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Stabilization of semilocal strings by dark scalar condensates

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Semilocal and electroweak strings are well known to be unstable against unwinding by the condensation of the second Higgs component in their cores. A large class of current models of dark matter contains dark scalar fields coupled to the Higgs sector of the Standard Model (Higgs portal) and/or dark U(1) gauge fields. It is shown that Higgs-portal-type couplings and a gauge kinetic mixing term of the dark U(1) gauge field have a significant stabilizing effect on semilocal strings in the "visible" sector.

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I. INTRODUCTION

Cosmic strings and their observational signatures have been studied for a long time as they are expected to form in the early Universe [1–5]. Even if by now it seems unlikely that cosmic strings could have significantly contributed to structure formation in the Universe, string-like excitations in the Standard Model (SM) continue to be of great interest not only from a theoretical point of view, but also because such objects may eventually leave observable signatures, e.g., in the Large Hadron Collider [6–8]. Remarkable string solutions have been uncovered in the bosonic sector of the Glashow-Salam-Weinberg (GSW) theory (in this paper we shall refer to a generalization of the electroweak sector of the SM, that allows its parameters to take on nonphysical values, as the GSW theory); for a review see Ref. [6]. A rather interesting class of models emerges by taking the $\theta_{\mathrm{W}} \to \pi/2$ limit of the GSW theory, where θ_{W} denotes the electroweak mixing angle. In this way, one obtains an Abelian Higgs model with an extended scalar sector having an $SU(2)_{\mathrm{global}}$ symmetry acting on the Higgs doublet; this is a prototype of semilocal models. Its string solutions are referred to as semilocal strings [6,9–11] and these are quite instructive to study as they are potentially important objects in the GSW theory. An important criterion for the physical relevance of string-type objects is their classical stability. Semilocal strings turned out to be stable only when the mass of the scalar particle is smaller than that of the (single) gauge boson, as shown in Refs. [10,11]. The stability of electroweak strings (whose progenitors are the semilocal ones) has been considered in Refs. [6,12–15]; it was found that for physically realistic values of θ_{W}, electroweak strings are unstable.

Moreover, there are good reasons to consider extended versions of the GSW theory with a dark sector (DS), motivated by the mystery of dark matter. In such extended models the question of the possible role of strings appears naturally. A minimalistic extension of the GSW theory is to couple a (dark) scalar field to the by now firmly established Higgs sector of the GSW theory (Higgs portal) [16,17], but there are also well-motivated extensions of the GSW theory containing U(1) gauge fields in the DS [18,19]. In Refs. [20–25] physical properties and possible observational signatures of cosmic strings in the DS (dark strings) have been considered. A more detailed investigation of string solutions in Abelian Higgs theories modeling a "visible" and a "dark" U(1) gauge sector was presented in Ref. [26]. In subsequent works [27,28] semilocal-type models with a "visible" and a "dark" U(1) gauge field spontaneously broken in both sectors have been investigated. It has been observed that the stability region of semilocal string solutions with a nonzero winding number in the DS can be extended as a function of the couplings between the visible and the DS. Higher winding vortices in the $\mathrm{U}(1) \times \mathrm{U}(1)$ model and its supersymmetric generalization have been considered in Refs. [29,30]. An earlier work on string solutions in a portal-type theory is Ref. [31]. In all these works only strings with nonzero winding in the DS have been considered, because of the known instabilities of "visible" semilocal strings.

The main goal of the present paper is to complement these studies on dark strings by concentrating on the influence of the DS on "visible" semilocal-type string solutions (i.e., with zero winding in the DS). We consider a $\mathrm{U}(1) \times \mathrm{U}(1)$ Abelian Higgs (AH) model, whose scalar sector consists of a complex Higgs doublet with (global) SU(2) symmetry coupled to a dark scalar field with a $\mathrm{U}(1) \times \mathrm{U}(1)$-symmetric potential, which is a simple generalization of the model of Witten [32]. We use this simplified model to study the effect of the DS on semilocal strings. It is convenient to distinguish between two symmetry-breaking patterns: either both the visible Higgs and the dark scalar field have nonzero vacuum expectation values (2VEV), or there is no symmetry breaking in the DS (1VEV). The 1VEV case is directly relevant to the Higgs portal (scalar phantom) model of Refs. [16,17], whereas when the DS contains gauge fields to model the interaction among the dark matter particles the symmetry must be broken in both the visible and dark sectors (2VEV case) [18–20].

Generically, semilocal strings are unstable with respect to condensation of the dark scalar field at their core [we shall refer to such strings as dark core (DC) ones]. In the absence of the gauge kinetic mixing, the DC strings investigated in the present paper correspond to embeddings of the solutions previously found in Refs. [33–35] into the $SU(2) \times U(1)$-symmetric semilocal model coupled to a DS. When the gauge kinetic mixing is different from zero the string solutions we consider here differ from those of Refs. [27,28] in that our strings have nontrivial winding only in the visible sector. Our main result is the stability of DC strings with respect to small perturbations for a rather large parameter domain.

It has to be pointed out that a number of mechanisms to stabilize semilocal strings have already been investigated. In Ref. [36], a stabilizing effect due to a bound state of an additional scalar field on semilocal and electroweak strings was found. In Ref. [37], it has been shown that a special (dilatonic-type) coupling between the gauge and scalar fields also has a stabilizing effect on semilocal strings.

In the complementary limit of the electroweak theory, $\theta_{\rm W} \to 0$, it has been demonstrated that quantum fluctuations of a heavy fermion doublet coupled to the string can also lead to stabilization in Refs. [38,39]. Stabilization of electroweak strings due to the interaction with thermal photons has been demonstrated in Ref. [40].

The plan of the paper is as follows. In Sec. II we introduce the models considered, followed by the discussion of visible straight string solutions in the 2VEV case and their stability properties in Sec. III. Next we analyze the 1VEV case in Sec. IV. We conclude in Sec. V. Some details have been relegated to various appendices: scalar masses in the 2VEV case to Appendix A, radial equations of vortices to Appendix B, and the linearization of the field equations about the vortices to Appendix C.

II. SIMPLE MODELS OF DARK MATTER

In Refs. [18,19], a unified model of dark matter was presented, which posits a DS with a U(1) gauge symmetry, spontaneously broken in order to avoid long-range interactions. The DS is modeled by an AH model (C_μ, χ), where the dark scalar field, χ, couples to the GSW theory through a Higgs portal coupling [16,17] and the dark gauge field C_μ through a gauge kinetic mixing term [41].

We consider a semilocal model coupled to a DS defined by the Lagrangian[1]

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}H_{\mu\nu}H^{\mu\nu} + \frac{\epsilon}{2}H_{\mu\nu}F^{\mu\nu} + D_\mu\Phi^\dagger D^\mu\Phi + (\tilde{D}_\mu\chi)^*\tilde{D}^\mu\chi - V(\Phi,\chi), \tag{1}$$

where $\Phi=(\phi_1,\phi_2)$, $D_\mu\Phi=(\partial_\mu - iA_\mu)\Phi$, $\tilde{D}_\mu\chi=(\partial_\mu - iqC_\mu)\chi$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and $H_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu$. The potential $V(\Phi,\chi)$ is a slight generalization of that in the Witten model [32],

$$V(\Phi,\chi) = \frac{\beta_1}{2}(|\Phi|^2 - 1)^2 + \frac{\beta_2}{2}|\chi|^4 + \beta'|\Phi|^2|\chi|^2 - \alpha|\chi|^2. \tag{2}$$

The parameters $\beta_1, \beta_2, \beta', \alpha$ are restricted by demanding that $V(\Phi,\chi) > 0$ for $|\Phi|^2, |\chi|^2 \to \infty$, resulting in $\beta_1 > 0, \beta_2 > 0$, and $\beta' > -\sqrt{\beta_1\beta_2}$. For a description of the vacua of $V(\Phi,\chi)$ we refer to Refs. [34,35]. The parameters β' and ϵ correspond to the Higgs portal and gauge kinetic mixing [41], respectively.

The above model (1) can be viewed as the $\theta_{\rm W} \to \pi/2$ limit of the GSW theory coupled to a DS; therefore, we shall refer to the fields Φ and A_μ as the "visible sector," and χ and C_μ as the DS. Apart from the local $U(1) \times U(1)$ it has a global SU(2) symmetry acting on the (complex) Higgs doublet, Φ, and we shall refer to Eq. (1) as the "semilocal-DS" model.

In the 2VEV case, for $\epsilon = 0$, the gauge boson masses are given as $m_A^2 = 2\eta_1^2$ and $m_C^2 = 2q^2\eta_2^2$, where the VEVs η_1 and η_2 expressed in terms of the parameters of the potential are listed in Appendix A [Eq. (A1)]. The scalar particles ϕ_1 and χ mix; the analysis thereof is presented in Appendix A. The field ϕ_2 remains massless (in the GSW theory, it is the would-be Goldstone boson corresponding to the longitudinal component of $W^\pm$). For a detailed analysis of the effects of the gauge kinetic mixing we refer to Refs. [21,41]. Unless the mass of the DS scalar χ is large ($m_\chi \gg 1$ TeV) compared to SM masses, $\epsilon \lesssim 10^{-3}$ [18,20]. In the 2VEV case, the dark sector Higgs and gauge bosons do not directly make up dark matter [18,19]. As a result, there are much less stringent experimental bounds on the model parameters, e.g., if the mixing of the visible sector and the dark sector Higgs particles is small enough, and the dark sector particles are heavy enough, the model is viable.

By setting $q = 0$, $C_\mu = 0$ we obtain a semilocal model coupled through the Higgs field to a dark scalar field (portal model). Assuming that there is an unbroken $\mathbb{Z}_2$ symmetry in the DS, the dark scalar cannot take on a VEV (1VEV case). The main interest of such portal models is their minimality in that the dark scalar field itself can be considered as a primary constituent of the dark matter. In the 1VEV case, the gauge boson mass is $m_A^2 = 2$, and the scalar masses are $m_{\phi_1}^2 = 2\beta_1$, $m_\chi^2 = \beta' - \alpha$. Due to the global SU(2) symmetry the field ϕ_2 stays massless. Experimental limits on the couplings can be found in Refs. [42–44]. We note that Higgs decays into the dark sector impose rather strong constraints on the coupling β', and the dark matter density imposes constraints on m_χ.

[1] We use the metric signature $+---$.

III. VISIBLE SEMILOCAL STRINGS WITH A BROKEN SYMMETRY IN THE DARK SECTOR

Straight string solutions in a two-component extended Abelian Higgs model with both fields having a nonzero VEV have been considered for two charged fields in Refs. [45–47] and for one charged and one neutral field in Refs. [34,35]. In the case of two electrically charged fields, unless the windings of the two scalar fields agree, the energy per unit length of such strings diverges logarithmically,[2] and their flux is fractional.

In the absence of the gauge kinetic mixing term ($\epsilon = 0$) the 2VEV vortices of Refs. [34,35] can be embedded in the model given by Eq. (1), by setting $\phi_2 = 0$. For $\epsilon \neq 0$ the angular component of the DS gauge field also becomes nonzero. The (straight) string solutions we consider are translationally symmetric in the z direction, and rotationally symmetric in the (x, y) plane, corresponding to the ansatz

$$\begin{aligned} \phi_1 &= f(r)e^{in\vartheta}, \qquad \chi = g(r), \\ A_\vartheta &= na(r), \qquad C_\vartheta = c(r), \end{aligned} \tag{3}$$

where r, ϑ are polar coordinates in the plane and the other field components (ϕ_2, A_r, A_z, C_r, C_z) vanish. Using the field equations (B1) one easily obtains that the energy (B2) is a monotonously increasing function of the dark charge q [see Eq. (B4)]. The derivative with respect to the gauge kinetic mixing is given by

$$\frac{\partial E}{\partial \epsilon} = -2\pi n \int_0^\infty \mathrm{d}r \frac{a'c'}{r}, \tag{4}$$

which vanishes at $\epsilon = 0$, since in that case the field equation for $c(r)$ in Eq. (B1) becomes homogeneous and a standard maximum principle argument implies $c(r) \equiv 0$. Expanding the fields in a power series of ϵ [see Eq. (B6)], the energy of the vortex can be written as

$$\begin{aligned} E &= E_0 + \epsilon^2 E_2 + O(\epsilon^3), \quad \text{where} \\ E_2 &= -2\pi(2n-1)\int_0^\infty \mathrm{d}r \frac{f_0^2(1-a_0)c_1}{r}. \end{aligned} \tag{5}$$

At $\beta_1 = 2$, $\beta_2 = 3$, $\beta' = 2$, $\alpha = 2.1$, and $q^2 = 1$, Eq. (5) yields an excellent approximation up to $\epsilon \lesssim 0.2$. Moreover, $E_0/(2\pi) = 0.906$ and the correction is $E_2/(2\pi) = -0.089$.

A further approximation is to consider the $q^2 \to 0$ limit [see Appendix B, in particular Eq. (B7)], in which case $c_1 \approx a_0$, simplifying the expression for E_2:

$$E_2 = -\pi(2n-1)\int_0^\infty r\mathrm{d}r \left(\frac{a_0'}{r}\right)^2 + \cdots. \tag{6}$$

[2]We assume that the potential vanishes at its minimum. This can be achieved by the subtraction of a constant from the potential V in Eq. (2).

Remarkably, E_2 in Eq. (6) is proportional to the magnetic energy of the unperturbed vortex. At $\beta_1 = 2$, $\beta_2 = 3$, $\beta' = 2$, $\alpha = 2.1$, and $q^2 = 0.1$, Eq. (6) yields $E_2/(2\pi) \approx -0.177$. For these parameter values, Eq. (5) gives $E_2/(2\pi) = -0.155$, which compares quite favorably.

Next we summarize the main results of our stability analysis of string (or vortex in the plane) solutions corresponding to the ansatz (3). The perturbation equations around the straight string solutions are given in Appendix C. Crucially, the fluctuation equations for $\delta\phi_2$ and $\delta\phi_2^*$ decouple from the other components (and also from each other). This decoupling is related to $\phi_2 \equiv 0$ for the background solution and to the coupling structure of the DS. The (only) known instabilities of the semilocal model (without a DS) have been found in the $\delta\phi_2$ sector. We argue that in the semilocal-DS model the only potential instabilities are expected to appear in the fluctuation equations for $\delta\phi_2$ (and for $\delta\phi_2^*$), at least for "not too large" values of β', simplifying considerably the stability analysis. Due to the translation symmetry in the t (z) variable the linearized equations for the corresponding vector-field components δA_0, δC_0 (δA_3, δC_3) decouple from each other and from the other components. Exploiting the symmetries of the background string solution, the linearized equations for the components $\Psi_\ell = (\delta\phi_1, \delta\phi_1^*, \delta A_i, \delta\chi, \delta\chi^*, \delta C_i)$ can be reduced to a coupled system of the form

$$\mathcal{M}_\ell \Psi_\ell = \Omega^2 \Psi_\ell, \ell = 0, 1, \ldots, \tag{7}$$

constituting a system of eight second-order radial ordinary differential equations for a given value of the angular momentum ℓ. For more details of the small fluctuation equations we refer to Appendix C, and Refs. [34,35,48–50].

The coupled perturbation system (7) is not expected to give rise to instabilities at least for not too large values of β', ϵ. When $\beta' = \epsilon = 0$ the string solution reduces to an Abrikosov-Nielsen-Olesen (ANO) one [51,52] in the visible sector, embedded into the semilocal-DS model with $\phi_2 \equiv 0$ and $\chi \equiv \eta_2$. Therefore, Eq. (7) decouples into the perturbations of the ANO vortex in the visible sector, and those of the vacuum in the dark sector. In the visible sector the lowest eigenvalues are well known to be positive [49] [e.g., for $\beta_1 = 2$, the lowest bound-state eigenvalue is $\Omega^2 = 1.76$, and the lowest continuum state is at $\Omega^2 = \min(2\beta_1, 2)$], while in the DS positivity is rather obvious as we are perturbing around a true vacuum state [continuum above $\Omega^2 = \min(2\beta_2\eta_2^2, 2q^2\eta_2^2)$]. Simple perturbation-theoretic arguments show that for $\beta' \ll 1$, $\epsilon \ll 1$ the spectrum remains positive. Therefore, in this paper we shall investigate only the decoupled fluctuation equations for $\delta\phi_2$, which can be written as

$$\begin{aligned} -\frac{1}{r}(rs_{2\ell}')' + Us_{2\ell} &= \Omega^2 s_{2\ell}, \\ U &= \frac{(na-\ell)^2}{r^2} + \beta_1(f^2-1) + \beta' g^2. \end{aligned} \tag{8}$$

TABLE I. Stabilization of 2VEV vortices: the value of α where the vortex becomes stable, and additionally the energy of the vortex at that value of α is displayed. The hidden sector charge is $q^2 = 1$.

			$\epsilon = 0$		$\epsilon = 0.1$		$\epsilon = 0.2$	
β_1	β_2	β'	α_s	$E/(2\pi)$	α_s	$E/(2\pi)$	α_s	$E/(2\pi)$
2	5	2	4.571	0.149	4.567	0.150	4.559	0.153
2	3	2	2.196	0.792	2.193	0.795	2.180	0.808
2	1.5	1.25	2.025	0.329	2.020	0.332	2.011	0.341

For $\ell = 0$, the potential in Eq. (8) has a negative valley close to the origin (the core of the vortex), while for $r \to \infty$ it is given as $(n-\ell)^2/r^2$. The existence of negative eigenvalues depends on the depth of the attractive valley. The stabilizing effect of the scalar condensate comes from making this attractive potential valley shallower. More quantitatively, for a given value of $\beta_1 > 1$, by increasing α (remember that $\alpha - \beta' > 0$) the negative eigenvalue approaches zero, and for some value $\alpha = \alpha_s(\beta_1, \beta_2, \beta')$ it actually reaches zero. For $\alpha > \alpha_s$, DC vortices are then stable. Quite importantly, a large value of the coupling α is also compatible with the experimental bounds on the model, which is quite promising for electroweak-dark strings.

Numerical data are presented in Table I. An unstable vortex and the potential in its perturbation equation (8) is shown in Fig. 1, and a stable one is shown in Fig. 2. As the parameters are tuned, the valley in the potential around the origin becomes shallow, and the bound mode disappears. Importantly, both the Higgs portal coupling and the gauge kinetic mixing act to stabilize semilocal vortices.

In Table I, some numerical data of DC vortices are given for $\alpha = \alpha_s$, i.e., at the value of α when the change of stability sets in. Note that larger values of ϵ correspond to lower values of α_s (i.e., a larger domain of stability). One may note that the values of α_s decrease on the order of $\mathcal{O}(10^{-2})$, while ϵ increases from 0 to 0.2. Therefore it may appear surprising that the change in the energy is rather small and positive, although $\partial E/\partial\epsilon < 0$ while ϵ changes considerably more than α. This effect can be accounted for by observing that the energy is rather more sensitive to a change in α than to one in ϵ, e.g., at $\beta_1 = 2$, $\beta_2 = 5$, $\beta' = 2$, $\alpha = 4.6$, $E_2 = -0.002 \times 2\pi$ and $\partial E/\partial\alpha \approx -0.344 \times 2\pi$. The relative smallness of $\partial E/\partial\epsilon = 2E_2\epsilon$ as compared to $\partial E/\partial\alpha$ can be understood from Eq. (6). Since $a(r) - 1$ is exponentially suppressed for large values of r the main contribution to the integral is expected to come from the region of $r < 1$; however, $f(r)^2 = \mathcal{O}(r^2)$ for $r \to 0$, accounting for the relative smallness of E_2. On the other hand, $\partial E/\partial\alpha = \mathcal{O}(1)$ [see Eq. (B5) in Appendix B].

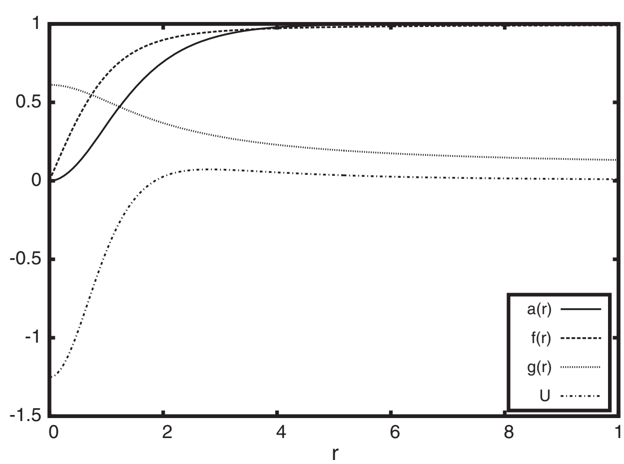

FIG. 1. An unstable 2VEV vortex and the potential in its perturbation equation (8): $\beta_1 = 2, \beta_2 = 3, \beta' = 2, \alpha = 2.011$, and $\epsilon = 0$. [For the notation, see Eqs. (3) and (8).]

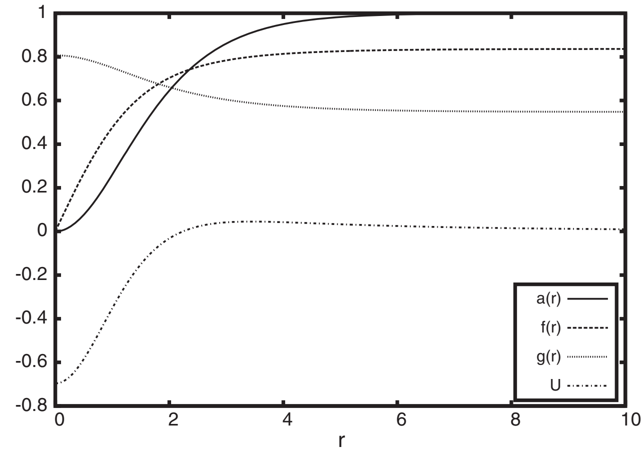

FIG. 2. A stable 2VEV vortex and the potential in its perturbation equation (8): $\beta_1 = 2$, $\beta_2 = 3$, $\beta' = 2$, $\alpha = 2.3$, and $\epsilon = 0$. [For the notation, see Eqs. (3) and (8).]

In Fig. 3(a) two-dimensional slices for $\beta_2 = 3$ and 5 of the domain of stability of DC vortices are depicted schematically. For values of (β_1, α) to the right of the curves, there exist stable DC vortices. Figure 3(a) shows that the domain of stability increases as α increases, and/or as β_2 decreases. Figure 3(b) shows additionally the curves separating stable and unstable vortices for fixed values of β'; these show that the domain of stability increases as β' increases. For better viewing, data points are connected with straight interpolating lines.

IV. SEMILOCAL STRINGS IN MODELS WITH PURELY SCALAR DARK MATTER

In Higgs portal models the DS contains only scalar fields, i.e., dark gauge fields are absent. Moreover, the VEV of the dark scalars is zero to ensure an unbroken $\mathbb{Z}_2$ symmetry. This case is referred to as the 1VEV case in this paper.

The string solutions we consider in this section correspond to the embedding of "condensate core" (CC) strings, with $\phi_1 = \phi^{(\mathrm{CC})}$, $A_\vartheta = A_\vartheta^{(\mathrm{CC})}$, $\chi = \chi^{(\mathrm{CC})}$, and $\phi_2 = 0$. CC

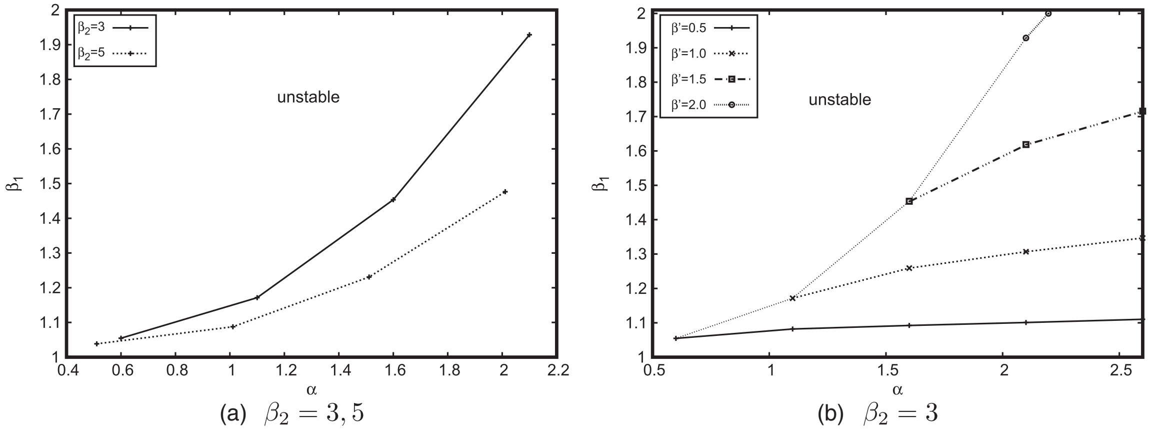

FIG. 3. (a) Schematic view of two two-dimensional slices of the domain of stability of DC vortices. (b) Contour plots of the boundary of the domain of stability of DC vortices.

strings have been studied in Refs. [34,35]. They are the zero-current limits of the superconducting strings of Ref. [31]. We refer to these solutions as DC strings. The linear stability analysis of DC strings is completely analogous to that of the 2VEV case in Sec. II. For more details of the perturbation equations, we refer to Appendix C.

Again, perturbations of the fields $\delta\phi_2$ and $\delta\phi_2^*$ decouple from all other components, and satisfy a Schrödinger-type equation [Eq. (8)]. The characteristics of the potential U are similar to the one in the 2VEV case; it has a repulsive (centrifugal) contribution determining its $r \to \infty$ asymptotics, and an attractive valley close to the origin, the depth of which depends on the background vortex. Therefore the existence of negative eigenvalues depends on the characteristics of the attractive potential valley near the vortex core in V [Eq. (2)]. Since the positive contribution $\beta' g^2$ is of crucial importance, it is illuminating to estimate its influence. We have found that its parameter dependence is well described qualitatively by approximating the condensate at the vortex core simply by a constant. The minimum of the potential for $\Phi = 0$ is at $|\chi| = \sqrt{\alpha/\beta_2}$. Thus in this crude approximation $\beta' g^2 = \beta' \alpha/\beta_2$. Therefore, the larger α and smaller β_2, the larger the domain of stability of CC vortices.

TABLE II. Stabilization of the strings by the condensate in the 1VEV case. The value of β_1 and the energy of the vortex at that value of β_1 is displayed. Embedded ANO strings are stable for $\beta_1 \leq 1$. The energy of the ANO vortex for $\beta = 2$ is $2\pi \times 1.1568$, and at $\beta_1 = 1$ it is 2π.

β_2	β'	α	β_{1s}	$E/(2\pi)$
3	2.3	2.05	1.615	1.0846
4	2.3	2.05	1.459	1.0630
5	2.3	2.05	1.367	1.0504
6	2.3	2.05	1.247	1.0299
2	2	1.85	1.805	1.1022

The reference solution of the semilocal model is the embedded ANO vortex, $\phi_1 = \phi^{(\mathrm{ANO})}$, $A_\vartheta = A_\vartheta^{(\mathrm{ANO})}$, $\chi = 0$, and $\phi_2 = 0$. It is also the unique z-independent string for a generic β_1. Embedded ANO vortices have been found to be unstable for $\beta_1 > 1$ [10,11]. We have found that DC vortices are stable for $\beta_1 < \beta_{1s}$, where, e.g., for $\beta_2 = 6$, $\beta' = 2.3$, and $\alpha = 2.05$, $\beta_{1s} = 1.247$. Some further numerical data are collected in Table II. As can be inferred from Table II, the domain of stability of semilocal strings gets significantly extended. A comparison of the potential U for embedded ANO and for DC vortices is shown in Fig. 4. However, the stable DC solutions we found are still not in the experimentally allowed parameter range (in the SM, $\beta_1 \approx 1.92$), and the Higgs portal coupling here is also too large [42–44].

The remaining set of fluctuation equations for the variables $\delta\phi$, $\delta\chi$, and δA_i have been investigated numerically in Refs. [34,35], and no instabilities have been found. This is in contrast with the reference semilocal (embedded

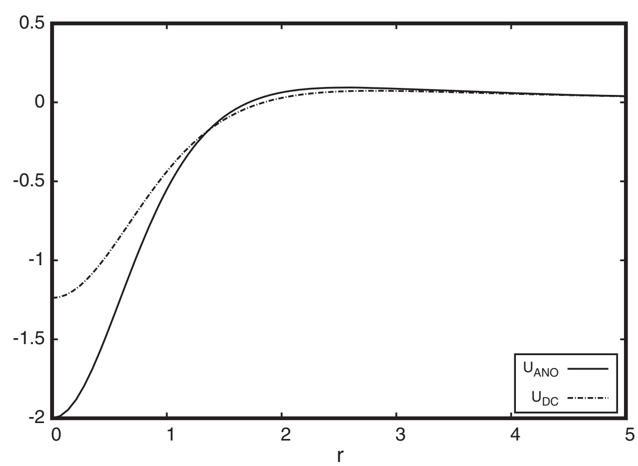

FIG. 4. The potentials in the Schrödinger-type equations for perturbations of ϕ_2 around ANO and DC vortices [Eq. (8)], with $\beta_{1,2} = 2$, $\beta' = 2.3$, and $\alpha = 2.05$.

ANO) vortex, which does have an instability in this sector. It has been shown that this does not persist for the CC vortex.

V. CONCLUSIONS

We have investigated the effect of a dark scalar field with the Higgs portal coupling and a U(1) gauge field with a gauge kinetic mixing term on semilocal strings with local U(1) and global SU(2) symmetries in the visible sector. The strings considered in this paper have unit winding number with respect to the visible U(1) and zero winding number with respect to the dark U(1). We have found that in a minimal Higgs portal model (with a single dark scalar field), semilocal strings get stabilized by a dark scalar condensate at the core of the string. By also considering a dark U(1) gauge field with a gauge kinetic mixing term, an additional stabilizing effect was found. These observations open up the possibility of the existence of classically stable dark core electroweak strings.

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APPENDIX A: SCALAR MASSES

To obtain scalar masses in the 2VEV case, we linearize the potential (2) about the vacuum $\phi_1 = \eta_1$, $\phi_2 = 0$, and $\chi = \eta_2$, with

$$\eta_1^2 = \frac{\beta_1\beta_2 - \alpha\beta'}{\beta_1\beta_2 - (\beta')^2}, \qquad \eta_2^2 = \frac{\beta_1(\alpha - \beta')}{\beta_1\beta_2 - (\beta')^2}. \tag{A1}$$

We also introduce the new variables $\delta\phi_1 = \phi_1^r + i\phi_1^i$ and $\delta\chi = \chi^r + i\chi^i$. The would-be Goldstone bosons—which are later gauged into the longitudinal components of the gauge fields—are then ϕ_1^i and χ^i. The propagating scalar particles are mixed out of ϕ_1^r and χ^r, and their mixing matrix is

$$M_S = \frac{1}{2}\begin{pmatrix} 4\beta_1\eta_1^2 & 4\beta'\eta_1\eta_2 \\ 4\beta'\eta_1\eta_2 & 4\beta_2\eta_2^2 \end{pmatrix}. \tag{A2}$$

The Higgs particle H and the dark Higgs particle K are related to these as [21]

$$\begin{pmatrix} \phi_1^r \\ \chi^r \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} H \\ K \end{pmatrix}, \tag{A3}$$

where

$$\tan 2\theta = \frac{2\beta'\eta_1\eta_2}{\beta_2\eta_2^2 - \beta_1\eta_1^2} = \frac{2M_{S12}}{M_{S22} - M_{S11}}.$$

The resulting scalar masses are

$$M_H^2 = M_{S11} - (M_{S22} - M_{S11})\sin^2\theta/\cos 2\theta,$$
$$M_K^2 = M_{S22} + (M_{S22} - M_{S11})\sin^2\theta/\cos 2\theta. \tag{A4}$$

The second semilocal component ϕ_2 remains massless.

APPENDIX B: RADIAL EQUATIONS

Inserting the ansatz (3) into the field equations corresponding to the Lagrangian (1) yields the radial equations

$$r\left(\frac{a'}{r}\right)' = \frac{2}{1-\epsilon^2}f^2(a-1) + \frac{2\epsilon}{1-\epsilon^2}q^2g^2c,$$
$$r\left(\frac{c'}{r}\right)' = \frac{2}{1-\epsilon^2}q^2g^2c + \frac{2\epsilon}{1-\epsilon^2}f^2(a-1),$$
$$\frac{1}{r}(rf')' = \left[\frac{n^2(1-a)^2}{r^2} + \beta_1(f^2-1) + \beta' g^2\right]f,$$
$$\frac{1}{r}(rg')' = \left[\frac{q^2c^2}{r^2} + \beta_2 g^2 - \alpha + \beta' f^2\right]g. \tag{B1}$$

The boundary conditions at the origin are demanded by the regularity of the fields in the plane, for $r \to 0$, $f \sim f^{(n)}r^n$, $g \to g(0)$, $a \sim a^{(2)}r^2$, and $c \sim c^{(2)}r^2$. For $r \to \infty$, we impose $a \to 1$, $c \to 0$, $f \to \eta_1$, and $g \to \eta_2$ in the 2VEV case, and $f \to 1$ and $g \to 0$ in the 1VEV one.

The energy density of a field configuration in the ansatz (3) is

$$\mathcal{E} = \frac{1}{2}\left[\left(\frac{na'}{r}\right)^2 + \left(\frac{c'}{r}\right)^2 - 2\epsilon n\frac{a'c'}{r^2}\right] + (f')^2 + (g')^2 + \frac{n^2(1-a)^2}{r^2}f^2 + \frac{q^2c^2}{r^2}g^2 + V(f,g), \tag{B2}$$

where

$$V(f,g) = \frac{\beta_1}{2}(f^2-1)^2 + \frac{\beta_2}{2}g^4 - \alpha g^2 + \beta' f^2g^2 - V_0,$$
$$V_0 = -\frac{1}{2}\frac{\beta_1(\alpha-\beta')^2}{\beta_1\beta_2 - (\beta')^2}. \tag{B3}$$

In Eq. (B3), V_0 is the term subtracted in the 2VEV case to set the potential to zero at its minimum. In the 1VEV case, no such term is necessary.

As the fields satisfy the Euler-Lagrange equations, in the derivatives of the energy with respect to the parameters of the model terms proportional to the implicit derivatives of the fields vanish, and only explicit terms remain, e.g.,

$$\frac{\partial E}{\partial q^2} = 2\pi\int_0^\infty \mathrm{d}r\frac{c^2g^2}{r} > 0. \tag{B4}$$

We explicitly spell out the derivative used in Sec. II in the 2VEV case,

$$\frac{\partial E}{\partial \alpha} = -2\pi \int_0^\infty r\mathrm{d}r(g^2 - \eta_2^2), \tag{B5}$$

where the second term is due to the subtraction of V_0 in Eq. (B3). In the 1VEV case, the derivative is the same as in Eq. (B5) without the subtraction of η_2^2.

A series expansion of the solutions in ϵ is as follows:

$$\begin{aligned} a &= a_0 + \epsilon^2 a_2 + O(\epsilon^3), \qquad f = f_0 + \epsilon^2 f_2 + O(\epsilon^3), \\ g &= g_0 + \epsilon^2 g_2 + O(\epsilon^3), \qquad c = \epsilon c_1 + O(\epsilon^3). \end{aligned} \tag{B6}$$

The resulting equations of motion are obtained for a_0, f_0, g_0 by setting $\epsilon = 0$ and $c = 0$ in the radial equations (B1). The leading-order correction c_1 satisfies

$$r\left(\frac{c_1'}{r}\right)' = 2q^2 g_0^2 c_1 + 2f_0^2(a_0 - 1), \tag{B7}$$

which can be approximated in the limit $q^2 \to 0$: in this limit, the right-hand side of Eq. (B7) becomes the same as that of the equation for a_0 [see Eq. (B1)], and therefore $c_1 \approx a_0$. Although the $q^2 \to 0$ limit is not uniform in r, the dominant contribution in the energy correction (6) is expected to come from the core, which is numerically verified.

APPENDIX C: LINEARIZED EQUATIONS

For the linearized field equations we use the formalism of Ref. [49] (see also Refs. [50,53,54] for applications to multicomponent vortices).

For the 2VEV case. in the linearized field equations we introduce a set of new variables for the gauge fields,

$$\begin{aligned} \delta A_\mu &= \frac{\delta K_\mu}{\sqrt{2}\sqrt{1-\epsilon}} + \frac{\delta L_\mu}{\sqrt{2}\sqrt{1+\epsilon}}, \\ \delta C_\mu &= \frac{\delta K_\mu}{\sqrt{2}\sqrt{1-\epsilon}} - \frac{\delta L_\mu}{\sqrt{2}\sqrt{1+\epsilon}}, \end{aligned} \tag{C1}$$

which diagonalize the gauge kinetic terms at the cost of introducing couplings between both gauge fields and both scalars,

$$\begin{aligned} e_- &= \frac{1}{\sqrt{2}\sqrt{1-\epsilon}}, \qquad e_+ = \frac{1}{\sqrt{2}\sqrt{1+\epsilon}}, \\ q_- &= \frac{q}{\sqrt{2}\sqrt{1-\epsilon}}, \qquad q_+ = \frac{q}{\sqrt{2}\sqrt{1+\epsilon}}. \end{aligned} \tag{C2}$$

The linearized equations assume a particularly simple form in the background field gauge [48,49],

$$\begin{aligned} F_K &= \partial_\mu \delta K^\mu + ie_-(\delta\Phi^\dagger\Phi - \Phi^\dagger\delta\Phi) + iq_-(\delta\chi^*\chi - \chi^*\delta\chi), \\ F_L &= \partial_\mu \delta L^\mu + ie_+(\delta\Phi^\dagger\Phi - \Phi^\dagger\delta\Phi) + iq_+(\delta\chi^*\chi - \chi^*\delta\chi). \end{aligned} \tag{C3}$$

The components $\delta K_{0,3}$ and $\delta L_{0,3}$ decouple from the rest of the variables due to the t, z independence of the background, satisfying

$$\begin{aligned} (\Box + U_{KK})\delta K_{0,3} + U_{KL}\delta L_{0,3} = 0, \\ (\Box + U_{LL})\delta L_{0,3} + U_{KL}\delta K_{0,3} = 0, \end{aligned} \tag{C4}$$

where

$$\begin{aligned} U_{KK} &= 2e_-^2\Phi^\dagger\Phi + 2q_-^2|\chi|^2, \\ U_{KL} &= 2e_-e_+\Phi^\dagger\Phi + 2q_-q_+|\chi|^2, \\ U_{LL} &= 2e_+^2\Phi^\dagger\Phi + 2q_+^2|\chi|^2. \end{aligned}$$

Infinitesimal gauge transformations act on the fields as

$$\begin{aligned} &\delta K_\mu \to \delta K_\mu + \partial_\mu \xi, \qquad \delta L_\mu \to \delta L_\mu + \partial_\mu \zeta, \\ &\delta\phi_a \to \delta\phi_a + i\phi_a(e_-\xi + e_+\zeta), \\ &\delta\chi \to \delta\chi + i\chi(q_-\xi + q_+\zeta). \end{aligned} \tag{C5}$$

Due to the residual gauge freedom allowed by the gauge fixing (C3), there are ghost modes, satisfying the equations

$$(\Box + U_{KK})\xi + U_{KL}\zeta = 0, \qquad (\Box + U_{LL})\zeta + U_{KL}\xi = 0, \tag{C6}$$

which agree with those of the 0,3 gauge field components [Eq. (C4)] and cancel part of the spectrum, including all modes in the $\delta K_{0,3} - \delta L_{0,3}$ sector; therefore, in what follows we omit these components.

The following ansatz is compatible with the field equations, due to the cylindrical symmetry of the background:

$$\begin{aligned} \delta\phi_1 &= \mathrm{e}^{i(\Omega t - kz)}\mathrm{e}^{i(n+\ell)\vartheta}s_{1,\ell}(r) + \mathrm{e}^{-i(\Omega t - kz)}\mathrm{e}^{i(n-\ell)\vartheta}s_{1,-\ell}(r), \\ \delta\phi_2 &= \mathrm{e}^{i(\Omega t - kz)}\mathrm{e}^{i\ell\vartheta}s_{2,\ell}(r) + \mathrm{e}^{-i(\Omega t - kz)}\mathrm{e}^{-i\ell\vartheta}s_{2,-\ell}(r), \\ \delta\chi &= \mathrm{e}^{i(\Omega t - kz)}\mathrm{e}^{i\ell\vartheta}h_\ell(r) + \mathrm{e}^{-i(\Omega t - kz)}\mathrm{e}^{-i\ell\vartheta}h_{-\ell}(r), \\ \delta K_+ &= \mathrm{e}^{i(\Omega t - kz)}\mathrm{e}^{i(\ell-1)\vartheta}it_\ell(r) + \mathrm{e}^{-i(\Omega t - kz)}\mathrm{e}^{-i(\ell+1)\vartheta}it_{-\ell}(r), \\ \delta K_- &= -\mathrm{e}^{i(\Omega t - kz)}\mathrm{e}^{i(\ell+1)\vartheta}it_{-\ell}^*(r) - \mathrm{e}^{-i(\Omega t - kz)}\mathrm{e}^{-i(\ell-1)\vartheta}it_\ell^*(r), \\ \delta L_+ &= \mathrm{e}^{i(\Omega t - kz)}\mathrm{e}^{i(\ell-1)\vartheta}iu_\ell(r) + \mathrm{e}^{-i(\Omega t - kz)}\mathrm{e}^{-i(\ell+1)\vartheta}iu_{-\ell}(r), \\ \delta L_- &= -\mathrm{e}^{i(\Omega t - kz)}\mathrm{e}^{i(\ell+1)\vartheta}iu_{-\ell}^*(r) - \mathrm{e}^{-i(\Omega t - kz)}\mathrm{e}^{-i(\ell-1)\vartheta}iu_\ell^*(r), \end{aligned} \tag{C7}$$

where $K_\pm = \exp(\mp i\vartheta)(K_r \mp iK_\vartheta/r)/\sqrt{2}$ and similarly for L (note that $K_\pm^* = K_\mp$). With the ansatz (C7), the perturbation equation assumes the form

$$\mathcal{M}_\ell \Psi_\ell = (\Omega^2 - k^2)\Psi_\ell, \tag{C8}$$

where $\Psi_\ell = (s_{1\ell}, s^*_{1-\ell}, s_{2\ell}, s^*_{2-\ell}, h_\ell, h^*_{-\ell}, t_\ell, t^*_{-\ell}, u_\ell, u^*_{-\ell})$. Note that the lowest eigenvalue corresponds to $k=0$; therefore, in what follows we shall only consider such perturbations. We write the 10×10 matrix operator $\mathcal{M}_\ell$ in Eq. (C8) as (suppressing all zero entries)

$$\mathcal{M}_\ell = \begin{pmatrix} D_1 & U_1 & & & V & V' & e_-A_1 & e_-A_1' & e_+A_1 & e_+A_1' \\ U_1 & \bar{D}_1 & & & V' & V & e_-A_1' & e_-A_1 & e_+A_1' & e_+A_1 \\ & & D_2 & & & & & & & \\ & & & \bar{D}_2 & & & & & & \\ V & V' & & & D_3 & U_2 & q_-A_2 & q_-A_2' & q_+A_2 & q_+A_2' \\ V' & V & & & U_2 & D_3 & q_-A_2' & q_-A_2 & q_+A_2' & q_+A_2 \\ e_-A_1 & e_-A_1' & & & q_-A_2 & q_-A_2' & D_4 & & U_{KL} & \\ e_-A_1' & e_-A_1 & & & q_-A_2' & q_-A_2 & & \bar{D}_4 & & U_{KL} \\ e_+A_1 & e_+A_1' & & & q_+A_2 & q_+A_2' & U_{KL} & & D_5 & \\ e_+A_1' & e_+A_1 & & & q_+A_2' & q_+A_2 & & U_{KL} & & \bar{D}_5 \end{pmatrix}. \tag{C9}$$

In Eq. (C9), the following notation is used:

$$\begin{aligned} &D_1 = -\nabla_r^2 + \frac{(n(1-a)+\ell)^2}{r^2} + W_1, \qquad \bar{D}_1 = -\nabla_r^2 + \frac{(n(1-a)-\ell)^2}{r^2} + W_1, \\ &D_2 = -\nabla_r^2 + \frac{(na-\ell)^2}{r^2} + W_2, \qquad \bar{D}_2 = -\nabla_r^2 + \frac{(na+\ell)^2}{r^2} + W_2, \\ &D_3 = -\nabla_r^2 + \frac{\ell^2}{r^2} + W_3, \\ &D_4 = D_K + \frac{(\ell-1)^2}{r^2}, \qquad \bar{D}_4 = D_K + \frac{(\ell+1)^2}{r^2}, \\ &D_5 = D_L + \frac{(\ell-1)^2}{r^2}, \qquad \bar{D}_5 = D_L + \frac{(\ell+1)^2}{r^2}, \end{aligned} \tag{C10}$$

with

$$D_K = -\nabla_r^2 + U_{KK}, \qquad D_c = -\nabla_r^2 + U_{LL},$$

and

$$\begin{aligned} &W_1 = \left(2\beta_1 + \frac{1}{1-\epsilon^2}\right) f^2 - \beta_1 + \beta' g^2, \qquad W_2 = \beta_1(f^2-1) + \beta' g^2, \\ &W_3 = \left(2\beta_2 + \frac{q^2}{1-\epsilon^2}\right) g^2 - \alpha + \beta' f^2, \qquad U_1 = \left(\beta_1 - \frac{1}{1-\epsilon^2}\right) f^2, \\ &U_2 = \left(\beta_2 - \frac{q^2}{1-\epsilon^2}\right) g^2, \qquad U_{KK} = \frac{2}{1-\epsilon} f^2 + \frac{2q^2}{1-\epsilon} g^2, \\ &U_{KL} = \frac{2}{\sqrt{1-\epsilon^2}} f^2 + \frac{2q^2}{\sqrt{1-\epsilon^2}} g^2, \qquad U_{LL} = \frac{2}{1+\epsilon} f^2 + \frac{2q^2}{1+\epsilon} g^2, \\ &A_1 = -\sqrt{2}\left(f' - \frac{nf}{r}(1-a)\right), \qquad V = \left(\beta' + \frac{\epsilon q}{1-\epsilon^2}\right) fg, \\ &A_1' = \sqrt{2}\left(f' + \frac{nf}{r}(1-a)\right), \qquad V' = \left(\beta' - \frac{\epsilon q}{1-\epsilon^2}\right) fg, \\ &A_2 = -\sqrt{2}(g' - qgc/r), \qquad A_2' = \sqrt{2}(g' + qgc/r). \end{aligned}$$

For $\epsilon = 0$, the ghost mode equations (C6) decouple. The visible sector case has been solved numerically; it has positive eigenvalues, which change slowly with the parameters. The DS case has a positive potential. Therefore, no modes corresponding to instabilities are canceled by ghosts.

The formulas presented above also apply for the 1VEV case by setting $\epsilon = q_- = 0$, replacing δK_μ with δA_μ, and dropping δL_μ and ζ altogether.

[1] T. W. B. Kibble, Topology of cosmic domains and strings, J. Phys. A **9**, 1387 (1976).

[2] A. Vilenkin and E. P. S. Shellard, *Cosmic Strings and Other Topological Defects* (Cambridge University Press, Cambridge, England, 1994).

[3] M. B. Hindmarsh and T. W. B. Kibble, Cosmic strings, Rep. Prog. Phys. **58**, 477 (1995).

[4] T. Vachaspati, L. Pogosian, and D. Steer, Cosmic strings, Scholarpedia **10**, 31682 (2015).

[5] C. Ringeval, Cosmic strings and their induced non-Gaussianities in the cosmic microwave background, Adv. Astron. **2010**, 380507 (2010).

[6] A. Achúcarro and T. Vachaspati, Semilocal and electroweak strings, Phys. Rep. **327**, 347 (2000).

[7] Y. Nambu, String-like configurations in the Weinberg-Salam theory, Nucl. Phys. **B130**, 505 (1977).

[8] K. Huang and R. Tipton, Vortex excitations in the Weinberg-Salam theory, Phys. Rev. D **23**, 3050 (1981).

[9] T. Vachaspati and A. Achúcarro, Semilocal cosmic strings, Phys. Rev. D **44**, 3067 (1991).

[10] M. Hindmarsh, Existence and stability of semilocal strings, Phys. Rev. Lett. **68**, 1263 (1992).

[11] M. Hindmarsh, Semilocal topological defects, Nucl. Phys. **B392**, 461 (1993).

[12] M. James, L. Perivolaropoulos, and T. Vachaspati, Stability of electroweak strings, Phys. Rev. D **46**, R5232 (1992).

[13] M. James, L. Perivolaropoulos, and T. Vachaspati, Detailed stability analysis of electroweak strings, Nucl. Phys. **B395**, 534 (1993).

[14] W. B. Perkins, *W* condensation in electroweak strings, Phys. Rev. D **47**, R5224 (1993).

[15] M. Goodband and M. Hindmarsh, Instabilities of electroweak strings, Phys. Lett. B **363**, 58 (1995).

[16] V. Silveira and A. Zee, Scalar phantoms, Phys. Lett. **161B**, 136 (1985).

[17] B. Patt and F. Wilczek, Higgs-field portal into hidden sectors, arXiv:hep-ph/0605188.

[18] N. Arkani-Hamed, D. P. Finkbeiner, T. R. Slatyer, and N. Weiner, A theory of dark matter, Phys. Rev. D **79**, 015014 (2009).

[19] N. Arkani-Hamed and N. Weiner, LHC signals for a superunified theory of dark matter, J. High Energy Phys. 12 (2008) 104.

[20] T. Vachaspati, Dark strings, Phys. Rev. D **80**, 063502 (2009).

[21] J. M. Hyde, A. J. Long, and T. Vachaspati, Dark strings and their couplings to the standard model, Phys. Rev. D **89**, 065031 (2014).

[22] A. J. Long, J. M. Hyde, and T. Vachaspati, Cosmic strings in hidden sectors: 1. Radiation of standard model particles, J. Cosmol. Astropart. Phys. 09 (**2014**) 030.

[23] A. J. Long and T. Vachaspati, Cosmic strings in hidden sectors: 2. Cosmological and astrophysical signatures, J. Cosmol. Astropart. Phys. 12 (**2014**) 040.

[24] C. Gomez-Sanchez and B. Holdom, Monopoles, strings, and dark matter, Phys. Rev. D **83**, 123524 (2011).

[25] M. Hindmarsh, R. Kirk, J. M. No, and S. M. West, Dark matter with topological defects in the inert doublet model, J. Cosmol. Astropart. Phys. 05 (**2015**) 048.

[26] B. Hartmann and F. Arbabzadah, Cosmic strings interacting with dark strings, J. High Energy Phys. 07 (2009) 068.

[27] Y. Brihaye and B. Hartmann, Effect of dark strings on semilocal strings, Phys. Rev. D **80**, 123502 (2009).

[28] A. Babeanu and B. Hartmann, Stability of superconducting strings coupled to cosmic strings, Phys. Rev. D **85**, 023518 (2012).

[29] P. Arias and F. A. Schaposnik, Vortex solutions of an Abelian Higgs model with visible and hidden sectors, J. High Energy Phys. 12 (**2014**) 011.

[30] P. Arias, E. Ireson, C. Núñez, and F. A. Schaposnik, $\mathcal{N} = 2$ SUSY Abelian Higgs model with hidden sector and BPS equations, J. High Energy Phys. 02 (2015) 156.

[31] P. Peter, Low-mass current-carrying cosmic strings, Phys. Rev. D **46**, 3322 (1992).

[32] E. Witten, Superconducting string, Nucl. Phys. **B249**, 557 (1985).

[33] P. Peter, Superconducting cosmic string: Equation of state for spacelike and timelike current in the neutral limit, Phys. Rev. D **45**, 1091 (1992).

[34] P. Forgács and Á. Lukács, Vortices with scalar condensates in two-component Ginzburg-Landau systems, Phys. Lett. B **762**, 271 (2016).

[35] P. Forgács and Á. Lukács, Vortices and magnetic bags in Abelian models with extended scalar sectors and some of their applications, Phys. Rev. D **94**, 125018 (2016).

[36] T. Vachaspati and R. Watkins, Bound states can stabilize electroweak strings, Phys. Lett. B **318**, 163 (1993).

[37] L. Perivolaropoulos and N. Platis, Stabilizing the semilocal string with a dilatonic Coupling, Phys. Rev. D **88**, 065017 (2013).

[38] H. Weigel, M. Quandt, and N. Graham, Stable charged cosmic strings, Phys. Rev. Lett. **106**, 101601 (2011).

[39] N. Graham, M. Quandt, and H. Weigel, Fermion energies in the background of a cosmic string, Phys. Rev. D **84**, 025017 (2011).

[40] M. Nagasawa and R. Brandenberger, Stabilization of the electroweak Z string in the early Universe, Phys. Rev. D **67**, 043504 (2003).

[41] B. Holdom, Two $U(1)$'s and ϵ charge shifts, Phys. Lett. **166B**, 196 (1986).

[42] D. E. Holz and A. Zee, Collisional dark matter and scalar phantoms, Phys. Lett. B **517**, 239 (2001).

[43] K. Cheung, Y.-L. S. Tsai, P.-Y. Tseng, T.-C. Yuan, and A. Zee, Global study of the simplest scalar phantom dark matter model, J. Cosmol. Astropart. Phys. 10 (2012) 042.

[44] A. Beniwal, F. Rajec, C. Savage, P. Scott, C. Weniger, M. White, and A. G. Williams, Combined analysis of effective Higgs portal dark matter models, Phys. Rev. D **93**, 115016 (2016).

[45] Á. Lukács, Twisted strings in extended Abelian Higgs models, in *What is Known and Unexpected at LHC: Proceedings of the International School of Subnuclear Physics, Erice-Sicily, Italy, 29 August–7 September 2010*, edited by A. Zichichi (World Scientific, Singapore, 2013).

[46] E. Babaev, Vortices with fractional flux in two-gap superconductors and in extended Faddeev model, Phys. Rev. Lett. **89**, 067001 (2002).

[47] E. Babaev and M. Speight, Semi-Meissner state and neither type-I nor type-II superconductivity in multicomponent systems, Phys. Rev. B **72**, 180502 (2005).

[48] J. Baacke and T. Daiber, One-loop corrections to the instanton transition in the two-dimensional Abelian Higgs model, Phys. Rev. D **51**, 795 (1995).

[49] M. Goodband and M. Hindmarsh, Bound states and instabilities of vortices, Phys. Rev. D **52**, 4621 (1995).

[50] P. Forgács and Á. Lukács, Instabilities of twisted strings, J. High Energy Phys. 12 (**2009**) 064.

[51] A. A. Abrikosov, The magnetic properties of superconducting alloys, J. Phys. Chem. Solids **2**, 3, 199 (1957).

[52] H. B. Nielsen and P. Olesen, Vortex-line models for dual strings, Nucl. Phys. **B61**, 45 (1973).

[53] J. Garaud and M. S. Volkov, Stability analysis of the twisted superconducting semilocal strings, Nucl. Phys. **B799**, 430 (2008).

[54] B. Hartmann and P. Peter, Can type II semilocal cosmic strings form?, Phys. Rev. D **86**, 103516 (2012).