PHYSICAL REVIEW D **95,** 043001 (2017)

Black hole mergers and the QCD axion at Advanced LIGO

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(Received 14 April 2016; revised manuscript received 7 June 2016; published 8 February 2017)

In the next few years, Advanced LIGO (aLIGO) may see gravitational waves (GWs) from thousands of black hole (BH) mergers. This marks the beginning of a new precision tool for physics. Here we show how to search for new physics beyond the standard model using this tool, in particular the QCD axion in the mass range $\mu_a \sim 10^{-14}$ to 10^{-10} eV. Axions (or any bosons) in this mass range cause rapidly rotating BHs to shed their spin into a large cloud of axions in atomic Bohr orbits around the BH, through the effect of superradiance (SR). This results in a gap in the mass vs spin distribution of BHs when the BH size is comparable to the axion's Compton wavelength. By measuring the spin and mass of the merging objects observed at LIGO, we could verify the presence and shape of the gap in the BH distribution produced by the axion. The axion cloud can also be discovered through the GWs it radiates via axion annihilations or level transitions. A blind monochromatic GW search may reveal up to 10^5 BHs radiating through axion annihilations, at distinct frequencies within ∼3% of $2\mu_a$. Axion transitions probe heavier axions and may be observable in future GW observatories. The merger events are perfect candidates for a targeted GW search. If the final BH has high spin, a SR cloud may grow and emit monochromatic GWs from axion annihilations. We may observe the SR evolution in real time.

DOI: 10.1103/PhysRevD.95.043001

I. INTRODUCTION

The LIGO detection of gravitational waves (GWs) [1] has opened a new window on the Universe. In the years to come, GWs from up to thousands of merger events will reveal a wealth of information about the hidden lives of black holes and neutron stars. We also have been given a new precision tool that may diagnose the presence of new bosonic particles [2]. When such a particle's Compton wavelength is comparable to the horizon size of a rotating BH, the superradiance effect [3–5] spins down the BH [6–9], populating bound Bohr orbits around the BH with an exponentially large number of particles [10,11]. Astrophysical BHs turn into nature's detectors probing bosons of mass between 10^{-20} and 10^{-10} eV. Stellar-mass BHs, such as those observed by aLIGO, correspond to the upper end of this mass range, which covers the parameter space for the QCD axion [12–14] with a decay constant f_a between the GUT and Planck scales.

The QCD axion was proposed more than 30 years ago to explain the smallness of the neutron electric dipole moment, and has been looked for ever since. However, SR is not limited to the QCD axion—it is an excellent probe of the string axiverse [2] as well as any other weakly interacting boson, such as a dark photon [15,16], that lies in the right mass range.

In this work, we assess how the potentially enormous amount of merger data collected by aLIGO in the next few years may be used to probe the effects of SR. A statistical analysis of the spins and masses of merging BHs can reveal the presence of an axion by the absence of rapidly rotating BHs. After a merger, the newly born BH may become a beacon of monochromatic GW radiation from axion annihilations, providing a unique opportunity to observe the time evolution of SR. Before we present the results of our analysis, we review the dynamics of SR and results from previous work.

II. BLACK HOLE SUPERRADIANCE AND ALL-SKY GW SEARCHES

Here we summarize the effects of superradiance on BH evolution (for a detailed discussion, see [10,11], as well as [17] for a review). We restrict ourselves to the study of weakly interacting spin-0 states, with the QCD axion as a primary example.

Axions with a large Compton wavelength compared to the size of the BH have an approximately hydrogenic spectrum of bound states around the BH with energies $\omega \approx \mu_a(1 - \frac{\alpha^2}{2n^2})$, where μ_a is the axion mass, $M_{\rm BH}$ the BH mass, and we define α to be the "fine-structure" constant of the gravitational "atom,"

$$\alpha \equiv G_N M_{\rm BH} \mu_a \sim 0.22 \left(\frac{M_{\rm BH}}{30\ M_\odot}\right)\left(\frac{\mu_a}{10^{-12}\ \mathrm{eV}}\right), \quad (1)$$

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with G_N Newton's constant [9,18]. Each state is uniquely characterized by the principal n, orbital ℓ, and magnetic m quantum numbers.

Such a state is superradiant (i.e. has an occupation number growing with time) if

$$\frac{\omega}{m} < \Omega_H, \tag{2}$$

where $\Omega_H = \frac{1}{2r_g}\frac{a_*}{1+\sqrt{1-a_*^2}}$ is the angular velocity of the event horizon, $r_g \equiv G_N M_{\rm BH}$, and $0 \le a_* < 1$ is the dimensionless BH spin.[1] The simplicity of Eq. (2) reflects that SR is a kinematic and thermodynamic phenomenon not unique to gravity [3,19].

When a spinning BH is born, the number of axions in superradiant levels grows exponentially, seeded by spontaneous emission. The growth rate is proportional to the value of the bound-state wave function at the horizon, $\Gamma_{\rm sr} \propto \alpha^{4\ell+4}\mu_a$. The fastest-growing level, generally one with the minimum ℓ, m such that Eq. (2) is satisfied, will extract energy and angular momentum from the BH until Eq. (2) is saturated. At that point, the bound state is occupied by $N_{\max} \sim \frac{\Delta a_*}{m} G M_{\rm BH}^2 \sim 10^{77}\frac{\Delta a_*}{0.1m}\left(\frac{M_{\rm BH}}{10\,M_\odot}\right)^2$ axions. For stellar-mass BHs, e-folding times are as fast as $\sim$100 sec, so energy extraction can occur faster than other processes such as accretion. For axion masses much smaller than the optimum values ($\alpha \ll 1$), the growth rate is much slower, while for much larger masses ($\alpha \gg 1$), satisfying Eq. (2) requires $l, m \gg 1$, again giving much slower growth. Thus, a given BH mass probes a range in mass around $\mu_a \sim r_g^{-1}$.

The process repeats for the next-fastest-growing level, until the time for the next level to grow is longer than the accretion timescale of the BH or the BH age. Axion self-interactions may modify this picture; a large occupation number in one level may affect the growth of the others, or lead to axion emission [10,11,20]. We consider masses small enough that, for the QCD axion, self-interactions are unimportant.

The absence of rapidly rotating old BHs is a signal that SR has taken place. The spin-mass distribution of BHs should be empty in the region affected by SR [2,10,11]. The handful of high-spin BH measurements in x-ray binaries already disfavor an axion in the mass range 6×10^{-13} eV to 2×10^{-11} eV [11].

Axions occupying the bound levels can produce monochromatic GWs in two ways. Axions can emit a graviton to transition between levels, or two axions can annihilate into a single graviton [2,11,21]. Annihilations probe axions of mass lighter than 10^{-11} eV; transition signals are largest for axion masses $\sim 10^{-11}$–10^{-10} eV. These signals are coherent, monochromatic, can last 10 years or more, and may be seen in blind searches for continuous GWs at aLIGO. Figure 1 summarizes and updates the findings of [11] for the prospects of those searches. Annihilations provide the most promising direct probe of SR; assuming exponentially falling BH mass distributions as in [11] we expect up to $\sim 10^4$ events at aLIGO coming from annihilations, while axion transitions become interesting for future detectors.[2] By updating the annihilation rates in [11] with the numerical results of [22], we find that the most optimistic assumptions about BH mass and spin distributions are already constrained from null continuous wave searches at initial LIGO [23–25].

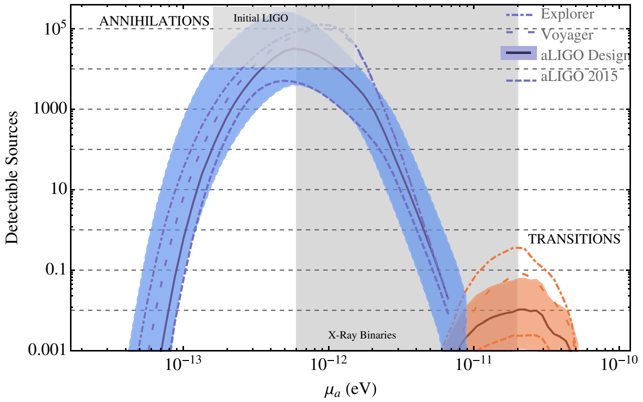

FIG. 1. Expected detectable sources in a blind monochromatic GW search, with sensitivity of current aLIGO (dashed), design aLIGO (solid), Voyager (wide-dashed) and Cosmic Explorer (dot-dashed) [26] for realistic mass and spin distributions, and BH formation rates ([11]). The shaded bands correspond to the range between pessimistic and optimistic BH distributions with design aLIGO (distributions as in [11], with the most narrow BH mass distribution removed as it is disfavored by the observation of GW150914). The coherent integration time is 2 days and total time 1 yr. The annihilation rate has been updated using the latest superradiance simulations [22]. Axion masses in the grayed-out region are disfavored by BH spin measurements [11]; the most optimistic distributions are disfavored by previous null LIGO searches [23–25].

In addition to individual monochromatic signals, there would be a stochastic GW background from unresolved sources. The individual signals considered in Fig. 1 would be concentrated in a narrow frequency range and stand well above plausible backgrounds. The stochastic background from axion SR could be detectable, but as individual signals would likely be seen first we defer a full discussion to future work.

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[1] The SR condition implies $\frac{\alpha}{m} < \frac{1}{2}$, which justifies the hydrogenic energy level approximation [18].

[2] Assuming a BH mass distribution falling as a power law at large mass results in an even higher number of annihilation events.

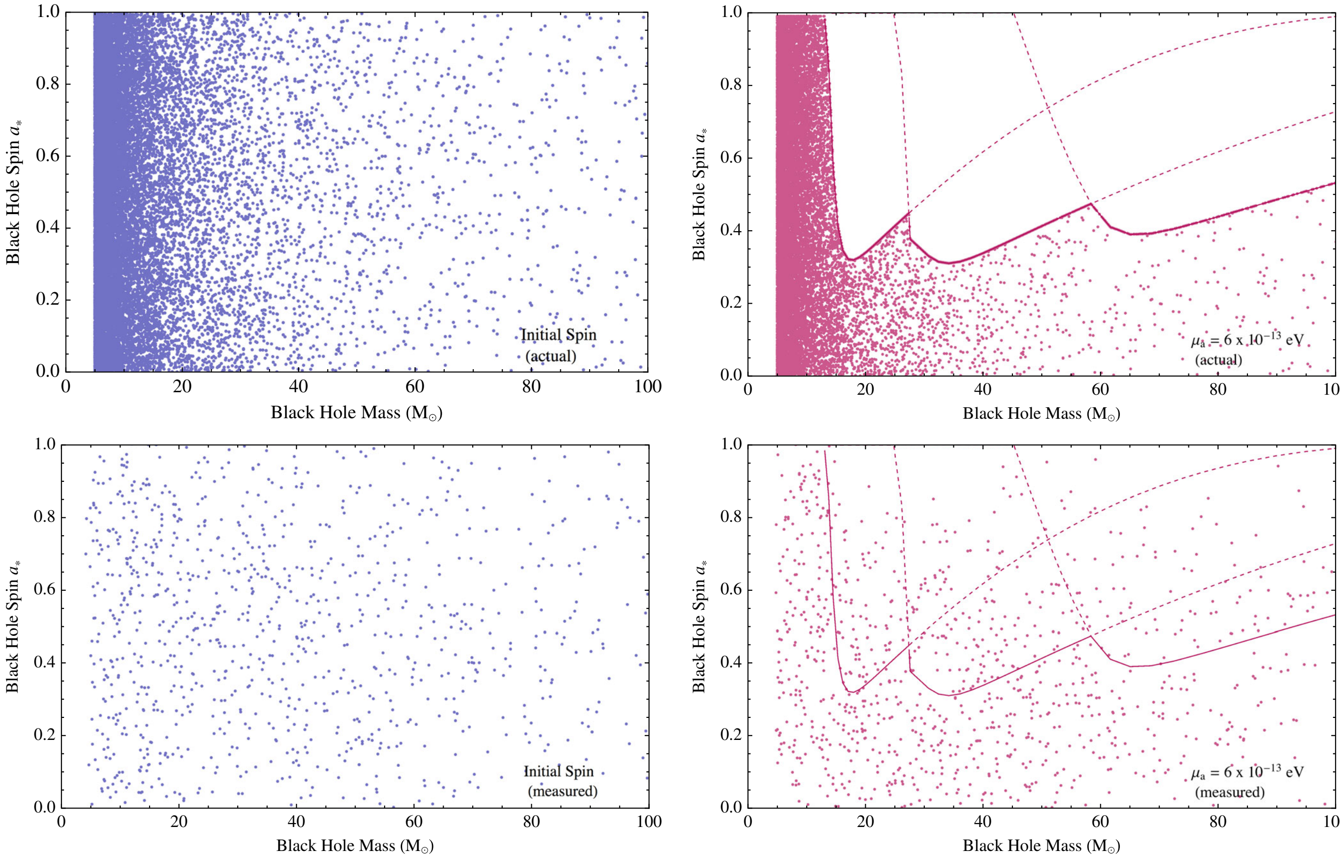

FIG. 2. Expected distribution of intrinsic (top) and measured (bottom) spins and masses of merging BHs in the absence (left) and the presence (right) of an axion of mass 6×10^{-13} eV, normalized to 1000 events detected at aLIGO. We assume $\sigma_M/M \sim 10\%$ measurement error in the mass and $\sigma_{a_*} \sim 0.25$ error in the spin [30,31]. We have assumed that all BBHs formed at a distance such that they take 10^{10} years to merge. The theoretical curves shown are boundaries of the regions where SR had at most 10^{10} years to spin down the BHs, and the effect of the companion BH does not significantly affect the SR rate.

III. STATISTICS OF BINARY BH MERGERS

At design sensitivity, aLIGO is expected to detect 80-1200 binary black hole (BBH) merger events per year [27–29], and measure the masses and spins of the merging BHs. A clear signature of superradiance is the absence of rapidly rotating old BHs in the range influenced by a given axion, and a large number of BHs populating the curve $\frac{\omega}{m} = \Omega_H$ for the last level that had time to grow, as illustrated in Fig. 2 (top). We show an example BH distribution with (right) and without (left) an axion. Unless otherwise specified, in what follows, we assume a flat BH spin distribution [30] and a power-law BH mass distribution $\rho(M) \propto M^{-2.35}$ [27] in the absence of an axion, as were assumed in LIGO analyses.

The histories of BH binaries affect the observed BH distribution in the mass-spin plane. If the BHs form in an existing binary system and merge quickly, then superradiant levels might not have had time to grow to maximal size. In addition, the gravitational perturbation of one black hole on the other's axion levels can mix superradiating levels with decaying ones, and may disrupt superradiance entirely [10,11]. On the other hand, if the binary was formed by capture [28], the initially isolated BHs are likely to have had time to superradiate without disruption.

For a given merger time, the BHs are spun down if SR is fast enough to fully populate the levels before the merger, and the gravitational perturbation is small enough such that the level-mixing effect on SR is negligible. Assuming equal mass BHs and initial separation giving $\tau_{\rm binary}$ time until the merger (assuming energy loss through GW emission only), the latter condition for the $\ell = m = 1$ level is [11],

$$\alpha \gtrsim 0.06 \left(\frac{M_{\rm BH}}{30\ M_\odot}\right)^{1/15} \left(\frac{10^{10}\ {\rm yr}}{\tau_{\rm binary}}\right)^{1/15}. \qquad (3)$$

In Fig. 2, level mixing is the limiting factor for the regions affected by $\ell = 1$, 2 levels, while $\ell = 3$ is limited by the level growth being slower than the binary merger time.[3]

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[3] The axion cloud is generally destroyed by annihilations or falling into the BH without spinning up the BH. Thus, the SR saturation lines are a good approximation to the BH's final spin.

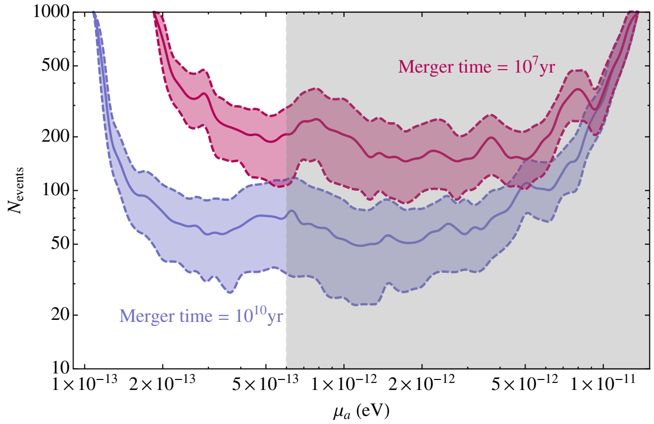

FIG. 3. Number of observed events required to show that the BH spin distribution varies with BH mass, assuming the presence of an axion of mass μ_a. Spin measurement errors of $\sigma_{a_*} = 0.25$ are assumed. Blue (red) curves correspond to BHs taking 10^{10} yrs (10^7 yrs) from formation to merger. The solid curves shows the median number of events required to reject the separable-distribution hypothesis at 2σ. The upper/lower dashed curves show the upper/lower quartiles, respectively. The test statistic used is the Kolmogorov-Smirnov distance between the spin distributions outside and inside a given BH mass range, maximized over choice of mass range. Shaded region is as in Fig. 1.

For Fig. 2, we have assumed that the BHs are formed in a binary, and take 10^{10} years to merge. This corresponds to the largest separation possible, and is the most optimistic scenario for spin-down, illustrating how strong a signal could be. The top panels of Fig. 2 present a sample spin-mass distribution of BBHs with and without an axion. In the bottom panels we present the corresponding distributions as seen by aLIGO, accounting for design detector sensitivity as a function of total merger mass [32] and mass and spin measurement uncertainties [30,31]. The large number of events shown make the lack of rapidly spinning BHs clear.[4]

Even with a relatively small number of events, it may be possible to infer that the mass-spin distribution has superradiancelike properties—for example, that the spin distribution varies with mass. Figure 3 shows the number of events at aLIGO needed to obtain 2σ evidence for such variation, under the assumptions explained in the caption. For axion masses between $\sim 2\times10^{-13}$ and 5×10^{-12} eV, we find that good evidence for a nonseparable mass-spin distribution may be obtained after observing $\mathcal{O}(50)$ events, probing axion masses below the x-ray binary bounds.

Different assumptions can change the required number of events by factors of a few. As shown in Fig. 3, reducing the assumed merger time from 10^{10} yrs to 10^7 yrs (the range suggested by BBH formation models [32]) increases the number of events necessary and decreases the range of axion masses probed. A pessimistic assumption of $\sigma_{a*} \sim 0.5$ requires ∼3–5 times as many events. Our error estimates are based on studies of intermediate mass BBHs; at design-sensitivity LIGO/Virgo detectors, one expects to obtain a 90% confidence interval of width $|\Delta a_*| < 0.8$ for total masses up to 600 $M_\odot$,[5] and a 10% error in mass determination for an order one fraction of primary black holes masses [31]. These estimates indicate a plausible range of variation—a comprehensive analysis, taking into account detailed mass and spin dependent measurement errors, would require full simulations.

Of course, dependence of the BH spin distribution on mass may come from astrophysical effects; if features are seen, more events would be required to trace out the superradiance contours with accuracy and determine an axion mass. Third-generation observatories can achieve much higher spin measurement precision (90% interval of $|\Delta a_*| < 0.1$ for a majority of events [33]) and confirm any features indicated by Advanced LIGO. In addition, if no features in the mass-spin distribution are seen, we cannot immediately exclude the presence of an axion since it may be that most formation histories did not allow for SR. Nevertheless, a statistical signal, especially along with other indications of an axion (e.g. the monochromatic GWs of Fig. 1), would be suggestive.

IV. DIRECT SIGNATURES

In addition to the wealth of aLIGO measurements of merging black holes, binary merger events provide a unique opportunity to observe the birth of a BH. This BH is the ideal point-source candidate to observe the evolution of the superradiant instability in real time.

For transitions, the levels responsible for an appreciable signal take over a thousand years to grow to large occupation numbers, so are uninteresting for a followup search. Axion annihilations are the most promising source of continuous GWs for targeted searches at aLIGO, with the first level taking from less than a month to up to 10 years to grow to maximum occupation number. Using the leading-order formula for the 2-axion to graviton annihilation rate $\Gamma_{\rm ann}$ from [34] (see [22] for numerical results), the peak GW strain at Earth from axion annihilations at distance d is [11]

$$h_{\rm ann} = \sqrt{\frac{4G_N\Gamma_{\rm ann}N_{\rm max}^2}{2\omega_a d^2}}$$
$$\approx 6\times10^{-23}\left(\frac{\alpha}{0.3}\right)^7\left(\frac{a_*}{0.9}\right)\left(\frac{M_{\rm BH}}{60\ M_\odot}\right)\left(\frac{1\ {\rm Mpc}}{d}\right) \qquad (4)$$

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[4] It may be possible to obtain better spin measurements for BHs in BH-NS mergers [30,31], but such events have not yet been observed.

[5] For the most pessimistic case of equal BH masses and misaligned spins; even better measurements are possible for dissimilar masses or aligned spins.

and lasts for

$$\tau_{\mathrm{ann}} \sim (\Gamma_{\mathrm{ann}} N_{\mathrm{max}})^{-1} \approx 0.1\ \mathrm{yr} \left(\frac{0.3}{\alpha}\right)^{15} \left(\frac{0.9}{a_*}\right) \left(\frac{M_{\mathrm{BH}}}{60\ M_\odot}\right). \tag{5}$$

Correlating these continuous wave emission properties with the spin and mass of the new BH will be a cross-check on SR predictions.

The reach of aLIGO to an optimal annihilation signal can be as large as 500 Mpc for an axion of mass 10^{-13} eV. The reach of aLIGO at design sensitivity for a typical event is close to 30 Mpc. In particular, the final BH of GW150914 with spin of ~ 0.7 would have had to be within 10 Mpc in order for axion annihilations to be observable.

In Fig. 4, we estimate the number of BBH merger products emitting observable monochromatic GWs per year as a function of the axion mass. The expected number of events is very sensitive to the spin and mass of the final BH; a linearly-increasing BH spin distribution increases the expected event rates by a factor of ~ 2 over a flat spin distribution. We estimate the spin of the final BH with [35], assuming equal, aligned initial spins and equal masses. If SR spun down the initial BHs before the merger, the final BH will generally not spin quickly enough for SR to produce an observable signal; for example, we estimate 10^{-3} events/yr. at $\mu_a = 2 \times 10^{-13}$ eV. Only merging BHs for which SR was inhibited can give rise to a signal observable at aLIGO with an appreciable rate, and Fig. 4 assumes this is the case for an $\mathcal{O}(1)$ fraction of events. There is, therefore, complementarity between the statistical and direct searches—either SR spins down enough of these to give a statistical signal or an appreciable fraction of postmerger BHs are spinning fast enough to give direct signals (assuming enough BHs are born with high spin).

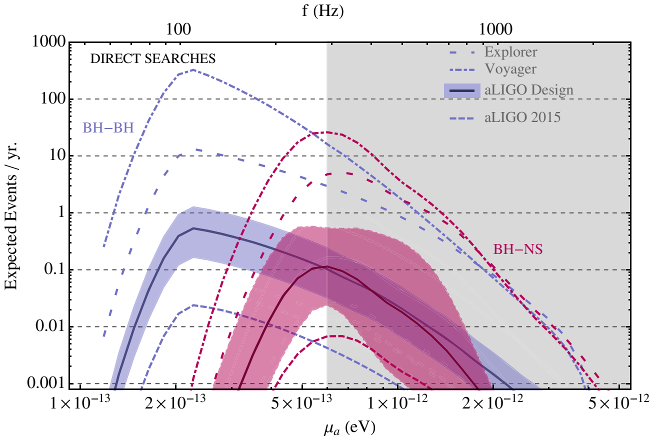

FIG. 4. Expected annual annihilation events for aLIGO and future observatories from products of BH-NS mergers (magenta) or BBH mergers of equal mass (blue). We assume the binary formation mechanism does not allow for superradiance. We take $a_* = 0$, $M = 1.4\ M_\odot$ for the NS and a power-law mass distribution and flat spin distribution of the merging BHs. The bands represent the merger rate uncertainty given the observed BBHs [27,29] and simulations for BH-NS (V4l and V2l in [36]). We assume a coherent integration time of 10 days for BBH and 1 year (or up to the duration of the signal) for BH-NS. Shaded region is as in Fig. 1.

Figure 4 also shows our expectations for BH-neutron star (NS) mergers, which have not been observed but are expected at aLIGO. For BH-NS (as well as NS-NS mergers), we use expected event rates from numerical simulations, 1–100 $\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$ [36]. Unlike BBHs, we expect an electromagnetic counterpart [37], allowing excellent sky positioning and extending the achievable coherence time for the monochromatic GW search to the full observation time. In addition, BHs produced during these events are lighter, allowing for searches for heavier axions.

Taking into account the uncertainty in the merger rates, the expected number of events ranges from 0.01–1 yr^{-1} for axion masses between $2 \times 10^{-13} - 2 \times 10^{-12}$ eV coming from BBHs and BH-NS binaries.

NS-NS mergers create the lightest BHs. These emit high frequency GWs at the edge of the aLIGO sensitivity curve, and the expected spin of the final BH is at most ~ 0.9 [38], leading to low event rates. At design sensitivity, the number of annihilation events is at best $\sim 10^{-3}$ yr^{-1} for an axion of mass 8×10^{-12} eV.

Unlike blind searches for isolated BHs, which are dominated by long signals from our galaxy, searches for post-merger signals are dominated by stronger signals at tens of Mpc. Thus, event rates will increase cubically with future strain sensitivity upgrades (Fig. 4), to as many as hundreds of events per year with projected Explorer sensitivity [26].

V. CONCLUSION

The earliest aLIGO signal for an axion is likely to come from monochromatic GWs in a full-sky survey (Fig. 1): a blind search for continuous waves can potentially discover up to $\sim 10^4$ distinct sources, all within a $\sim 3\%$ frequency range (as derived from the SR condition and bound-state energies). This would be strong evidence for a light boson of mass equal to half the observed frequency for annihilation signals. In contrast, monochromatic GWs produced by astrophysical objects, such as NSs, are unlikely to cluster within a few percent of a characteristic frequency. The presence of a unique frequency is a telltale sign of a new particle.

The statistical search can also lead to early evidence for an axion at aLIGO. The strength of the statistical evidence will depend on the formation history, axion mass, and the precision with which spins can be determined. With good precision, the experimental curve will approach the theoretical curve (Fig. 2, top right) and the evidence could be compelling. With less precise spin measurements, the possibility of a yet-unknown standard model mechanism which disfavors high-spin BHs in a certain mass range would have to be investigated.

Since the statistical searches and full-sky continuous wave searches probe a similar axion mass range (Figs. 1 and 3), there is the exciting possibility that these searches may independently indicate the presence of an axion of the same mass.

The targeted searches of recently formed BHs would be a way to look at the development of superradiance in real time. This tremendous possibility may have to wait until aLIGO upgrades. Future aLIGO upgrades will also make it more likely to observe signals from axion transitions, which would probe axion masses above 10^{-11} eV.

ACKNOWLEDGMENTS

We are grateful to Luis Lehner, Salvatore Vitale, and Keith Riles for many informative and clarifying discussions. Research at Perimeter Institute is supported by the Government of Canada through Industry Canada and by the Province of Ontario through the Ministry of Economic Development & Innovation. This work was partially supported by the National Science Foundation (NSF) under Grant No. PHYS-1316699 and by the NSF CAREER Award No. PHY-1352119.

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