Mills sector, including string models. Confinement at scales below 1 eV often does not solve the problem either, as constraints on the number of relativistic degrees of freedom at the time of BBN and CMB formation require hidden sectors no larger than $SU(5)$ [$SU(3)$] if the visible sector is comprised of the MSSM (the Standard Model); hidden sectors motivated by string theory are frequently much larger.

The above statements depend, however, on two critical assumptions: glueball stability and symmetric reheating. We studied whether relaxing either of these assumptions may solve the glueball problem.

In fact, string theory provides a natural avenue for relaxing the first assumption: glueballs in hidden sectors might decay into light string moduli or axions. If the glueball decay $\phi \to \chi\chi$ to moduli happens before the epoch of BBN, the dark glueball problem maps onto the more familiar cosmological moduli problem [47–51]. The latter is solved if the lightest modulus is sufficiently heavy (we have taken $m_\chi = 50$ TeV here), suggesting that gauge groups with $\Lambda \gtrsim \mathcal{O}(100$ TeV$)$ should produce an acceptable cosmology. But a more precise treatment of the effective field theory that governs glueball decays to moduli suggests much smaller decay rates, such that the glueball decays to moduli prior to BBN only if $\Lambda \gtrsim 10^8$ GeV. Thus, with other assumptions fixed, 17 orders of magnitude in the confinement scale (3 eV $\lesssim \Lambda \lesssim 10^{17}$ eV) remains afflicted by the dark glueball problem.

Alternatively the glueball could decay into axions, which are generically present in string constructions. If the axion has a mass $m_a \ll m_{gb}$ the glueball decays into a relativistic species that is subject to $\Delta N_{\rm eff}$ bounds. In the simplest case of a single relevant axion there are a very large number of degrees of freedom transferring entropy to a single field. This tends to make the $\Delta N_{\rm eff}$ constraints more severe, restricting the hidden sector to be no larger than $SU(3)$ in the case when the visible sector is comprised of the MSSM [no larger than $SU(2)$ if the visible sector is solely the Standard Model].

There are other bounds on glueball decays into axions: if the glueball does not decay into moduli before BBN commences, then it must decay into axions before BBN commences, otherwise the glueball spoils BBN in the same manner as in the cosmological moduli problem. Since axions can only appear in the low-energy effective Lagrangian through derivative interactions, a decay operator consistent with the underlying theory must be suppressed by the factor $(\Lambda/f_a)^8$. If one takes $f_a \simeq M_s \simeq 10^{16}$ GeV, as is typical in string theory, then none of the parameter space that is not rescued by decay to moduli ($\Lambda \lesssim 10^8$ GeV) can be rescued by glueball decay to axions, in which case the glueball still dominates the energy density of the Universe at the time of BBN.
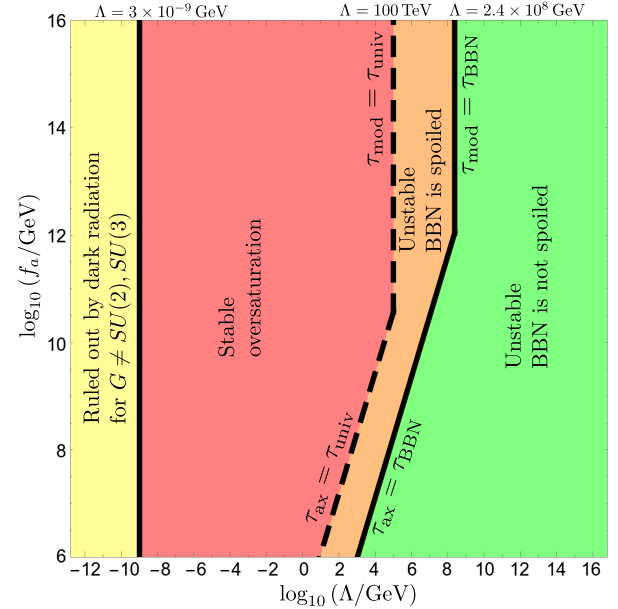

FIG. 4. The red and orange regions are ruled out by a stable oversaturating glueball relic and an unstable glueball that spoils BBN; the yellow region is ruled out for $G \neq SU(2), SU(3)$ by dark radiation constraints; the green region is allowed since the glueball is unstable and decays to moduli or axions prior to BBN.

These results apply in the democratic reheating scenario $\xi = 1$. For for a modulus decay operator suppression scale $M_s = M_{\rm GUT}$, the combined constraints from the glueball relic abundance, dark radiation, and nucleosynthesis are taken into account in Fig. 4. We emphasize that our results hold whether or not we assume low-scale supersymmetry, with only mild changes in the numerical values quoted here. Note that in Fig. 4 about two thirds of the parameter space is ruled out.

We would like to emphasize one of our major points. For one Yang-Mills sector the ruled out parameter space in Fig. 4 is only somewhat constraining. However, *many* hidden Yang-Mills sectors are typical in string theory, and in such models each glueball must fall within the allowed window. This is increasingly difficult to achieve as the number of hidden Yang-Mills sectors increases if the ultraviolet parameters are relatively well distributed by the physics of moduli stabilization. Thus, though only two thirds of the parameter space are ruled out for one Yang-Mills sector, the constraints are much more stringent for many such sectors.

We believe that this strongly motivates the study of asymmetric reheating models [52] in string theory, i.e. violating the second of our assumptions. In this case an initial condition, presumably through the dynamics of inflaton decay (or the decay of other scalar fields), ensures that the bulk of the reheating occurs in the visible sector. This may be quantified by a large visible to dark sector entropy density ratio $\xi = s/s'$ at the time of reheating.

Asymmetric reheating may aid the situation by relaxing ΔN_{eff} constraints or by lowering the glueball relic abundance. For the case of very low scale confinement ($\Lambda \ll 1$ eV) the bounds from ΔN_{eff} considerations are capable of accommodating larger rank hidden groups with a relatively mild preference for the visible sector—though the preference must become ever more acute if there are multiple such hidden sectors. By contrast, evading overproduction of glueball dark matter requires $\xi \gg 1$ in large regions of the realistic UV parameter space, often as large as $\xi = 10^{10} - 10^{20}$. Such a large asymmetric reheating is much larger than typically necessary to avoid dark radiation problems.

Of course, if a hidden sector is accompanied by gauge-charged matter that gives rise to baryon or meson states with masses below that of the glueball, then it may be possible (though not immediate) to avoid the dark glueball problem. But note that these fortuitous outcomes must arise for *all* problematic hidden sectors for the construction to be cosmologically viable.

In some cases out of equilibrium decays of other particles may dilute the glueball relic. If the associated $T'_{\text{rh}} < \Lambda$ then the cosmological assumptions we have used are violated and the glueball problem may be solved. However, if this $T'_{\text{rh}} > \Lambda$ then the cosmology we have studied is restarted, potentially with a different ξ, but this may still give rise to a dark glueball problem depending on the parameters.

We therefore conclude that there is ample opportunity for a typical string construction to experience a cosmologically fatal dark glueball problem. Our results motivate further research into the assurance of high confinement scales by moduli stabilization, and also the thorny issue of reheating, with an emphasis on establishing a strong preference for the visible sector.

[1] D. J. Gross, J. A. Harvey, E. J. Martinec, and R. Rohm, Phys. Rev. Lett. **54**, 502 (1985).

[2] L. J. Dixon, J. A. Harvey, C. Vafa, and E. Witten, Nucl. Phys. **B261**, 678 (1985).

[3] L. J. Dixon, J. A. Harvey, C. Vafa, and E. Witten, Nucl. Phys. **B274**, 285 (1986).

[4] L. E. Ibanez, H. P. Nilles, and F. Quevedo, Phys. Lett. B **187**, 25 (1987).

[5] L. E. Ibanez, J. Mas, H.-P. Nilles, and F. Quevedo, Nucl. Phys. **B301**, 157 (1988).

[6] O. Lebedev, H. P. Nilles, S. Raby, S. Ramos-Sanchez, M. Ratz, P. K. S. Vaudrevange, and A. Wingerter, Phys. Lett. B **645**, 88 (2007).

[7] M. Blaszczyk, S. Groot Nibbelink, M. Ratz, F. Ruehle, M. Trapletti, and P. K. S. Vaudrevange, Phys. Lett. B **683**, 340 (2010).

[8] I. Antoniadis, J. R. Ellis, J. S. Hagelin, and D. V. Nanopoulos, Phys. Lett. B **231**, 65 (1989).

[9] A. E. Faraggi, Int. J. Mod. Phys. A **14**, 1663 (1999).

[10] V. Braun, Y.-H. He, B. A. Ovrut, and T. Pantev, Phys. Lett. B **618**, 252 (2005).

[11] V. Bouchard and R. Donagi, Phys. Lett. B **633**, 783 (2006).

[12] L. B. Anderson, J. Gray, A. Lukas, and E. Palti, J. High Energy Phys. 06 (2012) 113.

[13] M. Cvetic, T. Li, and T. Liu, Nucl. Phys. **B698**, 163 (2004).

[14] F. Gmeiner, R. Blumenhagen, G. Honecker, D. Lust, and T. Weigand, J. High Energy Phys. 01 (2006) 004.

[15] R. Blumenhagen, V. Braun, T. W. Grimm, and T. Weigand, Nucl. Phys. **B815**, 1 (2009).

[16] T. P. T. Dijkstra, L. R. Huiszoon, and A. N. Schellekens, Phys. Lett. B **609**, 408 (2005).

[17] T. P. T. Dijkstra, L. R. Huiszoon, and A. N. Schellekens, Nucl. Phys. **B710**, 3 (2005).

[18] B. S. Acharya, Adv. Theor. Math. Phys. **3**, 227 (1999).

[19] D. D. Joyce, J. Diff. Geom. **43**, 291 (1996); **43**, 329 (1996).

[20] J. Halverson and D. R. Morrison, J. High Energy Phys. 04 (2016) 100.

[21] A. Grassi, J. Halverson, J. Shaneson, and W. Taylor, J. High Energy Phys. 01 (2015) 086.

[22] J. Halverson and W. Taylor, J. High Energy Phys. 09 (2015) 086.

[23] W. Taylor and Y.-N. Wang, J. High Energy Phys. 01 (2016) 137.

[24] W. Taylor and Y.-N. Wang, J. High Energy Phys. 12 (2015) 164.

[25] J. Halverson, arXiv:1603.01639.

[26] E. D. Carlson, M. E. Machacek, and L. J. Hall, Astrophys. J. **398**, 43 (1992).

[27] Y. Hochberg, E. Kuflik, T. Volansky, and J. G. Wacker, Phys. Rev. Lett. **113**, 171301 (2014).

[28] N. Bernal, X. Chu, C. Garcia-Cely, T. Hambye, and B. Zaldivar, J. Cosmol. Astropart. Phys. 03 (2016) 018.

[29] N. Bernal and X. Chu, J. Cosmol. Astropart. Phys. 01 (2016) 006.

[30] E. Kuflik, M. Perelstein, N. R. -L. Lorier, and Y.-D. Tsai, Phys. Rev. Lett. **116**, 221302 (2016).

[31] D. Pappadopulo, J. T. Ruderman, and G. Trevisan, Phys. Rev. D **94**, 035005 (2016).

[32] L. Forestell, D. E. Morrissey, and K. Sigurdson, Phys. Rev. D **95**, 015032 (2017).

[33] M. Farina, D. Pappadopulo, J. T. Ruderman, and G. Trevisan, J. High Energy Phys. 12 (2016) 039.

[34] A. E. Faraggi and M. Pospelov, Astropart. Phys. **16**, 451 (2002).

[35] J. L. Feng and Y. Shadmi, Phys. Rev. D **83**, 095011 (2011).

[36] K. K. Boddy, J. L. Feng, M. Kaplinghat, and T. M. P. Tait, Phys. Rev. D **89**, 115017 (2014).

[37] A. Soni and Y. Zhang, Phys. Rev. D **93**, 115025 (2016).

[38] G. D. Kribs and E. T. Neil, Int. J. Mod. Phys. A **31**, 1643004 (2016).

[39] P. A. R. Ade *et al.* (Planck Collaboration), Astron. Astrophys. **594**, A13 (2016).

[40] B. S. Acharya, S. A. R. Ellis, G. L. Kane, B. D. Nelson, and M. J. Perry, Phys. Rev. Lett. **117**, 181802 (2016).

[41] F. Denef and M. R. Douglas, J. High Energy Phys. 03 (2005) 061.

[42] M. Gomez-Reino and C. A. Scrucca, J. High Energy Phys. 05 (2006) 015.

[43] B. S. Acharya, G. Kane, and E. Kuflik, Int. J. Mod. Phys. A **29**, 1450073 (2014).

[44] B. Fields and S. Sarkar, arXiv:astro-ph/0601514.

[45] G. L. Kane, P. Kumar, B. D. Nelson, and B. Zheng, Phys. Rev. D **93**, 063527 (2016).

[46] J. L. Feng, H. Tu, and H.-B. Yu, J. Cosmol. Astropart. Phys. 10 (2008) 043.

[47] T. Banks, M. Berkooz, S. H. Shenker, G. W. Moore, and P. J. Steinhardt, Phys. Rev. D **52**, 3548 (1995).

[48] T. Banks, M. Berkooz, and P. J. Steinhardt, Phys. Rev. D **52**, 705 (1995).

[49] G. D. Coughlan, W. Fischler, E. W. Kolb, S. Raby, and G. G. Ross, Phys. Lett. **131B**, 59 (1983).

[50] B. de Carlos, J. A. Casas, F. Quevedo, and E. Roulet, Phys. Lett. B **318**, 447 (1993).

[51] M. Kawasaki and T. Moroi, Prog. Theor. Phys. **93**, 879 (1995).

[52] P. Adshead, Y. Cui, and J. Shelton, J. High Energy Phys. 06 (2016) 016.