## LHC multijet events as a probe for anomalous dimension-six gluon interactions

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Higher-dimensional multigluon interactions affect essentially all effective Lagrangian analyses at the LHC. We show that, contrary to common lore, such operators are best constrained in multijet production. Our limit on the corresponding new physics scale in the multi-TeV range exceeds the typical reach of global dimension-six Higgs boson and top analyses. This implies that the pure Yang-Mills operator can safely be neglected in almost all specific higher-dimensional analyses at Run II.

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With the first analyses of Run II data of the LHC appearing, effective Lagrangians [\[1,2\]](#page-4-0) are rapidly developing into the main physics framework describing searches for physics beyond the standard model (SM). Global analyses of Run I and early Run II data already exist for the Higgs and electroweak gauge sectors [\[3\]](#page-4-1) and for the top sector [\[4\]](#page-4-2), illustrating the power of this approach. The fact that essentially all production processes in all physics sectors involve incoming gluons poses a major, unsolved challenge to all such effective Lagrangian analyses: the pure Yang-Mills operator with its corresponding Wilson coefficient

<span id="page-0-0"></span>
$$
c_G \mathcal{O}_G = \frac{g_s c_G}{\Lambda^2} f_{abc} G_{ab}^{\rho} G_{b\lambda}^{\nu} G_{c\rho}^{\lambda}
$$
  
with 
$$
G_a^{\rho\nu} = \partial^{\rho} G_a^{\nu} - \partial^{\nu} G_a^{\rho} - ig_s f_{abc} G^{b\rho} G^{c\nu} \quad (1)
$$

correlates all such analyses [\[5,6\]](#page-4-3) and forces us into an unwieldy, if not unrealistic, global analysis of all LHC channels. The operator  $D^{\mu}G_{\mu\nu}D_{\rho}G^{\rho\nu}$  can lead to similar effects, but it can be removed from our operator basis through equations of motion, mapping it to four-quark operators [\[7\]](#page-4-4).

It is very well known that the contribution of  $\mathcal{O}_G$  to dijet production in gluon-gluon or gluon-quark scattering does not interfere with the standard model process [\[8\]](#page-4-5). Heavy quark production,  $gg \to t\bar{t}$ , is an exception, and it can be used to constrain  $c_G/\Lambda^2$  at the Tevatron [\[7\].](#page-4-4) However, the operator  $\mathcal{O}_G$  is only one of many operators contributing to top-pair production, giving marginalized Run I constraints top-pair production, giving marginalized Run I constraints<br>of the order  $\Lambda/\sqrt{c_G} \gtrsim 850$  GeV [\[4\].](#page-4-2) Alternative, but less<br>powerful search strategies include four-jet production at powerful search strategies include four-jet production at LEP [\[9\]](#page-4-6) and three-jet production at hadron colliders [\[10\]](#page-4-7), while the suggestion to constrain  $\mathcal{O}_G$  in a Higgs analysis [\[11\]](#page-4-8) lacks realism given the current reach of such a Higgs analysis [\[3\]](#page-4-1).

In this article we propose to search for effects of  $\mathcal{O}_G$  in a new channel, namely multijet production which we analyze for up to six hard jets. Our analytic understanding of inclusive and exclusive multijet production processes has matured [\[12\],](#page-4-9) and we can robustly and precisely simulate such processes [\[13\]](#page-4-10). In this note we rely on two wellcontrolled observables, namely the (exclusive) number of jets  $N_{\text{it}}$  and  $S_T$ , defined as the scalar sum of jet transverse momenta plus any missing transverse energy exceeding 50 GeV [\[14\],](#page-4-11)

$$
S_T = \left(\sum_{j=1}^{N_{\text{jets}}} E_{T,j}\right) + (E_T > 50 \text{ GeV})
$$
 (2)

The two observables allow the separation of two-jet production from events with a larger number of jets while simultaneously giving a measure of the energy scale tested in the partonic process.

Two-jet production from partonic processes such as  $q\bar{q} \rightarrow q'\bar{q}'$  serves as an excellent probe of four-quark<br>effective operators. Because this topology carries little effective operators. Because this topology carries little sensitivity to  $\mathcal{O}_G$  [\[8\]](#page-4-5) we impose the corresponding ATLAS limits on four-quark operators [\[15\]](#page-4-12) in our multijet analysis in order to limit the effect of these operators.

Our effective Lagrangian hypothesis is defined by following the standard approach of global effective Lagrangian analyses [\[3,4\]](#page-4-1) to test the dimension-six Lagrangian only as a well-defined hypothesis. The effect of the corresponding

<span id="page-1-0"></span>

FIG. 1.  $S_T$  distributions from CMS [\[14\]](#page-4-11) in various bins of exclusive/inclusive jet multiplicity  $N_{\text{jets}}$ , compared to our multijet-merged signal and background predictions including perturbative uncertainties.

dimension-six operators in generic multijet signatures scales like  $E^2/\Lambda^2$ , but the wide available energy range at the LHC sheds some doubt on the assumption that the effects of dimension-eight operators are systematically suppressed compared to dimension-six operators. We therefore treat the effects of higher-dimensional operators as theoretical uncertainties in the matching procedure of a given full model to the dimension-six Lagrangian [\[16\].](#page-4-13)

Multijet signature.—Our analysis of the dimension-six QCD Lagrangian is based on a CMS search for extradimensional black holes [\[14\]](#page-4-11), which to date is the only published 13 TeV analysis based on a sizeable data set and extending to a large number of jets without requiring any additional particles in the final state. Obviously, dedicated ATLAS or CMS analyses of multijet production in the light of dimension-six operators will improve upon our results. The background is completely dominated by QCD jet production, so just as in the original analysis we neglect non-QCD backgrounds.

For a robust description of the high-multiplicity QCD jet backgrounds, we employ the Catani-Krauss-Kuhn-Webber multijet merging within Sherpa [\[17,18\],](#page-4-14) with next-to-leadingorder matrix elements for dijet production and leading-order matrix elements for up to six jets in the final state. Our nominal choice for the factorization and renormalization scales is determined by a backwards clustering procedure scales is determined by a backwards clustering procedure<br>and the scale choice  $\sqrt{2}\mu_{r,f}^2 = 1/(s^{-1} + t^{-1} + u^{-1})$  for the  $2 \rightarrow 2$  core process [\[17\].](#page-4-14)

As shown in Fig. [1](#page-1-0), the observed  $S_T$  distributions are accurately described by our SM simulations. We estimate perturbative uncertainties through independent variation of both scales by a factor of 2 around the nominal values, omitting combinations where one scale is varied upwards and the other one downwards to avoid large logarithms. All differences between data and the SM simulation are within the estimated perturbative uncertainties. The minimal tension in the exclusive two-jet bin at low  $S_T$  only occurs after translating the original inclusive results into jet-exclusive distributions. They do not affect our analysis of the multijet rates and our constraints on higher-dimensional operators contributing to this process.

Our signal simulations including the operator  $\mathcal{O}_G$  are based on an implementation of the dimension-six operator of Eq. [\(1\)](#page-0-0) in FeynRules [\[19\]](#page-4-15). We employ the Ufo output format in order to facilitate event generation with Sherpa and its matrix element generator Comix [\[20,21\]](#page-4-16). For the purpose of implementing the new exotic color structures

<span id="page-2-0"></span>

FIG. 2. Effect of multiple occurrences of the dimension-six Yang-Mills operator in the multijet matrix elements.

that appear in the Feynman rules of the dimension-six operator, a code generator module for arbitrary color structures was implemented in Sherpa. This feature will become publicly available along with the next Sherpa release. The automatic generation of arbitrary Lorentz structures using Sherpa is described in Ref. [\[21\].](#page-4-17)

Just like the QCD background we compute the contributions of the dimension-six operator of Eq. [\(1\)](#page-0-0) using Catani-Krauss-Kuhn-Webber multijet merging techniques with leading-order matrix elements for up to five jets [\[17\]](#page-4-14). Formally, we can organize the effect of the higher dimension contributions in terms of the scale suppression in the multijet cross section. In this scheme, the leading interference terms with SM diagrams are proportional to  $1/\Lambda^2$ , while the dimension-six contributions squared contribute to  $1/\Lambda^4$  or higher, depending on the numerically relevant number of operator insertions.

In Fig. [2](#page-2-0) we show the new physics effects in the  $S_T$ distribution for large jet multiplicities. The effects due to interference terms proportional to  $1/\Lambda^2$  are negligible throughout the displayed range of  $S_T$ . Significant effects, however, arise from terms of order  $1/\Lambda^4$ . This dominance<br>of terms of order  $1/\Lambda^4$  over terms of order  $1/\Lambda^2$  can also of terms of order  $1/\Lambda^4$  over terms of order  $1/\Lambda^2$  can also be observed in top-pair production [\[4\].](#page-4-2) For  $S_T > \Lambda$ , the contributions due of terms of order  $1/\Lambda^6$  and beyond eventually become significant. This is to be expected, since  $S_T/\Lambda > 1$  in this region, thus spoiling the parametric suppression in  $1/\Lambda$  and leading to a breakdown of the effective field theory (EFT) approach. This might lead to problems in matching our effective Lagrangian results to a given full model. A standard solution to this problem is to truncate the  $S_T$  spectrum at  $S_T = \Lambda$ , thus avoiding the kinematic region in which the EFT breaks down. Such a cut is known to almost entirely remove the sensitivity to higher-dimensional operators for example in Higgs physics [\[3\]](#page-4-1). The sensitivity of the analysis presented here, however, is only very mildly affected by this cut, as is shown in what follows.

<span id="page-2-1"></span>Four-quark operator.—While multijet production at the LHC is dominated by gluon amplitudes, processes with quarks in the initial and final states still lead to visible effects. These processes are sensitive to the dimension-six contact interaction

$$
c_{q4}\mathcal{O}_{q4} = \pm \frac{c_{q4}}{\Lambda^2} \sum_{q,q'} (\bar{q}_L \gamma^\mu q_L)(\bar{q}'_L \gamma^\mu q'_L). \tag{3}
$$

While in principle the two operators in Eqs. [\(1\)](#page-0-0) and [\(3\)](#page-2-1) should be treated concurrently, we know from the amplitude structure that the number of jets  $N_{\text{jets}}$  separates their respective signal regions. For the four-quark operator the highest sensitivity can be obtained from two-jet correlations and we therefore use the state-of-the-art result from the comprehensive, multivariate ATLAS analysis [\[15\]](#page-4-12). Being formulated as an extension to resonance searches it does not include the higher-dimensional gluon operator, and one should therefore use the two-jet topology only. There, the ATLAS analysis gives

$$
\frac{\Lambda}{\sqrt{c_{q4}}} > 4.79...6.8 \text{ TeV}, \tag{4}
$$

<span id="page-2-3"></span>in the conventions of Eq. [\(3\)](#page-2-1) and depending on the assumed sign of the Wilson coefficient.

We estimate the impact of the four-quark operator on our Yang-Mills analysis by computing its effect on multijet production. In Fig. [3](#page-2-2) we show the impact of the four-quark operator within its allowed range of Eq. [\(4\)](#page-2-3) on the multijet signature. This result can be directly compared to the expected signal from  $\mathcal{O}_G$ , shown in Fig. [2](#page-2-0).

Comparing the two effects on the high-energy tail of the  $S_T$  distribution with an assumed new physics scale  $\Delta/\sqrt{c_G} \lesssim 5$  TeV we confirm that the four-quark effects<br>are strongly suppressed. We find that the two effects only are strongly suppressed. We find that the two effects only

<span id="page-2-2"></span>

FIG. 3. Effect of effective four-quark operators in our signal **FIG.** 3. Effect of effective four-quark operators in our signal region, with  $Λ/\sqrt{c_{q4}}$  set to the lower limits obtained by ATLAS [15] ATLAS [\[15\]](#page-4-12).

<span id="page-3-0"></span>

FIG. 4. Observed and expected signal confidence levels as a function of the integrated luminosity. We show the expected results for fixed numbers of  $N_{\text{jets}} = 2.3.4$  (left) and for  $N_{\text{jets}} \ge 5$  (right). An observed CL<sub>s</sub> below the dashed line indicates a signal confidence below 5% and allows for an exclusion of the dimension-six hypothesis.

become comparable when we increase the new physics become comparable when we increase the new phy scale in the Yang-Mills operator to  $\Lambda/\sqrt{c_G} \gtrsim 7$  TeV.

**Multigluon operator limit.**—Finally, we can use the  $S_T$ distributions in bins of  $N_{\text{jets}}$  to constrain the Yang-Mills operator  $\mathcal{O}_G$  in terms of a signal confidence  $CL_s$  as defined in  $[22]$ . In the calculation of  $CL<sub>s</sub>$  we take into account the dominant systematic uncertainties, which are inherent in our background predictions. In Fig. [4](#page-3-0) we show the expected our background predictions. In Fig. 4 we show the expected<br>signal confidence for  $\Lambda/\sqrt{c_G} = 5 \text{ TeV}$  as a function of the integrated luminosity collected at the LHC with  $\sqrt{s}$  = 13 TeV. In the left panel we see that indeed the sensitivity of the two-jet topology is poor. This also confirms that adding the Yang-Mills operator  $\mathcal{O}_G$  to the four-quark operator analysis of ATLAS does not affect the limit shown in Eq. [\(4\).](#page-2-3)

For higher jet multiplicities  $N_{\text{jets}} = 3$ , 4 the LHC reach slowly increases, and we expect to rule out reach slowly increases, and we expect to rule out  $\Lambda/\sqrt{c_G} < 5$  TeV based on an integrated luminosity of loss than 2 the Universe the by for strongest constraints less than 2 fb<sup>-1</sup>. However, the by far strongest constraints can be derived from the inclusive five-jet sample, with a required luminosity well below  $0.5$  fb<sup>-1</sup> for with a required<br> $\Lambda/\sqrt{c_G} = 5$  TeV.<br>In the convention

In the conventions of Eq.  $(1)$  we find a limit on the Yang-Mills operator  $\mathcal{O}_G$  of

$$
\frac{\Lambda}{\sqrt{c_G}} > 5.2 \text{ TeV} \quad \text{(observed)}
$$
  

$$
\frac{\Lambda}{\sqrt{c_G}} > 5.8 \text{ TeV} \quad \text{(expected)}, \tag{5}
$$

based on  $CL_s < 5\%$  (see the left panel of Fig. [5\)](#page-3-1). The difference between expected and observed limits corresponds to a deviation of just over 1 sigma and is, in part, due to a slight excess in the data between  $S_T = 5$  TeV and  $S_T = 6$  TeV, as shown in the lower right panel of Fig. [1](#page-1-0).

In Fig. [5](#page-3-1), we demonstrate that sensitivity of our analysis is not an artifact of the very large new physics effects in the region of  $S_T > \Lambda$ , where the applicability of the EFT is

<span id="page-3-1"></span>

FIG. 5. Observed and expected signal confidence levels for the Yang-Mills operator  $\mathcal{O}_G$  as a function of  $\Lambda/\sqrt{c_G}$ . The results shown in the left take into account the full  $S_{\alpha}$  distribution. The plot on the righ the left take into account the full S<sub>T</sub> distribution. The plot on the right-hand side shows the sensitivity of the analysis when truncating the distribution at  $S_n = \Lambda$ distribution at  $S_T = \Lambda$ .

questionable. We compare the expected and observed dependence of  $CL_s$  on  $\Lambda$  when taking into account all events and when taking into account only events with  $S_T < \Lambda$ . As can be seen in this figure, the expected sensitivity is only very mildly affected by this cut. The sensitivity is only very milidly affected by this cut. The<br>observed limit on  $\Lambda/\sqrt{c_G}$  is in fact stronger when avoiding<br>the region  $S \ge \Lambda$  due to the presence of a slight excess in the region  $S_T > \Lambda$ , due to the presence of a slight excess in<br>the data above  $S_T = 5$  TeV the data above  $S_T = 5$  TeV.

Conclusions.—The purely gluonic dimension-six operator  $\mathcal{O}_G$  is known to be a major problem for all effective Lagrangian analyses at Run II. We show, for the first time, that it can very effectively be constrained using multijet signatures at the LHC. Based on a CMS black-hole search with an integrated luminosity of 2.2 fb<sup>-1</sup> at 13 TeV search with an integrated we find a limit  $\Lambda / \sqrt{c_G}$  $\sqrt{c_G} > 5.2$  TeV. For an alternative definition  $\mathcal{O}_G = 1/\Lambda^2 f_{abc} G^3$  without the additional factor of  $g_s$ , we find  $Λ/\sqrt{c_g} > 4.7$  TeV.<br>The effect of four quark operator

The effect of four-quark operators on our analysis can be fully controlled by considering the two-jet and multijet signatures separately. Our analysis demonstrates that possible effects of this operator can be safely neglected in specific effective Lagrangian analyses for example of the gauge, Higgs, or top sectors.

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