Lepton flavor violating Higgs boson decay $h \rightarrow \mu\tau$ in the littlest Higgs model with T parity

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Inspired by the recent CMS $h \rightarrow \mu\tau$ excess, we calculate the lepton flavor violating Higgs decay $h \rightarrow \mu\tau$ in the littlest Higgs model with T parity (LHT). Under the constraints of $\ell_i \rightarrow \ell_j \gamma$, $Z \rightarrow \ell_i \overline{\ell}_j$ and Higgs data, we find that the branching ratio of $h \rightarrow \mu\tau$ can maximally reach $\mathcal{O}(10^{-4})$. We also investigate the correlation between $h \rightarrow \mu\tau$, $\tau \rightarrow \mu\gamma$, and $Z \rightarrow \mu\tau$, which can be used to test the LHT model at future $e^+ e^-$ colliders.

I. INTRODUCTION

The discovery of the Higgs boson at the Large Hadron Collider (LHC) [1] is a great step toward understanding the electroweak symmetry breaking (EWSB) mechanism. To ultimately establish its nature, a precise study of the Higgs boson properties, in particular the Higgs rare decays and productions [2], will be important tasks at LHC and future colliders.

In fact, CMS 8 TeV data has shown a 2.4 excess in searching for Higgs mediated lepton flavor violation (LFV) process $h \rightarrow \tau\mu$, which is interpreted to the branching ratio: $\mathrm{Br}(h \rightarrow \mu\tau) < 1.51 \times 10^{-2}(\mathrm{CMS})$ [3]. While ATLAS only observed a small excess in one of the signal regions and reported an upper limit: $\mathrm{Br}(h \rightarrow \mu\tau) < 1.43 \times 10^{-2}(\mathrm{ATLAS})$ [4]. From the available data it is premature to draw any definite conclusion and more data is needed to confirm its existence. However, the lepton flavor violating decay of the Higgs boson is widely predicted in various extensions of the standard model (SM), such as seesaw [5], supersymmetric (SUSY) [6], two-Higgs doublet model (2HDM) [7], 3-3-1 model [8], and other ones [9]. In the SM, the LFV process is extremely suppressed by Glashow-Iliopoulos-Maiani (GIM) mechanism [10] due to the smallness of neutrino mass. So, any observation of such decays would indicate the new physics beyond the SM.

As an extension of the SM, the littlest Higgs Model with T parity (LHT) is one of the popular candidates that can successfully solve the hierarchy problem. The LHT model predicts many new particles, such as heavy gauge bosons, mirror fermions, heavy scalars and heavy top partners. Moreover, the flavor structure of the LHT model is richer than the one of the SM, mainly due to the presence of the mirror fermions and their weak interactions with the ordinary fermions. It has been shown that the LHT model can give significant contributions to some LFV processes [11]. In this work, we investigate the LFV process $h \rightarrow \mu\tau$ in the LHT model under the current constraint of $\ell_i \rightarrow \ell_j \gamma$, $Z \rightarrow \ell_i \overline{\ell}_j$ and Higgs data.

The paper is organized as follows. In Sec. II we give a brief review of the LHT model related to our work. In Sec. III we calculate the LFV process $h \rightarrow \mu\tau$ in unitary gauge under current constraints. In Sec. IV we investigate the correlation between $h \rightarrow \mu\tau$, $\tau \rightarrow \mu\gamma$ and $Z \rightarrow \mu\tau$ in the LHT model. Finally, we draw our conclusions in Sec. V.

II. A BRIEF REVIEW OF THE LHT MODEL

The LHT model is based on an $SU(5)/SO(5)$ nonlinear σ model [12], where the $SU(5)$ global symmetry is broken down to $SO(5)$ at the scale $f \sim \mathcal{O}$ (TeV) by the vacuum expectation value (VEV) of the σ field, Σ_0, given by

$$\Sigma_0 = \langle \Sigma \rangle \begin{pmatrix} \mathbf{0}_{2\times2} & 0 & \mathbf{1}_{2\times2} \\ 0 & 1 & 0 \\ \mathbf{1}_{2\times2} & 0 & \mathbf{0}_{2\times2} \end{pmatrix}. \tag{1}$$

The VEV Σ_0 also breaks the gauged subgroup $[SU(2) \times U(1)]^2$ of the $SU(5)$ down to the SM electroweak $SU(2)_L \times U(1)_Y$. After EWSB, the new T-odd gauge bosons $W_H^\pm, Z_H, A_H$ acquires masses, given at $\mathcal{O}(v^2/f^2)$ by

$$M_{W_H} = M_{Z_H} = gf\left(1 - \frac{v^2}{8f^2}\right), \qquad M_{A_H} = \frac{g'f}{\sqrt{5}}\left(1 - \frac{5v^2}{8f^2}\right), \tag{2}$$

with g and g' being the SM $SU(2)$ and $U(1)$ gauge couplings, respectively. The T-even $W^\pm$ and Z bosons of the SM, whose masses at $\mathcal{O}(v^2/f^2)$ are given by

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$$M_W = \frac{gv}{2}\left(1 - \frac{v^2}{12f^2}\right), \qquad M_Z = \frac{gv}{2\cos\theta_W}\left(1 - \frac{v^2}{12f^2}\right),$$

$$M_A = 0. \tag{3}$$

Here, v represents the Higgs doublet VEV, which can be given by

$$v = \frac{f}{\sqrt{2}}\arccos\left(1 - \frac{v_{SM}^2}{f^2}\right) \simeq v_{SM}\left(1 + \frac{1}{12}\frac{v_{SM}^2}{f^2}\right), \tag{4}$$

where $v_{SM} = 246$ GeV is the SM Higgs VEV.

The implementation of T parity in the fermion sector requires the introduction of mirror fermions. Then, the T-odd mirror partners for each SM fermions are added and one can write down a Yukawa-type interaction to give masses to the mirror fermions

$$\mathcal{L}_{\text{mirror}} = -\kappa_{ij}f(\overline{\Psi}_2^i \xi + \overline{\Psi}_1^i \Sigma_0 \Omega \xi^\dagger \Omega)\Psi_R^j + \text{H.c.} \tag{5}$$

where $i, j = 1, 2, 3$ are the generation indices. After EWSB, the mirror leptons acquire masses, given at $\mathcal{O}(v^2/f^2)$ by

$$m_{\ell_H^i} = \sqrt{2}\kappa_i f, \qquad m_{\nu_H^i} = m_{\ell_H^i}\left(1 - \frac{v^2}{8f^2}\right), \tag{6}$$

where κ_i are the eigenvalues of the mass matrix κ.

As discussed in detail in Ref. [13], the existence of two Cabibbo-Kobayashi-Maskawa (CKM)-like unitary mixing matrices $V_{H\ell}$ and $V_{H\nu}$ is one of the important ingredients in the mirror lepton sector. Note that $V_{H\ell}$ and $V_{H\nu}$ are related through the Pontecorvo-Maki-Nakagata-Saki (PMNS) matrix:

$$V_{H\nu}^\dagger V_{H\ell} = V_{\text{PMNS}}^\dagger, \tag{7}$$

where in V_{PMNS} the Majorana phases are set to zero as no Majorana mass term has been introduced for right-handed neutrinos.

Follow Ref. [14], the matrix $V_{H\ell}$ can be parametrized with three mixing angles θ_{12}^ℓ, θ_{23}^ℓ, θ_{13}^ℓ and three complex phases δ_{12}^ℓ, δ_{23}^ℓ, δ_{13}^ℓ

$$V_{H\ell} = \begin{pmatrix} c_{12}^\ell c_{13}^\ell & s_{12}^\ell c_{13}^\ell e^{-i\delta_{12}^\ell} & s_{13}^\ell e^{-i\delta_{13}^\ell} \\ -s_{12}^\ell c_{23}^\ell e^{i\delta_{12}^\ell} - c_{12}^\ell s_{23}^\ell s_{13}^\ell e^{i(\delta_{13}^\ell - \delta_{23}^\ell)} & c_{12}^\ell c_{23}^\ell - s_{12}^\ell s_{23}^\ell s_{13}^\ell e^{i(\delta_{13}^\ell - \delta_{12}^\ell - \delta_{23}^\ell)} & s_{23}^\ell c_{13}^\ell e^{-i\delta_{23}^\ell} \\ s_{12}^\ell s_{23}^\ell e^{i(\delta_{12}^\ell + \delta_{23}^\ell)} - c_{12}^\ell c_{23}^\ell s_{13}^\ell e^{i\delta_{13}^\ell} & -c_{12}^\ell s_{23}^\ell e^{i\delta_{23}^\ell} - s_{12}^\ell c_{23}^\ell s_{13}^\ell e^{i(\delta_{13}^\ell - \delta_{12}^\ell)} & c_{23}^\ell c_{13}^\ell \end{pmatrix} \tag{8}$$

For the Yukawa interactions of the down-type quarks and charged leptons, one of the possible effective Lagrangians [15] is given by

$$\mathcal{L}_{\text{down}} = \frac{i\lambda_d}{2\sqrt{2}}f\epsilon_{ij}\epsilon_{xyz}[(\overline{\Psi}_2')_x \Sigma_{iy}\Sigma_{jz}X - (\overline{\Psi}_1'\Sigma_0)_x \tilde{\Sigma}_{iy}\tilde{\Sigma}_{jz}\tilde{X}]d_R, \tag{9}$$

where $\Psi_1' = (-\sigma_2 q_1, 0, 0_2)^T$, $\Psi_2' = (0_2, 0, -\sigma_2 q_2)^T$, i, $j = 1, 2$ and $x, y, z = 3, 4, 5$. Here X transforms into $\tilde{X}$ under T parity, and it is a singlet under $SU(2)_i(i = 1 - 2)$ and its $U(1)_i(i = 1 - 2)$ charges are $(Y_1, Y_2) = (1/10, -1/10)$. Usually, there are two possible choices for X: $X = (\Sigma_{33})^{-1/4}$ (denoted as Case A) and $X = (\Sigma_{33}^\dagger)^{1/4}$ (denoted as Case B), where Σ_{33} is the (3, 3) component of the nonlinear sigma model field Σ. At order $\mathcal{O}(v_{SM}^4/f^4)$, the corresponding corrections to the Higgs couplings with respect to their SM values are given by ($d \equiv d, s, b, \ell_i^\pm$)

$$\frac{g_{h\bar{d}d}}{g_{h\bar{d}d}^{SM}} = 1 - \frac{1}{4}\frac{v_{SM}^2}{f^2} + \frac{7}{32}\frac{v_{SM}^4}{f^4} \qquad \text{Case A}$$

$$\frac{g_{h\bar{d}d}}{g_{h\bar{d}d}^{SM}} = 1 - \frac{5}{4}\frac{v_{SM}^2}{f^2} - \frac{17}{32}\frac{v_{SM}^4}{f^4} \qquad \text{Case B}. \tag{10}$$

III. BRANCHING RATIO FOR $h \to \mu\tau$ IN THE LHT MODEL

In the LHT model, the relevant Feynman diagrams of the decay $h \to \mu\tau$ at one-loop level in unitary gauge are shown in Fig. 1, where the Goldstone bosons do not appear. We can see that the flavor violating interactions between SM charged leptons and mirror leptons are mediated by the heavy gauge bosons A_H, Z_H, $W_H^\pm$. According to our calculation, we find that the contributions of the self-energy diagram and the contributions of the vertex diagram are at different order, i.e. $\Gamma_{\text{vertex}} \propto \mathcal{O}(v^2/f^2)\Gamma_{\text{self}}$. To be clear, we show the relevant Feynman rules and the explicit expressions of the $h \to \mu\bar{\tau}$ invariant amplitudes in Appendix A and Appendix B, respectively. This implies that the dominant contributions come from the self-energy diagrams, so we can calculate the invariant amplitude up to $\mathcal{O}(v/f)$ and ignore the contributions of the vertex diagram. We checked the divergence in the self-energy diagrams and found that the divergent terms have been canceled. Each loop diagram is composed of some scalar loop functions [16], which are calculated by using LoopTools [17].

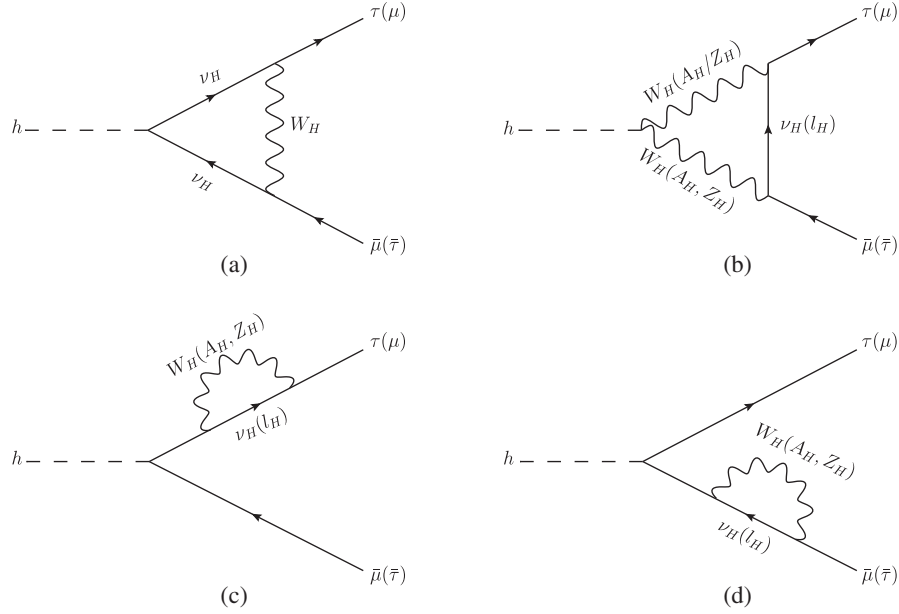In our numerical calculations, the SM parameters are taken as follows [18]

FIG. 1.   Feynman diagrams of the decay $h \to \mu^{\pm}\tau^{\mp}$ at one-loop level in unitary gauge. (a),(b) are vertex diagrams and (c),(d) are self-energy diagrams.

$$\sin^2\theta_W = 0.231, \qquad \alpha_e = 1/128, \qquad M_Z = 91.1876 \text{ GeV},$$

$$m_\mu = 105.66 \text{ MeV}, \qquad m_\tau = 1776.82 \text{ MeV},$$

$$m_h = 125 \text{ GeV}. \tag{11}$$

The LHT parameters related to our calculations are the scale f, the Yukawa couplings κ_i of the mirror neutrinos and the parameters in the matrices $V_{H\ell}$, $V_{H\nu}$. For the mirror neutrino masses, we assume

$$m_{\ell_H^1} = m_{\ell_H^2} = m_{\nu_H^1} = m_{\nu_H^2} = M_{12} = \sqrt{2}\kappa_{12}f,$$

$$m_{\ell_H^3} = m_{\nu_H^3} = M_3 = \sqrt{2}\kappa_3 f. \tag{12}$$

For the Yukawa couplings, the search for the monojet events at the LHC Run-1[19] give the constraint $\kappa_i \geq 0.6$. Considering the constraints in Ref. [20], we scan over the free parameters f, κ_{12} and κ_3 within the following region

$$500 \text{ GeV} \leq f \leq 2000 \text{ GeV}, \qquad 0.6 \leq \kappa_{12} \leq 3,$$

$$0.6 \leq \kappa_3 \leq 3.$$

For the parameters in the matrices $V_{H\nu}$, $V_{H\ell}$, we follow Ref. [21] to consider the two scenarios as follows:
  (i) Scenario I: $V_{H\nu} = I$, $V_{H\ell} = V^{\dagger}_{\text{PMNS}}$;
  (ii) Scenario II: $V_{H\ell} = V_{\text{CKM}}$.
where the PMNS matrix [22] and CKM matrix [18] are given by

$$V_{\text{PMNS}} = \begin{pmatrix} 0.822^{+0.010}_{-0.011} & 0.547^{+0.016}_{-0.015} & 0.155 \pm 0.008 \\ 0.451^{+0.014}_{-0.014} & 0.648^{+0.012}_{-0.014} & 0.614^{+0.019}_{-0.017} \\ 0.347^{+0.016}_{-0.014} & 0.529^{+0.015}_{-0.014} & 0.774^{+0.013}_{-0.015} \end{pmatrix}, \tag{13}$$

$$V_{\text{CKM}} = \begin{pmatrix} 0.97427 \pm 0.00014 & 0.22536 \pm 0.00061 & 0.00355 \pm 0.00015 \\ 0.22522 \pm 0.00061 & 0.97344 \pm 0.00015 & 0.0414 \pm 0.0012 \\ 0.00886^{+0.00033}_{-0.00032} & 0.0405^{+0.0011}_{-0.0012} & 0.99914 \pm 0.00005 \end{pmatrix}. \tag{14}$$

Furthermore, we will consider the constraint from the global fit of the current Higgs data and the electroweak precision observables (EWPOs) [23] as shown in Fig. 2.

In Fig. 3, we show the branching ratios of $h \to \mu\tau$ in the $\kappa_3 \sim f$ plane for two scenarios with excluded regions of Case A and Case B, where the $h \to \mu\bar{\tau}$ and $h \to \tau\bar{\mu}$ modes have been summed. From the left panel of Fig. 4, we can

see that the branching ratio of $h \to \mu\tau$ in scenario I can reach about 2×10^{-4} at 2σ level for Case A, which will become larger for Case B. From the right panel of Fig. 3, we can see that the branching ratio of $h \to \mu\tau$ in scenario II can reach over 4×10^{-7} at 2σ level, which is three orders of magnitude smaller than the one in scenario I. We can see that the behaviors for two scenarios are very different due to
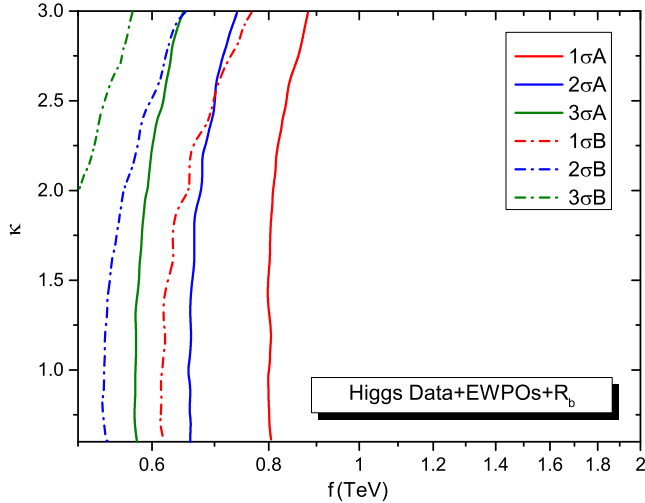
FIG. 2. Excluded regions (above each contour) in the $\kappa \sim f$ plane of the LHT model for Case A and Case B, where the parameter R is marginalized over. The solid (dash) lines from right to left respectively correspond to 1σ, 2σ, and 3σ exclusion limits for Case A(Case B).

the different selection of the matrix $V_{H\ell}$. From the two panels of Fig. 3, we can see that the large branching ratios mainly lie in the upper-left corner for scenario I and upper-left or lower-left corners for scenario II, where the scale f is small and the Yukawa coupling κ_3 is either too large or too small.

In Fig. 4, we show the branching ratios of $h \to \mu\tau$ in the $|M_3 - M_{12}| \sim f$ plane for two scenarios, respectively. We can see that the branching ratio of $h \to \mu\tau$ is insensitive to the mass splitting $|M_3 - M_{12}|$ values for scenario I. The largest branching ratios lie in the region of the contour figure with small f and $|M_3 - M_{12}|$ of 0–2 TeV. For

scenario II, we can see that the branching ratio of $h \to \mu\tau$ is enhanced by the increasing mass splitting $|M_3 - M_{12}|$. The largest branching ratios lie in the upper-left of the contour figure with small f and $|M_3 - M_{12}|$ of 1–2 TeV.

IV. CORRELATION BETWEEN $h \to \mu\tau$, $\tau \to \mu\gamma$, AND $Z \to \mu\tau$

The upper limits on the LFV processes $\tau \to \mu\gamma$ and $Z \to \mu\tau$ are set: $\mathrm{Br}(\tau \to \mu\gamma) < 4.4 \times 10^{-8}$ [24], $\mathrm{Br}(Z \to \mu\tau) < 1.69 \times 10^{-5}$ [25], which may further strengthen the bounds on the branching ratios of $h \to \mu\tau$.

In Fig. 5, we show the correlation between $\mathrm{Br}(h \to \mu\tau)$ and $\mathrm{Br}(\tau \to \mu\gamma)$ in two scenarios. The $\mathrm{Br}(\tau \to \mu\gamma)$ can easily be obtained from Eq. (3.21) in Ref. [26], where we take $\mathrm{Br}(\tau^- \to \nu_\tau \mu^- \bar{\nu}_\mu) = (17.41 \pm 0.04)\%$, and few other studies on such processes can be found in Ref. [27]. In Scenario I, we can see that a minority of points is outside the allowed range, which implies that the $V_{H\ell}$ matrix must be more hierarchical than V_{PMNS} in order to satisfy the present upper bounds on $h \to \mu\tau$ and $\tau \to \mu\gamma$. In Scenario II, we can see that all the points are in allowed range, this is because V_{CKM} matrix is much more hierarchical than V_{PMNS}.

In Fig. 6, we show the correlation between $\mathrm{Br}(h \to \mu\tau)$ and $\mathrm{Br}(Z \to \mu\tau)$ in two scenarios. The partial Z decay width $\Gamma(Z \to \mu\tau)$ can be calculated by using LoopTools, the relevant Feynman diagrams can be found in Ref. [28]. In Scenario I, we can see that all the points violate the current experimental bounds and the great majority of points exceed $\mathcal{O}(10^{-4})$. This will require that the $V_{H\ell}$ matrix must be more hierarchical than V_{PMNS}, unless the mirror lepton masses are quasidegenerate. In Scenario II, we can

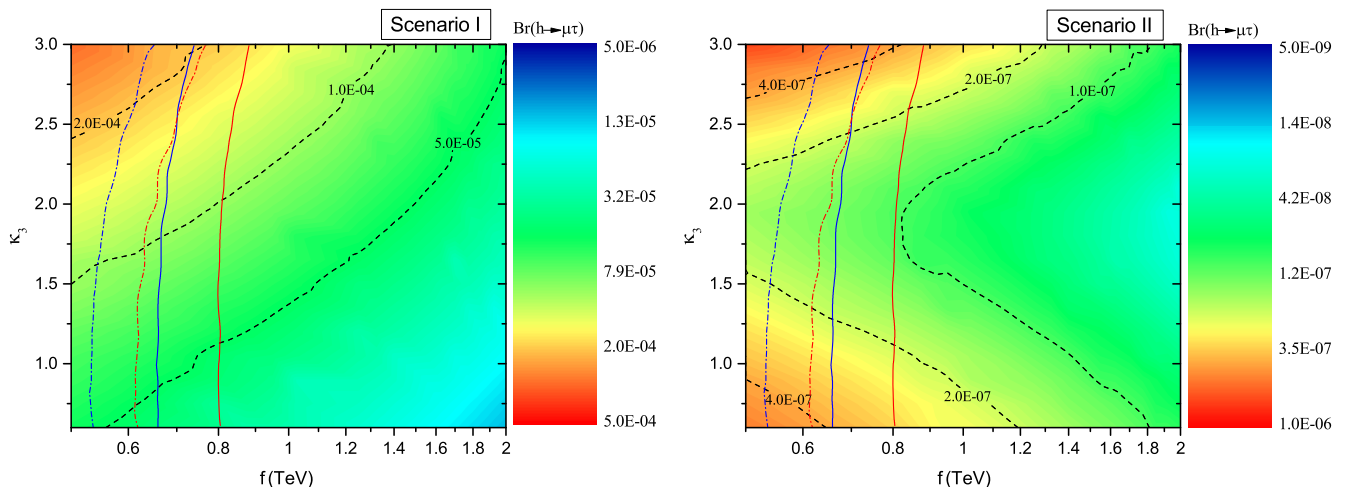FIG. 3. Branching ratios of $h \to \mu\tau$ in the $\kappa_3 \sim f$ plane for two scenarios with excluded regions of Case A and Case B, respectively. The red lines and blue lines respectively correspond to 1σ and 2σ exclusion limits as shown in Fig. 2.
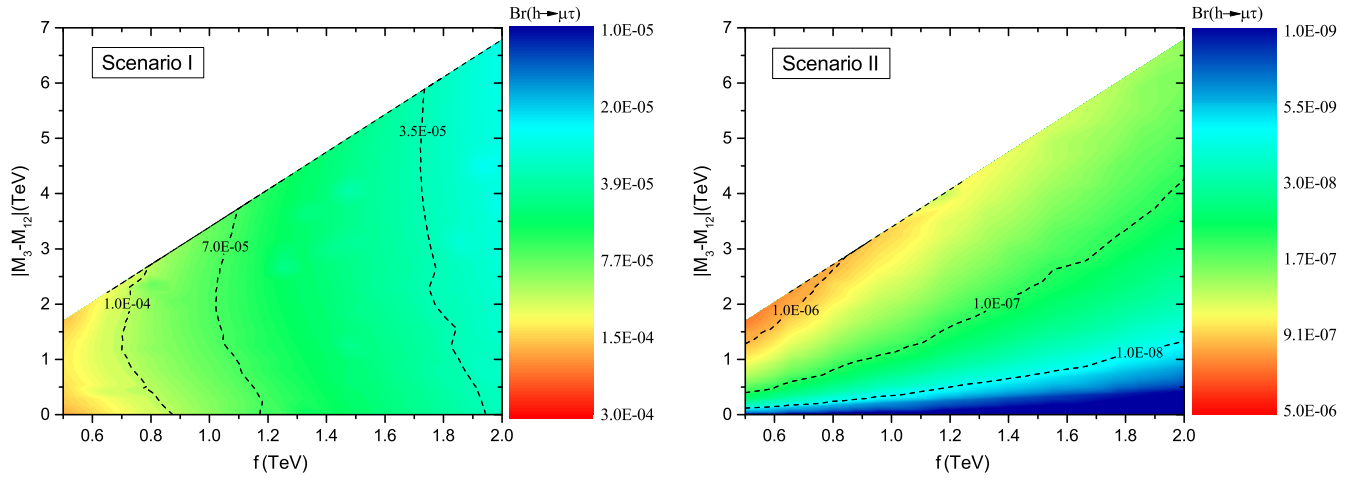
FIG. 4.   Branching ratios of $h \to \mu\tau$ in the $|M_3 - M_{12}| \sim f$ plane for two scenarios.

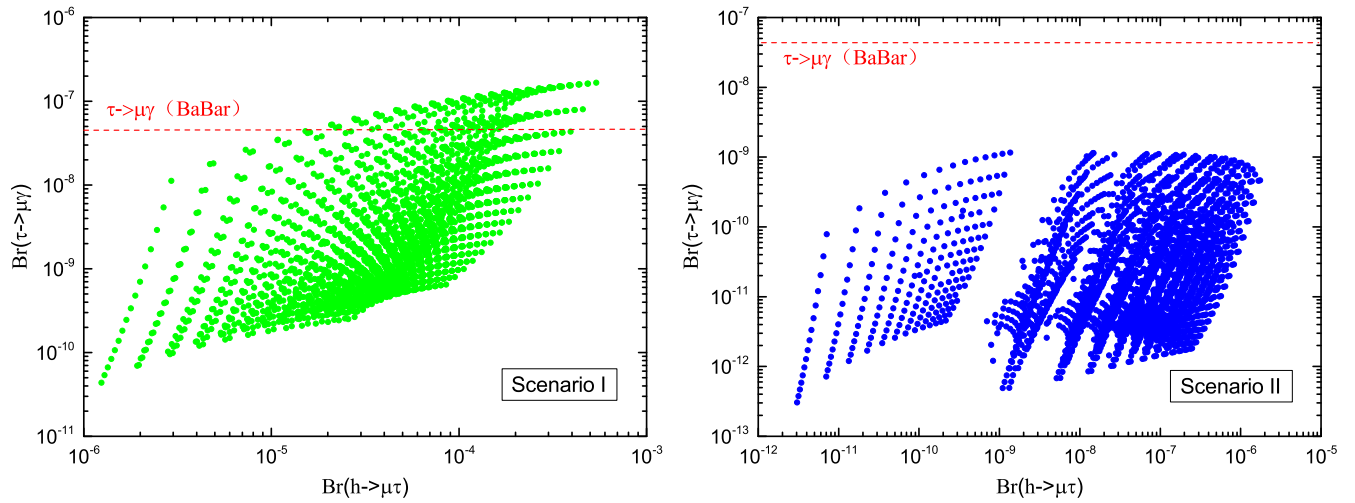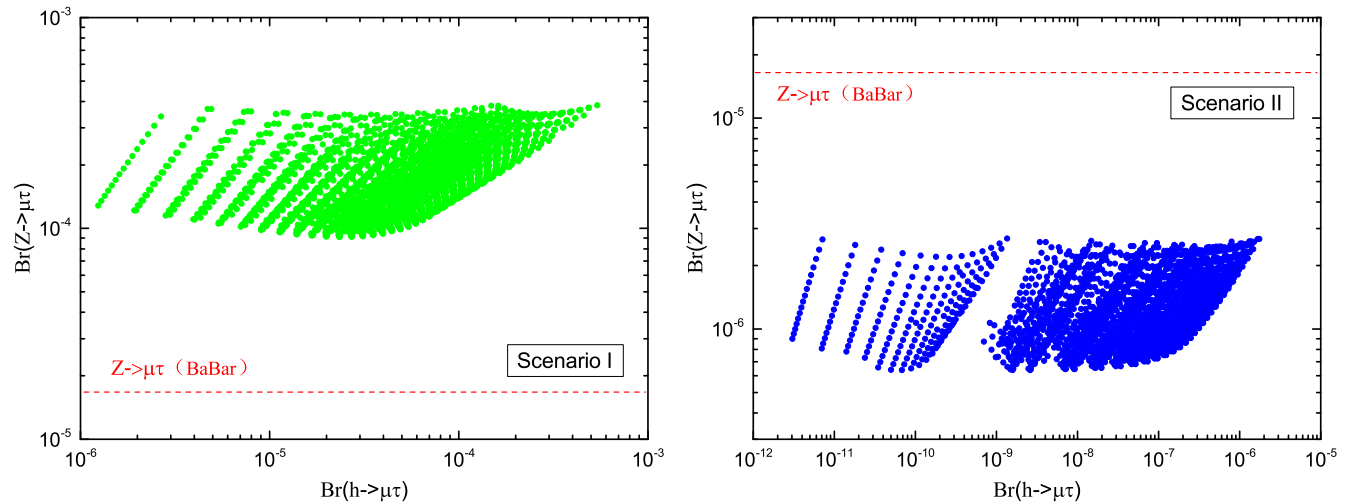FIG. 5.   Correlation between $\mathrm{Br}(h \to \mu\tau)$ and $\mathrm{Br}(\tau \to \mu\gamma)$ for two scenarios.

FIG. 6.   Correlation between $\mathrm{Br}(h \to \mu\tau)$ and $\mathrm{Br}(Z \to \mu\tau)$ for two scenarios.

see that all the points satisfy the current upper bounds due to the large hierarchy of the V_{CKM} matrix.

V. CONCLUSIONS

In this paper, we calculated LFV Higgs decay $h \to \mu\tau$ at one-loop level in the LHT model. According to the parameters in the mixing matrices, we considered two scenarios and found that the branching ratios for $h \to \mu\tau$ can respectively reach $\mathcal{O}(10^{-4})$ and $\mathcal{O}(10^{-7})$ under the current experimental constraints. We also investigated the correlation between $h \to \mu\tau$, $\tau \to \mu\gamma$, and $Z \to \mu\tau$, and found that the $Z \to \mu\tau$ can give a substantial constraint on the $h \to \mu\tau$, which required that the $V_{H\ell}$ matrix must have a very different hierarchy from V_{PMNS} matrix.

ACKNOWLEDGMENTS

APPENDIX A: FEYNMAN RULES

Interaction	Feynman rule	Interaction	Feynman rule
$H\bar{\nu}_H^j \nu_H^j$	$i\frac{m_{\nu_H^j}}{v}\frac{v^2}{4f^2}$	$HA_{H\alpha}A_{H\beta}$	$-\frac{i}{2}g'^2 v g_{\alpha\beta}$
$\bar{\ell}_H^j A_{H\alpha}\ell^k$	$-\frac{ig'}{10}(V_{H\ell})_{jk}\gamma_\alpha P_L$	$HZ_{H\alpha}Z_{H\beta}$	$-\frac{i}{2}g^2 v g_{\alpha\beta}$
$\bar{\ell}_H^j Z_{H\alpha}\ell^k$	$\frac{ig}{2}(V_{H\ell})_{jk}\gamma_\alpha P_L$	$HA_{H\alpha}Z_{H\beta}$	$-\frac{i}{2}g' g v g_{\alpha\beta}$
$\bar{\nu}_H^j W_{H\alpha}^+ \ell^k$	$-\frac{ig}{\sqrt{2}}(V_{H\ell})_{jk}\gamma_\alpha P_L$	$HW_{H\alpha}^+ W_{H\beta}^-$	$-\frac{i}{2}g^2 v g_{\alpha\beta}$

APPENDIX B: THE EXPRESSION OF THE $h \to \mu\bar{\tau}$ INVARIANT AMPLITUDES

They can be represented in form of 1-point, 2-point, and 3-point standard functions A, B_0, B_1, C_{ij}. Here the momenta p_μ, p_τ are assumed to be outgoing:

(1) The vertex diagram contribution:

$$\Gamma_{(a)}^{W_H \nu_H} = -\frac{g^2}{2}(V_{H\ell})_{j2}(V_{H\ell})_{j3}^* \frac{m_{\nu_H^j}^2 v}{4f^2}\frac{i}{16\pi^2}\left[4C_\beta\gamma^\beta + 2C_0(\not{p}_\mu - \not{p}_\tau) + \frac{2}{m_{W_H}^2}\tilde{C}_\beta\gamma^\beta + \frac{1}{m_{W_H}^2}C_{\beta\alpha}\gamma^\beta(\not{p}_\mu - \not{p}_\tau)\gamma^\alpha\right]P_L$$

$$C_\beta = C_\beta(p_\mu, -p_h, m_{W_H}, m_{\nu_H^j}, m_{\nu_H^j}),$$

$$\tilde{C}_\beta = p_{\mu\beta}[-B_0(-p_h, m_{\nu_H^j}, m_{\nu_H^j}) + m_{W_H}^2 C_{11}] - p_{h\beta}[B_1(-p_h, m_{\nu_H^j}, m_{\nu_H^j}) + m_{W_H}^2 C_{12}] \tag{B1}$$

$$\Gamma_{(b)}^{W_H W_H} = \frac{g^2}{2}\frac{g^2 v}{2}(V_{H\ell})_{j2}(V_{H\ell})_{j3}^* \frac{i}{16\pi^2}\left[2C_\beta\gamma^\beta + \frac{2}{m_{W_H}^2}\tilde{C}_\beta\gamma^\beta - \frac{m_{\nu_H^j}^4}{m_{W_H}^4}C_\beta\gamma^\beta - \frac{1}{m_{W_H}^4}(\tilde{B}_\beta\gamma^\beta - \gamma^\beta\not{p}_\mu\gamma^\nu B_{\mu\nu} + m_{\nu_H^j}^2 B_\beta\gamma^\beta)\right]P_L$$

$$B_\beta = B_\beta(-p_h, m_{W_H}, m_{W_H}), \quad \tilde{B}_\beta = -p_{h\beta}[m_{W_H}^2 B_1 - A_0(m_{W_H})],$$

$$C_\beta = C_\beta(p_\mu, -p_h, m_{\nu_H^j}, m_{W_H}, m_{W_H}),$$

$$\tilde{C}_\beta = p_{\mu\beta}[-B_0(-p_h, m_{W_H}, m_{W_H}) + m_{\nu_H^j}^2 C_{11}] - p_{h\beta}[B_1(-p_h, m_{W_H}, m_{W_H}) + m_{\nu_H^j}^2 C_{12}] \tag{B2}$$

$$\Gamma_{(b)}^{A_H A_H} = \frac{g'^2}{100}\frac{g'^2 v}{2}(V_{H\ell})_{j2}(V_{H\ell})_{j3}^* \frac{i}{16\pi^2}\left[2C_\beta\gamma^\beta + \frac{2}{m_{A_H}^2}\tilde{C}_\beta\gamma^\beta - \frac{m_{\ell_H^j}^4}{m_{A_H}^4}C_\beta\gamma^\beta - \frac{1}{m_{A_H}^4}(\tilde{B}_\beta\gamma^\beta - \gamma^\beta\not{p}_\mu\gamma^\nu B_{\mu\nu} + m_{\ell_H^j}^2 B_\beta\gamma^\beta)\right]P_L$$

$$B_\beta = B_\beta(-p_h, m_{A_H}, m_{A_H}), \quad \tilde{B}_\beta = -p_{h\beta}[m_{A_H}^2 B_1 - A_0(m_{A_H})],$$

$$C_\beta = C_\beta(p_\mu, -p_h, m_{\ell_H^j}, m_{A_H}, m_{A_H}),$$

$$\tilde{C}_\beta = p_{\mu\beta}[-B_0(-p_h, m_{A_H}, m_{A_H}) + m_{\ell_H^j}^2 C_{11}] - p_{h\beta}[B_1(-p_h, m_{A_H}, m_{A_H}) + m_{\ell_H^j}^2 C_{12}] \tag{B3}$$

$$\Gamma_{(b)}^{A_H Z_H} = \frac{g'^2}{20} \frac{g^2 v}{2} (V_{H\ell})_{j2}(V_{H\ell})^*_{j3} \frac{i}{16\pi^2} \left[-2C_\beta \gamma^\beta - \frac{1}{m_{A_H}^2} \tilde{C}_\beta \gamma^\beta - \frac{1}{m_{Z_H}^2} \tilde{C}_\beta \gamma^\beta + \frac{m_{\ell_H^j}^4}{m_{A_H}^2 m_{Z_H}^2} C_\beta \gamma^\beta \right.$$

$$\left. + \frac{1}{m_{A_H}^2 m_{Z_H}^2} (\tilde{B}_\beta \gamma^\beta - \gamma^\beta \not{p}_\mu \gamma^\nu B_{\mu\nu} + m_{\ell_H^j}^2 B_\beta \gamma^\beta) \right] P_L B_\beta = B_\beta(-p_h, m_{A_H}, m_{Z_H}),$$

$$\tilde{B}_\beta = -p_{h\beta}[m_{A_H}^2 B_1 - A_0(m_{Z_H})], \qquad C_\beta = C_\beta(p_\mu, -p_h, m_{\ell_H^j}, m_{A_H}, m_{Z_H}),$$

$$\tilde{C}_\beta = p_{\mu\beta}[-B_0(-p_h, m_{A_H}, m_{Z_H}) + m_{\ell_H^j}^2 C_{11}] - p_{h\beta}[B_1(-p_h, m_{A_H}, m_{Z_H}) + m_{\ell_H^j}^2 C_{12}] \tag{B4}$$

$$\Gamma_{(b)}^{Z_H A_H} = \frac{g'^2}{20} \frac{g^2 v}{2} (V_{H\ell})_{j2}(V_{H\ell})^*_{j3} \frac{i}{16\pi^2}$$

$$\times \left[-2C_\beta \gamma^\beta - \frac{1}{m_{A_H}^2} \tilde{C}_\beta \gamma^\beta - \frac{1}{m_{Z_H}^2} \tilde{C}_\beta \gamma^\beta + \frac{m_{\ell_H^j}^4}{m_{A_H}^2 m_{Z_H}^2} C_\beta \gamma^\beta + \frac{1}{m_{A_H}^2 m_{Z_H}^2} (\tilde{B}_\beta \gamma^\beta - \gamma^\beta \not{p}_\mu \gamma^\nu B_{\mu\nu} + m_{\ell_H^j}^2 B_\beta \gamma^\beta) \right] P_L$$

$$B_\beta = B_\beta(-p_h, m_{Z_H}, m_{A_H}), \tilde{B}_\beta = -p_{h\beta}[m_{Z_H}^2 B_1 - A_0(m_{A_H})], \qquad C_\beta = C_\beta(p_\mu, -p_h, m_{\ell_H^j}, m_{Z_H}, m_{A_H}),$$

$$\tilde{C}_\beta = p_{\mu\beta}[-B_0(-p_h, m_{Z_H}, m_{A_H}) + m_{\ell_H^j}^2 C_{11}] - p_{h\beta}[B_1(-p_h, m_{Z_H}, m_{A_H}) + m_{\ell_H^j}^2 C_{12}] \tag{B5}$$

$$\Gamma_{(b)}^{Z_H Z_H} = \frac{g^2}{4} \frac{g^2 v}{2} (V_{H\ell})_{j2}(V_{H\ell})^*_{j3} \frac{i}{16\pi^2}$$

$$\times \left[2C_\beta \gamma^\beta + \frac{2}{m_{Z_H}^2} \tilde{C}_\beta \gamma^\beta - \frac{m_{\ell_H^j}^4}{m_{Z_H}^4} C_\beta \gamma^\beta - \frac{1}{m_{Z_H}^4} (\tilde{B}_\beta \gamma^\beta - \gamma^\beta \not{p}_\mu \gamma^\nu B_{\mu\nu} + m_{\ell_H^j}^2 B_\beta \gamma^\beta) \right] P_L$$

$$B_\beta = B_\beta(-p_h, m_{Z_H}, m_{Z_H}), \tilde{B}_\beta = -p_{h\beta}[m_{Z_H}^2 B_1 - A_0(m_{Z_H})], \qquad C_\beta = C_\beta(p_\mu, -p_h, m_{\ell_H^j}, m_{Z_H}, m_{Z_H}),$$

$$\tilde{C}_\beta = p_{\mu\beta}[-B_0(-p_h, m_{Z_H}, m_{Z_H}) + m_{\ell_H^j}^2 C_{11}] - p_{h\beta}[B_1(-p_h, m_{Z_H}, m_{Z_H}) + m_{\ell_H^j}^2 C_{12}] \tag{B6}$$

(2) The self-energy diagram contribution:

$$\Gamma_{(c)}^{\mu A_H} = \frac{g'^2}{100} \frac{m_\mu}{v} (V_{H\ell})_{j2}(V_{H\ell})^*_{j3} \frac{1}{p_\tau^2 - m_\mu^2} \frac{i}{16\pi^2} \left(2m_\mu B_\beta \gamma^\beta P_L + \frac{m_\mu}{m_{A_H}^2} \tilde{B}_\beta \gamma^\beta P_L - 2\not{p}_\tau B_\beta \gamma^\beta P_L - \frac{1}{m_{A_H}^2} \not{p}_\tau \tilde{B}_\beta \gamma^\beta P_L \right)$$

$$B_\beta = B_\beta(p_\tau, m_{\ell_H^j}, m_{A_H}), \qquad \tilde{B}_\beta = p_{\tau\beta}[m_{\ell_H^j}^2 B_1 - A_0(m_{A_H})] \tag{B7}$$

$$\Gamma_{(c)}^{\mu Z_H} = \frac{g^2}{4} \frac{m_\mu}{v} (V_{H\ell})_{j2}(V_{H\ell})^*_{j3} \frac{1}{p_\tau^2 - m_\mu^2} \frac{i}{16\pi^2} \left(2m_\mu B_\beta \gamma^\beta P_L + \frac{m_\mu}{m_{Z_H}^2} \tilde{B}_\beta \gamma^\beta P_L - 2\not{p}_\tau B_\beta \gamma^\beta P_L - \frac{1}{m_{Z_H}^2} \not{p}_\tau \tilde{B}_\beta \gamma^\beta P_L \right)$$

$$B_\beta = B_\beta(p_\tau, m_{\ell_H^j}, m_{Z_H}), \qquad \tilde{B}_\beta = p_{\tau\beta}[m_{\ell_H^j}^2 B_1 - A_0(m_{Z_H})] \tag{B8}$$

$$\Gamma_{(c)}^{\mu W_H} = \frac{g^2}{2} \frac{m_\mu}{v} (V_{H\ell})_{j2}(V_{H\ell})^*_{j3} \frac{1}{p_\tau^2 - m_\mu^2} \frac{i}{16\pi^2} \left(2m_\mu B_\beta \gamma^\beta P_L + \frac{m_\mu}{m_{W_H}^2} \tilde{B}_\beta \gamma^\beta P_L - 2\not{p}_\tau B_\beta \gamma^\beta P_L - \frac{1}{m_{W_H}^2} \not{p}_\tau \tilde{B}_\beta \gamma^\beta P_L \right)$$

$$B_\beta = B_\beta(p_\tau, m_{\nu_H^j}, m_{W_H}), \qquad \tilde{B}_\beta = p_{\tau\beta}[m_{\nu_H^j}^2 B_1 - A_0(m_{W_H})] \tag{B9}$$

$$\Gamma_{(d)}^{\tau A_H} = \frac{g'^2}{100} \frac{m_\tau}{v} (V_{H\ell})_{j2}(V_{H\ell})^*_{j3} \frac{1}{p_\mu^2 - m_\tau^2} \frac{i}{16\pi^2} \left(2m_\tau B_\beta \gamma^\beta P_L + \frac{m_\tau}{m_{A_H}^2} \tilde{B}_\beta \gamma^\beta P_L + 2B_\beta \gamma^\beta \not{p}_\mu P_R + \frac{1}{m_{A_H}^2} \tilde{B}_\beta \gamma^\beta \not{p}_\mu P_R \right)$$

$$B_\beta = B_\beta(-p_\mu, m_{\ell_H^j}, m_{A_H}), \qquad \tilde{B}_\beta = -p_{\mu\beta}[m_{\ell_H^j}^2 B_1 - A_0(m_{A_H})] \tag{B10}$$

$$\Gamma_{(d)}^{\tau Z_H} = \frac{g^2}{4} \frac{m_\tau}{v} (V_{H\ell})_{j2} (V_{H\ell})_{j3}^* \frac{1}{p_\mu^2 - m_\tau^2} \frac{i}{16\pi^2} \left(2m_\tau B_\beta \gamma^\beta P_L + \frac{m_\tau}{m_{Z_H}^2} \tilde{B}_\beta \gamma^\beta P_L + 2B_\beta \gamma^\beta \not{p}_\mu P_R + \frac{1}{m_{Z_H}^2} \tilde{B}_\beta \gamma^\beta \not{p}_\mu P_R \right)$$

$$B_\beta = B_\beta(-p_\mu, m_{\ell_H^j}, m_{Z_H}), \tilde{B}_\beta = -p_{\mu\beta}[m_{\ell_H^j}^2 B_1 - A_0(m_{Z_H})] \tag{B11}$$

$$\Gamma_{(d)}^{\tau W_H} = \frac{g^2}{2} \frac{m_\tau}{v} (V_{H\ell})_{j2} (V_{H\ell})_{j3}^* \frac{1}{p_\mu^2 - m_\tau^2} \frac{i}{16\pi^2} \left(2m_\tau B_\beta \gamma^\beta P_L + \frac{m_\tau}{m_{W_H}^2} \tilde{B}_\beta \gamma^\beta P_L + 2B_\beta \gamma^\beta \not{p}_\mu P_R + \frac{1}{m_{W_H}^2} \tilde{B}_\beta \gamma^\beta \not{p}_\mu P_R \right)$$

$$B_\beta = B_\beta(-p_\mu, m_{\nu_H^j}, m_{W_H}), \qquad \tilde{B}_\beta = -p_{\mu\beta}[m_{\nu_H^j}^2 B_1 - A_0(m_{W_H})] \tag{B12}$$

[1] G. Aad *et al.* (ATLAS Collaboration), Phys. Lett. B **710**, 49 (2012); S. Chatrachyan *et al.* (CMS Collaboration), Phys. Lett. B **710**, 26 (2012).

[2] See examples, A. Falkowski and R. Vega-Morales, J. High Energy Phys. 12 (2014) 037; D. Curtin *et al.*, Phys. Rev. D **90**, 075004 (2014); C. Han, N. Liu, L. Wu, J. M. Yang, and Y. Zhang, Eur. Phys. J. C **73**, 2664 (2013); J. Cao, L. Wu, P. Wu, and J. M. Yang, J. High Energy Phys. 09 (2013) 043; J. Huang, T. Liu, L. T. Wang, and F. Yu, Phys. Rev. D **90**, 115006 (2014); J. Cao, C. Han, L. Wu, J. M. Yang, and M. Zhang, Eur. Phys. J. C **74**, 3058 (2014); C. Han, X. Ji, L. Wu, P. Wu, and J. M. Yang, J. High Energy Phys. 04 (2014) 003; S. L. Hu, N. Liu, J. Ren, and L. Wu, J. Phys. G **41**, 125004 (2014); L. Wu, J. High Energy Phys. 02 (2015) 061; D. Curtin and C. B. Verhaaren, J. High Energy Phys. 12 (2015) 072; L. Wu, J. M. Yang, C. P. Yuan, and M. Zhang, Phys. Lett. B **747**, 378 (2015); A. Kobakhidze, L. Wu, and J. Yue, J. High Energy Phys. 10 (2014) 100; J. High Energy Phys. 04 (2016) 011; H. Blusca-Maïto and A. Falkowski, Eur. Phys. J. C **76**, 514 (2016).

[3] V. Khachatryan *et al.* (CMS Collaboration), Phys. Lett. B **749**, 337 (2015).

[4] G. Aad *et al.* (ATLAS Collaboration), J. High Energy Phys. 11 (2015) 211.

[5] E. Arganda, M. J. Herrero, X. Marcano, and C. Weiland, Phys. Rev. D **91**, 015001 (2015).

[6] J. Cao, L. Wu, and J. M. Yang, Nucl. Phys. **B829**, 370 (2010); E. Arganda, M. J. Herrero, X. Marcano, and C. Weiland, Phys. Rev. D **93**, 055010 (2016); E. Arganda, M. J. Herrero, R. Morales, and A. Szynkman, J. High Energy Phys. 03 (2016) 055; A. Hammad, S. Khalil, and C. Un, arXiv:1605.07567.

[7] A. Crivellin, G. D'Ambrosio, and J. Heeck, Phys. Rev. Lett. **114**, 151801 (2015); N. Bizot, S. Davidson, M. Frigerio, and J. L. Kneur, J. High Energy Phys. 03 (2016) 073; F. J. Botella, G. C. Branco, M. Nebot, and M. N. Rebelo, Eur. Phys. J. C **76**, 161 (2016); D. Das and A. Kundu, Phys. Rev. D **92**, 015009 (2015).

[8] L. T. Hue, H. N. Long, T. T. Thuc, and T. Phong Nguyen, Nucl. Phys. **B907**, 37 (2016).

[9] S. Baek and Z.-F. Kang, J. High Energy Phys. 03 (2016) 106; S. Baek and K. Nishiwaki, Phys. Rev. D **93**, 015002

(2016); K. Cheung, W. Y. Keung, and P. Y. Tseng, Phys. Rev. D **93**, 015010 (2016); W. Altmannshofer, S. Gori, A. L. Kagan, L. Silvestrini, and J. Zupan, Phys. Rev. D **93**, 031301 (2016); X. G. He, J. Tandean, and Y. J. Zheng, J. High Energy Phys. 09 (2015) 093; I. Dorsner, S. Fajfer, A. Greljo, J. F. Kamenik, N. Kosnik, and Ivan Nisandzic, J. High Energy Phys. 06 (2015) 108; A. Crivellin, G. D'Ambrosio, and J. Heeck, Phys. Rev. D **91**, 075006 (2015); L. D. Lima, C. S. Machado, R. D. Matheus, and L. A. F. D. Prado, J. High Energy Phys. 11 (2015) 074; I. d. M. Varzielas, O. Fischer, and V. Maurer, J. High Energy Phys. 08 (2015) 080; C. F. Chang, C. H. V. Chang, C. S. Nugroho, and T. C. Yuan, Nucl. Phys. **B910**, 293 (2016); C. H. Chen and T. Nomura, Eur. Phys. J. C **76**, 353 (2016); K. Huitu, V. Keus, N. Koivunen, and O. Lebedev, J. High Energy Phys. 05 (2016) 026; M. Sher and K. Thrasher, Phys. Rev. D **93**, 055021 (2016).

[10] S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D **2**, 1285 (1970).

[11] A. Goyal, S. R. Choudhury, A. S. Cornell, A. Deandrea, N. Gaur, and A. Goyal, Phys. Rev. D **75**, 055011 (2007).

[12] H. C. Cheng and I. Low, J. High Energy Phys. 09 (2003) 051; 08 (2004) 061; I. Low, J. High Energy Phys. 10 (2004) 067; J. Hubisz and P. Meade, Phys. Rev. D **71**, 035016 (2005).

[13] J. Hubisz, S. J. Lee, and G. Paz, J. High Energy Phys. 06 (2006) 041.

[14] M. Blanke, A. J. Buras, A. Poschenrieder, S. Recksiegel, C. Tarantino, S. Uhlig, and A. Weiler, Phys. Lett. B **646**, 253 (2007).

[15] C. R. Chen, K. Tobe, and C. P. Yuan, Phys. Lett. B **640**, 263 (2006).

[16] G. 't Hooft and M. J. G. Veltman, Nucl. Phys. **B153**, 365 (1979).

[17] T. Hahn and M. Perez-Victoria, Comput. Phys. Commun. **118**, 153 (1999); T. Hahn, Nucl. Phys. B, Proc. Suppl. **135**, 333 (2004).

[18] K. A. Olive *et al.* (Particle Data Group Collaboration), Chin. Phys. C **38**, 090001 (2014).

[19] J. Hubisz, P. Meade, A. Noble, and M. Perelstein, J. High Energy Phys. 01 (2006) 135; M. Perelstein and J. Shao, Phys. Lett. B **704**, 510 (2011); G. Cacciapaglia,

A. Deandrea, J. Ellis, J. Marrouche, and L. Panizzi, Phys. Rev. D **87**, 075006 (2013); J. Reuter, M. Tonini, and M. de Vries, J. High Energy Phys. 02 (2014) 053; J. Reuter and M. Tonini, J. High Energy Phys. 02 (2013) 077.

[20] B. F. Yang, X. L. Wang, and J. Z. Han, Nucl. Phys. **B847**, 1 (2011); J. Hubisz, P. Meade, A. Noble, and M. Perelstein, J. High Energy Phys. 01 (2006) 135; A. Belyaev, C. R. Chen, K. Tobe, and C. P. Yuan, Phys. Rev. D **74**, 115020 (2006); Q. H. Cao and C. R. Chen, Phys. Rev. D **76**, 075007 (2007); J. Reuter and M. Tonini, J. High Energy Phys. 02 (2013) 077; X. F. Han, L. Wang, J. M. Yang, and J. Y. Zhu, Phys. Rev. D **87**, 055004 (2013); J. Reuter, M. Tonini, and M. de Vries, arXiv:1307.5010; C. C. Han, A. Kobakhidze, N. Liu, L. Wu, and B. F. Yang, Nucl. Phys. **B890**, 388 (2015).

[21] H. S. Hou, Phys. Rev. D **75**, 094010 (2007); J. Z. Han, B. Z. Li, and X. L. Wang, Phys. Rev. D **83**, 034032 (2011).

[22] Y. J. Zhang, X. Y. Zhang, and B.-Q. Ma, Phys. Rev. D **86**, 093019 (2012).

[23] B. F. Yang, G. F. Mi, and N. Liu, J. High Energy Phys. 10 (2014) 47; N. Liu, L. Wu, B. F. Yang, and M. C. Zhang, Phys. Lett. B **753**, 664 (2016); B. F. Yang, Z. Y. Liu, and N. Liu, arXiv:1603.04242.

[24] K. Hayasaka *et al.* (Belle Collaboration), Phys. Lett. B **666**, 16 (2008); B. Aubert *et al.* (*BABAR* collaboration), Phys. Rev. Lett. **104**, 021802 (2010).

[25] ATLAS Collaboration, Report No. CERN-EP-2016-055.

[26] M. Blanke, A. J Buras, B. Duling, A. Poschenrieder, and C. Tarantino, J. High Energy Phys. 05 (2007) 013.

[27] T. Goto, Y. Okada, and Y. Yamamoto, Phys. Rev. D **83**, 053011 (2011).

[28] J. Z. Han, X. L. Wang, and B. F. Yang, Nucl. Phys. **B843**, 383 (2011); C.-X. Yue, J.-Y. Liu, and S.-H. Zhu, Phys. Rev. D **78**, 095006 (2008).