PHYSICAL REVIEW D **95,** 035040 (2017)

Charged Higgs bosons from the 3-3-1 models and the $\mathcal{R}(D^{(*)})$ anomalies

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(Received 11 December 2016; published 28 February 2017)

Several anomalies in the semileptonic B-meson decays such as $\mathcal{R}(D^{(*)})$ have been reported by the *BABAR*, Belle, and LHCb collaborations recently. In this paper, we investigate the contributions of the charged Higgs bosons from the 3-3-1 models to the $\mathcal{R}(D^{(*)})$ anomalies. We find that, in a wide range of parameter space, the 3-3-1 models might give reasonable explanations to the $\mathcal{R}(D^{(*)})$ anomalies and other analogous anomalies of the B meson's semileptonic decays.

DOI: 10.1103/PhysRevD.95.035040

I. INTRODUCTION

In recent years, the discovery of heavy-quark spin-flavor symmetry and the formulation of the heavy-quark effective theory (HQET) [1] have shown that physical observables in the branching fractions of the semileptonic decays $B \to D^{(*)} l \nu_l$, which have drawn a lot of attention, could be rather reliably predicted within the Standard Model (SM) —especially at the zero-recoil point, which may allow a reliable determination of the Cabibbo-Kobayashi-Maskawa (CKM) element V_{cb} [2]. In the SM, semileptonic decays of B mesons proceed via first-order electroweak interactions and are mediated by the W boson [3]. Since the effect of new physics (NP) beyond the SM for these decays is induced by tree-level charged current, it is considered to be tiny.

There is an interesting phenomenon worthy of study: the expression can be written as

$$\mathcal{R}(D^{(*)}) = \left.\frac{\mathcal{B}(\overline{B} \to D^{(*)}\tau^- \overline{\nu}_\tau)}{\mathcal{B}(\overline{B} \to D^{(*)} l^- \overline{\nu}_l)}\right|_{l\in\{e,\mu\}},$$

where the SM predictions are given by $\mathcal{R}(D)_{\mathrm{SM}} = 0.300 \pm 0.008$ [4] and $\mathcal{R}(D^*)_{\mathrm{SM}} = 0.252 \pm 0.003$ [5]. However, the present experimental values measured by the *BABAR* [6], Belle [7], and LHCb [8] collaborations have recently observed anomalies in the ratios $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$. The average values given by the Heavy Flavor Averaging Group (HFAG) are $\mathcal{R}(D)_{\mathrm{avg}} = 0.397 \pm 0.040 \pm 0.028$ and $\mathcal{R}(D^*)_{\mathrm{avg}} = 0.316 \pm 0.016 \pm 0.010$ [9], which exceed the SM predictions by 1.9σ and 3.3σ, respectively. If one takes into account the $\mathcal{R}(D)$-$\mathcal{R}(D^*)$ correlation of -0.21, the tension with the SM expectation would be at the 4σ level [10]. These anomalies have caused wide concern, leading to many works, some of which were within model-independent frameworks [11–15], while others were in NP models. The works in NP models are classified as mediated by leptoquarks [2,11,12,16,17], charged Higgs bosons [10,17,18], charged vector bosons [11,19], and sparticles [20].

So far, the measurements of the ratios $\mathcal{R}(D^{(*)})$ in both the *BABAR* and Belle experiments used only the leptonic channels for the identification of τ. When one measures the branching fractions of the purely leptonic decay $B^+ \to \tau^+ \nu_\tau$, it will use both the leptonic and hadronic channels. Therefore, one kind of new ratios $\mathcal{R}_\tau(D^{(*)})$ is introduced, which are defined as

$$\mathcal{R}_\tau(D^{(*)}) = \frac{\mathcal{R}(D^{(*)})}{\mathcal{B}(B^+ \to \tau^+ \nu_\tau)}. \tag{1}$$

Then the systematics of τ detection tend to cancel, which provide us a more reliable test of the SM [21]. In the literature [2], the authors calculated the ratios $\mathcal{R}$ and $\mathcal{R}_\tau$ for the semileptonic decays $B \to D^{(*)}\tau\nu$, $B \to X_c \tau\nu$, and $B \to \pi\tau\nu$, which are induced by charged Higgs bosons in both the SM and the 2HDM-II. With the measurement of $B \to \tau\nu$ and refined measurements of observables in $B \to D\tau\nu$ in different experiments, the study of $\mathcal{R}(D^{(*)})$ anomalies will be an effective solution for us to explore the SM and NP models. On the other hand, NP models might yield a rational interpretation for the $\mathcal{R}(D^{(*)})$ anomalies.

Among the new physics models, the so-called 3-3-1 models [21–27] with gauge symmetry $SU(3)_C \times SU(3)_L \times U(1)_X$ are an interesting extension of the SM, which can explain why there are three family fermions and why there is quantization of electric charge. In the general 3-3-1 models, there are $3 \times 3 \times 2 = 18$ scalar states—namely, four pairs of singly charged states, five CP-odd states, and five CP-even states. After some of the states are absorbed into the gauge bosons, we have the physical singly charged Higgs bosons $H^\pm$, the CP-odd Higgs boson A, and the CP-even Higgs boson H_a. With the additional scalar bosons above, the 3-3-1 models may give us rich phenomenology. So far, the extended Higgs sectors have not

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yet been ruled out experimentally, and there have been already many works on the study of the Higgs bosons from the 3-3-1 models [28]. Since the charged Higgs bosons $H^{\pm}$ can couple to the SM quarks and leptons, we reasonably infer that the 3-3-1 models may give explanations for the $\mathcal{R}(D^{(*)})$ anomalies.

This paper is organized as follows: In Sec. II, we briefly review the essential features of the 3-3-1 models. The relevant couplings of the charged Higgs bosons to other particles are also discussed in that section. In Sec. III, we use the latest related experimental data to give constraints on the parameters of the 3-3-1 models. To make the study more comprehensive, we also consider two other useful semileptonic decay modes, $B \to X_c \tau \nu_\tau$ and $B \to \pi \tau \nu_\tau$, by similar methods. Our conclusions are given in Sec. IV.

II. THE BASIC CONTENT OF THE 3-3-1 MODELS

The 3-3-1 models are based on the gauge symmetry $SU(3)_C \times SU(3)_L \times U(1)_X$, in which the electric charge operator is defined as

$$Q = T_3 + \beta T_8 + XI, \tag{2}$$

where T_3 and T_8 are two of the eight generators of $SU(3)_L$, X is the new quantum number corresponding to $U(1)_X$, and the free parameter β defines the different particle structure and is used to label the particular type of the 3-3-1 models. To avoid the presence of exotic charges in the fermion and gauge boson sectors, we prefer to choose the scheme of $\beta = \pm 1/\sqrt{3}$.

In the 3-3-1 models, symmetry breaking is generally accomplished in two steps:

$$SU(3)_C \times SU(3)_L \times U(1)_X \underset{u}{\rightarrow} SU(3)_C \times SU(2)_L \times U(1)_X \underset{v_1, v_2}{\longrightarrow} SU(3)_C \times U(1)_X,$$

which can be realized via developing nonzero vacuum expectation values (VEVs) u, v_1, and v_2 for the neutral component fields of three triplets.

The quark content of the 3-3-1 models is generally described by

$$q_{mL} = \begin{pmatrix} u_m \\ -d_m \\ B_m \end{pmatrix}_L \sim (3^*, 3, 0),$$

$$q_{3L} = \begin{pmatrix} u_3 \\ d_3 \\ T_3 \end{pmatrix}_L \sim (3, 3, 1/3),$$

$$d^c \sim (3^*, 1, 1/3), \qquad u^c \sim (3^*, 1, -2/3),$$

$$T^c \sim (3^*, 1, -2/3), \qquad B_m^c \sim (3^*, 1, 1/3),$$

where $m = 1, 2$ and the numbers in the parentheses express their assigned quantum numbers of the group $SU(3)_C \times SU(3)_L \times U(1)_X$.

For the leptonic sector, each lepton family is arranged in triplets: the first two elements are the charged and neutral lepton, and the third element is a conjugate of the charged lepton or the neutral lepton. There are

$$\Psi_{nL} = \begin{pmatrix} e_n^- \\ \nu_n \\ N_n^0 \end{pmatrix}_L \sim (1, 3^*, -1/3),$$

$$\Psi_L = \begin{pmatrix} \nu_1 \\ e_1^- \\ E_1^- \end{pmatrix}_L \sim (1, 3, -2/3),$$

$$\Psi_{4L} = \begin{pmatrix} E_2^- \\ N_3^0 \\ N_4^0 \end{pmatrix}_L \sim (1, 3^*, -1/3),$$

$$\Psi_{5L} = \begin{pmatrix} N_5^0 \\ E_2^+ \\ e_3^+ \end{pmatrix}_L \sim (1, 3^*, 2/3),$$

$$e_n^c \sim (1, 1, 1), \qquad e_3^c \sim (1, 1, 1),$$

$$E_1^c \sim (1, 1, 1), \qquad E_2^c \sim (1, 1, 1),$$

where $n = 2, 3$. For this kind of 3-3-1 model, besides the ordinary gauge bosons γ, Z, and $W^{\pm}$, the new neutral gauge boson Z', as well as charged and neutral bileptons $V^{\pm}$ and X^0 are predicted.

The scalar fields of the 3-3-1 models are generally parametrized as

$$\Phi_1 = \begin{pmatrix} \phi_1^0 \\ \phi_1^+ \\ \eta_1^+ \end{pmatrix}, \qquad \Phi_2 = \begin{pmatrix} \phi_2^- \\ \phi_2^0 \\ \eta_2^0 \end{pmatrix}, \qquad \varphi = \begin{pmatrix} \eta_3^- \\ \eta_3^0 \\ \phi_3^0 \end{pmatrix}, \tag{3}$$

which correspond to $3 \times 3 \times 2 = 18$ scalar states—namely, four pairs of singly charged states, five CP-odd states, and five CP-even states. After eight are absorbed into the longitudinal components of two pairs of charged gauge bosons (W and W') and four neutral gauge bosons (Z, Z', Y_1, and Y_2), we have the physical singly charged Higgs bosons $H^{\pm}$, the CP-odd Higgs boson A, and the CP-even Higgs boson H_a [21]. The contributions from the 3-3-1 models to the $\mathcal{R}(D^{(*)})$ and $\mathcal{R}_\tau(D^{(*)})$ anomalies may be induced both by the charged Higgs bosons $H^{\pm}$ and by the new gauge boson W'. However, the mass of the new gauge boson W' is large, and the interactions between W' and the SM particles are faint. Compared with the contribution from the SM gauge boson W, the contribution from W' is tiny enough to be ignored. Therefore, we will concentrate on calculating the contributions from the charged Higgs bosons $H^{\pm}$ in the sections to follow.

The Yukawa coupling for the charged Higgs boson H^{+} from the 3-3-1 models is generally given by [21,26]

$$G_{\overline{u}dH^{+}} = -\frac{g}{2\sqrt{2}m_W}[V_{\rm CKM} - V_L^{u\dagger}\Delta V_L^d]^*_{ud}[m_d(1 - \gamma_5)\tan\beta + m_u(1+\gamma_5)\cot\beta], \quad (4)$$

where u and d refer to up- and down-type quarks; m_W, m_u, and m_d express the masses of the SM particles W, u and d; and $\Delta = {\rm diag}(0,0,1)$. V_L^u and V_L^d are the unitary matrices of the left-handed quarks, which are given by [21,26]

$$V_L^u = \begin{pmatrix} 0.975 & -0.223 & 1.86\times 10^{-3} \\ 0.222 & 0.974 & 0.0518 \\ -0.01340 & -0.0501 & 0.999 \end{pmatrix}, \quad (5)$$

$$V_L^d = \begin{pmatrix} 1.00 & 2.56\times 10^{-3} & 5.87\times 10^{-3} \\ -3.10\times 10^{-3} & 0.996 & 0.0941 \\ -5.61\times 10^{-3} & -0.0942 & 0.996 \end{pmatrix}, \quad (6)$$

where we have ignored CP violation, by which the experimental values of the elements of the CKM matrix defined as V_L^u (V_L^d) are reproduced. Because the most important factor in Eq. (4) is the $V_{\rm CKM}$ matrix, the numerical results are not much changed if different V_L^u and V_L^d are used. Besides, one can see from Eq. (4) that when the mass of the up-type quark is much smaller than that of the down-type quark, the main contribution of the coupling is coming from the down-type quark term, which contains the parameter $\tan\beta$, which applies to the decay $b\to c\tau\nu$. Namely, the main contribution of the coupling is coming from the b-quark term, since $m_c < m_b$. The decay rates for the charged Higgs bosons $H^{\pm}$ from the 3-3-1 models into the SM charged lepton and neutrino pairs are generally given by

$$\Gamma(H^+\to l^+\nu) = \frac{G_F m_{H^{\pm}}}{4\sqrt{2}\pi} m_l^2 \tan^2\beta\left(1-\frac{m_l^2}{m_{H^{\pm}}^2}\right)^2, \quad (7)$$

where m_l denotes the mass of the SM charged lepton, and $m_{H^{\pm}}$ denote the masses of the charged Higgs bosons $H^{\pm}$. The squared masses of $H^{\pm}$ are given by $m_{H^{\pm}}^2 = \frac{v^2}{2}\lambda_4 + M^2$, where $v=\sqrt{v_1^2+v_2^2} = (\sqrt{2}G_F)^{-1/2} \simeq 246$ GeV, where G_F is the Fermi constant, and λ_4 is one of the group parameters of the $SU(3)$ group. The value $\tan\beta = v_2/v_1$ is a scalar mixing angle, with which it is possible to obtain a natural fit for the observed neutrino hierarchical and mixing angles when $\tan\beta \gg \mathcal{O}(1)$ [29]. $M^2 = \mu u/(\sqrt{2}s_\beta c_\beta)$, s_β, and c_β are the shorthand notations for the angles β (i.e. $c_\beta = \cos\beta$, $s_\beta = \sin\beta$), and μ is the soft-breaking term involved in the Higgs potential. Since the charged Higgs bosons $H^{\pm}$ can couple to the SM quarks and leptons, we infer that the 3-3-1 models may give contributions to the $\mathcal{R}(D^{(*)})$ and $\mathcal{R}_\tau(D^{(*)})$ anomalies.

So far, the specific scope of masses of the charged Higgs bosons $H^{\pm}$ are not predicted by the 3-3-1 models. Considering that there are no experimental upper bounds on the mass of the charged Higgs, one generally expects to have $m_H < 1$ TeV in order to guarantee the perturbation theory remains valid [30]. In addition, the regions for $m_{H^+} \le 540$ GeV have already been excluded by $b\to s\gamma$ measurements at a 95% confidence level [31]. Therefore, in this paper, we will take 540 GeV $< m_{H^+} <$ 1000 GeV in the following numerical calculation.

III. THE CONTRIBUTIONS OF THE CHARGED HIGGS TO THE $\mathcal{R}(D^{(*)})$ AND $\mathcal{R}_\tau(D^{(*)})$ ANOMALIES

The expression for the branching fraction ${\rm Br}(B\to\tau\nu_\tau)$ in the SM is given by

$$\mathcal{B}^{\rm SM}(B^+\to\tau^+\nu_\tau) = \frac{G_F^2 m_B m_\tau^2}{8\pi}\left[1-\frac{m_\tau^2}{m_B^2}\right] f_B^2 |V_{ub}|^2 \tau_{B^+},$$

where τ_{B^+} is the lifetime of the B^+ meson, and m_B and m_τ are the masses of the B meson and τ lepton. In our numerical calculation, we will take $G_F = 1.1663787(6)\times 10^{-5}$ GeV^{-2}, $m_B = 5.27929\pm 0.00015$ GeV, $m_\tau = 1.77686\pm 0.00012$ GeV, $m_b = 4.20\pm 0.07$ GeV, $f_B = 0.191\pm 0.007$ GeV, $\tau_{B^+} = 1.638(4)$ ps^{-1}, and $|V_{ub}| = (3.61\pm 0.32)\times 10^{-3}$ [32–35].

In this section, we will calculate the contributions of the charged Higgs from the 3-3-1 models to the three kinds of decays, $B\to D^{(*)}\tau\nu_\tau$, $B\to X_c\tau\nu_\tau$, and $B\to\pi\tau\nu_\tau$. One can see from relevant expressions that the parameters $\tan\beta$ and m_{H^+} are important, which may have great influence on the numerical results. Besides considering the characteristics of the forms of the two parameters in the related calculation formula, we define a new parameter $r = \tan\beta/m_{H^+}$, which may reflect the synergy of the two parameters.

A. $\mathcal{R}(D^{(*)})$ and $\mathcal{R}_\tau(D^{(*)})$

The contributions from the 3-3-1 models to the $\mathcal{R}^{331}(D^{(*)})$ and $\mathcal{R}_\tau^{331}(D^{(*)})$ anomalies may be induced both by the charged Higgs bosons $H^{\pm}$ and by the new gauge boson W'. As we have discussed before, the contribution from the new gauge boson W' is tiny enough to be ignored. Therefore, we only investigate the contributions from the charged Higgs bosons $H^{\pm}$ in the following section.

After dropping the terms that are negligible, we have the compact expressions of the $\mathcal{R}^{331}(D)$ and $\mathcal{R}^{331}(D^*)$ ratios including the contributions of the 3-3-1 models, which can be approximately written as

$$\mathcal{R}(D^{(*)})_{331} \approx \mathcal{R}(D^{(*)})_{\rm SM} - A_{D^{(*)}}\frac{\tan^2\beta}{m_{H^+}^2} + B_{D^{(*)}}\frac{\tan^4\beta}{m_{H^+}^4}, \tag{8}$$

where $A(B)_{D^{(*)}}$ are coefficients determined by averaging over B^0 and B decays [6]. $A_D = -3.25 \pm 0.32\ \mathrm{GeV}^2$, $B_D = 16.9 \pm 2.0\ \mathrm{GeV}^4$, $A_{D^*} = -0.230 \pm 0.029\ \mathrm{GeV}^2$, $B_{D^*} = 0.643 \pm 0.085\ \mathrm{GeV}^4$, $\mathcal{R}_\tau^{\rm SM}(D) = (3.136 \pm 0.628) \times 10^3$, and $\mathcal{R}_\tau^{\rm SM}(D^*) = (2.661 \pm 0.512) \times 10^3$ [2]. One can see from Eq. (8) that the contributions from the 3-3-1 models to $\mathcal{R}(D^{(*)})$ anomalies are approximately proportional to the model parameter $\tan\beta$ (or $\frac{\tan\beta}{m_{H^+}}$).

The ratios $\mathcal{R}_\tau^{331}(D^{(*)})$ can be written as

$$\mathcal{R}_\tau^{331}(D^{(*)}) = \frac{\mathcal{R}^{331}(D^{(*)})}{\mathcal{B}(B^+ \to \tau^+\nu_\tau)} \approx \mathcal{R}_\tau^{\rm SM}(D^{(*)}) - \frac{A_{D^{(*)}}}{\mathcal{B}(B^+ \to \tau^+\nu_\tau)} \cdot \frac{\tan^2\beta}{m_{H^+}^2} + \frac{B_{D^{(*)}}}{\mathcal{B}(B^+ \to \tau^+\nu_\tau)} \cdot \frac{\tan^4\beta}{m_{H^+}^4}, \tag{9}$$

with

$$\mathcal{B}(B^+ \to \tau^+\nu_\tau)_{331} = \mathcal{B}(B^+ \to \tau^+\nu_\tau)_{\rm SM}\left[1 + (V_{\rm CKM} - V_L^{u\dagger}\Delta V_L^d)_{13}^2 \frac{m_B^2}{V_{ub}^2}\frac{\tan^2\beta}{m_{H^+}^2}\right]^2. \tag{10}$$

The values of all the parameters in Eqs. (9) and (10) have been given, which are not listed again.

The latest measured values of $\mathcal{R}(D^{(*)})$, $\mathcal{R}_\tau(D^{(*)})$ and $\mathcal{B}(B^+ \to \tau^+\nu_\tau)$ by *BABAR* and Belle are summarized in Table I [2,6,7].

One can see from Eqs. (8)–(10) that the values of the rates $\mathcal{R}(D^{(*)})$ and $\mathcal{R}_\tau(D^{(*)})$ depend on the model parameters m_{H^+} and $\tan\beta$. We use the above experimental data to bound the parameters of the 3-3-1 models, and find that the reasonable parameter values can explain the $\mathcal{R}(D^{(*)})$ anomalies. In order to see the allowed parameter space of the 3-3-1 models by *BABAR* and Belle data, we plot $\tan\beta$ as a function of m_{H^+} in Figs. 1(a) (left) and 1(b) (right), respectively. One can see from Fig. 1(a) that the largest parameter space is the blue region, which is coming from $\mathcal{R}(D)$ data. The smallest region, which also denotes the common allowed parameter space for the three data sets, is the yellow region, which is coming from $\mathcal{B}(B^+ \to \tau^+\nu_\tau)$ data. We found that the constraint on the parameter space from $\mathcal{B}(B^+ \to \tau^+\nu_\tau)$ data is the biggest. In the common space, the reasonable value of $\tan\beta$ is in the range of $27 \sim 145$. The conclusions of Fig. 1(b) are similar to those of Fig. 1(a), and the reasonable value of $\tan\beta$ is in the range of $0 \sim 108$ by Belle data.

In order to see the synergistic effect of the parameters $\tan\beta$ and m_{H^+} on the ratios $\mathcal{R}_\tau(D^{(*)})$, we plot $\mathcal{R}_\tau(D^{(*)})$ as a function of the parameter $r = \tan\beta/m_{H^+}$ in Fig. 2. In our numerical calculation, we assume $540\ \mathrm{GeV} \leq m_{H^+} \leq 1000\ \mathrm{GeV}$ for three typical values of $\tan\beta$ ($\tan\beta = 2$, 10, 20). We can see from Fig. 2 that it is possible for the 3-3-1 models to explain the $\mathcal{R}_\tau(D^{(*)})$ anomalies when $0 < \tan\beta \leq 10$.

B. $\mathcal{R}(X_c)$ and $\mathcal{R}_\tau(X_c)$

In this subsection, we discuss the inclusive semileptonic decay channels of the B meson, $B \to X_c\tau\nu_\tau$. The ratios $\mathcal{R}(X_c)$ and $\mathcal{R}_\tau(X_c)$ can be defined as

$$\mathcal{R}(X_c) = \frac{\mathcal{B}(B \to X_c\tau\bar{\nu})}{\mathcal{B}(B \to X_c e\bar{\nu})}, \qquad \mathcal{R}_\tau(X_c) = \frac{\mathcal{R}(X_c)}{\mathcal{B}(B^+ \to \tau^+\nu_\tau)}.$$

The SM prediction and experimental measurement values for $\mathcal{R}(X_c)$ and $\mathcal{R}_\tau(X_c)$ are summarized in Table II [21,36–38]. One can see from Table II that there are also similar anomalies in the $\mathcal{R}(X_c)$ and $\mathcal{R}_\tau(X_c)$ ratios. Thus, we calculate the contributions of the charged Higgs from the 3-3-1 models to the $\mathcal{R}(X_c)$ anomaly along with the $\mathcal{R}(D^{(*)})$ anomalies.

The expressions of the differential decay rate of the inclusive channel $B \to X_c\tau\nu_\tau$ have been given by HQET in the context of the SM [39]. Considering the contributions of the 3-3-1 models, this expression forms can be similarly given. To save the length of the paper, we will not present them here. We plot $\mathcal{R}_\tau^{331}(X_c)$ as a function of the parameter $r = \tan\beta/m_{H^+}$ for decentralized values of $\tan\beta$ and $540\ \mathrm{GeV} \leq m_{H^+} \leq 1000\ \mathrm{GeV}$ in Fig. 3. On account of the discussions above, we do not choose large values for $\tan\beta$ here. Our numerical results show that for $0 < \tan\beta \lesssim 20$ and $540\ \mathrm{GeV} \leq m_{H^+} \leq 1000\ \mathrm{GeV}$, the

TABLE I. The measured values of $\mathcal{R}(D)$, $\mathcal{R}(D^*)$ and $\mathcal{B}(B^+ \to \tau^+\nu_\tau)$ by *BABAR* and Belle.

	BABAR	Belle
$\mathcal{R}(D)$	$0.440 \pm 0.058 \pm 0.042$	$0.375 \pm 0.064 \pm 0.026$
$\mathcal{R}(D^*)$	$0.332 \pm 0.024 \pm 0.018$	$0.302 \pm 0.030 \pm 0.011$
$\mathcal{B}(B^+ \to \tau^+\nu_\tau)$	$1.83^{+0.53}_{-0.49} \times 10^{-4}$	$(1.25 \pm 0.28) \times 10^{-4}$
$\mathcal{R}_\tau(D)$	$(2.404 \pm 0.838) \times 10^3$	$(3.0 \pm 1.1) \times 10^3$
$\mathcal{R}_\tau(D^*)$	$(1.814 \pm 0.5282) \times 10^3$	$(2.416 \pm 0.794) \times 10^3$

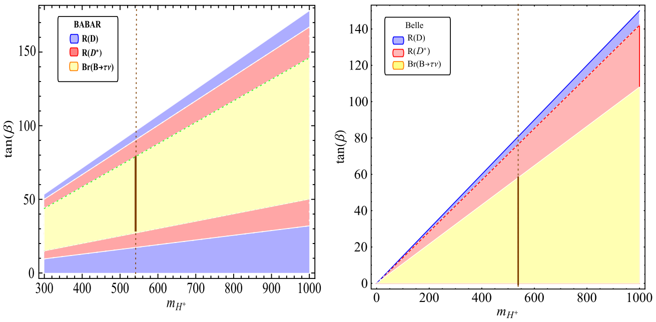

FIG. 1. For the 3-3-1 models, the allowed parameter spaces by *BABAR* (left) and Belle (right) data for $\mathcal{R}(D)$, $\mathcal{R}(D^*)$, and $\mathcal{B}(B^+ \to \tau^+\nu_\tau)$. The dotted vertical lines show $m_{H^+} = 540$ GeV.

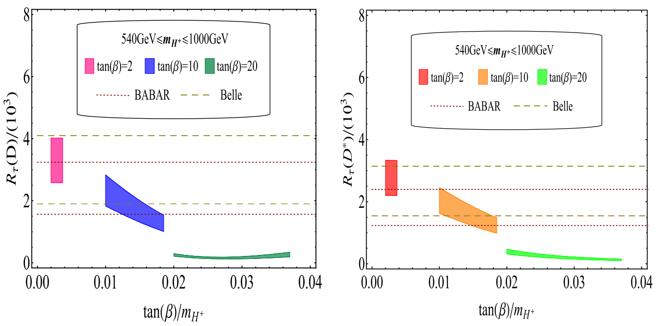

FIG. 2. Variations of $\mathcal{R}_\tau(D)$ (left) and $\mathcal{R}_\tau(D^*)$ (right) with the parameter $r = \tan\beta/m_{H^+}$ for three dispersive values of $\tan\beta$. The 1σ experimental ranges given by *BABAR* and Belle are shown as dotted and dashed horizontal lines.

3-3-1 models might give reasonable explanation to the $\mathcal{R}_\tau(X_c)$ anomaly.

In order to see the effect of the model parameter m_{H^+} on the ratio $\mathcal{R}(X_c)$, we plot $\mathcal{R}(X_c)$ as a function of m_{H^+} for four dispersive values of the parameter $\tan\beta$ in Fig. 4. The colorized thick lines actually denote the corresponding range of values, and the violet dot-dashed lines denote the experimental range $\mathcal{R}(X_c)_{\rm Exp}$, which is given by

$$\mathcal{R}(X_c)_{\rm Exp} = \frac{\mathcal{B}(b \to X_c\tau\bar{\nu})_{\rm LEP}}{\mathcal{B}(B \to X_c e\bar{\nu})_{\rm World\,Avg}},$$

where $\mathcal{B}(b \to X_c\tau\bar{\nu})_{\rm LEP} = (2.41 \pm 0.23)\%$ and $\mathcal{B}(B \to X_c e\bar{\nu})_{\rm World\,Avg} = (10.92 \pm 0.16)\%$. And we obtain

TABLE II. The SM prediction and experimental measurement values for $\mathcal{R}(X_c)$ and $\mathcal{R}_\tau(X_c)$.

	$\mathcal{R}(X_c)$	$\mathcal{B}(B^+ \to \tau^+\nu_\tau)$ ($\times 10^{-4}$)	$\mathcal{R}_\tau(X_c)$ ($\times 10^3$)
SM	0.225 ± 0.006	0.947 ± 0.182	$2.060^{+0.087}_{-0.083}$
BABAR	0.221 ± 0.021	$1.83^{+0.53}_{-0.49}$	$1.208^{+0.598}_{-0.361}$
Belle	0.221 ± 0.021	1.25 ± 0.28	$1.768^{+0.727}_{-0.461}$

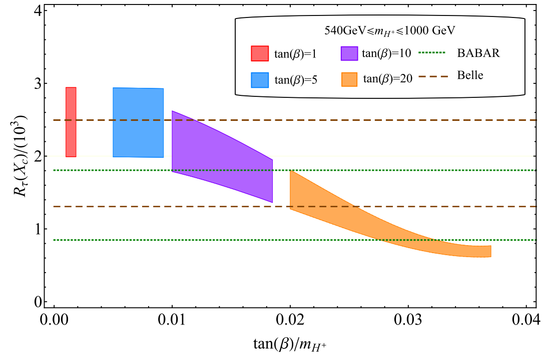

FIG. 3. Variations of $\mathcal{R}_\tau(X_c)$ with the parameter $r = \tan\beta/m_{H^+}$ for four typical values of $\tan\beta$. The 1σ experimental ranges are shown by the dotted (*BABAR*) and dashed (Belle) horizontal lines.

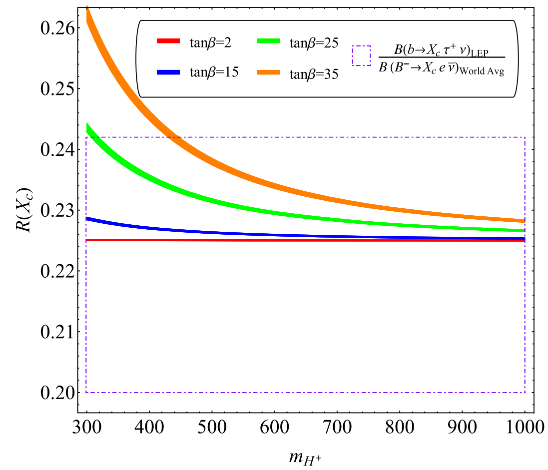

FIG. 4. Variations of $\mathcal{R}(X_c)$ with the mass parameter m_{H^+} for four typical $\tan\beta$ values. The experimental range is shown by violet dot-dashed lines.

$\mathcal{R}(X_c)_{\rm Exp} = 0.22^{+0.025}_{-0.023}$. One can see from Fig. 4 that $\mathcal{R}^{331}(X_c)$ is sensitive to both the parameters $\tan\beta$ and m_{H^+}. For $0 \le \tan\beta \le 35$ and $540\ \text{GeV} \le m_{H^+} \le 1000$ GeV, the value of $\mathcal{R}(X_c)$ is in the range 0.225∼0.2366, where the value 0.225 corresponds to the SM prediction.

C. $\mathcal{R}(\pi)$ and $\mathcal{R}_\tau^\pi$

In this subsection, we calculate the contributions of the charged Higgs bosons $H^\pm$ from the 3-3-1 models to the

TABLE III. The values of $\mathcal{R}(\pi)$ and $\mathcal{R}_\tau^\pi$ coming from the SM prediction and the upper limits based on the current experimental average.

	$\mathcal{R}(\pi)$	$\mathcal{R}_\tau^\pi$
SM	0.641 ± 0.016	0.733 ± 0.144
Exp.	<1.784	<2.62

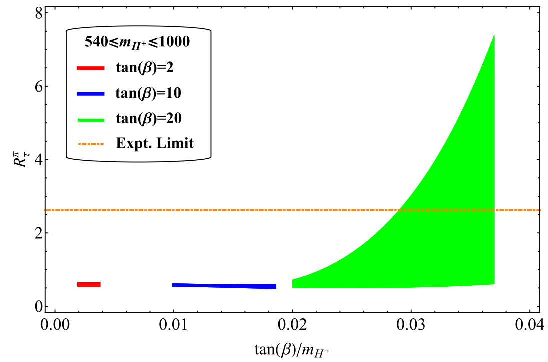

FIG. 5. Variations of $\mathcal{R}_\tau^\pi$ as a function of the parameter $r = \tan\beta/m_{H^+}$ for three dispersive values of $\tan\beta$ and $540 \le m_{H^+} \le 1000$. The experimental upper limit is shown by the orange dot-dashed horizontal line.

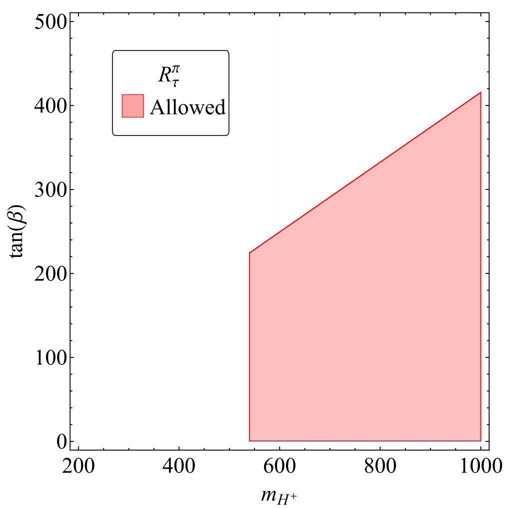

FIG. 6. Allowed parameter space for $\tan\beta$ and m_{H^+} obtained from the analysis of $\mathcal{R}_\tau^\pi$ in the 3-3-1 models.

semileptonic decay channels of the B meson, $B \to \pi\tau\nu_\tau$. The previous research of this mode [40,41] was with the SM leptons μ and e to extract the CKM element V_{ub} [32]. The observable is defined as

$$\mathcal{R}(\pi) = \frac{\mathcal{B}(B \to \pi\tau\bar{\nu}_\tau))}{\mathcal{B}(B \to \pi l\bar{\nu}_l)}\bigg|_{l\in\{e,\mu\}},$$

which is potentially sensitive to NP, where the dependence and the uncertainty due to V_{ub} are canceled. In order to keep the dependence on V_{ub} reserved, there is another kind of useful definition [21],

$$\mathcal{R}_\tau^\pi = \frac{\mathcal{B}(B \to \pi\tau\bar{\nu}_\tau)}{\mathcal{B}(B \to \tau\bar{\nu}_\tau)},$$

which makes minor adjustments to the form of Eq. (1). We summarize the present relevant data in Table III [21,40]. One can see from Table III that there are big deviations between the experimental data and the SM predictions. The method we use in this subsection is the same as what we used when we studied $\mathcal{R}(D^{(*)})$ and $\mathcal{R}_\tau(D^{(*)})$.

In Fig. 5, we plot $\mathcal{R}_\tau^\pi$ as a function of the parameter $r = \tan\beta/m_{H^+}$. We note that for $0 < \tan\beta \lesssim 20$ and $540\ \text{GeV} \le m_{H^+} \le 1000\ \text{GeV}$, the 3-3-1 models might give an explanation to the $\mathcal{R}_\tau^\pi$ anomaly. We also plot the allowed parameter space for $\tan\beta$ and m_{H^+} obtained from the analysis of $\mathcal{R}_\tau^\pi$ in Fig. 6.

IV. CONCLUSIONS

The so-called $\mathcal{R}(D^{(*)})$ anomalies have recently drawn wide attention. Many works have been done, some of which were within model-independent frameworks, while others were in NP models. They indicate that NP beyond the SM might give reasonable explanations for the $\mathcal{R}(D^{(*)})$ anomalies and analogous anomalies of other B-meson semileptonic decays.

The 3-3-1 models predict the charged Higgs bosons $H^\pm$, which can couple to the SM particles. Therefore, we have the motivation to study the charged Higgs bosons from the 3-3-1 models to the $\mathcal{R}(D^{(*)})$ anomalies and other analogous anomalies of the B-meson semileptonic decays. Furthermore, to make the study more comprehensive, we also calculate the contributions from the 3-3-1 models to two useful B-meson semileptonic decays, $B \to X_c\tau\nu_\tau$ and $B \to \pi\tau\nu_\tau$.

First, we calculate the contributions of the charged Higgs bosons $H^\pm$ to the $\mathcal{R}_{(\tau)}(D^{(*)})$ anomalies. Our numerical results show that there are large common allowed parameter spaces for both *BABAR* and Belle data. The biggest constraint on the 3-3-1 models' parameter space comes from $\mathcal{B}(B^+ \to \tau^+\nu_\tau)$ data. In the common spaces, the reasonable values of $\tan\beta$ are in the ranges of $27 \sim 145$ and $0 \sim 108$ for *BABAR* and Belle data, respectively. For $0 < \tan\beta \le 10$ and $540\ \text{GeV} \le m_{H^+} \le 1000\ \text{GeV}$, the 3-3-1 models might give a reasonable explanation for the $\mathcal{R}_\tau^{331}(D^{(*)})$ anomalies.

Then, we calculate the contributions to the $\mathcal{R}_{(\tau)}(X_c)$ anomalies. Our numerical results show that $\mathcal{R}_{(\tau)}(X_c)$ are sensitive to both the parameters $\tan\beta$ and m_{H^+}. The 3-3-1 models might give a reasonable explanation for the $\mathcal{R}_{(\tau)}(X_c)$ anomalies in the model parameter range of $0 < \tan\beta \lesssim 20$.

Finally, we investigate the contributions to the $\mathcal{R}(\pi)$ and $\mathcal{R}_\tau^\pi$ anomalies. Our numerical results show that for $0 < \tan\beta \lesssim 20$, the 3-3-1 models might give a reasonable explanation for the $\mathcal{R}(\pi)$ and $\mathcal{R}_\tau^\pi$ anomalies.

Combining the observations and our numerical results, we draw the following conclusions: In a wide range of parameter space, the 3-3-1 models may give reasonable explanations for the $\mathcal{R}(D^{(*)})$ anomalies and other analogous anomalies of B-meson semileptonic decays. With the measurement of $B \to \tau\nu$ and refined measurements of observables in $B \to D\tau\nu$ in different experiments, the study of $\mathcal{R}(D^{(*)})$ anomalies will be an effective solution for us to probe both the SM and NP models.

ACKNOWLEDGMENTS

This work was supported in part by the National Natural Science Foundation of China under Grant No. 11275088, and the Natural Science Foundation of the Liaoning Scientific Committee (No. 2014020151).

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