

Quark-gluon vertex: A perturbation theory primer and beyondR. Bermudez,¹ L. Albino,² L. X. Gutiérrez-Guerrero,³ M. E. Tejeda-Yeomans,⁴ and A. Bashir²¹*Departamento de Investigación en Física, Universidad de Sonora, Boulevard Luis Encinas J. y Rosales, Colonia Centro, Hermosillo, Sonora 83000, México*²*Instituto de Física y Matemáticas, Universidad Michoacana de San Nicolás de Hidalgo, Edificio C-3, Ciudad Universitaria, Morelia, Michoacán 58040, México*³*CONACyT-Mesoamerican Centre for Theoretical Physics, Universidad Autónoma de Chiapas, Carretera Zapata Km. 4, Real del Bosque (Terán), Tuxtla Gutiérrez 29040, Chiapas, México*⁴*Departamento de Física, Universidad de Sonora, Boulevard Luis Encinas J. y Rosales, Colonia Centro, Hermosillo, Sonora 83000, México*

(Received 27 November 2016; published 28 February 2017)

There has been growing evidence that the infrared enhancement of the form factors defining the full quark-gluon vertex plays an important role in realizing a dynamical breakdown of chiral symmetry in quantum chromodynamics, leading to the observed spectrum and properties of hadrons. Both the lattice and the Schwinger-Dyson communities have begun to calculate these form factors in various kinematical regimes of momenta involved. A natural consistency check for these studies is that they should match onto the perturbative predictions in the ultraviolet, where nonperturbative effects mellow down. In this article, we carry out a numerical analysis of the one-loop result for all the form factors of the quark-gluon vertex. Interestingly, even the one-loop results qualitatively encode most of the infrared enhancement features expected of their nonperturbative counter parts. We analyze various kinematical configurations of momenta: symmetric, on shell, and asymptotic. The on-shell limit enables us to compute anomalous chromomagnetic moment of quarks. The asymptotic results have implications for the multiplicative renormalizability of the quark propagator and its connection with the Landau-Khalatnikov-Fradkin transformations, allowing us to analyze and compare various *Ansätze* proposed so far.

DOI: [10.1103/PhysRevD.95.034041](https://doi.org/10.1103/PhysRevD.95.034041)**I. INTRODUCTION**

The nonperturbative study of the Schwinger-Dyson equation (SDE) for the quark propagator has suggested infrared enhancement of the running quark mass function $\mathcal{M}(p^2)$ through the dynamical breakdown of chiral symmetry [1–3]. Lattice studies have provided its confirmation [4–6]. It is also well known that the corresponding quark propagator breaches the axiom of reflection positivity and hence corresponds to a confined excitation [7,8]. However, it is important to note that the analytic structure of the quark propagator depends strongly on the details of the structure of the quark-gluon vertex, which makes the study of the latter all the more important [8]. Whereas in the infrared, i.e., $\mathcal{M}(p^2)|_{p^2 \rightarrow 0}$, the quark mass function obtains a constituent-like value of about 300–500 MeV, its perturbative limit is reproduced correctly in the ultraviolet domain. This running of the mass function has innumerable observable consequences in hadron physics; see, for example, recent reviews [7,9].

The quark propagator is intimately linked with the corresponding behavior of the gluon propagator and the quark-gluon vertex through the relevant SDEs as well as the symmetry relations of quantum chromodynamics (QCD), namely, the Slavnov-Taylor identities (STIs) [10,11], the transverse Takahashi identities [12–14], and

the generalized Landau-Khalatnikov-Fradkin transformations (LKFTs) [15]. Therefore, a knowledge of the gluon propagator and the quark-gluon vertex is vital to study their impact on the quark propagator and the dynamical mass generation.

In the last decade or so, valuable conclusions have been arrived at regarding the gluon propagator and a gluonic mass scale of about $(2-4)\Lambda_{\text{QCD}}$ associated with it in the infrared. The SDEs prediction for the massive gluon solution [16] has also been confirmed in modern lattice studies [17–21] which support a finite but infrared enhanced scalar form factor of the gluon propagator, the so-called decoupling solution. It is also in agreement with subsequent SDE results [22–24], exact renormalization group equations [25], the refined Gribov-Zwanziger formalism [26–29], and the earlier suggestion of Cornwall [30]. Even if one includes the effect of dynamical quarks [31–33], the qualitative behavior of the gluon propagator remains unaltered. The screening effect of the increasing number of flavors is reflected in the reduction in the infrared strength of the gluon propagator. Moreover, its feedback into the quark propagator [34] is similar to what is observed in quantum electrodynamics (QED) [35,36]. Interestingly, the analytic properties of the gluon propagator do not permit it to propagate freely [37]. Again, a transition of the associated scalar form factor to the

perturbative limit of $p^2 \gg \Lambda_{\text{QCD}}^2$ is faithfully achieved. All the above findings thus conform with the fact that individual quarks and gluons are confined within color singlet hadrons.

In addition to the gluon propagator, the quark-gluon vertex also feeds into the SDE of the quark propagator. Attempts have been initiated in lattice QCD to compute the form factors of the quark-gluon vertex for some symmetric kinematical configurations of momenta involved [38–41]. The SDEs can access any kinematical configuration of the external momenta with the same amount of effort. Studies have been carried out to see if the SDE truncations agree with the infrared enhancement of the form factors reported in the lattice computation [42–45]. A satisfactory agreement between the lattice and the SDE results reassures that these two approaches are complementary.

The theoretical and phenomenological implications of different form factors, defining the quark-gluon vertex, can hardly be overemphasized. For example, the mass splitting between the parity partners in low lying mesons, such as (π, σ) and (ρ, a_1) , can only be explained through incorporating the form factors proportional to the anomalous chromomagnetic/electromagnetic vector structure $q_\mu \sigma^{\mu\nu}$ [46]. The associated corrections cancel for the pseudoscalar and vector mesons but add in the scalar and axial vector channels [47], hence solving a long standing puzzle. On the other hand, the choice of the quark-gluon vertex (and quark-photon vertex, as the hadrons are probed through photons) is also critically important in studying the form factors of mesons [48,49] and baryons [50].

Just like the quark mass function and the gluon propagator, the form factors of the vertex should reduce to their perturbative Feynman expansion in the weak coupling limit. Recall that a truncation of the complete set of SDEs, which maintains gauge invariance and multiplicative renormalizability (MR) at every level of approximation, is perturbation theory. In QED, this fact has long been used to impose constraints on the *Ansatz* proposed for the fermion-boson vertex; see for example [51–60]. There are several one-loop results, available over the past three decades, which facilitate this task [51,61,62]. In this article, we shall employ the one-loop perturbative calculation of the quark-gluon vertex [63] to deduce a series of analytical and numerical requirements which any nonperturbative construction of this vertex must comply with in the weak coupling regime. Once it is achieved, the corresponding truncation scheme encodes a more reliable transition from infrared to the ultraviolet behavior of QCD.

This article is organized as follows: In Sec. II we present the general considerations regarding the construction of a physically meaningful and reliable quark-gluon vertex *Ansatz*. In Sec. III, analytical and numerical computations of the vertex form factors for the so-called symmetric limit (equal incoming, outgoing quark and gluon momenta squared) are presented. In Sec. IV, we provide analytical

results for the kinematical regime, where the momentum squared in one of the quark legs is much larger than in the other, namely, the asymptotic limit. In Sec. V, we discuss the physically relevant anomalous chromomagnetic moment of quarks in the on-shell limit where it is historically defined. Finally, in Sec. VI, we present our conclusions and final remarks.

II. THE QUARK-GLUON VERTEX: GENERAL CONSIDERATIONS

The quark-gluon vertex plays a fundamental role in perturbation theory and in the nonperturbative treatment of QCD and hadron physics. Therefore, we set out to study it in detail.

We start by expanding out this vertex in a tensor decomposition dictated by a necessary constraint of gauge invariance, i.e., the STI. We follow the procedure outlined in QED in [55,61,62] and adopted for QCD in [63]. The STI [10], which relates the quark-gluon vertex $\Gamma_\mu \equiv \Gamma_\mu(p, k, q)$ with the quark propagator, reads as follows:

$$q^\mu \Gamma_\mu = G(q^2) [\bar{H}(k, p, q) S^{-1}(k) - S^{-1}(p) H(p, k, q)], \quad (1)$$

where $q = k - p$ and $G(q^2)$ is the scalar function associated with the ghost propagator. The function H , and its “conjugated” function \bar{H} , are related to the auxiliary nontrivial vertices involving the complete four-point quark-quark-ghost-ghost vertex. k and p are the incoming and outgoing quark momenta, respectively, while q is the outgoing gluon momentum. Moreover, $S(k)$ is the full quark propagator, defined as

$$S(k) = \frac{F(k^2)}{k - \mathcal{M}(k^2)}, \quad (2)$$

where $F(k^2)$ is the so-called wave function renormalization, and $\mathcal{M}(k^2)$ is the running quark mass function. At the tree level, $F(k^2) = 1$ and $\mathcal{M}(k^2) = m$, the current quark mass. Similarly, we define the tree-level gluon propagator as

$$\Delta_{\mu\nu}(q^2) = -i \frac{1}{q^2} \left[g_{\mu\nu} - \xi \frac{q_\mu q_\nu}{q^2} \right], \quad (3)$$

where ξ is the covariant gauge parameter. $\xi = 0$ is the Feynman gauge while $\xi = 1$ corresponds to the Landau gauge.

Finally, the vector $\Gamma_\mu(p, k, q)$ stands for the fully dressed three-point quark-gluon vertex. Below we enlist the conditions which constrain its construction:

- (i) It must satisfy the STI. This implies that the requirement of gauge invariance fixes the longitudinal part of the quark-gluon vertex.

- (ii) The transverse part is constrained by the requirement of the MR of the massless quark propagator, the LKFTs and the transverse Takahashi identities.
- (iii) The tensor decomposition selected guarantees that every coefficient τ_i should be free of kinematic singularities when $k^2 \rightarrow p^2$ at the one-loop level in arbitrary covariant gauge and dimensions, [51,54,61,63]. We expect it to be true nonperturbatively too because the only singularities which arise are due to good dynamical reasons such as the mass poles for physical particles.
- (iv) The vertex must transform under the charge conjugation (C), parity (P), and time reversal (T) operations just as the bare vertex.
- (v) It should reduce to its perturbation theory Feynman expansion in the limit of weak coupling. Note that a truncation of the complete set of SDEs, which maintains gauge invariance and MR of a gauge theory at every level of approximation, is perturbation theory. Therefore, physically meaningful solutions of the SDEs must agree with perturbative results in the weak coupling regime. In this article, we use a one-loop perturbative calculation of the quark-gluon vertex [63] as a guiding principle to impose tight constraints on the quark-gluon vertex.

Starting from the STI, Eq. (1), we can decompose the vertex as a sum of longitudinal and transverse components [61]:

$$\Gamma_\mu(p, k, q) = \Gamma_\mu^L(p, k, q) + \Gamma_\mu^T(p, k, q), \quad (4)$$

where the longitudinal part $\Gamma_\mu^L(p, k, q)$ alone satisfies the STI, Eq. (1), and the transverse part, $\Gamma_\mu^T(p, k, q)$, is naturally constrained by the conditions

$$\begin{aligned} q^\mu \Gamma_\mu^T(p, k, q) &= 0, \\ \Gamma_\mu^T(p, p, 0) &= 0. \end{aligned} \quad (5)$$

This decomposition ensures that all ultraviolet (UV) divergences are encoded in the longitudinal component, which in turn is expressed as

$$\Gamma_\mu^L(p, k, q) = \sum_{i=1}^4 \lambda_i(p^2, k^2, q^2) L_\mu^i(p, k). \quad (6)$$

The longitudinal tensor basis acquires the form:

$$\begin{aligned} L_\mu^1 &= \gamma_\mu, \\ L_\mu^2(p, k) &= (\not{p} + \not{k})(p + k)_\mu, \\ L_\mu^3(p, k) &= -(p + k)_\mu, \\ L_\mu^4(p, k) &= -\sigma_{\mu\nu}(p + k)^\nu, \end{aligned} \quad (7)$$

where $\sigma_{\mu\nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu]$. The longitudinal form factors λ_i of Eq. (6)¹ are expressed in terms of the scalar functions associated with the quark, gluon, and ghost propagators and the four-point quark-ghost vertices [64]. On the other hand, the UV-finite transverse component is expanded out as

$$\Gamma_\mu^T(p, k, q) = \sum_{i=1}^8 \tau_i(p^2, k^2, q^2) T_\mu^i(p, k), \quad (8)$$

where the transverse scalar form factors, i.e., the functions τ_i , remain unknown, and the 8 transverse tensors are conveniently defined as

$$\begin{aligned} T_\mu^1(p, k) &= p_\mu(k \cdot q) - k_\mu(p \cdot q), \\ T_\mu^2(p, k) &= [p_\mu(k \cdot q) - k_\mu(p \cdot q)](\not{p} + \not{k}), \\ T_\mu^3(p, k) &= q^2 \gamma_\mu - q_\mu \not{q}, \\ T_\mu^4(p, k) &= q^2 [\gamma^\mu(\not{k} + \not{p}) - (k + p)^\mu] \\ &\quad + 2(k - p)^\mu \sigma_{\nu\lambda} p^\nu k^\lambda, \\ T_\mu^5(p, k) &= -\sigma_{\mu\nu} q^\nu, \\ T_\mu^6(p, k) &= \gamma_\mu(p^2 - k^2) + (p + k)_\mu \not{q}, \\ T_\mu^7(p, k) &= \frac{1}{2}(p^2 - k^2) [\gamma_\mu(\not{p} + \not{k}) - (p + k)_\mu] \\ &\quad - (p + k)_\mu \sigma_{\nu\lambda} p^\nu k^\lambda, \\ T_\mu^8(p, k) &= \gamma_\mu \sigma_{\nu\lambda} p^\nu k^\lambda - p_\mu \not{k} + k_\mu \not{p}. \end{aligned} \quad (9)$$

Note that these definitions explicitly obey the relations:

$$q^\mu T_\mu^i(p, k) = 0 \quad T_\mu(p, p) = 0. \quad (10)$$

It is worth noting that the above tensor basis guarantees a transverse vertex free of kinematical singularities when $k^2 \rightarrow p^2$. It is slightly different from the initial one put forward by Ball and Chiu [61]. They carried out a one-loop calculation of the electron-photon vertex in QED in the Feynman gauge. Guided by this calculation, they proposed the transverse basis, which ensured the corresponding form factors were independent of kinematic singularities. However, a later evaluation of the same vertex in an arbitrary covariant gauge by Kizilersü *et al.* [62] revealed that a modification of the basis was required to retain the absence of kinematic singularities for this general case. This consideration was later also extended to the case of finite temperature in [65].

In the next sections, we present one-loop perturbative results for the longitudinal and transverse vertex form factors for some special kinematics of interest. We employ the singularity free basis proposed in [62].

¹In perturbation theory, we will express $\lambda_1 \rightarrow 1 + \lambda_1$ for the sake of convenience, separating out the tree-level factor of 1.

III. THE SYMMETRIC LIMIT

Davydychev *et al.* [63] has provided one-loop quark-gluon vertex for arbitrary and distinct off-shell momenta in any gauge and dimensions. It provides us with an excellent platform to deduce results in different kinematical limits to provide a practical guide towards its possible non-perturbative constructions. We can also infer singularity structure of this vertex and its connection to the multiplicative renormalizability of the massless quark propagator.

In this section, we present analytical expressions as well as numerical computations for the longitudinal vertex in the symmetric case: $p^2 = k^2 = q^2$; see Fig. 1. These results provide a guide, for this kinematical configuration, to which all corresponding nonperturbative results should reduce when the coupling strength is sufficiently weak.

At one-loop perturbation theory, there are two diagrams which contribute to the longitudinal and transverse components of the vertex: Abelian (a) and non-Abelian (b), corresponding to the left and right diagrams in Fig. 1, respectively.

A. The longitudinal form factors

The longitudinal form factors are defined through Eqs. (6) and (7). $\lambda_1(p^2, k^2, q^2, 1/\epsilon)$ is the only one of these which is UV divergent at one loop. Note that the space-time dimension is defined as $n = 4 - 2\epsilon$; $n \rightarrow 4$ as $\epsilon \rightarrow 0$. We employ momentum subtraction renormalization scheme to define the renormalized vertex (identified by the subscript R below), such that at a large enough momentum scale $p^2 = -\mu^2$, tree-level perturbation theory is valid and hence, for the symmetric case,

$$\Gamma_R^\mu(p^2, p^2, p^2)|_{p^2=-\mu^2} \equiv \Gamma_R^\mu(p^2, -\mu^2)|_{p^2=-\mu^2} = \gamma^\mu. \quad (11)$$

This renormalization condition translates as

$$\lambda_{1R}(p^2, p^2, p^2)|_{p^2=-\mu^2} \equiv \lambda_{1R}(p^2, -\mu^2)|_{p^2=-\mu^2} = 0, \quad (12)$$

and determines the vertex renormalization constant $Z_{1F}^{-1}(\mu^2, \epsilon)$ as follows:

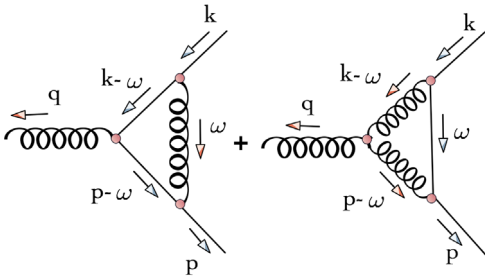


FIG. 1. The Abelian (left diagram) and non-Abelian (right diagram) contributions to the one-loop quark-gluon vertex.

$$\Gamma_R^\mu(p^2, -\mu^2) = Z_{1F}^{-1}(\mu^2, \epsilon)\Gamma_B^\mu(p^2, \epsilon), \quad (13)$$

where the subscript B specifies the bare quantities. In one-loop perturbation theory,

$$Z_{1F}(\mu^2, \epsilon) = 1 + \lambda_1^B(-\mu^2, \epsilon), \quad (14)$$

where the bare quantities depend upon the momentum scale p^2 and on the regulator ϵ , having the pole divergence $1/\epsilon$ for $\epsilon \rightarrow 0$. We convert the bare coupling into the renormalized one through the prescription: $g^2/4\pi = \alpha(\mu)\mathcal{Z}_\alpha(\mu^2, \epsilon)$. Note that as $\mathcal{Z}_\alpha(\mu^2, \epsilon) = 1 + \mathcal{O}(\alpha)$, we can write $g^2/4\pi = \alpha(\mu)$ to the one-loop order. Therefore, explicitly

$$Z_{1F} = 1 + \frac{1}{\epsilon} \left[(1 - \xi)C_a + \frac{3}{4}(2 - \xi)C_b \right] + \text{Fin}_Z, \quad (15)$$

where

$$C_a = \frac{\alpha(\mu)}{4\pi} \left(C_F - \frac{1}{2}C_A \right), \quad C_b = \frac{\alpha(\mu)}{4\pi} C_A.$$

Note that $C_F = (N^2 - 1)/2N$ is the eigenvalue of the Casimir operator in the fundamental representation of $SU(N)$, while $C_A = N$ is that in its adjoint representation. The term proportional to C_a corresponds to the Abelian QED-like diagram and the term involving C_b to the non-Abelian triple-gluon contribution [66]. Fin_Z is the finite part of the renormalization constant Z_{1F} and is given by

$$\begin{aligned} \text{Fin}_Z = & C_a(1 - \xi) \left\{ \frac{m^4 - \mu^4}{\mu^4} L(-\mu^2) - \frac{m^2}{\mu^2} + C_m \right\} \\ & - \frac{1}{4}C_b(2 - \xi) \left\{ -\frac{m^4 - m^2\mu^2 - 2\mu^4}{\mu^4} L(-\mu^2) + \frac{m^2}{\mu^2} \right. \\ & \left. - \ln\left(\frac{m^2}{\mu^2}\right) - (m^2 + \mu^2)\varphi_1(-\mu^2) - 2 - 3C_m \right\}, \quad (16) \end{aligned}$$

where $C_m = 1 - \ln(m^2) + \ln(4\pi) - \gamma_E$, γ_E being the Euler constant, and $L(p^2) = \ln(1 - p^2/m^2)$. Moreover, we have employed the following convention for the three-point integrals:

$$\begin{aligned} i\pi^{n/2}\varphi_1(k^2, p^2, q^2) &= \int \frac{d^n w}{(w^2 - m^2)(k - w)^2(p - w)^2}, \\ i\pi^{n/2}\varphi_2(k^2, p^2, q^2) &= \int \frac{d^n w}{w^2[(k - w)^2 - m^2][(p - w)^2 - m^2]}, \end{aligned}$$

and $\varphi_{1,2}(p^2) \equiv \varphi_{1,2}(p^2, p^2, p^2)$. Explicitly,

$$\begin{aligned}\varphi_1(p^2) &= \frac{1}{p^2\sqrt{3}} \left\{ 2\text{Cl}_2\left(\frac{\pi}{3}\right) + 2\text{Cl}_2\left(\frac{\pi}{3} + 2\theta\right) \right. \\ &\quad \left. + \text{Cl}_2\left(\frac{\pi}{3} - 2\theta\right) + \text{Cl}_2(\pi - 2\theta) \right\}, \\ \varphi_2(p^2) &= \frac{2}{p^2\sqrt{3}} \left\{ 2\text{Cl}_2\left(\frac{2\pi}{3}\right) + \text{Cl}_2\left(\frac{\pi}{3} + 2\tilde{\theta}\right) \right. \\ &\quad \left. + \text{Cl}_2\left(\frac{\pi}{3} - 2\tilde{\theta}\right) \right\},\end{aligned}$$

where Cl_2 are the Clausen functions with θ and $\tilde{\theta}$ defined as [see C(20–22) of [63]]

$$\begin{aligned}\tan \theta &= \frac{p^2 - 2m^2}{p^2\sqrt{3}}, \\ \tan \tilde{\theta} &= \sqrt{\frac{p^2 - 4m^2}{3p^2}}.\end{aligned}$$

In terms of these definitions, our evaluated analytical results for the longitudinal form factors in the symmetric limit are enlisted as follows.

Abelian contribution for λ 's:

$$\begin{aligned}\lambda_{1R}^a(p^2, -\mu^2) &= \frac{C_a(\xi - 1)}{p^4\mu^4} \{ p^4(m^4 - \mu^4)L(-\mu^2) \\ &\quad + \mu^4(p^4 - m^4)L(p^2) - m^2p^2\mu^2(p^2 + \mu^2) \},\end{aligned}\quad (17)$$

$$\lambda_2^a(p^2) = \frac{C_a(\xi - 1)}{2p^6} \{ p^2(2m^2 + p^2) + 2m^4L(p^2) \}, \quad (18)$$

$$\lambda_3^a(p^2) = \frac{C_a(\xi - 4)m}{p^4} \{ p^2 + m^2L(p^2) \}. \quad (19)$$

Non-Abelian contribution for λ 's:

$$\begin{aligned}\lambda_{1R}^b(p^2, -\mu^2) &= \frac{C_b(\xi - 2)}{4p^4\mu^4} \left\{ p^4(m^4 - m^2\mu^2 - 2\mu^4)L(-\mu^2) - \mu^4(m^4 + m^2p^2 - 2p^4)L(p^2) + p^4\mu^4 \ln\left(-\frac{p^2}{\mu^2}\right) \right. \\ &\quad \left. - \mu^2p^2[(\mu^2 + p^2)m^2 - \mu^2p^2(m^2 + \mu^2)\varphi_1(-\mu^2) + \mu^2p^2(m^2 - p^2)\varphi_1(p^2)] \right\},\end{aligned}\quad (20)$$

$$\begin{aligned}\lambda_2^b(p^2) &= \frac{C_b}{24p^6} \left\{ (2 + \xi)p^2 \left[p^2(2m^2 - p^2)\varphi_1(p^2) + 3p^2 - 2p^2 \ln\left(-\frac{p^2}{m^2}\right) + (m^2 + 2p^2)L(p^2) \right] \right. \\ &\quad \left. - 3(2 - \xi)[p^2(p^2 + 2m^2) + 2m^4L(p^2)] \right\},\end{aligned}\quad (21)$$

$$\begin{aligned}\lambda_3^b(p^2) &= \frac{C_b m}{8p^4[p^4 + m^4 - m^2p^2]} \left\{ [2(\xi - 6)m^6 + 3(\xi - 4)m^2p^2(p^2 - m^2) + \xi p^6]L(p^2) \right. \\ &\quad \left. + p^2 \left[2(\xi - 6)(p^4 + m^4 - m^2p^2) - \xi p^4 \ln\left(-\frac{p^2}{m^2}\right) \right] \right\}.\end{aligned}\quad (22)$$

Charge conjugation symmetry of the quark-gluon vertex implies $\lambda_4(p^2, k^2, q^2) = -\lambda_4(k^2, p^2, q^2)$. Therefore, it is naturally zero in the symmetric limit $k^2 = p^2$.

Although the perturbative result is valid only for large p^2 , we have taken the liberty to extrapolate it into the infrared for a comparative analysis and in the hunt for possible singularity structure. We can analytically calculate $\lambda_i^{a,b}(p^2 = 0)$. In the Landau gauge,

$$\lambda_{1R}^a(p^2 = 0) = \lambda_2^a(p^2 = 0) = \lambda_4^{a,b}(p^2 = 0) = 0,$$

$$\lambda_3^a(p^2 = 0) = \frac{3C_a}{2m} \rightarrow -0.020/\text{GeV},$$

$$\lambda_{1R}^b(p^2 = 0) = -\frac{C_b}{8\mu^4} \left\{ \mu^2(5\mu^2 - 2m^2) + 2\mu^2(m^2 + \mu^2)\varphi_1(-\mu^2) + 2(m^4 - m^2\mu^2 - 2\mu^4)L(-\mu^2) - 2\mu^4 \ln\left(\frac{\mu^2}{m^2}\right) \right\} \rightarrow 0.119,$$

$$\lambda_2^b(p^2 = 0) = \frac{C_b}{12m^2} \rightarrow 0.177/\text{GeV}^2,$$

$$\lambda_3^b(p^2 = 0) = \frac{3C_b}{4m} \rightarrow 0.183/\text{GeV},$$

where the numerical values have been evaluated in QCD for $\alpha = .118$, $m = .115$ GeV and $\mu = 2$ GeV.

There are several observations in place:

- (1) $\lambda_{1R}^a(p^2, -\mu^2)$ and $\lambda_2^a(p^2)$ identically vanish in the Landau gauge. This is consistent with the fact that so does the wave function renormalization $F(p^2)$ in the same gauge.
- (2) Therefore, for the sake of comparison, we plot $\lambda_{1R}^a(p^2, -\mu^2)$ and $\lambda_{1R}^b(p^2, -\mu^2)$ in the Feynman gauge, where both are explicitly nonzero. Note that the non-Abelian contribution is positive in the infrared and its magnitude there is about 95% more enhanced; see Fig. 2.
- (3) Note that even the one-loop calculation shows an infrared enhancement of $1 + \lambda_{1R}(p^2, -\mu^2)$, see Fig. 3, plotted in the Landau gauge, $\xi = 1$. However, expectedly this is only a small increase as compared to the nonperturbative effect observed in lattice and SDE studies; see for example [39,42]. The one-loop result is responsible for about a 10% infrared increase of $1 + \lambda_{1R}(p^2, -\mu^2)$ from its tree-level value, while the nonperturbative effects reveal more than a 100% rise.

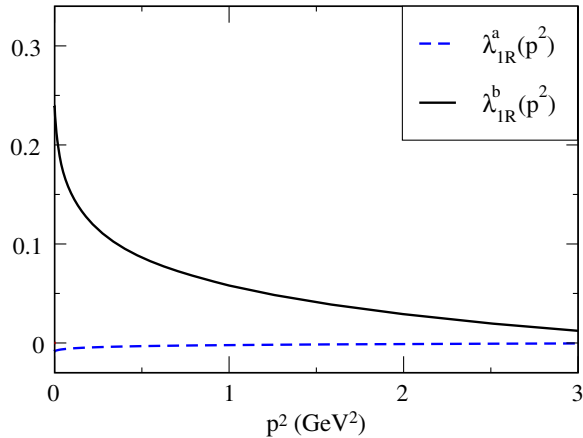


FIG. 2. One-loop form factors $\lambda_{1R}^{a,b}$ in the Feynman gauge.

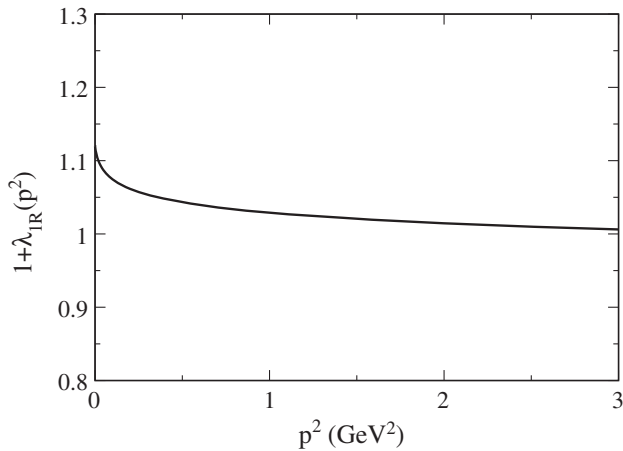


FIG. 3. Full one-loop form factor $1 + \lambda_{1R}$ in the Landau gauge.

- (4) We also check for the deep infrared behavior of one-loop $\lambda_{1R}(p^2, -\mu^2)$. A simple analytical and numerical check shows that this form factor saturates in this limit at the value ~ 0.119 . The same qualitative feature is also observed for $\lambda_2(p^2)$ and $\lambda_3(p^2)$, all plotted in Fig. 4. There is no infrared divergence in the symmetric case. This should be considered as a guideline for lattice studies for which we expect $p^2\lambda_2(p^2)$ to vanish in the infrared. Any nonzero value will imply a divergent infrared $\lambda_2(p^2)$ [39].
- (5) A variation of current mass for the quarks shows that the major infrared enhancement is for the lightest quarks and it diminishes with the increase of current

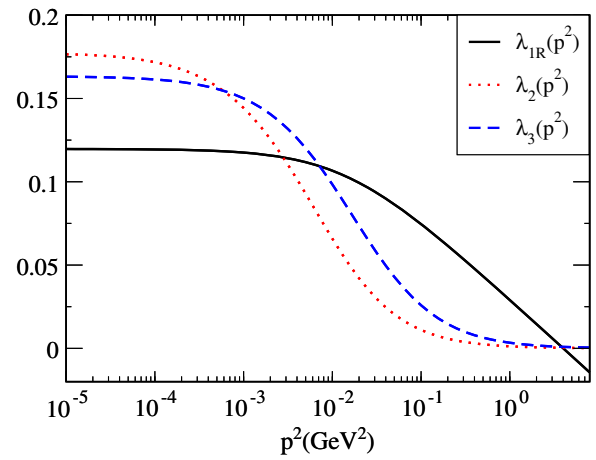


FIG. 4. One-loop form factors λ_{1R} , λ_2 , and λ_3 in the Landau gauge. We draw them until deep infrared to show that all of them saturate in that limit.

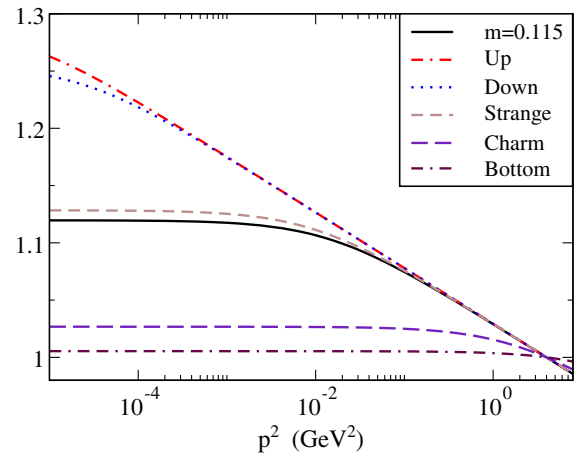


FIG. 5. One-loop form factor $1 + \lambda_{1R}$ in the Landau gauge for different quark masses. Smaller masses imply larger infrared enhancement just as in full nonperturbative QCD. All masses are given in GeV.

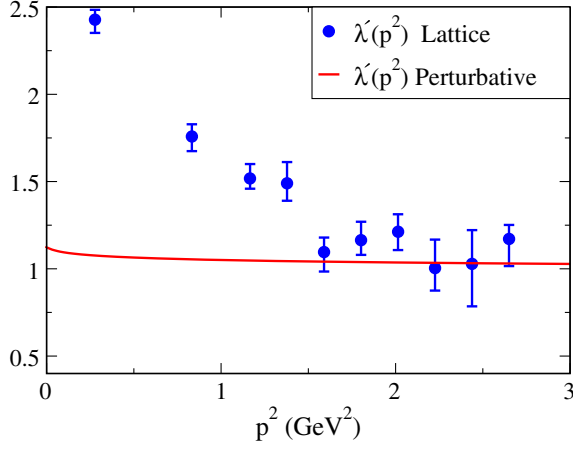


FIG. 6. One-loop form factor $\lambda'(p^2) = \lambda_{1R}(p^2) + p^2\tau_3(p^2)$ in the symmetric limit in the Landau gauge and its comparison with lattice results [40].

quark mass. Quantitatively, the rate of decrease goes as $\sim 7.2\% \rightarrow 7\% \rightarrow 2.2\% \rightarrow 0.3\%$ as we go from $u, d \rightarrow s \rightarrow c \rightarrow b$; see Fig. 5.

- (6) In Figs. 6 and 7, we compare one-loop results against the lattice as well as SDE results [39,40,42]. As mentioned earlier, the perturbative rise is only a very small percentage of the non-perturbative effects. We use $m = 0.115$ GeV and $m = 0.06$ GeV, respectively, in Fig. 6 and Fig. 7, to make a direct comparison with the lattice results.

B. The transverse form factors

On the other hand, the transverse vertex is defined via Eqs. (8) and (9). Just as for $\lambda_4(k^2, p^2, q^2)$, the symmetry of

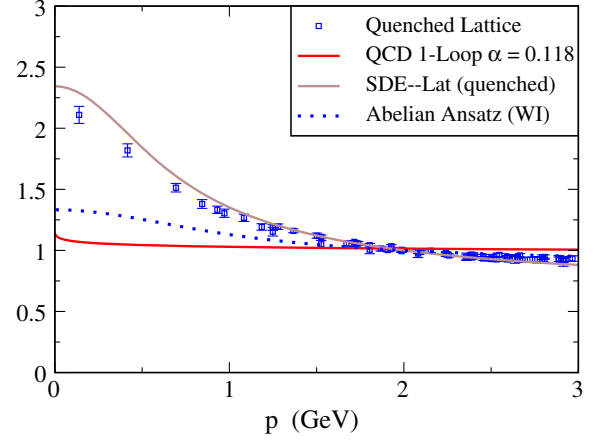


FIG. 7. One-loop form factor $1 + \lambda_{1R}(p^2, p^2, 0)$ in the soft photon limit in the Landau gauge and its comparison with lattice as well as SDE results.

the quark-gluon vertex under the interchange of quark and antiquark requires

$$\begin{aligned}\tau_4(k^2, p^2, q^2) &= -\tau_4(p^2, k^2, q^2), \\ \tau_6(k^2, p^2, q^2) &= -\tau_6(p^2, k^2, q^2).\end{aligned}\quad (23)$$

Therefore, in the symmetric limit, both of these form factors vanish identically. We now present explicit analytical results for the symmetric limit, and then carry out a numerical analysis. Moreover, we adopt the simplified notation $\tau_i^{a,b}(p^2, p^2, p^2) \equiv \tau_i^{a,b}$, and write out the $\tau_i^{a,b}$ explicitly as follows:

Abelian contribution for τ 's:

$$\begin{aligned}\tau_1^a &= \frac{2C_a m}{3p^6}(\xi - 4)\{p^2[3 - (2m^2 + p^2)\varphi_2(p^2) - 4f(p^2)] + (m^2 + 2p^2)L(p^2)\}, \\ \tau_2^a &= \frac{C_a}{3p^6}\{-4m^2(\xi - 1) - p^2(\xi - 5) + [4m^4(\xi - 1) - 2p^4]\varphi_2(p^2) + 4(\xi - 1)(2m^2 + p^2)f(p^2) + 2[2m^2 - (\xi - 1)p^2]L(p^2)\}, \\ \tau_3^a &= -\frac{C_a}{3p^6}\{3p^2[(\xi + 1)p^2 - (\xi - 1)m^2] + p^2[2(\xi - 1)m^4 + 4m^2p^2 + (1 - 2\xi)p^4]\varphi_2(p^2) \\ &\quad - 2p^2[(\xi + 1)p^2 - 2(\xi - 1)m^2]f(p^2) - (m^2 - p^2)[(\xi + 1)p^2 + (\xi - 1)m^2]L(p^2)\}, \\ \tau_4^a &= 0, \quad \tau_5^a = -\frac{C_a m \xi}{3p^4}\{p^2[3 + 2(m^2 - p^2)\varphi_2(p^2) + 4f(p^2)] + (5m^2 - 2p^2)L(p^2)\}, \\ \tau_6^a &= 0, \quad \tau_7^a = \frac{2C_a m \xi}{3p^6}\{p^2[1 + (2m^2 - p^2)\varphi_2(p^2) + 4f(p^2)] + (3m^2 - 2p^2)L(p^2)\}, \\ \tau_8^a &= -\frac{2C_a}{3p^4}\{p^2(2m^2 + p^2)\varphi_2(p^2) + 4p^2f(p^2) + 2(m^2 - p^2)L(p^2)\}.\end{aligned}\quad (24)$$

Non-Abelian contribution for τ 's:

$$\begin{aligned}
\tau_1^b &= \frac{C_b m}{24p^6(m^4 - m^2 p^2 + p^4)^2} \left\{ p^2 [8m^8(\xi - 6) + 4m^6 p^2(24 - 3\xi - \xi^2) - 6m^4 p^4(24 - 3\xi - 2\xi^2) \right. \\
&\quad + 2m^2 p^6(48 - 5\xi - 3\xi^2) - 2p^8(24 - 3\xi - 4\xi^2)] \ln \left(-\frac{p^2}{m^2} \right) + 2p^2(m^4 - m^2 p^2 + p^4) \\
&\quad \times [12m^2(m^2 - p^2)(\xi - 3) - 3p^4(12 - 4\xi + \xi^2) + 2(m^4 - m^2 p^2 + p^4)[2m^2(\xi - 6) + p^2(6 - 2\xi + \xi^2)]\varphi_1(p^2)] \\
&\quad + 2[4m^{10}(2\xi - 3) - m^8 p^2 \xi(13 + 2\xi) + 2m^6 p^4(6 + 7\xi + 4\xi^2) - m^4 p^6(48 + \xi + 6\xi^2) \\
&\quad \left. + 2m^2 p^8(18 - 2\xi - \xi^2) - p^{10}(24 - 5\xi - 2\xi^2)]L(p^2) \right\}, \\
\tau_2^b &= -\frac{C_b}{12p^6(m^4 - m^2 p^2 + p^4)} \left\{ 4p^2(m^4 - m^2 p^2 + p^4) - 2(\xi - 2)(\xi p^2 - 2m^2)(m^4 - m^2 p^2 + p^4) \right. \\
&\quad - [4(\xi - 2)m^6 + 8m^4 p^2 + (\xi^2 - 8)m^2 p^4 + 4\xi p^6] \ln \left(-\frac{p^2}{m^2} \right) \\
&\quad + [4(\xi - 2)m^4 + 2(\xi + 2)m^2 p^2 + (\xi - 2)(\xi + 1)p^4](m^4 - m^2 p^2 + p^4)\varphi_1(p^2) \\
&\quad \left. + [(-2\xi^2 + 5\xi - 6)m^6 + m^4 p^2(2\xi^2 - \xi + 6) + (\xi - 6)m^2 p^4 + 4\xi p^6]L(p^2) \right\}, \\
\tau_3^b &= \frac{C_b}{24p^6(m^4 - m^2 p^2 + p^4)} \left\{ 6p^2(m^4 - m^2 p^2 + p^4)[(2 + 3\xi - \xi^2)p^2 + (\xi - 2)m^2] \right. \\
&\quad - p^2[4(\xi - 2)m^6 - 2(\xi^2 - 3\xi - 6)m^4 p^2 + (5\xi^2 - 18\xi - 12)m^2 p^4 - 2(\xi^2 - 5\xi - 2)p^6] \ln \left(-\frac{p^2}{m^2} \right) \\
&\quad + p^2(m^4 - m^2 p^2 + p^4)[\xi^2 p^2(p^2 - 2m^2) + 4\xi(m^2 + p^2)^2 - 8(m^4 - m^2 p^2 + p^4)]\varphi_1(p^2) \\
&\quad \left. + 2(m^2 - p^2)[(\xi - 2)m^6 - 2\xi(\xi - 3)m^4 p^2 - \xi(\xi - 6)m^2 p^4 + (\xi^2 - 5\xi - 2)p^6]L(p^2) \right\}, \\
\tau_4^b &= \tau_6^b = 0, \\
\tau_5^b &= -\frac{C_b m}{12p^4(m^4 - m^2 p^2 + p^4)} \left\{ p^2 \xi [2m^4 - m^2 p^2(\xi - 4) + 2p^4(\xi - 2)] \ln \left(-\frac{p^2}{m^2} \right) \right. \\
&\quad + p^2(m^4 - m^2 p^2 + p^4)[6\xi + [p^2(18 - 8\xi + \xi^2) - 2m^2 \xi]\varphi_1(p^2)] \\
&\quad \left. + 2\xi(m^2 - p^2)[4m^4 + m^2 p^2(\xi - 4) + p^4(\xi - 2)]L(p^2) \right\}, \\
\tau_7^b &= \frac{C_b m \xi}{12p^6(m^4 - m^2 p^2 + p^4)^2} \left\{ -p^2(m^4 - m^2 p^2 + p^4)[-2m^4 + 2(1 - \xi)m^2 p^2 + p^4(\xi - 2)] \right. \\
&\quad - p^2[4m^8 - 6m^6 p^2 + 8m^4 p^4 - m^2 p^6(4 + \xi) + 2p^8(1 + \xi)] \ln \left(-\frac{p^2}{m^2} \right) \\
&\quad - p^2(m^4 - m^2 p^2 + p^4)[4m^6 - 4m^4 p^2 + 4m^2 p^4]\varphi_1(p^2) \\
&\quad \left. + 2(m^2 - p^2)[3m^8 + m^6 p^2(\xi - 6) - m^4 p^4(\xi - 7) + 2m^2 p^6(\xi - 2) + p^8(\xi + 1)]L(p^2) \right\}, \\
\tau_8^b &= \frac{C_b}{12p^4(m^4 - m^2 p^2 + p^4)} \left\{ p^2 [2m^4(\xi - 6) - (\xi^2 - 2\xi - 12)m^2 p^2 + 2p^4(\xi^2 - 3\xi - 6)] \ln \left(-\frac{p^2}{m^2} \right) \right. \\
&\quad + p^2(m^4 - m^2 p^2 + p^4)[(\xi^2 - 6\xi + 12)p^2 - 2(\xi - 6)m^2]\varphi_1(p^2) + 2(m^2 - p^2)[(\xi - 6)m^4 \\
&\quad \left. + (\xi^2 - 5\xi + 6)m^2 p^2 + (\xi^2 - 3\xi - 6)p^4]L(p^2) \right\}, \tag{25}
\end{aligned}$$

where function $f(p^2)$ is defined as

$$f(p^2) = \begin{cases} \sqrt{\frac{p^2-4m^2}{4p^2}} \ln \frac{\sqrt{p^2-4m^2} + \sqrt{p^2}}{\sqrt{p^2-4m^2} - \sqrt{p^2}}, & p^2 > 4m^2, \\ \sqrt{\frac{4m^2-p^2}{p^2}} \arctan \sqrt{\frac{p^2}{4m^2-p^2}}, & p^2 < 4m^2. \end{cases} \quad (26)$$

In the Landau gauge, numerical results for the Abelian components of the symmetric transverse form factors, Eq. (24), are presented in Fig. 8. One of the checks of their numerical evaluation is the deep infrared behavior, $\tau_i^a(p^2 = 0)$, which can be calculated analytically:

$$\begin{aligned} \tau_1^a(p^2 = 0) &= -\frac{C_a}{6m^3} \rightarrow 0.171/\text{GeV}^3, \\ \tau_2^a(p^2 = 0) &= -\frac{C_a}{18m^4} \rightarrow 0.497/\text{GeV}^4, \\ \tau_3^a(p^2 = 0) &= 4\frac{C_a}{6m^2} \rightarrow -0.078/\text{GeV}^2, \\ \tau_4^a(p^2 = 0) &= \tau_6^a(p^2 = 0) = 0, \\ \tau_5^a(p^2 = 0) &= 0, \\ \tau_7^a(p^2 = 0) &= -\frac{C_a}{6m^3} \rightarrow 0.171/\text{GeV}^3, \\ \tau_8^a(p^2 = 0) &= \frac{C_a}{m^2} \rightarrow -0.118/\text{GeV}^2, \end{aligned} \quad (27)$$

where, as before, the numerical values have been evaluated in QCD for $\alpha = .118$ and $m = .115$ GeV. These values match with the numerical computation of the plots displayed in Fig. 8 in the infrared limit. Note that all the τ_i^a converge to finite values in this limit. Therefore, we expect that for symmetric configuration of momenta, any QED construction (which is basically the Abelian version of

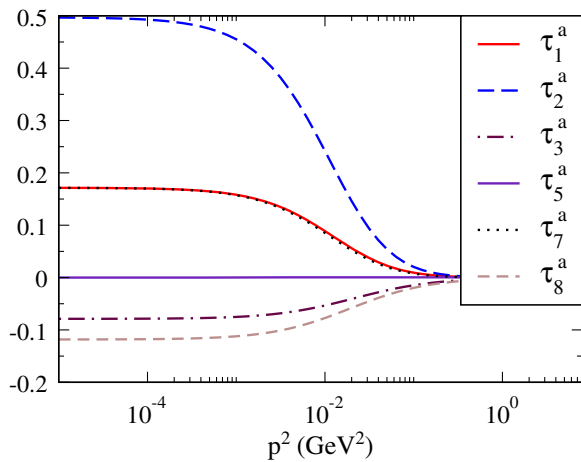


FIG. 8. Abelian $\tau_i^a(p^2)$ for the symmetric configuration of momenta $k^2 = p^2 = q^2$.

QCD) of the three-point vertex should not be singular in the infrared limit.

For the non-Abelian transverse coefficients, Eq. (25), the deep infrared limit, $\tau_i^b(p^2 \rightarrow 0)$, in the Landau gauge reads as

$$\begin{aligned} \tau_1^b(p^2 \rightarrow 0) &= \frac{C_b}{18m^3} \left[-4 + 3 \ln \left(\frac{p^2}{m^2} \right) \right], \\ \tau_2^b(p^2 \rightarrow 0) &= \frac{C_b}{24m^4} \left[1 + 2 \ln \left(\frac{p^2}{m^2} \right) \right], \\ \tau_3^b(p^2 \rightarrow 0) &= \frac{C_b}{144m^2} \left[-35 + 78 \ln \left(\frac{p^2}{m^2} \right) \right], \\ \tau_4^b(p^2 \rightarrow 0) &= 0, \\ \tau_5^b(p^2 \rightarrow 0) &= -\frac{C_b}{8m} \left[-7 + 10 \ln \left(\frac{p^2}{m^2} \right) \right], \\ \tau_6^b(p^2 \rightarrow 0) &= 0, \\ \tau_7^b(p^2 \rightarrow 0) &= \frac{C_b}{12m^3}, \\ \tau_8^b(p^2 \rightarrow 0) &= \frac{C_b}{8m^2} \left[1 + 10 \ln \left(\frac{p^2}{m^2} \right) \right]. \end{aligned}$$

The above results reveal a logarithmic divergence for the non-Abelian coefficients in the deep infrared regime, $p^2 \rightarrow 0$, which is absent in the Abelian counterpart. However, this is not in conflict with the requirement of no kinematical singularities because in the symmetric case, one also takes the photon momentum squared $\rightarrow 0$, which is the dynamical limit of taking the photon to be on shell. In Fig. 9, we can see that the product $p^2 \tau_i^b(p^2)$ is well behaved for infrared momenta. Therefore, this is what we opt to plot. Note that for several τ_i^b , the factor p^2 , necessary to suppress the logarithmic divergence, comes right from the tensor basis, Eq. (9).

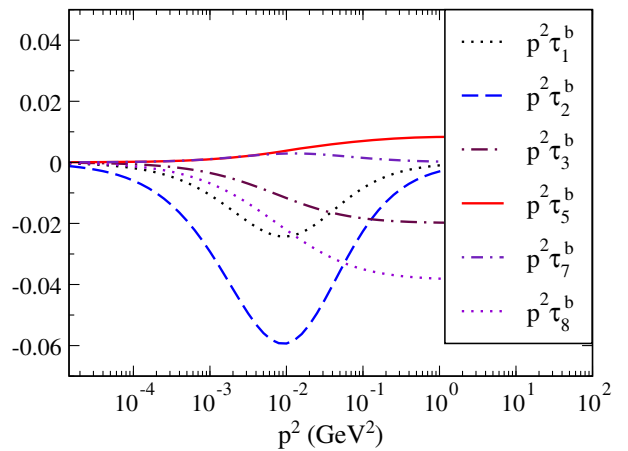


FIG. 9. Non-Abelian $\tau_i^b(p^2)$, weighted with p^2 for the symmetric configuration of momenta $k^2 = p^2 = q^2$.

We could think of redefining the transverse tensors to get rid of the divergence in some of the $\tau_i^b(p^2)$ in the symmetric limit, but it is not possible to get rid of the overall divergence arising from the three-gluon vertex configuration. So we resist the temptation to do it.

With all the results of this section, we have a complete guideline for any nonperturbative construction of the quark-gluon vertex to reduce to in the weak coupling regime of the symmetric configuration of momenta. Moreover, we have analyzed the infrared singularity structure of each component of the transverse vertex in this limit. We now focus our attention on the asymptotic limit of momenta, which has played a crucial role in implementing MR of the electron propagator in the massless limit.

IV. THE ASYMPTOTIC LIMIT

From the works in QED, we already know that the intricate structure of the quark-gluon vertex dictates MR of the electron and hence ensures LKFT for the two-point function are satisfied. Brown and Dorey [67] argue that an arbitrary construction of the electron-photon vertex does not satisfy the requirement of MR. It was realized that neither the bare vertex nor the Ball-Chiu vertex [61], which satisfies the Ward-Fradkin-Green-Takahashi identity [68–71], were good enough to fulfill the demands of MR. Since then, starting from the pioneering work by Curtis and Pennington [51], there have been improved attempts to incorporate the implications of LKFT in constructing a reliable electron-photon vertex *Ansatz* [52,55–58,60,72–74]. This owes itself to our better understanding of the LKFT [59,75–83].

The need for the same in QCD was realized in the work by Bloch, who constructs a model truncation which preserves MR, and reproduces the correct leading order perturbative behavior through assuming nontrivial cancellations involving the full quark-gluon vertex in the quark self-energy loop [84].

Note that the quark propagator beyond $\mathcal{O}(\alpha)$ involves gluon self-interactions. These interactions introduce the color factor C_A in the adjoint representation. The same is true for the transverse part of the quark-gluon vertex. In this section, we provide these transverse form factors for the asymptotic limit, $k^2 \gg p^2 \gg m^2$. For the Abelian part we have

$$\begin{aligned} k^4 \frac{\tau_1^a}{m} &= C_a (4 - \xi) \ln\left(\frac{p^2}{k^2}\right), & k^4 \tau_2^a &= \frac{C_a}{3} (2\xi - 1) \ln\left(\frac{p^2}{k^2}\right), \\ k^2 \tau_3^a &= \frac{C_a}{3} (2 - \xi) \ln\left(\frac{p^2}{k^2}\right), & k^4 \frac{\tau_4^a}{m} &= -\frac{C_a}{12} \xi \ln\left(\frac{p^2}{k^2}\right), \\ k^2 \frac{\tau_5^a}{m} &= -\frac{C_a}{6} \xi \ln\left(\frac{p^2}{k^2}\right), & k^2 \tau_6^a &= \frac{C_a}{6} (1 + \xi) \ln\left(\frac{p^2}{k^2}\right), \\ k^4 \frac{\tau_7^a}{m} &= \frac{C_a}{6} \xi \ln\left(\frac{p^2}{k^2}\right), & k^2 \tau_8^a &= C_a \ln\left(\frac{p^2}{k^2}\right). \end{aligned} \quad (28)$$

For the non-Abelian part, we find

$$\begin{aligned} k^4 \frac{\tau_1^b}{m} &= \frac{C_b}{12} [18 - 5\xi - \xi^2] \ln\left(\frac{p^2}{k^2}\right), \\ k^4 \tau_2^b &= \frac{C_b}{24} [-2 + 7\xi + \xi^2] \ln\left(\frac{p^2}{k^2}\right), \\ k^2 \tau_3^b &= \frac{C_b}{24} (1 - \xi)(4 - \xi) \ln\left(\frac{p^2}{k^2}\right), \\ k^4 \frac{\tau_4^b}{m} &= \frac{C_b}{48} \xi(3 - \xi) \ln\left(\frac{p^2}{k^2}\right), \\ k^2 \frac{\tau_5^b}{m} &= \frac{C_b}{24} [36 - 17\xi + 3\xi^2] \ln\left(\frac{p^2}{k^2}\right), \\ k^2 \tau_6^b &= \frac{C_b}{48} [2 - \xi + 2\xi^2] \ln\left(\frac{p^2}{k^2}\right), \\ k^4 \frac{\tau_7^b}{m} &= -\frac{C_b}{24} \xi(1 + \xi) \ln\left(\frac{p^2}{k^2}\right), \\ k^2 \tau_8^b &= -\frac{C_b}{8} [6 - 5\xi + \xi^2] \ln\left(\frac{p^2}{k^2}\right). \end{aligned} \quad (29)$$

In this asymptotic limit, the leading structures in the massless quark-gluon vertex are those proportional to τ_3 and τ_6 , whose corresponding basis vectors are proportional:

$$T_\mu^{3asy} = -T_\mu^{6asy} = k^2 \gamma_\mu - k k_\mu \equiv T_\mu, \quad (30)$$

thus revealing the linear dependence between them. Naturally, the leading behavior of the massless transverse vertex in this limit thus reads as

$$\begin{aligned} \Gamma_T^\mu(p, k, q) &\stackrel{k^2 \gg p^2}{\cong} (\tau_3 - \tau_6) T^\mu \\ &= -\alpha \frac{C_A(2 - \xi) - 8C_F(1 - \xi)}{64k^2\pi} \ln\frac{p^2}{k^2} T^\mu. \end{aligned} \quad (31)$$

We confirm this result numerically in Fig. 10.

This is the QCD generalization of the QED result already derived in [51]:

$$\begin{aligned} \Gamma_T^\mu(p, k, q) &\stackrel{k^2 \gg p^2}{\cong} (\tau_3 - \tau_6) T^\mu \\ &= \frac{\alpha(1 - \xi)}{8k^2\pi} \ln\frac{p^2}{k^2} T^\mu. \end{aligned} \quad (32)$$

We can carry out a similar analysis for the massive part. The leading contribution in this limit comes from $\tau_{4,5,7}$, whereas, τ_1 chips in with a subleading term. Moreover, it is worth noting that the following tensors are all proportional to each other:

$$\begin{aligned} T_\mu^{4asy} &= -k^2 T_\mu^{5asy} \\ &= -2T_\mu^{7asy} \\ &= k^2(\gamma_\mu k - k_\mu) \\ &= (k^2 \gamma_\mu - k_\mu k) k = T^\mu k. \end{aligned} \quad (33)$$

TABLE I. We compare different vertex *Ansätze* as regards the LKFT for the massless quark propagator. The first three columns define the vertex we consider. The letters correspond to the names of the authors. The fourth column shows whether the quark propagator is MR or not. The last column states the exact exponent of the quark propagator to determine if the vertex complies with the exact prediction of the LKFT for the leading log series, namely $\nu = C_F \alpha \xi / (4\pi)$.

Vertex	Structure	a_i	MR	ν
Bare	γ_μ	\dots	No	
BC [61]	$\Gamma_\mu = \sum_{i=1}^4 \lambda_i L_\mu = \Gamma_\mu^{BC}$	\dots	No	
CP [51]	$\Gamma_\mu = \Gamma_\mu^{BC} + \tau_6 T_\mu^6$	$a_2 = a_3 = a_8 = 0, a_6 = \frac{1}{2}$	Yes	$C_F \alpha \xi / (4\pi)$
BBCR [58]	$\Gamma_\mu = \Gamma_\mu^{BC} + \sum_{i=2,3,6,8} \tau_i T_\mu^i$	$a_6 = -\frac{1}{2}, a_2 + 2(a_3 + a_8) = -2$	Yes	Numerical
ABG [15]	$\Gamma_\mu = \Gamma_\mu^{BC} + \sum_{i=2,3,6,8} \tau_i T_\mu^i$	$a_6 = +\frac{1}{2}, a_2 + 2(a_3 + a_8) = 0$	Yes	$C_F \alpha \xi / (4\pi)$
QCLRS [85]	$\Gamma_\mu = \Gamma_\mu^{BC} + \sum_{i=2,3,6,8} \tau_i T_\mu^i$	$a_2 = a_6 = 0, a_3 = 1/2, a_8 = -1$	Yes	Numerical

One can readily see that the leading terms of $\tau_{4,5,7}$ add up to cancel. Hence, for small mass m , it is still the massless transverse part which is dominant.

Recently, there has been a vertex *Ansatz* proposed in [60]. The form of the vertex is similar to the ones used in Table I. However, the coefficients a_i depend explicitly on the angle between k and p and this dependence, in some cases, continues to persist even in the asymptotic limit $k^2 \gg p^2$, preventing a direct similar comparison.

The massless transverse part of the vertex is crucial in ensuring the MR of the quark propagator, as was discussed in detail in [15]. A popular choice of the vertex *Ansatz* consists in proposing the following form:

$$\tau_2(k^2, p^2) = \frac{a_2 \mathcal{D}_F(k^2, p^2)}{(k^2 + p^2)}, \quad (34a)$$

$$\tau_3(k^2, p^2) = a_3 \mathcal{D}_F(k^2, p^2), \quad (34b)$$

$$\tau_6(k^2, p^2) = a_6 \frac{(k^2 + p^2)}{(k^2 - p^2)} \mathcal{D}_F(k^2, p^2), \quad (34c)$$

$$\tau_8(k^2, p^2) = a_8 \mathcal{D}_F(k^2, p^2), \quad (34d)$$

where

$$\mathcal{D}_F(k^2, p^2) = \frac{1}{(k^2 - p^2)} \left[\frac{1}{F(k^2)} - \frac{1}{F(p^2)} \right].$$

Note that $\mathcal{D}_F(k^2, p^2)$ starts at one-loop perturbation theory and contains a multiplicative color factor C_F at that level, as expected from the one-loop calculation of the quark propagator. Based upon the choice of the a_i , we make contact with different choices of the quark-gluon vertex adopted in the literature. With such a choice of the Abelian-type vertex in QCD, different choices for a_i determine whether a MR solution is possible and if it correctly reproduces leading logarithm behavior to all orders for the quark wave function renormalization, as dictated by the generalized LKFT for QCD introduced in [15], namely, $F(p^2) \propto (p^2)^{\nu = C_F \alpha \xi / (4\pi)}$. We can compare and contrast

different vertex *Ansätze*, as explained in Table I, to see if they permit a MR solution and if the resulting anomalous dimension is $\nu = C_F \alpha \xi / (4\pi)$. The contribution of C_A begins at the next level in perturbation theory.

We now move onto discussing the on-shell limit in the next section.

V. THE ON-SHELL LIMIT

In this section we present some ‘‘physically’’ relevant results for the on-shell limit $p^2 = k^2 = m^2$ and $q^2 = 0$. The Dirac and Pauli form factors, $F_1(q^2)$ and $F_2(q^2)$, respectively, define the Gordon decomposition of the quark current as follows:

$$\begin{aligned} \bar{u}(p) \Gamma_\mu(p, k, q) |_{k^2=p^2=m^2} u(k) \\ = \bar{u}(p) \left\{ F_1(q^2) \gamma_\mu - \frac{F_2(q^2)}{2m} \sigma_{\mu\nu} q^\nu \right\} u(k), \end{aligned}$$

where the spinors, $\bar{u}(p)$ and $u(k)$, satisfy the Dirac equation:

$$\begin{aligned} \bar{u}(p) \not{p} &= m \bar{u}(p), \\ k u(k) &= m u(k). \end{aligned}$$

The anomalous chromomagnetic moment (ACM) of quarks can be identified as $F_2(q^2)$ for $q^2 \rightarrow 0$. The Abelian version of this decomposition with $C_F = 1$ and $C_A = 0$ is the electron-photon vertex of quantum electrodynamics. The great successes of the Dirac equation is the prediction of the magnetic moment of a charged fermion $\boldsymbol{\mu} = eg / (2m) \mathbf{S}$. The radiative corrections lead to [86]

$$\frac{e}{2m} \Rightarrow \left(1 + \frac{\alpha}{2\pi} \right) \frac{e}{2m}. \quad (35)$$

These corrections are now known to a much higher order in perturbation theory [87].

Note that the quark-gluon vertex differs from the electron-photon vertex already at one loop, by the contributions of an additional Feynman diagram, involving the triple-gluon

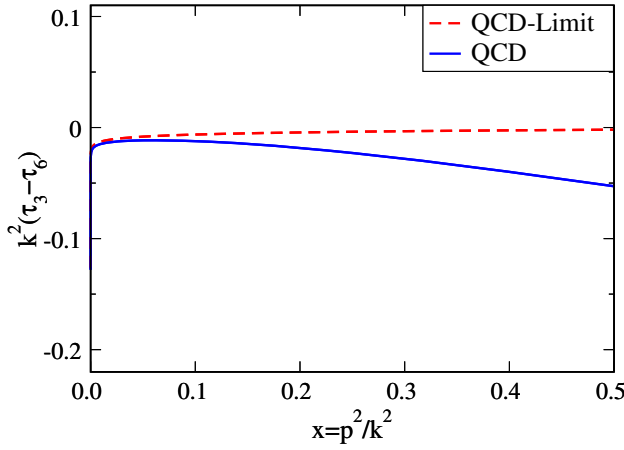


FIG. 10. The dimensionless combination $k^2(\tau_3 - \tau_6)$ of the transverse vertex. The solid (blue) line is the numerical evaluation of the one-loop result. The dashed (red) curve is the asymptotic result, valid for $x \rightarrow 0$ alone. As expected, the numerical result converges onto the asymptotic analytically obtained value for $x \rightarrow 0$.

vertex. In fact, apart from introducing additional color structure, this non-Abelian diagram introduces, at the one-loop level, a kinematical structure which is absent in the QED.

One is naturally tempted to calculate $F_2(q^2)$. It can be expressed in terms of the quark-gluon vertex form factors as follows:

$$F_2(q^2) = -2m\lambda_2^{\text{os}}(q^2) + \lambda_3^{\text{os}}(q^2) - \frac{1}{2}q^2\tau_1^{\text{os}}(q^2) - m^2\tau_2^{\text{os}}(q^2) - \tau_5^{\text{os}}(q^2) + \frac{1}{2}q^2\tau_7^{\text{os}}(q^2) - m\tau_8^{\text{os}}(q^2), \quad (36)$$

where we have introduced as a simplifying notation $\lambda_i^{\text{os}}, \tau_i^{\text{os}}(q^2) \equiv \lambda_i, \tau_i(m^2, m^2, q^2)$. At one-loop perturbation theory, in Landau gauge, the Abelian and non-Abelian contributions for the ACM are plotted in Fig. 11, as a function of gluon momenta q^2 , for a current quark mass $m = 0.115$ GeV, and $\alpha = 0.118$. These contributions can be analytically expressed as

$$F_2^a(q^2) = -\frac{8C_a m^2}{(q^2 - 4m^2)} f(q^2),$$

$$F_2^b(q^2) = \frac{2C_b m^2}{(q^2 - 4m^2)} \left\{ 8m^2 - 2q^2 - 6m^2 q^2 \varphi_1^{\text{os}}(q^2) + (8m^2 + q^2) \ln \left(-\frac{q^2}{m^2} \right) \right\}, \quad (37)$$

where we define $\varphi_1^{\text{os}}(q^2) \equiv \varphi_1(m^2, m^2, q^2)$.

It is straightforward to see that for the soft gluon limit, $q^2 = 0$, the Abelian contribution for the ACM reduces to the non-Abelian counterpart of Schwinger's

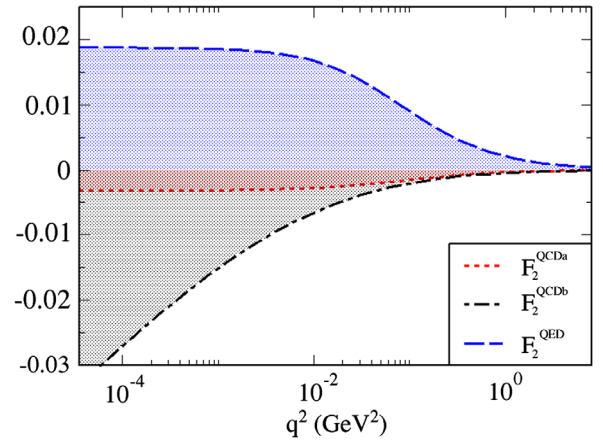


FIG. 11. $F_2(q^2)$ in the on-shell case.

result, $F_2^a(0) = -\alpha/12\pi$, already derived in [46]. On the other hand, the corresponding non-Abelian contribution vanishes for a massless current quark, $m = 0$, as reported in the same article. However, it yields a divergence [88] for a nonzero quark mass, $m \neq 0$. We find this divergence to be logarithmic. For deep infrared gluon momenta it behaves as $F_2^b(q^2 \rightarrow 0) = C_b \ln(-q^2/m^2)$. Of course, perturbation theory in QCD is not the way to explore deep infrared region. All perturbative conclusions will be taken over by nonperturbative effects, overshadowing this divergence.

VI. CONCLUSIONS

In this paper, we give a detailed numerical analysis of all the form factors defining the quark-gluon vertex at the one-loop level in different kinematical limits of interest: symmetric, asymptotic, and on shell. The symmetric limit of momenta is rather well behaved in the infrared region, where all the Abelian form factors converge to finite values. The non-Abelian form factors are only logarithmically divergent. Most noticeably, all the longitudinal form factors are infrared finite. Any nonperturbative construction of these form factors or their computation on the lattice must comply with this requirement. The on-shell limit enables us to compute anomalous chromomagnetic moment of quarks and confirm our numerical computation with the corresponding results known for QED and QCD, [46,86]. The triple-gluon contribution to the ACM of quarks is logarithmically divergent. We find exact analytical expression for this divergence. The asymptotic results have implications for the multiplicative renormalizability of the fermion propagator both in QED and QCD. This connection is exposed through the LKFT, allowing us to analyze various *Ansätze* put forward in the literature. Our study provides us with quantitatively detailed results for several kinematical limits of interest and hence a guideline to all nonperturbative constructions of the corresponding vertices as well as the lattice computations.

ACKNOWLEDGMENTS

This research is supported by CIC (UMSNH) and CONACyT Grants No. 4.10 and No. CB-2014-22117.

-
- [1] P. Maris and C. D. Roberts, *Phys. Rev. C* **56**, 3369 (1997).
- [2] M. S. Bhagwat and P. C. Tandy, *AIP Conf. Proc.* **842**, 225 (2006).
- [3] R. Williams, C. S. Fischer, and M. R. Pennington, *Phys. Lett. B* **645**, 167 (2007).
- [4] P. O. Bowman, U. M. Heller, and A. G. Williams, *Phys. Rev. D* **66**, 014505 (2002).
- [5] M. B. Parappilly, P. O. Bowman, U. M. Heller, D. B. Leinweber, A. G. Williams, and J. B. Zhang, *Phys. Rev. D* **73**, 054504 (2006).
- [6] S. Furui and H. Nakajima, *Phys. Rev. D* **73**, 074503 (2006).
- [7] A. Bashir, L. Chang, I. C. Cloët, B. El-Bennich, Y.-X. Liu, C. D. Roberts, and P. C. Tandy, *Commun. Theor. Phys.* **58**, 79 (2012).
- [8] C. S. Fischer, *J. Phys. G* **32**, R253 (2006).
- [9] I. G. Aznauryan *et al.*, *Int. J. Mod. Phys. E* **22**, 1330015 (2013).
- [10] A. A. Slavnov, *Teor. Mat. Fiz.* **10**, 153 (1972) [*Theor. Math. Phys.* **10**, 99 (1972)].
- [11] J. C. Taylor, *Nucl. Phys.* **B33**, 436 (1971).
- [12] Y. Takahashi, Canonical quantization and generalized Ward relations: Foundation of nonperturbative approach, Print-85-0421 (Alberta), 1985, p. 0019.
- [13] K.-I. Kondo, *Int. J. Mod. Phys. A* **12**, 5651 (1997).
- [14] H.-X. He, F. C. Khanna, and Y. Takahashi, *Phys. Lett. B* **480**, 222 (2000).
- [15] M. J. Aslam, A. Bashir, and L. X. Gutierrez-Guerrero, *Phys. Rev. D* **93**, 076001 (2016).
- [16] A. C. Aguilar and A. A. Natale, *J. High Energy Phys.* **08** (2004) 057.
- [17] A. Cucchieri and T. Mendes, *Proc. Sci.*, LAT2007 (2007) 297.
- [18] I. L. Bogolubsky, E. M. Ilgenfritz, M. Muller-Preussker, and A. Sternbeck, *Proc. Sci.*, LAT2007 (2007) 290 [arXiv:0710.1968].
- [19] A. Cucchieri and T. Mendes, *Proc. Sci.*, QCD-TNT09 (2009) 026 [arXiv:1001.2584].
- [20] I. L. Bogolubsky, E. M. Ilgenfritz, M. Muller-Preussker, and A. Sternbeck, *Phys. Lett. B* **676**, 69 (2009).
- [21] O. Oliveira and P. J. Silva, *Proc. Sci.*, LAT2009 (2009) 226 [arXiv:0910.2897].
- [22] A. Aguilar, D. Binosi, and J. Papavassiliou, *Phys. Rev. D* **78**, 025010 (2008).
- [23] P. Boucaud, J. P. Leroy, A. Le Yaouanc, J. Micheli, O. Pène, and J. Rodríguez-Quintero, *J. High Energy Phys.* **06** (2008) 099.
- [24] M. Pennington and D. Wilson, *Phys. Rev. D* **84**, 119901 (2011).
- [25] C. S. Fischer, A. Maas, and J. M. Pawłowski, *Ann. Phys. (Amsterdam)* **324**, 2408 (2009).
- [26] D. Dudal, S. P. Sorella, N. Vandersickel, and H. Vershelde, *Phys. Rev. D* **77**, 071501 (2008).
- [27] D. Dudal, J. A. Gracey, S. P. Sorella, N. Vandersickel, and H. Vershelde, *Phys. Rev. D* **78**, 065047 (2008).
- [28] D. Dudal, O. Oliveira, and N. Vandersickel, *Phys. Rev. D* **81**, 074505 (2010).
- [29] A. Cucchieri, D. Dudal, T. Mendes, and N. Vandersickel, *Phys. Rev. D* **85**, 094513 (2012).
- [30] J. M. Cornwall, *Phys. Rev. D* **26**, 1453 (1982).
- [31] P. O. Bowman, U. M. Heller, D. B. Leinweber, M. B. Parappilly, A. Sternbeck, L. von Smekal, A. G. Williams, and J.-b. Zhang, *Phys. Rev. D* **76**, 094505 (2007).
- [32] A. Ayala, A. Bashir, D. Binosi, M. Cristoforetti, and J. Rodríguez-Quintero, *Phys. Rev. D* **86**, 074512 (2012).
- [33] A. C. Aguilar, D. Binosi, and J. Papavassiliou, *Phys. Rev. D* **86**, 014032 (2012).
- [34] A. Bashir, A. Raya, and J. Rodríguez-Quintero, *Phys. Rev. D* **88**, 054003 (2013).
- [35] A. Bashir, C. Calcano-Roldan, L. X. Gutierrez-Guerrero, and M. E. Tejada-Yeomans, *Phys. Rev. D* **83**, 033003 (2011).
- [36] F. Akram, A. Bashir, L. X. Gutierrez-Guerrero, B. Masud, J. Rodríguez-Quintero, C. Calcano-Roldan, and M. E. Tejada-Yeomans, *Phys. Rev. D* **87**, 013011 (2013).
- [37] S. Strauss, C. S. Fischer, and C. Kellermann, *Phys. Rev. Lett.* **109**, 252001 (2012).
- [38] J. Skullerud and A. Kizilersu, *J. High Energy Phys.* **09** (2002) 013.
- [39] J. I. Skullerud, P. O. Bowman, A. Kizilersu, D. B. Leinweber, and A. G. Williams, *J. High Energy Phys.* **04** (2003) 047.
- [40] J.-I. Skullerud, P. O. Bowman, A. Kizilersu, D. B. Leinweber, and A. G. Williams, *Nucl. Phys. B, Proc. Suppl.* **2005** 244, **141**.
- [41] A. Kizilersu, D. B. Leinweber, J.-I. Skullerud, and A. G. Williams, *Eur. Phys. J. C* **50**, 871 (2007).
- [42] M. S. Bhagwat and P. C. Tandy, *Phys. Rev. D* **70**, 094039 (2004).
- [43] M. S. Bhagwat, A. Holl, A. Krassnigg, C. D. Roberts, and P. C. Tandy, *Phys. Rev. C* **70**, 035205 (2004).
- [44] F. J. Llanes-Estrada, C. S. Fischer, and R. Alkofer, *Nucl. Phys. B, Proc. Suppl.* **152**, 43 (2006).
- [45] R. Williams, *Eur. Phys. J.* **A51**, 57 (2015).
- [46] L. Chang, Y.-X. Liu, and C. D. Roberts, *Phys. Rev. Lett.* **106**, 072001 (2011).
- [47] L. Chang and C. D. Roberts, *Phys. Rev. Lett.* **103**, 081601 (2009).
- [48] K. Raya, L. Chang, A. Bashir, J. J. Cobos-Martinez, L. X. Gutierrez-Guerrero, C. D. Roberts, and P. C. Tandy, *Phys. Rev. D* **93**, 074017 (2016).

- [49] K. Raya, M. Ding, A. Bashir, L. Chang, and C. D. Roberts, [arXiv:1610.06575](#).
- [50] I. C. Cloet, C. D. Roberts, and A. W. Thomas, *Phys. Rev. Lett.* **111**, 101803 (2013).
- [51] D. C. Curtis and M. R. Pennington, *Phys. Rev. D* **42**, 4165 (1990).
- [52] A. Bashir and M. R. Pennington, *Phys. Rev. D* **50**, 7679 (1994).
- [53] A. Bashir, A. Kizilersu, and M. R. Pennington, *Phys. Rev. D* **57**, 1242 (1998).
- [54] A. Bashir, A. Kizilersu, and M. R. Pennington, [arXiv:hep-ph/9907418](#).
- [55] A. Bashir, Y. Concha-Sanchez, and R. Delbourgo, *Phys. Rev. D* **76**, 065009 (2007).
- [56] A. Kizilersu and M. Pennington, *Phys. Rev. D* **79**, 125020 (2009).
- [57] A. Bashir, A. Raya, and S. Sanchez-Madrigal, *Phys. Rev. D* **84**, 036013 (2011).
- [58] A. Bashir, R. Bermudez, L. Chang, and C. Roberts, *Phys. Rev. C* **85**, 045205 (2012).
- [59] L. A. Fernandez-Rangel, A. Bashir, L. X. Gutierrez-Guerrero, and Y. Concha-Sanchez, *Phys. Rev. D* **93**, 065022 (2016).
- [60] D. Binosi, L. Chang, J. Papavassiliou, S.-X. Qin, and C. D. Roberts, [arXiv:1609.02568](#) [*Phys. Rev. D* (to be published)].
- [61] J. S. Ball and T.-W. Chiu, *Phys. Rev. D* **22**, 2542 (1980).
- [62] A. Kizilersu, M. Reenders, and M. R. Pennington, *Phys. Rev. D* **52**, 1242 (1995).
- [63] A. I. Davydychev, P. Osland, and L. Saks, *Phys. Rev. D* **63**, 014022 (2000).
- [64] A. C. Aguilar and J. Papavassiliou, *Phys. Rev. D* **83**, 014013 (2011).
- [65] A. Ayala and A. Bashir, *Phys. Rev. D* **64**, 025015 (2001).
- [66] W. Celmaster and R. J. Gonsalves, *Phys. Rev. D* **20**, 1420 (1979).
- [67] N. Brown and N. Dorey, *Mod. Phys. Lett. A* **06**, 317 (1991).
- [68] J. C. Ward, *Phys. Rev.* **78**, 182 (1950).
- [69] E. S. Fradkin, *Zh. Eksp. Teor. Fiz.* **29**, 258 (1955) [*Sov. Phys. JETP* **2**, 361 (1956)].
- [70] H. S. Green, *Proc. Phys. Soc. London Sect. A* **66**, 873 (1953).
- [71] Y. Takahashi, *Nuovo Cimento* **6**, 371 (1957).
- [72] Z.-h. Dong, H. J. Munczek, and C. D. Roberts, *Phys. Lett. B* **333**, 536 (1994).
- [73] A. Bashir and M. R. Pennington, *Phys. Rev. D* **53**, 4694 (1996).
- [74] M. R. Pennington and S. Jia, *Phys. Rev. D* **94**, 116004 (2016).
- [75] S. Jia and M. R. Pennington, [arXiv:1610.06436](#).
- [76] N. Ahmadinia, A. Bashir, and C. Schubert, [arXiv:1609.03034](#).
- [77] N. Ahmadinia, A. Bashir, and C. Schubert, *Phys. Rev. D* **93**, 045023 (2016).
- [78] A. Bashir, A. Raya, and S. Sanchez-Madrigal, *J. Phys. A* **41**, 505401 (2008).
- [79] A. Bashir and A. Raya, *Few-Body Syst.* **41**, 185 (2007).
- [80] A. Bashir and A. Raya, *Nucl. Phys.* **B709**, 307 (2005).
- [81] A. Bashir and R. Delbourgo, *J. Phys. A* **37**, 6587 (2004).
- [82] A. Bashir and A. Raya, *Phys. Rev. D* **66**, 105005 (2002).
- [83] A. Bashir, *Phys. Lett. B* **491**, 280 (2000).
- [84] J. C. R. Bloch, *Phys. Rev. D* **66**, 034032 (2002).
- [85] S.-X. Qin, L. Chang, Y.-X. Liu, C. D. Roberts, and S. M. Schmidt, *Phys. Lett. B* **722**, 384 (2013).
- [86] J. S. Schwinger, *Phys. Rev.* **73**, 416 (1948).
- [87] T. Aoyama, M. Hayakawa, T. Kinoshita, and M. Nio, *Phys. Rev. D* **85**, 033007 (2012).
- [88] I. D. Choudhury and A. Lahiri, *Mod. Phys. Lett. A* **30**, 1550113 (2015).