$\mathcal{O}(\theta)$ Feynman rules for quadrilinear gauge boson couplings in the noncommutative standard model

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(Received 18 September 2016; published 28 February 2017)

We examine the electroweak gauge sector of the noncommutative standard model and, in particular, obtain the $\mathcal{O}(\theta)$ Feynman rules for all quadrilinear gauge boson couplings. Surprisingly, an electroweak-chromodynamics mixing appears in the gauge sector of the noncommutative standard model, where the photon as well as the neutral weak boson is coupled directly to three gluons. The phenomenological perspectives of the model in $W^-W^+ \to ZZ$ scattering are studied and it is shown that there is a characteristic oscillatory behavior in azimuthal distribution of scattering cross sections that can be interpreted as a direct signal of the noncommutative standard model. Assuming the integrated luminosity 100 fb^{-1}, the number of $W^-W^+ \to ZZ$ subprocesses are estimated for some values of noncommutative scale $\Lambda_{\rm NC}$ at different center of mass energies and the results are compared with predictions of the standard model.

DOI: 10.1103/PhysRevD.95.035034

I. INTRODUCTION

Over the last 15 years, there has been increasing interest in studying the noncommutative standard model as a candidate for beyond the Glashaw, Weinberg, and Salam model of particle physics [1–8]. This is partially because of the modern foundations of string theory, where in its context it was shown that noncommutative models occur in the description of low energy excitations of open strings in the presence of a constant background B-field [9]. On the other hand, noncommutative theories are of interest on their own as a nontrivial generalization of ordinary gauge theories on a deformed background, which is defined by commutation relations $[x^\mu, x^\nu] = i\theta^{\mu\nu}$, where x^μ denotes the spacetime 4-vector and $\theta^{\mu\nu}$ is a constant, real, and antisymmetric matrix of dimensions GeV^{-2} [10]. It is generally believed that signatures of noncommutative spacetime can be observed at string scale, typically on the order of Planck distance, where the quantum effects of gravitational fields become significant. Although the Planck scale (10^{19} GeV) is actually far from our direct access, assuming the possibility of the existence of large extra dimensions and given that the onset of string effects is at TeV scale, signatures of the noncommutative background are expected to be observable at a few TeV [11,12]. Today, there is a positive attitude, both experimentally and theoretically, about a new physics at TeV scale and intense experimental efforts [13,14], phenomenological studies [15,16], as well as many independent model buildings [17,18] are currently underway to find signs of the new physics beyond the standard model. Noncommutative extension of the standard model appears to be a suitable candidate for the new physics and it may finally be realized by nature at the TeV domain of energy. Nevertheless, the situation is uncertain and some alternative scenarios with compatible success, such as supersymmetry models [19,20] and D-branes [21,22], also have been suggested, all awaiting experimental confirmation.

Noncommutative field theories can be constructed by the Moyal-Weyl correspondence, where the usual product of functions is promoted to an associative star product that is defined as [23,24]

$$f(x) \star g(x) = f(x) \exp\left(\frac{i}{2}\theta^{\mu\nu}\frac{\overleftarrow{\partial}}{\partial x^\mu}\frac{\overrightarrow{\partial}}{\partial y^\nu}\right) g(y)|_{y=x}. \quad (1)$$

There are, however, two serious problems in constructing the noncommutative standard model based on this approach. The first and probably the most important difficulty in the Moyal-Weyl correspondence is the problem of charge quantization. That is, the possible charges for the matter fields are automatically restricted to the values $-1, 0, +1$. Secondly, it turns out that in the non-Abelian case, only noncommutative models with U(N) gauge symmetry are allowed in this approach [25,26]. An idea to resolve these problems was proposed by Chaichian *et al.* [27]. They built up a noncommutative $\mathrm{U}(3) \otimes \mathrm{U}(2) \otimes \mathrm{U}(1)$ gauge theory and then reduced it to the noncommutative $\mathrm{SU}(3) \otimes \mathrm{SU}(2) \otimes \mathrm{U}(1)$ model by breaking the original symmetry of the theory in an appropriate manner. The model, however, introduces some extra bosons (three vector and one scalar) in comparison with the standard model. An alternative solution that cures both the problems and at the same time preserves the particle content of the standard model is to use Seiberg-Witten maps for noncommutative gauge field $\hat{A}_\mu$ and the corresponding gauge transformation parameter $\hat{\Lambda}$ [9]. Under such a construction noncommutative objects are written as an infinite series on deformation quantity $\theta^{\mu\nu}$, which then, up to an arbitrary order in $\theta^{\mu\nu}$, can be expressed in terms of usual (commutative) fields and gauge parameters. Contrary to

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ordinary field theories because of the presence of the $\star$-product the commutation relations of noncommutative gauge fields as well as the gauge parameters do not close to the Lie algebra of the symmetry group. This problem can be circumvented by constructing noncommutative models based on the enveloping algebra of the gauge group. This idea was proposed by Jurčo *et al.* [28,29] and used to extend the Siberg-Witten maps to non-Abelian gauge fields as well as the gauges coupled to matter fields. Along these lines, Calmet *et al.* [30] introduced the minimal noncommutative standard model (mNCSM) and later developed it in the nonminimal extension (nmNCSM) (according to the freedom in choice of traces in the gauge sector) of the model [31,32]. The Seiberg-Witten construction by Jurčo and collaborators. [28] also has found applications in relation to gravitation and topology [33,34]. Recently, some of the geometric and topological implications of noncommutative Wilson loops have been studied in Ref. [35].

Noncommutative models have a rich phenomenological content and many interesting features. In particular, the noncommutative standard model introduces new interactions that are forbidden in the standard model. Such interactions can be used to test the model through rare events (see for example [36–38]) and may lead to a distinct phenomenology. Another remarkable feature of the model is that there are contributions from the Higgs part of the noncommutative action that enter directly into the pure gauge sector of the theory and can affect the electroweak gauge boson interactions. The Feynman rules for trilinear gauge boson couplings including contributions from the Higgs sector for both the minimal and nonminimal models have already been obtained [31]. Recently, the rules for the Higgs couplings with gauge bosons have also been completed [39]. Here, we obtain the $\mathcal{O}(\theta)$ Feynman rules for quadrilinear gauge boson couplings (QGCs) in both the minimal and nonminimal noncommutative standard model.

This paper is organized as follows. In Sec. II, we review briefly the minimum required basis of the noncommutative standard model. In particular, we emphasize the gauge and Higgs sectors of the model and identify the relevant interactions for QGCs. In Sec. III, we obtain the Feynman rules for all QGCs including the anomalous couplings of photon and the weak boson Z^0 to three gluons. In Sec. IV, we study phenomenological perspectives of the model in $W^-W^+ \to ZZ$ scattering and in Sec. V summarize the paper and outline the concluding remarks. We use a notation close to the original paper by Melić *et al.* [31] to make the next review section short and the results readily applicable for phenomenological studies.

II. SEIBERG-WITTEN MAPS AND THE NONCOMMUTATIVE STANDARD MODEL

To begin, let us recall that the action of the noncommutative standard model can be easily built up from the action of the standard model by replacing the normal products between fields with $\star$ ones, and the fields by their corresponding Seiberg-Witten maps. For the fermion field ψ and an arbitrary gauge field V_μ, up to the first order of deformation parameter $\theta^{\mu\nu}$ this means [28,30]

$$\psi \to \hat{\psi}[\psi, V] = \phi - \frac{1}{2}\theta^{\rho\sigma}V_\sigma\partial_\rho\phi + \frac{i}{8}\theta^{\rho\sigma}[V_\rho, V_\sigma], \quad (2)$$

$$V_\mu \to \hat{V}_\mu[V] = V_\mu + \frac{1}{4}\theta^{\rho\sigma}\{\partial_\rho V_\sigma + F_{\rho\sigma}, V_\sigma\}. \quad (3)$$

A hat on letters indicates the noncommutative objects. The bracket $\{,\}$ denotes the anticommutator of operators and $F_{\mu\nu}$ is the usual field strength tensor. The noncommutative field tensor is defined as $\hat{F}_{\mu\nu} = \partial_\mu\hat{V}_\nu - \partial_\nu\hat{V}_\mu - ig[\hat{V}_\mu, \hat{V}_\nu]_\star$ and $\star$-commutator means $\hat{V}_\mu \star \hat{V}_\nu - \hat{V}_\nu \star \hat{V}_\mu$. In order to construct the noncommutative standard model one can choose the gauge potential $V_\mu = g'\mathcal{A}_\mu Y + g\sum_{a=1}^{3} B_\mu^a T_{\rm L}^a + g_{\rm s}\sum_{b=1}^{8} G_\mu^b T_{\rm S}^b$, where $\mathcal{A}_\mu$, B_μ^a, and G_μ^b represent the fields associated respectively to $\mathrm{U_Y}(1)$, $\mathrm{SU_L}(2)$, and $\mathrm{SU_C}(3)$ gauge groups with corresponding coupling constants g', g, $g_{\rm s}$. Also, Y, $T_{\rm L}^a$, and $T_{\rm S}^b$ are generators of the relevant structure groups.

On the other hand, the noncommutative Higgs field $\hat{\Phi}$ is given by the hybrid Seiberg-Witten map as

$$\Phi \to \hat{\Phi}[\Phi, V, V'] = \Phi + \frac{1}{2}\theta^{\rho\sigma}V_\sigma\left[\partial_\rho\Phi - \frac{i}{2}(V_\rho\Phi - \Phi V'_\rho)\right] + \frac{1}{2}\theta^{\rho\sigma}\left[\partial_\rho\Phi - \frac{i}{2}(V_\rho\Phi - \Phi V'_\rho)\right]V'_\sigma. \quad (4)$$

Observe that the noncommutative Higgs field can be transformed under two different gauge groups on the left and the right corresponding respectively to gauge potentials V_μ and V'_μ [31]. The action of the noncommutative standard model can be formally written as

$$S_{\rm NCSM} = S_{\rm ferimion} + S_{\rm gauge} + S_{\rm Higgs} + S_{\rm Yukawa}. \quad (5)$$

The relevant expressions for each part of the above action have been obtained in [31]. For our purposes, it suffices to rewrite only the gauge and Higgs parts in detail.

A. Gauge sector

The gauge action is [31,32]

$$S_{\rm g} = -\frac{1}{2}\int d^4x \mathrm{Tr}\frac{1}{\mathbf{G^2}}\hat{F}_{\mu\nu} \star \hat{F}^{\mu\nu},$$
$$\frac{1}{g_I^2} = \mathrm{Tr}\frac{1}{\mathbf{G^2}}T_I^a T_I^a, \quad (6)$$

where, g_I's are usual (commutative) coupling constants g', g, $g_{\rm s}$. Here, the trace is over all the unitary and irreducible representations of the symmetry group and $\mathbf{G}$ is an operator that commutes with generators of the gauge group and determines the coupling constants of the model. It is in general a function of Y and Casimir operators of $\mathrm{SU_L}(2)$ and $\mathrm{SU_C}(3)$. Because the noncommutative fields are valued in the enveloping algebra of the gauge group, the trace in Eq. (6) is not unique and depends strongly on the choice of a representation for gauge fields [30,31]. All the representations that appear in the standard model are important and must be considered. Using the Seiberg-Witten map (3) and the $\star$-product prescription in (1) up to the first order in $\theta^{\mu\nu}$ we can rewrite the gauge action as

$$S_{\rm g} = -\frac{1}{2}\int d^4x \mathrm{Tr}\frac{1}{\mathbf{G^2}}F_{\mu\nu}F^{\mu\nu} + \theta^{\rho\sigma}\int d^4x \mathrm{Tr}\frac{1}{\mathbf{G^2}}\left(\frac{1}{4}F_{\rho\sigma}F_{\mu\nu} - F_{\rho\mu}F_{\sigma\nu}\right)F^{\mu\nu}. \quad (7)$$

1. Minimal noncommutative standard model

The simplest choice for the representation of gauge fields is the adjoint representation. In this case the trace is taken independently over generators of the symmetry groups, i.e., respectively over Y, $T^a_{\rm L}$, and $T^b_{\rm S}$. In this case the resulting (gauge) action remains as close as possible to that of the standard model. By substitution of the gauge potential V_μ in (7) and rearranging the fields we get

$$S^{\rm m}_{\rm g} = -\frac{1}{2}\int d^4x\left(\frac{1}{2}\mathcal{A}_{\mu\nu}\mathcal{A}^{\mu\nu} + B^a_{\mu\nu}B^{\mu\nu,a} + G^a_{\mu\nu}G^{\mu\nu,a}\right) + \frac{1}{4}g_{\rm s}d^{abc}\theta^{\rho\sigma}\int d^4x\left(\frac{1}{4}G^a_{\rho\sigma}G^b_{\mu\nu} - G^a_{\rho\mu}G^b_{\sigma\nu}\right)G^{\mu\nu,c}, \quad (8)$$

where

$$\mathcal{A}_{\mu\nu} = \partial_\mu\mathcal{A}_\nu - \partial_\nu\mathcal{A}_\mu, \quad (9a)$$

$$B^a_{\mu\nu} = \partial_\mu B^a_\nu - \partial_\nu B^a_\mu + g\epsilon^{abc}B^b_\mu B^c_\nu, \quad (9b)$$

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_{\rm s}f^{abc}G^b_\mu G^c_\nu. \quad (9c)$$

The $\mathcal{A}_\mu$ and B^a_μ fields can be expressed in terms of physical fields as usual using

$$B^1_\mu = \frac{W^+_\mu + W^-_\mu}{\sqrt{2}}, \qquad B^2_\mu = i\frac{W^+_\mu - W^-_\mu}{\sqrt{2}}, \quad (10a)$$

$$\mathcal{A}_\mu = \cos\theta_w A_\mu - \sin\theta_w Z_\mu, \quad (10b)$$

$$B^3_\mu = \sin\theta_w A_\mu + \cos\theta_w B_\mu. \quad (10c)$$

Here, A_μ is the photon field, Z_μ and $W^\pm_\mu$ are weak boson fields, and θ_w stands for the weak mixing angle. From Eq. (8) it follows that in the minimal noncommutative model and at leading order of $\theta^{\mu\nu}$, the electroweak part of the gauge action is the same as that of the standard model. The QCD sector, however, differs from its corresponding action in the standard model and has already been discussed in [32].

By substitution of field tensors (9b) and (9c) in (8) we can isolate the relevant parts of the gauge action to (electroweak) QGCs as

$$-\frac{1}{2}g^2\int d^4x\epsilon^{abc}\epsilon^{ab'c'}B^b_\mu B^c_\nu B^{\mu,b'}B^{\nu,c'}, \quad (11)$$

which then using (10b) and (10c) can be written as

$$\sim g^2\int d^4x(W^+_\mu W^{-\mu}W^+_\nu W^{-\nu} + \cdots + \sin^2\theta_w W^+_\mu W^{-\mu}A_\nu A^\nu + \cdots + \cos^2\theta_w W^+_\mu W^{-\mu}Z_\nu Z^\nu + \cdots + \sin\theta_w\cos\theta_w W^+_\mu W^{-\mu}A_\nu Z^\nu + \cdots), \quad (12)$$

where each line represents a typical number of interaction terms that differ from each other in indices and up to a numerical factor. From these interactions we obtain the gauge part of Feynman rules for $W^-W^+W^-W^+$, $W^-W^+\gamma\gamma$, W^-W^+ZZ, and $W^-W^+Z\gamma$ couplings in the context of the minimal model. Notice that because the minimal extension of the standard model leaves the electroweak gauge action invariant, these expressions are the same as the rules in the standard model. The rules are given in Sec. III.

2. Nonminimal noncommutative standard model

In the nonminimal model, the trace in (6) is chosen over all particles existent in the model (with different quantum numbers), which have covariant derivatives acting on them. In the standard model, there are five multiples of fermions for each generation and one Higgs multiplet (see Table I in [30,31]). The nonminimal gauge action up to the linear order in $\theta^{\mu\nu}$ is

$$S^{\rm nm}_{\rm g} = S^{\rm m}_{\rm g} + g'^3k_1\theta^{\rho\sigma}\int d^4x\left(\frac{1}{4}\mathcal{A}_{\rho\sigma}\mathcal{A}_{\mu\nu} - \mathcal{A}_{\mu\rho}\mathcal{A}_{\nu\sigma}\right)\mathcal{A}^{\mu\nu} + g'g^2k_2\theta^{\rho\sigma}\int d^4x\left[\left(\frac{1}{4}\mathcal{A}_{\rho\sigma}B^a_{\mu\nu} - \mathcal{A}_{\mu\rho}B^a_{\nu\sigma}\right)B^{\mu\nu,a} + \text{cyclic permutation of fields}\right] + g'g^2_{\rm s}k_3\theta^{\rho\sigma}\int d^4x\left[\left(\frac{1}{4}\mathcal{A}_{\rho\sigma}G^a_{\mu\nu} - \mathcal{A}_{\mu\rho}G^a_{\nu\sigma}\right)G^{\mu\nu,a} + \text{cyclic permutation of fields}\right]. \quad (13)$$

The constants k_i, $i = 1, 2, 3$ are model parameters that by using a set of constraints can be determined in terms of coupling constants of the model [36,37,40].

The pure electroweak QGCs arise from the following interactions:

$$g' g^2 k_2 \int d^4x \epsilon^{abc} \theta^{\rho\sigma} \Big[\mathcal{A}_{\rho\sigma} \partial_\mu B_\nu^a B^{\mu,b} B^{\nu,c} + \mathcal{A}_{\mu\nu} \partial_\rho B_\sigma^a B^{\mu,b} B^{\nu,c} + \frac{1}{2} \mathcal{A}_{\mu\nu} \Big(B_\rho^b B_\sigma^c \partial^\mu B^{\nu,a} - \frac{1}{2} B_\rho^b B_\sigma^c \partial^\nu B^{\mu,a} \Big) - \mathcal{A}_{\mu\rho} (\partial_\nu B_\sigma^a B^{\mu,b} B^{\nu,c} - \partial_\sigma B_\nu^a B^{\mu,b} B^{\nu,c} + B_\nu^b B_\sigma^c \partial^\mu B^{\nu,a} - B_\nu^b B_\sigma^c \partial^\nu B^{\mu,a}) - \mathcal{A}^{\mu\nu} (\partial_\mu B_\rho^a B_\nu^b B_\sigma^c - \partial_\rho B_\mu^a B_\nu^b B_\sigma^c + B_\mu^b B_\rho^c \partial_\nu B_\sigma^a - B_\mu^b B_\rho^c \partial_\sigma B_\nu^a) - \mathcal{A}^{\nu\sigma} (\partial_\mu B_\rho^a B^{\mu,b} B^{\nu,c} - \partial_\rho B_\mu^a B^{\mu,b} B^{\nu,c} + B_\mu^b B_\rho^c \partial^\mu B^{\nu,a} - B_\mu^b B_\rho^c \partial^\nu B^{\mu,a}) \Big]. \tag{14}$$

By inserting (10b) and (10c) in (14) we find

$$\sim g' g^2 k_2 \int d^4x \theta^{\rho\sigma} (\cos^2\theta_w \partial_\rho A_\sigma \partial_\mu W_\nu^- W^{+\mu} Z^\nu + \cdots + \sin\theta_w \cos\theta_w \partial_\rho A_\sigma \partial_\mu W_\nu^+ W^{-\mu} A^\nu + \cdots + \sin^2\theta_w \partial_\rho Z_\sigma \partial_\mu W_\nu^+ W^{-\mu} Z^\nu + \cdots. \tag{15}$$

An important point to be noted here is that the $W^- W^+ W^- W^+$ coupling is not affected by interactions in the gauge sector of the nonminimal noncommutative model because there is no interaction term containing four charged boson fields in (15).

On the other hand, in addition to pure elctroweak gauge couplings, because of the interactions involved in (13) there is also an electroweak-chromodynamics mixing in the gauge sector of the nonminimal model that arises from interactions in the last two lines of (13). They are

$$g' g_s^3 f^{abc} \int d^4x \theta^{\rho\sigma} [\mathcal{A}_{\rho\sigma} \partial_\mu G_\nu^a G^{\mu,b} G^{\nu,c} + \mathcal{A}_{\mu\rho} (\partial_\nu G_\sigma^a G^{\mu,b} G^{\nu,c} - \partial_\sigma G_\nu^a G^{\mu,b} G^{\nu,c} + \partial^\mu G^{\nu,a} G_\nu^b G_\sigma^c - \partial^\nu G^{\mu,a} G_\nu^b G_\sigma^c) + \mathcal{A}_{\mu\nu} (\partial^\mu G_\rho^a G^{\nu,b} G_\sigma^c - \partial_\rho G^{\mu,a} G^{\nu,b} G_\sigma^c + \partial^\nu G_\sigma^a G^{\mu,b} G_\rho^c - \partial_\nu G^{\sigma,a} G^{\mu,b} G_\rho^c) + \mathcal{A}_{\nu\sigma} (\partial_\mu G_\rho^a G^{\mu,b} G^{\nu,c} - \partial_\rho G_\mu^a G^{\mu,b} G^{\nu,c} + \partial^\mu G^{\nu,a} G_\mu^b G_\rho^c - \partial^\nu G^{\mu,a} G_\mu^b G_\rho^c)]. \tag{16}$$

Here a, b, c run from 1 to 8 for gluon fields G_μ. Proceeding as before, the relevant interactions for the coupling of photon to gluons are

$$\sim g' g_s^3 f^{abc} \int d^4x \theta^{\rho\sigma} [\cos\theta_w \partial_\rho A_\sigma \partial_\mu G_\nu^a G^{\mu,b} G^{\nu,c} + \cos\theta_w (\partial_\mu A_\rho \partial_\mu G_\sigma^a G^{\mu,b} G^{\nu,c} + \cdots)]. \tag{17}$$

Also, for Z^0 coupling to gluons we obtain

$$\sim g' g_s^3 f^{abc} \int d^4x \theta^{\rho\sigma} [\cos\theta_w \partial_\rho A_\sigma \partial_\mu G_\nu^a G^{\mu,b} G^{\nu,c} + \cos\theta_w (\partial_\mu A_\rho \partial_\mu G_\sigma^a G^{\mu,b} G^{\nu,c} + \cdots)]. \tag{18}$$

B. Higgs sector

The Higgs part of the noncommutative action is

$$S_{\text{Higgs}} = \int d^4x [(\hat{D}_\mu \hat{\Phi})^\dagger \star (\hat{D}^\mu \hat{\Phi}) - \mu^2 \hat{\Phi}^\dagger \star \hat{\Phi} - \lambda \hat{\Phi}^\dagger \star \hat{\Phi} \star \hat{\Phi}^\dagger \star \hat{\Phi}]. \tag{19}$$

Here, μ and λ are respectively the mass parameter and coupling constant. Also, $\hat{D}_\mu$ denotes the covariant derivative, which is defined for the noncommutative Higgs field as $\hat{D}_\mu = \partial_\mu \hat{\Phi} - i\hat{V}_\mu \star \hat{\Phi} + i\hat{\Phi} \star \hat{V}'_\mu$. The expansion of (19) using (1) and (4) yields

$$S_{\text{Higgs}} = \int d^4x [(\mathbf{D}_\mu \Phi)^\dagger (\mathbf{D}^\mu \Phi) - \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2] + \frac{1}{2} \theta^{\mu\nu} \int d^4x \Phi^\dagger \Big[\mathbf{U}_{\mu\nu} + \mathbf{U}_{\mu\nu}^\dagger + \frac{1}{2} \mu^2 F_{\mu\nu} - 2i\lambda \Phi (\mathbf{D}_\mu \Phi)^\dagger \mathbf{D}_\nu \Phi \Big] \Phi, \tag{20}$$

where $\mathbf{D}_\mu = \partial_\mu - i\mathbf{V}_\mu$ and the operator $\mathbf{U}_{\mu\nu}$ is

$$\mathbf{U}_{\mu\nu} = [\overleftarrow{\partial}^\varrho + i\mathbf{V}^\varrho] \Big[-\partial_\varrho \mathbf{V}_\mu \partial_\nu - \mathbf{V}_\mu \partial_\varrho \partial_\nu + \partial_\mu \mathbf{V}_\varrho \partial_\nu + i\mathbf{V}_\varrho \mathbf{V}_\mu \partial_\nu + \frac{i}{2} \mathbf{V}_\mu \mathbf{V}_\nu \partial_\varrho + \frac{i}{2} \partial_\varrho (\mathbf{V}_\mu \mathbf{V}_\nu) + \frac{1}{2} \mathbf{V}_\varrho \mathbf{V}_\mu \mathbf{V}_\nu + \frac{i}{2} \{\mathbf{V}_\mu, \partial_\nu \mathbf{V}_\varrho + \mathbf{F}_{\nu\varrho}\} \Big]. \tag{21}$$

Here, $\mathbf{V}_\mu$ is a 2×2 matrix that is defined as $\mathbf{V}_\mu = g' \mathcal{A}_\mu Y_\Phi \mathbf{1} + g B_\mu^a T_{\mathrm{L}}^a$. The explicit form of $\mathbf{V}_\mu$ is [31]

$$\mathbf{V}_\mu = \begin{pmatrix} eA_\mu + g\frac{(1-2\sin^2\theta_w)}{2\cos\theta_w} Z_\mu & \frac{g}{\sqrt{2}} W_\mu^+ \\ \frac{g}{\sqrt{2}} W_\mu^- & -\frac{g}{2\cos\theta_w} Z_\mu \end{pmatrix}. \tag{22}$$

Analysis of Eq. (20) reveals that the Higgs sector induces contributions into the pure gauge sector of the noncommutative standard model. Proceeding similarly as in [31] the interactions yielding to QGCs are those terms in (20) that contain multiplication of four $\mathbf{V}_\mu$ matrices, i.e.,

$$-\frac{1}{2} \theta^{\mu\nu} \int d^4x \Phi^\dagger (i\mathbf{V}_\varrho \mathbf{V}^\varrho \mathbf{V}_\mu \mathbf{V}_\nu + i\mathbf{V}_\varrho \mathbf{V}^\mu \mathbf{V}_\nu \mathbf{V}^\varrho - i\mathbf{V}_\varrho \mathbf{V}^\mu \mathbf{V}^\varrho \mathbf{V}_\nu + i\mathbf{V}_\mu \mathbf{V}_\nu \mathbf{V}_\varrho \mathbf{V}^\varrho - i\mathbf{V}_\mu \mathbf{V}_\varrho \mathbf{V}_\nu \mathbf{V}^\varrho + \text{Hermitian conjugate}) \Phi. \tag{23}$$

Using the explicit form of $\mathbf{V}_\mu$ and choosing the Higgs field to be in unitary gauge

$$\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ h(x)+v \end{pmatrix}, \qquad v = \sqrt{-\mu^2/\lambda} \tag{24}$$

after symmetry breaking, the Higgs-induced interactions into the pure gauge sector are found to be

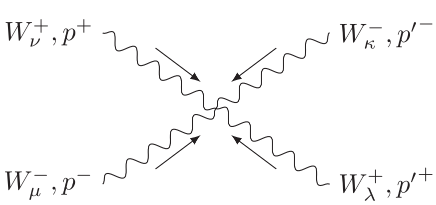
$$\begin{aligned} -i\frac{2M_W^2}{g}\theta^{\mu\nu}\int d^4x(&Z_\rho W^{-\rho}W_\mu^+ Z_\nu + \cdots \\ &+ W_\rho^- A^\rho A_\mu W_\nu^+ + \cdots \\ &+ W_\rho^- A^\rho Z_\mu W_\nu^+ + \cdots \\ &+ W_\rho^- W^{+\rho} W^{-\mu} W^{+\nu} + \cdots), \end{aligned} \tag{25}$$

where we have used $v = \frac{2M_W}{g}$ for later convenience. These interactions are of dimension 4 and hence momentum independent. Notice that the electroweak gauge action of the minimal noncommutative model is exactly the same as that of the standard model. In this case, only the Higgs-induced interactions can contribute to $\mathcal{O}(\theta)$ Feynman rules for electroweak QGCs.

III. FEYNMAN RULES FOR QGCS

The Feynman rule associated to a coupling diagram can be obtained straightforwardly by variation of the corresponding interactions. We used Eq. (12) to obtain the standard rules for $W^-W^+W^-W^+$, $W^-W^+\gamma\gamma$, W^-W^+ZZ, and $W^-W^+Z\gamma$. In the minimal model the effective interactions that contribute to $\mathcal{O}(\theta)$ Feynman rules arise from the Higgs-induced interactions, i.e., (25). In the nonminimal model, contrary to the minimal case, the effective interactions arise from the $\mathcal{O}(\theta)$ extension of the pure gauge action. We used Eq. (15) to get the rules of the nonminimal model for pure electroweak QGCs and Eqs. (17) and (18) to derive the associated vertex functions for γggg and $Zggg$ couplings. All momenta are assumed to be incoming into vertices. Here are the θ-expanded Feynman rules for all QGCs in the electroweak gauge sector of the noncommutative standard model.

(a) $W^-W^+W^-W^+$ coupling,



(i) Standard model

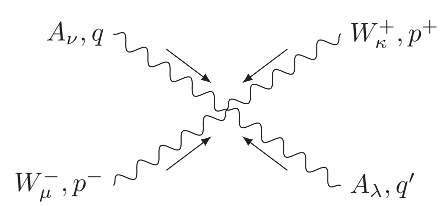
$$ig^2(2g_{\nu\lambda}g_{\mu\kappa} - g_{\nu\kappa}g_{\lambda\mu} - g_{\mu\nu}g_{\kappa\lambda}), \tag{26}$$

(ii) minimal/nonminimal model

$$\begin{aligned} \text{Eq. (26)} - \frac{3}{4}M_W^2 g^2(&\theta_{\kappa\lambda}g_{\mu\nu} + \theta_{\kappa\nu}g_{\mu\lambda} \\ &+ \theta_{\mu\lambda}g_{\nu\kappa} + \theta_{\mu\nu}g_{\kappa\lambda}). \end{aligned} \tag{27}$$

Equation (26) is the standard Feynman rule for the $W^-W^+W^-W^+$ coupling. It is derived from the gauge action of the standard model and is symmetric under substitutions $\mu \rightleftarrows \kappa$ and independently under $\nu \rightleftarrows \lambda$. It is also symmetric under simultaneous substitutions of $\mu \rightleftarrows \kappa$ and $\nu \rightleftarrows \lambda$. These symmetries can be inferred from the above coupling diagram because exchanging two W^- bosons with each other does not change the physical situation. A similar argument can be made also for exchange of W^+ bosons. The θ-dependent part of the rule (27) results from the Higgs-induced interactions (25) in the minimal case. It satisfies explicitly the symmetries of Eq. (26). As we mentioned earlier, the gauge sector of the nonminimal model does not contribute to the $W^-W^+W^-W^+$ coupling and expression (27) is the θ-expanded Feynman rule for both the minimal and nonminimal models.

(b) $W^-W^+\gamma\gamma$ coupling,



(i) Standard model

$$-ie^2(2g_{\nu\lambda}g_{\mu\kappa} - g_{\nu\kappa}g_{\lambda\mu} - g_{\mu\nu}g_{\kappa\lambda}), \tag{28}$$

(ii) minimal model

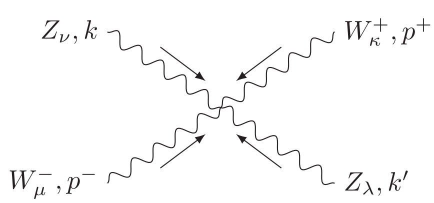
$$\text{Eq. (28)} - 2M_W^2 e^2(\theta_{\kappa\lambda}g_{\mu\nu} - \theta_{\nu\kappa}g_{\mu\lambda}), \tag{29}$$

(iii) nonminimal model

$$
\begin{aligned}
&\text{Eq. (29)} + g' g^2 k_2 \sin\theta_w \cos\theta_w (\{(-2\theta_{\alpha\rho} g_{\mu\kappa} g_{\nu\lambda} + 2\theta_{\alpha\rho} g_{\kappa\lambda} g_{\mu\nu}) q'_\rho \\
&+ [(-\theta_{\sigma\alpha} p^+_\sigma + \theta_{\sigma\alpha} p^-_\sigma) g_{\mu\lambda} + 2\theta_{\mu\alpha} q'_\lambda + 2\theta_{\alpha\lambda} q'_\mu - \theta_{\mu\alpha} p^-_\lambda - \theta_{\alpha\lambda} p^+_\mu \\
&+ (-p^+_\alpha - 2q'_\alpha - p^-_\alpha)\theta_{\mu\lambda}] g_{\nu\kappa} + [\theta_{\sigma\alpha} p^+_\sigma g_{\mu\nu} - 2\theta_{\alpha\nu} q'_\mu - 2\theta_{\mu\alpha} q'_\nu + \theta_{\alpha\nu} p^+_\mu + \theta_{\mu\alpha} p^+{}_\nu \\
&+ (2q'_\alpha - p^+_\alpha)\theta_{\mu\nu}] g_{\kappa\lambda} + [-\theta_{\sigma\alpha} p^-_\sigma g_{\nu\lambda} + 2\theta_{\alpha\nu} q'_\lambda - 2\theta_{\alpha\lambda} q'_\nu + \theta_{\alpha\lambda} p^-_\nu - \theta_{\alpha\nu} p^-_\lambda \\
&+ (-p^-{}_\alpha + 2q'_\alpha)\theta_{\nu\lambda}] g_{\mu\kappa} + [-\theta_{\alpha\kappa} p^-_\nu + \theta_{\alpha\nu} p^-_\kappa + \theta_{\alpha\kappa} p^+_\nu - \theta_{\alpha\nu} p^+_\kappa \\
&+ (p^-_\alpha - p^+_\alpha)\theta_{\nu\kappa}] g_{\mu\lambda} + [2\theta_{\alpha\kappa} q'_\mu + \theta_{\mu\alpha} p^-_\kappa - \theta_{\alpha\kappa} p^+_\mu - \theta_{\mu\alpha} p^+_\kappa \\
&+ (p^+_\alpha + p^-_\alpha + 2q'_\alpha)\theta_{\mu\kappa}] g_{\nu\lambda} + [-2\theta_{\alpha\kappa} q'_\lambda - \theta_{\alpha\lambda} p^-{}_\kappa + \theta_{\alpha\kappa} p^-_\lambda + \theta_{\alpha\lambda} p^+_\kappa \\
&+ (p^+_\alpha + p^-_\alpha + 2q'_\alpha)\theta_{\kappa\lambda}] g_{\mu\nu}\} q_\alpha + \{[(-\theta_{\sigma\rho} p^+_\sigma + \theta_{\sigma\rho} p^-_\sigma) g_{\mu\lambda} - \theta_{\mu\rho} p^-_\lambda - \theta_{\mu\lambda} p^-_\rho \\
&+ 2\theta_{\mu\rho} q_\lambda - \theta_{\rho\lambda} p^+_\mu - \theta_{\mu\lambda} p^+_\rho + 2\theta_{\rho\lambda} q_\mu] g_{\nu\kappa} + (\theta_{\nu\rho} p^+{}_\mu + \theta_{\mu\rho} p^-_\nu + \theta_{\mu\nu} p^+_\rho - \theta_{\mu\rho} p^+_\nu \\
&- 2\theta_{\nu\rho} q_\mu + \theta_{\mu\nu} p^-_\rho - \theta_{\sigma\rho} p^-_\sigma g_{\mu\nu}) g_{\kappa\lambda} + [\theta_{\sigma\rho} p^+_\sigma g_{\nu\lambda} + \theta_{\nu\lambda} p^+_\rho + \theta_{\nu\lambda} p^-_\rho + 2\theta_{\nu\rho} q_\lambda \\
&- \theta_{\nu\rho} p^-_\lambda + (-p^-_\nu + p^+_\nu)\theta_{\rho\lambda}] g_{\mu\kappa} + [\theta_{\nu\kappa} p^+_\rho - \theta_{\nu\kappa} p^-{}_\rho + (p^-_\kappa - p^+_\kappa)\theta_{\nu\rho} \\
&+ (-p^+_\nu + p^-_\nu)\theta_{\rho\kappa}] g_{\mu\lambda} + (\theta_{\rho\kappa} p^+_\mu - \theta_{\mu\kappa} p^+_\rho + \theta_{\mu\rho} p^+_\kappa - 2\theta_{\mu\rho} q_\kappa - 2\theta_{\rho\kappa} q_\mu) g_{\nu\lambda} \\
&+ (-\theta_{\rho\kappa} p^-_\lambda + \theta_{\rho\lambda} p^-_\kappa + 2\theta_{\rho\kappa} q_\lambda - 2\theta_{\rho\lambda} q_\kappa - \theta_{\kappa\lambda} p^-{}_\rho) g_{\mu\nu}\} q'_\rho + [(\theta_{\mu\rho} q'_\lambda + \theta_{\mu\rho} q_\lambda \\
&+ \theta_{\rho\lambda} q_\mu + \theta_{\rho\lambda} q'_\mu) p^+_\rho + (-\theta_{\sigma\mu} p^-_\sigma - \theta_{\sigma\mu} p^+_\sigma) q'_\lambda + (\theta_{\rho\lambda} q_\mu + \theta_{\rho\lambda} q'{}_\mu) p^-_\rho + (-\theta_{\sigma\mu} p^-_\sigma \\
&- \theta_{\sigma\mu} p^+_\sigma) q_\lambda] g_{\nu\kappa} + [(\theta_{\nu\rho} q'_\mu - \theta_{\mu\rho} q'_\nu + \theta_{\nu\rho} q_\mu) p^+{}_\rho + \theta_{\nu\rho} p^-_\rho q'_\mu + (-\theta_{\sigma\nu} p^-_\sigma \\
&- \theta_{\sigma\nu} p^+_\sigma) q'_\mu - \theta_{\sigma\nu} p^+{}_\sigma q_\mu + (\theta_{\sigma\mu} p^+{}_\sigma + \theta_{\sigma\mu} p^-_\sigma) q'{}_\nu] g_{\kappa\lambda} + [(-\theta_{\rho\lambda} q'_\nu - \theta_{\nu\rho} q'{}_\lambda) p^+_\rho \\
&+ (-\theta_{\nu\rho} p^-{}_\rho + \theta_{\sigma\nu} p^-_\sigma + \theta_{\sigma\nu} p^+_\sigma) q'_\lambda + (-\theta_{\nu\rho} q_\lambda - \theta_{\rho\lambda} q'{}_\nu) p^-_\rho + \theta_{\sigma\nu} p^-{}_\sigma q_\lambda] g_{\mu\kappa} \\
&+ [(-\theta_{\nu\rho} q_\kappa + \theta_{\rho\kappa} q'_\nu) p^+_\rho + (\theta_{\sigma\nu} p^+_\sigma + \theta_{\nu\rho} p^-_\rho - \theta_{\sigma\nu} p^-_\sigma) q_\kappa - \theta_{\rho\kappa} p^-_\rho q'_\nu] g_{\mu\lambda} \\
&+ [(-\theta_{\mu\rho} q_\kappa - \theta_{\rho\kappa} q'_\mu - \theta_{\rho\kappa} q_\mu) p^+_\rho + (\theta_{\sigma\mu} p^+_\sigma + \theta_{\sigma\mu} p^-_\sigma) q_\kappa - \theta_{\rho\kappa} p^-{}_\rho q_\mu] g_{\nu\lambda} \\
&+ (\theta_{\rho\kappa} q_\lambda - \theta_{\rho\lambda} q_\kappa) g_{\mu\nu} p^+_\rho + (\theta_{\rho\kappa} p^-_\rho q'_\lambda - \theta_{\rho\lambda} p^-_\rho q_\kappa + \theta_{\rho\kappa} p^-_\rho q_\lambda) g_{-\mu\nu} + (\theta_{\mu\lambda} p^+_\nu \\
&+ \theta_{\mu\nu} p^-_\lambda + \theta_{\nu\lambda} p^+{}_\mu + \theta_{\mu\lambda} p^-_\nu - 2\theta_{\mu\nu} q'_\lambda - 2\theta_{\nu\lambda} q'_\mu + 2\theta_{\mu\lambda} q'_\nu) q_\kappa + (-\theta_{\mu\nu} p^-_\kappa \\
&- \theta_{\mu\nu} p^+_\kappa - \theta_{\nu\kappa} p^+_\mu + 2\theta_{\nu\kappa} q_\mu - \theta_{\mu\kappa} p^-{}_\nu + \theta_{\mu\kappa} p^+_\nu) q'_\lambda + (-\theta_{\mu\nu} p^-_\kappa - 2\theta_{\mu\kappa} q'_\nu + \theta_{\mu\nu} p^+_\kappa \\
&- \theta_{\mu\kappa} p^-_\nu - \theta_{\mu\kappa} p^+{}_\nu + \theta_{\nu\kappa} p^+_\mu - 2\theta_{\nu\kappa} q'_\mu) q_\lambda + (-\theta_{\kappa\lambda} p^+_\nu - \theta_{\nu\lambda} p^-{}_\kappa + \theta_{\nu\kappa} p^-_\lambda + \theta_{\kappa\lambda} p^-_\nu \\
&- \theta_{\nu\lambda} p^+{}_\kappa) q'_\mu + (-\theta_{\nu\kappa} p^-_\lambda + \theta_{\nu\lambda} p^-_\kappa - \theta_{\kappa\lambda} p^-_\nu - 2\theta_{\kappa\lambda} q'\nu - \theta_{\kappa\lambda} p^+_\nu - \theta_{\nu\lambda} p^+_\kappa) q_\mu \\
&+ (\theta_{\mu\lambda} p^+_\kappa + \theta_{\kappa\lambda} p^+_\mu + \theta_{\mu\kappa} p^-_\lambda + \theta_{\mu\lambda} p^-_\kappa) q'{}_\nu).
\end{aligned}
\tag{30}
$$

Equation (28) for $W^- W^+ \gamma\gamma$ coupling is familiar from the standard model. The exchange of two photons leads to a topologically equivalent diagram. The associated rules are therefore required to be symmetric under substitutions $\nu \rightleftarrows \lambda$. Equations (28) and (29) obviously satisfy this requirement. The $\mathcal{O}(\theta)$ contribution of the rule (29) is derived from the Higgs sector induced interactions. Equation (30) represents the Feynman rule for the nonminimal extended model and contains a lengthy momentum dependent part. These terms are derived from (15). In this case, momenta must be simultaneously replaced with each other as ν, λ indices are substituted.

(c) $W^- W^+ ZZ$ coupling,



(i) Standard model

$$-ig^2\cos^2\theta_w(2g_{\nu\lambda}g_{\mu\kappa}-g_{\nu\kappa}g_{\lambda\mu}-g_{\mu\nu}g_{\kappa\lambda}), \tag{31}$$

(ii) minimal model

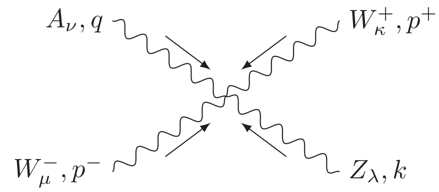
$$\begin{aligned}&\text{Eq. (31)}-\frac{M_W^2}{\cos^2\theta_w}g^2[-7\theta_{\mu\kappa}g_{\nu\lambda}-2\theta_{\kappa\lambda}g_{\mu\nu}+2\theta_{\nu\kappa}g_{\mu\lambda}+2\theta_{\mu\lambda}g_{\nu\kappa}+2\theta_{\mu\nu}g_{\kappa\lambda}\\&+\sin^2\theta_w(-3\theta_{\kappa\lambda}g_{\mu\nu}+3\theta_{\nu\kappa}g_{\mu\lambda}-\theta_{\mu\lambda}g_{\nu\kappa}-\theta_{\mu\nu}g_{\kappa\lambda}+4\theta_{\mu\kappa}g_{\nu\lambda})+\sin^4\theta_w(4\theta_{\kappa\lambda}g_{\mu\nu}-4\theta_{\nu\kappa}g_{\mu\lambda})],\end{aligned} \tag{32}$$

(iii) nonminimal model

$$\begin{aligned}&\text{Eq. (32)}+g'g^2k_2\sin\theta_w\cos\theta_w(\{(-2\theta_{\alpha\rho}g_{\mu\kappa}g_{\nu\lambda}+2\theta_{\alpha\rho}g_{\kappa\lambda}g_{\mu\nu})k'_\rho\\
&+[(-\theta_{\sigma\alpha}p^+_\sigma+\theta_{\sigma\alpha}p^-_\sigma)g_{\mu\lambda}+2\theta_{\mu\alpha}k'_\lambda+2\theta_{\alpha\lambda}k'_\mu-\theta_{\mu\alpha}p^-_\lambda-\theta_{\alpha\lambda}p^+_\mu\\
&+(-p^+_\alpha-2k'_\alpha-p^-_\alpha)\theta_{\mu\lambda}]g_{\nu\kappa}+[\theta_{\sigma\alpha}p^+_\sigma g_{\mu\nu}-2\theta_{\alpha\nu}k'_\mu-2\theta_{\mu\alpha}k'_\nu+\theta_{\alpha\nu}p^+_\mu+\theta_{\mu\alpha}p^+{}_\nu\\
&+(2k'_\alpha-p^+_\alpha)\theta_{\mu\nu}]g_{\kappa\lambda}+[-\theta_{\sigma\alpha}p^-_\sigma g_{\nu\lambda}+2\theta_{\alpha\nu}k'_\lambda-2\theta_{\alpha\lambda}k'_\nu+\theta_{\alpha\lambda}p^-_\nu-\theta_{\alpha\nu}p^-_\lambda\\
&+(-p^-{}_\alpha+2k'_\alpha)\theta_{\nu\lambda}]g_{\mu\kappa}+[-\theta_{\alpha\kappa}p^-_\nu+\theta_{\alpha\nu}p^-_\kappa+\theta_{\alpha\kappa}p^+_\nu-\theta_{\alpha\nu}p^+_\kappa\\
&+(p^-_\alpha-p^+_\alpha)\theta_{\nu\kappa}]g_{\mu\lambda}+[2\theta_{\alpha\kappa}k'_\mu+\theta_{\mu\alpha}p^-_\kappa-\theta_{\alpha\kappa}p^+_\mu-\theta_{\mu\alpha}p^+_\kappa\\
&+(p^+_\alpha+p^-_\alpha+2k'_\alpha)\theta_{\mu\kappa}]g_{\nu\lambda}+[-2\theta_{\alpha\kappa}k'_\lambda-\theta_{\alpha\lambda}p^-{}_\kappa+\theta_{\alpha\kappa}p^-_\lambda+\theta_{\alpha\lambda}p^+_\kappa\\
&+(p^+_\alpha+p^-_\alpha+2k'_\alpha)\theta_{\kappa\lambda}]g_{\mu\nu}\}k_\alpha+\{[(-\theta_{\sigma\rho}p^+_\sigma+\theta_{\sigma\rho}p^-_\sigma)g_{\mu\lambda}-\theta_{\mu\rho}p^-_\lambda-\theta_{\mu\lambda}p^-_\rho\\
&+2\theta_{\mu\rho}k_\lambda-\theta_{\rho\lambda}p^+_\mu-\theta_{\mu\lambda}p^+_\rho+2\theta_{\rho\lambda}k_\mu]g_{\nu\kappa}+(\theta_{\nu\rho}p^+{}_\mu+\theta_{\mu\rho}p^-_\nu+\theta_{\mu\nu}p^+_\rho-\theta_{\mu\rho}p^+_\nu\\
&-2\theta_{\nu\rho}k_\mu+\theta_{\mu\nu}p^-_\rho-\theta_{\sigma\rho}p^-_\sigma g_{\mu\nu})g_{\kappa\lambda}+[\theta_{\sigma\rho}p^+_\sigma g_{\nu\lambda}+\theta_{\nu\lambda}p^+_\rho+\theta_{\nu\lambda}p^-_\rho+2\theta_{\nu\rho}k_\lambda-\theta_{\nu\rho}p^-_\lambda\\
&+(-p^-_\nu+p^+_\nu)\theta_{\rho\lambda}]g_{\mu\kappa}+[\theta_{\nu\kappa}p^+_\rho-\theta_{\nu\kappa}p^-{}_\rho+(p^-_\kappa-p^+_\kappa)\theta_{\nu\rho}+(-p^+_\nu+p^-_\nu)\theta_{\rho\kappa}]g_{\mu\lambda}\\
&+(\theta_{\rho\kappa}p^+_\mu-\theta_{\mu\kappa}p^+_\rho+\theta_{\mu\rho}p^+_\kappa-2\theta_{\mu\rho}k_\kappa-2\theta_{\rho\kappa}k_\mu)g_{\nu\lambda}+(-\theta_{\rho\kappa}p^-_\lambda+\theta_{\rho\lambda}p^-_\kappa+2\theta_{\rho\kappa}k_\lambda\\
&-2\theta_{\rho\lambda}k_\kappa-\theta_{\kappa\lambda}p^-{}_\rho)g_{\mu\nu}\}k'_\rho+[(\theta_{\mu\rho}k'_\lambda+\theta_{\mu\rho}k_\lambda+\theta_{\rho\lambda}k_\mu+\theta_{\rho\lambda}k'_\mu)p^+_\rho\\
&+(-\theta_{\sigma\mu}p^-_\sigma-\theta_{\sigma\mu}p^+_\sigma)k'_\lambda+(\theta_{\rho\lambda}k_\mu+\theta_{\rho\lambda}k'{}_\mu)p^-_\rho+(-\theta_{\sigma\mu}p^-_\sigma-\theta_{\sigma\mu}p^+_\sigma)k_\lambda]g_{\nu\kappa}\\
&+[(\theta_{\nu\rho}k'_\mu-\theta_{\mu\rho}k'_\nu+\theta_{\nu\rho}k_\mu)p^+{}_\rho+\theta_{\nu\rho}p^-_\rho k'_\mu+(-\theta_{\sigma\nu}p^-_\sigma-\theta_{\sigma\nu}p^+_\sigma)k'_\mu-\theta_{\sigma\nu}p^+{}_\sigma k_\mu\\
&+(\theta_{\sigma\mu}p^+{}_\sigma+\theta_{\sigma\mu}p^-_\sigma)k'{}_\nu]g_{\kappa\lambda}+[(-\theta_{\rho\lambda}k'_\nu-\theta_{\nu\rho}k'{}_\lambda)p^+_\rho+(-\theta_{\nu\rho}p^-{}_\rho+\theta_{\sigma\nu}p^-_\sigma\\
&+\theta_{\sigma\nu}p^+_\sigma)k'_\lambda+(-\theta_{\nu\rho}k_\lambda-\theta_{\rho\lambda}k'{}_\nu)p^-_\rho+\theta_{\sigma\nu}p^-{}_\sigma k_\lambda]g_{\mu\kappa}+[(-\theta_{\nu\rho}k_\kappa+\theta_{\rho\kappa}k'_\nu)p^+_\rho\\
&+(\theta_{\sigma\nu}p^+_\sigma+\theta_{\nu\rho}p^-_\rho-\theta_{\sigma\nu}p^-_\sigma)k_\kappa-\theta_{\rho\kappa}p^-_\rho k'_\nu]g_{\mu\lambda}+[(-\theta_{\mu\rho}k_\kappa-\theta_{\rho\kappa}k'_\mu-\theta_{\rho\kappa}k_\mu)p^+_\rho\\
&+(\theta_{\sigma\mu}p^+_\sigma+\theta_{\sigma\mu}p^-_\sigma)k_\kappa-\theta_{\rho\kappa}p^-{}_\rho k_\mu]g_{\nu\lambda}+(\theta_{\rho\kappa}k_\lambda-\theta_{\rho\lambda}k_\kappa)g_{\mu\nu}p^+_\rho+(\theta_{\rho\kappa}p^-_\rho k'_\lambda\\
&-\theta_{\rho\lambda}p^-_\rho k_\kappa+\theta_{\rho\kappa}p^-_\rho k_\lambda)g_{_\mu\nu}+(\theta_{\mu\lambda}p^+_\nu+\theta_{\mu\nu}p^-_\lambda+\theta_{\nu\lambda}p^+{}_\mu+\theta_{\mu\lambda}p^-_\nu-2\theta_{\mu\nu}k'_\lambda\\
&-2\theta_{\nu\lambda}k'_\mu+2\theta_{\mu\lambda}k'_\nu)k_\kappa+(-\theta_{\mu\nu}p^-_\kappa-\theta_{\mu\nu}p^+_\kappa-\theta_{\nu\kappa}p^+_\mu+2\theta_{\nu\kappa}k_\mu-\theta_{\mu\kappa}p^-{}_\nu\\
&+\theta_{\mu\kappa}p^+_\nu)k'_\lambda+(-\theta_{\mu\nu}p^-_\kappa-2\theta_{\mu\kappa}k'_\nu+\theta_{\mu\nu}p^+_\kappa-\theta_{\mu\kappa}p^-_\nu-\theta_{\mu\kappa}p^+{}_\nu+\theta_{\nu\kappa}p^+_\mu-2\theta_{\nu\kappa}k'_\mu)k_\lambda\\
&+(-\theta_{\kappa\lambda}p^+_\nu-\theta_{\nu\lambda}p^-{}_\kappa+\theta_{\nu\kappa}p^-_\lambda+\theta_{\kappa\lambda}p^-_\nu-\theta_{\nu\lambda}p^+{}_\kappa)k'_\mu+(-\theta_{\nu\kappa}p^-_\lambda+\theta_{\nu\lambda}p^-_\kappa-\theta_{\kappa\lambda}p^-_\nu\\
&-2\theta_{\kappa\lambda}k'_\nu-\theta_{\kappa\lambda}p^+_\nu-\theta_{\nu\lambda}p^+_\kappa)k_\mu+(\theta_{\mu\lambda}p^+_\kappa+\theta_{\kappa\lambda}p^+_\mu+\theta_{\mu\kappa}p^-_\lambda+\theta_{\mu\lambda}p^-_\kappa)k'{}_\nu).\end{aligned} \tag{33}$$

The symmetry properties of (31)–(33) are exactly the same as Eqs. (28)–(30). The exchange of Z^0 bosons leaves the physical content of the diagram unchanged. The rules to be consistent with this requirement must be symmetric under substitutions $\nu \rightleftarrows \lambda$. Notice that in (30) momenta must be replaced with each other simultaneously as the indices are exchanged.

(d) $W^- W^+ Z\gamma$ coupling,



(i) Standard model

$$-ie^2 \cot\theta_w (2g_{\nu\lambda} g_{\mu\kappa} - g_{\nu\kappa} g_{\lambda\mu} - g_{\mu\nu} g_{\kappa\lambda}), \tag{34}$$

(ii) minimal model

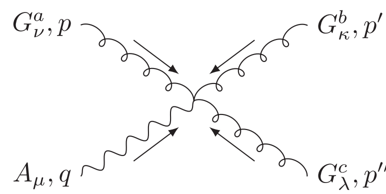
$$\text{Eq. (34)} + \frac{1}{2} M_W^2 eg[-6\theta_{\kappa\lambda} g_{\nu\mu} - 2\theta_{\kappa\mu} g_{\nu\lambda} - 3\theta_{\nu\kappa} g_{\lambda\mu} + 2\theta_{\mu\lambda} g_{\kappa\mu} + \theta_{\nu\mu} g_{\kappa\lambda} + (4\theta_{\kappa\lambda} g_{\nu\mu} - 4\theta_{\nu\kappa} g_{\lambda\mu})\sin^2\theta_w], \tag{35}$$

(iii) nonminimal model

$$\begin{aligned}
&\text{Eq. (35)} + g' g^2 k_2[\cos^2\theta_w((\theta_{\kappa\lambda} g_{\mu\nu} - \theta_{\mu\lambda} g_{\nu\kappa} - \theta_{\nu\lambda} g_{\mu\kappa} + \theta_{\mu\kappa} g_{\nu\lambda} + \theta_{\nu\kappa} g_{\mu\lambda}) q_\alpha{}^2 \\
&\quad + \{(-\theta_{\alpha\kappa} g_{\nu\lambda} + \theta_{\alpha\lambda} g_{\nu\kappa}) q_\mu + [2\theta_{\alpha\kappa} q_\lambda + (-\theta_{\alpha\rho} p^-_\rho - \theta_{\alpha\rho} k_\rho) g_{\kappa\lambda} - 2\theta_{\alpha\lambda} q_\kappa + \theta_{\alpha\kappa} k_\lambda \\
&\quad + (p^-_\kappa - k_\kappa)\theta_{\alpha\lambda} + (k_\alpha + p^-_\alpha)\theta_{\kappa\lambda}] g_{\mu\nu} + [\theta_{\alpha\rho} p^-{}_\rho g_{\mu\lambda} + (-p^-_\mu + k_\mu)\theta_{\alpha\lambda} \\
&\quad + (-p^-{}_\alpha - k_\alpha)\theta_{\mu\lambda} + \theta_{\mu\alpha} k_\lambda] g_{\nu\kappa} + [\theta_{\alpha\rho} k_\rho g_{\mu\kappa} + (-k_\mu - p^-_\mu)\theta_{\alpha\kappa} + (k_\alpha + p^-{}_\alpha)\theta_{\mu\kappa} \\
&\quad - \theta_{\mu\alpha} k_\kappa - \theta_{\mu\alpha} p^-_\kappa] g_{\nu\lambda} + (\theta_{\alpha\nu} p^-{}_\mu + \theta_{\alpha\nu} k_\mu + \theta_{\mu\alpha} p^-_\nu - \theta_{\mu\nu} p^-_\alpha + \theta_{\mu\nu} k_\alpha) g_{\kappa\lambda} \\
&\quad + (\theta_{\alpha\lambda} g_{\mu\kappa} - \theta_{\alpha\kappa} g_{\mu\lambda}) q_\nu + (\theta_{\nu\lambda} k_\alpha - \theta_{\alpha\nu} k_\lambda) g_{\mu\kappa} + (-\theta_{\nu\kappa} p^-_\alpha - \theta_{\alpha\nu} p^-_\kappa + \theta_{\alpha\kappa} p^-{}_\nu) g_{\mu\lambda}\} q_\alpha \\
&\quad + [-\theta_{\nu\kappa} q_\lambda + (\theta_{\rho\lambda} k_\rho + \theta_{\rho\lambda} p^-_\rho) g_{\nu\kappa} + (-2\theta_{\rho\kappa} p^-_\rho + \theta_{\rho\kappa} k_\rho) g_{\nu\lambda} + (\theta_{\nu\rho} p^-_\rho + \theta_{\nu\rho} k_\rho) g_{\kappa\lambda} \\
&\quad - \theta_{\kappa\lambda} q_\nu + \theta_{\nu\lambda} q_\kappa - \theta_{\nu\kappa} k_\lambda + (-k_\kappa - 2p^-_\kappa)\theta_{\nu\lambda} - \theta_{\kappa\lambda} p^-_\nu] q_\mu + [(\theta_{\rho\kappa} p^-_\rho - \theta_{\rho\kappa} k_\rho) q_\lambda \\
&\quad - \theta_{\rho\lambda} q_\kappa p^-_\rho] g_{\mu\nu} + [(\theta_{\mu\rho} k_\rho + \theta_{\mu\rho} p^-_\rho) g_{\nu\kappa} - \theta_{\mu\kappa} q_\nu - \theta_{\nu\rho} k_\rho g_{\mu\kappa} - \theta_{\mu\kappa} p^-_\nu + (k_\mu + p^-_\mu)\theta_{\nu\kappa} \\
&\quad + \theta_{\mu\nu} p^-_\kappa - \theta_{\mu\nu} k_\kappa] q_\lambda - \theta_{\mu\rho} q_\kappa p^-_\rho g_{\nu\lambda} - 2\theta_{\mu\rho} q_\nu k_\rho g_{\kappa\lambda} + (-\theta_{\mu\kappa} k_\lambda + \theta_{\mu\lambda} k_\kappa - \theta_{\rho\lambda} k_\rho g_{\mu\kappa} \\
&\quad - \theta_{\kappa\lambda} k_\mu + \theta_{\mu\lambda} q_\kappa) q_\nu - \theta_{\nu\rho} q_\kappa p^-{}_\rho g_{\mu\lambda} + (\theta_{\mu\lambda} p^-_\nu + \theta_{\nu\lambda} p^-_\mu) q_\kappa) \\
&\quad + \sin^2\theta_w(\{[(-\theta_{\sigma\eta} p^-_\sigma + \theta_{\sigma\eta} q_\sigma) g_{\kappa\lambda} + (-p^+_\lambda + q_\lambda)\theta_{\eta\kappa} + (p^+_\kappa - p^-_\kappa)\theta_{\eta\lambda} \\
&\quad + (k_\eta)\theta_{\kappa\lambda}] g_{\mu\nu} + [(-\theta_{\sigma\eta} p^+_\sigma + \theta_{\sigma\eta} p^-_\sigma) g_{\mu\lambda} + (p^-_\mu - q_\mu)\theta_{\eta\lambda} \\
&\quad + (p^+_\lambda - q_\lambda)\theta_{\mu\eta} + (-k_\eta)\theta_{\mu\lambda}] g_{\nu\kappa} + [(\theta_{\sigma\eta} p^+{}_\sigma - \theta_{\sigma\eta} q_\sigma) g_{\mu\kappa} + (p^-_\mu - q_\mu)\theta_{\eta\kappa} \\
&\quad + (p^-_\kappa - p^+{}_\kappa)\theta_{\mu\eta} + (k_\eta)\theta_{\mu\kappa}] g_{\nu\lambda} + [(p^+_\lambda - q_\lambda)\theta_{\eta\nu} + (q_\nu - p^+_\nu)\theta_{\eta\lambda} \\
&\quad + (p^+_\eta - q_\eta)\theta_{\nu\lambda}] g_{\mu\kappa} + [(p^-_\kappa - p^+_\kappa)\theta_{\eta\nu} + (p^+_\nu - p^-_\nu)\theta_{\eta\kappa} + (-p^+_\eta + p^-_\eta)\theta_{\nu\kappa}] g_{\mu\lambda} \\
&\quad + [(-p^-_\mu + q_\mu)\theta_{\eta\nu} + (-p^-_\nu + q_\nu)\theta_{\mu\eta} + (-q_\eta + p^-_\eta)\theta_{\mu\nu}] g_{\kappa\lambda}\} k_\eta + [(\theta_{\sigma\lambda} p^+_\sigma + \theta_{\sigma\lambda} q_\sigma \\
&\quad + \theta_{\sigma\lambda} p^-_\sigma) k_\kappa + (\theta_{\sigma\kappa} k_\sigma) k_\lambda] g_{\mu\nu} + [(\theta_{\sigma\lambda} k_\sigma) k_\mu - \theta_{\sigma\mu} k_\sigma k_\lambda] g_{\nu\kappa} \\
&\quad + [(-\theta_{\sigma\kappa} k_\sigma) k_\mu + (\theta_{\sigma\mu} k_\sigma) k_\kappa] g_{\nu\lambda} + [(\theta_{\sigma\nu} p^-_\sigma - \theta_{\sigma\nu} q_\sigma) g_{\kappa\lambda} + (p^+_\lambda - q_\lambda)\theta_{\nu\kappa} + (p^-_\kappa - p^+_\kappa)\theta_{\nu\lambda} \\
&\quad - (k_\nu)\theta_{\kappa\lambda}] k_\mu + [(-\theta_{\sigma\nu} p^-_\sigma + \theta_{\sigma\nu} p^+_\sigma) g_{\mu\lambda} + (-p^+_\lambda + q_\lambda)\theta_{\mu\nu} + (k_\nu)\theta_{\mu\lambda} + (-p^-_\mu + q_\mu)\theta_{\nu\lambda}] k_\kappa \\
&\quad + [(-\theta_{\sigma\nu} p^+_\sigma + \theta_{\sigma\nu} q_\sigma) g_{\mu\kappa} + (p^+_\kappa - p^-{}_\kappa)\theta_{\mu\nu} - (k_\nu)\theta_{\mu\kappa} + (-p^-_\mu + q_\mu)\theta_{\nu\kappa}] k_\lambda)].
\end{aligned} \tag{36}$$

The rule (34) is the standard model vertex function for the $W^-W^+Z\gamma$ coupling. In the present case there is no explicit exchange symmetry. Equation (35) represents the θ-expanded rule for the minimal noncommutative model. Its θ-dependent part is obtained from the Higgs-induced interactions. Equation (36) is the Feynman rule for the nonminimal extended model. The momentum dependent part of (36) is derived from the gauge action (15).

(e) γggg coupling,

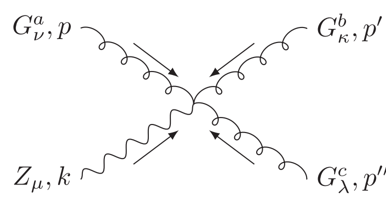


(i) Nonminimal model

$$\begin{aligned}
&g' g_s^3 \cos\theta_w f^{bac}(\{[2\theta_{\sigma\rho}p'_\sigma g_{\kappa\lambda} - 2\theta_{\sigma\rho}p''_\sigma g_{\kappa\lambda} + (2p'_\lambda - 2p_\lambda)\theta_{\rho\kappa}\\
&\quad + (-2p''_\kappa + 2p_\kappa)\theta_{\rho\lambda}]g_{\nu\mu} + [-2\theta_{\sigma\rho}p_\sigma g_{\nu\lambda} + 2\theta_{\sigma\rho}p''_\sigma g_{\nu\lambda} + (-2p'_\lambda + 2p_\lambda)\theta_{\nu\rho}\\
&\quad + (2p''_\nu - 2p'_\nu)\theta_{\rho\lambda}]g_{\kappa\mu} + [2\theta_{\sigma\rho}p_\sigma g_{\nu\kappa} - 2\theta_{\sigma\rho}p'_\sigma g_{\nu\kappa} + (2p''_\kappa - 2p_\kappa)\theta_{\nu\rho}\\
&\quad + (2p''_\nu - 2p'_\nu)\theta_{\rho\kappa}]g_{\lambda\mu} + ((-2p'_\lambda + 2p_\lambda)\theta_{\rho\mu} + (-2p_\mu + 2p'_\mu)\theta_{\rho\lambda})g_{\nu\kappa}\\
&\quad + [(2p''_\kappa - 2p_\kappa)\theta_{\rho\mu} + (-2p''_\mu + 2p_\mu)\theta_{\rho\kappa}]g_{\nu\lambda} + [(-2p''_\mu + 2p'_\mu)\theta_{\nu\rho}\\
&\quad + (2p'_\nu - 2p''_\nu)\theta_{\rho\mu}]g_{\kappa\lambda}\}q_\rho + [-2\theta_{\sigma\lambda}q_\sigma q_\kappa + 2\theta_{\sigma\kappa}q_\sigma q_\lambda + 2\theta_{\kappa\lambda}q_\sigma q_\sigma]g_{\nu\mu} + [2\theta_{\sigma\lambda}q_\sigma q_\nu\\
&\quad - 2\theta_{\sigma\nu}q_\sigma q_\lambda - 2\theta_{\nu\lambda}q_\sigma q_\sigma]g_{\kappa\mu} + [-2\theta_{\sigma\kappa}q_\sigma q_\nu + 2\theta_{\sigma\nu}q_\sigma q_\kappa + 2\theta_{\nu\kappa}q_\sigma q_\sigma]g_{\lambda\mu} + [-2\theta_{\sigma\mu}p'_\sigma g_{\kappa\lambda}\\
&\quad + 2\theta_{\sigma\mu}p''_\sigma g_{\kappa\lambda} + (2p'_\lambda - 2p_\lambda)\theta_{\kappa\mu} - 2q_\mu\theta_{\kappa\lambda} + (-2p''_\kappa + 2p_\kappa)\theta_{\lambda\mu}]q_\nu\\
&\quad + [2\theta_{\sigma\mu}p_\sigma g_{\nu\lambda} - 2\theta_{\sigma\mu}p''_\sigma g_{\nu\lambda} + (2p'_\lambda - 2p_\lambda)\theta_{\nu\mu} + 2q_\mu\theta_{\nu\lambda} + (2p''_\nu - 2p'_\nu)\theta_{\lambda\mu}]q_\kappa\\
&\quad + [-2\theta_{\sigma\mu}p_\sigma g_{\nu\kappa} + 2\theta_{\sigma\mu}p'_\sigma g_{\nu\kappa} + (-2p''_\kappa + 2p_\kappa)\theta_{\nu\mu} - 2q_\mu\theta_{\nu\kappa} + (2p''_\nu - 2p'_\nu)\theta_{\kappa\mu}]q_\lambda\\
&\quad + [(2\theta_{\kappa\mu}g_{\nu\lambda} - 2\theta_{\lambda\mu}g_{\nu\kappa})p_\sigma + (2\theta_{\lambda\mu}g_{\nu\kappa} - 2\theta_{\nu\mu}g_{\kappa\lambda})p'_\sigma - (2\theta_{\kappa\mu}g_{\nu\lambda} - 2\theta_{\nu\mu}g_{\kappa\lambda})p''_\sigma]q_\sigma).
\end{aligned} \tag{37}$$

The γggg coupling is forbidden in the standard model. The rule (37) is derived from (17) and is allowed only in the nonminimal noncommutative model. Because the exchange of gluons leaves the diagram topologically invariant, the rule must be symmetric under simultaneous substitutions of $\nu \rightleftarrows \kappa$, $a \rightleftarrows b$, $p \rightleftarrows p'$ and independently under $\nu \rightleftarrows \lambda$, $a \rightleftarrows c$, $p \rightleftarrows p''$ as well as under $\kappa \rightleftarrows \lambda$, $b \rightleftarrows c$, $p' \rightleftarrows p''$. Also, the cyclic symmetry $\nu \rightleftarrows \kappa \rightleftarrows \lambda$, $a \rightleftarrows b \rightleftarrows c$, $p \rightleftarrows p' \rightleftarrows p''$ must be satisfied.

(f) $Zggg$ coupling,



(i) Nonminimal model

$$\begin{aligned} &g' g_s^3 \sin\theta_w f^{abc}(\{[2\theta_{\sigma\rho} p'_\sigma g_{\kappa\lambda} - 2\theta_{\sigma\rho} p''_\sigma g_{\kappa\lambda} + (2p'_\lambda - 2p_\lambda)\theta_{\rho\kappa} \\ &+ (-2p''_\kappa + 2p_\kappa)\theta_{\rho\lambda}] g_{\nu\mu} [-2\theta_{\sigma\rho} p_\sigma g_{\nu\lambda} + 2\theta_{\sigma\rho} p''_\sigma g_{\nu\lambda} + (-2p'_\lambda + 2p_\lambda)\theta_{\nu\rho} \\ &+ (2p''_\nu - 2p'_\nu)\theta_{\rho\lambda}] g_{\kappa\mu} [2\theta_{\sigma\rho} p_\sigma g_{\nu\kappa} - 2\theta_{\sigma\rho} p'_\sigma g_{\nu\kappa} + (2p''_\kappa - 2p_\kappa)\theta_{\nu\rho} \\ &+ (2p''_\nu - 2p'_\nu)\theta_{\rho\kappa}] g_{\lambda\mu}((-2p'_\lambda + 2p_\lambda)\theta_{\rho\mu} + (-2p_\mu + 2p'_\mu)\theta_{\rho\lambda}) g_{\nu\kappa} \\ &+ [(2p''_\kappa - 2p_\kappa)\theta_{\rho\mu} + (-2p''_\mu + 2p_\mu)\theta_{\rho\kappa}] g_{\nu\lambda} + [(-2p''_\mu + 2p'_\mu)\theta_{\nu\rho} \\ &+ (2p'_\nu - 2p''_\nu)\theta_{\rho\mu}] g_{\kappa\lambda}\} k_\rho + [-2\theta_{\sigma\lambda} k_\sigma k_\kappa + 2\theta_{\sigma\kappa} k_\sigma k_\lambda + 2\theta_{\kappa\lambda} k_\sigma k_\sigma] g_{\nu\mu} + [2\theta_{\sigma\lambda} k_\sigma k_\nu \\ &- 2\theta_{\sigma\nu} k_\sigma k_\lambda - 2\theta_{\nu\lambda} k_\sigma k_\sigma] g_{\kappa\mu} + [-2\theta_{\sigma\kappa} k_\sigma k_\nu + 2\theta_{\sigma\nu} k_\sigma k_\kappa + 2\theta_{\nu\kappa} k_\sigma k_\sigma] g_{\lambda\mu} + [-2\theta_{\sigma\mu} p'_\sigma g_{\kappa\lambda} \\ &+ 2\theta_{\sigma\mu} p''_\sigma g_{\kappa\lambda} + (2p'_\lambda - 2p_\lambda)\theta_{\kappa\mu} - 2k_\mu \theta_{\kappa\lambda} + (-2p''_\kappa + 2p_\kappa)\theta_{\lambda\mu}] k_\nu \\ &+ [2\theta_{\sigma\mu} p_\sigma g_{\nu\lambda} - 2\theta_{\sigma\mu} p''_\sigma g_{\nu\lambda} + (2p'_\lambda - 2p_\lambda)\theta_{\nu\mu} + 2k_\mu \theta_{\nu\lambda} + (2p''_\nu - 2p'_\nu)\theta_{\lambda\mu}] k_\kappa \\ &+ [-2\theta_{\sigma\mu} p_\sigma g_{\nu\kappa} + 2\theta_{\sigma\mu} p'_\sigma g_{\nu\kappa} + (-2p''_\kappa + 2p_\kappa)\theta_{\nu\mu} - 2k_\mu \theta_{\nu\kappa} + (2p''_\nu - 2p'_\nu)\theta_{\kappa\mu}] k_\lambda \\ &+ [(2\theta_{\kappa\mu} g_{\nu\lambda} - 2\theta_{\lambda\mu} g_{\nu\kappa}) p_\sigma + (2\theta_{\lambda\mu} g_{\nu\kappa} - 2\theta_{\nu\mu} g_{\kappa\lambda}) p'_\sigma - (2\theta_{\kappa\mu} g_{\nu\lambda} - 2\theta_{\nu\mu} g_{\kappa\lambda}) p''_\sigma] k_\sigma). \end{aligned} \quad (38)$$

The $Zggg$ coupling is also forbidden in the standard model at tree level. This vertex function is derived from (18) and is allowed only in the nonminimal model. The symmetry properties of (38) are the same as that of the rule (37).

IV. DISCUSSION ON PHENOMENOLOGICAL PERSPECTIVES OF THE MODEL

To give an intuitive understanding of the model and, in particular, the Feynman rules developed in the previous section, let us consider the $W^- W^+ \to ZZ$ scattering. In the context of the standard model and at tree level, the scattering amplitude of the process is the sum over amplitudes of diagrams 1–4, in Appendix A. Using the analysis of relevant diagrams the scattering cross section is estimated approximately to be around 68 pb (10^{-12} barn). The amplitude for the contact interaction grows rapidly as the center of mass (c.m.) energy $\sqrt{s}$ increases. The amplitudes of the t-channel and the exchange diagrams give rise respectively to forward and backward scattering. On the other hand, the Higgs mediated diagram effectively tames the amplitude of the contact interaction and there is a strong cancellation in the high energy behavior of individual amplitudes. The total cross section ultimately reaches a nearly constant value at large c.m. energies. It is well known that only the scattering of longitudinally polarized bosons is responsible for the leading behavior of scattering amplitudes at the high energy limit. Then, let us define the kinematics of scattering as

$$p^\pm = (E, 0, 0, \pm p), \quad (39a)$$

$$k^\pm = (E, 0, \pm p\sin\theta, \pm p\cos\theta), \quad (39b)$$

$$\varepsilon^\pm(p) = \left(\frac{p}{M_W}, 0, 0, \pm\frac{E}{M_W}\right), \quad (39c)$$

$$\varepsilon^\pm(k) = \left(\frac{p}{M_Z}, 0, \pm\frac{E\sin\theta}{M_Z}, \pm\frac{E\cos\theta}{M_Z}\right). \quad (39d)$$

Here, θ and ϕ are respectively the polar and azimuthal angle. The momenta of incoming $W^\pm$ and outgoing Z^0 bosons in the c.m. reference frame are respectively denoted by $p^\pm$ and $k^\pm$. Also, $\varepsilon^\pm(p)$ and $\varepsilon^\pm(k)$ are used for polarization vectors of corresponding bosons. The general features of the scattering can be understood from Figs. 1–3. In the standard model, the azimuthal distribution of differential cross section, i.e., $\frac{d\sigma}{d\phi}$, is expected to be flat. The $\frac{d\sigma}{d\phi}$ distributions at $\theta = \frac{\pi}{2}$ have been shown in Fig. 1, for $\sqrt{s} = 1.0$, 1.5, and 2.0 TeV. Figure 2 displays the θ-integrated $\frac{d\sigma}{d\phi}$ distributions at the same c.m. energies. On the other hand, because of the t and u-channel diagrams the ϕ-integrated cross sections $\frac{d\sigma}{d\cos\theta}$ are expected to be very forward-backward distributions. The $\frac{d\sigma}{d\cos\theta}$ distributions have been shown in Fig. 3. For numerical evaluations some approximations were made. We neglected the decay width of intermediate bosons and assumed they were nearly stable particles. Also, the integrations on θ and ϕ were performed approximately because the subsequent similar integrations on $\frac{d\sigma_{\rm NC}}{d\Omega}$ ($d\Omega = \sin\theta d\theta d\phi$) could not be evaluated exactly in a reasonable computation time. We preferred to use the same level of accuracy for all of the numerical evaluations from the beginning. The masses of weak bosons are $M_W = 80.38$, $M_Z = 91.18$ and the Higgs mass is $M_H = 125.0$ GeV based on last issue of the Particle Data Group [41]. Furthermore, the parameter k_2 [see the rule (33)] is assumed to be 0.4 [40]. In our evaluations the total cross section approaches 28 pb at $\sqrt{s} = 1.5$ TeV. This is about 12.5% smaller than the exact value 32 pb [42].

Now, we consider the process in the context of the noncommutative standard model. The usual parametrization for the deformation quantity is $\theta^{\mu\nu} = c^{\mu\nu}/\Lambda_{\rm NC}^2$ where $c^{\mu\nu}$ is a dimensionless matrix of order unity and $\Lambda_{\rm NC}$ is the overall scale that characterizes the threshold at which noncommutative effects become relevant [43–45]. The $c^{\mu\nu}$ matrix is analogous to the (electromagnetic) field tensor in structure. However, it is not at all a tensor because its elements are assumed to be constant in all reference frames. Before proceeding to numerical analysis let us make some general remarks regarding calculation of scattering amplitudes.

First, two distinct cases should be discussed separately: The space-space or B-field-type noncommutativity that means the elements c^{ij} (i, j run from 1 to 3) are nonvanishing and space-time or E-field-type noncommutativity that means c^{0j} elements are nonzero. Two types may have some features in common. The later type has been known to have some problems concerning the unitary and causality considerations [46–48]. Here, we consider only the case of space-space noncommutativity.

Secondly, because the vertex functions for minimal and nonminimal extended models are different, their phenomenological perspectives should be discussed separately. In evaluation of amplitudes we used (32) and (33) (based on the model under consideration) and also the relevant θ-expanded rules developed in [31,39]. Notice that the amplitude of diagram 4 is equal in the minimal and nonminimal models because the Higgs couplings remain the same in both model extensions. This diagram may cause a distinction between two cases through interference contributions.

Lastly, as we mentioned earlier, the noncommutative model introduces new interactions that are forbidden in the standard model. Those that are relevant to our discussion are ZZZ and γZZ couplings [31]. By using these vertices, it is possible to add diagrams 5 and 6 into the standard Feynman graphs. Note that the latter diagram is allowed only in the nonminimal noncommutative extension of the standard model. The scattering amplitudes of these diagrams are of order (θ^2) and their interference contributions are much more important than their individual contributions to the total cross section.

A. B-type noncommutativity

Let us assume $\vec{\theta}_{\rm B} = \frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$ where $\theta_{\rm B}^k \equiv \epsilon^{ijk} c_{ij}$ for the case of B-field-type noncommutativity. This is a constant vector and aligns in a specific direction in space (in all reference frames). As the Earth rotates and revolves around the Sun, the direction of $\vec{\theta}_{\rm B}$ continuously changes and observable quantities are expected to show a specific time dependence. For those instants that our assumption on the direction of $\vec{\theta}_{\rm B}$ is satisfied the $\frac{d\sigma_{\rm NC}}{d\phi}$ and $\frac{d\sigma_{\rm NC}}{d\cos\theta}$ distributions show a characteristic oscillatory behavior. We have shown the numerical results of the noncommutative standard model in Figs. 4–15 in Appendix B. The figures correspond respectively to $\frac{d\sigma_{\rm NC}}{d\phi}$ at $\theta = \frac{\pi}{2}$, the θ-integrated $\frac{d\sigma_{\rm NC}}{d\phi}$, and ϕ-integrated $\frac{d\sigma_{\rm NC}}{d\cos\theta}$ distributions for different values of $\Lambda_{\rm NC}$. Figure 4 exhibits the azimuthal distributions $\frac{d\sigma_{\rm NC}}{d\phi}$ at $\theta = \frac{\pi}{2}$ and $\sqrt{s} = 1.0$ TeV for $\Lambda_{\rm NC} = 0.6$, 0.8, 1.2, 1.5, 1.8 TeV and ∞ in the context of the minimal noncommutative model. Figure 5 represents the integrated $\frac{d\sigma_{\rm NC}}{d\phi}$ distributions and Fig. 6 is the integrated $\frac{d\sigma_{\rm NC}}{d\cos\theta}$ distribution at the same c.m. energy for different scales. The evolution of differential cross sections by variation of the noncommutativity scale can be easily understood from these graphics. We see that $\frac{d\sigma_{\rm NC}}{d\phi}$ distributions show an oscillatory behavior with crests at $\phi = \frac{3\pi}{4}, \frac{7\pi}{4}$. As the noncommutativity scale increases, the crests smoothly collapse and disappear at large enough scales. In particular, in the limit of $\Lambda_{\rm NC} = \infty$, we recover the results of the standard model (see blue curves in Figs. 1–3). The $\frac{d\sigma_{\rm NC}}{d\cos\theta}$ distributions show a symmetric pattern around $\theta = \frac{\pi}{2}$ with two local maxima at $\theta = \frac{\pi}{4}, \frac{3\pi}{4}$ as in Fig. 6. The dashed curve is for $\Lambda_{\rm NC} = 0.6$ TeV. Others are, however, hidden because they are tiny at $\sqrt{s} = 1.0$ TeV. From the phenomenological point of view the appearance of local maxima at $\theta = \frac{\pi}{4}, \frac{3\pi}{4}$ means that particles are scattered more likely either in forward direction from $\theta = 0$ to $\theta = \frac{\pi}{4}$ or in backward direction from $\theta = \frac{3\pi}{4}$ to $\theta = \pi$. Note that $\frac{d\sigma_{\rm NC}}{d\cos\theta}$ distributions are forward-backward symmetric as in the standard model. In Figs. 7–9, we have shown the same distributions at $\sqrt{s} = 1.5$ TeV. The general features of azimuthal distributions are as those in Figs. 4 and 5 except that in the present case crests are much sharper for $\Lambda_{\rm NC} = 0.6$ while others are hidden. Observe that the local maxima for $\Lambda_{\rm NC} = 0.8$ are visible in Fig. 7. By comparing the results we can conclude immediately that for a given scale $\Lambda_{\rm NC}$ as the center of mass energy increases crests become sharp and much stronger (compare Figs. 4–6 respectively with Figs. 5–7). Again, by increasing $\Lambda_{\rm NC}$, the oscillation amplitudes collapse and disappear as before.

Next, we consider the nonminimal noncommutative standard model. Figures 10–12 display the numerical results of the nonminimal model at $\sqrt{s} = 1.5$ TeV. The phenomenological implications of the minimal and nonminimal models can be easily understood and compared using Figs. 7–12. In the context of the nonminimal model, azimuthal distributions up to a numerical factor of order 10^5 are essentially the same as those in the minimal model. The $\frac{d\sigma_{\rm NC}}{d\cos\theta}$ distributions are, however, much different from similar distributions in the minimal case both in shape and

TABLE I. Number of signals in the mNCSM, nmNCSM, and SM at integrated luminosity 100 fb^{-1}.

Model	$\sqrt{s}$ (TeV)	Λ_{NC} (TeV)	No. of events
		0.6	1.130×10^{7}
mNCSM	1.0	1.2	3.651×10^{6}
		1.8	3.158×10^{6}
		0.6	1.921×10^{9}
mNCSM	1.5	1.2	1.080×10^{8}
		1.8	6.200×10^{8}
		0.6	5.543×10^{12}
nmNCSM	1.5	1.2	1.790×10^{11}
		1.8	1.036×10^{11}
		0.6	1.054×10^{13}
nmNCSM	2.0	1.2	3.427×10^{12}
		1.8	1.972×10^{11}
	1.0	∞	2.960×10^{6}
SM	1.5	∞	2.862×10^{6}
	2.0	∞	2.763×10^{6}

scale. In this case, $\frac{d\sigma_{\mathrm{NC}}}{d\cos\theta}$ distributions show a strong peak at $\theta = \frac{\pi}{2}$ and outgoing Z^0 bosons are expected to scatter most likely around $\theta = \frac{\pi}{2}$. Let us recall that Z^0 bosons are distributed symmetrically in forward and backward directions. Figures 13–15 exhibit the expected results of the nonminimal model at $\sqrt{s} = 2.0$ TeV. Again, as Λ_{NC} increases, the characteristic oscillations of distributions are suppressed and disappear at $\Lambda_{\mathrm{NC}} = \infty$.

B. Estimation of the number of events in the noncommutative model

The number of events, i.e., the number of $W^+W^- \to ZZ$ (subprocess) scatterings, can be used to give a direct sense of the implications of the noncommutative model. Assuming the integrated luminosity 100 fb^{-1}, we estimated the number of signals in the context of the SM, mNCSM, as well as nmNCSM for some values of Λ_{NC} at c.m. energies $\sqrt{s} = 1.0$, 1.5, and 2.0 TeV in Table I. The last three lines ($\Lambda_{\mathrm{NC}} = \infty$) correspond to predictions of the standard model.

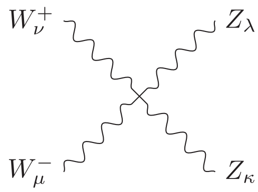
V. SUMMARY AND CONCLUSION

We examined the gauge sector of both the minimal and nonminimal noncommutative standard model and obtained the $\mathcal{O}(\theta)$ Feynman rules for all QGCs. It was found that the Higgs part of the action induces contributions in the electroweak gauge sector of the noncommutative standard model. In the minimal case and up to the leading order of the deformation quantity, the electroweak gauge sector of the model is the same as that of the standard model and only the Higgs sector induced interactions contribute to $\mathcal{O}(\theta)$ Feynman rules for gauge boson couplings. These contributions are of dimension 4 and momentum independent. In contrast, in the nonminimal case the gauge sector of the model contributes to QGCs through dimension-6 and momentum dependent interactions. Also, two anomalous couplings appear in the nonminimal model where the photon as well as the neutral weak boson are coupled directly to three gluons. Such an electroweak-chromodynamics mixing is forbidden in the standard model at tree level. We studied the phenomenological implications of the model in $W^-W^+ \to ZZ$ scattering and showed that noncommutativity of spacetime manifests itself through a characteristic oscillatory behavior in the azimuthal distribution of differential cross sections. In particular, for the case of space-space noncommutativity we evaluated scattering cross sections at $\sqrt{s} = 1.0$, 1.5, 2.0 TeV for Λ_{NC} from 0.6 to 1.8 TeV and found that the $\frac{d\sigma_{\mathrm{NC}}}{d\phi}$ distributions at $\theta = \frac{\pi}{2}$ as well as the integrated $\frac{d\sigma_{\mathrm{NC}}}{d\phi}$ distributions show a sinusoidal behavior with crests at $\phi = \frac{3\pi}{4}, \frac{7\pi}{4}$. For a given c.m. energy as Λ_{NC} increases crests smoothly collapse and disappear at large scales. On the other hand, for a fixed value of Λ_{NC} by increasing the c.m. energy crests become much stronger and appear as sharp peaks. Also, the $\frac{d\sigma_{\mathrm{NC}}}{d\cos\theta}$ distributions show a symmetric pattern around $\theta = \frac{\pi}{2}$. However, the patterns for minimal and nonminimal models are different in shape. In the minimal model, two separate crests appear at $\theta = \frac{\pi}{4}, \frac{3\pi}{4}$ while in the nonminimal case there is a central peak at $\theta = \frac{\pi}{2}$ and curves smoothly disappear in forward-backward directions. Analysis of $\frac{d\sigma_{\mathrm{NC}}}{d\cos\theta}$ distributions indicates that in the minimal model the number of events, in comparison with the standard model, increases considerably in the backward direction from $\theta = 0$ to $\theta = \frac{\pi}{4}$ and also in forward direction from $\theta = \frac{3\pi}{4}$ to $\theta = \pi$ while from $\theta = \frac{\pi}{4}$ to $\theta = \frac{3\pi}{4}$ this remains essentially the same. In contrast, in the nonminimal case the number of events is expected to increase from $\theta = \frac{\pi}{4}$ to $\theta = \frac{3\pi}{4}$ with a sharp maximum at $\theta = \frac{\pi}{2}$. However, in both the models the scattering is forward-backward symmetric. Assuming the integrated luminosity 100 fb^{-1}, we estimated the number of $W^-W^+ \to ZZ$ scatterings in both the minimal and nonminimal noncommutative models for some values of Λ_{NC} at c.m. energies $\sqrt{s} = 1.0$, 1.5, 2.0 TeV and compared the results with predictions of the standard model. The number of events is expected to increase by a factor of order 10^1 up to 10^5 in some cases.

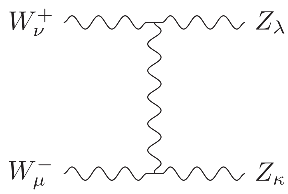
APPENDIX A: FEYNMAN DIAGRAMS

In the standard model, there are four diagrams that contribute to $W^-W^+ \to ZZ$ scattering. These are contact coupling, t-channel, u-channel, and Higgs-mediated s-channel diagrams. In the context of the noncommutative standard model the scattering amplitude of these diagrams is evaluated using the θ-expanded vertex functions. Also, two new diagrams 5 and 6 are allowed in the noncommutative extended model. Observe that the last diagram contributes only in the nonminimal model.

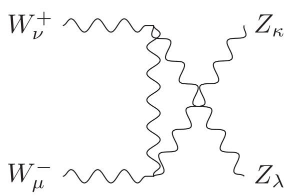
(1) Contact



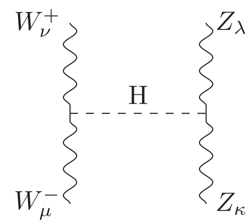
(2) t channel



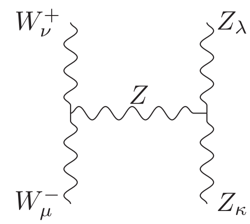
(3) u channel,



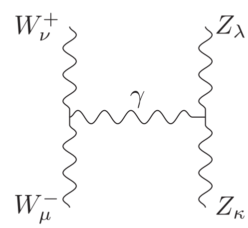
(4) s channel,



(5) s channel,



(6) s channel,



APPENDIX B: FIGURES

See Figs. 1–15.

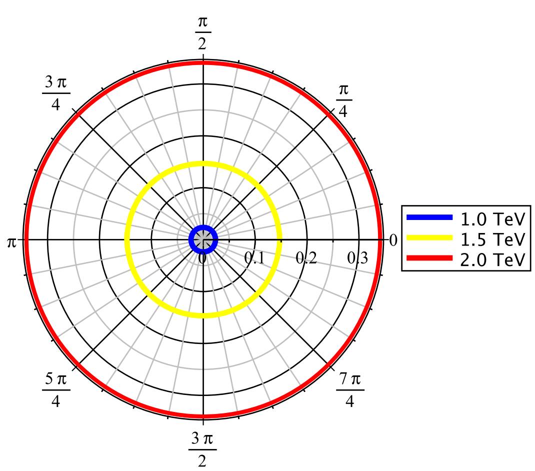


FIG. 1. Differential distributions $\frac{d\sigma}{d\phi}$ at $\theta = \frac{\pi}{2}$ in the SM for $\sqrt{s} = 1.0,\ 1.5,\ 2.0$ TeV.

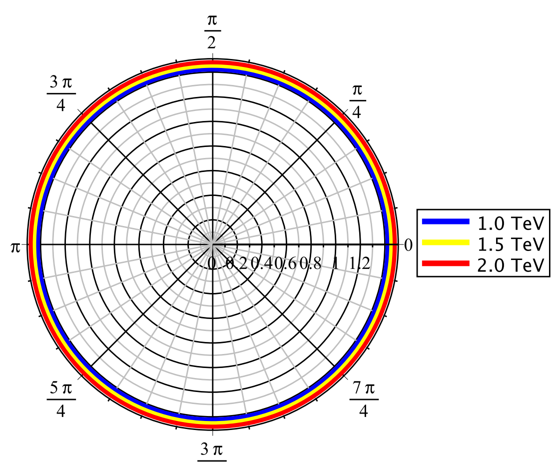


FIG. 2. The θ-integrated $\frac{d\sigma}{d\phi}$ distributions in the SM for $\sqrt{s} = 1.0,\ 1.5,\ 2.0$ TeV.

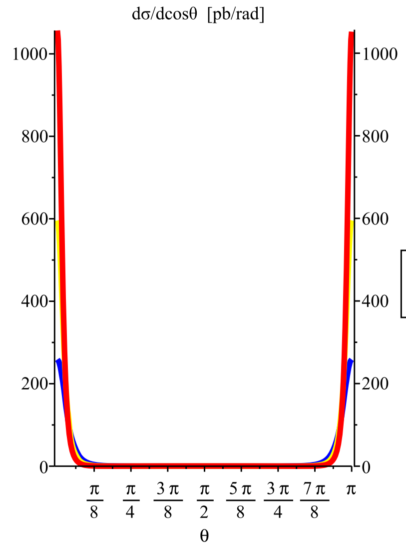


FIG. 3. The ϕ-integrated $\frac{d\sigma}{d\cos\theta}$ distributions in the SM for $\sqrt{s} = 1.0,\ 1.5,\ 2.0$ TeV.

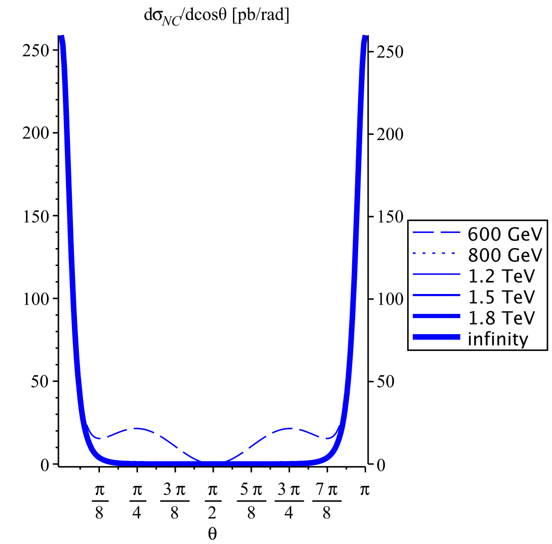


FIG. 6. The ϕ-integrated $\frac{d\sigma_{\rm NC}}{d\cos\theta}$ distributions in the mNCSM for $\sqrt{s} = 1.0$ TeV.

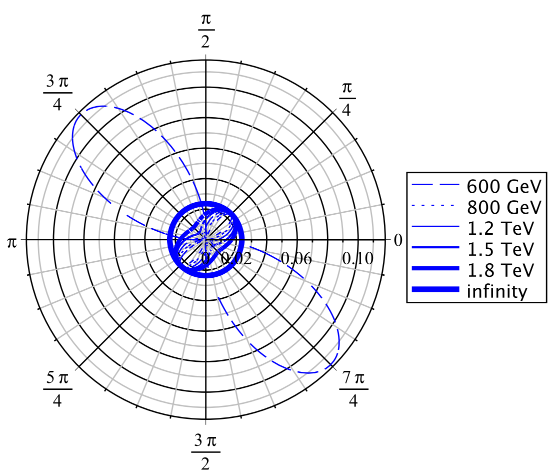


FIG. 4. Differential distributions $\frac{d\sigma_{\rm NC}}{d\phi}$ at $\theta = \frac{\pi}{2}$ in the mNCSM for $\sqrt{s} = 1.0$ TeV.

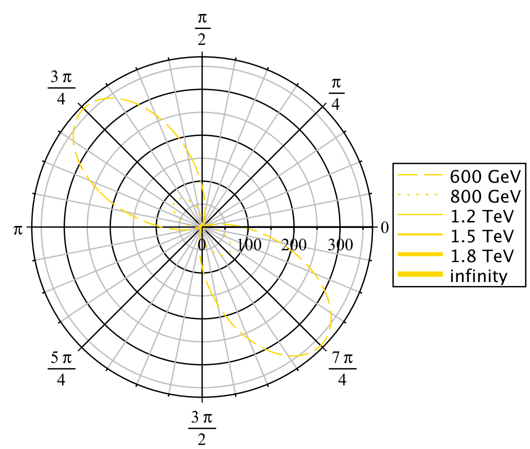


FIG. 7. Differential distributions $\frac{d\sigma_{\rm NC}}{d\phi}$ at $\theta = \frac{\pi}{2}$ in the mNCSM for $\sqrt{s} = 1.5$ TeV.

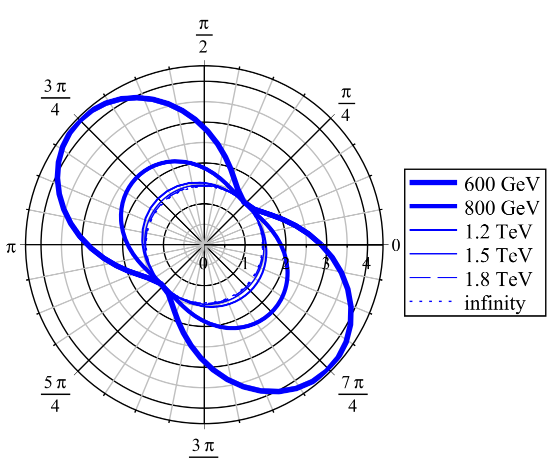


FIG. 5. The θ-integrated $\frac{d\sigma_{\rm NC}}{d\phi}$ distributions in the mNCSM for $\sqrt{s} = 1.0$ TeV.

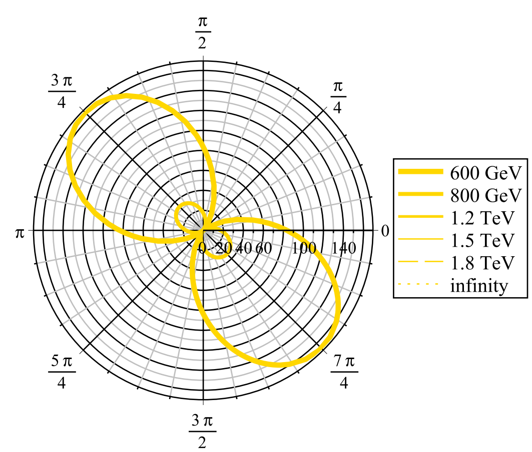


FIG. 8. The θ-integrated $\frac{d\sigma_{\rm NC}}{d\phi}$ distributions in the mNCSM for $\sqrt{s} = 1.5$ TeV.

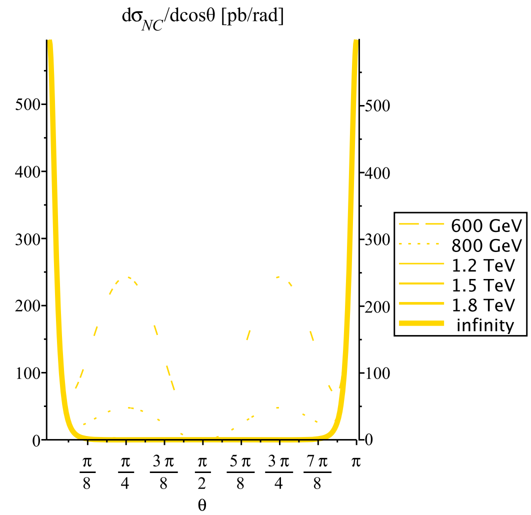


FIG. 9. The ϕ-integrated $\frac{d\sigma_{\rm NC}}{d\cos\theta}$ distributions in the mNCSM for $\sqrt{s} = 1.5$ TeV.

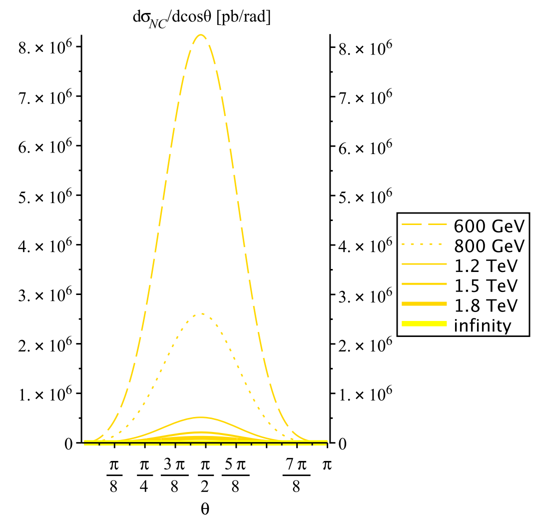


FIG. 12. The ϕ-integrated $\frac{d\sigma_{\rm NC}}{d\cos\theta}$ distributions in the nmNCSM for $\sqrt{s} = 1.5$ TeV.

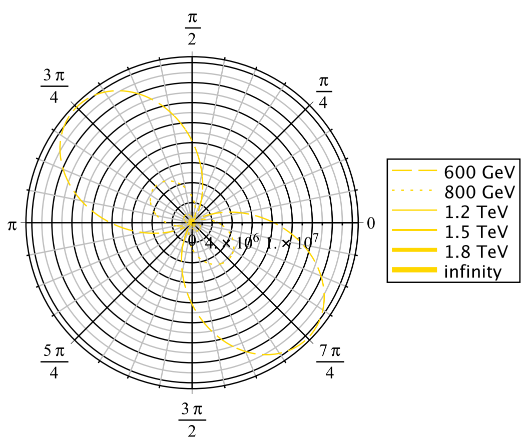


FIG. 10. Differential distributions $\frac{d\sigma_{\rm NC}}{d\phi}$ at $\theta = \frac{\pi}{2}$ in the nmNCSM for $\sqrt{s} = 1.5$ TeV.

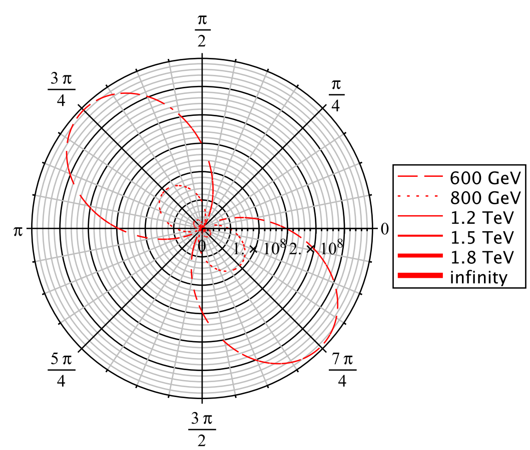


FIG. 13. Differential distributions $\frac{d\sigma_{\rm NC}}{d\phi}$ at $\theta = \frac{\pi}{2}$ in the nmNCSM for $\sqrt{s} = 2.0$ TeV.

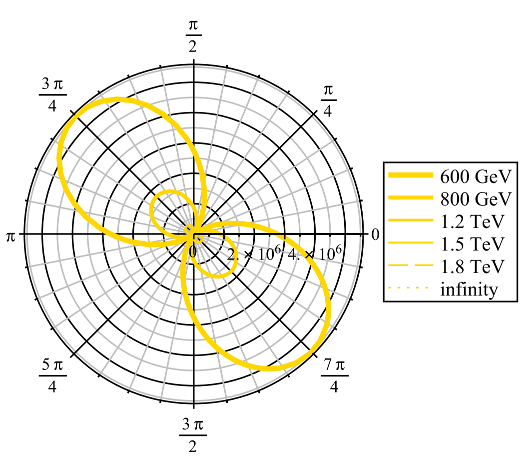


FIG. 11. The θ-integrated $\frac{d\sigma_{\rm NC}}{d\phi}$ distributions in the nmNCSM for $\sqrt{s} = 1.5$ TeV.

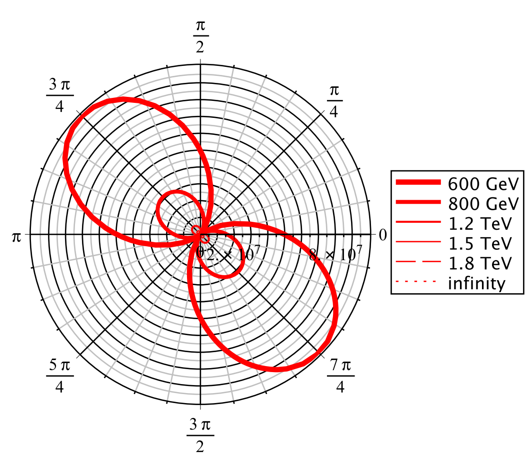


FIG. 14. The θ-integrated $\frac{d\sigma_{\rm NC}}{d\phi}$ distributions in the nmNCSM for $\sqrt{s} = 2.0$ TeV.

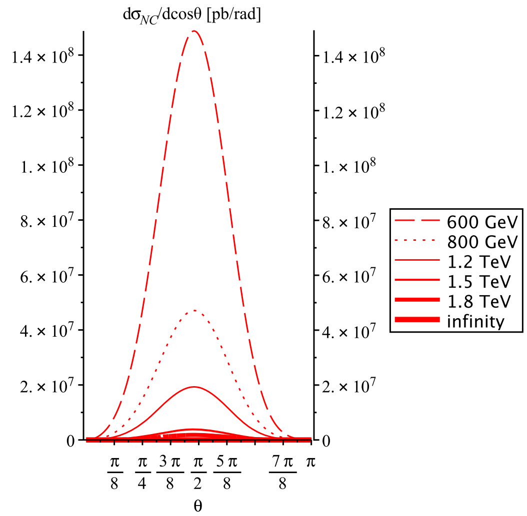


FIG. 15. The ϕ-integrated $\frac{d\sigma_{\rm NC}}{d\cos\theta}$ distributions in the nmNCSM for $\sqrt{s} = 2.0$ TeV.

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