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Collider signals of W' and Z' bosons in the gauge-Higgs unification

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In the $SO(5)\times U(1)$ gauge-Higgs unification (GHU), Kaluza-Klein (KK) excited states of charged and neutral vector bosons, $W^{(1)}$, $W_R^{(1)}$, $Z^{(1)}$, $\gamma^{(1)}$, and $Z_R^{(1)}$, can be observed as W' and Z' signals in collider experiments. In this paper we evaluate the decay rates of the W' and Z', and s-channel cross sections mediated by W' and Z' bosons with final states involving the standard model (SM) fermion pair ($\ell\nu$, $\ell\bar{\ell}$, $q\bar{q}'$), WH, ZH, WW, and WZ. W' and Z' resonances appear around 6.0 TeV (8.5 TeV) for $\theta_H = 0.115$ (0.0737) where θ_H is the Aharonov-Bohm phase in the fifth dimension in GHU. For decay rates we find $\Gamma(W'\to WH)\simeq\Gamma(W'\to WZ)$ $(W'=W^{(1)},\ W_R^{(1)})$, $\Gamma(W^{(1)}\to WH, WZ)\sim\Gamma(W_R^{(1)}\to WH, WZ)$, $\Gamma(Z^{(1)}\to ZH)\simeq\sum_{Z'=Z^{(1)},\gamma^{(1)}}\Gamma(Z'\to WW)$, and $\Gamma(Z_R^{(1)}\to ZH)\simeq\Gamma(Z_R^{(1)}\to WW)$. W' and Z' signals of GHU can be best found at the LHC experiment in the processes $pp\to W'(Z')+X$ followed by $W'\to t\bar{b}$, WH, and $Z'\to e^+e^-$, $\mu^+\mu^-$, ZH near the W' and Z' resonances. For the lighter Z' ($\theta_H = 0.115$) case, with forthcoming 30 fb^{-1} data of the 13 TeV LHC experiment we expect about ten $\mu^+\mu^-$ events for the invariant mass range 3000 to 7000 GeV, though the number of the events becomes much smaller when $\theta_H = 0.0737$. In the process with WZ in the final state, it is confirmed that the leading contributions in the amplitude from the longitudinal polarizations of W and Z in the s-, t-, and u-channels cancel with each other so that the unitarity is preserved, provided that all KK excited states in the intermediate states are taken into account. Deviation of the WWZ coupling from the SM is very tiny. Exotic partners $t_T^{(1)}$ and $b_Y^{(1)}$ of the top and bottom quarks with electric charge $+5/3$ and $-4/3$ have mass $M_{t_T^{(1)},b_Y^{(1)}} = 4.6$ TeV (5.4 TeV) for $\theta_H = 0.115$ (0.0737), becoming the lightest non-SM particles in GHU which can be singly produced in collider experiments.

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I. INTRODUCTION

At the LHC, W' and Z' are searched with various decay modes: decay to lepton pairs ($\ell\bar{\ell}$, $\ell\nu$, $\nu\bar{\nu}$) [1–6], a pair of top and bottom [7–9], $q\bar{q}$-dijet [10,11], $WH(\to\ell\nu b\bar{b})$ and $ZH(\to\ell\bar{\ell}b\bar{b})$ [12–15], and WW and WZ [16–22]. In the models with extra dimensions, W' and Z' appear as Kaluza-Klein (KK) excited states of charged and neutral vector bosons. The couplings of W' and Z' to fields in the standard model (SM) depend on the details of the models. For example, in the minimal universal extra dimension (mUED) model [23], the conservation of KK number forbids tree-level couplings of KK-excited states to the SM fields so that production of W' or Z' in the collider experiment is highly suppressed.

Nonvanishing couplings of W' and Z' to the SM fields in models with extra-dimensions originate from the violation of the KK number conservation which reflects the translational invariance in the direction of the extra dimensional space. The KK number conservation is broken by, for instance, domain-wall like bulk mass terms, brane localized mass terms to the fermion, or warped extra dimensions. It is well known that in the models in which SM fields live in the warped bulk space, couplings among fermions and KK-excited gauge bosons are nonvanishing and can be large. In the minimal $SU(3)$ gauge-Higgs unification (GHU) model [24] there are no W' or Z' couplings to the SM fields. In a custodially protected warped extra dimensional model [25], WW and WH diboson signals have been studied.

The GHU provides a natural scenario for solving the gauge hierarchy problem. Many advances have been made in this direction recently [24,26–71]. In the present paper, we evaluate couplings of KK excited gauge bosons to the SM gauge boson, Higgs boson, and fermions, and study collider signals of W' and Z' in the $SO(5)\times U(1)$ GHU model which naturally incorporates the Higgs boson of mass $m_H =$ 125 GeV and gives almost the same phenomenology as the SM at low energies [46–49]. In the previous work [47], we reported that in hadron collider experiments large Z' signals are expected due to the large couplings of right-handed fermions to the KK-excited gauge bosons [72]. In the present paper we evaluate cross sections not only with fermionic final states but also with bosonic WH, ZH, WW and WZ final states.

In the warped space, in general, it is difficult to evaluate couplings among various fields as they should be calculated in their mass eigenstates. One remarkable feature of the GHU is that the Higgs vacuum expectation value (VEV) can be eliminated by a large gauge-transformation and its effect is transmitted to the change in the boundary conditions so that one can obtain mass eigenstates easily. In the previous work [49], it is found that KK nonconserving couplings $HW^{(m)}W^{(n)}$ and $ZW^{(m)}W^{(n)}$ ($m \neq n$), including $HWW^{(n)}$ and $ZWW^{(n)}$, are nonvanishing, and we used them in the calculation of the $H \to Z\gamma$ decay rate.

This paper is organized as follows. In Sec. II, the model is introduced. In Sec. III decay width and cross sections formulas are given. In Sec. IV we introduce effective theories to qualitatively describe salient relations among various decay widths. In Sec. V, couplings and decay widths of W' and Z' are evaluated in GHU. In Sec. VI cross sections are evaluated. W' and Z' signals in pp collision experiment at LHC are explored. We also show how the unitarity in the process $d\bar{u} \to WZ$ is ensured by including contributions from KK states of vector bosons and fermions in the intermediate states. Section VII is devoted to summary. In Appendix A $SO(5)$ generators and basis functions in the analysis are summarized. In Appendices B and C masses and wave functions for bosonic and fermionic KK states are given, respectively. In Appendix D fermion couplings to vector bosons and the Higgs boson are summarized, whereas cubic vector couplings and Higgs couplings are given in Appendix E. In Appendices F and G, formulas for decay widths and scattering cross sections are summarized, respectively.

II. MODEL

We consider five-dimensional (5D) gauge theory in the Randall-Sundrum space-time, whose metric is

$$ds^2 = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 = G_{MN}dx^M dx^N, \qquad (2.1)$$

where $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$. Here $\sigma(y) = k|y|$ for $-L \leq y \leq L$ and $\sigma(y + 2L) = \sigma(y)$ is satisfied. k is the AdS_5 curvature. $y = 0$ and $y = L$ boundaries are referred to as the UV (ultraviolet) and IR (infrared) branes, respectively. The Kaluza-Klein (KK) mass scale is given by

$$m_{KK} \equiv \frac{\pi k}{e^{kL} - 1}, \qquad (2.2)$$

and when $kL \gtrsim 5$, it is followed by $m_{KK} \ll k$, L^{-1}.

This space-time has symmetric under the Z_2 reflection $y \to -y$, and fundamental region of the extra dimension is given by $0 \leq y \leq L$. In this region we introduce a new coordinate $z = e^{ky}$, with which the metric becomes

$$ds^2 = \frac{1}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + \frac{dz^2}{k^2}\right). \qquad (2.3)$$

Note that $\partial_y = kz\partial_z$ and $V_y = kzV_z$ for a vector field V_M.

In the 5D bulk space there are $SO(5)$ and $U(1)_X$ gauge fields, four $SO(5)$-vector fermions per generation Ψ_a^g ($a = 1, 2, 3, 4$, $g = 1, 2, 3$), and N_F $SO(5)$-spinor fermions Ψ_{F_i} ($i = 1, \ldots, N_F$). We note that each of Ψ_1 and Ψ_2 is an $SU(3)$-color triplet.

The bulk part of the action is given by

$$\begin{aligned} S_{\mathrm{bulk}} = \int d^5x\sqrt{-G}\Big\{&-\frac{1}{2}\mathrm{tr}G^{MR}G^{NS}F^{(A)}_{MN}F^{(A)}_{RS} \\ &-\frac{1}{4}G^{MR}G^{NS}F^{(B)}_{MN}F^{(B)}_{RS} + \frac{1}{2\xi_{(A)}}(f^{(A)}_{\mathrm{gf}})^2 \\ &+\frac{1}{2\xi_{(B)}}(f^{(B)}_{\mathrm{gf}})^2 + \mathcal{L}^{(A)}_{GH} + \mathcal{L}^{(B)}_{GH} \\ &+\sum_{g=1}^{3}\sum_{a=1}^{4}\bar{\Psi}^g_a\mathcal{D}(c^g_a)\Psi^g_a + \sum_{i=1}^{N_F}\bar{\Psi}_{F_i}\mathcal{D}(c_{F_i})\Psi_{F_i}\Big\}, \end{aligned} \qquad (2.4)$$

$$\begin{aligned}\mathcal{D}(c_a) \equiv \Gamma^A {e_A}^M\bigg(&\partial_M + \frac{1}{8}\Omega_{MBC}[\Gamma^B, \Gamma^C] - ig_A A_M \\ &- ig_B Q_{X,a} - c_a k\epsilon(y)\bigg),\end{aligned} \qquad (2.5)$$

where $\epsilon(y) \equiv \sigma'/k$ is a sign function. Γ^M denotes gamma matrices which is defined by $\{\Gamma^M, \Gamma^N\} = 2\eta^{MN}$ ($\eta^{55} = +1$). ${e_A}^M$ is an inverse fielbein, and Ω_{MBC} is the spin connection. $F^{(A)}_{MN} = \partial_M A_N - \partial_N A_M - ig_A[A_M, A_N]$ and $F^{(B)}_{MN} = \partial_M B_N - \partial_N B_M$. g_A and g_B are 5D gauge couplings of $SO(5)$ and $U(1)_X$, respectively. $g_w \equiv g_A/\sqrt{L}$ is the four-dimensional (4D) $SO(5)$ coupling. $f^{(A)}_{\mathrm{gf}}$ and $f^{(B)}_{\mathrm{gf}}$ are gauge-fixing functions, and $\xi_{(A)}$ and $\xi_{(B)}$ are corresponding gauge parameters. $\mathcal{L}^{(A)}_{GH}$ and $\mathcal{L}^{(B)}_{GH}$ denote ghost Lagrangians.

Bulk fermions are $SO(5)$-vectors. For the third generation, they are given by

$$\begin{aligned}\Psi_1 &= \left(\begin{pmatrix} T & t \\ B & b\end{pmatrix}, t'\right) = ((Q_1, q), t') = (\check{\Psi}^q_1, t'), \\ \Psi_2 &= \left(\begin{pmatrix} U & X \\ D & Y\end{pmatrix}, b'\right) = ((Q_2, Q_3), b') = (\check{\Psi}^q_2, b'), \\ \Psi_3 &= \left(\begin{pmatrix} \nu_\tau & L_{1X} \\ \tau & L_{1Y}\end{pmatrix}, \tau'\right) = ((\ell, L_1), \tau'), = (\check{\Psi}^\ell_3, \tau'), \\ \Psi_4 &= \left(\begin{pmatrix} L_{2X} & L_{3X} \\ L_{2Y} & L_{3Y}\end{pmatrix}, \nu'_\tau\right) = ((L_2, L_3), \nu'_\tau), = (\check{\Psi}^\ell_4, \nu'_\tau),\end{aligned} \qquad (2.6)$$

where $SO(4)$ vector is embedded in $(\mathbf{2}, \mathbf{2})$-representation of $SU(2)_L \times SU(2)_R$ by

$$\begin{aligned}\begin{pmatrix}\psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22}\end{pmatrix} &= \frac{1}{\sqrt{2}}(\psi_4 + i\vec{\sigma}\cdot\vec{\psi})i\sigma_2 \\ &= \begin{pmatrix}-i\psi_1 - \psi_2 & i\psi_3 + \psi_4 \\ i\psi_3 - \psi_4 & i\psi_1 - \psi_2\end{pmatrix}.\end{aligned} \qquad (2.7)$$

The brane part of the action consists of scalar part $S^{\Phi}_{\mathrm{brane}}$ and brane-fermion part $S^{\chi}_{\mathrm{brane}}$. The scalar part is given by

$$S^{\Phi}_{\rm brane} = \int d^5x\sqrt{-G}\delta(y)[-(D_\mu\Phi)^\dagger(D^\mu\Phi) - \lambda_\Phi(|\Phi|^2-w^2)^2],$$
$$D_\mu\Phi = \left(\partial_\mu - i\left\{g_A\sum_{a_R=1}^{3}A_\mu^{a_R}T^{a_R}+\frac{1}{2}g_BB_\mu\right\}\right)\Phi. \tag{2.8}$$

The fermion part of the brane action is

$$\begin{aligned}
S^{\chi}_{\rm brane} &= \int d^5x\sqrt{-G}\delta(y)\{\mathcal{L}_q+\mathcal{L}_\ell\},\\
\mathcal{L}_q &\equiv \sum_{g=1}^{3}\sum_{\alpha=1}^{3}(\hat\chi^{q,g\dagger}_{\alpha R}i\bar\sigma^\mu D_\mu\hat\chi^{q,g}_{\alpha R})\\
&\quad - i\sum_{g,g'=1}^{3}[\kappa_1^{q,gg'}\hat\chi^{q,g\dagger}_{1R}\check\Psi^{q,g'}_{1L}\tilde\Phi+\tilde\kappa^{q,gg'}\hat\chi^{q,g\dagger}_{2R}\check\Psi^{q,g'}_{1L}\Phi\\
&\quad + \kappa_2^{q,gg'}\hat\chi^{q,g\dagger}_{2R}\check\Psi^{g'}_{2L}\tilde\Phi+\kappa_3^{q,gg'}\hat\chi^{q,g\dagger}_{3R}\check\Psi^{g'}_{2L}\Phi-(\text{H.c.})],\\
\mathcal{L}_\ell &\equiv \sum_{g=1}^{3}\sum_{\alpha=1}^{3}(\hat\chi^{\ell,g\dagger}_{\alpha R}i\bar\sigma^\mu D_\mu\hat\chi^{\ell,g}_{\alpha R})\\
&\quad - i\sum_{g,g'=1}^{3}[\tilde\kappa^{\ell,gg'}\hat\chi^{\ell,g\dagger}_{3R}\check\Psi^{\ell,g'}_{3L}\tilde\Phi+\kappa_1^{\ell,gg'}\hat\chi^{\ell,g\dagger}_{1R}\check\Psi^{\ell,g'}_{3L}\Phi\\
&\quad + \kappa_2^{\ell,gg'}\hat\chi^{\ell,g\dagger}_{2R}\check\Psi^{\ell,g'}_{4L}\tilde\Phi+\kappa_3^{\ell,gg'}\hat\chi^{\ell,g\dagger}_{3R}\check\Psi^{\ell,g'}_{4L}\Phi-(\text{H.c.}),
\end{aligned} \tag{2.9}$$

$$D_\mu\hat\chi = \left(\partial_\mu - ig_A\sum_{a_L=1}^{3}A_\mu^{a_L}T^{a_L}-iQ_Xg_BB_\mu\right)\hat\chi,$$
$$\tilde\Phi \equiv i\sigma_2\Phi^*, \tag{2.10}$$

where

$$\hat\chi^q_{1R} = \begin{pmatrix}\hat T_R\\ \hat B_R\end{pmatrix}_{7/6},\qquad \hat\chi^q_{2R} = \begin{pmatrix}\hat U_R\\ \hat D_R\end{pmatrix}_{1/6},\qquad \hat\chi^q_{3R} = \begin{pmatrix}\hat X_R\\ \hat Y_R\end{pmatrix}_{-5/6}, \tag{2.11}$$

$$\hat\chi^\ell_{1R} = \begin{pmatrix}\hat L_{1XR}\\ \hat L_{1YR}\end{pmatrix}_{-3/2},\qquad \hat\chi^\ell_{2R} = \begin{pmatrix}\hat L_{2XR}\\ \hat L_{2YR}\end{pmatrix}_{1/2},\qquad \hat\chi^\ell_{3R} = \begin{pmatrix}\hat L_{3XR}\\ \hat L_{3YR}\end{pmatrix}_{-1/2}, \tag{2.12}$$

are right-handed brane fermions. We note that each of $\hat\chi^q_{1R}$, $\hat\chi^q_{2R}$, and $\hat\chi^q_{3R}$ is an $SU(3)$-color triplet. We also have introduced 3×3 Yukawa coupling matrices $\kappa^q_{1,2,3}$, $\kappa^\ell_{1,2,3}$, $\tilde\kappa^q$, and $\tilde\kappa^\ell$.

A. Orbifold symmetry breaking

We impose Z_2 boundary conditions at boundaries $y=y_i$, $y_0\equiv0$, $y_1\equiv L$.

$$\begin{pmatrix}A_\mu\\ A_y\end{pmatrix}(x,y_i-y) = P_i\begin{pmatrix}A_\mu\\ -A_y\end{pmatrix}(x,y_i+y)P_i^{-1}, \tag{2.13}$$

$$\begin{pmatrix}B_\mu\\ B_y\end{pmatrix}(x,y_i-y) = \begin{pmatrix}B_\mu\\ -B_y\end{pmatrix}(x,y_i+y), \tag{2.14}$$

$$\Psi_a(x,y_i-y) = \gamma_5P_i\Psi(x,y_i+y)_a, \tag{2.15}$$

where

$$P_0^{\rm vec} = P_1^{\rm vec} = \text{diag}(-1,-1,-1,-1,+1),$$
$$P_0^{\rm sp} = P_1^{\rm sp} = \begin{pmatrix}\mathbf{1} & \\ & \mathbf{-1}\end{pmatrix} \tag{2.16}$$

in the vector and spinor representations, respectively. These boundary conditions break $SO(5)$ to $SO(4)\simeq SU(2)_L\times SU(2)_R$. $A_\mu^{a_L,a_R}$ and $A_y^{\hat a}$ are even function against reflections at $y=y_i$ and can have their zero modes. Zero modes of $A_\mu^{a_L,a_R}$ are the gauge fields of unbroken $SO(4)$ symmetry.

B. Symmetry breaking by brane scalar

Once Φ develops a VEV

$$\langle\Phi\rangle = \begin{pmatrix}0\\ w\end{pmatrix}, \tag{2.17}$$

$SU(2)_R\times U(1)_X$ symmetry is broken to $U(1)_Y$. After Φ develops a VEV, the boundary conditions in the original gauge are given by

$$\begin{aligned}
&\text{at } z=1:\ \partial_zA_\mu^{a_L} = \left(\partial_z-\frac{\kappa}{2k}\right)A_\mu^{1_R,2_R}\\
&\qquad = \left(\partial_z-\frac{\kappa'}{2k}\right)A_\mu^{3'_R} = \partial_zB_\mu^{Y'} = 0,\\
&A_\mu^{\hat a} = A_\mu^{\hat 4} = 0,\\
&A_z^{a_L} = A_z^{a_R}+B_z = 0,\quad \partial_z\left(\frac{1}{z}A_z^{\hat a}\right) = \partial_z\left(\frac{1}{z}A_z^{\hat 4}\right) = 0,\\
&\text{at } z=z_L:\ \partial_zA_\mu^{a_L} = \partial_zA_\mu^{a_R} = \partial_zB_\mu^X = 0,\\
&A_\mu^{\hat a} = A_\mu^{\hat 4} = 0,\\
&A_z^{a_L} = A_z^{a_R} = B_z = 0,\qquad \partial_z\left(\frac{1}{z}A_z^{\hat a}\right) = \partial_z\left(\frac{1}{z}A_z^{\hat 4}\right) = 0.
\end{aligned} \tag{2.18}$$

Here we have defined

$$\kappa \equiv \frac{g_A^2 w^2}{4} = \frac{g_w^2 L w^2}{4}, \qquad \kappa' \equiv \frac{(g_A^2 + g_B^2) w^2}{4}, \tag{2.19}$$

and

$$\begin{pmatrix} A_M^{3_R'} \\ B_M^{Y'} \end{pmatrix} \equiv \begin{pmatrix} c_\phi & -s_\phi \\ s_\phi & c_\phi \end{pmatrix} \begin{pmatrix} A_M^{3_R} \\ B_M \end{pmatrix},$$
$$c_\phi \equiv \cos\phi = \frac{g_A}{\sqrt{g_A^2 + g_B^2}},$$
$$s_\phi \equiv \sin\phi = \frac{g_B}{\sqrt{g_A^2 + g_B^2}}, \tag{2.20}$$

where a mixing angle ϕ is defined by $\tan\phi \equiv g_B/g_A$. KK modes of $A_\mu^{1,2_R}$ and $A_{\mu'}^{3_R}$ with $m_n \ll w$ obey effectively Dirichlet boundary conditions on the UV brane: $A_\mu^{1_R,2_R} = A_\mu^{3_R'} = 0$ at $z = 1$.

For fermions, nonvanishing $\langle\Phi\rangle$ also induces brane mass terms given by

$$S_{\rm brane}^{\rm mass} = \int d^5x \sqrt{-G}\delta(y) \{\mathcal{L}_{\rm quark}^{\rm mass} + \mathcal{L}_{\rm lepton}^{\rm mass}\}, \tag{2.21}$$

$$\mathcal{L}_{\rm quark}^{\rm mass} = \sum_{g,g'=1}^{3} \bigg[-\sum_{\alpha=1}^{3} i\mu_\alpha^{q,gg'} (\hat\chi_{\alpha R}^{q,g\dagger} Q_{\alpha L}^{g'} - Q_{\alpha L}^{g'\dagger} \hat\chi_{\alpha R}^{q,g}) - i\tilde\mu^{q,gg'} (\hat\chi_{2R}^{q,g\dagger} q_L^{g'} - q_L^{g'\dagger} \hat\chi_{2R}^{q,g}) \bigg], \tag{2.22}$$

$$\mathcal{L}_{\rm lepton}^{\rm mass} = \sum_{g,g'=1}^{3} \bigg[-\sum_{\alpha=1}^{3} i\mu_\alpha^{\ell,gg'} (\hat\chi_{\alpha L}^{\ell,g\dagger} L_{\alpha L}^{g'} - L_{\alpha L}^{g'\dagger} \hat\chi_{\alpha R}^{\ell,g}) - i\tilde\mu^{\ell,gg'} (\hat\chi_{3R}^{\ell,g\dagger} \ell_L^{g'} - \ell_L^{g'\dagger} \hat\chi_{3R}^{\ell,g}) \bigg], \tag{2.23}$$

where

$$\frac{\mu_\alpha^{q,gg'}}{\kappa_\alpha^{q,gg'}} = \frac{\tilde\mu^{q,gg'}}{\tilde\kappa^{q,gg'}} = \frac{\mu_\alpha^{\ell,gg'}}{\kappa_\alpha^{\ell,gg'}} = \frac{\tilde\mu^{\ell,gg'}}{\tilde\kappa^{\ell,gg'}} = w. \tag{2.24}$$

For $w \gg m_{KK}$, and $\mu_\alpha, \tilde\mu \gg \sqrt{m_{KK}}$, exotic fermions couple to brane fermions to become very heavy so that only quarks and leptons remain at low energy.

Since the gauge field A_y plays the role of the Higgs boson, the Yukawa couplings of quarks and leptons are diagonal in the flavor space, and flavor mixing can be induced by nondiagonal brane mass terms. For simplicity we assume that all brane mass terms are flavor-diagonal:

$$\mu_\alpha^{q,gg'} = \delta^{gg'} \mu_\alpha^q, \qquad \mu_\alpha^{\ell,gg'} = \delta^{gg'} \mu_\alpha^\ell, \qquad \alpha = 1,2,3,$$
$$\tilde\mu^{q,gg'} = \delta^{gg'} \tilde\mu^q, \qquad \tilde\mu^{\ell,gg'} = \delta^{gg'} \tilde\mu^\ell. \tag{2.25}$$

C. Electroweak symmetry breaking

$A_z^{\hat a}$ ($a = 1$, 2, 3 and 4) have their zero modes:

$$A_z^{\hat a}(x,z) = \phi^a(x) \sqrt{\frac{2}{k(z_L^2 - 1)}} \cdot z + \ldots, \tag{2.26}$$

(where "$\cdots$" includes higher-KK modes) and can develop a VEV. We assume that A_z develops a VEV in the direction of $T^{\hat 4}$ and we parametrize it by $\langle\phi^a\rangle = v_W \delta^{4a}$. Then we define the Wilson-line phase parameter θ_H by

$$\exp\left[\frac{i}{2}\theta_H (2\sqrt{2} T^{\hat 4})\right] = \exp\left[ig_A \int_0^{z_L} \langle A_z\rangle dz\right], \tag{2.27}$$

so that we obtain

$$\theta_H = \frac{1}{2} g_A v_W \sqrt{\frac{z_L^2 - 1}{k}} \sim \frac{g_2 v_W}{2} \frac{\pi\sqrt{kL}}{m_{KK}}, \tag{2.28}$$

where $g_w \equiv g_A/\sqrt{L}$ is the 4D $SO(4) \simeq SU(2)_L \times SU(2)_R$ gauge coupling constant. We also have a formula of W-boson mass.

$$m_W \simeq \frac{m_{KK}}{\pi\sqrt{kL}} |\sin\theta_H|, \tag{2.29}$$

and for $\theta_H \ll 1$, $m_W = \frac{1}{2} g_w v_W$ is obtained. This may be compared with the SM formula $m_W = \frac{1}{2} g_w v_H$, $v_H = 246$ GeV.

To solve the equations of motion, we move to the twisted gauge in which $\langle\tilde A_z\rangle = 0$. This is achieved by

$$\tilde A_M = \Omega A_M \Omega^{-1} + \frac{i}{g_A} \Omega(\partial_M \Omega^{-1}), \qquad \tilde\Psi = \Omega\Psi, \tag{2.30}$$

$$\Omega = \exp[i\theta(z)\sqrt{2} T^{\hat 4}], \qquad \theta(z) = \theta_H \frac{z_L^2 - z^2}{z_L^2 - 1}. \tag{2.31}$$

Using Ω and making use of $SO(5)$ algebra, we find gauge transformation as

$$A_M^{a_L} = \frac{1}{\sqrt{2}} \{\tilde A_M^{a_+} + \tilde A_M^{a_-} \cos\theta(z) - \tilde A_M^{\hat a} \sin\theta(z)\},$$
$$A_M^{a_R} = \frac{1}{\sqrt{2}} \{\tilde A_M^{a_+} - \tilde A_M^{a_-} \cos\theta(z) + \tilde A_M^{\hat a} \sin\theta(z)\},$$
$$A_M^{\hat a} = \tilde A_M^{a_-} \sin\theta(z) + \tilde A_M^{\hat a} \cos\theta(z), \qquad a = 1,2,3,$$
$$A_\mu^{\hat 4} = \tilde A_\mu^{\hat 4}, \qquad A_z^{\hat 4} = \tilde A_z^{\hat 4} - \frac{\sqrt{2}}{g_A} \theta'(z), \tag{2.32}$$

where $\tilde A_M^{a_\pm} \equiv (\tilde A_M^{a_L} \pm \tilde A_M^{a_R})/\sqrt{2}$.

D. Kaluza-Klein towers

1. Gauge bosons

$SO(5)\times U(1)_X$ gauge fields are decomposed into Kaluza-Klein towers given by

$$A_\mu(x^\mu,z)+B_\mu(x^\mu,z)T^B=\hat{W}_\mu+\hat{W}_\mu^\dagger+\hat{W}_{R\mu}+\hat{W}_{R\mu}^\dagger+\hat{Z}_\mu+\hat{Z}_{R\mu}+\hat{A}_\mu^\gamma+\hat{A}_\mu^{\hat{4}}, \tag{2.33}$$

where T^B is a $U(1)_X$ generator. $\hat{W}$, $\hat{Z}$, and $\hat{A}^\gamma$ are KK towers for W, Z bosons and photons in the SM, respectively. We note that each of $\hat{W}$ and $\hat{Z}$ towers contains two KK towers so that there are eleven KK towers in (2.33).

Each tower has an expansion of the form

$$\begin{aligned}
\hat{A}_\mu^C&=\sum_n A_\mu^{(n)}(x^\mu)\{h^L_{A^{(n)}}(z)T^{-_L}+h^R_{A^{(n)}}(z)T^{-_R}+\hat{h}_{A^{(n)}}(z)T^{\hat{-}}\},\\
\hat{A}_\mu^N&=\sum_n A_\mu^{(n)}(x^\mu)\{h^L_{A^{(n)}}(z)T^{3_L}+h^R_{A^{(n)}}(z)T^{3_R}\\
&\quad+\hat{h}_{A^{(n)}}(z)T^{\hat{3}}+h^B_{A^{(n)}}(z)T^B\},\\
\hat{A}_\mu^{\hat{4}}&=\sum_n A_\mu^{(n)\hat{4}}(x)h_{A^{(n)\hat{4}}}(z)T^{\hat{4}},
\end{aligned} \tag{2.34}$$

for $\hat{A}^C=\hat{W}$, $\hat{W}_R$ and $\hat{A}^N=\hat{Z}$, $\hat{Z}_R$ and $\hat{A}^\gamma$. $T^\pm=(T^1\pm iT^2)/\sqrt{2}$. Explicit forms of KK towers of gauge fields are summarized in Appendix B and also found in [47].

2. A_z and B_z

A_z and B_z are expanded, in the twisted gauge, as

$$\begin{aligned}
\tilde{A}_z(x,z)&=\sum_{a=1}^3\hat{G}^a+\sum_{a=1}^3\hat{D}^a+\hat{H},\\
B_z(x,z)&=\hat{B}=\sum_{n=1}^\infty B^{(n)}(x)u_{B^{(n)}}(z)T^B.
\end{aligned} \tag{2.35}$$

$\hat{D}$ and $\hat{G}$ towers are expanded as

$$\begin{aligned}
\hat{S}^-&=\sum_n S^{-(n)}(x)\{u^L_{S^{(n)}}(z)T^{-_L}+u^R_{S^{(n)}}(z)T^{-_R}+\hat{u}_{S^{(n)}}(z)T^{\hat{-}}\},\\
\hat{S}^3&=\sum_n S^{3(n)}(x)\{u^L_{S^{(n)}}(z)T^{3_L}+u^R_{S^{(n)}}(z)T^{3_R}+\hat{u}_{S^{(n)}}T^{\hat{3}}\},\\
S&=D,G,
\end{aligned} \tag{2.36}$$

whereas $\hat{H}$ is expanded as

$$\hat{H}=\sum_{n=0}^\infty H^{(n)}(x)u_{H^{(n)}}(z)T^{\hat{4}}. \tag{2.37}$$

$H^{(0)}$ corresponds to the SM Higgs boson. $\hat{D}$ contain two KK towers so that $\tilde{A}_z$ contain ten KK towers. We note that KK modes other than $H^{(0)}$ are Nambu-Goldstone bosons and eaten by KK excited states of corresponding vector bosons.

3. Fermions

Bulk fermions are also decomposed into KK towers as follows. For example, quark bulk fermions Ψ_1 and Ψ_2 in the third generation are decomposed into

$$\begin{aligned}
(\Psi_1+\Psi_2)&=\hat{t}_{T(5/3)}+\hat{t}_{(2/3)}+\hat{t}_{B(2/3)}+\hat{t}_{U(2/3)}\\
&\quad+\hat{b}_{(-1/3)}+\hat{b}_{D(-1/3)}+\hat{b}_{X(-1/3)}+\hat{b}_{Y(-4/3)},
\end{aligned} \tag{2.38}$$

where numbers in subscripts denote the electric charge of fields. $\hat{t}$ and $\hat{b}$ are towers for top and bottom quarks, respectively, whereas others are towers for non-SM exotic partners of quarks. We note that each of $\hat{t}$ and $\hat{b}$ contains two KK towers. In all Ψ_1 and Ψ_2 contain ten KK towers of fermions. In the same way KK towers of quarks of the first and second generations are expressed as $\hat{u}_T+\hat{u}+\ldots$, $\hat{d}+\hat{d}_D+\ldots$, and so on.

Lepton bulk fermions Ψ_3 and Ψ_4 (in the third generation) are decomposed as

$$\begin{aligned}
(\Psi_3+\Psi_4)&=\hat{\nu}_{\tau(0)}+\hat{\tau}_{(-1)}+\hat{\tau}_{1X(-1)}+\hat{\tau}_{1Y(-2)}\\
&\quad+\hat{\nu}_{\tau 2X(+1)}+\hat{\nu}_{\tau 2Y(0)}+\hat{\nu}_{\tau 3X(0)}+\hat{\tau}_{3Y(-1)},
\end{aligned} \tag{2.39}$$

where $\hat{\tau}$ and $\hat{\nu}_\tau$ are towers for tau and tau-neutrino, respectively, and others are towers for non-SM exotic lepton partners.

Hereafter we work with rescaled bulk fermions $\check{\Psi}\equiv z^2\Psi$. One can find mass spectra of KK towers corresponding to $\hat{t}_T$, $\hat{b}_Y$, $\hat{t}$, and $\hat{b}$ in Refs. [39,42]. For the fermions with $Q_{\rm EM}=+2/3$, integrating brane fermions and utilizing orbifold boundary conditions, boundary conditions at $z=1_+$ in the original gauge are given by

$$\begin{aligned}
\frac{\mu_2}{2k}[\mu_2\check{U}_L+\tilde{\mu}\check{t}_L]-D_+^{(2)}\check{U}_L&=0,\\
\frac{\mu_1^2}{2k}\check{B}_L-D_+^{(1)}\check{B}_L&=0,\\
\frac{\tilde{\mu}}{2k}[\mu_2\check{U}_L+\tilde{\mu}\check{t}_L]-D_+^{(1)}\check{t}_L&=0,\\
\check{t}'_L&=0,
\end{aligned} \tag{2.40}$$

where $D_\pm^{(a)}\equiv\pm(d/dz)+(c_a/z)$. Bulk fermions in the twisted gauge, $\tilde{U}$, $\tilde{t}$, $\tilde{B}$, and $\tilde{t}'$ are obtained by gauge transformation

$$\begin{aligned}
\begin{pmatrix}(\tilde{t}-\tilde{B})/\sqrt{2}\\ \tilde{t}'\end{pmatrix}&=\begin{pmatrix}\cos\theta(z) & \sin\theta(z)\\ -\sin\theta(z) & \cos\theta(z)\end{pmatrix}\begin{pmatrix}(t-B)/\sqrt{2}\\ t'\end{pmatrix},\\
\tilde{U}&=U,\\
\tilde{t}+\tilde{B}&=t+B,
\end{aligned} \tag{2.41}$$

where $\theta(z)=\theta_H(z_L^2-z^2)/(z_L^2-1)$. We express KK expansion of bulk fermions in the twisted gauge as

$$\begin{pmatrix} \breve{\tilde{U}} \\ \breve{\tilde{t}} \\ \breve{\tilde{B}} \\ \breve{\tilde{t}}' \end{pmatrix} = \sqrt{k}\sum_n \psi_L^{(n)}(x) \begin{pmatrix} a_U C_L^{(2)} \\ a_t C_L^{(1)} \\ a_B C_L^{(1)} \\ a_{t'} S_L^{(1)} \end{pmatrix}(z,\lambda_n) + \sqrt{k}\sum_n \psi_R^{(n)}(x) \begin{pmatrix} a_U S_R^{(2)} \\ a_t S_R^{(1)} \\ a_B S_R^{(1)} \\ a_{t'} C_R^{(1)} \end{pmatrix}(z,\lambda_n), \qquad (2.42)$$

where $C_L^{(a)}(z,\lambda_n) = C_L(z;\lambda_n, c_a)$ and so on. $\psi_L^{(n)}$ and $\psi_R^{(n)}$ are 4D left- and right-handed fermions with the mass $m_n = k\lambda_n$, respectively. When we assume that $\mu_1^2, \mu_2^2, \tilde{\mu}^2$, $\tilde{\mu}\mu_2 \gg k\lambda_n$, then the boundary conditions at $z = 1$ are simplified as follows

$$\breve{B}_L = 0, \qquad \mu_2 \breve{U}_L + \tilde{\mu}\breve{t}_L = 0,$$
$$\tilde{\mu} D_+^{(2)} \breve{U}_L - \mu_2 D_+^{(1)} \breve{t}_L = 0, \qquad \breve{t}'_L = 0. \qquad (2.43)$$

Substituting left-handed KK modes in (2.42) into the above conditions, we can obtain KK masses and corresponding eigenstates. KK fermions with $Q_{\rm EM} = -1/3$ are also obtained in a similar way.

Masses and couplings of fermions are summarized in Appendices C and D, respectively. It is seen in Tables XII and XIII, that $t_T^{(1)}$ and $b_Y^{(1)}$, which are exotic partners of the top and bottom quarks with electric charge $+5/3$ and $-4/3$, respectively, have mass $M_{t_T^{(1)}, b_Y^{(1)}} = 4.6$ TeV (5.4 TeV) for $\theta_H = 0.115$ (0.0737). $t_T^{(1)}$ and $b_Y^{(1)}$ are the lightest non-SM states in GHU which can be singly produced in the colliders.

III. DECAY WIDTH AND CROSS SECTIONS

The relevant parts of the Lagrangian for the present study consist of cubic interactions among vector and Higgs bosons

$$\begin{aligned}
\mathcal{L}_{\rm eff}^{\rm boson} &= \mathcal{L}_{W'WZ} + \mathcal{L}_{W'WH} + \mathcal{L}_{Z'WW} + \mathcal{L}_{Z'ZH},\\
\mathcal{L}_{W'WZ} &= \sum_{W'=W^{(1)},W_R^{(1)}} i g_{W'WZ}(\eta^{\alpha\gamma}\eta^{\beta\delta} - \eta^{\alpha\delta}\eta^{\beta\gamma})(W'^-_\gamma Z_\delta \partial_\alpha W^+_\beta + W^-_\gamma Z_\delta \partial_\alpha W'^+_\beta\\
&\quad + Z_\gamma W'^+_\delta \partial_\alpha W^-_\beta + Z_\gamma W^+_\delta \partial_\alpha W'^-_\beta + W'^+_\gamma W^-_\delta \partial_\alpha Z_\beta + W^+_\gamma W'^-_\delta \partial_\alpha Z_\beta),\\
\mathcal{L}_{W'WH} &= g_{W^{(1)}WH}[H W^+_\mu W^{-(1)\mu} + H W^-_\mu W^{+(1)\mu}],\\
\mathcal{L}_{Z'WW} &= \sum_{Z'=Z^{(1)},\gamma^{(1)},Z_R^{(1)}} i g_{Z'WW}(\eta^{\alpha\gamma}\eta^{\beta\delta} - \eta^{\alpha\delta}\eta^{\beta\gamma})(W^-_\gamma Z'_\delta \partial_\alpha W^+_\beta + W^-_\gamma Z'_\delta \partial_\alpha W^+_\beta\\
&\quad + Z'_\gamma W^+_\delta \partial_\alpha W^-_\beta + Z'_\gamma W^+_\delta \partial_\alpha W^-_\beta + W^+_\gamma W^-_\delta \partial_\alpha Z'_\beta + W^+_\gamma W^-_\delta \partial_\alpha Z'_\beta),\\
\mathcal{L}_{Z'ZH} &= \sum_{Z'=Z^{(1)},\gamma^{(1)},Z_R^{(1)}} g_{Z'ZH}[H Z_\mu Z'^\mu], \qquad (3.1)
\end{aligned}$$

and the W' and Z' couplings to fermions

$$\begin{aligned}
\mathcal{L}_{\rm eff}^{\rm fermion} &= \mathcal{L}_{W'f\bar{f}} + \mathcal{L}_{Z'f\bar{f}},\\
\mathcal{L}_{W'f\bar{f}} &= \sum_{W'=W^{(1)},W_R^{(1)}} \left\{ \sum_{(\ell,\nu_\ell)} \left[g^L_{W'\ell\nu} W'^-_\mu \bar{\ell}\gamma^\mu \frac{1-\gamma_5}{2}\nu_\ell + (\text{H.c.})\right]\right.\\
&\quad \left. + \sum_{\rm color}\sum_{(U,D)} \left[g^L_{W'UD} W'^-_\mu \bar{D}\gamma^\mu \frac{1-\gamma_5}{2} U + (\text{H.c.})\right] + (L\to R, \gamma_5 \to -\gamma_5)\right\},\\
\mathcal{L}_{Z'f\bar{f}} &= \sum_{Z'=Z^{(1)},\gamma^{(1)},Z_R^{(1)}} \left\{ \sum_{l=\ell,\nu_\ell} \left[g^L_{Z'l} Z'^-_\mu \bar{l}\gamma^\mu \frac{1-\gamma_5}{2}\nu_l + (\text{H.c.})\right]\right.\\
&\quad \left. + \sum_{\rm color}\sum_{q=u,d,s,c,b,t} \left[g^L_{Z'q} Z'^-_\mu \bar{q}\gamma^\mu \frac{1-\gamma_5}{2} q + (\text{H.c.})\right], + (L\to R, \gamma_5 \to -\gamma_5)\right\}. \qquad (3.2)
\end{aligned}$$

The couplings $\mathcal{L}_{\rm eff}^{\rm bosons}$ and $\mathcal{L}_{\rm eff}^{\rm fermions}$ are summarized in Appendix E and Appendix D, respectively. Z' couplings to fermions are also given in Ref. [47].

Formulas for decay widths of W' and Z' are summarized in Appendix F. When $M_{W'}, M_{Z'} \gg M_W, M_Z, M_H$, partial decay widths of $W' = W^{(1)}$ and $Z' = Z^{(1)}, \gamma^{(1)}, Z_R^{(1)}$ are approximately given by

$$\begin{aligned}
\Gamma(W' \to WH) &\simeq \frac{M_{W'}}{192\pi}\left(\frac{g_{HW'W}}{M_W}\right)^2, \\
\Gamma(Z' \to ZH) &\simeq \frac{M_{Z'}}{192\pi}\left(\frac{g_{HZ'Z}}{M_Z}\right)^2, \\
\Gamma(W' \to WZ) &\simeq \frac{M_{W'}}{192\pi} g_{W'WZ}^2 \frac{M_{W'}^4}{M_W^2 M_Z^2}, \\
\Gamma(Z' \to W^+W^-) &\simeq \frac{M_{Z'}}{192\pi} g_{Z'WW}^2 \frac{M_{Z'}^4}{M_W^4},
\end{aligned} \tag{3.3}$$

and

$$\begin{aligned}
\Gamma(W' \to f\bar{f}') &\simeq N_c \frac{M_{W'}}{24\pi}(|g_{W'ff'}^L|^2 + |g_{W'ff'}^R|^2), \\
\Gamma(Z' \to f\bar{f}) &\simeq N_c \frac{M_{Z'}}{24\pi}(|g_{Z'f}^L|^2 + |g_{Z'f}^R|^2),
\end{aligned} \tag{3.4}$$

where N_c is the number of color of fermions f, f'. For later use, we define ratios of partial decay widths as follows.

$$\frac{\Gamma(W' \to WH)}{\Gamma(W' \to WZ)} \simeq \frac{\frac{g_{HW'W}^2}{M_W^2}}{g_{W'WZ}^2 \frac{M_{W'}^4}{M_W^2 M_Z^2}} = \frac{g_{HW'W}^2}{g_{W'WZ}^2}\frac{M_Z^2}{M_{W'}^4} \equiv \eta_{W'}, \tag{3.5}$$

$$\frac{\Gamma(Z' \to ZH)}{\Gamma(Z' \to WW)} \simeq \frac{\frac{g_{HZ'Z}^2}{M_Z^2}}{g_{Z'WW}^2 \frac{M_{Z'}^4}{M_W^4}} = \frac{g_{HZ'Z}^2}{g_{Z'WW}^2}\frac{M_W^4}{M_{Z'}^4 M_Z^2} \equiv \eta_{Z'}. \tag{3.6}$$

Formulas of s-channel cross sections mediated by W' or Z' are summarized in Appendix G. When $\sqrt{s} \gg M_W, M_Z, M_H$, cross sections for the processes $f\bar{f}' \to \{W, W^{(1)}\} \to WH$ and $f\bar{f} \to \{Z, Z^{(1)}, Z_R^{(1)}\} \to ZH$ are approximately given by

$$\begin{aligned}
&\sigma(f\bar{f}' \to \{W, W^{(1)}\} \to WH) \\
&\simeq \frac{1}{N_c^i}\frac{1}{384\pi}\frac{s}{M_W^2}\left\{\sum_{V=W,W^{(1)}}\frac{g_{HVW}^2[|g_{Vff'}^L|^2 + |g_{Vff'}^R|^2]}{(s-M_V^2)^2 + M_V^2\Gamma_V^2} + 2\mathrm{Re}\left[\frac{g_{HWW}g_{HW^{(1)}W}[(g_{Wff'}^L)(g_{W^{(1)}ff'}^L)^* + (g_{Wff'}^R)(g_{W^{(1)}ff'}^R)^*]}{[(s-M_W^2) + iM_W\Gamma_W][(s-M_{W^{(1)}}^2) - iM_{W^{(1)}}\Gamma_{W^{(1)}}]}\right]\right\},
\end{aligned} \tag{3.7}$$

$$\begin{aligned}
&\sigma(f\bar{f} \to \{Z, Z^{(1)}, Z_R^{(1)}\} \to ZH) \\
&\simeq \frac{1}{N_c^i}\frac{1}{384\pi}\frac{s}{M_Z^2}\left\{\sum_{V=Z,Z^{(1)},Z_R^{(1)}}\frac{g_{HVZ}^2[|g_{Vf}^L|^2 + |g_{Vf}^R|^2]}{(s-M_V^2)^2 + M_V^2\Gamma_V^2} + \sum_{\substack{V_1,V_2=Z,Z^{(1)},Z_R^{(1)} \\ V_1\neq V_2}}\mathrm{Re}\left[\frac{g_{HV_1Z}g_{HV_2Z}[(g_{V_1f}^L)(g_{V_2f}^L)^* + (g_{V_1f}^R)(g_{V_2f}^R)^*]}{[(s-M_{V_1}^2) + iM_{V_1}\Gamma_{V_1}][(s-M_{V_2}^2) - iM_{V_2}\Gamma_{V_2}]}\right]\right\}.
\end{aligned} \tag{3.8}$$

For processes $f\bar{f} \to WW$ and $f\bar{f}' \to WZ$, careful treatments are necessary. Each amplitude contains not only s- channel diagrams but also t- and u- channel diagrams. Therefore the total amplitude $\mathcal{M}$ will be given by

$$\begin{aligned}
\mathcal{M} &= \mathcal{M}^{\rm SM} + \mathcal{M}^{NP}, \\
\mathcal{M}^{\rm SM} &= \mathcal{M}_s^{\rm SM} + \mathcal{M}_t^{\rm SM} + \mathcal{M}_u^{\rm SM}, \\
\mathcal{M}^{NP} &= \mathcal{M}_s^{NP} + \mathcal{M}_t^{NP} + \mathcal{M}_u^{NP},
\end{aligned} \tag{3.9}$$

where $\mathcal{M}_{s,t,u}^{\rm SM}$ and $\mathcal{M}_{s,t,u}^{NP}$ are s-, t- and u-channel amplitudes for the SM fields and new physics parts, respectively. The square of the total amplitude is given by

$$|\mathcal{M}|^2 = |\mathcal{M}^{\rm SM}|^2 + |\mathcal{M}^{NP}|^2 + (\text{interference}), \tag{3.10}$$

where the interference terms contain products of $\mathcal{M}^{\rm SM(*)}$ and $\mathcal{M}^{NP(*)}$. When the energy of the initial state $\sqrt{s}$ is much larger than the EW scale, each of $\mathcal{M}_{s,t,u}^{\rm SM}$ grows due to the longitudinal part of the vector bosons in the final state. In the SM it is known that the growing contributions from longitudinal parts cancels with each other precisely, and that the unitarity of the amplitude $\mathcal{M}^{\rm SM}$ is protected. In our model, couplings among SM fields are very close to those of the SM values, so that $\mathcal{M}^{\rm SM}$ is well behaved. In the vicinity of $M_{W'}$ and $M_{Z'}$ productions, $|\mathcal{M}_s^{NP}|^2$ dominates over the interference terms. The relevant formulas for cross sections for the SM part $|\mathcal{M}^{\rm SM}|^2$ are found in [73,74]. For the NP part near the resonance $\sqrt{s} \sim M_{W'}, M_{Z'} \gg M_W, M_Z, M_H$, one can neglect small $M_{t,u}^{NP}$. In the precesses $f\bar{f} \to WW$ and $f\bar{f}' \to WZ$, we approximate $|\mathcal{M}|^2$ by $|\mathcal{M}^{\rm SM}|^2 + |\mathcal{M}_s^{NP}|^2$, though the interference term needs to be included for more rigorous treatment. Cross sections of s-channel processes $f\bar{f}' \to \{W^{(n)}\} \to WZ$ and $f\bar{f} \to \{\gamma^{(n)}, Z^{(n)}, Z_R^{(n)}\} \to W^+W^-$ are given in Appendix G. Hence the NP part of the cross sections are approximately given by

$$\sigma(f\bar{f}' \to \{W^{(n)}\} \to WZ) \simeq \frac{1}{N_c^i}\frac{1}{384\pi}\frac{s^3}{M_W^2 M_Z^2}\Bigg\{\sum_{\substack{V=W^{(n)}\\ n\geq 1}} \frac{[|g_{Vff'}^L|^2 + |g_{Vff'}^R|^2] g_{VWZ}^2}{(s-M_V^2)^2 + M_V^2\Gamma_V^2} + \sum_{\substack{(V_1,V_2)=(W^{(n)},W^{(m)})\\ 1\leq n<m}} 2{\rm Re}\left[\frac{g_{V_1WZ}g_{V_2WZ}[(g_{V_1ff'}^L)(g_{V_2ff'}^L)^* + (g_{V_1ff'}^R)(g_{V_2ff'}^R)]}{[(s-M_{V_1}^2) + iM_{V_1}\Gamma_{V_1}][(s-M_{V_2}^2) - iM_{V_2}\Gamma_{V_2}]}\right]\Bigg\}, \tag{3.11}$$

$$\sigma(f\bar{f} \to \{\gamma^{(n)}, Z^{(n)}, Z_R^{(n)}\} \to W^+W^-) \simeq \frac{1}{N_c^i}\frac{1}{384\pi}\frac{s^3}{M_W^4}\Bigg\{\sum_{\substack{V=Z^{(n)},\gamma^{(n)},Z_R^{(n)}\\ n\geq 1}} \frac{g_{VWW}^2[|g_{Vf}^L|^2 + |g_{Vf}^R|^2]}{(s-M_V^2)^2 + M_V^2\Gamma_V^2} + \sum_{\substack{(V_1,V_2)=\{Z^{(n)},\gamma^{(n)},Z_R^{(n)}\}\\ n\geq 1\\ M_{V_1}<M_{V_2}}} 2{\rm Re}\left[\frac{g_{V_1WW}g_{V_2WW}[(g_{V_1f}^L)(g_{V_2f}^L)^* + (g_{V_1f}^R)(g_{V_2f}^R)^*]}{[(s-M_{V_1}^2) + iM_{V_1}\Gamma_{V_1}][(s-M_{V_2}^2) - iM_{V_2}\Gamma_{V_2}]}\right]\Bigg\}. \tag{3.12}$$

We note that at around the resonance points $s \simeq M_{W^{(1)}}, M_{Z^{(1)}}, M_{Z_R^{(1)}}$, we see that the ratios of the cross sections behave

$$\frac{\sigma(f\bar{f}' \to W^{(1)} \to WH)}{\sigma(f\bar{f}' \to W^{(1)} \to WZ)}\Bigg|_{s=M_{W^{(1)}}} \simeq \frac{\frac{M_{W^{(1)}}^2}{M_W^2} g_{HW^{(1)}W}^2}{\frac{M_{W^{(1)}}^6}{M_W^2 M_Z^2} g_{W^{(1)}WZ}^2} = \frac{g_{HW^{(1)}W}^2}{g_{W^{(1)}WZ}^2}\frac{M_Z^2}{M_{W^{(1)}}^4} = \eta_{W^{(1)}}, \tag{3.13}$$

and

$$\frac{\sigma(f\bar{f}' \to Z' \to ZH)}{\sigma(f\bar{f}' \to Z' \to WW)}\Bigg|_{s=M_{Z'}} \simeq \frac{\frac{M_{Z'}^2}{M_Z^2} g_{HZ'Z}^2}{\frac{M_{Z'}^6}{M_W^4} g_{Z'WW}^2} = \frac{g_{HZ^{(1)}Z}^2}{g_{Z^{(1)}WW}^2}\frac{M_W^4}{M_Z^2 M_{Z'}^4} = \eta_{Z'}, \tag{3.14}$$

for $Z' = Z^{(1)}, Z_R^{(1)}$. It is observed that ratios (3.5) and (3.6) coincide with (3.13) and (3.14), respectively.

For SM fermions $f^{(\prime)}$, $F^{(\prime)}$, the cross section of the process $f\bar{f}' \to \{W, W^{(1)}\} \to F\bar{F}'$ is given by

$$\begin{aligned}\sigma(f\bar{f}' \to \{W, W^{(1)}\} \to F\bar{F}') &\simeq \frac{N_c^f}{N_c^i}\frac{s}{48\pi}\Bigg\{\sum_{V=W,W^{(1)}} \frac{[|g_{Vff'}^L|^2 + |g_{Vff'}^R|^2][|g_{VFF'}^L|^2 + |g_{VFF'}^R|^2]}{(s-M_V^2)^2 + M_V^2\Gamma_V^2} \\ &\quad + 2{\rm Re}\left[\frac{K(f,f')K(F,F')}{[(s-M_W^2) + iM_W\Gamma_W][(s-M_{W^{(1)}}^2) - iM_{W^{(1)}}\Gamma_{W^{(1)}}]}\right]\Bigg\}, \\ K(f,f') &\equiv (g_{Wff'}^L)(g_{W^{(1)}ff'}^L)^* + (g_{Wff'}^R)(g_{W^{(1)}ff'}^R)^*,\end{aligned} \tag{3.15}$$

where N_c^f is the number of color of final-state fermions. A similar formula for processes $f\bar{f} \to Z, Z' \to f'\bar{f}'$ is found in [47].

IV. EFFECTIVE THEORIES

Before jumping to calculate the couplings numerically, we formulate effective theories which yield qualitative understandings of various couplings and decay widths.

A. 4D $SO(4) \times U(1)$ model

Let us consider a 4D $SO(4) \times U(1)_X$ model with two scalars Φ_R and Φ_H. Gauge couplings of $SO(4) \simeq SU(2)_L \times SU(2)_R$ and $U(1)_X$ are denoted by g_w and g_b, respectively. Φ_R is $(\mathbf{0}, \mathbf{2})_{-1/2}$ in $SU(2)_L \times SU(2)_R \times U(1)_X$ and Φ_H is a $SO(4)$-vector $(\mathbf{2}, \bar{\mathbf{2}})$ corresponding to the Higgs boson. When Φ_R develops a VEV $\langle \Phi_R \rangle = (0, \mu)/\sqrt{2}$, the $SU(2)_R \times U(1)_X$ symmetry is broken to $U(1)_Y$. We take $\mu = \mathcal{O}(m_{KK})$. On the other hand nonvanishing Φ_H breaks $SU(2)_L \times SU(2)_R$ to $SU(2)_V$ whose generators are $(T^{a_L} + T^{a_R})/\sqrt{2}$. With both $\langle \Phi_R \rangle$ and $\langle \Phi_H \rangle$ nonvanishing, $SO(4) \times U(1)_X$ symmetry is broken to $U(1)_{\rm em}$. The mass matrices of gauge bosons in $(A_\mu^{a_L}, A_\mu^{a_R})$ $(a = 1, 2)$ and $(A_\mu^{3_L}, A_\mu^{3_R}, B_\mu)$ are given by

$$
\begin{aligned}
\mathcal{M}_C &= \begin{pmatrix} M_{LL}^2 & -M_{LL}^2 \\ -M_{LL}^2 & M_{LL}^2 + M_{RR}^2 \end{pmatrix}, \\
\mathcal{M}_N &= \begin{pmatrix} M_{LL}^2 & -M_{LL}^2 & 0 \\ -M_{LL}^2 & M_{LL}^2 + M_{RR}^2 & -M_{RB}^2 \\ 0 & -M_{RB}^2 & M_{BB}^2 \end{pmatrix}, \\
M_{LL}^2 &= \frac{g_w^2 v^2}{4}, \qquad M_{RR}^2 = \frac{g_w^2 \mu^2}{4}, \\
M_{RB}^2 &= \frac{g_w g_b \mu^2}{4}, \qquad M_{BB}^2 = \frac{g_b^2 \mu^2}{4},
\end{aligned} \tag{4.1}
$$

respectively. $\mathcal{M}_C$ has two eigenvalues corresponding to the mass-squared of W and W_R bosons, respectively, which are given by

$$
\begin{aligned}
M_W^2 &= \bar{M}_W^2 (1 + \mathcal{O}(v^2/\mu^2)), \qquad \bar{M}_W \equiv \frac{g_w v}{2}, \\
M_{W_R}^2 &= \frac{g_w^2 \mu^2}{4} (1 + \mathcal{O}(v^2/\mu^2)).
\end{aligned} \tag{4.2}
$$

$\mathcal{M}_N$ has three eigenvalues, which correspond to mass-squared of the photon, Z-boson, and Z_R-boson, respectively. Here

$$
\begin{aligned}
M_\gamma &= 0, \\
M_Z^2 &= \frac{g_w^2 v^2}{4} \cdot \frac{g_w^2 + 2 g_b^2}{g_w^2 + g_b^2} (1 + \mathcal{O}(v^2/\mu^2)) \\
&= \bar{M}_Z^2 (1 + \mathcal{O}(v^2/\mu^2)), \qquad \bar{M}_Z \equiv \frac{g_w v}{2 \cos\theta_W}, \\
M_{Z_R}^2 &= \frac{(g_w^2 + g_b^2)\mu^2}{4} (1 + \mathcal{O}(v^2/\mu^2)) \\
&= \frac{g_w^2 \mu^2}{4} \frac{\cos^2\theta_W}{\cos^2\theta_W - \sin^2\theta_W} (1 + \mathcal{O}(v^2/\mu^2)).
\end{aligned} \tag{4.3}
$$

Diagonalizing these mass matrices we obtain mass eigenstates W, W' and Z, Z', γ. Mixing matrices are given by

$$
\begin{aligned}
&\begin{pmatrix} A_\mu^{a_L} \\ A_\mu^{a_R} \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} W_\mu^a \\ W_{R\mu}^a \end{pmatrix}, \quad a = 1, 2, \\
&\begin{pmatrix} A_\mu^{3_L} \\ A_\mu^{3_R} \\ B_\mu \end{pmatrix} = \left(\vec{v}_\gamma, \vec{v}_- \cos\theta_N + \vec{v}_+ \sin\theta_N, \vec{v}_+ \cos\theta_N - \vec{v}_- \sin\theta_N \right) \\
&\qquad \times \begin{pmatrix} \gamma_\mu \\ Z_\mu \\ Z_{R\mu} \end{pmatrix}, \\
&\vec{v}_\gamma = \begin{pmatrix} \sin\theta_W \\ \sin\theta_W \\ \sqrt{\cos 2\theta_W} \end{pmatrix}, \quad \vec{v}_- = \begin{pmatrix} \cos\theta_W \\ -\frac{\sin^2\theta_W}{\cos\theta_W} \\ -\tan\theta_W \sqrt{\cos 2\theta_W} \end{pmatrix}, \\
&\vec{v}_+ = \begin{pmatrix} 0 \\ \frac{\sqrt{\cos 2\theta_W}}{\cos\theta_W} \\ -\tan\theta_W \end{pmatrix},
\end{aligned} \tag{4.4}
$$

where mixing angles are determined so as to mixing matrices properly diagonalize mass matrices:

$$
\begin{aligned}
\tan(2\theta_C) &= \frac{2v^2}{\mu^2}, \\
\tan(2\theta_N) &= \frac{2 g_w^3 v^2 \sqrt{g_w^2 + 2 g_b^2}}{w^2 (g_w^2 + g_b^2)^2 - 2 g^2 g_b^2 v^2}.
\end{aligned} \tag{4.5}
$$

The weak mixing angle is given by

$$
\sin\theta_W = \frac{g_b}{\sqrt{g_w^2 + 2 g_b^2}}, \tag{4.6}
$$

so that γ_μ couples to $W^\pm$ bosons with $e = g_w \sin\theta_W$.

Now we can calculate couplings among these mass eigenstates. For vector-boson trilinear $V_1 V_2 V_3$ couplings and $V_1 V_2 H$ couplings, we obtain

$$
\begin{aligned}
g_{WW\gamma} &= g_w \sin\theta_W = e, \\
g_{WWZ} &= g_w \cos\theta_W + \mathcal{O}(v^4/\mu^4), \\
g_{Z_R WW} &= -g_w \frac{(\cos^2\theta_W - \sin^2\theta_W)^{3/2}}{\cos^3\theta_W}\frac{v^2}{\mu^2}, \\
g_{ZW_R W} &= -g_w \frac{1}{\cos\theta_W}\frac{v^2}{\mu^2},
\end{aligned} \tag{4.7}
$$

and

$$
\begin{aligned}
g_{HWW} &= g_w \bar{M}_W (1 + \mathcal{O}(v^2/\mu^2)), \\
g_{HZZ} &= \frac{g_w}{\cos\theta_W}\bar{M}_Z (1 + \mathcal{O}(v^2/\mu^2)), \\
g_{HW_R W} &= -g_w \bar{M}_W (1 + \mathcal{O}(v^2/\mu^2)), \\
g_{HZ_R Z} &= -g_w \bar{M}_Z \frac{\sqrt{\cos^2\theta_W - \sin^2\theta_W}}{\cos\theta_W}(1 + \mathcal{O}(v^2/\mu^2)),
\end{aligned} \tag{4.8}
$$

where $\bar{M}_W = g_w v/2$ and $\bar{M}_Z = \bar{M}_W/\cos\theta_W$ are used.

Hence we see that (a) g_{ZWW} is very close to its SM-value $g_w \cos\theta_W$ and the deviation from the SM-value will be suppressed by a factor $\mathcal{O}(v^4/m_{KK}^4)$, (b) Deviations of g_{HZZ} and g_{HWW} couplings from their SM values are both suppressed by v^2/m_{KK}^2, (c) $g_{Z'WW}$ and $g_{ZW_R W}$ are suppressed by a factor $\mathcal{O}(v^2/\mu^2)$.

$$
g_{W_R WZ}, g_{Z_R WW} \sim g_{WWZ} \cdot \frac{v^2}{\mu^2}, \tag{4.9}
$$

(d) In contrast, values of HWW_R and HZZ_R approximately equal to HWW and HZZ couplings in magnitudes but opposite in signs, respectively.

$$
g_{HW_R W} \sim -g_{HWW}, \qquad g_{HZ'Z} \sim -g_{HZZ}. \tag{4.10}
$$

(e) For the decay widths of W_R and Z_R one finds

$$
\frac{\Gamma(W_R \to ZW)}{\Gamma(W_R \to WH)} \simeq \left(\frac{g_{ZW_R W}}{g_{HW_R W}}\frac{M_{W_R}^2}{\bar{M}_Z}\right)^2 = 1 + \mathcal{O}(v^2/\mu^2), \tag{4.11}
$$

$$
\frac{\Gamma(Z_R \to WW)}{\Gamma(Z_R \to ZH)} \simeq \left(\frac{g_{Z_R WW}}{g_{HZ'Z}}\frac{M_{Z_R}^2 \bar{M}_Z}{\bar{M}_W^2}\right)^2 = 1 + \mathcal{O}(v^2/\mu^2). \tag{4.12}
$$

We will find in the following section that the relation (4.11) will be satisfied, and that (4.12) needs to be generalized to incorporate KK-γ and KK-Z bosons.

B. 5D $SO(5) \times U(1)_X$ GHU in the flat-space limit

We also explore the flat-space limit of the warped $SO(5) \times U(1)_X$ GHU by taking $kL \to 0$ limit while keeping $L = \pi R$ finite [71]. A similar model is seen in [70]. In this limit $m_{KK} = 1/R$. The W-boson mass and the AB phase θ_H are related with each other by $\sqrt{2}\sin(m_W \pi R) = \sin\theta_H$. The couplings among vector bosons and Higgs are summarized as follows. Vector boson trilinear couplings are given by

$$
\begin{aligned}
& g_{\gamma^{(0)}W^{(n)}W^{(m)}} = \delta_{mn} e, \qquad g_{\gamma^{(1)}W^{(n)}W^{(m)}} = \delta_{mn} 2\sqrt{2} e, \\
& g_{Z^{(0)}W^{(0)}W^{(0)}} = g_w \cos\theta_W + \mathcal{O}(m_W^4 R^4), \\
& g_{Z^{(1)}W^{(0)}W^{(0)}}, \\
& g_{Z^{(0)}W^{(1)}W^{(0)}} = \mathcal{O}(m_W^3 R^3), \\
& g_{Z_R^{(1)}W^{(0)}W^{(0)}} = \frac{8\sqrt{2}\sqrt{\cos 2\theta_W}}{\pi\cos\theta_W} g_w m_W^2 R^2 + \mathcal{O}(m_W^4 R^4), \\
& g_{Z^{(0)}W_R^{(1)}W^{(0)}} = \frac{8\sqrt{2}}{\pi} g_w m_W m_Z R^2 + \mathcal{O}(m_W^4 R^4).
\end{aligned} \tag{4.13}
$$

Higgs vector-boson couplings are given by

$$
\begin{aligned}
& g_{HW^{(m)}W^{(n)}} = \delta_{mn}(-1)^n g_w m_{W^{(n)}}, \\
& g_{HZ^{(m)}Z^{(n)}} = \delta_{mn}(-1)^n \frac{g_w}{\cos\theta_W} m_{Z^{(n)}}, \\
& g_{HZ_R^{(1)}Z^{(0)}} = -\frac{2\sqrt{2}\sqrt{\cos 2\theta_W}}{\pi\cos\theta_W} g_w m_Z [1 + \mathcal{O}(m_W^2 R^2)], \\
& g_{HW_R^{(1)}W^{(0)}} = -\frac{2\sqrt{2} g_w}{\pi} m_W [1 + \mathcal{O}(m_W^2 R^2)], \\
& g_{H\gamma^{(n)}Z^{(m)}}, \\
& g_{H\gamma^{(n)}Z_R^{(m)}} = 0.
\end{aligned} \tag{4.14}
$$

Here we note that $m_{W_R^{(1)}} = m_{Z_R^{(1)}} = 1/(2R)$.

It seems that W and W_R, Z and Z_R are mixed in an ordinary manner as seen in the 4D model, whereas W and $W^{(n)}$, Z and $Z^{(n)}$ are very weakly mixed. This will be understood by mass-mixings in the original gauge. Gauge fields for charged bosons are $A_\mu^{(n)\pm_L}, A_\mu^{(m)\pm_R}, A_\mu^{(m)\hat{\pm}}$, ($a = 1, 2$, $n = 0, 1, 2, \cdots$ and $m = 1, 2, \cdots$). For up to first KK excited states, the mass matrix in the ($A_\mu^{(0)\pm_L}, A_\mu^{(1)\pm_R}, A_\mu^{(1)\pm_L}, A_\mu^{(1)\hat{\pm}}$) basis is given by

$$
\begin{pmatrix}
M_v^2 & -M_v^2 & 0 & 0 \\
-M_v^2 & M_v^2 + M_R^2 & -M_v'^2 & 0 \\
0 & -M_v'^2 & M_v^2 + M_L^2 & 0 \\
0 & 0 & 0 & 2M_v^2 + M_X^2
\end{pmatrix}, \tag{4.15}
$$

where

TABLE I. Input parameters. Masses of Z boson, leptons, and quarks in the unit of GeV.

M_Z	$\sin^2\theta_W$		$m_e(M_Z)$	$m_\mu(M_Z)$	$m_\tau(M_Z)$
91.1876	0.23126		0.487×10^{-3}	0.103	1.75
$m_u(M_Z)$	$m_d(M_Z)$	$m_s(M_Z)$	$m_c(M_Z)$	$m_b(M_Z)$	$m_t(M_Z)$
1.27×10^{-3}	2.90×10^{-3}	0.055	0.619	2.89	171

TABLE II. Aharonov-Bohm phase θ_H, the bulk mass parameter of dark fermions c_F, AdS_5 curvature k, Kaluza-Klein scale m_{KK} and masses of first KK gauge bosons are given in the unit of GeV with respect to z_L for $N_F=4$ are summarized. $Z^{(1)}$, $W^{(1)}$, and $\gamma^{(1)}$ are almost degenerate. Their mass differences is 1–2 GeV.

z_L	θ_H [rad.]	c_F	k [GeV]	m_{KK} [TeV]	$m_{Z^{(1)}}$ [TeV]	$m_{W^{(1)}}$ [TeV]	$m_{\gamma^{(1)}}$ [TeV]	$m_{Z_R^{(1)}}=m_{W_R^{(1)}}$ [TeV]
10^5	0.115	0.3321	2.36×10^8	7.41	6.00	6.00	6.01	5.67
10^4	0.0737	0.2561	3.29×10^7	10.3	8.52	8.52	8.52	7.92

$$M_v^2=\frac{1}{4}g_w^2v^2,\qquad M_R^2=\frac{1}{4R^2},\qquad M_L^2=M_X^2=\frac{1}{R^2},$$
$$M_v'^2=\frac{1}{4}g_w^2v^2\frac{1}{\pi R}\int_0^{2\pi R}\cos\left(\frac{\pi R-y}{2R}\right)\cos\left(\frac{\pi R-y}{R}\right)dy. \tag{4.16}$$

$A_\mu^{(0)\pm_L}$ and $A_\mu^{(1)\pm_R}$ have a mixing term, whereas there is no mixing between $A_\mu^{(0)\pm_L}$ and $A_\mu^{(1)\pm_L}$ in the mass matrix due to the KK-number conservation. Therefore mixing between $A_\mu^{(0)\pm_L}$ and $A_\mu^{(1)\pm_L}$ is induced only through both $A_\mu^{(0)\pm_L}-A_\mu^{(1)\pm_R}$ and $A_\mu^{(1)\pm_L}-A_\mu^{(1)\pm_R}$ mixing terms so that the mixing angle is suppressed.

C. Comment on the warped $SO(5)\times U(1)_X$ GHU

In the flat GHU case, $A_\mu^{(0)a_L}-A_\mu^{(1)a_L}$ mass terms vanish. This is because the Higgs wave function is constant along the extra dimension. The mass term, which is written as an overlap integral of wave functions of $A_\mu^{(0)\pm_L}$, $A_\mu^{(1)\pm_L}$ and the Higgs boson, vanishes by the orthonormality conditions of wave functions. In the warped case, the Higgs wave function is not constant along the direction of the extra dimension. Therefore $A_\mu^{(0)a_L}-A_\mu^{(1)a_L}$ mass terms do not vanish.

V. COUPLINGS AND DECAY WIDTHS

Input parameters used in the numerical study are summarized in Table I. The W boson mass at the tree level becomes $M_W^{\rm tree}=79.9$ GeV. In the following study, we have adopted model parameters $N_F=4$ and $z_L=10^4$, 10^5. With $(N_F,\ z_L)$ given bulk mass parameter of dark fermions, c_F, is determined such that the resultant Higgs mass becomes $M_H=125$ GeV. This procedure determines the value of θ_H and the bulk mass parameters of quarks and leptons.(See for the details [46,47].) The resultant values of θ_H and k, m_{KK} and first-KK gauge boson masses are tabulated in Table II. We set fermion bulk mass parameters as $c_1^g=c_2^g\equiv\{c_u,c_c,c_t\}$ and $c_3^g=c_4^g\equiv\{c_e,c_\mu,c_\tau\}$. These parameters are tuned so that fermion masses coincide with values in Table I. These are listed in Table III.

A. Couplings

The couplings among vector bosons V, Higgs H, and SM fermions $f_{\rm SM}$ are evaluated from overlap integrals where functions of V, H, and $f_{\rm SM}$ are inserted. The detailed formulas are given in Appendices E and D, and Refs. [47,49].

In Table IV, the left-handed couplings of SM fermions to the W boson and its KK excited states are tabulated. Similar results for $\theta_H=\pi/2$ are found in Ref. [42]. The right-hand couplings vanish within the accuracy of numerical calculation. In this model couplings of the SM fermions to the W_R boson vanish.

It is seen that couplings to $W^{(0)}$ (the W boson) are slightly larger than the SM value $g_w/\sqrt{2}$ for light quarks and leptons, whereas those for top and bottom quarks is slightly smaller. Further the couplings of SM fermions except for t and b to $W^{(1)}$ are smaller than couplings to $W^{(0)}$ and their signs are opposite. $W^{(1)}t\bar{b}$ coupling is larger than the SM value.

The difference between $W^{(1)}ud$ and $W^{(1)}tb$ couplings is understood as follows. The couplings among left-handed up- and down-sector fermions $(U,D)=(u,d)$, (t,b) and

TABLE III. Fermion bulk mass parameters for quarks and leptons.

z_L	c_u	c_c	c_t	c_e	c_μ	c_τ
10^5	1.55	1.05	0.227	1.72	1.22	0.950
10^4	1.82	1.19	0.0366	2.04	1.41	1.07

TABLE IV. Masses and couplings of $W^{(n)}$ to left-handed SM fermions.

	$N_F = 4$, $z_L = 10^5$ ($\theta_H = 0.115$)				
	$n = 0$	1	2	3	4
$m_{W^{(n)}}$ [GeV]	79.9	6004	9034	13378	16538
$g^L_{W^{(n)}\ell\nu}/(g_w/\sqrt{2})$					
$(\ell,\nu) = (e,\nu_e)$	1.00019	−0.3455	−0.02507	0.2510	0.01937
(μ,ν_μ)	1.00019	−0.3455	−0.02507	0.2510	0.01937
(τ,ν_τ)	1.00019	−0.3452	−0.02505	0.2507	0.01934
$g^L_{W^{(n)}UD}/(g_w/\sqrt{2})$					
$(U,D) = (u,d)$	1.00019	−0.3455	−0.02507	0.2510	0.01937
(c,s)	1.00019	−0.3454	−0.02506	0.2510	0.01936
(t,b)	0.9993	1.2970	0.06527	−0.4342	−0.03110

	$N_F = 4$, $z_L = 10^4$ ($\theta_H = 0.0737$)				
	$n = 0$	1	2	3	4
$m_{W^{(n)}}$ [GeV]	79.9	8520	12624	18852	23112
$g^L_{W^{(n)}\ell\nu}/(g_w/\sqrt{2})$					
$(\ell,\nu) = (e,\nu_e)$	1.00009	−0.3904	−0.01861	0.2901	0.01461
(μ,ν_μ)	1.00009	−0.3904	−0.01861	0.2901	0.01461
(τ,ν_τ)	1.00009	−0.3901	−0.01858	0.2896	0.01457
$g^L_{W^{(n)}UD}/(g_w/\sqrt{2})$					
$(U,D) = (u,d)$	1.00009	−0.3904	−0.01861	0.2901	0.01461
(c,s)	1.00009	−0.3904	−0.01860	0.2900	0.01460
(t,b)	0.9995	1.7517	0.04516	−0.2925	−0.01490

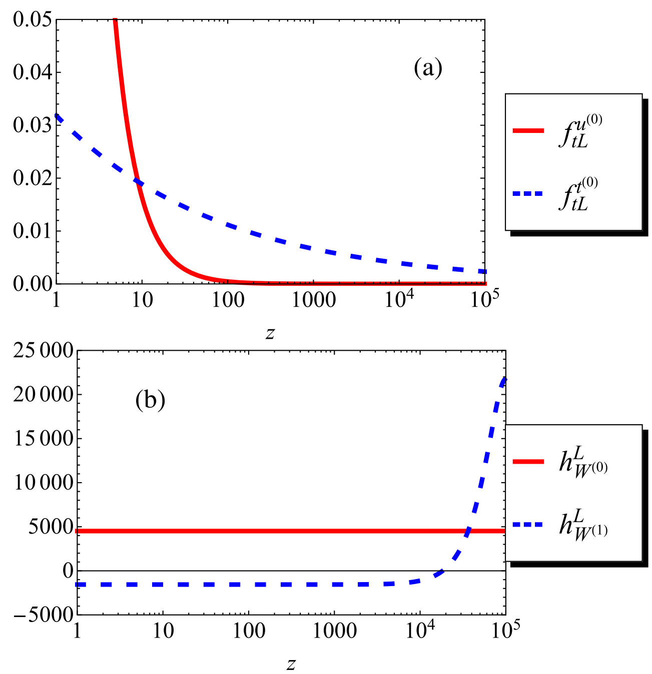

FIG. 1. Behavior of dominant component of the wave functions for fermions and $W^{(n)}$ bosons for $N_F = 4, z_L = 10^5$ ($\theta_H = 0.115$). (a) Red-solid and blue-dashed lines are $f^{t^{(0)}}_{tL}$ for the top and $f^{u^{(0)}}_{tL}$ for the up quark, respectively. (b) Red-solid and blue-dashed lines are $h^L_{W^{(0)}}$ for $W^{(0)}$ and $h^L_{W^{(1)}}$ for $W^{(1)}$ bosons, respectively.

W boson $g^L_{W^{(1)}UD}$ are given by the overlapping (D4). The integration in (D4) is dominated by the term $h^L_{W^{(n)}} f^D_{bL} f^U_{tL}$ ($(U,D) = (u^{(0)}, d^{(0)}), (t^{(0)}, b^{(0)})$). In Fig. 1, the leading part of bulk wave functions for fermions and gauge bosons are plotted. In Fig. 1(a), we see that $f^{u^{(0)}}_{tL}$ decreases much faster than $f^{t^{(0)}}_{tL}$. In particular, near $z = z_L$, $f^{t^{(0)}}_{tL}$ has small but sizable value whereas the value of $f^{u^{(0)}}_{tL}$ almost vanishes. In Fig. 1(b), $h^L_{W^{(0)}}$ is almost constant, whereas $h^L_{W^{(1)}}$ has negative values for small values of z and has large positive value near $z = z_L$. In Fig. 2, we plot overlapping of wave functions of up- and down-type fermions and $W^{(1)}$. In the figure, overlapping of wave functions of light quarks and

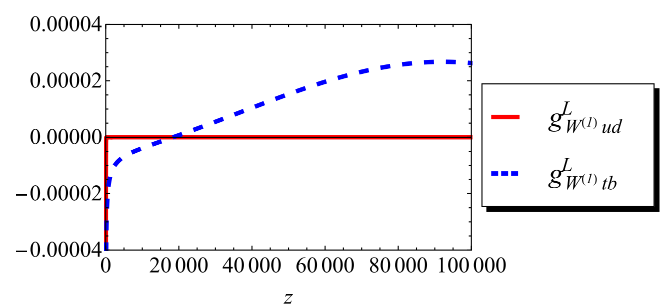

FIG. 2. Integrand of the coupling $g^L_{W^{(1)}UD}$ in (D4). Red-solid and blue-dashed lines are for $(U,D) = (t,b)$ and (u,d), respectively.

TABLE V. Couplings of $W^{(n)}$, $Z^{(n)}$, $\gamma^{(n)}$, and $Z_R^{(n)}$ to WH, ZH in the unit of GeV.

	$N_F = 4,\ z_L = 10^5$ ($\theta_H = 0.115$)				
	$n = 0$	1	2	3	4
$g_{W^{(n)}WH}/(g_w \cos\theta_H)$	80.0	255	2.57	45.4	0.220
$g_{Z^{(n)}ZH}/(g_w \cos\theta_H/\cos\theta_W)$	91.2	291	3.35	51.8	0.286
$g_{W_R^{(n)}WH}/g_w$	—	266	50.5	20.6	11.1
$g_{Z_R^{(n)}ZH}/(g_w/\cos\theta_W)$	—	223	42.3	17.2	9.27
	$N_F = 4\ z_L = 10^4$ ($\theta_H = 0.0737$)				
	$n = 0$	1	2	3	4
$g_{W^{(n)}WH}/(g_w \cos\theta_H)$	80.0	225	1.89	39.2	0.169
$g_{Z^{(n)}ZH}/(g_w \cos\theta_H/\cos\theta_W)$	91.2	257	2.46	44.8	0.220
$g_{W_R^{(n)}WH}/g_w$	—	238	45.1	18.4	9.89
$g_{Z_R^{(n)}ZH}/(g_w/\cos\theta_W)$	—	199	37.7	15.4	8.27

$W^{(1)}$ has large negative value only near the UV brane ($z = 1$). On the other hand, the overlapping of heavy quarks and $W^{(1)}$ takes negative value for small values of z but becomes positive for larger z. This difference in the integrand results in the differences in the signs and magnitudes of $g^L_{W^{(1)}tb}$, $g^L_{W^{(1)}ud}$, and $g_{W^{(0)}ud}$.

In Table V, HWW' ($W' = W^{(n)}$, $W_R^{(n)}$) and HZZ' ($Z' = Z^{(n)}$, $Z_R^{(n)}$) couplings are tabulated. We note that $H\gamma^{(n)}Z^{(m)}$ and $H\gamma^{(n)}Z_R^{(m)}$ couplings vanish. We find that g_{WWH} and g_{ZZH} couplings are given by the SM values multiplied by $\cos\theta_H$. $g_{W^{(1)}WH}$ and $g_{W_R^{(1)}WH}$ are a few times larger than g_{WWH}, and similar relations hold among $g_{Z_R^{(1)}ZH}$, $g_{Z^{(1)}ZH}$, and g_{ZZH}. All $g_{W'^{(n)}WH}$ and $g_{Z'(n)ZH}$ couplings become small as n becomes larger.

In Table VI, trilinear vector-boson couplings are tabulated. The g_{WWZ} coupling is very close to its SM value $g_w \cos\theta_W$. The deviation is one part in 10^8. $g_{\gamma WW}$ is exactly e, which reflects the unbroken $U(1)_{\rm em}$ gauge symmetry. Couplings among the first KK and two SM vector bosons are suppressed by a factor of $\mathcal{O}(10^{-4})$, which is close to the square of the ratio of the weak boson mass to the 1st KK boson mass.

B. Decay width

In Tables VII and VIII, decay widths of W and W_R boson are tabulated, respectively. Since the $W^{(1)}$ boson couples equally to the light SM fermions except for b and t quarks, partial decay widths to light SM fermions are almost identical besides the QCD color factor. The $W^{(1)}$ coupling to t and b quarks is larger than the couplings to other

TABLE VI. Trilinear vector-boson couplings of $W^{(n)}$, $Z^{(n)}$, $\gamma^{(n)}$, and $Z_R^{(n)}$ to W^+W^-, WZ.

	$N_F = 4,\ z_L = 10^5$ ($\theta_H = 0.115$)				
	$n = 0$	1	2	3	4
$g_{W^{(n)}WZ}/(g_w \cos\theta_W)$	0.9999998	-7.35×10^{-4}	-9.63×10^{-7}	-2.64×10^{-5}	-1.60×10^{-7}
$g_{Z^{(n)}WW}/(g_w \cos\theta_W)$	0.9999998	-3.96×10^{-4}	1.62×10^{-8}	-1.42×10^{-5}	-1.17×10^{-7}
$g_{\gamma^{(n)}WW}/e$	1	-1.13×10^{-3}	-4.04×10^{-5}	-7.63×10^{-6}	-2.07×10^{-6}
$g_{Z_R^{(n)}WW}/g_w$	—	5.51×10^{-4}	1.99×10^{-5}	3.28×10^{-6}	9.53×10^{-7}
$g_{W_R^{(n)}WZ}/g_w$	—	7.51×10^{-4}	2.71×10^{-5}	4.47×10^{-6}	1.30×10^{-6}
	$N_F = 4,\ z_L = 10^4$ ($\theta_H = 0.0737$)				
	$n = 0$	1	2	3	4
$g_{W^{(n)}WZ}/(g_w \cos\theta_W)$	0.99999995	-3.32×10^{-4}	-3.88×10^{-7}	-1.15×10^{-5}	-6.04×10^{-8}
$g_{Z^{(n)}WW}/(g_w \cos\theta_W)$	0.99999995	-1.73×10^{-4}	-1.46×10^{-8}	-6.18×10^{-6}	-4.43×10^{-8}
$g_{\gamma^{(n)}WW}/e$	1	-4.95×10^{-4}	-1.76×10^{-5}	-3.46×10^{-6}	-9.22×10^{-7}
$g_{Z_R^{(n)}WW}/g_w$	—	2.53×10^{-4}	9.10×10^{-6}	1.51×10^{-6}	4.37×10^{-7}
$g_{W_R^{(n)}WZ}/g_w$	—	3.45×10^{-4}	1.24×10^{-5}	2.05×10^{-6}	5.96×10^{-7}

TABLE VII. Partial and total decay width of $W^{-(1)}$ in the unit of GeV.

$N_F = 4,\ z_L = 10^5\ (\theta_H = 0.115)$									
mode	$e^-\bar{\nu}_e$	$\mu^-\bar{\nu}_\mu$	$\tau^-\bar{\nu}_\tau$	$d\bar{u}$	$s\bar{c}$	$b\bar{t}$	W^-Z	W^-H	total
Γ	2.00	2.00	1.99	5.99	5.98	84.7	42.7	42.1	187

$N_F = 4,\ z_L = 10^4\ (\theta_H = 0.0737)$									
mode	$e^-\bar{\nu}_e$	$\mu^-\bar{\nu}_\mu$	$\tau^-\bar{\nu}_\tau$	$d\bar{u}$	$s\bar{c}$	$b\bar{t}$	W^-Z	W^-H	total
Γ	3.63	3.63	3.62	10.88	10.88	219	46.9	47.2	346

TABLE VIII. Partial and total decay widths of $W_R^{-(1)}$ in the unit of GeV.

$N_F = 4,\ z_L = 10^5\ (\theta_H = 0.115)$			
SM fermions	W^-Z	W^-H	total
0	43.4	43.4	86.8

$N_F = 4,\ z_L = 10^4\ (\theta_H = 0.0737)$			
SM fermions	W^-Z	W^-H	total
0	48.7	48.8	97.4

fermion pairs. The $W^{(1)}$ decay to tb dominates over decay to other fermion pairs. Partial decay widths of $W^{(1)}$ to WZ and WH are almost identical:

$$\begin{aligned}
&\Gamma(W^{(1)} \to WH) \simeq \Gamma(W^{(1)} \to WZ), \qquad \therefore \eta_{W^{(1)}} \simeq 1, \\
&\Gamma(W_R^{(1)} \to WH) \simeq \Gamma(W_R^{(1)} \to WZ), \qquad \therefore \eta_{W_R^{(1)}} \simeq 1.
\end{aligned} \tag{5.1}$$

We also find

$$\begin{aligned}
&\Gamma(W^{(1)} \to WH) \sim \Gamma(W_R^{(1)} \to WH), \\
&\Gamma(W^{(1)} \to WZ) \sim \Gamma(W_R^{(1)} \to WZ).
\end{aligned} \tag{5.2}$$

Since the $W_R^{(n)}$ boson does not couple to SM fermions, and $W_R^{(1)}$ decay only to the SM bosons.

In Tables IX, X, and XI, decay widths of $Z^{(1)}$, $\gamma^{(1)}$, and $Z_R^{(1)}$ are tabulated, respectively. Compared with $W^{(1)}$ and $W_R^{(1)}$, $\gamma^{(1)}$ and $Z_R^{(1)}$ have large total widths

$$\Gamma_{\gamma^{(1)}}/M_{\gamma^{(1)}} = \begin{cases} 0.151 & \text{for } N_F = 4, z_L = 10^5 (\theta_H = 0.115) \\ 0.125 & \text{for } N_F = 4, z_L = 10^4 (\theta_H = 0.0737) \end{cases} \tag{5.3}$$

$$\Gamma_{Z_R^{(1)}}/M_{Z_R^{(1)}} = \begin{cases} 0.129 & \text{for } N_F = 4, z_L = 10^5 (\theta_H = 0.115) \\ 0.133 & \text{for } N_F = 4, z_L = 10^4 (\theta_H = 0.0737). \end{cases} \tag{5.4}$$

TABLE IX. Partial and total decay widths of $Z^{(1)}$ in the unit of GeV.

$N_F = 4,\ z_L = 10^5\ (\theta_H = 0.115)$														
e^+e^-	$\mu^+\mu^-$	$\tau^+\tau^-$	$\nu_e\bar{\nu}_e$	$\nu_\mu\bar{\nu}_\mu$	$\nu_\tau\bar{\nu}_\tau$	$u\bar{u}$	$c\bar{c}$	$t\bar{t}$	$d\bar{d}$	$s\bar{s}$	$b\bar{b}$	W^+W^-	ZH	total
40.4	35.7	32.1	1.30	1.30	1.30	53.3	46.1	48.5	15.7	13.8	45.7	16.1	54.8	406

$N_F = 4,\ z_L = 10^4\ (\theta_H = 0.0737)$														
e^+e^-	$\mu^+\mu^-$	$\tau^+\tau^-$	$\nu_e\bar{\nu}_e$	$\nu_\mu\bar{\nu}_\mu$	$\nu_\tau\bar{\nu}_\tau$	$u\bar{u}$	$c\bar{c}$	$t\bar{t}$	$d\bar{d}$	$s\bar{s}$	$b\bar{b}$	W^+W^-	ZH	total
48.1	42.5	38.0	2.36	2.36	2.36	64.4	55.5	84.4	20.3	18.1	106.8	17.8	61.1	564

TABLE X. Partial and total decay width of $\gamma^{(1)}$ in the unit of GeV.

$N_F = 4,\ z_L = 10^5\ (\theta_H = 0.115)$														
e^+e^-	$\mu^+\mu^-$	$\tau^+\tau^-$	$\nu_e\bar{\nu}_e$	$\nu_\mu\bar{\nu}_\mu$	$\nu_\tau\bar{\nu}_\tau$	$u\bar{u}$	$c\bar{c}$	$t\bar{t}$	$d\bar{d}$	$s\bar{s}$	$b\bar{b}$	W^+W^-	ZH	total
133.0	117.4	105.7	0	0	0	171.0	147.2	93.0	42.8	36.8	23.3	39.3	0	909

$N_F = 4,\ z_L = 10^4\ (\theta_H = 0.0737)$														
e^+e^-	$\mu^+\mu^-$	$\tau^+\tau^-$	$\nu_e\bar{\nu}_e$	$\nu_\mu\bar{\nu}_\mu$	$\nu_\tau\bar{\nu}_\tau$	$u\bar{u}$	$c\bar{c}$	$t\bar{t}$	$d\bar{d}$	$s\bar{s}$	$b\bar{b}$	W^+W^-	ZH	total
158.9	140.2	125.2	0	0	0	204.6	175.0	101.0	51.1	43.8	25.3	43.6	0	1068

TABLE XI. Partial and total decay widths of $Z_R^{(1)}$ in the unit of GeV.

$N_F = 4,\ z_L = 10^5\ (\theta_H = 0.115)$					
e^+e^-	$\mu^+\mu^-$	$\tau^+\tau^-$	$\nu_e\bar{\nu}_e$	$\nu_\mu\bar{\nu}_\mu$	$\nu_\tau\bar{\nu}_\tau$
71.3	63.8	58.0	$\mathcal{O}(10^{-16})$	$\mathcal{O}(10^{-11})$	$\mathcal{O}(10^{-7})$
$u\bar{u}$	$c\bar{c}$	$t\bar{t}$	$d\bar{d}$	$s\bar{s}$	$b\bar{b}$
92.0	80.4	146.2	23.0	20.1	111.9
W^+W^-	ZH				total
30.5	30.7				729
$N_F = 4,\ z_L = 10^4\ (\theta_H = 0.0737)$					
e^+e^-	$\mu^+\mu^-$	$\tau^+\tau^-$	$\nu_e\bar{\nu}_e$	$\nu_\mu\bar{\nu}_\mu$	$\nu_\tau\bar{\nu}_\tau$
83.9	75.2	68.1	$\mathcal{O}(10^{-13})$	$\mathcal{O}(10^{-10})$	$\mathcal{O}(10^{-6})$
$u\bar{u}$	$c\bar{c}$	$t\bar{t}$	$d\bar{d}$	$s\bar{s}$	$b\bar{b}$
108.5	94.6	268.4	27.1	23.7	240.3
W^+W^-	ZH				total
34.1	34.3				1058

From the tables, one finds that

$$\Gamma(Z^{(1)} \to HZ) \simeq \sum_{Z'=Z^{(1)},\gamma^{(1)}} \Gamma(Z' \to W^+W^-), \quad (5.5)$$

$$\Gamma(Z_R^{(1)} \to HZ) \simeq \Gamma(Z_R^{(1)} \to W^+W^-). \quad (5.6)$$

$\gamma^{(1)}$ and $Z^{(1)}$ are almost degenerate. The relation (5.5) follows from the relation among Higgs-vector boson and trilinear vector boson couplings

$$\frac{g^2_{\gamma^{(1)}ZH} + g^2_{Z^{(1)}ZH}}{M_Z^2} \simeq (g^2_{\gamma^{(1)}WW} + g^2_{Z^{(1)}WW}) \frac{M^4_{Z'}}{M^4_Z}, \quad (5.7)$$

where $M_{\gamma^{(1)}} \simeq M_{Z^{(1)}} \equiv M_{Z'}$ and $g_{\gamma^{(1)}ZH} = 0$.

VI. CROSS SECTION

In this section we evaluate cross sections in pp-collisions for various final states. In the numerical evaluation we use CTEQ5 parton distribution functions [75].

A. $W' \to tb$, $\mu\nu$ and $Z' \to \ell^+\ell^-$

In Fig. 3, the differential cross sections of processes $pp \to \{W^-, W^{(1)-}\} \to b\bar{t}, \mu^-\bar{\nu}$ are plotted. For light fermion doublet paris $\ell\bar{\nu}$, $d\bar{u}$, $s\bar{c}$ in the final state, due to the flipped-signs of the couplings to $W^{(1)}$, a clear deficit of cross section just below the resonance $M_{\mu\nu} \sim M_{W^{(1)}}$ is observed. For processes with $b\bar{t}$ final state, a deficit of cross section is observed above the resonance $M_{tb} > M_{W^{(1)}}$, since the $W^{(1)}\bar{t}b$ coupling has opposite sign relative to the $W^{(1)}\bar{u}d$ coupling.

When the final state contains a neutrino, the transverse momentum distribution $d\sigma/dp_T$ with respect to the transverse momentum of charged lepton, p_T, gives information on the mass of W'. The transverse-momentum distribution at parton-level is given in Appendix G. The p_T distribution in pp collision is given by

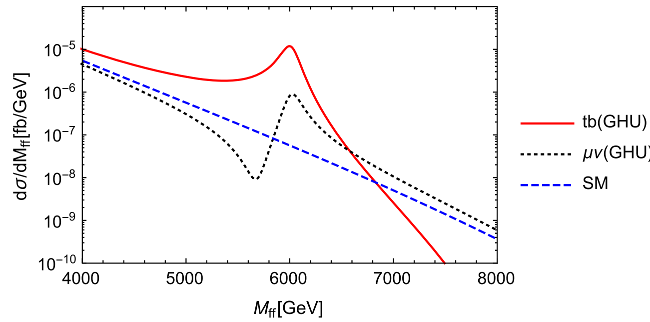

FIG. 3. $pp(d\bar{u}) \to \{W^-, W^{(1)-}\} \to b\bar{t}, \mu^-\bar{\nu}_\mu$ differential cross sections $d\sigma/dM_{\rm ff}$ at $\sqrt{s_{pp}} = 14$ TeV for $N_F = 4$, $z_L = 10^5$ ($\theta_H = 0.115$). $M_{\rm ff}$ is the invariant mass of (b, $\bar{t}$) or (μ, $\bar{\nu}$). Red-solid, black-dotted lines are for the $b\bar{t}$ and $\mu^-\bar{\nu}$ final states in GHU. Blue-dashed line is the cross section in the SM. Cross sections for the processes $pp \to W^{(1)-} \to f\bar{f}'$, $(f, f') = (e^-, \nu_e), (\tau^-, \nu_\tau)$ are almost identical with that of $\mu^-\bar{\nu}$, whereas cross section for $(f, f') = (d, u), (s, c)$ final states is three times as large as that of $\mu^-\bar{\nu}$ due to the color factor.

$$\frac{d\sigma(pp \to e^-\bar{\nu} + X)}{dp_T}(p_T) = \int_0^1 d\tau \left\{ \frac{dp_T(d\bar{u} \to e^-\bar{\nu})}{dp_T}(s_{pp}\tau, p_T) \cdot \frac{dL_{d\bar{u}}}{d\tau}(\tau) \right\}, \quad (6.1)$$

where the parton luminosity $dL_{d\bar{u}}/d\tau$ is given in terms of parton distribution functions $f_q(x_1, Q)$ by

$$\begin{aligned} \frac{dL_{d\bar{u}}}{d\tau}(\tau) = &\int_0^1 dx_1 \int_0^1 dx_2 [f_d(x_1, Q) f_{\bar{u}}(x_2, Q) \\ &+ f_d(x_2, Q) f_{\bar{u}}(x_1, Q)] \delta(\tau - x_1 x_2), \\ Q = &\ s_{pp}\tau. \end{aligned} \quad (6.2)$$

In Fig. 4 the p_T-distribution $d\sigma(pp \to e^-\bar{\nu})/dp_T$ is plotted. In the figure, Jacobian peak at $p_T = M_{M^{(1)}}/2 \simeq 3$ TeV is observed.

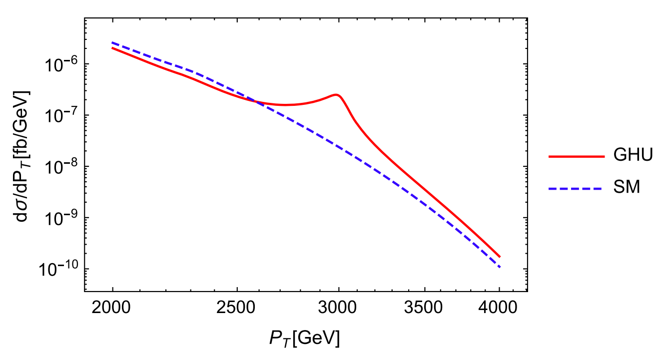

FIG. 4. $d\sigma(pp \to e^-\bar{\nu}_e)/dp_T$ as a function of p_T at $\sqrt{s_{pp}} = 14$ TeV for $N_F = 4$, $z_L = 10^5$ ($\theta_H = 0.115$). Red-solid and blue-dashed lines are for the GHU model and the SM, respectively.

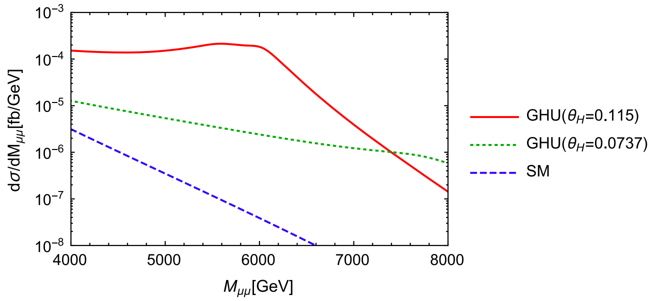

FIG. 5. Differential cross section $d\sigma/dM_{\mu\mu}$ of the process $pp(u\bar{u}, d\bar{d}) \to \{\gamma, Z, Z^{(1)}, \gamma^{(1)}, Z_R^{(1)}\} \to \mu^+\mu^-$ at $\sqrt{s} = 14$ TeV. $M_{\mu\mu}$ is the invariant mass of $\mu^+\mu^-$. Solid [red], dotted [green], and dashed [blue] lines are the Z' resonance in the GHU for $N_F = 4$, $z_L = 10^5$ ($\theta_H = 0.115$), $N_F = 4$, $z_L = 10^4$ ($\theta_H = 0.0737$), and the SM, respectively. The cross section for the e^+e^- final state is identical to the $\mu^+\mu^-$ final state.

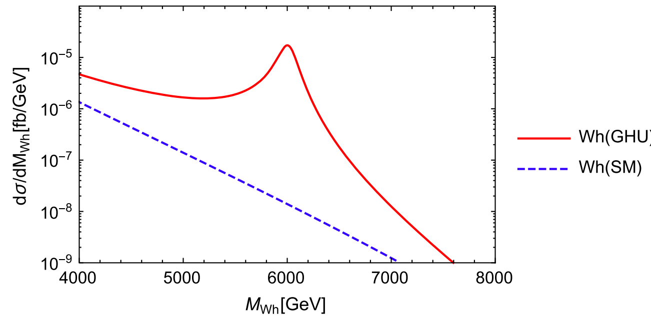

FIG. 6. Differential cross section $d\sigma/M_{Wh}$ of the process $pp(d\bar{u}) \to \{W^-, W^{(1)-}\} \to W^-H$ at $\sqrt{s_{pp}} = 14$ TeV for $N_F = 4$, $z_L = 10^5$ ($\theta_H = 0.115$). M_{Wh} is the invariant mass of WH. Red-solid and blue-dashed lines show cross sections in GHU and in the SM, respectively.

For the process $pp(u\bar{u}, d\bar{d}) \to Z' \to \ell^+\ell^-$, we show the plot of the differential cross section $d\sigma/dM_{\mu\mu}$ in Fig. 5. In the plot the updated decay widths of Z' ($Z' = \gamma^{(1)}, Z^{(1)}$ and $Z_R^{(1)}$) has been used, which takes bosonic final states (W^+W^- and ZH) into account, and is $\mathcal{O}(10\%)$ bigger than that used in the previous paper [47]. We stress that the rate of Z' production is rather large, and it is promising to see the Z' events at the current LHC Run 2. Since in this model Z' bosons have large widths, at the early stage of LHC experiment, sporadic events of high-energy $\mu^+\mu^-$ final states will be seen. For $\theta_H = 0.115$ ($M_{Z^{(1)},\gamma^{(1)}} \sim 6.0$ TeV, $M_{Z_R^{(1)}} \sim 5.7$ TeV), with the 30 fb^{-1} and $\sqrt{s_{pp}} = 13$ TeV data, expected numbers of events in GHU N_{GHU} and SM signal $N_{\rm SM}$, and significance S are $N_{GHU}/N_{\rm SM}(S) =$ 8.4/3.6(2.2), 3.9/0.26(3.7), 2.6/0.02(4.4), 2.9/0.004(6.0), 0.65/0.0002(3.0) and $0.01/1 \times 10^{-5}(0.34)$ for bins (GeV) [2000, 3000], [3000, 4000], [4000, 5000], [5000, 6000], [6000, 7000] and [7000, 8000], respectively. In this case, an excess of high-energy ($M_{\mu\mu} \gtrsim 3000$ GeV) events is expected. For smaller θ_H (heavier $M_{Z'}$), the signals becomes smaller and more data is required for confirming/rejecting the model. For $\theta_H = 0.0737$ ($M_{Z^{(1)},\gamma^{(1)}} \sim 8.5$ TeV, $M_{Z_R^{(1)}} \sim 7.9$ TeV), with the 1000 fb^{-1} and $\sqrt{s_{pp}} = 14$ TeV data, $N_{GHU}/N_{\rm SM}(S) = 140/155(1.21)$, 23/12(2.7), 8.5/1.3(4.2), 3.7/0.14(4.12), 0.7/0.006(2.3), and 0.3/0.0004(1.7) for bins (GeV) [2000, 3000], [3000, 4000], [4000, 5000], [5000, 6000], [6000, 7000], and [7000, 8000], respectively.

B. $W' \to WH$ and $Z' \to ZH$

In Figs. 6 and 7, differential cross sections of processes $pp \to \{W^-, W^{(1)-}\} \to W^-H$ and $pp \to \{Z, Z^{(1)}, Z_R^{(1)}\} \to ZH$ are plotted, respectively. Compared with the WH mode, cross section for ZH mode is bigger and the width is wider. It is due to the fact that $Z^{(1)}$ and $Z_R^{(1)}$ have large couplings to the right-handed quarks, and their widths are large.

C. $W' \to WZ$ and $Z' \to WW$

In Figs. 8 and 9, differential cross sections of the processes $pp \to \{\gamma, Z, Z^{(1)}, \gamma^{(1)}, Z_R^{(1)}\} \to W^+W^-$ and $pp \to \{W^-, W^{(1)-}\} \to W^-Z$ are plotted, respectively. For the WZ final states, the signal of the resonance of W' is a few times larger than that of the SM. For the WW final states, the contribution from Z' resonances is much smaller than the SM cross section so that the signal is hard to see.

We comment that there are no s-channel processes with ZZ final states mediated by vector bosons. The process mediated by KK gravitons [76,77] can be ignored, as the couplings of KK gravitons to the SM fields are suppressed by $k/M_{\rm Pl} \ll 1$, where $M_{\rm Pl}$ is the Planck mass.

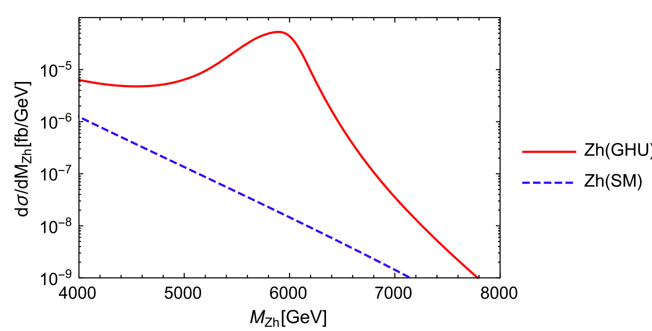

FIG. 7. Differential cross section $d\sigma/dM_{Zh}$ of the process $pp(u\bar{u}, d\bar{d}) \to \{Z, Z^{(1)}, Z_R^{(1)}\} \to ZH$ cross section at $\sqrt{s_{pp}} =$ 14 TeV for $N_F = 4$, $z_L = 10^5$ ($\theta_H = 0.115$). M_{Zh} is the invariant mass of ZH. Red-solid and blue-dashed lines show cross sections in GHU and in the SM, respectively.

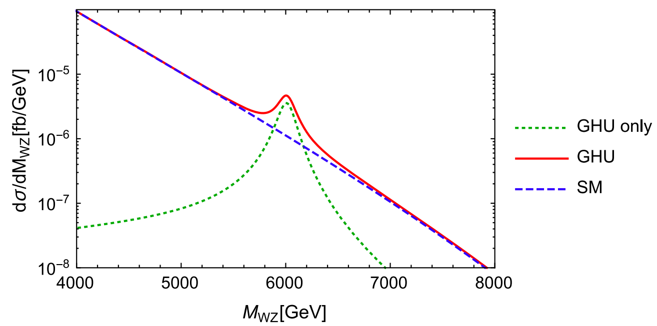

FIG. 8. Differential cross section $d\sigma/dM_{WZ}$ of the process $pp(d\bar{u}) \to \{W^-, W^{(1)-}\} \to W^- Z$ at $\sqrt{s_{pp}} = 14\,\mathrm{TeV}$ for $N_F = 4$, $z_L = 10^5$ ($\theta_H = 0.115$). M_{WZ} is the invariant mass of WZ. Green-dotted line shows the s-channel W' signals in GHU model. Blue-dashed line shows the SM prediction including s-, t-, and u-channels [74]. Red-solid line is the sum of SM and GHU signals.

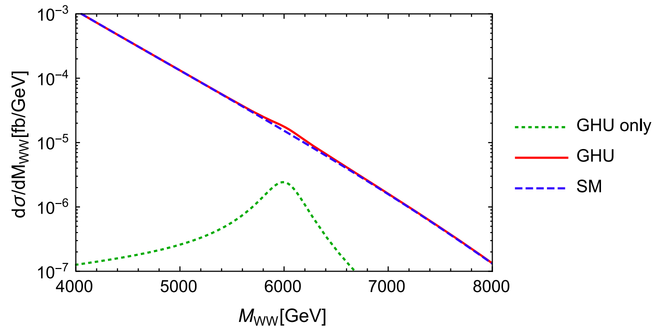

FIG. 9. Differential cross section $d\sigma/dM_{WW}$ of the process $pp(u\bar{u}, d\bar{d}) \to \{\gamma, Z, Z^{(1)}, \gamma^{(1)}, Z_R^{(1)}\} \to W^+W^-$ at $\sqrt{s_{pp}} = 14$ TeV for $N_F = 4$, $z_L = 10^5$ ($\theta_H = 0.115$). M_{WW} is the invariant mass of W^+W^-. Green-dotted line shows the s-channel W' signals in GHU model. Blue-dashed line shows the SM prediction including s-, t-, and u-channels [73]. Red-solid line is the sum of SM and GHU signals.

D. Unitarity in $f\bar{f}' \to WZ$

It is important to see how the unitarity is ensured when vector bosons are involved in the final states. The unitarity in the W boson scattering, $WW \to WW$ in the gauge-Higgs unification has been studied in [41] by using position-space propagators. In the present paper we are considering $f\bar{f} \to V' \to WW, WZ$. In these processes, t- and u-channel amplitudes must be included to cancel the growing part of the s-channel amplitude at $\sqrt{s} \gg m_{V'}$ (See Fig. 10).

Let us consider the process $d(p_1)\bar{u}(p_2) \to W^-(k_1)Z(k_2)$. When the initial states are given by $d_R\bar{u}_L$, there are no s-channel contribution as $W^{(n)}$ do not couple to the right-handed quarks. For the final state bosons with longitudinal polarization, $\epsilon^\mu(k_1) \simeq k_1^\mu/M_W$ and $\epsilon^\nu(k_2) \simeq k_2^\nu/M_Z$, t- and u-channel amplitudes at very high-energy $\sqrt{s} \gg m_{KK}$ are expressed as

$$\begin{aligned}\mathcal{M}_t &\sim \frac{1}{M_W M_Z}\sum_{\mathcal{U}} g^R_{Wd\mathcal{U}} g^R_{Zu\mathcal{U}} \bar{v}(p_2)\not{k}_2 P_R \\ &\quad\times \frac{(\not{p}_1 - \not{k}_1)}{(p_1 - k_1)^2}\not{k}_1 P_R u(p_1) \\ &= \frac{1}{2M_W M_Z}\sum_{\mathcal{U}} g^R_{Wd\mathcal{U}} g^R_{Zu\mathcal{U}} \bar{v}(p_2)(\not{k}_2 - \not{k}_1) P_R u(p_1),\end{aligned} \tag{6.3}$$

$$\begin{aligned}\mathcal{M}_u &\sim \frac{1}{M_W M_Z}\sum_{\mathcal{D}} g^R_{Zd\mathcal{D}} g^R_{Wu\mathcal{D}} \bar{v}(p_2)\not{k}_1 P_R \\ &\quad\times \frac{(\not{p}_1 - \not{k}_2)}{(p_1 - k_2)^2}\not{k}_2 P_R u(p_1) \\ &= \frac{1}{2M_W M_Z}\sum_{\mathcal{D}} g^R_{Zd\mathcal{D}} g^R_{Wu\mathcal{D}} \bar{v}(p_2)(\not{k}_1 - \not{k}_2) P_R u(p_1),\end{aligned} \tag{6.4}$$

where $P_{L/R} \equiv (1 \mp \gamma_5)/2$, and $\mathcal{U}$ and $\mathcal{D}$ denote KK-excited states with $Q_{\mathrm{EM}} = +2/3$ and $-1/3$, respectively. Here we have retained contributions only from the first KK states of fermions, as the $Wu\mathcal{D}^{(n)}$, $Wd\mathcal{U}^{(n)}$, $Zu\mathcal{U}^{(n)}$, and $Zd\mathcal{D}^{(n)}$ couplings ($n \geq 2$), etc., are all negligibly small.

In order for the growing parts of $\mathcal{M}_t$ and $\mathcal{M}_u$ to cancel with each other, the relation

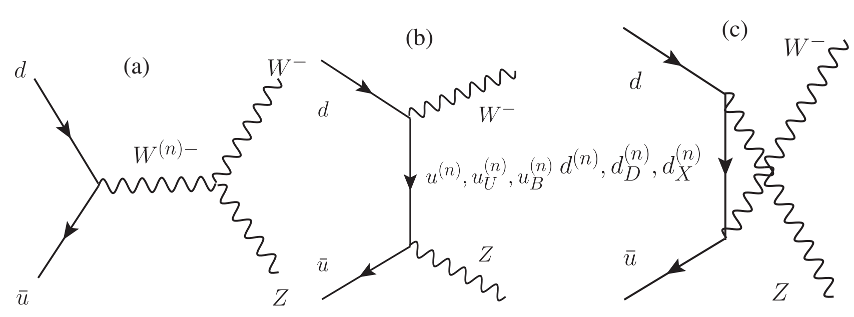

FIG. 10. Diagrams of $d\bar{u} \to WZ$ at a energy scale above M_{KK}. (a), (b), and (c) represent s-, t-, and u-channels, respectively. $W^{(n)}$ is the nth KK state of W, whereas $\mathcal{D}^{(n)}$ and $\mathcal{U}^{(n)}$ are the nth KK states of d and u, and partners $d^{(n)}_{D,X}$, $u^{(n)}_{U,B}$, respectively.

$$\sum_{\mathcal{U}} g^R_{W\mathcal{U}d} g^R_{Zu\mathcal{U}} \simeq \sum_{\mathcal{D}} g^R_{Z\mathcal{D}d} g^R_{Wu\mathcal{D}} \tag{6.5}$$

should be satisfied. With the values in Tables XIV–XVIII, one finds that in (6.5)

$$\begin{aligned}(\text{L.H.S}) &\simeq g^R_{Wdu^{(1)}} g^R_{Zuu^{(1)}} + g^R_{Wdu^{(1)}_B} g^R_{Zuu^{(1)}_B} \\ &= [(-2.95)\cdot(-0.65) + (1.29) \\ &\quad \cdot (-1.48)] \times 10^{-4} g_w^2/\sqrt{2},\end{aligned} \tag{6.6}$$

$$\begin{aligned}(\text{R.H.S}) &\simeq g^R_{dd^{(1)}} g^R_{Wud^{(1)}} + g^R_{Zdd^{(1)}_X} g^R_{Wud^{(1)}_X} \\ &= [(1.48)\cdot(-1.3) + (0.65) \\ &\quad \cdot (2.95)] \times 10^{-4} g_w^2/\sqrt{2}.\end{aligned} \tag{6.7}$$

We observe that the relation (6.5) is well satisfied. We note that

$$\begin{aligned} g^R_{Wdu^{(1)}} &= -g^R_{Wud^{(1)}_X}, & g^R_{Zuu^{(1)}} &= -g^R_{Zdd_X^{(1)}}, \\ g^R_{Wdu^{(1)}_B} &= -g^R_{Wud^{(1)}}, & g^R_{Zdd^{(1)}} &= -g^R_{Zuu_B^{(1)}}. \end{aligned} \tag{6.8}$$

When the initial states are given by $d_L \bar{u}_R$, there are contributions from s-channel amplitudes. The condition for the cancellations is given by (cf. Chapter 21 of [78])

$$\sum_{n=0}^{\infty} g^L_{W^{(n)}ud} g_{W^{(n)}WZ} \simeq \sum_{\mathcal{U}'} g^L_{Wd\mathcal{U}'} g^L_{Zu\mathcal{U}'} - \sum_{\mathcal{D}'} g^L_{Zd\mathcal{D}'} g^L_{Wu\mathcal{D}'}, \tag{6.9}$$

where $\mathcal{U}'$ and $\mathcal{D}'$ represent all SM and non-SM fermions in the first generation with $Q_{\rm EM} = 2/3$ and $-1/3$, respectively. From Tables XIV, XV, XVII, and XVIII in Appendix D, one finds that $Wu\mathcal{D}^{(n)}$, $Wd\mathcal{U}^{(n)}$, $Zu\mathcal{U}^{(n)}$, and $Zd\mathcal{D}^{(n)}$ couplings ($n \geq 1$) are all small. Hence the right-hand side of (6.9) will be approximately given by

$$(\text{R.H.S}) \simeq g_{Wdu}(g_{Zuu} - g_{Zdd}) = 0.877163 \cdot g_w^2/\sqrt{2}, \tag{6.10}$$

The left-hand side is approximately given, with use of Tables. IV and VI, by

$$(\text{L.H.S}) \simeq \sum_{n=0}^{4} g_{W^{(n)}ud} g_{W^{(n)}WZ} = 0.877162 \cdot g_w^2/\sqrt{2}. \tag{6.11}$$

It is recognized that (6.9) is also quantitatively well-satisfied.

In an analogous way one can confirm the unitarity of the amplitude of $f\bar{f} \to W^+W^-$. In this case KK bosons of γ, Z, and Z_R are involved in the s-channel amplitudes.

VII. SUMMARY

In this paper we have studied the collider signals of W' and Z' in the $SO(5)\times U(1)_X$ gauge-Higgs unification.

First we evaluated the couplings of W' and Z' to the SM fields. We found that the $W^{(1)}$ couplings to light fermions and to top-bottom are different in signs, which is explained from the different behavior of wave functions of fields along the extra dimension.

Next we evaluated the decay rates of neutral and charged KK vector bosons. The total decay widths of Z' are large. $\Gamma_{Z'}/M_{Z'} = 15\%$, 6.6% and 13% for $Z' = \gamma^{(1)}$, $Z^{(1)}$ and $Z_R^{(1)}$, respectively. On the other hand, $W^{(1)}$ has a narrow total width: $\Gamma_{W^{(1)}}/M_{W^{(1)}} \simeq 3\%$. Several interesting relations among decay modes (5.1), (5.2), (5.5), and (5.6) are found. In the warped space $W^{(1)}$ and $W_R^{(1)}$ can decay to WH and WZ. Decay width of W' to WH and WZ are all nearly equal with each other. For Z' it is found that $\Gamma(Z^{(1)}\to ZH) \simeq \Gamma(Z^{(1)}\to WW) + \Gamma(\gamma^{(1)}\to WW)$ and $\Gamma(Z_R\to ZH) \simeq \Gamma(Z_R \to WW)$. These properties of W_R and Z_R are qualitatively understood in terms of the 4D $SO(4)\times U(1)$ model introduced in Sec. IV.

Further we have numerically evaluated the s-channel cross sections of W' and Z' in the LHC. We studied not only processes with fermionic final states but also bosonic WH, ZH, WW and WZ final states. W' and Z' signals of GHU can be found at the LHC experiment in the processes $pp \to W'(Z') + X$, $W' \to tb, WH$, and $Z' \to e^+e^-, \mu^+\mu^-, ZH$ near the W' and Z' resonances. For $\theta_H = 0.115$ ($M_{Z^{(1)},\gamma^{(1)}} \simeq 6.0$ TeV and $M_{Z_R^{(1)}} \simeq 5.7$ TeV), with the data of 30 fb^{-1}, $\sqrt{s_{pp}} = 13$ TeV at LHC, an excess of the events of $\mu^+\mu^-$ with invariant mass is expected. (e.g. expected signal[background] is 3.9 [0.29] events for the bin (GeV) [3000, 4000]).

In the process with WZ in the final state, it is found that in the amplitude the leading contributions from the longitudinal polarizations of W and Z in the s-, t-, and u-channels cancel with each other so that the unitarity is preserved, provided that both KK vector bosons and KK fermions in the intermediate states are taken into account. We have confirmed numerically that this cancellation of the leading terms in the amplitude with 6 digits of precision by taking into account contributions of up to the 4th level of KK excited states.

We also found that the non-SM 1st KK excited state of fermions can be much lighter than other KK states. Especially the 1st KK excited top and bottom partners ($t^{(1)}_{U,B,T}$ and $b^{(1)}_{D,X,Y}$) are the lightest non-SM particles and can be singly produced in colliders. It is seen in Tables XII and XIII that $t^{(1)}_{U,B,T}$ and

$b^{(1)}_{D,X,Y}$, which are exotic partners of the top and bottom quarks respectively, have mass $M_{t^{(1)}_{U,B,T}, b^{(1)}_{D,X,Y}} = 4.6$ TeV (5.4 TeV) for $\theta_H = 0.115$ (0.0737). $t^{(1)}_T$ and $b^{(1)}_Y$ have electric charges $+5/3$ and $-4/3$ and can be observed in the processes $t + W^+ \to t_T \to t + W^+$ and $b + W^- \to b_Y \to b + W^-$ in colliders [79–81].

The gauge-Higgs unification scenario is promising. It gives many predictions to be tested at LHC and future colliders. The 4D Higgs boson appears as the gauge boson in the extra dimension. The gauge hierarchy problem is solved. The AB phase θ_H is the important parameter in GHU. Many of the physical quantities are determined by θ_H. The universal relations among θ_H and m_{KK}, Higgs cubic and quartic couplings have been found. Corrections to the decay rates for $H \to \gamma\gamma$, $Z\gamma$ due to infinitely many KK states turn out finite and small. Z' and W' are predicted around 6–8 TeV. Discovery of Z' and W' is most awaited.

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APPENDIX A: BASIC FORMULAS

1. $SO(5)$ generators

The $SO(5)$ generators in the spinor-representation are given by

$$T^{a_L} = \frac{1}{2}\begin{pmatrix} \sigma^a & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}, \qquad T^{a_R} = \frac{1}{2}\begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \sigma^a \end{pmatrix},$$
$$T^{\hat{a}} = \frac{1}{2\sqrt{2}}\begin{pmatrix} & i\sigma^a \\ -i\sigma^a & \end{pmatrix}, \qquad T^{\hat{4}} = \begin{pmatrix} & \mathbf{1} \\ \mathbf{1} & \end{pmatrix}, \tag{A1}$$

and $\mathrm{tr}(T^a T^b) = \frac{1}{2}\delta^{ab}$ is satisfied. Here σ^a ($a = 1, 2, 3$) are Pauli matrices. T^{a_L} and T^{a_R} are generators for $SU(2)_L$ and $SU(2)_R$ subgroups, respectively.

2. Bulk wave functions

a. Gauge boson bulk functions

Bulk functions of gauge bosons $C = C(z;\lambda)$ and $S = S(z;\lambda)$ are defined as solutions of

$$\left(\frac{d^2}{dz^2} - \frac{1}{z}\frac{d}{dz} + \lambda^2\right)\begin{pmatrix} C \\ S \end{pmatrix} = 0, \tag{A2}$$

with boundary conditions

$$C = z_L, \qquad S = 0, \qquad C' = 0, \qquad S' = \lambda \quad \text{at } z = z_L. \tag{A3}$$

Here $C' \equiv (d/dz)C$ etc. The solutions are given by

$$\begin{aligned} C(z;\lambda) &= +\frac{\pi}{2}\lambda z z_L F_{1,0}(\lambda z, \lambda z_L), \\ C'(z;\lambda) &= +\frac{\pi}{2}\lambda^2 z z_L F_{0,0}(\lambda z, \lambda z_L), \\ S(z;\lambda) &= -\frac{\pi}{2}\lambda z F_{1,1}(\lambda z, \lambda z_L), \\ S'(z;\lambda) &= -\frac{\pi}{2}\lambda^2 z F_{0,1}(\lambda z, \lambda z_L), \end{aligned} \tag{A4}$$

where $F_{\alpha,\beta}(u,v) \equiv J_\alpha(u)Y_\beta(v) - Y_\alpha(u)J_\beta(v)$ and $J_\alpha(x)$ and $Y_\alpha(x)$ are Bessel functions of the 1st and 2nd kind, respectively. C, S and C', S' satisfy

$$CS' - SC' = \lambda. \tag{A5}$$

b. Fermion bulk functions

Fermion bulk functions $C_{L/R}(z;\lambda,c)$, $S_{L/R}(z;\lambda,c)$ are defined by

$$\begin{aligned} C_L(z;\lambda,c) &= +\frac{\pi}{2}\lambda\sqrt{z z_L} F_{c+\frac{1}{2},c-\frac{1}{2}}(\lambda z, \lambda z_L), \\ S_L(z;\lambda,c) &= -\frac{\pi}{2}\lambda\sqrt{z z_L} F_{c+\frac{1}{2},c+\frac{1}{2}}(\lambda z, \lambda z_L), \\ C_R(z;\lambda,c) &= +\frac{\pi}{2}\lambda\sqrt{z z_L} F_{c-\frac{1}{2},c+\frac{1}{2}}(\lambda z, \lambda z_L), \\ S_R(z;\lambda,c) &= +\frac{\pi}{2}\lambda\sqrt{z z_L} F_{c-\frac{1}{2},c-\frac{1}{2}}(\lambda z, \lambda z_L). \end{aligned} \tag{A6}$$

These satisfy

$$D_+\begin{pmatrix} C_L \\ S_L \end{pmatrix} = \lambda\begin{pmatrix} S_R \\ C_R \end{pmatrix}, \qquad D_-\begin{pmatrix} C_R \\ S_R \end{pmatrix} = \lambda\begin{pmatrix} S_L \\ C_L \end{pmatrix},$$
$$D_\pm(c) \equiv \pm\frac{d}{dz} + \frac{c}{z},$$
$$C_L C_R - S_L S_R = 1, \tag{A7}$$

and

$$C_R = C_L = 1, \qquad S_R = S_L = 0, \quad \text{at } z = z_L. \tag{A8}$$

In particular, for $c = 0$ we have

$$\begin{aligned} C_L(z;\lambda,0) &= C_R(z;\lambda,0) = \cos(\lambda(z - z_L)), \\ S_L(z;\lambda,0) &= -S_R(z;\lambda,0) = \sin(\lambda(z - z_L)). \end{aligned} \tag{A9}$$

APPENDIX B: GAUGE BOSON WAVE FUNCTIONS

Wave functions for a charged vector boson $V_C = W^{(n)}, W_R^{(m)}$ ($n = 0, 1, 2, \cdots$, $m = 1, 2, \cdots$) are given by

$$\begin{pmatrix} h^L_{V_C} \\ h^R_{V_C} \\ \hat{h}_{V_C} \end{pmatrix} = \begin{pmatrix} v^L_{V_C} C(z;\lambda_{V_C}) \\ v^R_{V_C} C(z;\lambda_{V_C}) \\ \hat{v}_{V_C}\hat{S}(z;\lambda_{V_C}) \end{pmatrix}, \tag{B1}$$

where

$$\begin{pmatrix} v^L_{V_C} \\ v^R_{V_C} \\ \hat{v}_{V_C} \end{pmatrix} = \frac{1}{\sqrt{r_{V_C}}} \begin{cases} \begin{pmatrix} \frac{1+c_H}{\sqrt{2}} \\ \frac{1-c_H}{\sqrt{2}} \\ -s_H \end{pmatrix} & V_C = W^{(n)} \\ \frac{1}{\sqrt{1+c_H^2}} \begin{pmatrix} \frac{+1-c_H}{\sqrt{2}} \\ \frac{-1-c_H}{\sqrt{2}} \\ 0 \end{pmatrix} & V_C = W_R^{(m)} \end{cases} \tag{B2}$$

with $C = C(z; \lambda_{V_C})$ etc. $c_H, s_H \equiv \cos\theta_H, \sin\theta_H$. We have defined

$$\hat{S}(z;\lambda) \equiv \frac{C(1;\lambda)}{S(1;\lambda)} S(z;\lambda). \tag{B3}$$

The mass spectrum $\{m_{V_C} = k\lambda_{V_C}\}$ is determined by

$$\begin{aligned} 2SC'(1, \lambda_{W^{(n)}}) + \lambda_{W^{(n)}} s_H^2 = 0, \\ C(1; \lambda_{W_R^{(n)}}) = 0, \end{aligned} \tag{B4}$$

and normalization factors are given by

$$\begin{aligned} r_{W_R^{(n)}} &= \int_1^{z_L} \frac{dz}{kz} C(z; \lambda_{W_R^{(n)}})^2, \\ r_{W^{(n)}} &= \int_1^{z_L} \frac{dz}{kz} \{(1+c_H^2) C(z; \lambda_{W^{(n)}})^2 + s_H^2 \hat{S}(z; \lambda_{W^{(n)}})^2\}. \end{aligned} \tag{B5}$$

Wave functions for the photon $\gamma = \gamma^{(0)}$ is given by

$$\begin{pmatrix} h^L_{\gamma^{(0)}} \\ h^R_{\gamma^{(0)}} \\ h^B_{\gamma^{(0)}} \end{pmatrix} = \frac{1}{\sqrt{(1+s_\phi^2)L}} \begin{pmatrix} s_\phi \\ s_\phi \\ c_\phi \end{pmatrix}, \qquad \hat{h}_{\gamma^{(0)}} = 0, \tag{B6}$$

where $s_\phi \equiv \sin\phi$ and $c_\phi \equiv \cos\phi$. Wave functions for a massive neutral vector boson $V = Z^{(n)}$, $\gamma^{(m)}$, and $Z_R^{(m)}$ ($n = 0, 1, 2, \cdots$, $m = 1, 2, \cdots$) are given by

$$\begin{pmatrix} h^L_V \\ h^R_V \\ \hat{h}_V \\ h^B_V \end{pmatrix} = \begin{pmatrix} v^L_V C(z;\lambda_V) \\ v^R_V C(z;\lambda_V) \\ \hat{v}_V \hat{S}(z;\lambda_V) \\ v^B_V C(z;\lambda_V) \end{pmatrix}, \tag{B7}$$

where

$$\begin{pmatrix} v^L_V \\ v^R_V \\ \hat{v}_V \\ v^B_V \end{pmatrix} = \frac{1}{\sqrt{r_V}} \begin{cases} \frac{1}{\sqrt{1+s_\phi^2}} \begin{pmatrix} \frac{(1+s_\phi^2)(1+c_H)-2s_\phi^2}{\sqrt{2}} \\ \frac{(1+s_\phi^2)(1-c_H)-2s_\phi^2}{\sqrt{2}} \\ -(1+s_\phi^2)s_H \\ -\sqrt{2}s_\phi c_\phi \end{pmatrix} & V = Z^{(n)} \\ \frac{1}{\sqrt{1+s_\phi^2}} \begin{pmatrix} s_\phi \\ s_\phi \\ 0 \\ c_\phi \end{pmatrix} & V = \gamma^{(m)}, \\ \frac{1}{\sqrt{1+(1+2t_\phi^2)c_H^2}} \begin{pmatrix} \frac{+1-c_H}{\sqrt{2}} \\ \frac{-1-c_H}{\sqrt{2}} \\ 0 \\ \sqrt{2}t_\phi c_H \end{pmatrix} & V = Z_R^{(m)}, \end{cases} \tag{B8}$$

where $t_\phi = \tan\phi$. The mass spectrum $\{m_V = k\lambda_V\}$ is determined by

$$\begin{aligned} C'(1; \lambda_{\gamma^{(m)}}) = 0, \\ 2SC'(1; \lambda_{Z^{(n)}}) + (1+s_\phi^2)\lambda_{Z^{(n)}} \sin^2\theta_H = 0, \\ C(1; \lambda_{Z_R^{(m)}}) = 0, \end{aligned} \tag{B9}$$

and normalization factors are given by $r_V = \int_1^{z_L} \frac{dz}{kz} \mathcal{F}_V$ where

$$\mathcal{F}_V = \begin{cases} C(z;\lambda_V)^2 & V = Z_R^{(m)}, \gamma^{(m)}, \\ c_\phi^2 C(z;\lambda_V)^2 + (1+s_\phi^2)[c_H^2 C(z;\lambda_V)^2 + s_H^2 \hat{S}(z;\lambda_V)^2] & V = Z^{(n)}. \end{cases} \tag{B10}$$

APPENDIX C: MASSES AND WAVE FUNCTIONS OF *SO*(5)-VECTOR FERMIONS

1. Quark sector

a. $Q_{\rm em} = +5/3$ quark partners (t_T)

$(T^{3_L}, T^{3_R}) = (+\frac{1}{2}, +\frac{1}{2})$ of $\Psi_1^{q,g=3}$ state has an expansion

$$T(x,z) = \sqrt{k} z^2 \sum_{n=1}^{\infty} \left\{ t_{T,L}^{(n)}(x) \frac{1}{\sqrt{r_{t_{T,L}^{(n)}}}} C_L(z, \lambda_{t_T^{(n)}}, c_1) + t_{T,R}^{(n)}(x) \frac{1}{\sqrt{r_{t_{T,R}^{(n)}}}} S_R(z, \lambda_{t_T^{(n)}}, c_1) \right\}, \tag{C1}$$

where $\lambda_{t_T^{(n)}} = m_{t_T^{(n)}}/k$. The KK mass $m_{t_T^{(n)}}$ is determined by

$$C_L(1, \lambda_{t_T^{(n)}}, c_1) = 0. \tag{C2}$$

Normalization factors $r_{t^{(n)}_{T,L/R}}$ are determined so that they satisfy

$$\frac{1}{r_{t^{(n)}_{T,L}}} \int_1^{z_L} C_L(z, \lambda_T^{(n)}, c_1)^2 dz = \frac{1}{r_{t^{(n)}_{T,R}}} \int_1^{z_L} S_R(z, \lambda_T^{(n)}, c_1)^2 dz = 1, \tag{C3}$$

and one finds $r_{t^{(n)}_{T,L}} = r_{t^{(n)}_{T,R}}$.

b. $Q_{\rm em} = -4/3$ quark partners (b_Y)

$(T^{3_L}, T^{3_R}) = (-\frac{1}{2}, -\frac{1}{2})$ of $\Psi_2^{q,g=3}$ state has an expansion

$$Y(x,z) = \sqrt{k} z^2 \sum_{n=1}^{\infty} \left\{ b^{(n)}_{Y,L}(x) \frac{1}{\sqrt{r_{b^{(n)}_{Y,L}}}} C_L(z, \lambda_{b_Y^{(n)}}, c_2) + b^{(n)}_{Y,R}(x) \frac{1}{\sqrt{r_{b^{(n)}_{Y,R}}}} S_R(z, \lambda_{b_Y^{(n)}}, c_2) \right\}, \tag{C4}$$

where $\lambda_{b_Y^{(n)}} = m_{b_Y^{(n)}}/k$ and $m_{b_Y^{(n)}}$ is the KK mass, which is determined by

$$C_L(1, \lambda_{b_Y^{(n)}}, c_2) = 0. \tag{C5}$$

Factors $r_{b^{(n)}_{Y,L/R}}$ are normalized so that they satisfy

$$\frac{1}{r_{b^{(n)}_{Y,L}}} \int_1^{z_L} C_L(z, \lambda_{b_Y^{(n)}}, c_2)^2 dz = \frac{1}{r_{b^{(n)}_{Y,R}}} \int_1^{z_L} S_R(z, \lambda_{b_Y^{(n)}}, c_2)^2 dz = 1, \tag{C6}$$

and one finds that $r_{b^{(n)}_{Y,L}} = r_{b^{(n)}_{Y,R}}$.

c. $Q_{\rm em} = +2/3$ quark and its partners (t, t_B, t_U)

$(T^{3_L}, T^{3_R}) = (+\frac{1}{2}, -\frac{1}{2})$, $(-\frac{1}{2}, +\frac{1}{2})$ and (0,0) of Ψ_1 states B, t, t' together with $(+\frac{1}{2}, +\frac{1}{2})$ of Ψ_2 state U have $Q_{\rm em} = +2/3$ states. For the third generation $\Psi^{q,g=3}_{a=1,2}$ contain $\hat{t}$, $\hat{t}_B$, and $\hat{t}_U$. We have an expansion as follows.

$$\begin{pmatrix} U \\ t \\ B \\ t' \end{pmatrix}(x,z) = \sqrt{k} z^2 \sum_{n=0}^{\infty} \left\{ t_L^{(n)}(x) \frac{1}{\sqrt{r_{t_L^{(n)}}}} \begin{pmatrix} a_U^{(t)} C_L^{(t^{(n)})}(z) \\ a_t^{(t)} C_L^{(t^{(n)})}(z) \\ a_B^{(t)} C_L^{(t^{(n)})}(z) \\ a_{t'}^{(t)} \hat{S}_L^{(t^{(n)})}(z) \end{pmatrix} + t_R^{(n)}(x) \frac{1}{\sqrt{r_{t_R^{(n)}}}} \begin{pmatrix} a_U^{(t)} S_R^{(t^{(n)})}(z) \\ a_t^{(t)} S_R^{(t^{(n)})}(z) \\ a_B^{(t)} S_R^{(t^{(n)})}(z) \\ a_{t'}^{(t)} \hat{C}_R^{(t)}(z) \end{pmatrix} \right\}$$
$$+ \sqrt{k} z^2 \sum_{n=1}^{\infty} \left\{ t_{B,L}^{(n)}(x) \frac{1}{\sqrt{r_{t^{(n)}_{B,L}}}} \begin{pmatrix} a_U^{(t_B)} C_L^{(t_B^{(n)})}(z) \\ a_t^{(t_B)} C_L^{(t_B^{(n)})}(z) \\ a_B^{(t_B)} C_L^{(t_B^{(n)})}(z) \\ a_{t'}^{(t_B)} \hat{S}_L^{(t_B^{(n)})}(z) \end{pmatrix} + t_{B,R}^{(n)}(x) \frac{1}{\sqrt{r_{t^{(n)}_{B,R}}}} \begin{pmatrix} a_U^{(t_B)} S_R^{(t_B^{(n)})}(z) \\ a_t^{(t_B)} S_R^{(t_B^{(n)})}(z) \\ a_B^{(t_B)} S_R^{(t_B^{(n)})}(z) \\ a_{t'}^{(t_B)} \hat{C}_R^{(t_B^{(n)})}(z) \end{pmatrix} \right\}$$
$$+ \sqrt{k} z^2 \sum_{n=1}^{\infty} \left\{ t_{U,L}^{(n)}(x) \frac{1}{\sqrt{r_{t^{(n)}_{U,L}}}} \begin{pmatrix} a_U^{(t_U)} C_L^{(t_U^{(n)})}(z) \\ a_t^{(t_U)} C_L^{(t_U^{(n)})}(z) \\ a_B^{(t_U)} C_L^{(t_U^{(n)})}(z) \\ a_{t'}^{(t_U)} \hat{S}_L^{(t_U^{(n)})}(z) \end{pmatrix} + t_{U,R}^{(n)}(x) \frac{1}{\sqrt{r_{t^{(n)}_{U,R}}}} \begin{pmatrix} a_U^{(t_U)} S_R^{(t_U^{(n)})}(z) \\ a_t^{(t_U)} S_R^{(t_U^{(n)})}(z) \\ a_B^{(t_U)} S_R^{(t_U^{(n)})}(z) \\ a_{t'}^{(t_U)} \hat{C}_R^{(t_U^{(n)})}(z) \end{pmatrix} \right\}, \tag{C7}$$

where $C_L^{(t_B^{(n)})}(z) \equiv C_L(z, \lambda_{t_B^{(n)}}, c)$, $\lambda_{t_B^{(n)}} = m_{t_B^{(n)}}/k$ etc. We have defined

$$\{\hat{S}_L(z,\lambda,c), \hat{C}_R(z,\lambda,c)\} \equiv \frac{C_L(1,\lambda,c)}{S_L(1,\lambda,c)} \{S_L(z,\lambda,c), C_R(z,\lambda,c)\}. \tag{C8}$$

KK masses $m_{t^{(n)}}$, $m_{t_B^{(n)}}$, and $m_{t_U^{(n)}}$ are determined by

$$s_H^2 \frac{(\mu_2^q)^2}{(\mu_2^q)^2 + (\tilde{\mu}^q)^2} + 2S_R S_L(z=1;\lambda_{t^{(n)}}, c) = 0, \qquad c_1 = c_2 \equiv c, \tag{C9}$$

and

$$C_L(1, \lambda_{t_B^{(n)}}, c_1) = 0, \qquad C_L(1, \lambda_{t_U^{(n)}}, c) = 0, \tag{C10}$$

respectively. Common coefficients are given by

$$\begin{pmatrix} a_U^{(t)} \\ a_t^{(t)} \\ a_B^{(t)} \\ a_{t'}^{(t)} \end{pmatrix} = \begin{pmatrix} -\sqrt{2}\tilde{\mu}/\mu_2 \\ (1+c_H)/\sqrt{2} \\ (1-c_H)/\sqrt{2} \\ -s_H \end{pmatrix}, \qquad \begin{pmatrix} a_U^{(t_B)} \\ a_t^{(t_B)} \\ a_B^{(t_B)} \\ a_{t'}^{(t_B)} \end{pmatrix} = \begin{pmatrix} 0 \\ (c_H-1)/\sqrt{2} \\ (c_H+1)/\sqrt{2} \\ 0 \end{pmatrix}, \qquad \begin{pmatrix} a_U^{(t_U)} \\ a_t^{(t_U)} \\ a_B^{(t_U)} \\ a_{t'}^{(t_U)} \end{pmatrix} = \begin{pmatrix} 1+c_H^2 \\ (\tilde{\mu}/\mu_2)(1+c_H) \\ (\tilde{\mu}/\mu_2)(1-c_H) \\ 0 \end{pmatrix}. \tag{C11}$$

Normalization factors $r_{f_L^{(n)}}$, $r_{f_R^{(n)}}$ $(f = t, t_B, t_U)$ are determined by

$$\begin{aligned} &\frac{1}{r_{f_L^{(n)}}} \int_1^{z_L} \{[(a_U^{(f)})^2 + (a_t^{(f)})^2 + (a_B^{(f)})^2](C_L^{(f^{(n)})})^2 + (a_{t'}^{(f)})^2 (\hat{S}_L^{(f^{(n)})})^2]\} dz \\ &= \frac{1}{r_{f_R^{(n)}}} \int_1^{z_L} \{[(a_U^{(f)})^2 + (a_t^{(f)})^2 + (a_B^{(f)})^2](S_R^{(f^{(n)})})^2 + (a_{t'}^{(f)})^2 (\hat{C}_R^{(f^{(n)})})^2]\} dz = 1, \end{aligned} \tag{C12}$$

and one finds that $r_{f_L^{(n)}} = r_{f_R^{(n)}}$ are satisfied.

d. $Q_{\rm em} = -1/3$ quark and its partners (b, b_D, b_X)

$(T^{3_L}, T^{3_R}) = (-\frac{1}{2}, -\frac{1}{2})$ of Ψ_1 states b together with $(+\frac{1}{2}, -\frac{1}{2})$, $(-\frac{1}{2}, +\frac{1}{2})$, and (0,0) of Ψ_2 states X, D, b' have $Q_{\rm em} = -1/3$ states. For the third generation the corresponding towers are $\hat{b}$, $\hat{b}_D$, and $\hat{b}_X$. Hence we have an expansion

$$\begin{aligned} \begin{pmatrix} b \\ X \\ D \\ b' \end{pmatrix}(x,z) = \sqrt{k}z^2 \sum_{n=0}^{\infty} &\left\{ b_L^{(n)}(x) \frac{1}{\sqrt{r_{b_{X,L}^{(n)}}}} \begin{pmatrix} a_b^{(b)} C_L^{(b^{(n)})}(z) \\ a_X^{(b)} C_L^{(b^{(n)})}(z) \\ a_D^{(b)} C_L^{(b^{(n)})}(z) \\ a_{b'}^{(b)} \hat{S}_L^{(b^{(n)})}(z) \end{pmatrix} + b_R^{(n)}(x) \frac{1}{\sqrt{r_{b_{X,R}^{(n)}}}} \begin{pmatrix} a_b^{(b)} S_R^{(b^{(n)})}(z) \\ a_X^{(b)} S_R^{(b^{(n)})}(z) \\ a_D^{(b)} S_R^{(b^{(n)})}(z) \\ a_{b'}^{(b)} \hat{C}_R^{(b^{(n)})}(z) \end{pmatrix} \right\} \\ + \sqrt{k}z^2 \sum_{n=1}^{\infty} &\left\{ b_{X,L}^{(n)}(x) \frac{1}{\sqrt{r_{b_{X,L}^{(n)}}}} \begin{pmatrix} a_b^{(b_X)} C_L^{(b_X^{(n)})}(z) \\ a_X^{(b_X)} C_L^{(b_X^{(n)})}(z) \\ a_D^{(b_X)} C_L^{(b_X^{(n)})}(z) \\ a_{b'}^{(b_X)} \hat{S}_L^{(b_X^{(n)})}(z) \end{pmatrix} + b_{X,R}^{(n)}(x) \frac{1}{\sqrt{r_{b_{X,R}^{(n)}}}} \begin{pmatrix} a_b^{(b_X)} S_R^{(b_X^{(n)})}(z) \\ a_X^{(b_X)} S_R^{(b_X^{(n)})}(z) \\ a_D^{(b_X)} S_R^{(b_X^{(n)})}(z) \\ a_{b'}^{(b_X)} \hat{C}_R^{(b_X^{(n)})}(z) \end{pmatrix} \right\} \\ + \sqrt{k}z^2 \sum_{n=1}^{\infty} &\left\{ b_{D,L}^{(n)}(x) \frac{1}{\sqrt{r_{b_{D,L}^{(n)}}}} \begin{pmatrix} a_b^{(b_D)} C_L^{(b_D^{(n)})}(z) \\ a_X^{(b_D)} C_L^{(b_D^{(n)})}(z) \\ a_D^{(b_D)} C_L^{(b_D^{(n)})}(z) \\ a_{b'}^{(b_D)} \hat{S}_L^{(b_D^{(n)})}(z) \end{pmatrix} + b_{D,R}^{(n)}(x) \frac{1}{\sqrt{r_{b_{D,R}^{(n)}}}} \begin{pmatrix} a_b^{(b_D)} S_R^{(b_D^{(n)})}(z) \\ a_X^{(b_D)} S_R^{(b_D^{(n)})}(z) \\ a_D^{(b_D)} S_R^{(b_D^{(n)})}(z) \\ a_{b'}^{(b_D)} \hat{C}_R^{(b_D^{(n)})}(z) \end{pmatrix} \right\}, \end{aligned} \tag{C13}$$

where $\lambda_{b_X^{(n)}} \equiv m_{b_X^{(n)}}/k$ etc. Mass spectra $m_{b^{(n)}}$, $m_{b_X^{(n)}}$, $m_{b_D^{(n)}}$ are determined by

$$s_H^2 \frac{(\tilde{\mu}^q)^2}{(\mu_2^q)^2 + (\tilde{\mu}^q)^2} + 2S_R S_L(1;\lambda_{b^{(n)}}, c) = 0, \quad c_1 = c_2 \equiv c \tag{C14}$$

and

$$C_L(1;\lambda_{b_X^{(n)}}, c_1) = 0, \qquad C_L(1;\lambda_{b_D^{(n)}}, c) = 0. \tag{C15}$$

Combining (C9) and (C14), one finds

$$\left(\frac{\tilde{\mu}^q}{\mu_2^q}\right)^2 = -\left\{1 + \frac{s_H^2}{2S_L S_R(1;\lambda_{t^{(n)}}, c)}\right\} = -\left\{1 + \frac{s_H^2}{2S_L S_R(1;\lambda_{b^{(n)}}, c)}\right\}^{-1}, \tag{C16}$$

and c and $\tilde{\mu}^q/\mu_2^q$ are determined from the masses of top and bottom quarks. Common coefficients are given by

$$\begin{pmatrix} a_b^{(b)} \\ a_X^{(b)} \\ a_D^{(b)} \\ a_{b'}^{(b)} \end{pmatrix} = -\begin{pmatrix} -\sqrt{2}\mu_2/\tilde{\mu} \\ (1-c_H)/\sqrt{2} \\ (1+c_H)/\sqrt{2} \\ s_H \end{pmatrix}, \quad \begin{pmatrix} a_b^{(b_D)} \\ a_X^{(b_D)} \\ a_D^{(b_D)} \\ a_{b'}^{(b_D)} \end{pmatrix} = \begin{pmatrix} (\tilde{\mu}/\mu_2)(1+c_H^2) \\ 1-c_H \\ 1+c_H \\ 0 \end{pmatrix}, \quad \begin{pmatrix} a_b^{(b_X)} \\ a_X^{(b_X)} \\ a_D^{(b_X)} \\ a_{b'}^{(b_X)} \end{pmatrix} = \begin{pmatrix} 0 \\ (1+c_H)/\sqrt{2} \\ (1-c_H)/\sqrt{2} \\ 0 \end{pmatrix}, \tag{C17}$$

Factors $r_{f_{L/R}^{(n)}}$ $(f = b, b_X, b_D)$ are normalized so that

$$\frac{1}{r_{f_L^{(n)}}}\int_1^{z_L} \{[(a_b^{(f)})^2 + (a_X^{(f)})^2 + (a_D^{(f)})^2](C_L^{(f)})^2 + (a_{b'}^{(f)})^2(\hat{S}_L^{(f)})^2]\}dz = \frac{1}{r_{f_R^{(n)}}}\int_1^{z_L} \{[(a_b^{(f)})^2 + (a_X^{(f)})^2 + (a_D^{(f)})^2](S_R^{(f)})^2 + (a_{b'}^{(f)})^2(\hat{C}_R^{(f)})^2]\}dz = 1, \tag{C18}$$

and one finds $r_{f_L^{(n)}} = r_{f_R^{(n)}}$ are satisfied.

TABLE XII. Masses of KK fermions for $N_F = 4$, $z_L = 10^5$ ($\theta_H = 0.115$) in the unit of TeV. $M_{q_x^{(n)}}$ is the mass of the nth KK excited state of exotic partners of q-quark (see text).

n	1	2	3	4
$M_{u^{(n)}}$	9.19	12.23	16.71	20.00
$M_{d^{(n)}}$	9.19	12.23	16.71	20.00
$M_{t^{(n)}}$	6.62	8.17	13.99	15.62
$M_{b^{(n)}}$	6.64	8.15	14.01	15.60
$M_{u_x^{(n)}} = M_{d_x^{(n)}}$	9.19	16.71	24.15	31.58
$M_{t_x^{(n)}} = M_{b_x^{(n)}}$	4.64	11.99	19.38	26.78

In Table XII and XIII, masses of KK fermions are tabulated. In tables masses of exotic partners of up- and down-type quarks are

$$M_{u_U^{(n)}} = M_{u_B^{(n)}} = M_{u_T^{(n)}} \equiv M_{u_x^{(n)}}, \qquad M_{d_D^{(n)}} = M_{d_X^{(n)}} = M_{d_Y^{(n)}} \equiv M_{d_x^{(n)}}, \tag{C19}$$

and $M_{u_x^{(n)}} = M_{d_x^{(n)}}$ are satisfied.

Here we note that KK masses of exotics largely depend on their bulk mass parameters. In particular, since the bulk mass parameter of top and bottom quarks approaching to zero for smaller z_L (θ_H), the mass spectrum for exotic partners of top and bottom quarks are approximately given by $C_L(1;\lambda_{f^{(1)}}, c_t = 0) = \cos((z_L - 1)m_{f^{(1)}}/k) = 0$ so that

$$m_{f^{(1)}} \simeq \frac{m_{KK}}{2}, \qquad f = t_{T,U,B}, b_{Y,X,D}. \tag{C20}$$

2. Lepton sector

For charged lepton, neutrino, and their exotic partners, KK states are given as follows.

a. $Q_{\rm em} = +1$ and -2 lepton partners

$(T^{3_L}, T^{3_R}) = (-\frac{1}{2}, -\frac{1}{2})$ of Ψ_3, L_{1Y}, and $(\frac{1}{2}, \frac{1}{2})$ of Ψ_4, L_{2X}, have $Q_{\rm em} = -2$ and $+1$, respectively. For the third generation, they are expanded as

TABLE XIII. Same as Table XII but for $N_F = 4$, $z_L = 10^4$ ($\theta_H = 0.0737$).

n	1	2	3	4
$M_{u^{(n)}}$	14.02	18.24	24.61	29.16
$M_{d^{(n)}}$	14.02	18.24	24.61	29.16
$M_{t^{(n)}}$	10.11	10.59	20.45	20.95
$M_{b^{(n)}}$	10.18	10.52	20.52	20.88
$M_{u_x^{(n)}} = M_{d_x^{(n)}}$	14.02	24.61	35.06	45.46
$M_{t_x^{(n)}} = M_{b_x^{(n)}}$	5.40	15.73	26.07	36.42

$$L_{2X}(x,z) = \sqrt{k}z^2 \sum_{n=1}^{\infty} \left\{ \nu^{(n)}_{\tau 2X,L}(x) \frac{1}{\sqrt{r_{\nu^{(n)}_{\tau 2X,L}}}} C_L(z;\lambda_{\nu^{(n)}_{\tau 2X}}, c_4) + \nu^{(n)}_{\tau 2X,R}(x) \frac{1}{\sqrt{r_{\nu^{(n)}_{\tau 2X,R}}}} S_R(z;\lambda_{\nu^{(n)}_{\tau 2X}}, c_4) \right\}, \quad (C21)$$

$$L_{1Y}(x,z) = \sqrt{k}z^2 \sum_{n=1}^{\infty} \left\{ \tau^{(n)}_{1Y,L}(x) \frac{1}{\sqrt{r_{\tau^{(n)}_{1Y,L}}}} C_L(z;\lambda_{\tau^{(n)}_{1Y}}, c_3) + \tau^{(n)}_{1Y,R}(x) \frac{1}{\sqrt{r_{\tau^{(n)}_{1Y,R}}}} S_R(z;\lambda_{\tau^{(n)}_{1Y}}, c_3) \right\}, \quad (C22)$$

where the KK masses are given by $(m_{\nu^{(n)}_{\tau 2X}}, m_{\tau^{(n)}_{1Y}}) = k(\lambda^{(n)}_{\nu_{\tau 2X}}, \lambda_{\tau^{(n)}_{1Y}})$ and determined by

$$C_L(1;\lambda_{\nu^{(n)}_{\tau 2X}}, c_4) = C_L(1;\lambda_{\tau^{(n)}_{1Y}}, c_3) = 0. \quad (C23)$$

Normalization factors are determined by

$$\frac{1}{r_{f^{(n)}_L}} \int_1^{z_L} C_L(z;\lambda_{f^{(n)}}, c)^2 dz = \frac{1}{r_{f^{(n)}_R}} \int_1^{z_L} S_R(z;\lambda_{f^{(n)}}, c)^2 dz = 1, \quad (C24)$$

for $(f,c) = (\tau_{1Y}, c_3)$, $(\nu_{\tau 2X}, c_4)$.

b. $Q_{\mathrm{em}} = -1$ charged lepton and its partners

$(T^{3_L}, T^{3_R}) = (-\frac{1}{2}, \frac{1}{2})$, $(\frac{1}{2}, -\frac{1}{2})$, and (0,0) of Ψ_3 together with $(-\frac{1}{2}, -\frac{1}{2})$ of Ψ_4 are $Q_{\mathrm{em}} = -1$ states. They are expanded as

$$\begin{pmatrix} L_{3Y} \\ L_{1X} \\ \tau \\ \tau' \end{pmatrix}(x,z) = \sqrt{k}z^2 \sum_f \sum_n \left\{ f^{(n)}_L \frac{1}{\sqrt{r_{f^{(n)}_L}}} \begin{pmatrix} a^{(f)}_{3Y} C^{f^{(n)}}_L(z) \\ a^{(f)}_{1X} C^{f^{(n)}}_L(z) \\ a^{(f)}_{\tau} C^{f^{(n)}}_L(z) \\ a^{(f)}_{\tau'} \hat{S}^{f(n)}_L(z) \end{pmatrix} + f^{(n)}_R \frac{1}{\sqrt{r_{f^{(n)}_R}}} \begin{pmatrix} a^{(f)}_{3Y} S^{f^{(n)}}_R(z) \\ a^{(f)}_{1X} S^{f^{(n)}}_R(z) \\ a^{(f)}_{\tau} S^{f^{(n)}}_R(z) \\ a^{(f)}_{\tau'} \hat{C}^{f(n)}_R(z) \end{pmatrix} \right\}, \quad (C25)$$

where $f = \tau$, τ_{1X}, and τ_{3Y} and $C^{f^{(n)}}_L(z) \equiv C_L(z;\lambda_{f^{(n)}}, c)$ etc. Common coefficients are given by

$$\begin{pmatrix} a^{(f)}_{3Y} \\ a^{(f)}_{1X} \\ a^{(f)}_{\tau} \\ a^{(f)}_{\tau'} \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}\tilde{\mu}}{\mu_3} \\ \frac{1-c_H}{\sqrt{2}} \\ \frac{1+c_H}{\sqrt{2}} \\ s_H \end{pmatrix}, \quad \begin{pmatrix} 0 \\ \frac{c_H+1}{\sqrt{2}} \\ \frac{c_H-1}{\sqrt{2}} \\ 0 \end{pmatrix} \text{ and } \begin{pmatrix} 1+c_H^2 \\ (1-c_H)\frac{\tilde{\mu}^\ell}{\mu^\ell_3} \\ (1+c_H)\frac{\tilde{\mu}^\ell}{\mu^\ell_3} \\ 0 \end{pmatrix}, \quad (C26)$$

for $f = \tau$, τ_{1X}, and τ_{3Y}, respectively. The mass of $\tau^{(n)}$ is given by $m_{\tau^{(n)}} = k\lambda_{\tau^{(n)}}$ where $\lambda_{\tau^{(n)}}$ are determined by

$$s_H^2 \frac{(\mu^\ell_3)^2}{(\mu^\ell_3)^2 + (\tilde{\mu}^\ell)^2} + 2S_L S_R(z=1;\lambda_{\tau^{(n)}}, c_\ell) = 0, \quad (C27)$$

and $\tau^{(0)}$ corresponds to the tau lepton. For $\tau^{(n)}_{\mathcal{E}}$ ($\mathcal{E} = 3Y$ and $1X$, $n = 1, 2, \cdots$), the KK masses are determined by

$$C_L(1;\lambda_{\tau^{(n)}_{\mathcal{E}}}, c_\ell) = 0, \qquad m_{\tau^{(n)}_{\mathcal{E}}} \equiv k\lambda_{\tau^{(n)}_{\mathcal{E}}}. \quad (C28)$$

Normalization factors are determined by

$$\frac{1}{r_{f^{(n)}_L}} \int_1^{z_L} \{[(a^{(f)}_{3Y})^2 + (a^{(f)}_{1X})^2 + (a^{(f)}_{\tau})^2](C^{(f)}_L)^2 + (a^{(f)}_{\tau'})^2(\hat{S}^{(f)}_L)^2]\} dz = \frac{1}{r_{f^{(n)}_R}} \int_1^{z_L} \{[(a^{(f)}_{3Y})^2 + (a^{(f)}_{1X})^2 + (a^{(f)}_{\tau})^2](S^{(f)}_R)^2 + (a^{(f)}_{\tau'})^2(\hat{C}^{(f)}_R)^2]\} dz = 1, \quad (C29)$$

where $f = \tau, \tau_{3Y}$ and $\tau_{\tau_{1X}}$. One finds $r_{f^{(n)}_L} = r_{f^{(n)}_R}$ are satisfied.

c. $Q_{\mathrm{em}} = 0$ neutrino and its partners

$(T^{3_L}, T^{3_R}) = (\frac{1}{2}, \frac{1}{2})$ of Ψ_3, and $(\frac{1}{2}, -\frac{1}{2})$ and $(-\frac{1}{2}, \frac{1}{2})$ and (0,0) of Ψ_4 are $Q_{\mathrm{em}} = 0$ states. They are expanded as

$$\begin{pmatrix} \nu \\ L_{3X} \\ L_{2Y} \\ \nu' \end{pmatrix}(x,z) = \sqrt{k}z^2 \sum_f \sum_n \left\{ f^{(n)}_L \frac{1}{\sqrt{r_{f^{(n)}_L}}} \begin{pmatrix} a^{(f)}_{\nu} C^{f^{(n)}}_L(z) \\ a^{(f)}_{3X} C^{f^{(n)}}_L(z) \\ a^{(f)}_{2Y} C^{f^{(n)}}_L(z) \\ a^{(f)}_{\nu'} \hat{S}^{f(n)}_L(z) \end{pmatrix} + f^{(n)}_R \frac{1}{\sqrt{r_{f^{(n)}_R}}} \begin{pmatrix} a^{(f)}_{\nu} S^{f^{(n)}}_R(z) \\ a^{(f)}_{3X} S^{f^{(n)}}_R(z) \\ a^{(f)}_{2Y} S^{f^{(n)}}_R(z) \\ a^{(f)}_{\nu'} \hat{C}^{f(n)}_R(z) \end{pmatrix} \right\}, \quad (C30)$$

where $f = \nu,\ \nu_{\tau 3X}$, and $\nu_{\tau 2Y}$. $C_L^{f^{(n)}} = C_L(z;\lambda_{f^{(n)}},c)$, $c_3 = c_4 \equiv c^\ell$ etc. Common coefficients are given by

$$\begin{pmatrix} a_\nu^{(f)} \\ a_{3X}^{(f)} \\ a_{2Y}^{(f)} \\ a_{\nu'}^{(f)} \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}\mu_3^\ell}{\tilde{\mu}^\ell} \\ -\frac{1+c_H}{\sqrt{2}} \\ -\frac{1-c_H}{\sqrt{2}} \\ s_H \end{pmatrix}, \quad \begin{pmatrix} 0 \\ \frac{1-c_H}{\sqrt{2}} \\ \frac{1+c_H}{\sqrt{2}} \\ 0 \end{pmatrix}, \text{ and } \begin{pmatrix} (1+c_H^2)\frac{\tilde{\mu}^\ell}{\mu_3^\ell} \\ 1+c_H \\ 1-c_H \\ 0 \end{pmatrix} \tag{C31}$$

for $f = \tau,\ \nu_{\tau 2Y}$, and $\nu_{\tau 3X}$, respectively. KK masses of $\nu_\tau^{(n)}$, $m_{\nu_\tau^{(n)}} \equiv k\lambda_{\nu_\tau^{(n)}}$, $(n = 0, 1, 2, \cdots)$ are determined by

$$s_H^2 \frac{(\tilde{\mu}^\ell)^2}{(\mu_3^\ell)^2 + (\tilde{\mu}^\ell)^2} + 2S_L S_R(z = 1; \lambda_{\nu^{(n)}}, c_\ell) = 0, \tag{C32}$$

and $\nu_\tau^{(0)}$ corresponds to the tau neutrino. From (C27) and (C32), one finds

$$\begin{aligned} \left(\frac{\mu_3^\ell}{\tilde{\mu}^\ell}\right)^2 &= -\left\{1 + \frac{s_H^2}{2S_L S_R(1;\lambda_{\nu_\tau^{(n)}}, c)}\right\} \\ &= -\left\{\frac{s_H^2}{2S_L S_R(1;\lambda_{\tau^{(n)}}, c)}\right\}^{-1}, \end{aligned} \tag{C33}$$

and c and $\tilde{\mu}^\ell/\mu_3^\ell$ are determined from the masses of τ and ν_τ. For $\nu_{\tau\mathcal{N}}^{(n)}$ ($\mathcal{N} = 3X$ and $1Y$, $n = 1, 2, \cdots$), the KK masses are determined by

$$C_L(1;\lambda_{\nu_{\tau\mathcal{N}}}^{(n)}, c_\ell) = 0, \qquad m_{\nu_{\tau\mathcal{N}}^{(n)}} \equiv k\lambda_{\nu_{\tau\mathcal{N}}^{(n)}}. \tag{C34}$$

Normalization factors are determined by

$$\begin{aligned} &\frac{1}{r_{f_L^{(n)}}}\int_1^{z_L}\{[(a_\nu^{(f)})^2 + (a_{3X}^{(f)})^2 + (a_{2Y}^{(f)})^2](C_L^{(f)})^2 \\ &\quad + (a_{\nu'}^{(f)})^2(\hat{S}_L^{(f)})^2]\}dz \\ &= \frac{1}{r_{f_R^{(n)}}}\int_1^{z_L}\{[(a_\nu^{(f)})^2 + (a_{3X}^{(f)})^2 + (a_{2Y}^{(f)})^2](S_R^{(f)})^2 \\ &\quad + (a_{\nu'}^{(f)})^2(\hat{C}_R^{(f)})^2]\}dz = 1, \end{aligned} \tag{C35}$$

for $f = \nu,\ \nu_{3X}$, and ν_{2Y}. One finds $r_{f_L^{(n)}} = r_{f_R^{(n)}}$ are satisfied.

APPENDIX D: FERMION COUPLINGS

The KK expansions (C1), (C4), (C7) (C13), (C21), (C22), (C25), and (C30) are written in the form of

$$\begin{aligned} T(x,z) &= \sqrt{k}z^2\sum_{n=1}^{\infty}[t_{T,L}^{(n)}(x)f_{TL}^{t_T^{(n)}}(z) + t_{T,R}^{(n)}(x)f_{TR}^{(t_T^{(n)})}(z)], \\ Y(x,z) &= \sqrt{k}z^2\sum_{n=1}^{\infty}[b_{Y,L}^{(n)}(x)f_{YL}^{b_Y^{(n)}}(z) + b_{Y,R}^{(n)}(x)f_{YR}^{(b_Y^{(n)})}(z)], \end{aligned} \tag{D1}$$

$$\begin{pmatrix} U \\ t \\ B \\ t' \end{pmatrix}(x,z) = \sqrt{k}z^2 \sum_{t_u = t, t_B, t_U}\sum_n \left\{ t_{u,L}(x)\begin{pmatrix} f_{UL}^{t_u^{(n)}}(z) \\ f_{tL}^{t_u^{(n)}}(z) \\ f_{BL}^{t_u^{(n)}}(z) \\ f_{t'L}^{t_u^{(n)}}(z) \end{pmatrix} + t_{u,R}(x)\begin{pmatrix} f_{UR}^{t_u^{(n)}}(z) \\ f_{tR}^{t_u^{(n)}}(z) \\ f_{BR}^{t_u^{(n)}}(z) \\ f_{t'R}^{t_u^{(n)}}(z) \end{pmatrix}\right\}, \tag{D2}$$

$$\begin{pmatrix} b \\ X \\ D \\ b' \end{pmatrix}(x,z) = \sqrt{k}z^2 \sum_{b_d = b, b_X, b_D}\sum_n \left\{ b_{d,L}(x)\begin{pmatrix} f_{bL}^{b_d^{(n)}}(z) \\ f_{XL}^{b_d^{(n)}}(z) \\ f_{DL}^{b_d^{(n)}}(z) \\ f_{b'L}^{t_u^{(n)}}(z) \end{pmatrix} + b_{d,R}(x)\begin{pmatrix} f_{bR}^{b_d^{(n)}}(z) \\ f_{XR}^{b_d^{(n)}}(z) \\ f_{DR}^{b_d^{(n)}}(z) \\ f_{b'R}^{b_d^{(n)}}(z) \end{pmatrix}\right\}. \tag{D3}$$

In terms of these wave functions we write gauge-boson couplings and Yukawa couplings as follows.

1. Vector boson couplings

a. $\bar{\psi}_n^{(-1/3)} V^- \psi_m^{(2/3)}$ and $\bar{\psi}_n^{(-1)} V^- \psi_m^{(0)}$ couplings

For $b_d = b, b_D, b_X$, $t_u = t, t_U, t_B$, and $V^- = W^-, W_R^-$ we have

$$\begin{aligned} &\int \frac{dz}{kz^4} \bar{\Psi}_1 [\gamma^\mu g_A A_\mu + \bar{\Psi}_2 \gamma^\mu g_A A_\mu \Psi_2] \\ &\supset \bar{b}_{dL}^{(n)} V_\mu^- t_{uL}^{(m)} \cdot g_w \sqrt{L} \int_1^{z_L} dz \frac{1}{\sqrt{2}} \{ h_V^L [f_{bL}^{b_b^{(n)}} f_{tL}^{t_u^{(m)}} + f_{DL}^{b_d^{(n)}} f_{UL}^{t_u^{(m)}}] + h_V^R [f_{bL}^{b_d^{(n)}} f_{BL}^{t_u^{(m)}} + f_{XL}^{b_d^{(n)}} f_{UL}^{t_u^{(m)}}] + \hat{h}_V [f_{bL}^{b_d^{(n)}} f_{t'L}^{t_u^{(m)}} - f_{b'L}^{b_d^{(n)}} f_{UL}^{t_u^{(m)}}] \} + \text{H.c.} \\ &\equiv \bar{b}_{dL}^{(n)} V_\mu^- t_{uL}^{(m)} \cdot g_{V d_d^{(n)} t_u^{(m)}}^L + \text{H.c.} \end{aligned} \tag{D4}$$

and right-handed couplings with replacements $L \to R$ in spinors and their wave functions.

For leptons couplings we obtain $\bar{\ell} V^- \nu$ couplings from the above formula with replacements

$$b_d \to \tau_{\mathcal{E}}, \qquad t_u \to \nu_{\tau\mathcal{N}}, \qquad (b, b_D, b_X, b') \to (\tau, \tau_{3Y}, \tau_{1X}, \tau'), \qquad (t, t_U, t_B, t') \to (\nu_\tau, \nu_{\tau 2Y}, \nu_{\tau 3X}, \nu'_\tau). \tag{D5}$$

b. $\bar{\psi}_n^{(2/3)} V_\mu \psi_m^{(2/3)}$ and $\bar{\psi}_n^{(-1/3)} V_\mu \psi_m^{(-1/3)}$

For up-type quarks and their exotic partners $t_u, t_{u'} = t, t_B, t_U$, down-type quarks and their exotic partners $b_d, b_{d'} = b, b_D, b_X$, and neutral vector boson $V = \gamma^{(l)}, Z^{(l)}, Z_R^{(l)}$, we have $\bar{t}_u V t_{u'}$ and $\bar{b}_d V b_{d'}$ couplings as

$$\begin{aligned} &g_A \bar{\Psi}_1 \gamma^\mu \left[A_\mu + \left(\frac{2}{3}\right) \frac{g_B}{g_A} B_\mu \right] \Psi_1 + g_A \bar{\Psi}_2 \gamma^\mu \left[A_\mu + \left(-\frac{1}{3}\right) \frac{g_B}{g_A} B_\mu \right] \Psi_2 \\ &\supset \bar{t}_{u'L}^{(n)} \gamma^\mu V_\mu t_{uL}^{(m)} \cdot g_w \sqrt{L} \int_1^{z_L} dz \bigg\{ \frac{1}{2} (h_V^L - h_V^R) [-f_{BL}^{t_{u'}^{(n)}} f_{BL}^{t_u^{(m)}} + f_{tL}^{t_{u'}^{(n)}} f_{tL}^{t_u^{(m)}}] + \frac{1}{2} (h_V^L + h_V^R) f_{UL}^{t_{u'}^{(n)}} f_{UL}^{t_u^{(m)}} \\ &\quad + \frac{1}{2} \hat{h}_V [f_{t'L}^{t_{u'}^{(n)}} (f_{BL}^{t_u^{(m)}} + f_{tL}^{t_u^{(m)}}) + (f_{BL}^{t_{u'}^{(n)}} + f_{tL}^{t_{u'}^{(n)}}) f_{t'L}^{t_u^{(m)}}] \\ &\quad + \frac{g_B}{g_A} h_V^B \left[\frac{2}{3} (f_{tL}^{t_{u'}^{(n)}} f_{tL}^{t_u^{(m)}} + f_{BL}^{t_{u'}^{(n)}} f_{BL}^{t_u^{(m)}} + f_{t'L}^{t_{u'}^{(n)}} f_{t'L}^{t_u^{(m)}}) - \frac{1}{3} (f_{UL}^{t_{u'}^{(n)}} f_{UL}^{t_u^{(m)}}) \right] \bigg\} + \text{H.c.} \\ &\quad + \bar{b}_{d'L}^{(n)} \gamma^\mu V_\mu b_{dL}^{(m)} \cdot g_w \sqrt{L} \int_1^{z_L} dz \bigg\{ \frac{1}{2} (h_V^L - h_V^R) [-f_{DL}^{b_{d'}^{(n)}} f_{DL}^{b_d^{(m)}} + f_{XL}^{d_{b'}^{(n)}} f_{XL}^{d_b^{(m)}}] - \frac{1}{2} (h_V^L + h_V^R) f_{bL}^{b_{d'}^{(n)}} f_{bL}^{b_d^{(m)}} \\ &\quad + \frac{1}{2} \hat{h}_V [f_{b'L}^{b_{d'}^{(n)}} (f_{XL}^{b_d^{(m)}} + f_{DL}^{b_d^{(m)}}) + (f_{XL}^{b_{d'}^{(n)}} + f_{DL}^{b_{d'}^{(n)}}) f_{b'L}^{b_d^{(m)}}] \\ &\quad + \frac{g_B}{g_A} h_V^B \left[-\frac{1}{3} (f_{XL}^{b_{d'}^{(n)}} f_{XL}^{b_d^{(m)}} + f_{DL}^{b_{d'}^{(n)}} f_{DL}^{b_d^{(m)}} + f_{b'L}^{b_{d'}^{(n)}} f_{b'L}^{b_d^{(m)}}) + \frac{2}{3} (f_{bL}^{b_{d'}^{(n)}} f_{bL}^{b_d^{(m)}}) \right] \bigg\} + \text{H.c.} \\ &\equiv \bar{t}_{u'L}^{(n)} \gamma^\mu V_\mu t_{uL}^{(m)} \cdot g_{V t_{u'}^{(n)} t_u^{(m)}}^L + \text{H.c.} + \bar{b}_{d'L}^{(n)} \gamma^\mu V_\mu b_{dL}^{(m)} \cdot g_{V b_{d'}^{(n)} b_d^{(m)}}^L + \text{H.c.} \end{aligned} \tag{D6}$$

and right-handed couplings. Lepton couplings $\bar{\psi}^{(-1)} V_\mu \psi^{(-1)}$ and $\bar{\psi}^{(0)} V_\mu \psi^{(0)}$ are obtained from the above formula with replacements (D5).

We note that photon wave functions (B6) which can be rewritten as

$$h_{\gamma^{(0)}}^L = h_{\gamma^{(0)}}^R = \frac{g_B}{g_A} h_{\gamma^{(0)}}^B = \frac{\sin\theta_W}{\sqrt{L}}, \qquad \hat{h}_{\gamma^{(0)}} = 0 \tag{D7}$$

yield proper electromagnetic couplings $Q_{\text{em}} e \bar{\psi} \gamma^\mu A_\mu^\gamma \psi$.

c. $\bar{\psi}_n^{(2/3)} V_\mu^- \psi_m^{(5/3)}$ and $\bar{\psi}_n^{(0)} V_\mu^- \psi_m^{(+1)}$

For t_T, $t_u = t, t_U, t_B$ and $V^- = W^-, W_R^-$ we have

$$\int \frac{dz}{kz^4} \bar{\Psi}_1 \gamma^\mu g_A A_\mu \Psi_1 \supset \bar{t}_{uL}^{(n)} \gamma^\mu V_\mu^- t_{TL}^{(m)} \cdot g_w \sqrt{L} \int_1^{z_L} dz \frac{1}{\sqrt{2}} \left\{ h_V^L [f_{BL}^{t_u^{(n)}} f_{TL}^{t_T^{(m)}}] + h_V^R [f_{tL}^{t_u^{(n)}} f_{TL}^{t_T^{(m)}}] - \hat{h}_V [f_{t'L}^{t_u^{(n)}} f_{TL}^{t_T^{(m)}}] \right\} + \text{H.c.}$$
$$\equiv \bar{t}_{uL}^{(n)} \gamma^\mu V_\mu^- t_{TL}^{(m)} \cdot g^L_{V t_u^{(n)} t_T^{(m)}} + \text{H.c.} \tag{D8}$$

and corresponding right-handed couplings.

Lepton couplings are obtained from the above formula with replacements (D5) and

$$t_T \to \nu_{\tau 2X}, \qquad f_T \to f_{\nu_{\tau 2X}}. \tag{D9}$$

d. $\bar{\psi}_n^{(-1/3)} V_\mu^+ \psi_m^{(-4/3)}$ and $\bar{\psi}_n^{(-1)} V_\mu^+ \psi_m^{(-2)}$ couplings

For b_Y, $b_d = b, b_X, b_D$ and $V^+ = W^+, W_R^+$, we have

$$\int \frac{dz}{kz^4} \bar{\Psi}_2 \gamma^\mu g_A A_\mu \Psi_2 \supset \bar{b}_{dL}^{(n)} V_\mu^+ b_{YL}^{(m)} \cdot g_w \sqrt{L} \int_1^{z_L} dz \frac{1}{\sqrt{2}} \{ h_V^L [f_{XL}^{b_d^{(n)}} f_{YL}^{b_Y^{(m)}}] + h_V^R [f_{DL}^{b_d^{(n)}} f_{YL}^{b_Y^{(m)}}] + \hat{h}_V [f_{b'L}^{b_d^{(n)}} f_{YL}^{b_Y^{(m)}}] \} + \text{H.c.}$$
$$\equiv \bar{b}_{dL}^{(n)} V_\mu^+ b_{YL}^{(m)} \cdot g^L_{V b_d^{(n)} b_Y^{(m)}} + \text{H.c.} \tag{D10}$$

and corresponding right-handed couplings.

Lepton couplings are obtained from the above formula with replacements (D5) and

$$b_Y \to \tau_{1Y}, \qquad f_Y \to f_{1Y}. \tag{D11}$$

e. Numerical values

In Tables XIV–XVIII, KK fermions' couplings to W and Z bosons for $N_F = 4$, $z_L = 10^5$ ($\theta_H = 0.115$) are tabulated.

2. Higgs Yukawa couplings

Yukawa couplings among Higgs $H^{(k)}$ and quark-sector fermions are read from

$$\sum_{i=1,2} \int_1^{z_L} \sqrt{-g} e_5^z i \Psi_i \Gamma^m (-i g_A A_z) \Psi_i$$
$$\supset H^{(k)} (\bar{t}_{uL}^{(m)} t_{u'R}^{(n)} + \bar{t}_{u'R}^{(n)} t_{uL}^{(m)})(x)$$
$$\times \frac{1}{2} g_w \sqrt{L} \int_1^{z_L} u_{H^{(k)}}(z) [f_{t'L}^{t_u^{(m)}} (f_{BR}^{t_{u'}^{(n)}} - f_{tR}^{t_{u'}^{(n)}}) - (f_{BL}^{t_u^{(m)}} - f_{tL}^{t_u^{(m)}}) f_{t'R}^{t_{u'}^{(n)}}] dz - H^{(k)} (\bar{t}_{uR}^{(m)} t_{u'L}^{(n)} + \bar{t}_{u'L}^{(n)} t_{uR}^{(m)})(x)$$
$$\times \frac{1}{2} g_w \sqrt{L} \int_1^{z_L} u_{H^{(k)}}(z) [f_{t'R}^{t_u^{(m)}} (f_{BL}^{t_{u'}^{(n)}} - f_{tL}^{t_{u'}^{(n)}}) - (f_{BR}^{t_u^{(m)}} - f_{tR}^{t_u^{(m)}}) f_{t'L}^{t_{u'}^{(n)}}] dz, \tag{D12}$$

and when $t_u^{(m)} = t_{u'}^{(n)}$, one obtains

$$H^{(n)}(x) (\bar{t}_{uL}^{(m)} t_{uR}^{(m)} + \bar{t}_{uR}^{(m)} t_{uL}^{(m)}) \frac{1}{2} g_w \sqrt{L} \int_1^{z_L} u_{H^{(n)}} [f_{t'L}^{t_u^{(m)}} (f_{BR}^{t_u^{(m)}} - f_{tR}^{t_u^{(m)}}) - (f_{BL}^{t_u^{(m)}} - f_{tL}^{t_u^{(m)}}) f_{t'R}^{t_u^{(m)}}] dz. \tag{D13}$$

For the Higgs boson $H = H^{(0)}$, the wave function is given by

$$u_H(z) = u_{H^{(0)}}(z) = \sqrt{\frac{2}{k(z_L^2 - 1)}} z. \tag{D14}$$

TABLE XIV. Couplings of the W boson to the down-quark and KK excited up-type states in unit of $g_w/\sqrt{2}$ for $N_F = 4$, $z_L = 10^5$ ($\theta_H = 0.115$).

	$n = 1$	2	3	4
$g^L_{Wu^{(0)}d^{(n)}}/(g_w/\sqrt{2})$	-4.28×10^{-9}	-2.98×10^{-7}	-5.77×10^{-8}	9.24×10^{-6}
$g^R_{Wu^{(0)}d^{(n)}}/(g_w/\sqrt{2})$	-1.30×10^{-2}	-3.67×10^{-13}	-2.16×10^{-3}	-1.87×10^{-13}
$g^L_{Wu^{(0)}d_D^{(n)}}/(g_w/\sqrt{2})$	1.59×10^{-8}	-3.54×10^{-9}	2.52×10^{-9}	-1.58×10^{-9}
$g^R_{Wu^{(0)}d_D^{(n)}}/(g_w/\sqrt{2})$	2.95×10^{-2}	4.91×10^{-3}	1.62×10^{-3}	7.27×10^{-4}
$g^L_{Wu^{(0)}d_X^{(n)}}/(g_w/\sqrt{2})$	8.90×10^{-8}	-1.22×10^{-7}	1.50×10^{-7}	-1.73×10^{-7}
$g^R_{Wu^{(0)}d_X^{(n)}}/(g_w/\sqrt{2})$	1.23×10^{-14}	-9.31×10^{-15}	7.87×10^{-15}	-6.97×10^{-15}

TABLE XV. Couplings of the W boson to the up-quark and KK excited down-type states in unit of $g_w/\sqrt{2}$ for $N_F = 4$, $z_L = 10^5$ ($\theta_H = 0.115$).

	$n = 1$	2	3	4
$g^L_{Wd^{(0)}u^{(n)}}/(g_w/\sqrt{2})$	-3.36×10^{-8}	-1.21×10^{-7}	6.54×10^{-9}	3.46×10^{-8}
$g^R_{Wd^{(0)}u^{(n)}}/(g_w/\sqrt{2})$	-2.95×10^{-2}	-3.68×10^{-13}	-4.92×10^{-3}	-1.87×10^{-13}
$g^L_{Wd^{(0)}u_{B^{(n)}}}/(g_w\sqrt{2})$	1.44×10^{-24}	-2.48×10^{-24}	-1.99×10^{-24}	-4.23×10^{-24}
$g^R_{Wd^{(0)}u_B^{(n)}}/(g_w/\sqrt{2})$	5.21×10^{-31}	-4.15×10^{-31}	-4.21×10^{-32}	-2.99×10^{-31}
$g^L_{Wd^{(0)}u_U^{(n)}}/(g_w/\sqrt{2})$	1.59×10^{-8}	-3.54×10^{-9}	2.52×10^{-9}	-1.58×10^{-9}
$g^R_{Wd^{(0)}u_U^{(n)}}/(g_w/\sqrt{2})$	1.29×10^{-2}	2.15×10^{-3}	7.12×10^{-4}	3.19×10^{-4}

TABLE XVI. Couplings of the W boson to the up- or down-quark and KK excite states with non-SM electric charges in unit of $g_w/\sqrt{2}$ for $N_F = 4$, $z_L = 10^5$ ($\theta_H = 0.115$).

	$n = 1$	2	3	4
$g^L_{Wu^{(0)}u_T^{(n)}}/(g_w/\sqrt{2})$	-1.74×10^{-7}	3.86×10^{-9}	-2.75×10^{-9}	1.72×10^{-9}
$g^R_{Wu^{(0)}u_T^{(n)}}/(g_w/\sqrt{2})$	-3.22×10^{-2}	-5.37×10^{-3}	-1.78×10^{-3}	-7.94×10^{-4}
$g^L_{Wd^{(0)}d_Y^{(n)}}/(g_w/\sqrt{2})$	-3.96×10^{-8}	8.82×10^{-9}	-6.29×10^{-9}	3.93×10^{-9}
$g^R_{Wd^{(0)}d_Y^{(n)}}/(g_w/\sqrt{2})$	-3.22×10^{-2}	-5.37×10^{-3}	-1.78×10^{-3}	-7.94×10^{-4}

TABLE XVII. Couplings of the Z boson to the up quark and KK excited states in unit of $g_w/\cos\theta_W$ for $N_F = 4$, $z_L = 10^5$ ($\theta_H = 0.115$).

	$n = 0$	1	2	3	4
$g^L_{Zu^{(0)}u^{(n)}}/(g_w/\cos\theta_W)$	0.345912	-2.26×10^{-9}	-5.04×10^{-8}	5.61×10^{-11}	1.82×10^{-8}
$g^R_{Zu^{(0)}u^{(n)}}/(g_w/\cos\theta_W)$	−0.154263	-6.47×10^{-3}	-8.43×10^{-6}	-1.08×10^{-3}	-1.20×10^{-7}
$g^L_{Zu^{(0)}u_B^{(n)}}/(g_w/\cos\theta_W)$	–	-8.65×10^{-9}	1.93×10^{-9}	-1.37×10^{-9}	8.59×10^{-10}
$g^R_{Zu^{(0)}u_B^{(n)}}/(g_w/\cos\theta_W)$	–	-1.61×10^{-2}	-2.68×10^{-3}	-8.85×10^{-4}	-3.96×10^{-4}
$g^L_{Zu^{(0)}u_U^{(n)}}/(g_w/\cos\theta_W)$	–	-7.98×10^{-9}	1.78×10^{-9}	-1.27×10^{-9}	7.92×10^{-10}
$g^R_{Zu^{(0)}u_U^{(n)}}/(g_w/\cos\theta_W)$	–	-1.48×10^{-2}	-2.47×10^{-3}	-8.17×10^{-4}	-3.65×10^{-4}

TABLE XVIII. Couplings of the Z boson to the down quark and KK excited states in unit of $g_w/\cos\theta_W$ for $N_F = 4$, $z_L = 10^5$ ($\theta_H = 0.115$).

	$n = 0$	1	2	3	4
$g^L_{Zd^{(0)}d^{(n)}}/(g_w/\cos\theta_W)$	−0.423018	4.46×10^{-8}	-3.32×10^{-7}	-4.12×10^{-8}	5.27×10^{-7}
$g^R_{Zd^{(0)}d^{(n)}}/(g_w/\cos\theta_W)$	0.0771316	1.48×10^{-2}	4.22×10^{-6}	2.47×10^{-3}	6.00×10^{-8}
$g^L_{Zd^{(0)}d_X^{(n)}}/(g_w/\cos\theta_W)$	–	-1.77×10^{-8}	4.74×10^{-8}	-6.02×10^{-8}	7.13×10^{-8}
$g^R_{Zd^{(0)}d_X^{(n)}}/(g_w/\cos\theta_W)$	–	1.62×10^{-2}	2.69×10^{-3}	8.91×10^{-4}	3.99×10^{-4}
$g^L_{Zd^{(0)}d_D^{(n)}}/(g_w/\cos\theta_W)$	–	8.00×10^{-9}	-1.78×10^{-9}	1.27×10^{-9}	-7.94×10^{-10}
$g^R_{Zd^{(0)}d_D^{(n)}}/(g_w/\cos\theta_W)$	–	6.51×10^{-3}	1.08×10^{-3}	3.58×10^{-4}	1.60×10^{-4}

APPENDIX E: BOSON COUPLINGS

1. Vector boson trilinear couplings

The $V^{(l)}W^{(m)}W^{(n)}$ couplings for $V = \gamma, Z, Z_R$ are contained in

$$\begin{aligned}
&\int_1^{z_L}\frac{dz}{kz}\left(-\frac{1}{4}\right)\mathrm{Tr}[F_{\mu\nu}F_{\rho\sigma}]\eta^{\mu\rho}\eta^{\nu\sigma}\\
&\quad\supset ig_A\int_1^{z_L}\frac{dz}{kz}\mathrm{Tr}[(\partial_\mu\hat{V}_\nu-\partial_\nu\hat{V}_\mu)[\hat{W}^+_\rho,\hat{W}^-_\sigma]+(\partial_\mu\hat{W}^-_\nu-\partial_\nu\hat{W}^-_\mu)[\hat{V}_\rho,\hat{W}^+_\sigma]+(\partial_\mu\hat{W}^+_\nu-\partial_\nu\hat{W}^+_\mu)[\hat{V}_\rho,\hat{W}^-_\sigma]]\eta^{\mu\rho}\eta^{\nu\sigma}\\
&\quad\supset i\sum_{m,n}g_{V^{(l)}W^{(m)}W^{(n)}}\eta^{\mu\rho}\eta^{\nu\sigma}\{(\partial_\mu Z^{(l)}_\nu-\partial_\nu Z^{(l)}_\mu)W^{+(m)}_\rho W^{-(n)}_\sigma\\
&\qquad-(\partial_\mu W^{+(m)}_\nu-\partial_\nu W^{+(m)}_\mu)Z^{(l)}_\rho W^{-(n)}_\sigma+(\partial_\mu W^{-(n)}_\nu-\partial_\nu W^{-(n)}_\mu)Z^{(l)}_\rho W^{+(m)}_\sigma\}
\end{aligned}\tag{E1}$$

so that one finds that

$$g_{V^{(l)}W^{(m)}W^{(n)}} = g_w\sqrt{L}\int_1^{z_L}\frac{dz}{kz}\left\{h^L_{V^{(l)}}\left(h^L_{W^{(m)}}h^L_{W^{(n)}}+\frac{\hat{h}_{W^{(m)}}\hat{h}_{W^{(n)}}}{2}\right)+h^R_{V^{(l)}}\left(h^R_{W^{(m)}}h^R_{W^{(n)}}+\frac{\hat{h}_{W^{(m)}}\hat{h}_{W^{(n)}}}{2}\right)\right.$$
$$\left.+\hat{h}_{V^{(l)}}\left(\frac{h^L_{W^{(m)}}\hat{h}_{W^{(n)}}+h^R_{W^{(m)}}\hat{h}_{W^{(n)}}+\hat{h}_{W^{(m)}}h^L_{W^{(n)}}+\hat{h}_{W^{(m)}}h^R_{W^{(n)}}}{2}\right)\right\}. \tag{E2}$$

Here $C_{W^{(m)}} = C(z;\lambda_{W^{(m)}})$ etc.

$V^{(l)}W^{(m)}W_R^{(n)}$ and $V^{(l)}W_R^{(m)}W_R^{(n)}$ couplings are obtained from above expression with replacements $W^{(n)} \to W_R^{(n)}$.

2. *HZZ* and *HZZ$_R$* couplings

The Higgs coupling $HZ^{(m)}Z^{(n)}$ is contained in the $\mathrm{Tr}F_{\mu z}F^{\mu z}$ term

$$-ig_Ak^2\int_1^{z_L}\frac{dz}{kz}\mathrm{Tr}[(\partial_z\hat{Z}_\mu)[\hat{H},\hat{Z}_\nu]]\eta^{\mu\nu} \supset -\frac{1}{2}\sum_n g_{HZ^{(m)}Z^{(n)}}HZ^{(m)}_\mu Z^{(n)}_\nu\eta^{\mu\nu} - \sum_{m<n} g_{HZ^{(m)}Z^{(n)}}HZ^{(m)}_\mu Z^{(n)}_\nu\eta^{\mu\nu} \tag{E3}$$

so that

$$g_{HZ^{(m)}Z^{(n)}} = -g_Ak^2\int_1^{z_L}\frac{dz}{kz}\frac{1}{2}u_H(z)[-(\partial_z\hat{h}_{Z^{(m)}})(h^L_{Z^{(n)}}-h^R_{Z^{(n)}})+\partial_z(h^L_{Z^{(n)}}-h^R_{Z^{(n)}})\hat{h}_{Z^{(m)}}+(m\leftrightarrow n)], \tag{E4}$$

where the $u_H(z)$ is given in (D14).

Similarly, the Higgs coupling $HZ^{(m)}Z_R^{(n)}$ is contained in the $\mathrm{Tr}F_{\mu z}F^{\mu z}$ term

$$-ig_Ak^2\int_1^{z_L}\frac{dz}{kz}\{\mathrm{Tr}[(\partial_z\hat{Z}_\mu)[\hat{H},\hat{Z}_{R\nu}]]+\mathrm{Tr}[(\partial_z\hat{Z}_{R\mu})[\hat{H},\hat{Z}_\nu]]\}\eta^{\mu\nu} \supset -\sum_{m,n} g_{HZ^{(m)}Z_R^{(n)}}HZ^{(m)}_\mu Z^{(n)}_{R\nu}\eta^{\mu\nu} \tag{E5}$$

so that

$$g_{HZ^{(m)}Z_R^{(n)}} = -g_Ak^2\int_1^{z_L}\frac{dz}{kz}\frac{1}{2}u_H(z)[-(\partial_z\hat{h}_{Z^{(m)}})(h^L_{Z_R^{(n)}}-h^R_{Z^{(n)}R})-\hat{h}_{Z^{(m)}}\partial_z(h^L_{Z_R^{(n)}}-h^R_{Z^{(n)}R})]. \tag{E6}$$

HWW and HWW_R couplings are seen in [49].

APPENDIX F: DECAY WIDTH

For a heavy charged vector boson W', the $W'\to WH$ decay width is given by

$$\Gamma(W'\to WH) = \frac{M_{W'}}{192\pi}\left(\frac{g_{W'WH}}{M_W}\right)^2\left(1+\frac{10M_W^2-2M_H^2}{M_{W'}^2}+\frac{M_W^4+M_H^4-2M_W^2M_H^2}{M_{W'}^4}\right), \tag{F1}$$

and $\Gamma(Z'\to ZH)$ is obtained from the above expression by replacements of W with Z. The decay width for $W'\to WZ$ is given by

$$\Gamma(W'\to ZW) = \frac{M_{W'}}{192\pi}g^2_{W'WZ}\frac{M^4_{W'}}{M_W^2M_Z^2}\left(1-\frac{(M_Z+M_W)^2}{M^2_{W'}}\right)\left(1-\frac{(M_Z-M_W)^2}{M^2_{W'}}\right)$$
$$\times\left(1+\frac{10(M_Z^2+M_W^2)}{M^2_{W'}}+\frac{M_Z^4+M_W^4+10M_Z^2M_W^2}{M^4_{W'}}\right), \tag{F2}$$

and $\Gamma(Z'\to W^+W^-)$ is obtained by replacements $W'\to Z'$ and $Z\to W$.

For the decay of a heavy fermion F (mass m_F) to a light fermion f (mass m_f) and a vector boson V (mass m_V), the decay rate is given by

$$\Gamma(F\to fV)=\frac{m_F}{32\pi}\sqrt{\lambda(1,m_f/m_F,m_V/m_F)}\left\{\frac{(g^L_{VFf})^2+(g^L_{VFf})^2}{m_V^2m_F^2}[(m_F^2-m_f^2)^2+m_V^2(m_F^2+m_f^2)-2m_V^2]-12g_Lg_R\frac{m_f}{m_F}\right\}, \tag{F3}$$

where

$$\lambda(A,B,C)\equiv A^4+B^4+C^4-2(A^2B^2+B^2C^2+C^2A^2), \tag{F4}$$

where $g^{L/R}_{VFf}$ are the left- and right- handed coupling of $\bar{f}FV$.

For decay widths of exotic fermions $t_T^{(n)}$ and $b_Y^{(n)}$, we have

$$\begin{aligned}\Gamma(t_T^{(n)}\to tW^+)=&\frac{1}{32\pi}M_{t_T^{(n)}}\sqrt{\lambda(1,M_t/M_{t_T^{(n)}},M_W/M_{T^{(n)}})}\\&\times\left\{\frac{(g^L)^2+(g^R)^2}{M_W^2M^2_{t_T^{(n)}}}[(M^2_{t_T^{(n)}}-M_t^2)^2+M_W^2(M^2_{t_T^{(n)}}+M_t^2)-2M_W^4]-12g^Lg^R\frac{M_t}{M_{t_T^{(n)}}}\right\},\end{aligned} \tag{F5}$$

where

$$g^{L,R}=g^{L,R}_{Wt_T^{(n)}t}, \tag{F6}$$

are left- and right-hand couplings of $t_T^{(n)}$ to tW^+. Decay width for $b_Y^{(n)}$ to bW^- are obtained by replacements

$$(t_T^{(n)},t,W^+)\to(b_Y^{(n)},W^-,b). \tag{F7}$$

APPENDIX G: CROSS SECTION

Cross sections of processes $f\bar{f}'\to W'\to WH$ and $f\bar{f}\to Z'\to ZH$ in the center-of-mass frame are given as follows. For the process $f(p_1)\bar{f}(p_2)\to V'\to V(k_1)H(k_2)$, the differential cross section is given by

$$\frac{d\sigma}{d\cos\theta}=\frac{1}{64\pi}\frac{|\boldsymbol{k}|}{s\sqrt{s}}g^2_{HV'V}(|g^L_{V'f}|^2+|g^R_{V'f}|^2)\frac{2M_V^2-|\boldsymbol{k}|^2(\cos^2\theta-1)}{M_V^2}\frac{s}{(s-M_{V'^2})^2+M^2_{V'}\Gamma^2_{V'}}, \tag{G1}$$

where θ is the angle between $\boldsymbol{p}_1$ and $\boldsymbol{k}_1$.

$$|\boldsymbol{k}|\equiv|\boldsymbol{k}_1|=|\boldsymbol{k}_2|=\frac{\sqrt{\lambda(M_{V'},M_V,M_H)}}{2M_{V'}}, \tag{G2}$$

is the momentum of a final state particle. Integrating with respect to θ, and taking interferences among intermediate bosons into account we obtain

$$\begin{aligned}\sigma(f\bar{f}'\to W,W'\to WH)=&\frac{1}{N_c^i}\frac{1}{48\pi}\frac{|\boldsymbol{k}|}{\sqrt{s}}\frac{3M_W^2+|\boldsymbol{k}|^2}{M_W^2}\left\{\sum_{V=W,W^{(1)}}\frac{g^2_{HVW}[|g^L_{Vff'}|^2+|g^R_{Vff'}|^2]}{(s-M_V^2)^2+M_V^2\Gamma_V^2}\right.\\&\left.+2\mathrm{Re}\left[\frac{g_{HWW}g_{HW^{(1)}W}[(g^L_{Wff'})(g^L_{W^{(1)}ff'})^*+(g^R_{Wff'})(g^R_{W^{(1)}ff'})^*]}{[(s-M_W^2)+iM_W\Gamma_W][(s-M^2_{W^{(1)}})-iM_{W^{(1)}}\Gamma_{W^{(1)}}]}\right]\right\},\end{aligned} \tag{G3}$$

$$\sigma(f\bar{f} \to \gamma, Z, Z' \to ZH) = \frac{1}{N_c^i}\frac{1}{48\pi}\frac{|\boldsymbol{k}|}{\sqrt{s}}\frac{3M_Z^2 + |\boldsymbol{k}|^2}{M_Z^2}\left\{\sum_{V=Z,Z^{(1)},\gamma^{(1)},Z_R^{(1)}} \frac{g_{HVZ}^2[|g_{Vf}^L|^2 + |g_{Vf}^R|^2]}{(s-M_V^2)^2 + M_V^2\Gamma_V^2}\right. \\ \left. + \sum_{\substack{V_1,V_2=Z,Z^{(1)},\gamma^{(1)},Z_R^{(1)} \\ V_1\neq V_2}} \mathrm{Re}\left[\frac{g_{HV_1Z}g_{HV_2Z}[(g_{V_1f}^L)(g_{V_2f}^L)^* + (g_{V_1f}^R)(g_{V_2f}^R)^*]}{[(s-M_{V_1}^2) + iM_{V_1}\Gamma_{V_1}][(s-M_{V_2}^2) - iM_{V_2}\Gamma_{V_2}]}\right]\right\}, \tag{G4}$$

where N_c^i is the number of colors of initial-state fermions.

For the process $f(p_1)\bar{f}'(p_2) \to W(k_1)Z(k_2)$, we adopt the approximation in which the interference term between the SM part and NP part is dropped. The cross section formulas in the SM are found in [74]. The differential cross section mediated by heavy charged vector bosons W' in the center-of-mass frame is given by

$$\frac{d\sigma}{dt}(f\bar{f} \to W' \to WZ) = \frac{1}{64\pi s^2}\cdot 4s^2 A(t,u)\left\{\sum_{\substack{W'\in\{W^{(n)}\} \\ W'\neq W}} \frac{g_{W'WZ}^2(|g_{W'ff'}^L|^2 + |g_{W'ff'}^R|^2)^2}{(s-M_{W'}^2)^2 + M_{W'}^2\Gamma_{W'}^2}\right. \\ \left. + \sum_{\substack{W_1,W_2\in\{W^{(n)}\} \\ M_W \ll M_{W_1} < M_{W_2}}} 2\mathrm{Re}\left[\frac{g_{W_1WZ}g_{W_2WZ}[(g_{W_1ff'}^L)(g_{W_2ff'}^L)^* + (g_{W_1ff'}^R)(g_{W_2ff'}^R)^*]}{[(s-M_{W_1}^2) + iM_{W_1}\Gamma_{W_1}][(s-M_{W_2}) - iM_{W_2}\Gamma_{W_2}]}\right]\right\}, \tag{G5}$$

where s, t, and u are Mandelstam variables and $A(t,u)$ is given in [74] by

$$A(t,u) = \left(\frac{ut}{M_Z^2M_W^2} - 1\right)\left[\frac{1}{4} - \frac{M_Z^2 + M_W^2}{2s} + \frac{(M_W^2 + M_Z^2)^2 + 8M_W^2M_Z^2}{4s^2}\right] \\ + \left(\frac{M_W^2 + M_Z^2}{M_W^2M_Z^2}\right)\left[\frac{s}{2} - M_W^2 - M_Z^2 + \frac{(M_W^2 - M_Z^2)^2}{2s}\right]. \tag{G6}$$

Here $t_{\min} \le t \le t_{\max}$, $t_{\min,\max} = \frac{1}{2}(M_W^2 + M_Z^2 - s) \pm \frac{1}{2}s\beta$, $\beta = |\boldsymbol{k}|/(\sqrt{s}/2)$ with $|\boldsymbol{k}| = |\boldsymbol{k}_{1,2}| = \sqrt{\lambda(M_{W'}, M_W, M_Z)}/2M_{W'}$. Integrating $d\sigma/dt$ with respect to t and using

$$\int_{t_{\min}}^{t_{\max}} A(t,u)dt = \frac{s^3\beta^3}{24M_W^2M_Z^2}\left[1 + \frac{10(M_W^2 + M_Z^2)}{s} + \frac{M_W^4 + M_Z^4 + 10M_W^2M_Z^2}{s^2}\right], \tag{G7}$$

we obtain

$$\sigma(ff' \to W' \to WZ) = \frac{1}{384\pi}\frac{s^3\beta^3}{M_W^2M_Z^2}\left[1 + \frac{10(M_W^2 + M_Z^2)}{s} + \frac{M_W^4 + M_Z^4 + 10M_W^2M_Z^2}{s^2}\right] \\ \times\left\{\sum_{\substack{W'\in\{W^{(n)}\} \\ M_{W'}\gg M_W}} \frac{g_{W'WZ}^2(|g_{W'ff'}^L|^2 + |g_{W'ff'}^R|^2)^2}{(s-M_{W'}^2)^2 + M_{W'}^2\Gamma_{W'}^2} \sum_{\substack{W_1,W_2\in\{W^{(n)}\} \\ M_W\ll M_{W_1}<M_{W_2}}}\right. \\ \left. + 2\mathrm{Re}\left[\frac{g_{W_1WZ}g_{W_2WZ}[(g_{W_1ff'}^L)(g_{W_2ff'}^L)^* + (g_{W_1ff'}^R)(g_{W_2ff'}^R)^*]}{[(s-M_{W_1}^2) + iM_{W_1}\Gamma_{W_1}][(s-M_{W_2}) - iM_{W_2}\Gamma_{W_2}]}\right]\right\}. \tag{G8}$$

Formulas for the processes $f\bar{f} \to Z' \to W^+W^-$ ($Z' = Z^{(n)}, \gamma^{(n)}, Z_R^{(n)}$, $n = 1, 2, \cdots$) can be obtained from the above formulas by replacements $W' \to Z'$ and $Z \to W$.

For the process $f\bar{f}' \to \{V_i\} \to F\bar{F}$ where $f^{(\prime)}$, $F^{(\prime)}$ are massless fermions and V_i are vector bosons, differential cross section is given by

$$\begin{aligned}\frac{d\sigma}{d\cos\theta}(f\bar{f}\to\{V_i\}\to F\bar{F}') = \frac{N_c^f}{N_c^i}\frac{s}{128\pi}\Bigg\{&\sum_i \frac{1}{(s-M_{V_i}^2)^2+M_{V_i}^2\Gamma_{V_i}^2}\\
&\times[(|g^L_{V_i ff'}|^2+|g^R_{V_i ff'}|^2)(|g^L_{V_i FF'}|^2+|g^R_{V_i FF'}|^2)(1+\cos^2\theta)+2(|g^L_{V_i ff'}|^2\\
&-|g^R_{V_i ff'}|^2)(|g^L_{V_i FF'}|^2-|g^R_{V_i FF'}|^2)\cos\theta]\\
&+2\mathrm{Re}\sum_{i>j}\frac{1}{(s-M_{V_i}^2)^2+M_{V_i}^2\Gamma_{V_i}^2}\frac{1}{(s-M_{V_i}^2)^2+M_{V_i}^2\Gamma_{V_i}^2}\\
&\times[(g^L_{V_i ff'}g^{L*}_{V_j ff'}+g^R_{V_i ff'}g^{R*}_{V_j ff'})(g^L_{V_i FF'}g^{L*}_{V_j FF'}+g^R_{V_i FF'}g^{R*}_{V_j FF'})(1+\cos^2\theta)\\
&+2(g^L_{V_i ff'}g^{L*}_{V_j ff'}-g^R_{V_i ff'}g^{R*}_{V_j ff'})(g^L_{V_i FF'}g^{L*}_{V_j ff'}-g^R_{V_i FF'}g^{R*}_{V_j FF'})\cos\theta]\Bigg\},\end{aligned}\tag{G9}$$

where θ is the scattering angle. The corresponding distribution in the transverse momentum $p_T\equiv(\sqrt{s}/2)\sin\theta$ is obtained by

$$\frac{d\sigma}{dp_T}(p_T,s)=\frac{d\sigma}{d\cos\theta}\bigg|_{\cos\theta=\sqrt{1-4p_T^2/s}}\cdot\frac{4p_T}{s\sqrt{1-4p_T^2/s}}.\tag{G10}$$

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