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CP-violating decays of the pseudoscalars η and η' and their connection to the electric dipole moment of the neutron

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Using the present upper bound on the neutron electric dipole moment, we give an estimate for the upper limit of the CP-violating couplings of the $\eta(\eta')$ to the nucleon. Using this result, we then derive constraints on the CP-violating $\eta(\eta')\pi\pi$ couplings, which define the two-pion CP-violating decays of the η and η' mesons. Our results are relevant for the running and planned measurements of rare decays of the η and η' mesons by the GlueX Collaboration at JLab and the LHCb Collaboration at CERN.

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I. INTRODUCTION

The CP violation (CPV) is crucial for understanding the observed baryon asymmetry of the Universe (BAU). In the Standard Model (SM), CP is explicitly broken by the complex phase of the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix and by the θ term of QCD. Up to date, experimentally CPV has only been observed in K- and B-meson mixing and hadronic decays [1], which are perfectly compatible with the CKM phase. On the other hand, the CPV of SM origin is by far insufficient for the explanation of the BAU. The missing amount of CPV is believed to arise from non-SM sources.

Apart from the above-mentioned CPV observables, there are others with distinct sensitivity to different sources of CPV. Among them, the electric dipole moments (EDMs) of the neutron, leptons, and atoms have attracted special attention [2–5]. In particular, the neutron EDM is weakly sensitive to the CKM phase, but strongly sensitive to the θ term, constraining the latter to be unnaturally small. This smallness is elegantly explained by the famous Peccei and Quinn mechanism [6,7].

Various beyond-the-SM contributions to the EDM have been studied in the literature, for example, the R-parity violating supersymmetry [8–11] and meson-cloud effects in the nucleon [12,13]. For a review on EDMs as probes of new physics, see, e.g., Ref. [14].

There is also an extensive experimental program, both for the measurements of the EDMs and looking for rare CPV decays with increasing sensitivity. In particular, searches for rare η and η' decays have been performed by the LHCb Collaboration at CERN [15] and are planned by the GlueX experiment at JLab (Hall D) [16]. In the present paper, we focus on $\eta(\eta') \to \pi\pi$. As will be shown, the CPV $\eta(\eta')\pi\pi$ couplings underlying these decays also contribute to the neutron EDM. Thus, the current experimental limits on the neutron EDM [1]

$$|d_n| \le 2.9 \times 10^{-26} e \text{ cm}, \qquad 90\% \text{ C.L.}, \tag{1}$$

will allow us to derive new indirect upper bounds on the branching ratios of these CPV decays. The current direct experimental 90% C.L. upper limits [1,15] are

$$\frac{\Gamma(\eta\to\pi\pi)}{\Gamma_\eta^{\rm tot}} < \begin{cases} 1.3\times 10^{-5} & \text{for } \pi^+\pi^- \\ 3.5\times 10^{-4} & \text{for } \pi^0\pi^0 \end{cases},$$
$$\frac{\Gamma(\eta'\to\pi\pi)}{\Gamma_{\eta'}^{\rm tot}} < \begin{cases} 1.8\times 10^{-5} & \text{for } \pi^+\pi^- \\ 4.0\times 10^{-4} & \text{for } \pi^0\pi^0 \end{cases}. \tag{2}$$

Here, $\Gamma_\eta^{\rm tot} = (1.31 \pm 0.05)$ keV and $\Gamma_{\eta'}^{\rm tot} = (0.198 \pm 0.009)$ MeV are the total decay widths.

In Ref. [2], the size of the neutron electric dipole moment (EDM) was estimated on the basis of a CPV chiral Lagrangian that couples the light pseudoscalars to the neutron, modulo the CPV phase. At leading order, only the contributions of the charged mesons survive, for which there is no experimental input on the size of their CPV couplings. In order to relate the neutron EDM to the couplings with the $\eta(\eta')$, next-to-leading order chiral Lagrangians must be taken into account.

This is one of the aims of the present work. We carry out the analysis of the EDM within fully covariant chiral perturbation theory (ChPT) and with the explicit inclusion of intermediate spin-3/2 states, namely the $\Delta(1232)$ resonance. The latter couples strongly to the nucleon and is therefore expected to give important contributions to processes that lie in energies close to the resonance mass. Note, that the contribution of higher Δ-resonance states is suppressed by their coupling with the nucleon and pion. We use the extended on-mass shell (EOMS) scheme [17,18] for renormalization. It is relativistic, satisfies analyticity, and usually converges faster than nonrelativistic approaches.

The paper is organized as follows. In Sec. II, we construct the Lagrangian for the CP-violating coupling of the $\eta(\eta')$ to the pions, in order to connect it with the branching ratio of the reaction. In Sec. III, we use this input to construct the CP-violating coupling of the $\eta(\eta')$ to the nucleon. The CP-conserving coupling is discussed in Sec. IV, with the usual chiral Lagrangian considerations. In Sec. V, we give a brief overview on the couplings with vector mesons. The calculation of an estimate for the neutron EDM with these tools is shown in Sec. VI. By comparing the result with the experimental constraint on the EDM, we extract an estimate for the $\eta(\eta')\to\pi\pi$ branching ratio upper limits. Finally, in Sec. VII, we summarize the work and give our conclusions.

II. THE CP-VIOLATING $\eta(\eta')\to\pi\pi$ DECAY

The effective Lagrangian describing the CP-violating $\eta(\eta')\pi\pi$ coupling is given by [5]

$$\mathcal{L}^{\mathrm{CP}}_{H\pi\pi} = f_{H\pi\pi} M_H H \vec{\pi}^2, \tag{3}$$

with $H=\eta,\eta'$, M_H the mass of the $\eta(\eta')$ meson, and $f_{H\pi\pi}$ the coupling constant of $\eta(\eta')$ to the pions. Thus, the decay width is given by

$$\Gamma = \frac{n_\Gamma |\vec{p}_\pi|}{8\pi M_H^2}|\mathcal{M}_{H\pi\pi}|^2 = n_\Gamma \frac{\sqrt{M_H^2-4M_\pi^2}}{4\pi}|f_{H\pi\pi}|^2, \tag{4}$$

where n_Γ is an additional final-state factor, which equals $1/2$ for the $\pi^0\pi^0$ and 1 for the $\pi^+\pi^-$ channel. Using the limits from Eq. (2), we obtain upper limits for the coupling constants.

Here, we choose to calculate the charged and neutral channels separately, and to keep only the lower result as the global upper limit

$$|f_{\eta\pi\pi}| < 2.1\times10^{-5}, \qquad |f_{\eta'\pi\pi}| < 2.2\times10^{-4}. \tag{5}$$

III. THE CP-VIOLATING COUPLINGS OF THE η AND η' TO THE NUCLEON

With the previous considerations, one can obtain an estimate for the CP-violating coupling of the $\eta(\eta')$ to the nucleon

$$\mathcal{L}^{CP}_{HNN} = g^{CP}_{HNN} H\bar{N}N, \tag{6}$$

with the ansatz that the coupling is made via pion loops as shown in Fig. 1.

The chiral Lagrangians to describe the couplings appearing in those loops are

$$\mathcal{L}^{(1)}_N = \bar{\Psi}\left(\mathrm{i}\not{D} - m + \frac{g_0}{2}\not{u}\gamma_5\right)\Psi,$$
$$\mathcal{L}^{(1)}_{\Delta\pi N} = \frac{\mathrm{i}h_A}{2F_0 M_\Delta}\bar{\Psi}T^a\gamma^{\mu\nu\lambda}(\partial_\mu\Delta_\nu)(\partial_\lambda\pi^a) + \mathrm{H.c.}, \tag{7}$$

where Ψ is the nucleon doublet $(p,n)^T$ with mass m, and Δ the isospin-3/2 quadruplet $(\Delta^{++},\Delta^+,\Delta^0,\Delta^-)^T$ with mass M_Δ. The covariant derivative is given by

$$\mathrm{D}_\mu = (\partial_\mu + \Gamma_\mu), \qquad \Gamma_\mu = \frac{1}{2}[u^\dagger(\partial_\mu - \mathrm{i}r_\mu)u + u(\partial_\mu - \mathrm{i}l_\mu)u^\dagger]. \tag{8}$$

The meson fields appear through

$$u^2 = U = \exp\left(\frac{\mathrm{i}\hat{\pi}}{F_0}\right), \qquad \hat{\pi} = \vec{\pi}\,\vec{\tau} = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix},$$
$$u_\mu = \mathrm{i}[u^\dagger(\partial_\mu - \mathrm{i}r_\mu)u - u(\partial_\mu - \mathrm{i}l_\mu)u^\dagger], \tag{9}$$

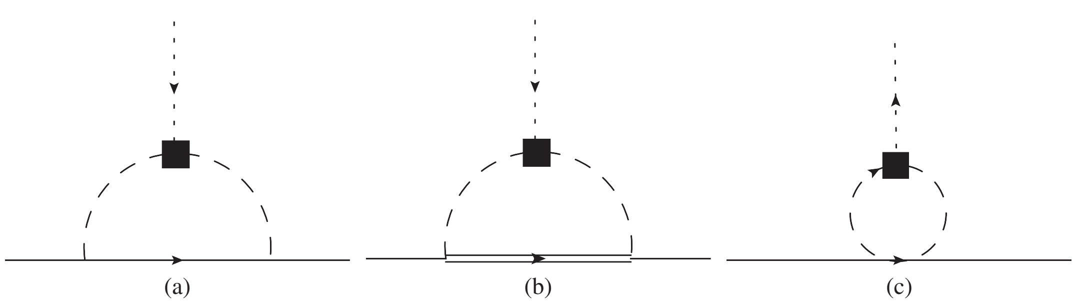

FIG. 1. Loops that can contribute to the CP-violating coupling of the $\eta(\eta')$ to the nucleon: (a) $\eta(\eta')$-nucleon loop diagram, (b) $\eta(\eta')$-$\Delta(1232)$ loop diagram, (c) tadpole $\eta(\eta')$ loop diagram. The single solid lines stand for nucleons, the double lines for the Δ, the dashes for pions, and the dotted lines for the $\eta(\eta')$. The black box at the $H\pi\pi$ vertex indicates the CP-violating coupling.

with r_μ and l_μ being right- and left-handed external fields, and F_0 is the meson-decay constant. At leading chiral order, the low-energy constant (LEC) g_0 corresponds to the physical axial-vector coupling constant $g_A = 1.27$. Furthermore, we use the notation

$$\gamma^{\mu\nu\lambda} = \frac{1}{2}\{\gamma^{\mu\nu}, \gamma^{\lambda}\}, \qquad \gamma^{\mu\nu} = \frac{1}{2}[\gamma^{\mu}, \gamma^{\nu}]. \tag{10}$$

The coupling h_A can be obtained from the Δ width, leading to the value $h_A = 2.85$ [19]. The conventions and definitions for the isospin operators T^i follow Ref. [20],

$$T^1 = \frac{1}{\sqrt{6}}\begin{pmatrix} -\sqrt{3} & 0 & 1 & 0 \\ 0 & -1 & 0 & \sqrt{3} \end{pmatrix},$$
$$T^2 = \frac{-\mathrm{i}}{\sqrt{6}}\begin{pmatrix} \sqrt{3} & 0 & 1 & 0 \\ 0 & 1 & 0 & \sqrt{3} \end{pmatrix},$$
$$T^3 = \sqrt{\frac{2}{3}}\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}. \tag{11}$$

We would like to stress, that the contribution of higher Δ-resonance states is suppressed by their coupling with the nucleon and pion. In particular, the $\Delta \to N + \pi$ decay rate in terms of the coupling constant h_A is given by [19]

$$\Gamma(\Delta \to N + \pi) = \frac{h_A^2}{48\pi F_\pi^2}(P^*)^3 \frac{E_N + m}{M_\Delta}, \tag{12}$$

where $P^* = \lambda^{1/2}(M_\Delta^2, m^2, M_\pi^2)/(2M_\Delta)$ and $E_N = (M_\Delta^2 + m^2 - M_\pi^2)/(2M_\Delta)$ are the magnitude of three-momentum and energy of the nucleon in the Δ rest frame, M_π is the pion mass, and $\lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$ is the Källen triangle kinematical function. Next, using the data (central values) for the decay widths $\Gamma(\Delta^{++}(1232) \to p + \pi^+) = 115$ MeV and $\Gamma(\Delta^{++}(1600) \to p + \pi^+) \approx 50$ MeV, we get for the ratio of the corresponding couplings squared

$$R = \left(\frac{h_A(1600)}{h_A(1232)}\right)^2 \simeq 0.05. \tag{13}$$

This means that the contribution of the $\Delta(1600)$ state relative to the $\Delta(1232)$ in the diagram of Fig. 1(b) is strongly suppressed.

From isospin considerations, it should be clear that there is no contribution from Fig. 1(c), due to the cancellation of the π^+ and the π^- loop. The loops in Figs. 1(a) and 1(b) do contribute, though. With the Lagrangians introduced in Eq. (7) and the vertex from Eq. (3), the loop in Fig. 1(a) reads

$$g_{HNN}^{CP} = -\mathrm{i} I_{N\pi}^2 \frac{g_A^2 f_{H\pi\pi} M_H}{F_\pi^2} \times \int \frac{\mathrm{d}^d z}{(2\pi)^d} \frac{(\not{z}+\not{k})\gamma_5(\not{p}-\not{z}+m)\not{z}\gamma_5}{[(k+z)^2 - M_\pi^2][z^2 - M_\pi^2][(p-z)^2 - m^2]}, \tag{14}$$

where k is the momentum of the $\eta(\eta')$, and $I_{N\pi}$ is the isospin factor. It is $1/2$ for the $\pi^0 n$ loop and $\sqrt{2}/2$ for $\pi^- p$. The incoming nucleon momentum is given by p. To estimate the coupling, we use the approximation where the external nucleon legs are on shell. When simplifying this integral with the help of Feynman parameters and dimensional regularization, we obtain the result,

$$g_{HNN}^{CP} = \frac{g_A^2 I_{N\pi}^2}{F_\pi^2} f_{H\pi\pi} M_H \int_0^1 \mathrm{d}f_a \int_0^{1-f_a} \mathrm{d}f_b \times \Big[-3m(f_b+1)\lambda_2(\Delta_{HNN}) + m\frac{2+f_b}{2}\rho_2(\Delta_{HNN}) + (f_a(f_b+2)(f_a+f_b-1)k^2 m + f_b^3 m^3)\lambda_3(\Delta_{HNN})\Big]. \tag{15}$$

In the last expression, f_a and f_b are Feynman parameters, and

$$\lambda_2(\Delta) = \frac{1}{16\pi^2}\left[\frac{2}{\epsilon} - \log\left(\frac{\Delta}{\mu^2}\right) + \log(4\pi) - \gamma_E\right],$$
$$\rho_2(\Delta) = \frac{2}{16\pi^2}, \qquad \lambda_3(\Delta) = \frac{1}{16\pi^2\Delta}, \tag{16}$$

with $\epsilon = 4 - d$. Here, d is the Minkowski-space dimension, and the renormalization scale μ is set to the nucleon mass. For this diagram, we have

$$\Delta_{HNN} = M_\pi^2(1-f_b) - f_a k^2(1-f_a-f_b) + m^2 f_b^2. \tag{17}$$

For the purpose of comparison, we extract the heavy-baryon limit from Eq. (15) by taking the leading order of the Taylor expansion around the small parameter m^{-1}. When choosing a vanishing $k^2 = 0$ for the H, the CP-violating coupling has the compact form

$$g_{HNN}^{CP,HB} = \frac{3 f_{H\pi\pi} g_A^2 m M_H (\gamma_E - 2 - \log(4\pi) - \frac{2}{\epsilon})}{32\pi^2 F_\pi^2}. \tag{18}$$

This result is in agreement with the previous calculations of Ref. [5] after some typos are corrected.

The divergences are absorbed with the $\overline{\mathrm{MS}}$ scheme: terms proportional to $\frac{2}{\epsilon} + \log(4\pi) - \gamma_E$ are subtracted. Setting $k^2 = 0$, we obtain a compact result for Eq. (15),

$$g_{HNN}^{CP} = -\frac{3f_{H\pi\pi}g_A^2 M_H}{16\pi^2 F_\pi^2 m}\left\{M_\pi^2 \log\left(\frac{m}{M_\pi}\right) + m^2 + \frac{M_\pi(M_\pi^2 - 3m^2)}{\sqrt{4m^2 - M_\pi^2}} \times \left[\arctan\left(\frac{M_\pi}{\sqrt{4m^2 - M_\pi^2}}\right) + \arctan\left(\frac{2m^2 - M_\pi^2}{\sqrt{4m^2 M_\pi^2 - M_\pi^4}}\right)\right]\right\}. \tag{19}$$

As for Fig. 1(b), with a Δ intermediate state, the coupling reads

$$g_{HNN,\Delta}^{CP} = \mathrm{i} I_{N\Delta\pi}^2 \frac{h_A^2 f_{H\pi\pi} M_H}{F_\pi^2 M_\Delta^2} \int \frac{\mathrm{d}^d z}{(2\pi)^d} \frac{(p^\alpha - z^\alpha) z^\delta \gamma_{\alpha\beta\delta} S_\Delta^{\beta\beta'}(p-z)(p^{\alpha'} - z^{\alpha'})(z^{\delta'} + k^{\delta'})\gamma_{\alpha'\beta'\delta'}}{[(k+z)^2 - M_\pi^2][z^2 - M_\pi^2][(p-z)^2 - M_\Delta^2]}, \tag{20}$$

where the isospin factor $I_{N\Delta\pi}$ is $1/6$ for the $\pi^0\Delta^0$ loop and $1/3$ for the combination of $\pi^-\Delta^+$ and $\pi^+\Delta^-$. The Δ propagator is

$$S_\Delta^{\alpha\beta}(p) = \frac{\not p + M_\Delta}{p^2 - M_\Delta^2 + \mathrm{i}\varepsilon}\left[-g^{\alpha\beta} + \frac{1}{D-1}\gamma^\alpha\gamma^\beta + \frac{1}{(D-1)M_\Delta}(\gamma^\alpha p^\beta - \gamma^\beta p^\alpha) + \frac{D-2}{(D-1)M_\Delta^2} p^\alpha p^\beta\right]. \tag{21}$$

When putting the external nucleons on shell and choosing $k^2 = 0$, we obtain

$$\begin{aligned} g_{HNN,\Delta}^{CP} = \frac{f_{H\pi\pi} h_A^2 m^2 M_H}{1152\pi^2 F_\pi^2 M_\Delta^2}\Bigg\{ & -\frac{6(m^2 + M_\pi^2 - M_\Delta^2)}{m} + 6(2m + 3M_\Delta)\log\left(\frac{M_\Delta^2}{m^2}\right) + 4m \\ & - \frac{6(2m^4 + 3m^3 M_\Delta + m^2(2M_\Delta^2 - 6M_\pi^2) + m(3M_\Delta^3 - 3M_\pi^2 M_\Delta) + 2(M_\pi^2 - M_\Delta^2)^2)}{m^3} \\ & + \frac{1}{m^5\sqrt{-m^4 + 2m^2(M_\pi^2 + M_\Delta^2) - (M_\pi^2 - M_\Delta^2)^2}} \\ & \times [6(2m^4 - 5m^3 M_\Delta + m^2(6M_\Delta^2 - 4M_\pi^2) + 5mM_\Delta(M_\pi^2 - M_\Delta^2) + 2(M_\pi^2 - M_\Delta^2)^2) \\ & \times (m^2 + 2mM_\Delta - M_\pi^2 + M_\Delta^2)^2] \times \left[\arctan\left(\frac{-m^2 - M_\pi^2 + M_\Delta^2}{\sqrt{-m^4 + 2m^2(M_\pi^2 + M_\Delta^2) - (M_\pi^2 - M_\Delta^2)^2}}\right)\right. \\ & \left. - \arctan\left(\frac{m^2 - M_\pi^2 + M_\Delta^2}{\sqrt{-m^4 + 2m^2(M_\pi^2 + M_\Delta^2) - (M_\pi^2 - M_\Delta^2)^2}}\right)\right] \\ & + \frac{3}{m^5}\log(\frac{M_\pi^2}{M_\Delta^2})[2m^6 + 3m^5 M_\Delta + 6m^4 M_\pi^2 + 6m^3 M_\pi^2 M_\Delta + 6m^2 M_\pi^2(M_\Delta^2 - M_\pi^2) \\ & - 3mM_\Delta(M_\pi^2 - M_\Delta^2)^2 + 2(M_\pi^2 - M_\Delta^2)^3]\Bigg\}. \end{aligned} \tag{22}$$

Using the pion-decay ratio $F_\pi = 92.4$ MeV and the upper bounds on the CP-violating couplings $f_{H\pi\pi}$ as introduced in Eq. (5), one obtains the following upper limits,

$$\begin{aligned} |g_{\eta NN}^{CP}| &= 2.8\cdot 10^{-5}, & |g_{\eta' NN}^{CP}| &= 5.1\cdot 10^{-4}, \\ |g_{\eta NN}^{CP,HB}| &= 3.9\cdot 10^{-5}, & |g_{\eta' NN}^{CP,HB}| &= 7.1\cdot 10^{-4}, \\ |g_{\eta NN,\Delta}^{CP}| &= 7.9\cdot 10^{-6}, & |g_{\eta' NN,\Delta}^{CP}| &= 1.4\cdot 10^{-4}. \end{aligned} \tag{23}$$

One can see from the numerical result that it is important to take the Δ loop into account, as its contribution is larger than 20% of the nucleon's. Furthermore, although the magnitude of the heavy-baryon calculation is similar in size to the fully covariant one, one can see that there is a sizeable change of around 30% in the numerical value due to this nonrelativistic approximation.[1]

[1]These couplings, without the Δ contribution, had been calculated previously in the heavy-baryon ChPT (HBChPT) approach, in Ref. [5]. A direct comparison of the numerical results has little meaning because of some errors in the formulas. Also, now we have experimentally better constrained values for the branching ratios of Eq. (2) [1,15].

IV. THE *CP*-CONSERVING COUPLING OF THE η AND η' TO THE NUCLEON

The *CP*-conserving coupling of the $\eta(\eta')$ to the nucleon is given by

$$\mathcal{L}_{HNN} = -\mathrm{i}\frac{g_{HNN}}{2F_\eta} H\bar{N}k\!\!\!/\gamma_5 N, \qquad (24)$$

with k the H momentum, and $\bar{N}$ and N the outgoing and incoming nucleon states, respectively. In this calculation, the decay constants F_η and $F_{\eta'}$ are taken from Ref. [21], where they have been calculated taking into account the mixing of the quark nonstrange F_q and strange F_s leptonic decay constants,

$$F_\eta = 1.37F_\pi, \qquad F_{\eta'} = 1.16F_\pi. \qquad (25)$$

The physical η and η' are a mixing of the singlet and the octet states. Thus, the coupling $\pi^0 nn$ is given by $-g_A = -(F+D)$, while the ηnn and $\eta' nn$ vertices have the couplings $g_{\eta nn} = (D+F)\cos\psi + \sqrt{2}(F-D)\sin\psi$ and $g_{\eta' nn} = \sqrt{2}(D-F)\cos\psi + (F+D)\sin\psi$, respectively. The lower-case n refers to the neutron. The mixing angle ψ between the η and the η' has been estimated in many works [21–27] to be in a range between 38° [26] from $\eta \to e^+e^-\gamma$ decay data and 45° [22] in a ChPT analysis. The more recent results tend to have values close to 40°, which we use in the following. We also take the physical-average values for $F = 0.47$ and $D = 0.8$ [28].

V. COUPLINGS OF VECTOR MESONS

In the present work, we also study the effects of loops containing vector mesons coupling to the $\eta(\eta')$ and to the nucleon. The relevant pieces of the Lagrangians describing this type of couplings are [29,30]

$$\mathcal{L}_{\gamma HV} = \frac{e}{4} g_{\gamma HV}\epsilon_{\mu\nu\alpha\beta}F^{\mu\nu}V^{\alpha\beta}H, \qquad (26)$$

$$\mathcal{L}_{VNN} = \bar{N}\left(g_v\gamma^\mu + g_t\frac{\sigma^{\mu\nu}}{2m}\partial_\nu\right)V_\mu\tau_V N, \qquad (27)$$

where the coupling $g_{\gamma HV}$ is parametrized as $g_{\gamma HV} = \lambda_{HV}/M_H$. The values of the dimensionless couplings g_v^V, ratio g_t^V/g_v^V, λ_{HV} are taken from Refs. [1,5,30] and are summarized in Table I. Here, $\tau_V = \tau_3$ for $V = \rho^0$ and $\tau_V = \mathbb{1}_2$ for $V = \omega$. The λ_{HV} couplings are fixed by data [1] on rates of the $\rho(\omega) \to \eta\gamma$ and $\eta' \to \rho(\omega)\gamma$ decays, which are related as

$$\Gamma(V \to \eta + \gamma) = \frac{\alpha}{24}\lambda_{\eta V}^2 \frac{M_V^3}{M_H^2}\left[1 - \frac{M_H^2}{M_V^2}\right]^2,$$

$$\Gamma(\eta' \to V + \gamma) = \frac{\alpha}{8}\lambda_{\eta' V}^2 M_{\eta'}\left[1 - \frac{M_V^2}{M_{\eta'}^2}\right]^2. \qquad (28)$$

TABLE I. Parameters for the vector-meson coupling Lagrangians.

V	g_v^V	g_t^V/g_v^V	$\lambda_{\eta V}$	$\lambda_{\eta' V}$
ρ^0	2.4	6.1	0.87	1.27
ω	16	0	0.25	0.40

The electromagnetic field couples via the usual definition $F^{\mu\nu} = \partial^\mu\mathcal{A}^\nu - \partial^\nu\mathcal{A}^\mu$ and $V^{\mu\nu} = \partial^\mu V^\nu - \partial^\nu V^\mu$. The propagator of a vector-meson field with momentum k and mass m_V is taken as

$$\frac{1}{k^2 - m_V^2}\left(-g^{\alpha\beta} + \frac{k^\alpha k^\beta}{m_V^2}\right). \qquad (29)$$

Here, we want to remark that the values for the couplings are poorly known, for which reason they are an important source of uncertainty for the results. Furthermore, in higher orders, they have a dependency on the virtuality of the vector meson, which we ignore in the leading-order calculations that follow.

VI. CALCULATION OF THE NUCLEON EDM

The EDM is extracted from the amplitude coupling the photon to the nucleon. In our case, as the amplitude always involves a *CP*-violating vertex, only one form factor containing the EDM appears. Therefore, the *CP*-violating part of the vector current J^μ between baryon states reads

$$\langle B(p')|J^\mu_{\mathrm{CPV}}|B(p)\rangle = \bar{u}(p')\frac{\mathrm{i}\sigma^{\mu\nu}\gamma_5 q_\nu}{4m}F_{\mathrm{EDM}}(q^2)u(p), \qquad (30)$$

where q_ν is the photon momentum, ϵ_μ its polarization, and $\sigma^{\mu\nu} = \mathrm{i}\gamma^{\mu\nu}$. At the point where $q^2 = 0$, the form factor reduces to the electric dipole moment $F_{\mathrm{EDM}}(0) = \tilde{d}_N$. In our model, the CPV comes from the loops of Fig. 2.

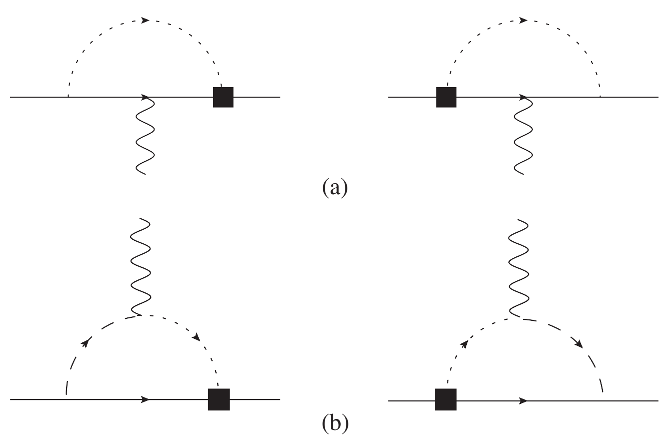

FIG. 2. Loops that can contribute to the neutron EDM, via: (a) an $\eta(\eta')$ propagating in the loop, (b) an $\eta(\eta')$ and a vector meson propagating in the loop. The solid line represents the neutron, the dotted line the $\eta(\eta')$, the dashed lines are vector-meson contributions, and the wavy line corresponds to the photon. Again, the black box stands for a *CP*-violating vertex.

In Fig. 2(a), the photon couples to the nucleon that propagates inside the loop. In the particular case of the neutron, the leading-order coupling to the photon vanishes, for which reason only next-to-leading order terms contribute. The second-order nucleon Lagrangian is needed to describe such a vertex at lowest nonvanishing order for the neutron, which reduces to

$$\mathcal{L}^{(2)}_{\gamma nn} = \sigma^{\mu\nu} F_{\mu\nu} \frac{e\kappa_n}{4m}, \tag{31}$$

where $\kappa_n = -1.913$ is the neutron magnetic moment in units of $\frac{e}{2m}$.

A direct coupling of the photon to an $\eta(\eta')$ propagating inside a loop is not possible, due to this meson's vanishing charge. Nevertheless, as is depicted in Fig. 2(b), it is possible to achieve a coupling via a vector-meson exchange, which here we also perform for the sake of comparison with Ref. [5] and for an estimate of the importance of its effect.

The amplitude of Fig. 2(a) reads

$$\frac{e\kappa_n g_{HNN}\bar{g}_{HNN}}{8mF_H}\int\frac{\mathrm{d}^d z}{(2\pi)^d}\frac{1}{[z^2-M_H^2][(p-z)^2-m^2][(p+q-z)^2-m^2]}[(\not{p}+\not{q}-\not{z}+m)(\not{q}\not{\epsilon}-\not{\epsilon}\not{q})(\not{p}-\not{z}+m)\not{z}\gamma_5$$
$$-\not{z}\gamma_5(\not{p}+\not{q}-\not{z}+m)(\not{q}\not{\epsilon}-\not{\epsilon}\not{q})(\not{p}-\not{z}+m)], \tag{32}$$

which for the dipole moment $\tilde{d}_n$ in units of $\frac{e}{2m}$ at $q^2=0$ leads to

$$\tilde{d}_{n,a} = \frac{m\kappa_n\bar{g}_{HNN}g_{HNN}}{F_H}\int_0^1\mathrm{d}f_b\int_0^{1-f_b}\mathrm{d}f_a\{(6f_a-5)\lambda_2(\Delta_{\mathrm{EDM,a}})+(2-f_a)\rho_2(\Delta_{\mathrm{EDM,a}})$$
$$-2m^2[f_a(f_a^2-2)+2(1+(f_a-1)f_a)f_b+(f_a-2)f_b^2]\lambda_3(\Delta_{\mathrm{EDM,a}})\}. \tag{33}$$

Together with the definitions in Eq. (16), we choose the notation

$$\Delta_{\mathrm{EDM,a}} = M_H^2(1-f_a-f_b)+m^2(f_a+f_b^2). \tag{34}$$

After integration, the analytical expression is also quite compact,

$$\tilde{d}_{n,a} = \frac{\kappa_n\bar{g}_{HNN}g_{HNN}}{32\pi^2F_Hm^3}\left\{m^4-3m^2M_H^2+(3M_H^4-6m^2M_H^2)\log\left(\frac{M_H}{m}\right)-\frac{3M_H^3}{\sqrt{4m^2-M_H^2}}(M_H^2-4m^2)\right.$$
$$\left.\times\left[\arctan\left(\frac{M_H}{\sqrt{4m^2-M_H^2}}\right)+\arctan\left(\frac{2m^2-M_H^2}{M_H\sqrt{4m^2-M_H^2}}\right)\right]\right\}. \tag{35}$$

As for the amplitude in Fig. 2(b), it is given by

$$\frac{e\lambda_{HV}\tau_V\bar{g}_{HNN}}{M_H}\int\frac{\mathrm{d}^d z}{(2\pi)^d}\left[\frac{(\not{z}+m)}{[(p-z)^2-M_V^2][z^2-m^2][(p'-z)^2-M_H^2]}q_\mu\epsilon_\nu(p-z)_\alpha\mathrm{i}\epsilon^{\mu\nu\alpha\beta}\left(-g_{\beta\beta'}+\frac{(p-z)_\beta(p-z)_{\beta'}}{M_V^2}\right)\right.$$
$$\times\left(g_v^V\gamma^{\beta'}-\frac{g_t^V}{4m}(p-z)^{\alpha'}[\gamma^{\beta'},\gamma^{\alpha'}]\right)-\frac{(g_v^V\gamma^{\beta'}+\frac{g_t^V}{4m}(p'-z)^{\alpha'}[\gamma^{\beta'},\gamma^{\alpha'}])(-g_{\beta'\beta}+\frac{(p'-z)_{\beta'}(p'-z)_\beta}{M_V^2})}{[(p-z)^2-M_H^2][z^2-m^2][(p'-z)^2-M_V^2]}$$
$$\left.\times q_\mu\epsilon_\nu(p'-z)_\alpha\mathrm{i}\epsilon^{\mu\nu\alpha\beta}(\not{z}+m)\right]. \tag{36}$$

For this loop diagram, the analytical result has the very simple form

$$\tilde{d}_{n,b} = 2\frac{\lambda_{HV}\tau_V m\bar{g}_{HNN}}{M_H}\int_0^1\mathrm{d}f_b\int_0^{1-f_b}\mathrm{d}f_a$$
$$\times\{(g_v^V-g_t^V)[2\lambda_2(\Delta_{\mathrm{EDM,b}})+3\rho_2(\Delta_{\mathrm{EDM,b}})]\},$$

where

$$\Delta_{\mathrm{EDM,b}} = m^2(1-f_a-f_b)^2+M_H^2f_b+m_V^2f_a.$$

Note that, for each of the two diagrams in Fig. 2(b) separately, there are also pieces of the type $\lambda_3(\Delta_{\mathrm{EDM,b}})$, but they cancel each other. Integrating over the Feynman parameters yields

$$\tilde{d}_{n,b} = \frac{\bar{g}_{HNN}(g_t^V - g_v^V)\lambda_{HV}\tau_V}{24\pi^2 m^3 M_H(M_H^2 - m_V^2)} \Bigg\{ m^2(M_H^4 - m_V^4) + M_H^3(4m^2 - M_H^2)^{3/2} \times \left[\arctan\left(\frac{M_H}{\sqrt{4m^2 - M_H^2}}\right) - \arctan\left(\frac{M_H^2 - 2m^2}{M_H\sqrt{4m^2 - M_H^2}}\right)\right] - M_V^3(4m^2 - M_V^2)^{3/2}\left[\arctan\left(\frac{M_V}{\sqrt{4m^2 - M_V^2}}\right) - \arctan\left(\frac{M_V^2 - 2m^2}{M_V\sqrt{4m^2 - M_V^2}}\right)\right] + M_H^4(6m^2 - M_H^2)\log\left(\frac{M_H}{m}\right) - M_V^4(6m^2 - M_V^2)\log\left(\frac{M_V}{m}\right)\Bigg\}. \tag{37}$$

The numerical results are summarized in Table II. The vector-meson contributions of Fig. 2(b) turn out to be of the same order of magnitude as the loops in Fig. 2(a). This is to be expected, even though the vector mesons are higher-mass states. For Fig. 2(a), the Lagrangian of the first chiral order does not allow a coupling of the photon to the neutron. Therefore, this contribution is suppressed, and the vector-meson contributions become equally important. The sum of all the contributions yields a total value for the dipole moment of $d_n^{\rm tot} = 4.2 \times 10^{-18} e$ cm. Note that this value takes into account the new result for the η' two-pion decay [15]. Therefore, it is smaller by approximately a factor of $\sqrt{3}$, when compared to values obtained from the η' two-pion decays in Ref. [1]. Considering the current experimental upper limit of $2.9 \times 10^{-26} e$ cm for the neutron dipole moment, the ratio between expectation and measurement is of the order of 10^8. This means that the present upper limit for the decay ratio of the $\eta(\eta')$ into two pions gives a large overestimation of the CP-violating coupling constant. In fact, in order for the results to be compatible with the experimental constraint on the neutron dipole moment, the branching ratio would have to be at least 8 orders of magnitude smaller.

It is interesting to confront these results with those in Ref. [2]. There, as mentioned, the size of the neutron EDM was estimated within a similar framework as presented here, but by considering a CP-violating vertex in the coupling of the charged mesons to the baryons and calculating their induced contributions to the EDM at leading chiral order. There, up to a factor including the unknown CP-violating phase θ, the EDM was estimated to be of the order of $10^{-16} e$ cm. The fact that we get an estimate approximately 2 orders of magnitude smaller is in good agreement with that calculation, knowing that for the neutral mesons considered here, the diagrams that contribute are of the next chiral order. It is important to keep in mind that the values shown in Table II are not to be seen as predictions for the neutron EDM, but as estimates for the order of magnitude of the $\eta(\eta')$ branching ratios into two pions. Other processes, which are beyond the scope of this paper, give additional contributions to the neutron EDM. These are, e.g., pieces obtained from the CP-violating decay of the $\eta(\eta')$ into four pions or processes that do not conserve flavor via the quark-mixing matrix. Furthermore, as mentioned in Sec. V, some of the coupling constants used here are poorly known, and the results depend on the renormalization scheme used. Nevertheless, due to the very large discrepancy between the experimental constraint on the EDM and the one calculated from the current upper limits for the CP-violating branching ratios, the results are still rigorous enough to be instructive. The conclusions made here remain, even if other processes are to be additionally considered, or if the coupling constants are to have different sizes.

TABLE II. Contributions to the upper limit of the neutron EDM, from the current experimental upper limits of the η and η' branching ratios into two pions [1,15]. The units are e cm.

	η	η'
Fig. 2(a)	2.7×10^{-20}	1.5×10^{-18}
Fig. 2(b)	2.0×10^{-19}	2.5×10^{-18}

VII. SUMMARY

In the present paper, we calculated the nucleon EDM originated by a CP-violating coupling to the $\eta(\eta')$ meson. In particular, we focused on the result for the neutron, as its experimental upper limit is very small, $2.9 \times 10^{-26} e$ cm. This limit sets a very strong constraint on observables related to it. More specifically, if a neutron EDM is to exist, then CPV has to occur. Therefore, here the goal was to give an estimate of the size of this violation.

This was achieved by constructing a CP-violating coupling of the η to the nucleon via loops that include an $\eta(\eta')\pi\pi$ vertex. While there are experimental results for the upper limit of the $\eta(\eta') \to \pi\pi$ decay ratio, here we wanted to test if this constraint is indeed compatible with the limit on the neutron EDM. The Δ-isobar contributions were taken into account as well, leading to a correction to the CP-violating $\eta(\eta')NN$ vertex larger than 20%.

We considered two possible sources for the neutron EDM. In one case, the photon coupled to the neutron within a loop with a CP-violating ηNN vertex. In the other, vector-meson contributions were considered as well. The two contributions turned out to be of a similar size.

In total, we obtained a constraint on the CP-violating $\eta(\eta') \to \pi\pi$ decay ratio roughly 8 orders of magnitude smaller than measured in the experiment so far. This is a very instructive result, since it gives an estimate on symmetry violations in nature, where experimental results are not yet achievable.

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