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Backward charmonium production in πN collisions

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The QCD collinear factorization framework allows one to describe exclusive backward production of a J/ψ meson in pion-nucleon collisions in terms of pion-to-nucleon transition distribution amplitudes. We calculate the scattering amplitude at the leading order in the strong coupling constant and estimate the cross section of this reaction in the backward kinematical region for a medium energy pion beam available at the J-Parc experimental facility.

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I. INTRODUCTION

Besides leptoproduction experiments, hadronic facilities open complementary accesses to the study of the partonic content of hadrons. Indeed the collinear factorization theorems of QCD allow one to define universal hadronic matrix elements which enter scattering amplitudes in both lepton-nucleon and meson-nucleon reactions in specific kinematical conditions. This statement is true for both inclusive and exclusive reactions. In the last two decades, we have witnessed tremendous progress in the understanding of deeply virtual Compton scattering and deep meson electroproduction within this framework. The detailed study of generalized parton distributions (GPDs) [1], the relevant hadronic matrix elements, is a major goal of modern hadronic physics. Timelike processes such as the timelike Compton scattering [2] and exclusive Drell-Yan production in πN collisions [3] obey the same factorization properties and allow one to access the same GPDs. Exclusive charmonium production has also been addressed in the same framework [4].

The extension of the collinear factorization approach to other processes such as backward virtual Compton scattering and backward meson electroproduction has been advocated [5–7]—although the corresponding factorization theorems have not yet been rigorously proven. This leads to the definition of new hadronic matrix elements of three quark operators on the light cone, the nucleon-to-meson transition distribution amplitudes (TDAs) [8]. To motivate the validity of such a factorization regime, one requires the existence of a large scale Q, which may be taken as the virtuality of the photon quantifying the electromagnetic probe or the mass of a heavy quark in the case of heavy quarkonium production. This large scale plays the role of the factorization scale and determines the magnitude of the QCD coupling constant α_s.

The intense pion beam available at the Japan Proton Accelerator Research Complex (J-Parc) (the pion beam momentum $P_\pi \sim 10$–20 GeV; center-of-mass energy squared $W^2 = m_N^2 + m_\pi^2 + 2E_\pi m_N \approx 2m_N P_\pi$) opens the possibility to study hard exclusive reactions such as lepton pair or charmonium production in πN collisions. This will provide new ways of testing the universality of GPDs and TDAs. The recent feasibility study [9] of forward lepton pair production suggests that GPDs can be accessed there. Here, we address the complementary case of backward charmonium production, the perturbative QCD description of which involves pion-to-nucleon TDAs. This process can be seen as the cross-channel counterpart of nucleon-antinucleon annihilation into heavy quarkonium in association with a pion. The description of this latter process within the collinear factorization approach involving nucleon-to-pion TDAs was studied in Ref. [10]. The cross section estimates performed for the kinematical conditions of P̄ANDA@GSI-FAIR lead to large enough production rates to be experimentally accessed [11,12].

II. KINEMATICS OF THE REACTION

In the present paper, we consider the reaction

$$\pi^-(p_\pi) + N^p(p_1) \to J/\psi(p_\psi) + N^n(p_2). \tag{1}$$

The πN center-of-mass energy squared $s = (p_\pi + p_1)^2 \equiv W^2$ and the charmonium mass squared M_ψ^2 introduce the natural hard scale. In complete analogy with our analysis of the nucleon-antinucleon annihilation process [13,14], we assume that this reaction admits a factorized description in the near-backward kinematical regime (see Fig. 1), where $|u| \equiv |\Delta^2| = |(p_2 - p_\pi)^2| \ll W^2, M_\psi^2$. This corresponds to the final nucleon moving almost in the direction of the initial pion in πN center-of-mass system (CMS).

The z-axis is chosen along the direction of the pion beam in the meson-nucleon CMS frame. We introduce the light-cone vectors p, n satisfying $2p \cdot n = 1$. The Sudakov decomposition of the relevant momenta reads

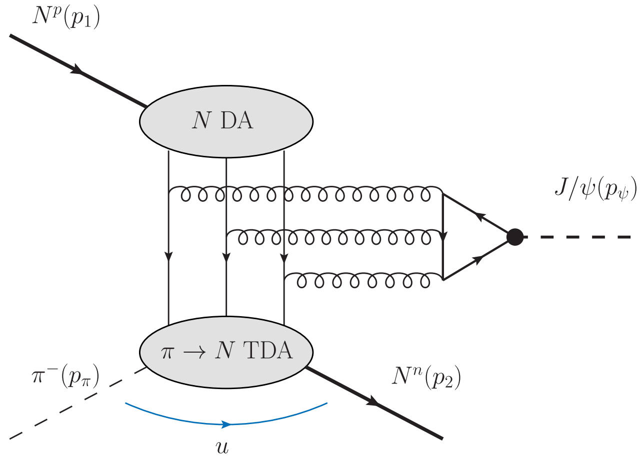

FIG. 1. Collinear factorization of the $\pi^-(p_\pi) + N^p(p_1) \to N^n(p_2) + J/\psi(p_\psi)$ reaction in the u-channel regime. DA stands for the distribution amplitude of the incoming nucleon; $\pi \to N$ TDA stands for the transition distribution amplitude from a pion to a nucleon.

$$\begin{aligned} p_\pi &= (1+\xi)p + \frac{m_\pi^2}{1+\xi}n; \\ p_1 &= \frac{2(1+\xi)m_N^2}{W^2+\Lambda(W^2,m_N^2,m_\pi^2)-m_N^2-m_\pi^2}p \\ &\quad + \frac{W^2+\Lambda(W^2,m_N^2,m_\pi^2)-m_N^2-m_\pi^2}{2(1+\xi)}n; \\ \Delta &\equiv (p_2-p_\pi) = -2\xi p + \left(\frac{m_N^2-\Delta_T^2}{1-\xi} - \frac{m_\pi^2}{1+\xi}\right)n + \Delta_T; \\ p_\psi &= p_1 - \Delta; \qquad p_2 = p_\pi + \Delta, \end{aligned} \tag{2}$$

where

$$\Lambda(x,y,z) = \sqrt{x^2+y^2+z^2-2xy-2xz-2yz} \tag{3}$$

is the Mandelstam function and m_N and m_π stand respectively for the nucleon and pion masses. The transverse direction in (2) is defined with respect to the z direction, and ξ is the skewness variable

$$\xi \equiv -\frac{(p_2-p_\pi)\cdot n}{(p_2+p_\pi)\cdot n}. \tag{4}$$

Within the collinear factorization framework, we neglect both the pion and nucleon masses with respect to M_ψ and W and set $\Delta_T = 0$ within the coefficient function. This results in the approximate expression for the skewness variable (4):

$$\xi \simeq \frac{M_\psi^2}{2W^2 - M_\psi^2}. \tag{5}$$

However, the approximation (5) can potentially affect the definition of the physical domain of the reaction (1) determined by the requirement $\Delta_T^2 \le 0$, where

$$\Delta_T^2 = \frac{1-\xi}{1+\xi}\left(u - 2\xi\left[\frac{m_\pi^2}{1+\xi} - \frac{m_N^2}{1-\xi}\right]\right). \tag{6}$$

To improve the approximate kinematical formulas in the vicinity of the threshold, it is sometimes convenient to keep partly the finite mass corrections resulting in the modified expression for skewness variable

$$\begin{aligned} \xi &= \frac{M_\psi^2 - m_N^2 - u}{W^2+\Lambda(W^2,m_N^2,m_\pi^2)+u-M_\psi^2-m_\pi^2} \\ &\quad + O\left(\frac{m_N^4}{W^4}\right) + O\left(\frac{m_N^2 u}{W^4}\right) + O\left(\frac{m_N^2 m_\pi^2}{W^4}\right). \end{aligned} \tag{7}$$

In order to control the validity of the kinematic approximations (5), (7), it is instructive to consider the exact kinematics of the reaction (1) in the πN CMS frame. In this frame, the relevant momenta read

$$\begin{aligned} p_\pi &= \left(\frac{W^2+m_\pi^2-m_N^2}{2W}, \vec{p}_\pi\right); \\ p_\psi &= \left(\frac{W^2+M_\psi^2-m_N^2}{2W}, -\vec{p}_2\right); \\ p_1 &= \left(\frac{W^2+m_N^2-m_\pi^2}{2W}, -\vec{p}_\pi\right); \\ p_2 &= \left(\frac{W^2+m_N^2-M_\psi^2}{2W}, \vec{p}_2\right), \end{aligned} \tag{8}$$

where

$$|\vec{p}_\pi| = \frac{\Lambda(W^2,m_N^2,m_\pi^2)}{2W}; \quad |\vec{p}_2| = \frac{\Lambda(W^2,m_N^2,M_\psi^2)}{2W}. \tag{9}$$

The CMS scattering angle θ_u^* is defined as the angle between $\vec{p}_\pi$ and $\vec{p}_2$:

$$\cos\theta_u^* = \frac{2W^2(u-m_N^2-m_\pi^2)+(W^2+m_\pi^2-m_N^2)(W^2+m_N^2-M_\psi^2)}{\Lambda(W^2,m_N^2,m_\pi^2)\Lambda(W^2,m_N^2,M_\psi^2)}. \tag{10}$$

The transverse momentum transfer squared (6) is then given by

$$\Delta_T^2 = -\frac{\Lambda^2(W^2,M_\psi^2,m_N^2)}{4W^2}(1-\cos^2\theta_u^*), \tag{11}$$

and the physical domain for the reaction (1) is defined from the requirement that $\Delta_T^2 \le 0$:

(i) In particular, the backward kinematics regime $\theta_u^* = 0$ corresponds to $\vec{p}_2$ along $\vec{p}_\pi$, which means that J/ψ is produced along $-\vec{p}_\pi$ i.e. in the backward direction. In this case, u reaches its maximal value

$$u_{\max} \equiv \frac{2\xi(m_\pi^2(\xi-1)+m_N^2(\xi+1))}{\xi^2-1} = m_N^2+m_\pi^2-\frac{(W^2+m_\pi^2-m_N^2)(W^2+m_N^2-M_\psi^2)}{2W^2} + 2|\vec{p}_\pi||\vec{p}_\psi|. \quad (12)$$

At the same time, $t = (p_2 - p_1)^2$ reaches its minimal value $t_{\min}$ ($W^2 + u_{\max} + t_{\min} = 2m_N^2 + m_\pi^2 + M_\psi^2$).

Note that u is negative, and therefore $|u_{\max}|$ is the minimal possible absolute value of the momentum transfer squared. It is for $u \sim u_{\max}$ that one may expect to satisfy the requirement $|u| \ll W^2$, M_ψ^2 which is crucial for the validity of the factorized description of (1) in terms of $\pi \to N$ TDAs and nucleon distribution amplitudes (DAs).

(ii) Another limiting value $\theta_u^* = \pi$ corresponds to $\vec{p}_2$ along $-\vec{p}_\pi$ i.e. J/ψ produced in the forward direction. In this case, u reaches its minimal value

$$u_{\min} = m_N^2 + m_\pi^2 - \frac{(W^2+m_\pi^2-m_N^2)(W^2+m_N^2-M_\psi^2)}{2W^2} - 2|\vec{p}_\pi||\vec{p}_\psi|. \quad (13)$$

At the same time, $t = (p_2 - p_1)^2$ reaches its maximal value $t_{\max}$. The factorized description in terms of $\pi \to N$ TDAs does not apply in this case as $|u|$ turns to be of order of W^2.

III. HARD PART OF THE $\pi^- + N^p \to J/\psi + N^n$ AMPLITUDE

The calculation of the $\pi^- + N^p \to J/\psi + N^n$ scattering amplitude follows the same main steps as the classical calculation of the $J/\psi \to p + \bar{p}$ decay amplitude [15–18]. Assuming the factorization of small and large distance dynamics, the hard part of the amplitude is computed within perturbative QCD (pQCD). Large distance dynamics is encoded within the matrix elements of QCD light-cone operators between the appropriate hadronic states.

The leading order amplitude of the $J/\psi N^n$ production subprocess of (1) is, up to the reverse of the direction of fermionic lines, given by the sum of the same three diagrams presented on Fig. 2 of Ref. [10].

For the calculation of these diagrams, we apply the collinear approximation, neglecting both the nucleon and pion masses and assuming $\Delta_T = 0$. Therefore, the Sudakov decomposition (2) reads as

$$p_\pi \simeq (1+\xi)p; \qquad p_1 \simeq \frac{s}{(1+\xi)}n;$$
$$p_\psi \simeq 2\xi p + \frac{s}{(1+\xi)}n; \qquad \Delta \simeq -2\xi p. \quad (14)$$

Also, throughout our calculation, we set $M_\psi \simeq 2m_c \equiv \bar{M}$ with $\bar{M} = 3$ GeV.

Below, we summarize our conventions for the relevant light-cone matrix elements encoding the soft dynamics:

(i) The leading twist-3 uud π^--to-neutron ($\pi^- \to n$) TDAs are defined from the Fourier transform

$$\mathcal{F} \equiv (p\cdot n)^3 \int \left[\prod_{j=1}^{3} \frac{d\lambda_j}{2\pi}\right] e^{i\sum_{k=1}^{3} x_k \lambda_k (p\cdot n)} \quad (15)$$

of the $n\pi^-$ matrix element of the trilinear antiquark operator on the light cone

$$M_{\rho\tau\chi}^{(\pi^-\to n)}(\lambda_1 n, \lambda_2 n, \lambda_3 n) = \langle n(p_2)|\varepsilon_{c_1c_2c_3}\bar{u}_\rho^{c_1}(\lambda_1 n)\bar{u}_\tau^{c_2}(\lambda_2 n)\bar{d}_\chi^{c_3}(\lambda_3 n)|\pi^-(p_\pi)\rangle. \quad (16)$$

Namely,

$$4\mathcal{F}M_{\rho\tau\chi}^{(\pi^-\to n)}(\lambda_1 n, \lambda_2 n, \lambda_3 n) = \delta(x_1+x_2+x_3-2\xi)\, i\frac{f_N}{f_\pi} \times \sum_{\substack{\text{Dirac}\\\text{structures}}} s_{\rho\tau,\chi}^{(\pi\to N)} H_s^{(\pi^-\to n)}(x_1,x_2,x_3,\xi,\Delta^2), \quad (17)$$

where the sum goes over the eight leading twist Dirac structures $s_{\rho\tau,\chi}^{(\pi\to N)}$ built of p, n, Δ_T, charge conjugation matrix C and the Dirac spinor $\bar{U}(p_2)$. The explicit form of these Dirac structures is worked out in the Appendix, which contains also the relation of the parametrization of the leading twist $\pi^- \to n$ TDAs to the conventional $n \to \pi^-$ TDAs introduced in Ref. [19].

(ii) For the leading twist antinucleon DAs, we employ the standard parametrization of Ref. [17] involving three invariant functions V^p, A^p and T^p (see also Appendix B of Ref. [14]).

(iii) For the light-cone wave function of J/ψ heavy quarkonium, we use the so-called nonrelativistic wave function suggested in Ref. [17].

The leading order and leading twist amplitude of the reaction (1) admits the following parametrization[1],

$$\mathcal{M}_\lambda^{s_1s_2} = \mathcal{C}\frac{1}{\bar{M}^5}[\bar{U}(p_2,s_2)\hat{\mathcal{E}}^*(\lambda)\gamma_5 U(p_1,s_1)\mathcal{I}(\xi,\Delta^2) - \frac{1}{m_N}\bar{U}(p_2,s_2)\hat{\mathcal{E}}^*(\lambda)\hat{\Delta}_T\gamma_5 U(p_1,s_1)\mathcal{I}'(\xi,\Delta^2)], \quad (18)$$

[1]We adopt Dirac's "hat" notation $\hat{v} \equiv v_\mu \gamma^\mu$.

where $\mathcal{E}$ is the charmonium polarization vector and $\bar{U}$, U stand for the nucleon Dirac spinors.

The calculation of the three Born order diagrams yields the same result for the integral convolutions $\{\mathcal{I},\mathcal{I}'\}(\xi,\Delta^2)$ as for $J/\psi\pi$ production in $\bar{p}p$ annihilation up to the obvious replacement of nucleon-to-pion ($N\to\pi$) TDAs with pion-to-nucleon ($\pi\to N$) TDAs introduced in (17). The explicit expressions for $\mathcal{I}$ and $\mathcal{I}'$ can be found in Eqs. (13) and (15) of Ref. [10]. The overall numerical factor $\mathcal{C}$ in (18) is expressed as

$$\mathcal{C}=(4\pi\alpha_s)^3\frac{f_N^2 f_\psi}{f_\pi}\frac{10}{81}, \tag{19}$$

where α_s stands for the strong coupling, $f_\pi=93$ MeV is the pion weak decay constant, f_ψ determines the normalization of the wave function of heavy quarkonium, and f_N determines the value of the nucleon wave function at the origin. The normalization constant f_ψ is extracted from the charmonium leptonic decay width $\Gamma(J/\psi\to e^+e^-)$. With the values quoted in Ref. [20], we get $|f_\psi|=416\pm 5$ MeV.

IV. ESTIMATES OF THE CROSS SECTION

To work out the cross section formula, we square the amplitude (18) and average (sum) over spins of initial (final) nucleons. Staying at the leading twist accuracy, we account for the production of transversely polarized J/ψ. Summing over the transverse polarizations, we find

$$|\bar{\mathcal{M}}_T|^2=\sum_{\lambda_T}\left(\frac{1}{2}\sum_{s_1 s_2}\mathcal{M}_{\lambda_T}^{s_1 s_2}(\mathcal{M}_{\lambda_T}^{s_1 s_2})^*\right). \tag{20}$$

The leading twist differential cross section of $\pi+N\to J/\psi+N$ then reads

$$\begin{aligned}\frac{d\sigma}{d\Delta^2}&=\frac{1}{16\pi\Lambda^2(s,m_N^2,m_\pi^2)}|\bar{\mathcal{M}}_T|^2\\&=\frac{1}{16\pi\Lambda^2(s,m_N^2,m_\pi^2)}\frac{1}{2}|\mathcal{C}|^2\frac{2(1+\xi)}{\xi\bar{M}^8}\\&\quad\times\left(|\mathcal{I}(\xi,\Delta^2)|^2-\frac{\Delta_T^2}{m_N^2}|\mathcal{I}'(\xi,\Delta^2)|^2\right),\end{aligned} \tag{21}$$

where $\Lambda(x,y,z)$ is defined in (3).

In order to get a rough estimate of the cross section, we use the simple nucleon exchange model for $\pi\to N$ TDAs suggested in Ref. [22]. We do not expect that the inclusion of the spectral part for $\pi\to N$ TDAs [21,23] would be essential to draw a conclusion on the feasibility of the relevant experiment. The refinement of the present description will be done in course of availability of precise experimental data.

The predictions of the cross-channel nucleon exchange model of Ref. [22] for $n\to\pi^-$ TDAs within the parametrization (A1) are summarized in Eqs. (25)–(27) of Ref. [10]. Employing the results of the Appendix, we conclude that $\pi^-\to n$ TDAs within this model are expressed as

$$\begin{aligned}&\{V_{1,2},A_{1,2},T_{1,2,3}\}^{(\pi^-\to n)}(x_{1,2,3},\xi,\Delta^2)|_{N(940)}\\&\quad=\sqrt{2}\{V_{1,2},A_{1,2},T_{1,2,3}\}^{(p\to\pi^0)}(-x_{1,2,3},-\xi,\Delta^2)|_{N(940)};\\&T_4^{(\pi^-\to n)}(x_{1,2,3},\xi,\Delta^2)|_{N(940)}=0.\end{aligned} \tag{22}$$

Note that nucleon-to-pion TDAs within the cross-channel nucleon exchange model have purely the Efremov-Radyushkin-Brodsky-Lepage (ERBL)-like support. As the result, the convolution integrals $\mathcal{I},\mathcal{I}'$ within this model turn to be real since the poles in the corresponding integrands are located either on the crossover trajectories which separate the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP)-like and the ERBL-like support regions of $\pi\to N$ TDAs[2] or within the DGLAP-like support regions.

The convolution integrals $\mathcal{I}$, $\mathcal{I}'$ within the simple nucleon pole model (22) are expressed as

$$\begin{aligned}\mathcal{I}(\xi,\Delta^2)|_{N(940)}&=-\sqrt{2}\frac{f_\pi g_{\pi NN}m_N(1+\xi)}{(\Delta^2-m_N^2)(1-\xi)}M_0;\\\mathcal{I}'(\xi,\Delta^2)|_{N(940)}&=-\sqrt{2}\frac{f_\pi g_{\pi NN}m_N}{(\Delta^2-m_N^2)}M_0,\end{aligned} \tag{23}$$

where M_0 is given by Eq. (19) of Ref. [10]. Note that the integral convolution M_0 also occurs in the well-known expression for the $J/\psi\to\bar{p}p$ decay width within the pQCD approach [18]:

$$\Gamma_{J/\psi\to p\bar{p}}=(\pi\alpha_s)^6\frac{1280 f_\psi^2 f_N^4}{243\pi\bar{M}^9}|M_0|^2. \tag{24}$$

As the phenomenological input for the cross section estimate, we may employ different solutions for the leading twist nucleon DAs V^p, A^p, T^p. Similarly to the case of the charmonium decay width, our result depends strongly both on the form of the input leading twist nucleon DAs and the value of α_s.

Analogously to Ref. [10], we have chosen to present our results for the $\pi^-p\to J/\psi n$ cross section with the value of α_s fixed by the requirement that the given phenomenological solution reproduces the experimental $J/\psi\to N\bar{N}$ decay width from the pQCD expression of Ref. [18]. The corresponding values of α_s for several phenomenological solutions for the leading twist nucleon DA are summarized in the Table 1 of Ref. [10].

Phenomenological solutions for nucleon DAs that are largely concentrated in the end point regions such as the Chernyak-Ogloblin-Zhitnitsky [18] or King-Sachrajda [24] require smaller values of $\alpha_s\sim 0.25$ to reproduce the experimental value of $\Gamma_{J/\psi\to p\bar{p}}$. The solutions which are close to the asymptotic form of the nucleon DA $\phi_{\rm as}^N(y_{1,2,3})\equiv V_{\rm as}^p(y_{1,2,3})-A_{\rm as}^p(y_{1,2,3})=120y_1y_2y_3$ like the

[2]For the definition of the ERBL-like and DGLAP-like support regions of TDAs, see Ref. [21].

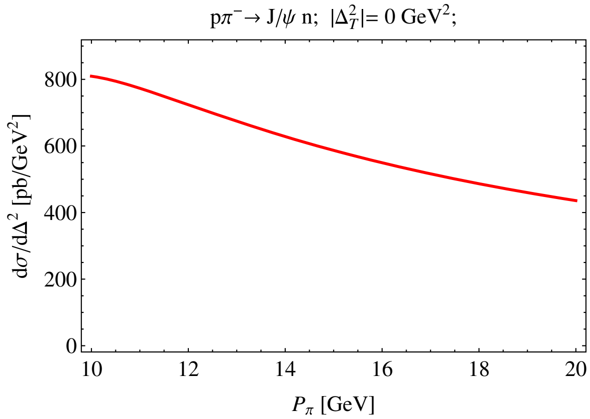

FIG. 2. Differential cross section $\frac{d\sigma}{d\Delta^2}$ for $\pi^- p \to J/\psi n$ as a function of the pion beam momentum P_π ($W^2 \approx 2m_N P_\pi$) for $\Delta_T^2 = 0$. Our estimate is based on the nucleon exchange contribution to the $\pi \to N$ TDAs and is independent of the specific form of the leading twist nucleon DA [see Eq. (25) and the corresponding discussion].

Bolz-Kroll [25] and Braun-Lenz-Wittmann next-to-leading order model [26] require rather large values of $\alpha_s \sim 0.4$ to reproduce the experimental value of $\Gamma_{J/\psi \to p\bar{p}}$ with Eq. (24). We refer the reader to Refs. [27,28] for the discussion and critics of the available phenomenological models of the leading twist nucleon DAs.

Note that with the aforementioned assumptions the predictions of the cross-channel nucleon exchange model for $\pi \to N$ TDAs (22) for the cross section (21) can be seen as being independent of the particular form of the phenomenological solution for the leading twist nucleon DAs. The cross section can be expressed through the experimental value [20] of the decay width $\Gamma_{J/\psi \to p\bar{p}} = 0.197 \pm 0.009$ KeV:

$$\left.\frac{d\sigma}{d\Delta^2}\right|_{N(940)} = \frac{40}{27} \frac{g_{\pi NN}^2 \Gamma_{J/\psi \to p\bar{p}} \bar{M} m_N^2}{(m_N^2 - \Delta^2)^2 \Lambda^2(W^2, m_N^2, m_\pi^2)} \frac{1+\xi}{\xi} \times \left[\frac{(1+\xi)^2}{(1-\xi)^2} - \frac{\Delta_T^2}{m_N^2}\right]. \quad (25)$$

Let us stress that we employ this model only as the first very rough cross section estimate intended for the preliminary feasibility studies at J-Parc kinematical conditions. Once the experimental data would be available, its description within the suggested factorization mechanism would require both the detailed discussion on the form of the leading twist nucleon DAs and $\pi \to N$ TDAs as well as the estimation of the possible higher twist and α_s corrections.

On Fig. 2, we show our estimates of the differential cross section $\frac{d\sigma}{d\Delta^2}$ for $\pi^- p \to J/\psi n$ (21) as a function of the pion beam momentum P_π for the exactly backward charmonium production ($\Delta_T^2 = 0 \Leftrightarrow \theta_\pi^* = 0$). The range of the pion beam momentum $10\ \mathrm{GeV} \le P_\pi \le 20\ \mathrm{GeV}$ corresponds to the J-Parc experimental setup.

On Fig. 3, we show the differential cross section $\frac{d\sigma}{d\Delta^2}$ for $\pi^- p \to J/\psi n$ as a function of $|u - u_{\max}|$ for $|u| \le 1\ \mathrm{GeV}^2$, where $u_{\max}$ is the threshold value (12) of the momentum transfer squared. We show the result for several values of the pion beam energy P_π. In order to better describe the cross section behavior for $\Delta_T^2 \neq 0$, we employ the exact value (12) for the maximal cross-channel momentum transfer squared.

On Fig. 4, we show the characteristic center-of-mass angular distribution for the $d\sigma/d\Delta^2$ cross section for the backward factorization regime visualized on the polar plot with the polar angle being the pion CMS scattering angle θ_π^* [see Eq. (10) for the definition]. We present the ratio

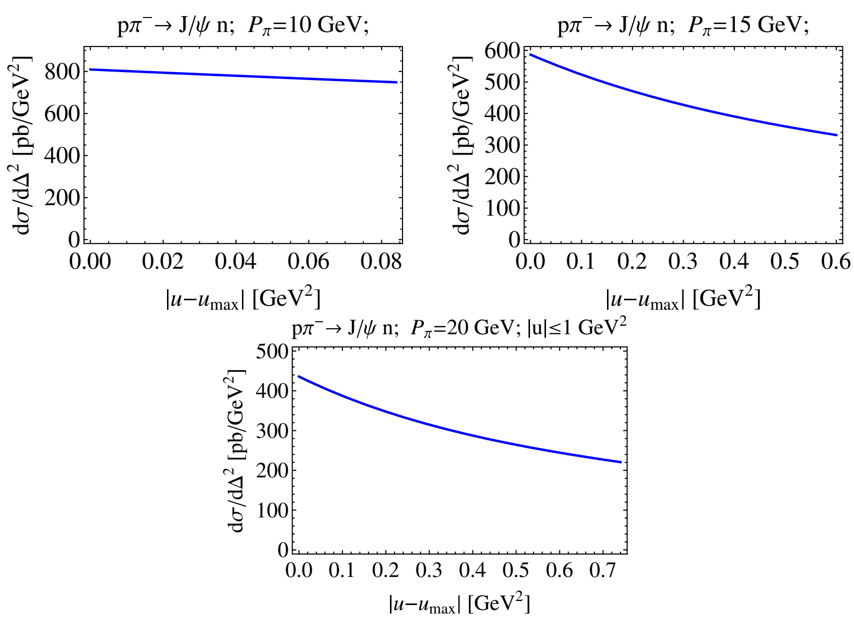

FIG. 3. Differential cross section $\frac{d\sigma}{d\Delta^2}$ for $\pi^- p \to J/\psi n$ as a function of $|u - u_{\max}|$ for $P_\pi = 10, 15, 20$ GeV for $|u| \le 1\ \mathrm{GeV}^2$.

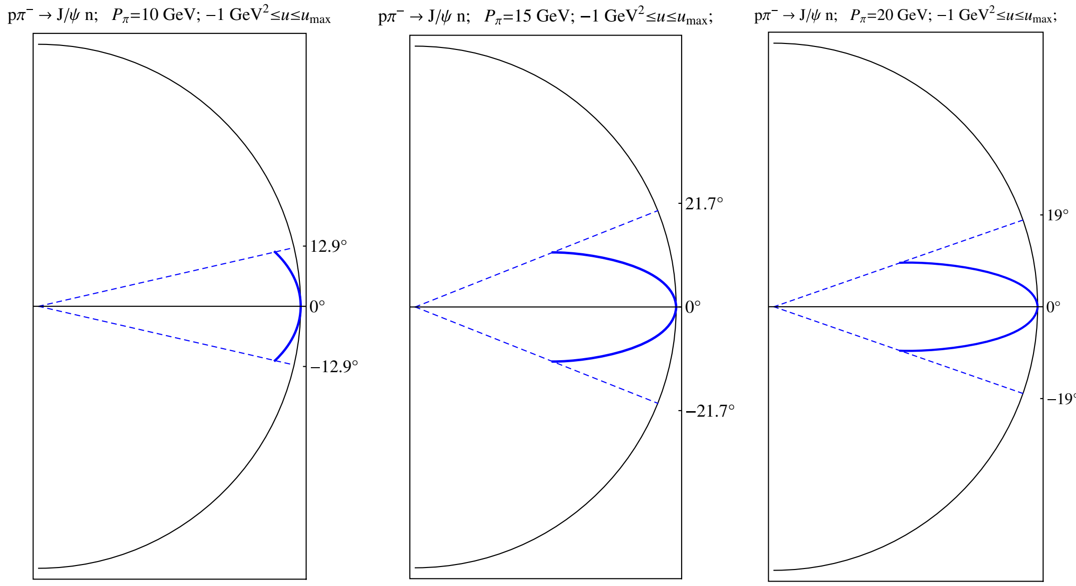

FIG. 4. Angular distribution for the $d\sigma/d\Delta^2$ cross section for the near-backward charmonium production ($\theta^*_\pi \simeq 0$) for $-1\ \mathrm{GeV}^2 \leq \Delta^2 \leq u_{\max}$. Dashed lines show the effect of the cutoff $|\Delta^2| \leq 1\ \mathrm{GeV}^2$ for the values of the pion CMS scattering angle θ^*_π.

$$\frac{\frac{d\sigma}{d\Delta^2}(W^2, \theta^*_\pi)}{\frac{d\sigma}{d\Delta^2}(W^2, \theta^*_\pi = 0)} \tag{26}$$

as a function of θ^*_π showing the result for $P_\pi = 10$, 15, 20 GeV and for $-1\ \mathrm{GeV}^2 \leq \Delta^2 \leq u_{\max}$, where $u_{\max}$ is the threshold value (12) of the momentum transfer squared. With the dashed lines, we show the effect of the cutoff $|\Delta^2| \leq 1\ \mathrm{GeV}^2$ for the values of the CMS scattering angle θ^*_π [see the discussion around Eq. (10)].

Since these rates are certainly within the experimental reach of the J-Parc experiment, the study of the reaction (1) will provide a valuable universality test for the TDA approach since the same TDAs also arise in the description of $N\bar{N} \to \gamma^*\pi$ [14], $N\bar{N} \to J/\psi\pi$ [11,12] and backward pion electroproduction off a nucleon $\gamma^* N \to \pi N$ [23].

V. CONCLUSIONS

In this paper, we address the reaction $\pi^- + N^p \to J/\psi + N^n$ which may be studied in the J-Parc facility. We argue that this reaction may be analyzed within the pQCD framework. It will not only help to quantitatively disentangle resonance production from the universal hadronic background but also will provide valuable information on the hadronic structure encoded in pion-to-nucleon TDAs. Pion-to-nucleon TDAs supply complementary information with respect to partonic distributions diagonal in quantum numbers such as common parton distributions and GPDs.

Within the kinematical range accessible at J-Parc, we provide the predictions for the $\pi^- + N^p \to J/\psi + N^n$ cross section using a simple nucleon pole model for $\pi \to N$ TDAs. The obtained cross section estimates give hope of experimental accessibility of the reaction. A specific feasibility study similar to that recently performed for accessing $N \to \pi$ TDAs at $\bar{\mathrm{P}}$ANDA [11,12,29] should be performed with J-Parc experimental efficiencies, as it has been done for exclusive forward lepton pair production at J-Parc [9].

It is worth mentioning that the mass of the charm quark may not be large enough for our leading order (in α_s) and leading twist analysis to be sufficient to describe the data. More work is certainly needed to go beyond the Born approximation for the hard amplitude, in particular because the timelike nature of the hard probe is often accompanied by large $O(\alpha_s)$ corrections [30].

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APPENDIX: CROSSING $\pi \to N$ TDAs TO $N \to \pi$ TDAs

We have the parametrization for the nucleon-to-pion ($N \to \pi$) TDAs defined through the Fourier transform of the πN matrix element of the trilinear quark operator on the light cone. The parametrization involves eight invariant functions, each being the function of three longitudinal momentum fractions, the skewness variable, the momentum transfer squared as well as of the factorization scale.

Throughout this paper, we make use of the parametrization of Ref. [19], where only three invariant functions turn out to be relevant in the $\Delta_T = 0$ limit. Let us consider the neutron-to-π^- uud TDA,

$$\begin{aligned}
&4(p\cdot n)^3 \int \left[\prod_{j=1}^{3}\frac{d\lambda_j}{2\pi}\right] e^{i\sum_{k=1}^{3}\tilde{x}_k\lambda_k(p\cdot n)} \langle \pi^-(p_\pi)|\varepsilon_{c_1c_2c_3}u_\rho^{c_1}(\lambda_1 n)u_\tau^{c_2}(\lambda_2 n)d_\chi^{c_3}(\lambda_3 n)|n(p_N,s_N)\rangle \\
&= \delta(\tilde{x}_1+\tilde{x}_2+\tilde{x}_3-2\tilde{\xi})\, i\frac{f_N}{f_\pi}[V_1^{(n\to\pi^-)}(\tilde{x}_{1,2,3},\tilde{\xi},\tilde{\Delta}^2)(\hat{p}C)_{\rho\tau}(U^+)_\chi \\
&\quad + A_1^{(n\to\pi^-)}(\tilde{x}_{1,2,3},\tilde{\xi},\tilde{\Delta}^2)(\hat{p}\gamma^5 C)_{\rho\tau}(\gamma^5 U^+)_\chi + T_1^{(n\to\pi^-)}(\tilde{x}_{1,2,3},\tilde{\xi},\tilde{\Delta}^2)(\sigma_{p\mu}C)_{\rho\tau}(\gamma^\mu U^+)_\chi \\
&\quad + m_N^{-1}V_2^{(n\to\pi^-)}(\tilde{x}_{1,2,3},\tilde{\xi},\tilde{\Delta}^2)(\hat{p}C)_{\rho\tau}(\hat{\tilde{\Delta}}_T U^+)_\chi + m_N^{-1}A_2^{(n\to\pi^-)}(\tilde{x}_{1,2,3},\tilde{\xi},\tilde{\Delta}^2)(\hat{p}\gamma^5 C)_{\rho\tau}(\gamma^5\hat{\tilde{\Delta}}_T U^+)_\chi \\
&\quad + m_N^{-1}T_2^{(n\to\pi^-)}(\tilde{x}_{1,2,3},\tilde{\xi},\tilde{\Delta}^2)(\sigma_{p\tilde{\Delta}_T}C)_{\rho\tau}(U^+)_\chi + m_N^{-1}T_3^{(n\to\pi^-)}(\tilde{x}_{1,2,3},\tilde{\xi},\tilde{\Delta}^2)(\sigma_{p\mu}C)_{\rho\tau}(\sigma^{\mu\tilde{\Delta}_T}U^+)_\chi \\
&\quad + m_N^{-2}T_4^{(n\to\pi^-)}(\tilde{x}_{1,2,3},\tilde{\xi},\tilde{\Delta}^2)(\sigma_{p\tilde{\Delta}_T}C)_{\rho\tau}(\hat{\tilde{\Delta}}_T U^+)_\chi] \\
&\equiv \delta(\tilde{x}_1+\tilde{x}_2+\tilde{x}_3-2\tilde{\xi})\, i\frac{f_N}{f_\pi}\sum_{\substack{\text{Dirac}\\ \text{structures}}} s_{\rho\tau,\chi}^{(N\to\pi)} H_s^{(n\to\pi^-)}(\tilde{x}_1,\tilde{x}_2,\tilde{x}_3,\tilde{\xi},\tilde{\Delta}^2). \qquad \text{(A1)}
\end{aligned}$$

We adopt Dirac's "hat" notation $\hat{v}\equiv v_\mu\gamma^\mu$; $\sigma^{\mu\nu}=\frac{1}{2}[\gamma^\mu,\gamma^\nu]$; $\sigma^{v\mu}\equiv v_\lambda\sigma^{\lambda\mu}$; C is the charge conjugation matrix, and $U^+=\hat{p}\,\hat{n}\,U(p_N,s_N)$ is the large component of the nucleon spinor. $f_\pi = 93$ MeV is the pion weak decay constant, and f_N determines the value of the nucleon wave function at the origin.

Note that the $N\to\pi$ TDA (A1) is defined with respect to its natural kinematical variables. Namely the cross-channel momentum transfer is $\tilde{\Delta}=p_\pi - p_N$, and the skewness parameter $\tilde{\xi}$ is defined from the longitudinal momentum transfer between the pion and nucleon,

$$\tilde{\xi}\equiv -\frac{(p_\pi - p_N)\cdot n}{(p_\pi + p_N)\cdot n}$$

[i.e. it differs by the sign from the definition (4) natural for the reaction (1)].

Now, we would like to express pion-to-nucleon ($\pi\to N$) TDAs through ($N\to\pi$) TDAs occurring in (A1). For this issue, we apply the Dirac conjugation (complex conjugation and convolution with γ_0 matrices in the appropriate spinor indices) for both sides of Eq. (A1) and compare the result to the definition of $\pi\to N$ TDAs (17):

$$\begin{aligned}
&-4(p\cdot n)^3 \int \left[\prod_{j=1}^{3}\frac{d\lambda_j}{2\pi}\right] e^{-i\sum_{k=1}^{3}\tilde{x}_k\lambda_k(p\cdot n)} \langle n(p_N,s_N)|\varepsilon_{c_1c_2c_3}\bar{u}_\rho^{c_1}(\lambda_1 n)\bar{u}_\tau^{c_2}(\lambda_2 n)\bar{d}_\chi^{c_3}(\lambda_3 n)|\pi^-(p_\pi)\rangle \\
&= -\delta(\tilde{x}_1+\tilde{x}_2+\tilde{x}_3-2\tilde{\xi})\, i\frac{f_N}{f_\pi}\sum_s \underbrace{(\gamma_0^T)_{\tau\tau'}[s_{\rho'\tau',\chi'}^{(N\to\pi)}]^\dagger(\gamma_0)_{\rho'\rho}(\gamma_0)_{\chi'\chi}}_{s_{\rho\tau,\chi}^{(\pi\to N)}} H_s^{(N\to\pi)}(\tilde{x}_1,\tilde{x}_2,\tilde{x}_3,\tilde{\xi},\Delta^2). \qquad \text{(A2)}
\end{aligned}$$

For the relevant Dirac structures, we get

$$\begin{aligned}
(v_1^{(\pi\to N)})_{\rho\tau,\chi} &= (C\hat{p})_{\rho\tau}\bar{U}_\chi^+; \\
(a_1^{(\pi\to N)})_{\rho\tau,\chi} &= (C\hat{p}\gamma_5)_{\rho\tau}(\bar{U}^+\gamma_5)_\chi; \\
(t_1^{(\pi\to N)})_{\rho\tau,\chi} &= -(C\sigma_{p\mu})_{\rho\tau}(\bar{U}^+\gamma_\mu)_\chi; \\
(v_2^{(\pi\to N)})_{\rho\tau,\chi} &= (C\hat{p})_{\rho\tau}(\hat{\tilde{\Delta}}_T\bar{U}^+)_\chi = -(C\hat{p})_{\rho\tau}(\hat{\Delta}_T\bar{U}^+)_\chi; \\
(a_2^{(\pi\to N)})_{\rho\tau,\chi} &= (C\hat{p}\gamma_5)_{\rho\tau}(\bar{U}^+\hat{\tilde{\Delta}}_T\gamma_5)_\chi = -(C\hat{p}\gamma_5)_{\rho\tau}(\bar{U}^+\hat{\Delta}_T\gamma_5)_\chi \\
(t_2^{\pi\to N)})_{\rho\tau,\chi} &= -(C\sigma_{p\tilde{\Delta}_T})_{\rho\tau}(\bar{U}^+)_\chi = (C\sigma_{p\Delta_T})_{\rho\tau}(\bar{U}^+)_\chi; \\
(t_3^{(\pi\to N)})_{\rho\tau,\chi} &= (C\sigma_{p\mu})_{\rho\tau}(\bar{U}^+\sigma_{\mu\tilde{\Delta}_T})_\chi = -(C\sigma_{p\mu})_{\rho\tau}(\bar{U}^+\sigma_{\mu\Delta_T})_\chi; \\
(t_4^{(\pi\to N)})_{\rho\tau,\chi} &= -(C\sigma_{p\tilde{\Delta}_T})_{\rho\tau}(\bar{U}^+\hat{\tilde{\Delta}}_T)_\chi = -(C\sigma_{p\Delta_T})_{\rho\tau}(\bar{U}^+\hat{\Delta}_T)_\chi, \qquad \text{(A3)}
\end{aligned}$$

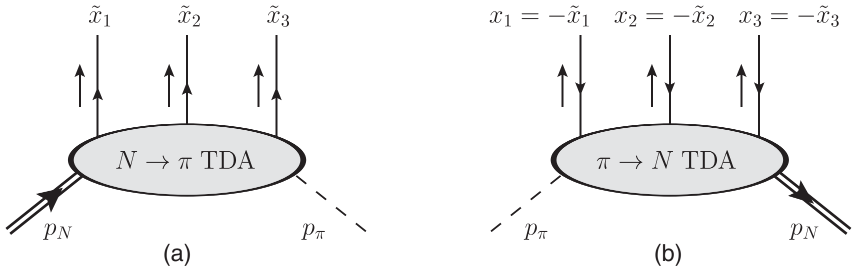

FIG. 5. Small arrows show the direction of the longitudinal momentum flow in the ERBL-like regime for the following: (a) The longitudinal momentum flow for $N \to \pi$ TDAs defined in (A1). The longitudinal momentum transfer is $(p_\pi - p_N) \cdot n \equiv \tilde{\Delta} \cdot n$. (b): The longitudinal momentum flow for $\pi \to N$ TDAs defined in (17). The longitudinal momentum transfer is $(p_N - p_\pi) \cdot n \equiv \Delta \cdot n$. Arrows on the nucleon and quark (antiquark) lines show the direction of flow of the baryonic charge.

where we switch to the definition of momentum transfer natural for the reaction (1): $\tilde{\Delta} \to -\Delta$. $\bar{U}^+ \equiv \bar{U}(p_N)\hat{n}\hat{p}$ stands for the large component of the $\bar{U}(p_N)$ Dirac spinor.

The flow of the longitudinal momentum for $N \to \pi$ TDAs defined as in Eq. (A1) and $\pi \to N$ TDAs defined as in Eq. (17) is presented on Fig. 5. By switching to the variables $\xi = -\tilde{\xi}$ and $x_i = -\tilde{x}_i$ natural for the reaction (1) and $\tilde{\Delta}^2 \to \Delta^2$ and comparing (A2) to (17), we conclude that

$$\{V_{1,2}^{(\pi^- \to n)}, A_{1,2}^{(\pi^- \to n)}, T_{1,2,3,4}^{(\pi^- \to n)}\}(x_{1,2,3}, \xi, \Delta^2) = \{V_{1,2}^{(n \to \pi^-)}, A_{1,2}^{(n \to \pi^-)}, T_{1,2,3,4}^{(n \to \pi^-)}\}(-x_{1,2,3}, -\xi, \Delta^2). \quad \text{(A4)}$$

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