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Revisiting the $B^0 \to \pi^0\pi^0$ decays in the perturbative QCD approach

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We recalculate the branching ratio and CP asymmetry for $\bar{B}^0(B^0) \to \pi^0\pi^0$ decays in the perturbative QCD approach. In this approach, we consider all the possible diagrams including nonfactorizable contributions and annihilation contributions. We obtain $\mathrm{Br}(\bar{B}^0(B^0) \to \pi^0\pi^0) = (1.17^{+0.11}_{-0.12}) \times 10^{-6}$. Our result is in agreement with the latest measured branching ratio of $B^0 \to \pi^0\pi^0$ by the Belle and HFAG collaborations. We also predict large direct CP asymmetry and mixing CP asymmetry in $B^0 \to \pi^0\pi^0$ decays, which can be tested by the coming Belle-II experiments.

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I. INTRODUCTION

The detailed study of B meson decays is a key source of testing the standard model (SM), exploring CP violation, and in searching for possible new physics beyond the SM. The theoretical studies of B meson decays have been explored widely in the literature, especially the nonleptonic two-body branching ratios and their CP asymmetries. Although we have achieved great success in explaining many decay branching ratios, there are still some puzzles remaining. One of the challenges is that the measured branching ratio [1–3] for the decay of the B meson to neutral pion pairs $B^0 \to \pi^0\pi^0$ is significantly larger than the theoretical predictions obtained in the QCD factorization approach (QCDF) [4–7] or a perturbative QCD approach (PQCD) [8].

For a long time, the factorization approach [9] was the method we widely use to estimate the decays [10,11]. Although the way is an easy method at predictions of branching ratios and in accord with experiments in most cases, there are still some unclear theoretical points. In order to study the nonleptonic B decays better, QCD factorization [12] and perturbative QCD approach [13] were invented. The basic idea of the PQCD method is that the transverse momenta k_T of valence quarks are considered in the calculations of hadronic matrix elements, and then for B meson decays, nonfactorizable spectator and annihilation contributions are all calculable in the framework of k_T factorization, where three energy scales m_W, m_B, and $t \approx \sqrt{m_B \Lambda_{\mathrm{QCD}}}$ are involved [8,13,14].

The branching ratio of $B^0 \to \pi^0\pi^0$ has been measured, whose data [15] are

$$\begin{pmatrix} (1.83 \pm 0.21 \pm 0.13) \times 10^{-6}; (BABAR), \\ (0.90 \pm 0.12 \pm 0.10) \times 10^{-6}; (\text{Belle}) \\ (1.17 \pm 0.13) \times 10^{-6}, (\text{HFAG}) \end{pmatrix}. \quad (1)$$

In the last ten or more years, many theoretical teams have calculated this decay in different approaches. Beneke and Neubert made the analysis of $B^0 \to \pi^0\pi^0$ decay based on QCD factorization in 2003 [5]. Recently, Qin Chang [16], Xin Liu [17] and Cong-Feng Qiao *et al.* [18] recalculated this decay model using a different method. The next-leading-order (NLO) contributions from the vertex corrections, the quark loops, and the magnetic penguins have also been calculated in the literature [19–22]. By comparing their results, we find that the agreement between the theoretical predictions and the experimental data is still not satisfactory, so we revisit the decays of $B^0 \to \pi^0\pi^0$ in this paper. We use the PQCD approach to recalculate this decay directly; nonfactorizable contributions and annihilation contributions are all taken into account. Our theoretical formulas about the decay $\bar{B}^0 \to \pi^0\pi^0$ in the PQCD framework are given in the next section. In Sec. III we give the numerical results and discussions of the branching ratio and CP asymmetries. In the end, we give a short summary in Sec. IV.

II. THE FRAMEWORK AND PERTURBATIVE CALCULATIONS

For the considered $\bar{B}^0 \to \pi^0\pi^0$ decays, the corresponding weak effective Hamiltonian can be given as [23].

$$H_{\mathrm{eff}} = \frac{G_F}{\sqrt{2}} \left\{ V_{ud}^* V_{ub} [C_1(\mu) O_1(\mu) + C_2(\mu) O_2(\mu)] - V_{td}^* V_{tb} \left[\sum_{i=3}^{10} C_i(\mu) O_i(\mu) \right] \right\} + \mathrm{H.c.}, \quad (2)$$

where G_F is the Fermi constant, $C_i(\mu) (i = 1, \ldots, 10)$ are Wilson coefficients at the renormalization scale μ, and $O_i (i = 1, \ldots, 10)$ are the following four-quark operators:

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(1) current-current (tree) operators

$$O_1 = (\bar{u}_\alpha u_\alpha)_{V-A}(\bar{d}_\beta b_\beta)_{V-A},$$
$$O_2 = (\bar{u}_\alpha b_\alpha)_{V-A}(\bar{d}_\beta u_\beta)_{V-A}; \tag{3}$$

(2) QCD penguin operators

$$O_3 = (\bar{d}_\alpha b_\alpha)_{V-A}\sum_q(\bar{q}_\beta q_\beta)_{V-A},$$
$$O_4 = (\bar{d}_\alpha b_\beta)_{V-A}\sum_q(\bar{q}_\beta q_\alpha)_{V-A},$$
$$O_5 = (\bar{d}_\alpha b_\alpha)_{V-A}\sum_q(\bar{q}_\beta q_\beta)_{V+A},$$
$$O_6 = (\bar{d}_\alpha b_\beta)_{V-A}\sum_q(\bar{q}_\beta q_\alpha)_{V+A}; \tag{4}$$

(3) electroweak penguin operators

$$O_7 = \frac{3}{2}(\bar{d}_\alpha b_\alpha)_{V-A}\sum_q e_q(\bar{q}_\beta q_\beta)_{V+A},$$
$$O_8 = \frac{3}{2}(\bar{d}_\alpha b_\beta)_{V-A}\sum_q e_q(\bar{q}_\beta q_\alpha)_{V+A},$$
$$O_9 = \frac{3}{2}(\bar{d}_\alpha b_\alpha)_{V-A}\sum_q e_q(\bar{q}_\beta q_\beta)_{V-A},$$
$$O_{10} = \frac{3}{2}(\bar{d}_\alpha b_\beta)_{V-A}\sum_q e_q(\bar{q}_\beta q_\alpha)_{V-A}. \tag{5}$$

Here α and β are $SU(3)$ color indices. Then the calculation of the decay amplitude is made by computing the hadronic matrix elements of the local operators.

In the PQCD, the soft (Φ), hard (H), and harder (C) dynamics characterized by different scales make up the decay amplitude. It is conceptually written as follows:

$$\text{Amplitude} \sim \int d^4k_1 d^4k_2 d^4k_3 \mathrm{Tr}[C(t)\Phi_{\bar{B}^0}(k_1)\Phi_{\pi^0}(k_2) \times \Phi_{\pi^0}(k_3)H(k_1,k_2,k_3,t)], \tag{6}$$

where k_i are the momenta of light quarks included in each meson, and Tr denotes the trace over Dirac and color indices. The Wilson coefficient $C(t)$ results from the radiative corrections at short distance. The nonperturbative part is absorbed into wave function Φ_M, which is universal and channel independent. H describes the four-quark operator and the quark pair produced by a gluon whose scale is at the order of M_B, so this hard part H can be perturbatively calculated.

We consider the B meson to be at rest for simplicity and assume the light final state pion meson moving along the direction of $n = (1, 0, 0_T)$ and $v = (0, 1, 0_T)$. It is convenient to use the light-cone coordinate (P^+, P^-, P_T) to describe the meson's momenta, where $P^\pm = \frac{1}{\sqrt{2}}(p^0 \pm p^3)$, $P_T = (p^1, p^2)$. Working at the rest frame of the $\bar{B}^0$ meson, the momenta of $\bar{B}^0$, π^0, and π^0 can be written as follows:

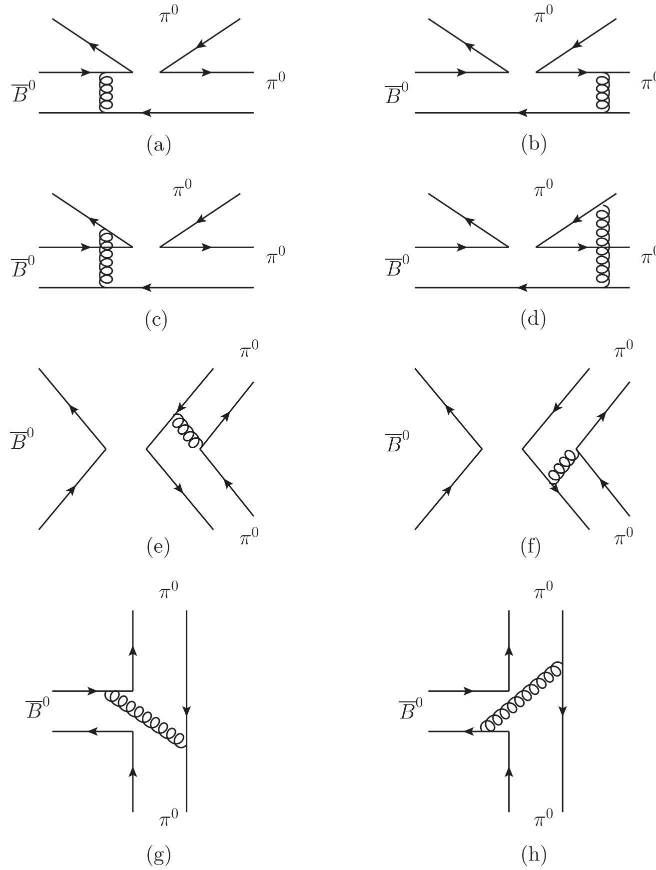

FIG. 1. Typical Feynman diagrams contributing to the $\bar{B}^0 \to \pi^0\pi^0$ decays in the PQCD approach at leading order.

$$P_1 = \frac{M_B}{\sqrt{2}}(1, 1, 0_T),$$
$$P_2 = \frac{M_B}{\sqrt{2}}(0, 1, 0_T),$$
$$P_3 = \frac{M_B}{\sqrt{2}}(1, 0, 0_T). \tag{7}$$

Putting the light (anti) quark momenta in $\bar{B}^0$, π^0, and π^0 as k_1, k_2, k_3, respectively, we can choose

$$k_1 = (x_1 p_1^+, 0, k_{1T}),$$
$$k_2 = (0, x_2 p_2^-, k_{2T}),$$
$$k_3 = (x_3 p_3^+, 0, k_{3T}). \tag{8}$$

Then, integrating over k_1^+, k_2^-, k_3^+ in Eq. (6) leads to

$$\text{Amplitude} \sim \int d^4x_1 d^4x_2 d^4x_3 b_1 db_1 b_2 db_2 b_3 db_3 \times \mathrm{Tr}[C(t)\Phi_{\bar{B}^0}(x_1, b_1)\Phi_{\pi^0}(x_2, b_2)\Phi_{\pi^0}(x_3, b_3) \times H(x_i, b_i, t)]e^{-S(t)}, \tag{9}$$

where b_i is the conjugate space coordinate of k_{iT}, and t the largest energy scale in H. The exponential Sudakov factor $e^{-S(t)}$ comes from higher order radiative corrections to wave functions and hard amplitudes; it suppresses the soft dynamics effectively [24] and thus makes a reliable perturbative calculation of the hard part H.

Figure 1 shows the lowest order diagrams to be calculated in the PQCD approach for $\bar{B}^0 \to \pi^0\pi^0$ decay. The sum contributions of the nonfactorizable diagrams (a) and (b) that come from the operator O_2 are

$$\begin{aligned}\mathcal{M}_a = &\frac{-1}{\sqrt{2N_c}} 32\pi C_F M_B^2 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \Phi_B(x_1,b_1)\{[(x_2-2)\Phi_\pi^A(x_2)\Phi_\pi^A(x_3) + r_\pi(1-2x_2)\Phi_\pi^T(x_2)\Phi_\pi^A(x_3)\\ &+ r_\pi(1-2x_2)\Phi_\pi^P(x_2)\Phi_\pi^A(x_3)]\alpha_s(t_a^1)h_a^1(x_1,x_2,x_3,b_1,b_2)\exp[-S_B(t_a^1) - S_\pi(t_a^1) - S_\pi(t_a^1)]C(t_a^1)\\ &- 2r_\pi\Phi_\pi^P(x_2)\Phi_\pi^A(x_3)\alpha_s(t_a^2)h_a^2(x_1,x_2,x_3,b_1,b_2)\exp[-S_B(t_a^2) - S_\pi(t_a^2) - S_\pi(t_a^2)]C(t_a^2)\},\end{aligned} \tag{10}$$

where $C_F = 4/3$ is the group factor of the $SU(3)_c$ gauge group and $r_\pi = M_{0\pi}/M_B$. The wave function Φ_M, the functions $h_a^{1,2}(x_1,x_2,x_3,b_1,b_2)$, and the Sudakov factor $S_X(t_i)(X = \bar{B}^0, \pi^0, \pi^0)$ are given in the appendix.

The total contribution for the nonfactorizable diagrams (c) and (d) is

$$\begin{aligned}\mathcal{M}_c = &\frac{-1}{\sqrt{2N_c}} 32\pi C_F M_B^2 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \Phi_B(x_1,b_3)\{[\Phi_\pi^A(x_2)\Phi_\pi^A(x_3)(1-x_1-x_3)\\ &+ r_\pi\Phi_\pi^P(x_2)\Phi_\pi^A(x_3)(1-x_2) + r_\pi\Phi_\pi^T(x_2)\Phi_\pi^A(x_3)(1-x_2)]\alpha_s(t_c^1)h_c^1(x_1,x_2,x_3,b_2,b_3)\\ &\times\exp[-S_B(t_c^1) - S_\pi(t_c^1) - S_\pi(t_c^1)]C(t_c^1) + [-\Phi_\pi^A(x_2)\Phi_\pi^A(x_3)(1+x_3-x_1-x_2) - r_\pi\Phi_\pi^P(x_2)\Phi_\pi^A(x_3)(1-x_2)\\ &+ r_\pi\Phi_\pi^T(x_2)\Phi_\pi^A(x_3)(1-x_2)]\alpha_s(t_c^2)h_c^2(x_1,x_2,x_3,b_2,b_3)\exp[-S_B(t_c^2) - S_\pi(t_c^2) - S_\pi(t_c^2)]C(t_c^2)\}.\end{aligned} \tag{11}$$

The factorizable annihilation diagrams (e) and (f) that come from the operators O_1, O_3, O_4, O_5, O_6, O_7, O_8, O_9, O_{10} involve only two light meson wave functions. M_e is for $(V-A)(V-A)$ and $(V-A)(V+A)$-type operators, and M_e^P is for $(1+\gamma_5)(1-\gamma_5)$-type operators,

$$\begin{aligned}\mathcal{M}_e = &8S\pi C_F M_B^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \{[-\Phi_\pi^A(x_2)\Phi_\pi^A(x_3)x_2 - 2r_\pi^2\Phi_\pi^P(x_2)\Phi_\pi^P(x_3)(1+x_2)\\ &+ 2r_\pi^2\Phi_\pi^T(x_2)\Phi_\pi^P(x_3)(x_2-1)]\alpha_s(t_e^1)h_e^1(x_2,x_3,b_2,b_3)\exp[-S_\pi(t_e^1) - S_\pi(t_e^1)]C(t_e^1)\\ &+ [\Phi_\pi^A(x_2)\Phi_\pi^A(x_3)x_3 + 2r_\pi^2\Phi_\pi^P(x_2)\Phi_\pi^P(x_3)(1+x_3) + 2r_\pi^2\Phi_\pi^P(x_2)\Phi_\pi^T(x_3)(1-x_3)]\\ &\times\alpha_s(t_e^2)h_e^2(x_2,x_3,b_2,b_3)\exp[-S_\pi(t_e^2) - S_\pi(t_e^2)]C(t_e^2)\},\end{aligned} \tag{12}$$

$$\begin{aligned}\mathcal{M}_e^P = &8S\pi C_F M_B^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \{[-r_\pi\Phi_\pi^P(x_2)\Phi_\pi^A(x_3)x_2 - r_\pi\Phi_\pi^T(x_2)\Phi_\pi^A(x_3)x_2\\ &- 2r_\pi\Phi_\pi^A(x_2)\Phi_\pi^P(x_3)]\alpha_s(t_e^1)h_e^1(x_2,x_3,b_2,b_3)\exp[-S_\pi(t_e^1) - S_\pi(t_e^1)]C(t_e^1) + [-2r_\pi\Phi_\pi^P(x_2)\Phi_\pi^A(x_3)\\ &- r_\pi\Phi_\pi^A(x_2)\Phi_\pi^P(x_3)x_3 - r_\pi\Phi_\pi^A(x_2)\Phi_\pi^T(x_3)x_3]\alpha_s(t_e^2)h_e^2(x_2,x_3,b_2,b_3)\exp[-S_\pi(t_e^2) - S_\pi(t_e^2)]C(t_e^2)\},\end{aligned} \tag{13}$$

where $S = 2$ comes from the requirement of the identity principle. The nonfactorizable annihilation diagrams (g) and (h) come from the operators O_4, O_6, O_8, O_{10}. M_g is the contribution containing the operator of type $(V-A)(V-A)$, and M_g^P is the contribution containing the operator of type $(1+\gamma_5)(1-\gamma_5)$.

$$\begin{aligned}\mathcal{M}_g = &\frac{1}{\sqrt{2N_c}} 32S\pi C_F M_B^2 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \Phi_B(x_1,b_1)\{[(x_1+x_3)\Phi_\pi^A(x_2)\Phi_\pi^A(x_3)\\ &+ r_\pi^2(2+x_1+x_2+x_3)\Phi_\pi^P(x_2)\Phi_\pi^P(x_3) - r_\pi^2\Phi_\pi^P(x_2)\Phi_\pi^T(x_3)(x_2-x_1-x_3) + r_\pi^2\Phi_\pi^T(x_2)\Phi_\pi^P(x_3)(x_1+x_3-x_2)\\ &- r_\pi^2\Phi_\pi^T(x_2)\Phi_\pi^T(x_3)(2-x_1-x_2-x_3)]\alpha_s(t_g^1)h_g^1(x_1,x_2,x_3,b_1,b_2)\exp[-S_B(t_g^1) - S_\pi(t_g^1) - S_\pi(t_g^1)]C(t_g^1)\\ &+ [-\Phi_\pi^A(x_2)\Phi_\pi^A(x_3)x_2 + r_\pi^2\Phi_\pi^P(x_2)\Phi_\pi^P(x_3)(x_1-x_2-x_3) - r_\pi^2\Phi_\pi^P(x_2)\Phi_\pi^T(x_3)(x_1-x_3+x_2)\\ &- r_\pi^2\Phi_\pi^T(x_2)\Phi_\pi^P(x_3)(x_1-x_3+x_2) + r_\pi^2\Phi_\pi^T(x_2)\Phi_\pi^T(x_3)(x_1-x_2-x_3)]\alpha_s(t_g^2)\\ &\times h_g^2(x_1,x_2,x_3,b_1,b_2)\exp[-S_B(t_g^2) - S_\pi(t_g^2) - S_\pi(t_g^2)]C(t_g^2)\},\end{aligned} \tag{14}$$

$$\mathcal{M}_g^P = \frac{-1}{\sqrt{2N_c}} 32S\pi C_F M_B^2 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \Phi_B(x_1, b_1)\{[-\Phi_\pi^A(x_2)\Phi_\pi^A(x_3)x_2 - r_\pi^2(2 + x_1 + x_2 + x_3)\Phi_\pi^P(x_2)\Phi_\pi^P(x_3) - r_\pi^2\Phi_\pi^P(x_2)\Phi_\pi^T(x_3)(x_2 - x_1 - x_3) + r_\pi^2\Phi_\pi^T(x_2)\Phi_\pi^P(x_3)(x_1 + x_3 - x_2) + r_\pi^2\Phi_\pi^T(x_2)\Phi_\pi^T(x_3)(x_1 + x_2 + x_3 - 2)]\alpha_s(t_g^1)h_g^1(x_1, x_2, x_3, b_1, b_2)\exp[-S_B(t_g^1) - S_\pi(t_g^1) - S_\pi(t_g^1)]C(t_g^1) + [-\Phi_\pi^A(x_2)\Phi_\pi^A(x_3)(x_1 - x_3) - r_\pi^2\Phi_\pi^P(x_2)\Phi_\pi^P(x_3)(x_1 - x_2 - x_3) - r_\pi^2\Phi_\pi^P(x_2)\Phi_\pi^T(x_3)(x_1 - x_3 + x_2) + r_\pi^2\Phi_\pi^T(x_2)\Phi_\pi^P(x_3)(x_1 + x_2 - x_3) - r_\pi^2\Phi_\pi^T(x_2)\Phi_\pi^T(x_3)(x_2 + x_3 - x_1)]\alpha_s(t_g^2)h_g^2(x_1, x_2, x_3, b_1, b_2) \times \exp[-S_B(t_g^2) - S_\pi(t_g^2) - S_\pi(t_g^2)]C(t_g^2)\}. \tag{15}$$

The total decay amplitude of $\bar{B}^0 \to \pi^0\pi^0$ is then

$$\bar{\mathcal{A}}(\bar{B}^0 \to \pi^0\pi^0) = V_{ud}^* V_{ub}[C_1\mathcal{M}_e f_B + C_2(\mathcal{M}_a + \mathcal{M}_c)] - V_{td}^* V_{tb}\left[\left(2C_3 + \frac{5}{3}C_4 + 2C_5 + \frac{2}{3}C_6 + \frac{1}{2}C_7 + \frac{1}{6}C_8 + \frac{1}{2}C_9 - \frac{1}{3}C_{10}\right)\mathcal{M}_e f_B + \left(C_6 - \frac{1}{2}C_8\right)\mathcal{M}_e^P f_B + \left(2C_4 + \frac{1}{2}C_{10}\right)\mathcal{M}_g + \left(2C_6 + \frac{1}{2}C_8\right)\mathcal{M}_g^P\right] \tag{16}$$

and the decay width is expressed as

$$\Gamma(\bar{B}^0 \to \pi^0\pi^0) = \frac{G_F^2 M_B^3}{128\pi}|\bar{\mathcal{A}}(\bar{B}^0 \to \pi^0\pi^0)|^2. \tag{17}$$

The decay amplitude of the charge conjugate channel for $\bar{B}^0 \to \pi^0\pi^0$ can be obtained by replacing $V_{ud}^* V_{ub}$ with $V_{ud}V_{ub}^*$ and $V_{td}^* V_{tb}$ with $V_{td}V_{tb}^*$ in Eq. (16). The decay amplitude of $\bar{B}^0 \to \pi^0\pi^0$ in Eq. (16) can be parametrized as

$$\bar{\mathcal{A}} = V_{ud}^* V_{ub} T - V_{td}^* V_{tb} P = V_{ud}^* V_{ub} T[1 + z e^{i(-\alpha+\delta)}], \tag{18}$$

where $z = |V_{td}^* V_{tb}/V_{ud}^* V_{ub}||P/T|$, and $\delta = \arg(P/T)$ is the relative strong phase between tree diagrams T and penguin diagrams P. z and δ can be calculated from PQCD.

Similarly, the decay amplitude for $B^0 \to \pi^0\pi^0$ can be parametrized as

$$\mathcal{A} = V_{ub}^* V_{ud} T - V_{tb}^* V_{td} P = V_{ub}^* V_{ud} T[1 + z e^{i(\alpha+\delta)}]. \tag{19}$$

III. NUMERICAL EVALUATION AND DISCUSSIONS OF RESULTS

The parameters that have been used in numerical calculation [1,2,25–27] are shown in Table I.

We leave the Cabibbo-Kobayashi-Maskawa (CKM) phase angle α as a free parameter to explore the branching ratio and CP asymmetry. From Eqs. (18) and (19), we get the averaged decay width for $\bar{B}^0(B^0) \to \pi^0\pi^0$,

$$\Gamma(\bar{B}^0(B^0) \to \pi^0\pi^0) = \frac{G_F^2 M_B^3}{128\pi}\left(\frac{|\mathcal{A}|^2}{2} + \frac{|\bar{\mathcal{A}}|^2}{2}\right) = \frac{G_F^2 M_B^3}{128\pi}|V_{ud}^* V_{ub} T|^2 \times [1 + 2z\cos(\alpha)\cos(\delta) + z^2]. \tag{20}$$

Using the above parameters, we get $z = 0.52$ and $\delta = 106°$ in PQCD. Equation (20) is a function of the CKM angle α. In Fig. 2, we plot the averaged branching ratio of the decay $\bar{B}^0(B^0) \to \pi^0\pi^0$ with respect to the parameter α. Since the CKM angle α is constrained as α around 85° [26],

$$\alpha = (85.4^{+3.9}_{-3.8})°, \tag{21}$$

we can arrive from Fig. 2,

$$1.15 \times 10^{-6} < \mathrm{Br}(\bar{B}^0(B^0) \to \pi^0\pi^0) < 1.18 \times 10^{-6}, \quad \text{for } 80° < \alpha < 90°. \tag{22}$$

The number $z = |V_{td}^* V_{tb}/V_{ud}^* V_{ub}||P/T| = 0.52$ means that the amplitude of penguin diagrams is about 0.52 times

TABLE I. The values of parameters adopted in numerical evaluation.

Parameters	$\Lambda_{\mathrm{QCD}}^{f=4}$	m_W	m_B	f_π	f_B	$m_{0\pi}$	τ_{B^0}	$\lvert V_{ud}^* V_{ub}\rvert$	$\lvert V_{td}^* V_{tb}\rvert$
Values	0.25 GeV	80.41 GeV	5.280 GeV	0.13 GeV	0.21 GeV	1.4 GeV	1.55×10^{-12} s	0.00346	0.00885

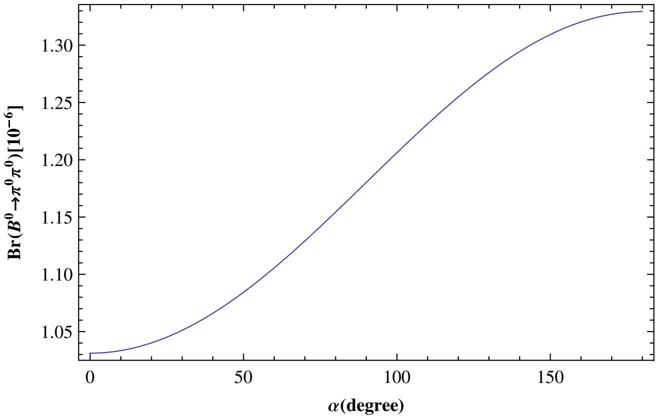

FIG. 2. The averaged branching ratio of $\bar{B}^0(B^0) \to \pi^0\pi^0$ decay as a function of CKM angle α.

that of tree diagrams, which shows that though the tree contribution dominates this decay, the penguin contribution cannot be ignored, i.e., there are large contributions from both tree diagrams and penguin diagrams.

Besides the phase angle α, the major theoretical errors come from the uncertainties of $\omega_b = 0.4 \pm 0.04$ GeV, $f_B = 0.21 \pm 0.02$ GeV, and the Gegenbauer moment $a_2^\pi = 0.25 \pm 0.15$. Taking into account the uncertainties of these parameters, we find

$$\mathrm{Br}(\bar{B}^0(B^0) \to \pi^0\pi^0) = [1.17^{+0.09}_{-0.08}(\omega_b)^{+0.05}_{-0.07}(f_B)^{+0.02}_{-0.06}(a_2^\pi)] \times 10^{-6}. \quad (23)$$

When all important theoretical errors from different sources, including those from the uncertainty of phase angle α, are added in quadrature, we get $\mathrm{Br}(\bar{B}^0(B^0) \to \pi^0\pi^0) = (1.17^{+0.11}_{-0.12}) \times 10^{-6}$.

In the literature, there already exist a lot of studies on $B^0 \to \pi^0\pi^0$ decay. We offer some recent works devoted to the resolution of the challenge.

(a) In Ref. [16], Qin Chang *et al.* perform a global fit on the spectator scattering and annihilation parameters $X_H(\rho_H, \phi_H)$, $X_A^i(\rho_A^i, \phi_A^i)$ and $X_A^f(\rho_A^f, \phi_A^f)$ for the available experimental data for $B_{u,d} \to \pi\pi, \pi K$ and $K\bar{K}$ decays in the QCDF framework. They obtained large $B^0 \to \pi^0\pi^0$ branching ratios $(1.67^{+0.33}_{-0.30}) \times 10^{-6}$ and $(2.13^{+0.43}_{-0.38}) \times 10^{-6}$ for different scenarios.

(b) In Ref. [17], Xin Liu *et al.* investigate the Glauber-gluon effect on the $B \to \pi\pi$ and $\rho\rho$ decays based on the k_T factorization theorem; they observed significant modification of the $B^0 \to \pi^0\pi^0$ branching ratio through a transverse-momentum-dependent wave function for the pion with a weak falloff in parton transverse momentum k_T. They get the branching ratio of $B^0 \to \pi^0\pi^0$ 0.61×10^{-6}.

(c) In Ref. [18], Cong-Feng Qiao *et al.* give a possible solution to the $B \to \pi\pi$ puzzle using the principle of maximum conformality (PMC). They applied the PMC procedure to the QCDF analysis with the goal of eliminating the renormalization scale ambiguity and achieving an accurate pQCD prediction that is independent of theoretical conventions. They found that the pQCD prediction is highly sensitive to the choice of the renormalization scale that enters the decay amplitude; they obtained $\mathrm{Br}(B_d \to \pi^0\pi^0) = (0.98^{+0.44}_{-0.31}) \times 10^{-6}$ by applying the principle of maximum conformality. However, we find that the PQCD prediction is not sensitive to the choice of the renormalization scale for this decay based on our calculation. In our approach, we set the renormalization scale $\mu = t$ (the largest energy scale in H) to diminish the large logarithmic radiative corrections and minimize the NLO contributions to the form factors. By changing the hard scale t from $0.9t$ to $1.3t$, we find that the branching ratio of $B^0 \to \pi^0\pi^0$ changes a little. The choice of the renormalization scale is not a main reason for the $B^0 \to \pi^0\pi^0$ puzzle, even when the NLO contributions are taken into account [28].

(d) In Ref. [28], Ya-Lan Zhang *et al.* performed a systematic study for the $B \to (\pi^+\pi^-, \pi^+\pi^0, \pi^0\pi^0)$ decays in the PQCD factorization approach with the inclusion of all currently known NLO contributions from various sources. They got the NLO PQCD prediction for the $B^0 \to \pi^0\pi^0$ branching ratio $\mathrm{Br}(B^0 \to \pi^0\pi^0) = [0.23^{+0.08}_{-0.05}(\omega_b)^{+0.05}_{-0.04}(f_B)^{+0.04}_{-0.03}(a_2^\pi)] \times 10^{-6}$; it is still much smaller than the measured data.

(e) In Ref. [29], Hai-Yang Cheng *et al.* used flavor SU(3) symmetry to analyze the data of charmless B meson decays to two pseudoscalar mesons (PP) and one vector and one pseudoscalar meson (VP). They found that the color-suppressed tree amplitude is larger than previously known and has a strong phase of $-70°$ relative to the color favored tree amplitude in the PP sector; this large color-suppressed tree amplitude results in the large $B^0 \to \pi^0\pi^0$ branching ratios $1.43 \pm 0.55 \times 10^{-6}$ and $1.88 \pm 0.42 \times 10^{-6}$ for a different scheme.

There were some works on $B^0 \to \pi^0\pi^0$ decay in the framework of the PQCD approach before [8,27,28]; we list these numerical values in Table II. Reference [8] has the earliest PQCD calculations for $B^0 \to \pi^0\pi^0$ decay at the leading order (LO); Hsiang-nan Li *et al.* considered partial NLO contributions in Ref. [27]. Based on the work of Refs. [8,27], Ya-Lan Zhang *et al.* calculated all currently known NLO contributions from various sources in Ref. [28]. As shown in Table II, one can see that the NLO contributions are much larger than LO contributions for $B^0 \to \pi^0\pi^0$ decay in previous works. Despite this, it is still much smaller than the experimental data. In this work, we recalculate the $B^0 \to \pi^0\pi^0$ decay in the framework of

TABLE II. The pQCD predictions for the CP-averaged branching ratios (in unit of 10^{-6}).

Channel	LO [8]	NLO [27]	NLO [28]	LO (this work)	QCDF [5]	*BABAR* data [15]	Belle data [15]	HFAG data [15]
$B^0 \to \pi^0\pi^0$	0.12	0.29	0.23	$1.17^{+0.11}_{-0.12}$	0.3	$1.83 \pm 0.21 \pm 0.13$	$0.90 \pm 0.12 \pm 0.10$	1.17 ± 0.13

the PQCD approach at LO. Our result is much larger than that of previous predictions [8,27,28]; there are two reasons for the difference. For the operator $O_1 = (\bar{u}_\alpha u_\alpha)_{V-A}(\bar{d}_\beta b_\beta)_{V-A}$, it can contribute not only to nonfactorizable diagrams (a) and (b), but to factorizable annihilation diagrams (e) and (f) (see Fig. 1) as well. We find the largest contributions come from the factorizable annihilation diagrams (e) and (f), which come from tree operator O_1 and penguin operators O_3, O_4, O_5, O_6, O_7, O_8, O_9, O_{10}. In previous PQCD works [8,27,28], first, the contributions of the factorizable annihilation diagrams (e) and (f) coming from tree operator O_1 had not been taken into account; the authors only considered the nonfactorizable diagrams (a) and (b) (small contributions) for operator O_1; second, for O_3, O_4, O_9, O_{10} operators, previous calculations [8] showed that their contributions cancel between diagrams (e) and (f); however, we recalculate it and find that their contributions cannot be canceled between diagrams (e) and (f), as shown in Eqs. (12) and (13). If we get rid of the contributions of $\mathcal{M}_e$ and $\mathcal{M}_e^P$ terms in Eq. (16), our result is $\mathrm{Br}(\bar{B}^0(B^0) \to \pi^0\pi^0) \sim 0.26 \times 10^{-6}$, which is consistent with previous PQCD predictions [8,27,28]. The hard scale t in Eq. (9) characterizes the size of NLO contributions; by changing the hard scale t from $0.9t$ to $1.3t$, we find the branching ratio of $B^0 \to \pi^0\pi^0$ changes about10%, which means although the NLO diagrams may make significant contributions to $B^0 \to \pi^0\pi^0$ decay [27,28], the LO contributions still dominate this decay. Because there are identical particles in the final state for this decay, one must consider the identical principle. Usually the decay width receives a symmetry factor $1/2$ due to the identical particles in the final state, but in our calculations, we have calculated the symmetrized Feynman diagrams and all these contributions have been included in the total decay amplitude formula (16), and hence there is no need to add an extra factor in decay width. In our recalculations, we consider all the possible diagram contributions, including nonfactorizable contributions and annihilation contributions. We obtain the branching ratio of $B^0 \to \pi^0\pi^0$ $(1.17^{+0.11}_{-0.12}) \times 10^{-6}$, which is still smaller than the *BABAR* result [15], but it is consistent with the Belle and HFAG results [15]. More experimental and theoretical efforts should be made to resolve the $B^0 \to \pi^0\pi^0$ puzzle.

In SM, the CKM phase angle is the origin of CP violation. Using Eqs. (18) and (19), the direct CP violating parameter is

$$\mathcal{A}_{CP}^{\mathrm{dir}} = \frac{|\bar{\mathcal{A}}|^2 - |\mathcal{A}|^2}{|\bar{\mathcal{A}}|^2 + |\mathcal{A}|^2} = \frac{2z\sin(\alpha)\sin(\delta)}{1 + 2z\cos(\alpha)\cos(\delta) + z^2}. \tag{24}$$

It is approximately proportional to CKM angle $\sin(\alpha)$, strong phase $\sin(\delta)$, and the relative size z between the penguin contribution and tree contribution. We show the direct CP asymmetry $\mathcal{A}_{CP}^{\mathrm{dir}}$ in Fig. 3. One can see from this figure that the direct CP asymmetry parameter of $\bar{B}^0(B^0) \to \pi^0\pi^0$ can be as large as -83% to -82% when $80° < \alpha < 90°$. The large direct CP asymmetry is also a result of large contributions from both tree diagrams and penguin diagrams in these decays.

For the neutral B^0 decays, the $\bar{B}^0 - B^0$ mixing is very complex. Following notations in the previous literature [30], we define the mixing induced CP violation parameter as

$$a_{\epsilon+\epsilon'} = \frac{-2\mathrm{Im}(\lambda_{CP})}{1 + |\lambda_{CP}|^2}, \tag{25}$$

where

$$\lambda_{CP} = \frac{V_{tb}^* V_{td} \langle \pi^0\pi^0 | H_{\mathrm{eff}} | \bar{B}^0 \rangle}{V_{tb} V_{td}^* \langle \pi^0\pi^0 | H_{\mathrm{eff}} | B^0 \rangle}. \tag{26}$$

Using Eqs. (18) and (19), we can derive this as

$$\lambda_{CP} = e^{2i\alpha} \frac{1 + ze^{i(\delta-\alpha)}}{1 + ze^{i(\delta+\alpha)}}. \tag{27}$$

If z is a very small number, i.e., the penguin diagram contribution is suppressed comparing with the tree diagram contribution, the mixing induced CP asymmetry parameter $a_{\epsilon+\epsilon'}$ is proportional to $-\sin 2\alpha$, which is a good place for the CKM angle α measurement. However as we have already mentioned, z is not very small. We give the mixing

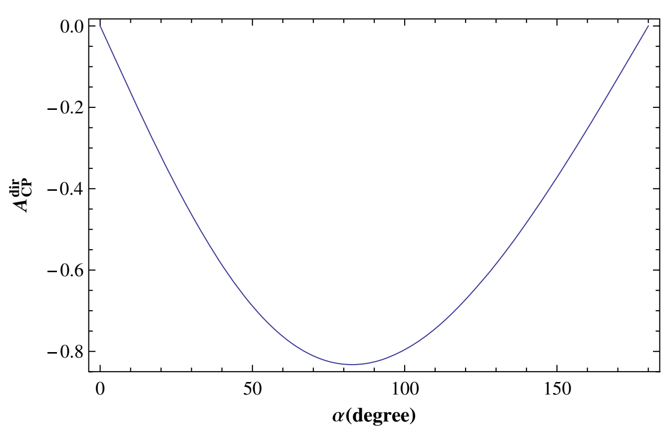

FIG. 3. Direct CP violation parameter of $\bar{B}^0(B^0) \to \pi^0\pi^0$ decay as a function of CKM angle α.

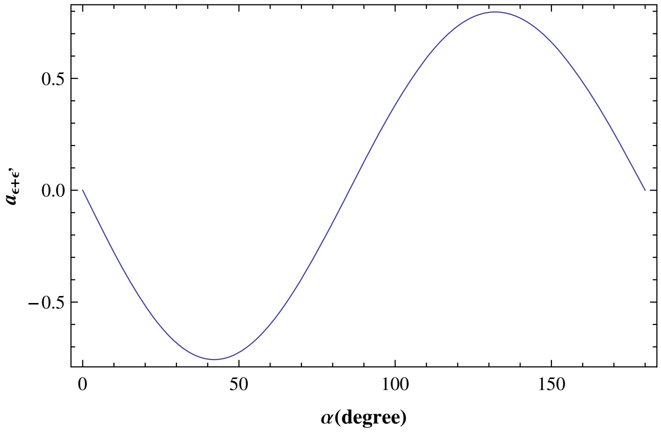

FIG. 4. Mixing of the CP violation parameter of $\bar{B}^0(B^0) \to \pi^0\pi^0$ decay as a function of CKM angle α.

CP asymmetry in Fig. 4; one can see that $a_{\epsilon+\epsilon'}$ is not a simple $-\sin 2\alpha$ behavior because of the so-called penguin pollution. It is close to 6% when the angle is near 85°. At present, there are no CP asymmetry measurements in experiment but the possible large CP violation we predict for $\bar{B}^0(B^0) \to \pi^0\pi^0$ decays might be observed in the coming Belle-II experiments.

IV. SUMMARY

In this work, we recalculate the branching ratio and CP asymmetries of the decays $\bar{B}^0(B^0) \to \pi^0\pi^0$ in the PQCD approach at LO. From our calculations, we find the branching ratio of $B^0 \to \pi^0\pi^0$ $(1.17^{+0.11}_{-0.12}) \times 10^{-6}$, much larger than that of previous predictions [8], and there are large CP violations in this process, which may be measured in the coming Belle-II experiments. The branching ratio we get is still smaller than the *BABAR* result [15], but it is consistent with the latest Belle and HFAG results [15].

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APPENDIX: FORMULAS FOR THE CALCULATIONS USED IN THE TEXT

We present the explicit expressions of the formulas used in Sec. II in the appendix. The expressions of the meson distribution amplitudes Φ_M are given at first. For the B^0 meson wave function, we use the function [8,14,31]

$$\phi_B(x,b) = N_B x^2(1-x)^2 \exp\left[-\frac{1}{2}\left(\frac{x m_B}{\omega_b}\right)^2 - \frac{\omega_b^2 b^2}{2}\right]. \quad \text{(A1)}$$

The parameter $\omega_b = 0.4$ GeV is constrained by other charmless B decays [8,14,31]. For the π meson's wave function, the distribution amplitude Φ_π^A for the twist-2 wave function and the distribution amplitude Φ_π^P and Φ_π^T of the twist-3 wave functions are taken from [27,32–34]

$$\begin{aligned}
\Phi_\pi^A(x) &= \frac{3f_\pi}{\sqrt{2N_c}} x(1-x) \times [1 + a_1^\pi C_1^{\frac{3}{2}}(2x-1) \\
&\quad + a_2^\pi C_2^{\frac{3}{2}}(2x-1) + a_4^\pi C_4^{\frac{3}{2}}(2x-1)], \\
\Phi_\pi^P(x) &= \frac{f_\pi}{2\sqrt{2N_c}}\left[1 + \left(30\eta_3 - \frac{5}{2}\rho_\pi^2\right) C_2^{\frac{1}{2}}(2x-1)\right. \\
&\quad \left. - 3\left\{\eta_3\omega_3 + \frac{9}{20}\rho_\pi^2(1+6a_2^\pi)\right\} C_4^{\frac{1}{2}}(2x-1)\right], \\
\Phi_\pi^T(x) &= \frac{f_\pi}{2\sqrt{2N_c}}(1-2x)\left[1 + 6\left(5\eta_3 - \frac{1}{2}\eta_3\omega_3\right.\right. \\
&\quad \left.\left. - \frac{7}{20}\rho_\pi^2 - \frac{3}{5}\rho_\pi^2 a_2^\pi\right)(1 - 10x + 10x^2)\right], \qquad \text{(A2)}
\end{aligned}$$

where a_i^π are the Gegenbauer moments, and the mass ratio $\rho_\pi = m_\pi/m_{0\pi}$. The Gegenbauer polynomials are defined by [27].

$$\begin{aligned}
C_2^{\frac{1}{2}}(t) &= \frac{1}{2}(3t^2 - 1), \\
C_4^{\frac{1}{2}}(t) &= \frac{1}{8}(35t^4 - 30t^2 + 3), \\
C_2^{\frac{3}{2}}(t) &= \frac{3}{2}(5t^2 - 1), \\
C_4^{\frac{3}{2}}(t) &= \frac{15}{8}(21t^4 - 14t^2 + 1), \\
C_1^{\frac{3}{2}}(t) &= 3t, \qquad \text{(A3)}
\end{aligned}$$

and the Gegenbauer moments and other parameters are adopted from Refs. [27,35],

$$\begin{aligned}
&a_1^\pi = 0, \qquad a_2^\pi = 0.25, \qquad a_4^\pi = -0.015, \\
&\rho_\pi = m_\pi/m_{0\pi}, \qquad \eta_3 = 0.015, \qquad \omega_3 = -3.0, \qquad \text{(A4)}
\end{aligned}$$

with $m_{0\pi}$ being the chiral mass of the pion.

$S_{\bar{B}^0}$, S_{π^0}, S_{π^0} used in the decay amplitudes are defined as

$$S_{\bar{B}^0}(t) = s(x_1 P_1^+, b_1) + 2\int_{\frac{1}{b_1}}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma(\alpha_s(\bar{\mu})), \quad \text{(A5)}$$

$$\begin{aligned}
S_{\pi^0}(t) &= s(x_2 P_2^-, b_2) + s((1-x_2)P_2^-, b_2) \\
&\quad + 2\int_{\frac{1}{b_2}}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma(\alpha_s(\bar{\mu})), \qquad \text{(A6)}
\end{aligned}$$

$$S_{\pi^0}(t) = s(x_3 P_3^+, b_3) + s((1-x_3)P_3^+, b_3) + 2\int_{\frac{1}{b_3}}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \gamma(\alpha_s(\bar{\mu})), \tag{A7}$$

where the so-called Sudakov factor $s(Q,b)$ resulting from the resummation of double logarithms is given as [36,37]

$$s(Q,b) = \int_{\frac{1}{b}}^{Q} \frac{d\mu}{\mu}\left[\ln\left(\frac{Q}{\mu}\right)A(\alpha(\bar{\mu})) + B(\alpha_s(\bar{\mu}))\right] \tag{A8}$$

with

$$A = C_F\frac{\alpha_s}{\pi} + \left[\frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27}n_f + \frac{2}{3}\beta_0 \ln\left(\frac{e^{\gamma_E}}{2}\right)\right]\left(\frac{\alpha_s}{\pi}\right)^2, \tag{A9}$$

$$B = \frac{2}{3}\frac{\alpha_s}{\pi}\ln\left(\frac{e^{2\gamma_E - 1}}{2}\right); \tag{A10}$$

here $\gamma_E = 0.57722\cdots$ is the Euler constant; n_f is the active quark flavor number.

The functions $h_i (i = a, c, e, g)$ come from the Fourier transformation of propagators of the virtual quark and gluon in the hard part calculations. They are given as follows:

$$\begin{aligned} h_a^j(x_1, x_2, x_3, b_1, b_2) &= \{\theta(b_1 - b_2) I_0(M_B\sqrt{x_1(1-x_2)}b_2) \\ &\quad \times K_0(M_B\sqrt{x_1(1-x_2)}b_1) + (b_1 \leftrightarrow b_2)\} \\ &\quad \times \begin{pmatrix} (K_0(M_B F_{a(j)} b_1), & \text{for } F_{a(j)}^2 > 0 \\ \frac{\pi i}{2} \mathrm{H}_0^{(1)}(M_B\sqrt{|F_{a(j)}^2|}b_1), & \text{for } F_{a(j)}^2 < 0 \end{pmatrix}, \end{aligned} \tag{A11}$$

where $F_{a(j)}$'s are defined by

$$\begin{aligned} F_{a(1)}^2 &= 1 - x_2, \\ F_{a(2)}^2 &= x_1. \end{aligned} \tag{A12}$$

$$\begin{aligned} h_c^j(x_1, x_2, x_3, b_2, b_3) &= \Big\{\theta(b_2 - b_3) I_0(M_B\sqrt{x_1(1-x_2)}b_3) \\ &\quad \times K_0(M_B\sqrt{x_1(1-x_2)}b_2) + (b_2 \leftrightarrow b_3)\Big\} \\ &\quad \times \begin{pmatrix} (K_0(M_B F_{c(j)} b_3), & \text{for } F_{c(j)}^2 > 0 \\ \frac{\pi i}{2} \mathrm{H}_0^{(1)}(M_B\sqrt{|F_{c(j)}^2|}b_3), & \text{for } F_{c(j)}^2 < 0 \end{pmatrix}, \end{aligned} \tag{A13}$$

where $F_{c(j)}$'s are defined by

$$\begin{aligned} F_{c(1)}^2 &= x_1 + x_2 + x_3 - x_1x_2 - x_2x_3 - 1, \\ F_{c(2)}^2 &= x_1 - x_3 - x_1x_2 + x_2x_3. \end{aligned} \tag{A14}$$

$$\begin{aligned} h_e^1(x_2, x_3, b_2, b_3) &= S_t(x_2) K_0(M_B\sqrt{x_2x_3}b_3) \\ &\quad \times \{\theta(b_2 - b_3) I_0(M_B\sqrt{x_2}b_2) K_0(M_B\sqrt{x_2}b_3) \\ &\quad + (b_2 \leftrightarrow b_3)\}, \end{aligned} \tag{A15}$$

$$\begin{aligned} h_e^2(x_2, x_3, b_2, b_3) &= S_t(x_3) K_0(M_B\sqrt{x_2x_3}b_2) \\ &\quad \times \{\theta(b_2 - b_3) I_0(M_B\sqrt{x_3}b_3) K_0(M_B\sqrt{x_3}b_2) \\ &\quad + (b_2 \leftrightarrow b_3)\}. \end{aligned} \tag{A16}$$

$$\begin{aligned} h_g^j(x_1, x_2, x_3, b_1, b_2) &= \{\theta(b_1 - b_2) I_0(M_B\sqrt{x_2x_3}b_1) K_0(M_B\sqrt{x_2x_3}b_2) \\ &\quad + (b_1 \leftrightarrow b_2)\} \\ &\quad \times \begin{pmatrix} (K_0(M_B F_{g(j)} b_1), & \text{for } F_{g(j)}^2 > 0 \\ \frac{\pi i}{2} \mathrm{H}_0^{(1)}(M_B\sqrt{|F_{g(j)}^2|}b_1), & \text{for } F_{g(j)}^2 < 0 \end{pmatrix}, \end{aligned} \tag{A17}$$

where $F_{g(j)}$'s are defined by

$$\begin{aligned} F_{g(1)}^2 &= x_1 + x_2 + x_3 - x_1x_2 - x_2x_3, \\ F_{g(2)}^2 &= x_1x_2 - x_2x_3. \end{aligned} \tag{A18}$$

We adopt the parametrization for $S_t(x)$ contributing to the factorizable diagrams [38]

$$S_t(x) = \frac{2^{1+2c}\Gamma(\frac{3}{2}+c)}{\sqrt{\pi}\Gamma(1+c)}[x(1-x)]^c, \tag{A19}$$

where the parameter $c = 0.3$. The hard scale t in the amplitudes is taken as the largest energy scale in H to kill the large logarithmic radiative corrections,

$$t_a^1 = \max\left(M_B\sqrt{|F^2_{a(1)}|}, M_B\sqrt{x_1(1-x_2)}, \frac{1}{b_1}, \frac{1}{b_2}\right), \qquad t_a^2 = \max\left(M_B\sqrt{|F^2_{a(2)}|}, M_B\sqrt{x_1(1-x_2)}, \frac{1}{b_1}, \frac{1}{b_2}\right),$$
$$t_c^1 = \max\left(M_B\sqrt{|F^2_{c(1)}|}, M_B\sqrt{x_1(1-x_2)}, \frac{1}{b_2}, \frac{1}{b_3}\right), \qquad t_c^2 = \max\left(M_B\sqrt{|F^2_{c(2)}|}, M_B\sqrt{x_1(1-x_2)}, \frac{1}{b_2}, \frac{1}{b_3}\right),$$
$$t_e^1 = \max\left(M_B\sqrt{x_2}, \frac{1}{b_2}, \frac{1}{b_3}\right), \qquad t_e^2 = \max\left(M_B\sqrt{x_3}, \frac{1}{b_2}, \frac{1}{b_3}\right),$$
$$t_g^1 = \max\left(M_B\sqrt{|F^2_{g(1)}|}, M_B\sqrt{x_2x_3}, \frac{1}{b_1}, \frac{1}{b_2}\right), \qquad t_g^2 = \max\left(M_B\sqrt{|F^2_{g(2)}|}, M_B\sqrt{x_2x_3}, \frac{1}{b_1}, \frac{1}{b_2}\right).$$

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